

Simulation and Modeling of Hypersonic Turbulent Boundary Layers Subject to Favorable Pressure Gradients due to Streamline Curvature

Gary L. Nicholson* and Lian Duan†
The Ohio State University, Columbus, OH 43210

Meelan M. Choudhari‡
NASA Langley Research Center, Hampton, VA 23681

Rodney D.W. Bowersox§
Texas A&M University, College Station, TX 77843

Direct numerical simulations (DNS) of favorable-pressure-gradient turbulent boundary layers are presented for a nominal freestream Mach number of 5, with the objective of assessing the limitations of the currently available Reynolds-averaged Navier-Stokes (RANS) models. The wall geometry and flow conditions of the DNS are representative of the experimental data for a Mach 4.9 turbulent boundary layer that was tested on a two-dimensional planar convex wall model in the high-speed blow-down wind tunnel located at the National Aerothermochemistry Laboratory at Texas A&M University. The favorable pressure gradient is induced by the streamwise surface curvature. The DNS results will be compared with experimental measurements and RANS computations for the same flow conditions and wall geometry. The DNS datasets will also be analyzed to provide an improved physical understanding of hypersonic boundary-layer turbulence subject to mechanical nonequilibrium, including the validity of Morkovin's hypothesis and the various available compressibility transformations, as well as to obtain turbulence statistics that are relevant to RANS modeling, including the distribution of eddy viscosity, the budgets of turbulent kinetic energy and Reynolds stresses, and the velocity-temperature correlations. The RANS predictions will be obtained by using both the commonly used one- and two-equation models that assume a constant turbulent Prandtl number and an algebraic, variable Prandtl number model for the turbulent energy flux.

Nomenclature

B_q	=	wall heat transfer rate, $B_q = q_w / (\rho_w C_p u_\tau T_w)$, dimensionless
C_f	=	wall skin friction coefficient, $C_f = \tau_w / (\frac{1}{2} \rho_\infty U_\infty^2)$, dimensionless
C_h	=	wall heat flux coefficient, $C_h = q_w / (\rho_\infty C_p U_\infty (T_r - T_w))$, dimensionless
C_p	=	heat capacity at constant pressure, J/(K·kg)
C_v	=	heat capacity at constant volume, J/(K·kg)
E	=	total energy per unit mass, $E \equiv e + \frac{1}{2} u_i u_i$, J/kg
H	=	shape factor, $H \equiv \delta^* / \theta$, dimensionless
M	=	Mach number, $M \equiv u/a$, dimensionless
M_τ	=	friction Mach number, $M_\tau = u_\tau / (\gamma R T_w)^{1/2}$, dimensionless
N_f	=	number of fields used to accumulate statistics, dimensionless
Pr	=	molecular Prandtl number, $Pr = 0.71$, dimensionless
Pr_t	=	turbulent Prandtl number, $Pr_t \equiv (\overline{\rho u'' w''} (\partial \tilde{T} / \partial z)) / (\overline{\rho w'' T'''} (\partial \tilde{u} / \partial z))$, dimensionless
R	=	ideal gas constant, $R = 287$, J/(K·kg)

*Graduate Student, Department of Mechanical and Aerospace Engineering, Student Member, AIAA

†Associate Professor, Department of Mechanical and Aerospace Engineering, Senior Member, AIAA

‡Aerospace Technologist, Computational AeroSciences Branch, M.S. 128. Fellow, AIAA

