

Acoustic Radiation from Supersonic Turbulent Boundary Layers in a Rectangular Test Section

Junji Huang^{*} and Lian Duan[†]
The Ohio State University, Columbus, OH 43210

Meelan M. Choudhari[‡] and Nathaniel Hildebrand[§]
NASA Langley Research Center, Hampton, VA 23681

Direct numerical simulation (DNS) of acoustic radiation from the spatially evolving turbulent boundary layers within a rectangular test section is conducted for a supersonic potential core with a Mach number of $M_\infty = 2.5$ and a local Karman number of $Re_\tau \approx 510$ at the location where statistics are reported. The emphasis is on comparing the radiated pressure fluctuations in a geometrically confined environment with those radiated from a single flat wall in an unconfined setting. As a risk reduction effort, a preliminary DNS of acoustic radiation within a two-dimensional test section (i.e., a double-wall configuration) was conducted and compared with those of a single flat wall in this abstract. The DNS data shows that the acoustic fluctuations dominate in the core region of the channel; the superposition of acoustic radiation from the uncorrelated turbulent fluctuations along the two walls leads to an acoustic intensity in the core region that is approximately twice the freestream acoustic intensity above a single wall. This finding is qualitatively consistent with the measurements by Laufer [1], indicating that the intensity of pressure fluctuations from different tunnel walls can be superimposed and that the mean square pressure fluctuation in the core region of the test section is approximately equal to that radiated from a single tunnel-wall boundary layer multiplied by the number of radiating boundary layers. Consistent with the equal contributions to the total acoustic radiation from both walls, the computed two-point correlation of the pressure fluctuations with respect to a reference location along the center plane is a nearly symmetric function of the wall-normal coordinate. The prominent structures associated with the acoustic fluctuations exhibit a preferred orientation in the free stream, similar to the findings for a single flat plate [2]. The power spectral density (PSD) and bulk propagation speed of the acoustic fluctuations in the core region are not affected by the confined channel environment. In the full paper, a similar study will be performed for a rectangular test section configuration with acoustic radiation

^{*}Graduate Student, Mechanical and Aerospace Department, Student Member, AIAA.

[†]Associate Professor, Mechanical and Aerospace Department, Senior Member, AIAA.

[‡]Aerospace Technologist, Computational AeroSciences Branch, M.S. 128, Fellow, AIAA.

[§]Aerospace Technologist, Computational AeroSciences Branch, Member, AIAA.

from uncorrelated turbulent fluctuations along four side walls.

Nomenclature

H	=	shape factor, $H = \delta^*/\theta$, dimensionless
(L_x, L_y, L_z)	=	Computational domain size along (x, y, z) directions, m
M	=	Mach number, dimensionless
Pr	=	Prandtl number, $Pr = 0.71$, dimensionless
P_∞	=	freestream mean pressure, Pa
R	=	ideal gas constant, $R = 287$, J/(K·kg)
Re_θ	=	Reynolds number based on momentum thickness and freestream viscosity, $Re_\theta = \rho_\infty U_\infty \theta / \mu_\infty$, dimensionless
$Re_{\delta 2}$	=	Reynolds number based on momentum thickness and wall viscosity, $Re_{\delta 2} = \rho_\infty U_\infty \theta / \mu_w$, dimensionless
Re_τ	=	Reynolds number based on shear velocity and wall viscosity, $Re_\tau = \rho_w U_\infty \delta / \mu_w$, dimensionless
T	=	temperature, K
T_r	=	recovery temperature, $T_r = T_\infty (1 + 0.89 \frac{\gamma-1}{2} M_\infty^2)$
U_∞	=	freestream velocity, m/s
p	=	pressure, Pa
u	=	streamwise velocity, m/s
u_τ	=	friction velocity, m/s
v	=	spanwise velocity, m/s
w	=	wall-normal velocity, m/s
x	=	streamwise direction of the right-hand Cartesian coordinate
y	=	spanwise direction of the right-hand Cartesian coordinate
z	=	wall-normal direction of the right-hand Cartesian coordinate
Δx	=	streamwise separation used to evaluate spatial correlations
Δy	=	spanwise separation in spanwise direction
Δt	=	time delay
z_τ	=	viscous length, $z_\tau = \nu_w / u_\tau$, m
β	=	Clauser pressure-gradient parameter, $\beta = \delta^* / \tau_w dP_\infty / dx$, dimensionless
γ	=	specific heat ratio, dimensionless
δ	=	boundary layer thickness, m

δ^*	= displacement thickness, m
θ	= momentum thickness, m
κ	= thermal conductivity, $\kappa = \mu C_p / Pr$, W/(m·K)
μ	= dynamic viscosity, $\mu = 1.458 \times 10^{-6} \frac{T^{3/2}}{T+110.4}$, kg/(m·s)
ν	= kinematic viscosity, $\nu = \mu / \rho$, m ² ·s
ρ	= density, kg/m ³
ω	= vorticity, s ⁻¹
α	= expansion angle, deg

Subscripts

i	= inflow station for the domain of direct numerical simulations
rms	= root mean square
w	= wall variables
∞	= freestream variables
t	= stagnation quantities

Superscripts

+	= inner wall units
$\overline{(\cdot)}$	= averaged variables
$(\cdot)'$	= perturbation from averaged variable

I. Introduction

The present study is part of a broader effort to investigate the acoustic radiation from supersonic and hypersonic turbulent boundary layers, motivated by the need to characterize the spectral as well as structural contents of the freestream disturbance environment in conventional (i.e., “noisy”) high-speed wind tunnels. An in-depth knowledge of the acoustic disturbance environment in the free stream of conventional high-speed wind tunnels could enable more accurate extrapolation of wind-tunnel boundary-layer transition data to flight, thereby expanding the utility of such wind tunnels for studying flows in which laminar-turbulent transition plays an important role [1, 3, 4]. Previous DNS have focused on the acoustic radiation from a spatially developing turbulent boundary layer over a single flat wall. Specifically, Duan et al. [2, 5] and Zhang et al. [6] explored the effects of freestream Mach number on the acoustic radiation from flat-plate turbulent boundary layers, with the nominal freestream Mach number varying from 2.5 to 14; Zhang et al. [7] followed this work with a study of the effect of wall cooling on boundary-layer induced acoustic radiation. These flat-plate simulations have the benefits of isolating more easily the acoustic radiation from a single surface, thus facilitating a comprehensive understanding of the freestream disturbance field and its dependence on

boundary-layer parameters (e.g., freestream Mach number, wall temperature, Reynolds number). DNS has also been applied to investigate the acoustic radiation from turbulent boundary layers developing along the inner walls of an axisymmetric nozzle [8–10]. These simulations permitted the characterization of noise generation and reverberation within an axisymmetric nozzle. An overview of the recent investigations related to the characterization of freestream disturbances may be found in Duan et al. [11].

Many operating hypersonic wind tunnels (including the NASA 20-Inch Mach 6 Wind Tunnel) have a rectangular test section, and therefore, are nonaxisymmetric in nature. The freestream disturbances in these tunnels represent the net effect of acoustic radiation from the boundary layers along the four nozzle walls and the corner flows in between. The spatio-temporal characteristics of freestream disturbances within a rectangular test section are determined by the outcome of the acoustic reverberation process within the confined environment, and the nature of this reverberation in a nonaxisymmetric nozzle is expected to be different from that inside an axisymmetric wind tunnel. To further understand the wind-tunnel acoustic environment in nonaxisymmetric nozzles, the current research effort seeks to simulate the acoustic radiation within a rectangular test section. Here, we report DNS of acoustic radiation in a rectangular configuration with four independent, spatially evolving flat-plate turbulent boundary layers with an edge Mach number of $M_\infty = 2.5$, similar to that studied by Duan et al. [2] for the single flat-wall configuration. Of principal interest is the comparison between the freestream pressure fluctuations in a nonaxisymmetric confined environment with those radiated from a single flat wall in an unconfined setting. Inter alia, an attempt is made to understand the role of the previous flat-plate acoustic simulations in the context of investigating the noise environment within a nonaxisymmetric wind tunnel. The flat-wall test section considered herein represents a simpler geometric configuration than the quasi-2D nozzle considered in the previous simulations by Deegan et al. [12] and Hildebrand et al. [13] for a hypersonic flow ($M \approx 6$) inside the test section.

As effort of risk reduction, this abstract presents a DNS of acoustic radiation in a double-wall (channel) configuration with two independent, spatially evolving flat-plate turbulent boundary layers with an edge Mach number of $M_\infty = 2.5$, similar to that studied by Duan et al. [2] for the single flat-wall configuration. Comparisons of the freestream pressure fluctuations between the single-wall and double-wall configurations were performed to highlight any influences of flow configurations.

II. Flow Conditions and Numerical Methodology

A. Flow Conditions

To simulate acoustic radiation within an enclosed environment while simultaneously facilitating a direct comparison with acoustic radiation from a single flat wall in an unconfined setting, we consider a nominally straight channel at Mach 2.5, with two independent turbulent boundary layers developing spatially over the top and bottom flat walls, respectively.