§Ford 1 Professor and Department Head of Aerospace Engineering. Fellow, AIAA

Re_θ	= Reynolds number based on momentum thickness and freestream viscosity, $Re_\theta \equiv \rho_\infty u_\infty \theta / \mu_\infty$, dimensionless
Re_{δ_2}	= Reynolds number based on momentum thickness and wall viscosity, $Re_{\delta_2} \equiv \rho_\infty u_\infty \theta / \mu_w$, dimensionless
Re_τ	= Reynolds number based on shear velocity and wall viscosity, $Re_\tau \equiv \rho_w u_\tau \delta / \mu_w$, dimensionless
Re_τ^*	= semilocal Reynolds number, $Re_\tau^* \equiv \sqrt{\tau_w / \rho_\infty} \delta / \nu_\infty$, dimensionless
S_{ij}	= rate-of-strain tensor, $S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, Hz
T	= temperature, K
T_r	= recovery temperature, $T_r = T_\infty (1 + 0.89 \frac{\gamma-1}{2} M_\infty^2)$, K
T_f	= time spanned to accumulate statistics, s
U_∞	= freestream velocity, m/s
a	= speed of sound, m/s
e	= internal energy per unit mass, $e \equiv C_v T$, J/kg
h	= enthalpy per unit mass, $h \equiv C_p T$, J/kg
$\tilde{k}$	= turbulent kinetic energy, $\tilde{k} \equiv \overline{u_i'' u_i''} / 2$, J/kg
p	= pressure, Pa
q	= conductive heat flux, W/m ²
q_i^T	= turbulent heat flux term, $q_i^T = \overline{\rho h'' u_i''}$, W/m ²
q_w	= wall heat flux, W/m ²
s	= entropy, J/(kg·K)
u	= streamwise velocity, m/s
u_τ	= friction velocity, $u_\tau \equiv \sqrt{\tau_w / \rho_w}$, m/s
u_τ^*	= density weighted velocity scale, $u_\tau^* \equiv \sqrt{\tau_w / \rho} = u_\tau \sqrt{\rho_w / \rho}$, m/s
v	= spanwise velocity, m/s
w	= wall-normal velocity, m/s
x	= streamwise direction of the right-handed Cartesian coordinate, m
y	= spanwise direction of the right-handed Cartesian coordinate, m
z	= wall-normal direction of the right-handed Cartesian coordinate, m
z_τ	= viscous length, $z_\tau = \nu_w / u_\tau$, m
z_τ^*	= semilocal length scale, $z_\tau^* \equiv \bar{\mu} / (\bar{\rho} u_\tau^*)$, m
β	= Clauser pressure gradient parameter, $\beta = (dp/dx)(\delta^* / \tau_w)$, dimensionless
β_{inc}	= incompressible Clauser pressure gradient parameter, $\beta = (dp/dx)(\delta_{inc}^* / \tau_w)$, dimensionless
γ	= specific heat ratio, $\gamma = C_p / C_v$, dimensionless
δ	= boundary layer thickness (based on 99.5% of the freestream total enthalpy), m
δ^*	= displacement thickness, m
κ	= von Kármán constant $\kappa = 0.41$, dimensionless
θ	= momentum thickness, m
ϑ_i^T	= turbulent heat flux term, $\vartheta_i^T = \overline{\rho e'' u_i''}$, W/m ²
μ	= dynamic viscosity, $\mu = 1.458 \times 10^{-6} T^{3/2} / (T + 110.4)$, kg/(m·s)
ν	= kinematic viscosity, $\nu = \mu / \rho$, m ² /s
ρ	= density, kg/m ³
σ	= Reynolds stress, Pa
τ	= shear stress, Pa

Subscripts

i	= inflow station for the domain of direct numerical simulations
inc	= incompressible variables
rms	= root mean square
w	= wall variables
∞	= freestream variables

Superscripts

T	= turbulent quantities in RANS equations
-----	--

- $+$ = variable in inner wall units, $(\cdot)^+ \equiv (\cdot)/z_\tau$
- $\overline{(\cdot)}$ = standard (Reynolds) averaged variable
- $\widetilde{(\cdot)}$ = density-weighted (Favre) averaged variable, $\widetilde{(\cdot)} \equiv \overline{\rho(\cdot)}/\bar{\rho}$
- $(\cdot)'$ = fluctuations around standard averages
- $(\cdot)''$ = fluctuations around Favre averages

I. Introduction

All realistic flight vehicles exhibit some degree of streamline curvature that induces favorable and/or adverse pressure gradients. For hypersonic flight, in particular, the combination of curvature driven pressure gradients and compressibility effects such as bulk dilatation can distort the turbulent motions within the boundary layer, causing a restructuring of the boundary-layer turbulence [1]. Such effects associated with a mechanical nonequilibrium pose a significant challenge even to the modern turbulence models for high-speed flows, especially because a majority of these models are based on compressible flow extensions to models that were originally developed for incompressible flows, and because those models have not accounted for intrinsically high-speed phenomena such as bulk dilatation. As such, there is a need to better understand how pressure gradients drive the transport of boundary-layer turbulence in the hypersonic regime, so that the underlying physical processes can be accurately modelled and, hence, also utilized to the advantage of vehicle designs.