The channel has a slightly increasing height with an expansion angle of 0.3° to account for the boundary layer growth in the downstream direction. At the selected expansion angle, the resulting Clauser pressure-gradient parameter is $|\beta| < 0.05$ over the downstream portion of the channel where statistics of turbulent and acoustic fields are evaluated. The small values of the Clauser parameter β in the present simulation may be compared with the range of $0 < \beta < 1.68$ in the simulations of Lee and Sung [14], who investigated the effect of adverse pressure gradient on turbulent boundary layers, designating $\beta = 1.68$ as a strong pressure gradient and the case of $\beta = 0.73$ as a moderate pressure gradient. The above comparison indicates that the pressure gradient effect in the present simulation is relatively insignificant and that a near-equilibrium state should have been reached in the channel-wall turbulent boundary layers [15]. This claim is further supported in the subsequent results wherein we demonstrate a satisfactory agreement between the computed variation in skin friction coefficient C_f as a function of Re_θ and other established results in the literature. The accompanying variation in the shape factor H and the form parameter provides corroborating evidence to justify the nominally zero pressure gradient of the boundary layer. By choosing such a double-wall geometry, the effects of surface curvature and pressure gradient in the streamwise direction can be neglected to the leading order, thus avoiding extraneous complexity in the simulation. In addition to the DNS of acoustic radiation within a double-wall configuration (Case Double Wall), a DNS of acoustic radiation from a single flat surface in the unconfined setting (Case Single Wall) is conducted under the same flow conditions. The results of the two cases are compared to highlight the effects of an enclosed environment. Table 1 outlines the freestream and wall-temperature conditions for both simulations. The wall surface temperature is approximately equal to the the estimated recovery temperature based on a recovery factor of 0.89.

Table 1 Freestream conditions for Mach 2.5 DNS of turbulent boundary layers.

M_∞	$U_\infty(\text{m/s})$	$\rho_\infty(\text{kg/m}^3)$	$T_\infty(\text{K})$	$T_w(\text{K})$
2.5	823.6	0.1	270.0	568.0

B. Numerical Method

The three-dimensional compressible Navier-Stokes equations are solved numerically. The working fluid is assumed to be a perfect gas, and the usual constitutive relations for a Newtonian fluid are used. The viscous stress tensor is linearly related to the rate-of-strain tensor, and the heat flux vector is linearly related to the temperature gradient through Fourier's law. Sutherland's law is used for the coefficient of viscosity, μ . The coefficient of thermal conductivity κ is computed from $\kappa = \mu C_p / Pr$, where $Pr = 0.71$. The inviscid fluxes of the governing equations are computed via a sixth-order split convective finite difference (SCFD) scheme, proposed by Pirozzoli [16]. This scheme is simple to implement in existing finite-difference compressible flow solvers and runs more than twice as fast as the conventional WENO method. Numerical experiments performed by Pirozzoli et al. [17] had also established the robustness of split convective discretization, along with an order of accuracy of up to 10 for supersonic boundary-layer edge Mach numbers

up to Mach 4. The viscous fluxes are discretized using a fourth-order central difference scheme, and time integration is performed using a third-order low-storage Runge-Kutta scheme [18].

C. Simulation Setup

Two simulations representing, respectively, the double-wall case (denoted henceforth as Double Wall) and the single-wall case (Single Wall) have been carried out in the present work. Figure 1 shows the general computational set-up for the DNS of Case Double Wall. As mentioned before, both bottom and top walls are kept open at a fixed angle of $\alpha = 0.3^\circ$ with respect to the center plane. The domain size for the computations is $(78.4\delta_i, 11.7\delta_i, 11.7\delta_i)$ in the streamwise, spanwise, and wall-normal directions, respectively, where δ_i is the mean boundary layer thickness based on $u/U_\infty = 0.99$ at the inflow boundary. The streamwise domain length, $L_x = 78.4\delta_i$, is selected so that the Karman number, Re_τ , reaches approximately 510 and 508 near the end of the domain (where most of the analysis is carried out) for the Single Wall and Double Wall cases, respectively. For reference, the boundary layer thickness δ at the selected downstream location for statistical analysis corresponds to $\delta/\delta_i \approx 1.97$ and 1.99 for the above two cases. The spanwise domain size is large enough for the spanwise pressure correlation at the wall to become nearly zero, as shown in Figure 2. However, the correlation in the free stream has a small nonzero negative value at a spanwise separation equal to one half of the domain width, indicating a low-level coherence at the largest scales of the radiated acoustic field. Further investigation of the effects of spanwise domain width at the flow conditions of interest was presented in an earlier study [2], which has verified that the nonzero correlation at the largest scales of the radiated acoustic field has negligible influence on the statistical features of the freestream acoustic field that are reported in the present paper. For Case Double Wall, the distance between the two flat plates near the end of the computational domain corresponds to $H/\delta \approx 6$. The plate distance has been selected to be large enough so that an inflow sponge (Eq. 3) provides adequate damping to the artificial inflow shock. The selected value of plate distance H/δ is also comparable to that in the test section of a hypersonic wind tunnel [12, 13]. Numerical experiments with varying H/δ have shown negligible differences in the conclusions drawn herein when a simulation is done with a larger plate distance H/δ .

The number of grid points in the streamwise, spanwise, and wall-normal directions are 2350, 600, and 581, respectively. Table 2 lists the details of the domain length, grid dimensions, and grid resolution in all three directions for each case. Grids are uniformly spaced in the streamwise and spanwise directions, with grid spacings of $\Delta x^+ = 8.9$ and $\Delta y^+ = 5.2$, respectively. The ‘+’ superscript denotes nondimensionalization by the viscous length scale $z_\tau = \nu_w/u_\tau$ at the reference location where statistics are collected. The grid in the wall-normal direction is symmetric about the center plane at $z/\delta_i \approx 6$, and is clustered in the boundary layer with $\Delta z^+ = 0.6$ at the wall, and kept uniform at $\Delta z^+ = 9.5$ in the free stream all the way up to the center plane of the domain in the Double-Wall case. The computational grid resolution is similar to the reported values in the previous simulations of acoustic radiation from supersonic and hypersonic turbulent boundary layers that employed comparable numerical algorithms [2, 5, 6]. Additional evidence to demonstrate

satisfactory numerical accuracy of the current simulation is presented in the following sections, wherein the computed flow statistics are compared with both experiments and other high-quality simulations at similar flow conditions.

Table 2 Grid resolution and domain size for the DNS with $\delta_i = 4.0$ mm, $z_\tau = 0.015$ mm.

Case	L_x/δ_i	L_y/δ_i	L_z/δ_i	$N_x \times N_y \times N_z$	Δx^+	Δy^+	Δz_{min}^+	Δz_{max}^+
Double Wall	76.4	11.7	11.7	$2350 \times 600 \times 581$	8.9	5.2	0.6	9.0
Single Wall	57.2	11.7	41.0	$1760 \times 600 \times 574$	8.9	5.2	0.6	8.7

The overall numerical procedure used for the present set of DNS is very similar to the well established process for single-wall simulations [2, 5, 6]. A significant difference for the Double Wall simulation involves the generation of an initial flow field, which is now generated from two independent single-wall turbulent boundary layers by swapping one of them to the top symmetric part. At the wall(s), no-slip conditions are applied for the three velocity components, and an isothermal condition is used for the temperature, setting $T_w = 568$ K. Unsteady nonreflecting boundary conditions based on Thompson [19] are imposed at the outlet boundary and periodic boundary conditions are used in the spanwise direction. The inflow boundary condition is prescribed by means of a recycling-rescaling method adapted from Xu & Martin [20]. A dynamic spanwise shifting [21] is added to improve the low-frequency characteristics of the generated inflow turbulence. A spatial filter, which is used to avoid the recycling of acoustic disturbances within the freestream region is also added at the inlet boundary, and it has been shown to have no direct impact on the boundary layer flow [2]. The rescaling station is located at a distance of $25\delta_i$ downstream of the inflow plane, as indicated in Figure 1. For comparison, Simen et al. [22] showed that the rescaling plane used to synthesize the inflow turbulence should be at least $20 - 30\delta_i$ to avoid spurious flow periodicities. The recycling process for the Double-Wall configuration is based on the multiwall extension proposed by Boles et al [23]. The extension applies the standard single wall recycling/rescaling procedure, and then computes the recycled quantity via weighted contributions from the pair of grid points that are located at equal distance from the the two walls, specifically,

$$q = \sum_{n=1}^2 W_n q_n \quad (1)$$

$$W_n = \frac{1}{d_n^2} \left(\sum_{m=1}^2 \frac{1}{d_m^2} \right)^{-1} \quad (2)$$

where q_n indicates the quantity computed for wall n , and d_n indicates the perpendicular distance of a given point to wall n .

The inflow treatment mentioned above, combined with the filter in the core-flow region, induces a weak shock wave emanating from the inflow station (see Figure 4a) To prevent the artificial inflow shock from interacting with the boundary layer along the opposite wall, a sponge zone is imposed near the center of the double-wall configuration.

The sponge zone is established by adding a restoring body force vector, $-\sigma(\mathbf{x})(q - q_{ref})$, to the right-hand side of the governing equations. The effect of the sponge zone is to drive the computed solution of q to match the reference solution q_{ref} . The parameter (sponge strength) σ , which is a function of x and z only, takes the form of Eq. 3 [24].

$$\sigma(x, z) = a \left(1.0 - \frac{|z - z_c|}{h_{sponge}} \right)^2, \quad 0 \leq x/\delta_i \leq 17.5, \quad 2.5 \leq z/\delta_i \leq 9.2, \quad (3)$$

where z is the wall-normal coordinate, z_c is the center plane height, and h_{sponge} is the length of the sponge region in z direction. In the current simulation, we have taken $h_{sponge}/\delta_i = 6.7$. The constant a is set to $1e-5$, which is found to provide a gentle damping of the shock wave over the current sponge region. In the full paper, the influence of the inflow sponge on computed turbulent and acoustic fields will be further investigated to justify the choice of the sponge layer parameters.