The existing literature on the effect of streamline curvature on hypersonic boundary-layer turbulence is rather limited, whether in regard to measurements, modeling, or numerical simulations. Most previous studies of fully turbulent boundary layers (TBLs) negotiating pressure gradients focused on low-speed boundary layers (see, for example, Refs. [2–8]), with considerably fewer studies involving supersonic boundary layers and even less targeting the hypersonic regime. Early experiments of high-speed favorable-pressure-gradient (FPG) and adverse-pressure-gradient (APG) TBLs were largely limited to flow visualizations and single-point measurements of velocity fluctuations at moderately supersonic speeds, obtained via hot-wire anemometry and/or laser Doppler velocimetry ($M_\infty < 3$) [9–15]. Detailed and global measurements of turbulence organization at supersonic and hypersonic speeds had not been conducted until more recent studies based on particle image velocimetry (PIV) [1, 16–24]. The available measurements have yielded valuable insights into the effect of pressure gradients on the behavior, distribution, and scaling of the mean velocity and the Reynolds stresses associated with the turbulence field. Specifically, FPG has a stabilizing influence on the Reynolds stresses, resulting in up to 70 – 90% decrease of the axial turbulence intensities; a strong FPG can even cause a sign reversal in the Reynolds shear stress in the outer part of the boundary layer. APG destabilizes the boundary layer, causing an increase in the Reynolds stresses, including both normal and shear stresses. The previous studies also showed that an increase in freestream speed from a nominal Mach number of 3 to Mach 5 resulted in systematic differences in the transverse fluctuations and Reynolds shear stresses [25]. Thus far, however, none of the existing experiments has been able to provide sufficiently well resolved, global measurements of both velocity and thermodynamic fields (including in the immediate vicinity of the wall), so as to facilitate a systematic investigation of the turbulence scaling laws and to enable an evaluation of the individual terms in the exact equations of turbulent kinetic energy (TKE) and Reynolds-stress transport. A thorough characterization of turbulence scaling laws and the budget terms of TKE and Reynolds stresses is critical to accurate, physics-based modeling for this class of flows. While high-fidelity simulations such as DNS can provide detailed, global distributions of turbulent fluctuations, all of the existing DNS have been conducted at freestream Mach numbers of less than 3 [26–31], where the compressibility effects are expected to be too weak to cause any significant deviation of the turbulence statistics from its incompressible behavior.

One of the most important foundations for our current understanding of high-speed turbulence is the Morkovin’s hypothesis, which postulates that the structure of turbulence in a zero-pressure-gradient, high-speed TBL is largely the same as that in an incompressible flow [15]. An important consequence of Morkovin’s hypothesis involves the so-called “compressibility transformations” that transform the mean velocity and Reynolds stress profiles in a compressible boundary layer to equivalent incompressible profiles by accounting for the mean property variations across the thickness of the boundary layer. A classical example of such transformations is the density-weighted velocity scaling of Van Driest [32]. Another consequence of Morkovin’s hypothesis is the analogy between the temperature and velocity fields that leads to velocity-temperature relations such as the classical Walz formula [33] and the strong Reynolds analogy (SRA) [34–36]. Recently, the current authors [37, 38] provided a comprehensive assessment of the performance of several well-known compressibility transformations ranging from the classical, Morkovin’s scaling to the SRA and other generalizations. By using a DNS database for spatially evolving, zero-pressure-gradient (ZPG) TBLs over a broad range

of nominal freestream Mach numbers ($2.5 < M_\infty < 14$) and for a wall-to-recovery temperature ratio between 0.18 and 1.0, they confirmed the accuracy of the compressibility transformations and the SRA for ZPG TBLs in the high Mach number, cold-wall regime. The above-mentioned studies appear to support Morkovin’s hypothesis that, at least for wall turbulence under equilibrium, there exist few, if any, dynamic differences across the Mach number regime. However, a similar assessment of the compressibility transformations has not been conducted for boundary-layer turbulence under nonequilibrium conditions, such as those subject to mechanical distortion by wall curvature and pressure gradients. Consequently, the extent to which the existing compressibility transformations may be extrapolated to APG and FPG hypersonic TBLs is largely unclear. This important technical gap will be targeted in the present research.