Figure 4 shows the contours of the mean pressure and the instantaneous (or RMS) fluctuation field. The mean pressure field clearly indicates the inflow induced shock and its reflections from both walls in the downstream region. However, the strength of this system of shocks is relatively weak. The inflow shock is the strongest, with a pressure ratio of approximately 1.1, and the subsequent, reflected shocks are even weaker. In contrast to the mean pressure field, the pressure fluctuation field does not indicate any visible trace of rapid amplification across these shocks, either in an instantaneous snapshot or in the rms distribution.

The set up of Case Single Wall employs a streamwise domain length of $L_x/\delta_i = 57.2$, which is shorter than the domain length in the double-wall case, but long enough for the turbulent boundary layer to reach $Re_\tau \approx 510$ by the end of domain. The height is set to $L_z/\delta_i = 41.0$, and the nonreflecting outlet boundary condition of Thompson [19] is applied on the top boundary. The grid resolutions in all three directions are identical to those used in the case Double Wall, except that the wall-normal grid begins to stretch out beyond $z/\delta_i = 6$.

III. Results

In the results that follow, the statistics and frequency spectra of both the boundary layer turbulence and the acoustic field in the free stream are evaluated in the vicinity of a reference plane at x_a . Averages are first calculated over each instantaneous snapshot of the flow field within a streamwise window of $[x_a - \delta_i, x_a + \delta_i]$ and the entire spanwise domain; then, an ensemble average over such snapshots is calculated across a time interval of approximately $235\delta_i/U_\infty$. Flow statistics over the full set of snapshots is compared with that obtained using the snapshots across just one half of the total interval and negligible differences ($< 1\%$) are observed between the two, indicating a satisfactory statistical convergence of the computed flow characteristics. Power spectra are calculated using the Welch method [25] with eight segments and 50% overlap. A Hanning window is used for weighting the data prior to the fast Fourier transform (FFT) processing. The sampling frequency corresponds to $f_s \approx 24.2U_\infty/\delta_i$ (or, equivalently, $\Delta t_s^+ \approx 0.53$), and the duration of

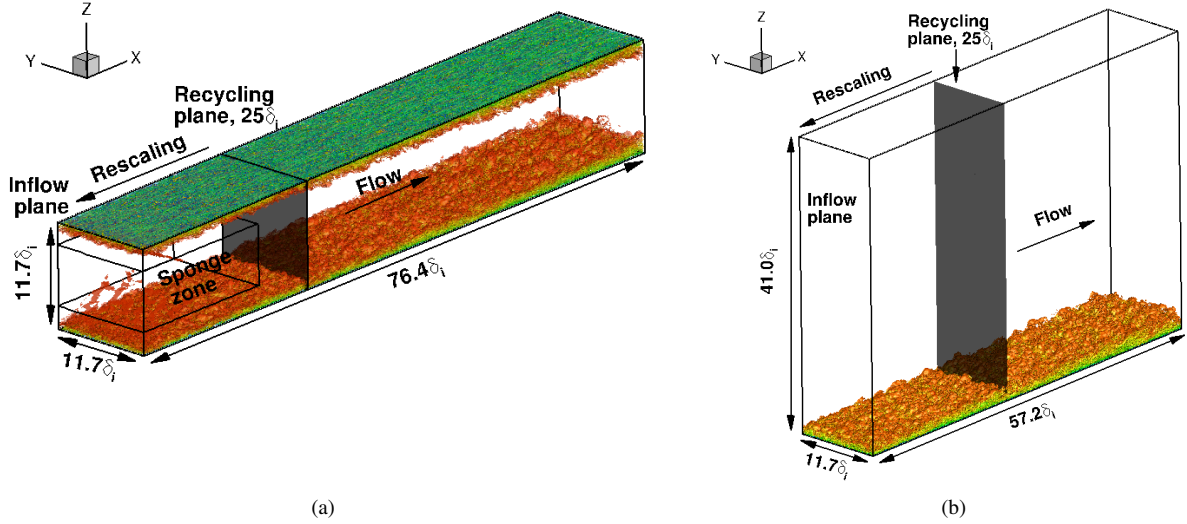


Fig. 1 Computational domain set up for the DNS case (a) Double Wall, and (b) Single Wall. The reference length δ_i is the inflow boundary layer thickness. An instantaneous flow is shown in the domain visualized by iso-surface of the magnitude of density gradient, $|\Delta\rho|\delta_i/\rho_\infty = 0.98$, colored by the streamwise velocity component. x , y , and z are the streamwise, spanwise, and wall-normal coordinate, respectively. The black edge box near the inflow region represents the sponge zone, which extends from $0 \leq x/\delta_i \leq 17.5$ in the streamwise direction, and $2.5 \leq z/\delta_i < 9.2$ in the wall-normal direction.

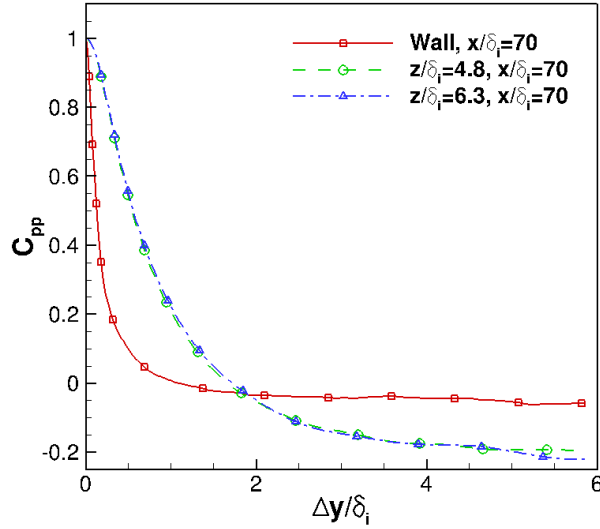


Fig. 2 The correlation coefficient of pressure fluctuations as a function of spanwise separation at the wall and in the free stream at $x = 70\delta_i$.

an individual segment is approximately $52.5\delta_i/U_\infty$ (i.e., $t_{segment}^+ = 675$). Negligible difference in spectral estimation is observed within the reported frequency range, when the overall time record is subdivided in 4 segments instead of the baseline number of 8 segments.

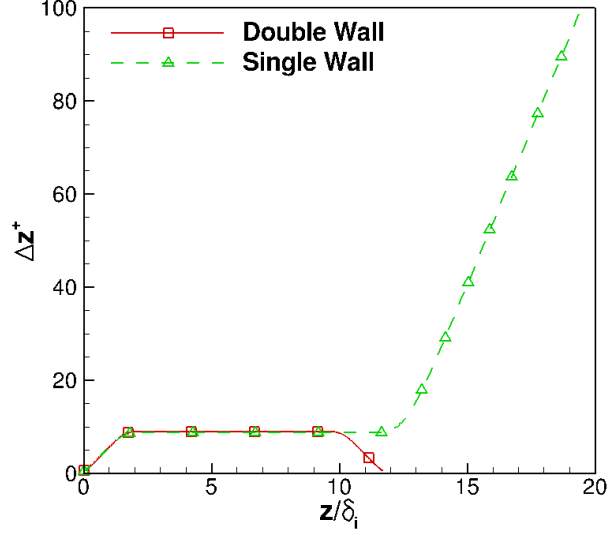


Fig. 3 The wall-normal grid distribution. The grid spacing Δz^+ is normalized by the viscous length scale, $z_\tau = 0.015$ mm.

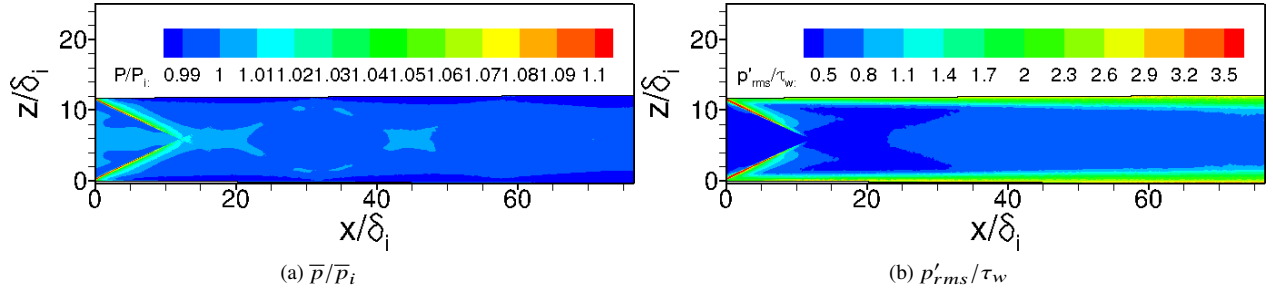


Fig. 4 Contours of (a) normalized mean pressure with contour levels varying from 0.99 to 1.1 with increment of 0.01, and (b) normalized rms pressure fluctuations with contour levels varying from 0.5 to 3.5 with increment of 0.3 for Case Double Wall.

Table 3 lists the value of the boundary layer parameters at the reference location (x_a), including the momentum thickness θ , shape factor $H = \delta^*/\theta$, boundary layer thickness δ , viscous length z_τ , friction velocity u_τ , and three different Reynolds number parameters, Re_θ , Re_τ , and Re_{δ^2} . The values are comparable to those of the previous DNS data [2]. Although the difference between Re_τ values is believed to be insignificant, we will minimize such a difference by resampling at closer values in the final paper.

A. Boundary layer statistics

Figure 5a compares the van Driest II transformed skin friction coefficient of the current DNS cases to the empirical friction law of Smits et al. [26] and a DNS by Pirozzoli et al. [27] at Mach 2. The van Driest II transformation [28] is used to transform the compressible skin friction to that in an equivalent incompressible flow to allow a comparison with the established skin friction correlations for low-speed flows, specifically,

Table 3 Boundary layer properties at the station x_a selected for the statistical analysis.

Case	x_a/δ_i	Re_θ	Re_τ	Re_{δ_2}	θ (mm)	H	δ (mm)	z_τ (mm)	u_τ (m s ⁻¹)
Double Wall	70.0	2840	469	1660	0.59	4.1	7.26	0.015	39.7
Single Wall	48.8	3275	537	1914	0.68	3.9	8.31	0.015	39.6

$$C_{fi} = F_c C_f, \quad Re_{\theta i} = \frac{\mu_\infty}{\mu_w} Re_\theta = Re_{\delta_2} \quad (4)$$

where $Re_{\theta i}$ is the Reynolds number based on the momentum thickness of the equivalent incompressible boundary layer, and F_c is the van Driest II scaling factor, given as:

$$F_c = \frac{\bar{T}_w/T_\infty - 1}{\arcsin^2 \alpha}, \quad \alpha = \frac{\bar{T}_w/T_\infty - 1}{\sqrt{\bar{T}_w/T_\infty (\bar{T}_w/T_\infty - 1)}} \quad (5)$$

The incompressible friction law [26] used for comparison is

$$C_{fi} = 0.024 Re_{\theta i}^{-1/4}. \quad (6)$$

Figure 5a shows that the van Driest II transformed skin friction coefficients for Cases Single Wall and Double Wall match well with each other and also with the above incompressible friction law and the supersonic DNS by Pirozzoli et al. [27].

Figure 5b plots the van Driest transformed mean velocity profile, $\bar{U}_{VD} = \frac{1}{u_\tau} \int_0^{\bar{U}} (\bar{T}_w/\bar{T})^{1/2} dU$, at the reference streamwise location for the two DNS cases, along with pertinent data from the previous literature. The figure shows that the van Driest transformed velocity profiles of the two DNS cases collapse well with those from the reference datasets. The DNS profiles in the figure show a clearly linear viscous zone, $u_{VD}^+ = z^+$, and a logarithmic region where $u_{VD} = 1/\kappa \log(z^+) + C$ with $\kappa = 0.41$ and $C = 5.2$.

Figure 5c shows the wall-normal distributions of the normalized, density weighted turbulence intensity [27]. Both DNS cases are in good agreement with the data from the existing literature [2, 5, 29]. The Mach number fluctuations, shown in Figure 5d, are also the same for both Double Wall and Single Wall cases, indicating that there is little or no effect of the Double-Wall configuration on the turbulent fluctuations within the boundary layer.

B. Pressure fluctuations

Figure 6a plots the wall-normal profiles of normalized pressure fluctuations at three different streamwise locations, $x/\delta_i = 37.5, 42.5$, and 47.5 , respectively. The agreement between all three profiles indicates that both the boundary layer turbulence and the freestream acoustic field have become approximately homogeneous within this downstream portion

of the computational box. Figure 6b compares the wall-normal distributions of the normalized pressure fluctuation at the reference location. Within the boundary layer, the distribution of the pressure fluctuations is almost the same for case Double Wall and case Single Wall, indicating that the reflected acoustic radiation by the turbulent boundary layer from the other wall has a negligible effect on the wall pressure fluctuation. The value of pressure fluctuations for the Single Wall case plateaus at 0.43 in the free stream. However, the pressure fluctuation for the Double Wall case has a higher freestream value of approximately 0.61. The ratio of the freestream pressure fluctuations in the Double Wall and Single Wall cases corresponds to $(p'_{rms})_{DoubleWall} / (p'_{rms})_{SingleWall} \approx \sqrt{2}$, or equivalently, the intensity of the freestream acoustic fluctuations for case Double Wall is nearly 2 times that in the Single Wall case. This finding is consistent with the observations by Laufer [1], who reported that the acoustic fluctuations radiated from multiple tunnel walls can be superimposed in a mean-square sense and, hence, that the intensity of pressure fluctuations in the core region of the tunnel test section is approximately equal to that radiated from a single tunnel-wall boundary layer multiplied by the number of radiating walls. An analogous result was also obtained for a Mach 6 test section in our previously reported simulations [12, 13].

C. Frequency spectra of pressure fluctuations

The frequency spectrum of the pressure fluctuations is defined as

$$\Phi_p(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{p'(x, y, z, t)p'(x, y, z, t + \tau)} e^{-i\omega\tau} d\tau. \quad (7)$$

Figure 7 shows a comparison involving the power spectral density (PSD) of pressure fluctuations at the respective reference locations from cases Double Wall and Single Wall, respectively. In each case, the spectrum is normalized by the mean square pressure fluctuation so that the integral of the spectrum over the full range of the abscissa is equal to unity. Results are shown for the pressure spectra at the wall as well as in the free stream. In each case, the spectral variation is plotted in both outer and inner scales. The degree of agreement between the spectra for the unconfined (Single Wall) and confined (Double Wall) cases indicates that the geometric confinement has relatively small influence on the normalized frequency spectrum of pressure fluctuation at the wall and also in the core flow (i.e., freestream) region. Thus, the primary difference between the pressure spectra for the two configurations corresponds to a nearly two-fold increase in the spectral level of freestream fluctuations in the Double Wall case. It is worth noting, however, that for sufficiently low frequencies ($\omega\delta/U_\infty < 0.5$), the normalized acoustic spectra for the Single-Wall and Double-Wall cases indicate noticeable differences. At these low frequencies, the acoustic wavelength becomes comparable to the height of the test section so that acoustic reflections from the opposite wall are expected to become significant. Analysis of coherence between the acoustic fluctuations just outside of the two boundary layers will be presented in the final paper. Figure 8 further compares the premultiplied spectrum for the single-wall and double-wall cases. Similar to the

spectra from Figure 7, the two cases show similar energy distribution over the resolvable frequency range.

D. Two-point correlation of pressure fluctuations

Two-point space-time correlation is used next to illustrate the spatial size and orientation of statistically significant pressure structures, as well as the propagation and evolution of such structures. The space-time correlation of the pressure fluctuation field is defined as

$$C_{pp}(\Delta x, \Delta y, z, z_{ref}, \Delta t) = \frac{\overline{p'(x, y, z_{ref}, t) p'(x + \Delta x, y + \Delta y, z, t + \Delta t)}}{\left(\overline{p'^2(x, y, z_{ref}, t)}\right)^{1/2} \left(\overline{p'^2(x + \Delta x, y + \Delta y, z, t + \Delta t)}\right)^{1/2}} \quad (8)$$

where Δx , Δy , and Δt are the streamwise and spanwise spatial separations and the time delay, respectively.

Figure 9 shows the contours of streamwise-spanwise correlation coefficient at zero time delay, $C_{pp}(\Delta x, \Delta y, z_{ref}, z_{ref}, 0)$, within a specified constant- z plane for the Single Wall and Double Wall cases. The correlation structure at the wall ($z_{ref}/\delta = 0$) is approximately circular when the spatial separation is less than approximately 15 percent of the local boundary layer thickness, but it becomes increasingly elongated in the spanwise direction for larger separation distances. In the free stream ($z_{ref}/\delta = 3.17$), the extent of the pressure correlation contours increases in both streamwise and spanwise directions in comparison with those at the wall. The contours show similar patterns for both the single-wall and double-wall cases, indicating that the presence of the second wall does not affect the shape of the correlation contours within a fixed constant- z plane.

Figure 10 shows the contours of correlation coefficient in a streamwise wall-normal plane for multiple reference locations along the wall-normal axis in the Double Wall case. For reference location at the wall, the contours of constant $C_{pp}(\Delta x, 0, z, z_{ref})$ are inclined at approximately 80° with respect to the downstream direction, but the inclination angle gradually decreases as the reference location moves out of the boundary layer. For reference locations within the potential core, i.e., inside the local free stream, the inclination angle is approximately 40° , which is nearly the same as that of the single-wall case [2]. More significant, for the reference locations in the freestream region, correlation contours for the double-wall case represent a superposition of two sets of elliptical contours. The elliptical contours traversing the first and third quadrants correspond to upward propagating acoustic waves emanating from the turbulent fluctuations near the lower wall, whereas those traversing the second and fourth quadrants denote the downward inclined pressure waves radiating from the upper wall boundary layer. On the centerline ($z_{ref}/\delta = 3.17$), the computed contours of pressure correlation are nearly symmetric with respect to the wall-normal coordinate, representing the nominally equal contributions to the local acoustic radiation from the boundary layers along the bottom and top walls, respectively. Similar acoustic structures consisting of upward and downward propagating acoustic waves within the freestream region are shown in an instantaneous flow field, where the freestream acoustic field is visualized by the gray contours of the density gradient (Figure 11). Such a trend is consistent with the statistically significant spatial acoustic structures in the

free stream illustrated by the two-dimensional slice of line contours of the spatial correlation coefficient in Figure 10. Figure 12 further compares the characteristics of the streamwise correlation, $C_{pp}(\Delta x, 0, z, z_{ref})$, for the single-wall and double-wall cases, respectively, both at the wall and in the free stream. For z_{ref} at the wall, the contours of constant $C_{pp}(\Delta x, 0, z, z_{ref})$ are nearly the same for both cases, indicating that the presence of the second wall has little influence on the pressure structures within the boundary layer. For z_{ref} in the free stream ($z_{ref} = 3.17$), however, correlation contours for the double-wall case represent a superposition of upward and downward propagating acoustic waves while those for the single-wall case correspond to upward propagating acoustic waves only.

Figures 14a and 14b show the contours of space-time correlation at the wall ($z_{ref}/\delta = 0$) and in the free stream ($z_{ref}/\delta = 3.17$), respectively, for Cases Double Wall and Single Wall. The narrow band of $C_{pp}(\Delta x, \Delta t)$ contours at both z_{ref} locations indicates a strong downstream propagation of the pressure fluctuations. The shape and skewness of the space-time correlation contours are approximately the same for the two DNS cases, indicating that the presence of the second wall does not affect the propagation and evolution of pressure structures within the boundary layer and in the free stream.

Figure 15a plots the propagation speed U_c as a function of streamwise spatial separations Δx at several different wall-normal heights for Case Double Wall. For a given Δx , the propagation speed $U_c = \Delta x/\Delta t$ can be defined by using the time lag Δt , where

$$\left. \frac{\partial C_{pp}(\Delta x, 0, z_{ref}, z_{ref}, r_t)}{\partial r_t} \right|_{r_t = \Delta t} = 0. \quad (9)$$

For $\Delta x/\delta > 0.5$, U_c/U_∞ is approximately 0.75 at the wall, and 0.4 in the free stream. The propagation speed within the core (i.e., freestream) region is significantly smaller than the local mean velocity, resulting in the supersonic relative Mach number of $M_r = (U_\infty - U_c)/a_\infty \approx 1.5$. Such a trend is consistent with the classic theory of “eddy Mach-wave radiation” and the early measurements of freestream noise by Laufer [1]. Figure 15b shows that the propagation speeds in cases Double-Wall and Single-Wall are nearly the same, regardless of whether one considers signal propagation along the wall or in the freestream region.

IV. Conclusion

In this abstract, direct numerical simulation of acoustic radiation is conducted in a double-wall (channel) configuration with two independent spatially-developing flat-plate turbulent boundary layers at a freestream Mach number of 2.5 and a friction Reynolds number of approximately 510. To investigate the effect of geometric confinement, the global acoustic field in the double-wall configuration is compared to that induced by a flat wall in an unconfined setting at a similar freestream Mach number and Reynolds number. The DNS data showed that the intensity of freestream acoustic fluctuations in the double-wall configuration is higher in comparison with that over a single flat wall in an unconfined setting, with the pressure fluctuations in the free stream satisfying $\overline{(p'^2)}_{DoubleWall} \approx 2\overline{(p'^2)}_{SingleWall}$

or $(p'_{rms})_{DoubleWall} \approx \sqrt{2}(p'_{rms})_{SingleWall}$. This finding is similar to our earlier findings for the acoustic radiation inside a Mach 6 test section downstream of a converging-diverging nozzle. The geometric confinement has negligible influence on the frequency spectrum and bulk propagation speed of the acoustic disturbances within the freestream region, except for low enough frequencies where the acoustic wavelength becomes comparable to the height of the test section. For both the single-wall and double-wall cases, the prominent structures associated with acoustic fluctuations within the freestream region, as visualized by instantaneous density gradient, exhibit a preferred orientation similar to that predicted by the classic theory of “eddy Mach-wave radiation” and the early measurements of freestream noise by Laufer.

The full paper will extend the current two-dimensional (i.e., spanwise periodic) simulations to three-dimensional ones that include spanwise side-wall effects and investigate the effect of geometric confinement on turbulent and acoustic fields.

Acknowledgments

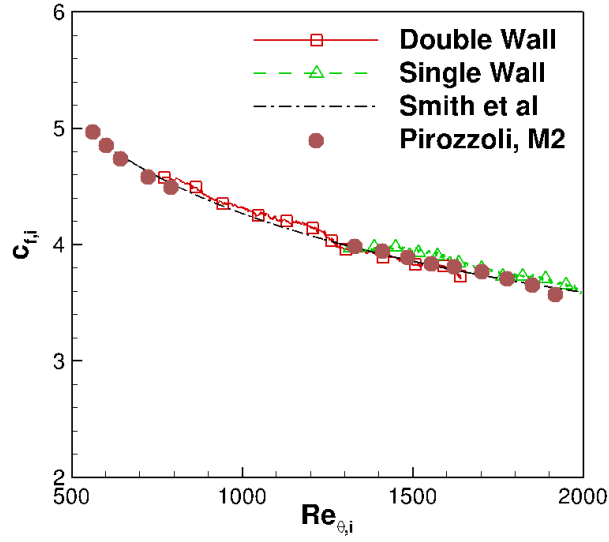
This material is based on the work supported by the Office of Naval Research under grant N00014-17-1-2347, managed by Dr. Eric Marineau. Computational resources were provided by the DoD High Performance Computing Modernization Program and the Ohio Supercomputer Center. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the funding agencies or the U.S. Government.

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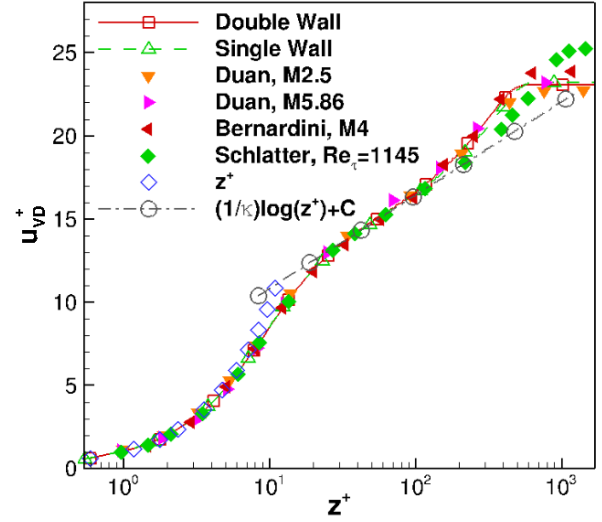
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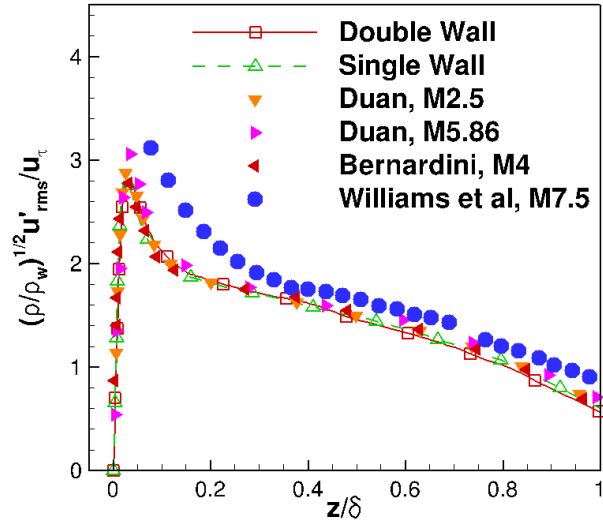
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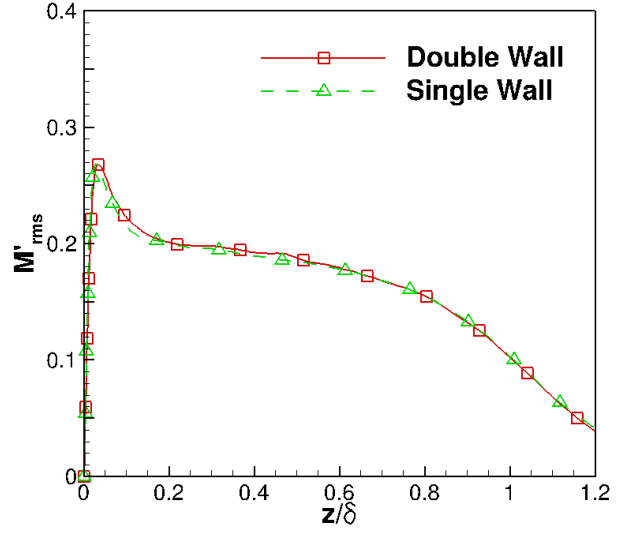
(a)



(b)



(c)



(d)

Fig. 5 (a) van Driest II transformed friction coefficient; (b) van Driest transformed mean velocity profile ($\kappa = 0.41$, $C = 5.2$); (c) turbulence intensities; and (d) fluctuating Mach number at the reference location.

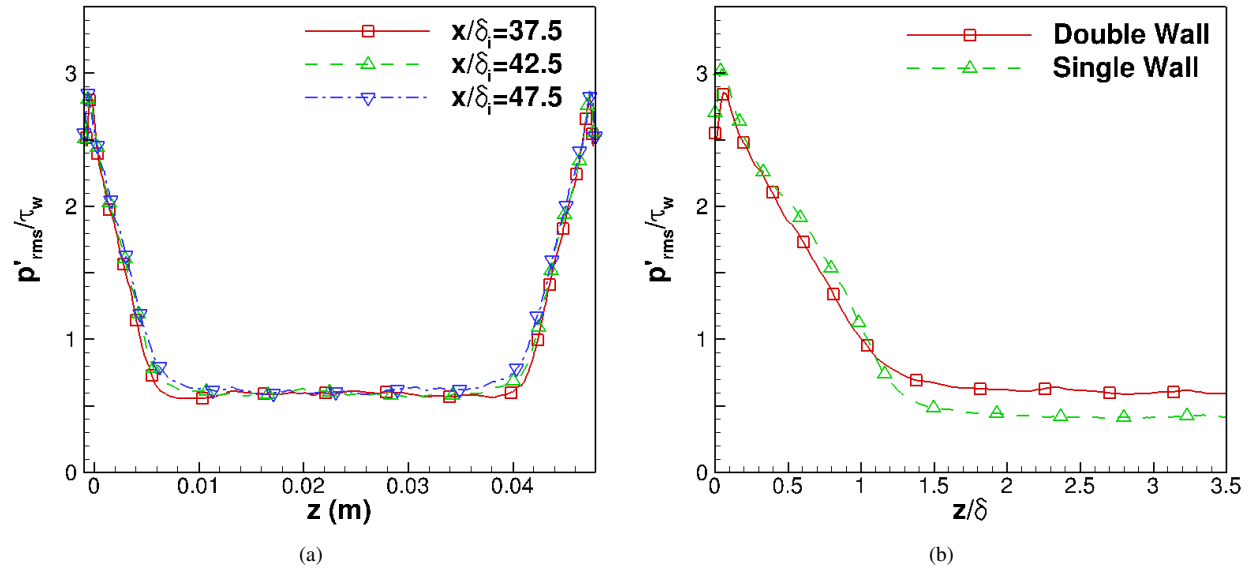


Fig. 6 (a) Wall-normal distributions of the normalized r.m.s. pressure fluctuation at selected downstream locations for case Double Wall, and (b) wall-normal distribution of the normalized rms pressure fluctuation for DNS case Double Wall and Single Wall at $x/\delta_i = 47.5$.

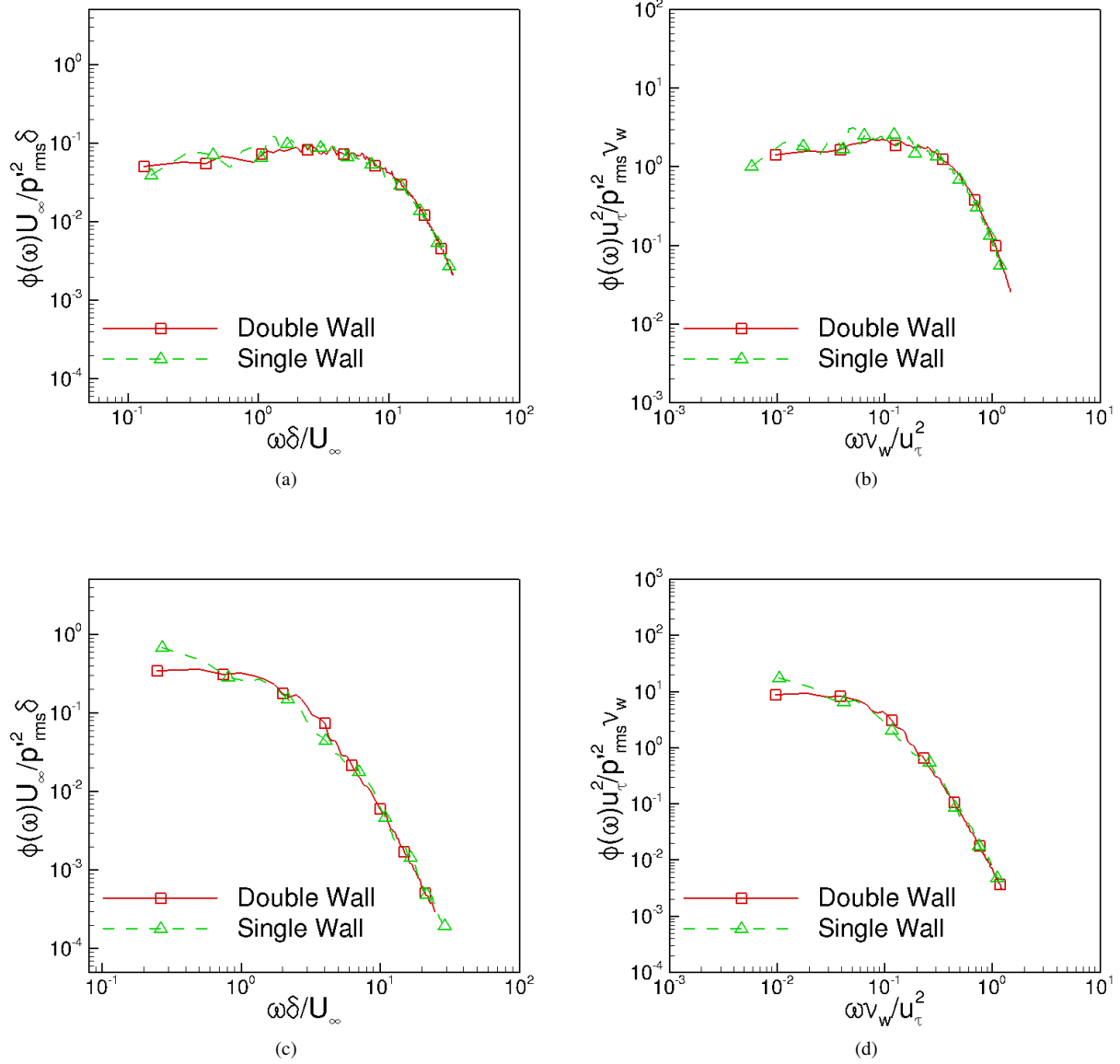


Fig. 7 Frequency spectrum of computed pressure signal at selected heights for DNS Cases Double Wall and Single Wall (a,b) at the wall and (c,d) in the free stream. Parts (a),(c) correspond to outer scales, whereas parts (b),(d) correspond to the inner scales. For Case Double Wall, the freestream height is selected at the symmetric plane, $z/\delta = 3.17$. Single Wall case corresponds to $L_y/\delta_i = 11.7$.

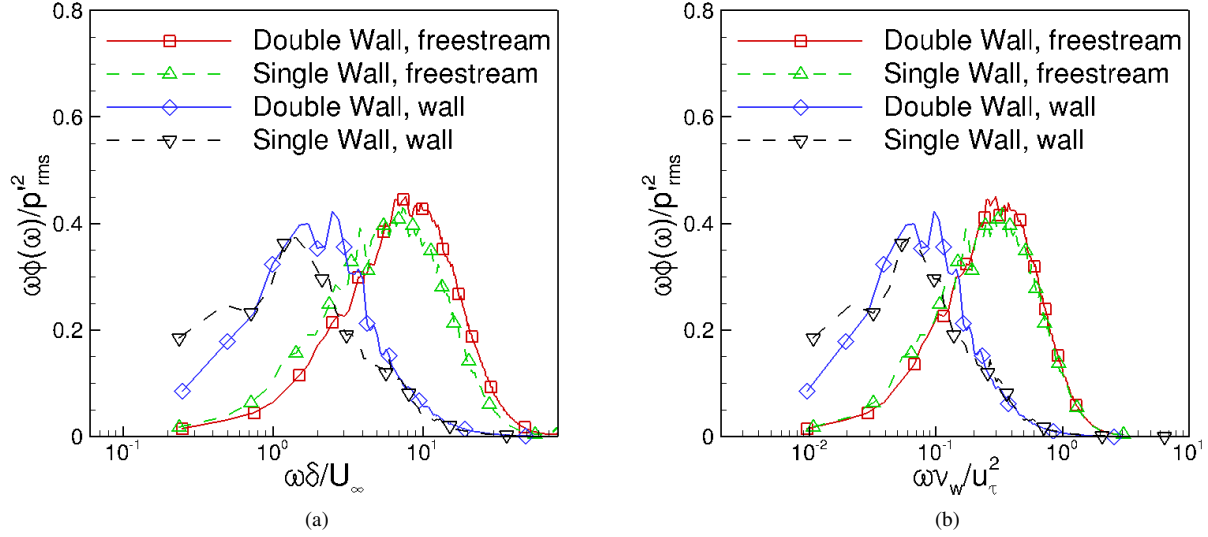


Fig. 8 Comparison in premultiplied power spectrum density of pressure signals at selected heights above the wall between Cases Double Wall and Single Wall. For Case Double Wall, the freestream height is selected at the symmetric plane, $z/\delta = 3.17$. Single Wall case corresponds to $L_y/\delta_i = 11.7$.

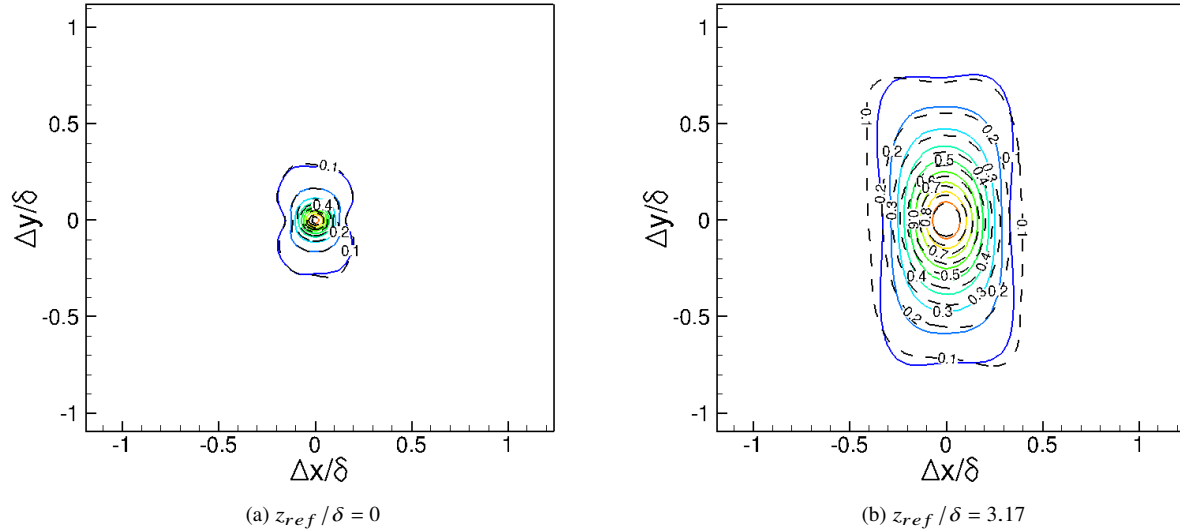


Fig. 9 Contours of constant streamwise-spanwise correlation coefficient of the pressure signal $C_{pp}(\Delta x, \Delta y, z_{ref}, z_{ref}, 0)$ within a fixed constant- z plane for Cases Double Wall (colored solid line) and Single Wall (black dashed line). (a) At the wall ($z_{ref}/\delta = 0$); (b) in the free stream ($z_{ref}/\delta = 3.17$). Single Wall case corresponds to $L_y/\delta_i = 11.7$.

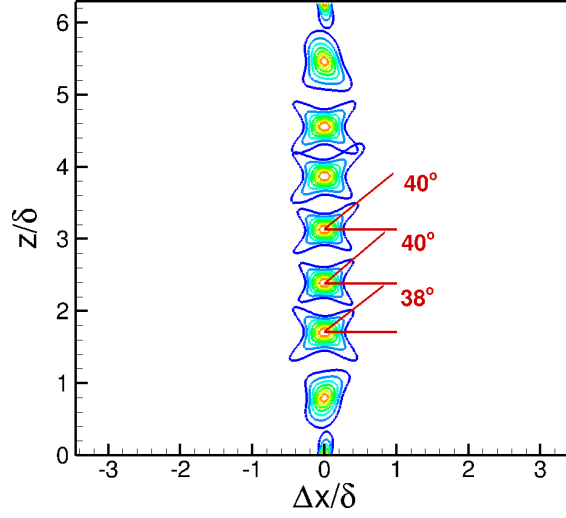


Fig. 10 Contours of constant streamwise wall-normal correlation coefficient of the pressure signal $C_{pp}(\Delta x, 0, z, z_{ref}, 0)$ at multiple selected reference heights ($z_{ref}/\delta = 0, 0.79, 1.70, 2.39, 3.17, 3.87, 4.56, 5.46, 6.26$) for Case Double Wall. Contour levels vary from 0.3 to 0.9 with increment of 0.1.

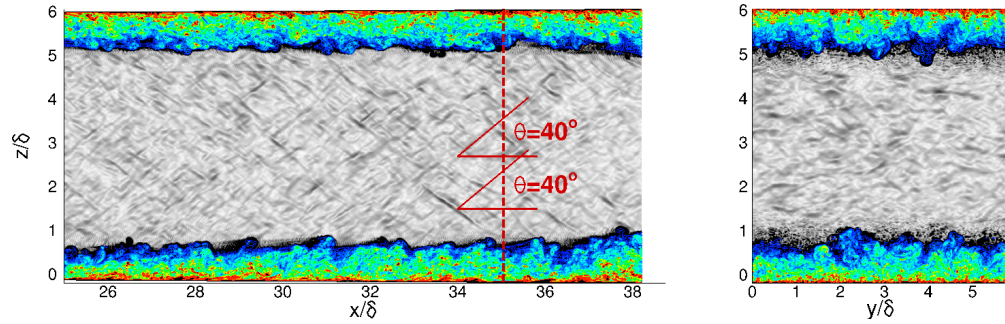


Fig. 11 Numerical schlieren image based on instantaneous flow field: (left) streamwise wall-normal (x-z) plane, and (right) spanwise wall-normal (y-z) plane. The vertical dashed line indicates the streamwise location of the selected spanwise wall-normal plane visualized in the left figure.

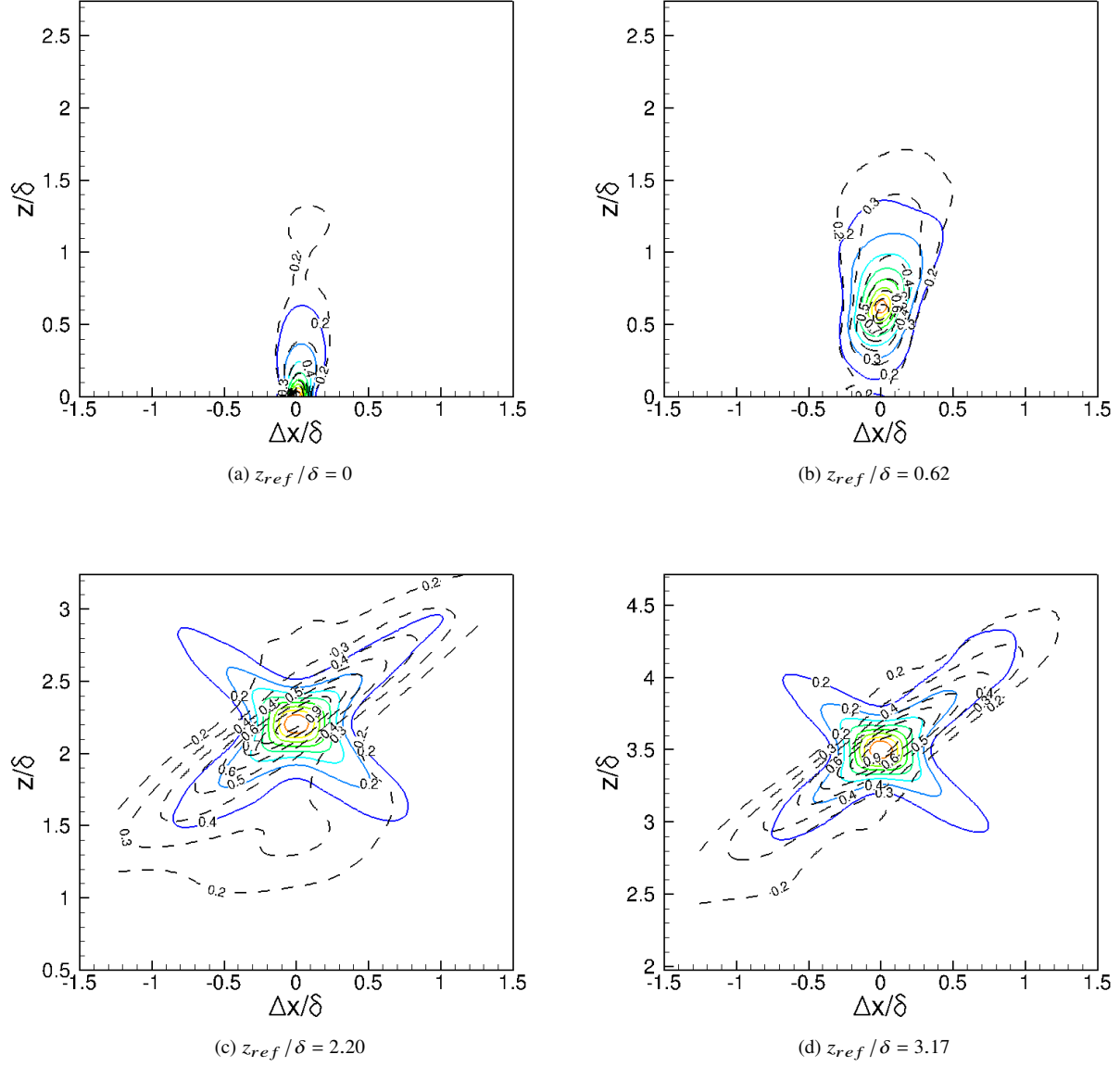


Fig. 12 Comparison of streamwise wall-normal pressure correlation structure $C_{pp}(\Delta x, 0, z, z_{ref}, 0)$ at multiple selected reference heights z_{ref} between Case Double Wall (colored solid line) and Case Single Wall (black dashed line). The Single Wall case corresponds to $L_y/\delta_i = 11.7$.

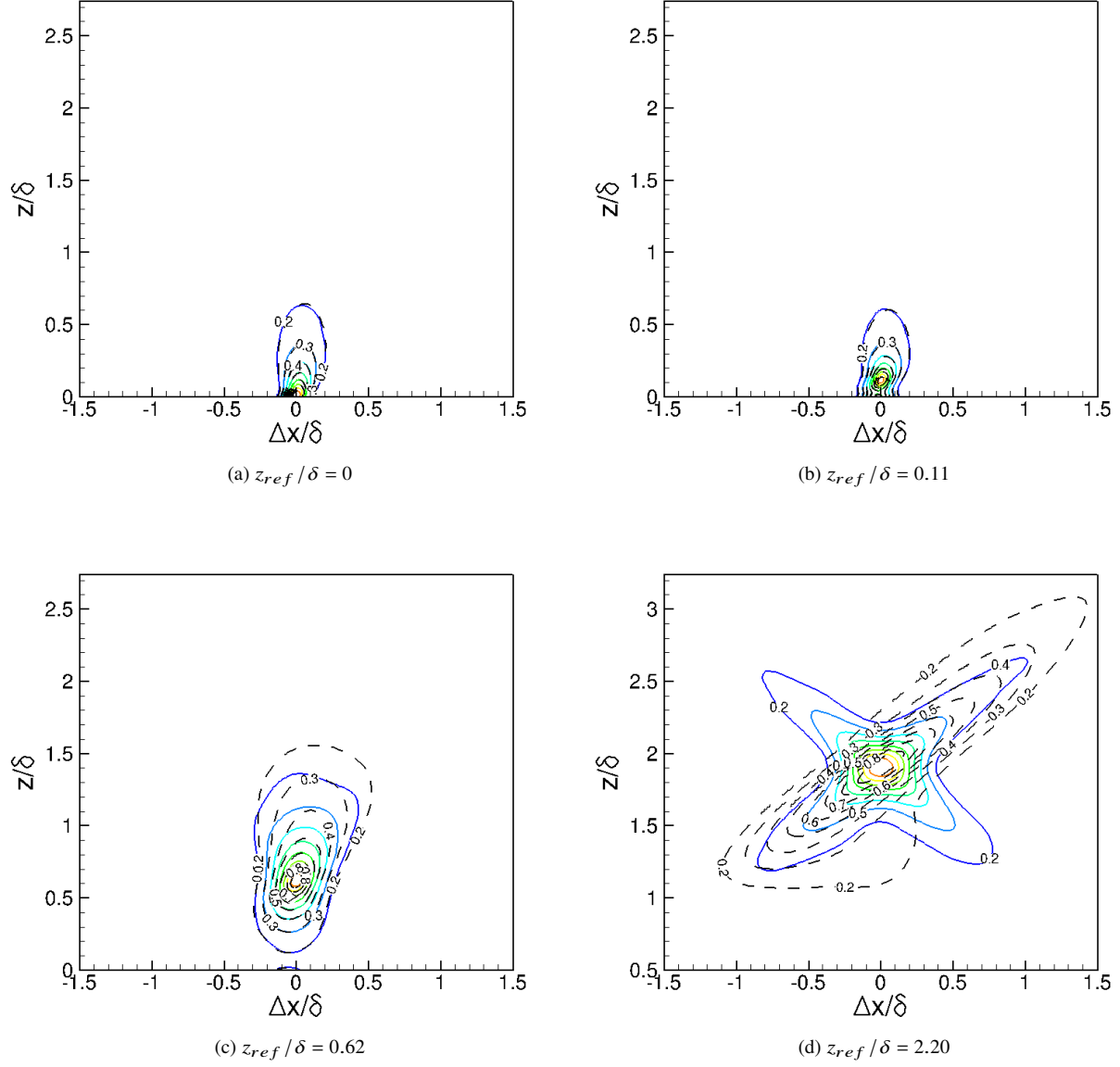


Fig. 13 Comparison of streamwise wall-normal pressure correlation structure $C_{pp}(\Delta x, 0, z, z_{ref}, 0)$ at multiple selected reference heights z_{ref} between Case Double Wall (colored solid line) and Case Single Wall (black dashed line). The Single Wall case corresponds to $L_y/\delta_i = 15.6$ (Duan et al., JFM 2014).

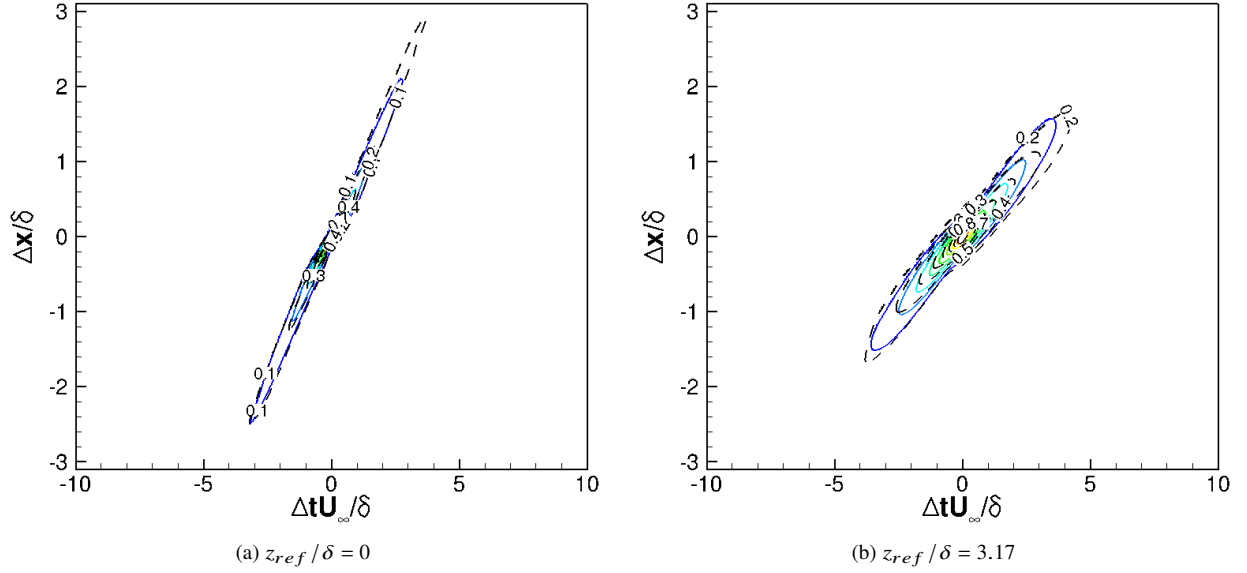


Fig. 14 Contours of constant space-time correlation coefficient of the pressure signal $C_{pp}(\Delta x, 0, z_{ref}, z_{ref}, \Delta t)$ at the wall ($z_{ref} = 0$) and in the free stream ($z_{ref} = 3.17$) for Cases Double Wall (colored solid line) and Single Wall (black dashed line). Single Wall case corresponds to $L_y/\delta_i = 5.4$.

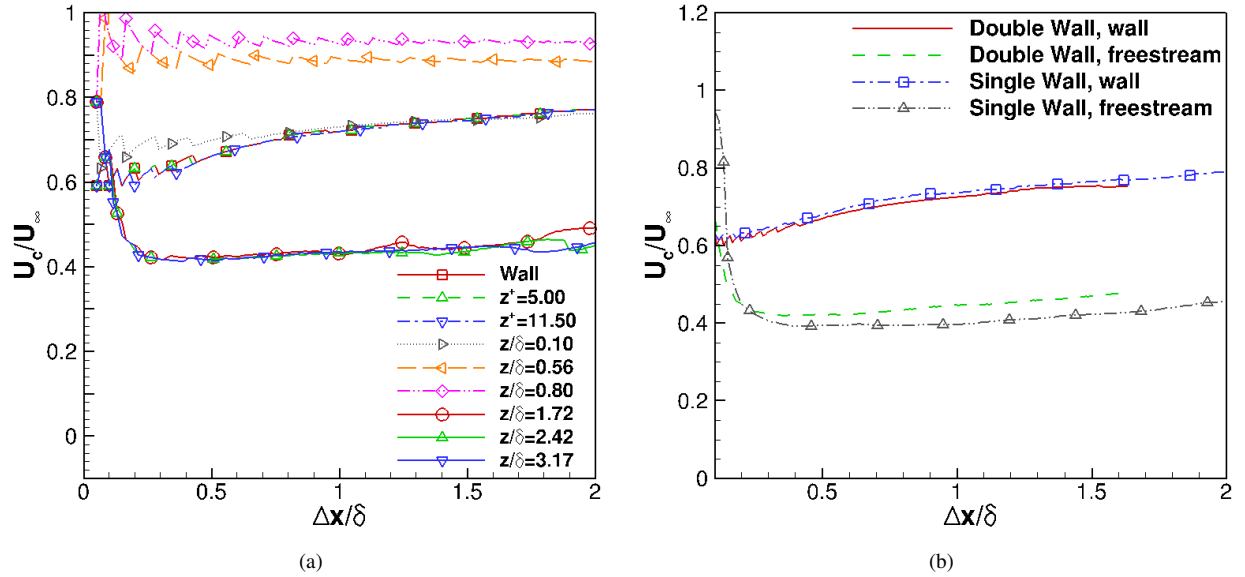


Fig. 15 (a) Convection velocity at selected heights for Double Wall as a function spatial separation for Case Double Wall; (b) comparison of convection velocity for case Double Wall and case Single Wall at the wall and in the free stream. Single Wall case corresponds to $L_y/\delta_i = 5.4$.