One of the primary objectives of the current paper is to conduct benchmark quality DNS of FPG TBLs at a hypersonic Mach number to obtain detailed, global distributions of turbulent fluctuations. Specifically, the DNS will simulate flow conditions that are representative of the experimental data for a Mach 4.9 curvature-driven FPG TBL that was tested at the Texas A&M University (TAMU). The DNS will be validated against the experimental measurements obtained at TAMU. The resulting DNS datasets will then be used to provide an assessment of the Morkovin’s hypothesis, including the compressibility transformations and the SRA for hypersonic TBLs developing under a nonzero pressure gradient. The DNS will also be post-processed to provide the desired terms from the TKE and Reynolds-stress budgets and other turbulence quantities that are relevant to RANS modeling.

In addition to characterizing the physics of high-speed turbulence subject to streamline-curvature-driven FPG, a second objective of the current paper is to use the resulting DNS datasets to inform the future development of turbulence models for high-speed flows and to assess the potential limitations of the currently available turbulence models. Standard one- or two-equation Reynolds-averaged Navier-Stokes (RANS) models such as the Spalart-Allmaras (S-A) Model and the Menter’s Shear Stress Transport (SST) Model have been developed largely on the basis of studies at subsonic or moderately supersonic Mach numbers and with adiabatic walls. Therefore, it is not surprising that multiple researchers [39–41] have found that, even for the simplest hypersonic configurations involving zero-pressure-gradient boundary layers, the predictions of surface heat flux based on these standard one- or two-equation RANS models are significantly less accurate than the predictions obtained with simpler, algebraic turbulence models with some form of compressibility corrections. To improve the RANS predictions of heat transfer rates for high-speed shear flows, Bowersox [42, 43] derived an algebraic energy flux (AEF) model by truncating the transport equation for the turbulent heat flux. The newly developed turbulent energy flux model showed significant improvements, up to 20%, in the temperature predictions over the constant Prandtl number model for boundary-layer flows over moderately cooled walls (i.e., $T_w/T_r > 0.3$) [42–44]. More recently, the present authors extended the assessment of Bowersox’s AEF model for zero-pressure-gradient TBLs at Mach 11 and 14 with a highly-cooled wall [38]. They found that Bowersox’s algebraic energy flux model improves the prediction of the temperature profiles over the commonly used constant- Pr_t formulation while maintaining a good prediction of the velocity field. A similar assessment of the AFE model will be extended to FPG TBLs in the current paper.

The abstract is structured as follows. The flow conditions are outlined in Section II. Section III presents preliminary RANS results for TAMU’s experiments of streamline-curvature-driven FPG TBL at Mach 4.9. A summary of the planned work is given in Section IV.

II. Flow Conditions and Numerical Methodology

The present work targets both DNS and RANS computations of the streamline-curvature-driven FPG TBLs with an incoming Mach number of 4.9. The flow conditions as well as the wall geometry have been selected to match those from the experiment by Tichenor et al. [1] in the National Aerothermochemistry Laboratory (NAL) at the Texas A&M University. The wall geometry in their experiment consisted of a tunnel wall with a changeable floor insert that could be varied from a flat wall with zero pressure gradient to a curved surface inducing a favorable pressure gradient (see the bottom part of Figure 1a). Table 1 outlines the freestream and wall-temperature conditions for the cases of interest during the present work. Case M5ZPG simulates a baseline ZPG TBL over a flat plate, and Case M5FPG simulates a FPG TBL developing spatially over a convex wall geometry that is prescribed by the following polynomial profile:

$$z_w(x) = A(x - x_1)^3 + B(x - x_1)^2 \quad (m) \quad (1)$$

where $A = 11.07$ and $B = -2.507$ are the polynomial coefficients, and $x_1 = 0.2454$ m is the axial location where the curvature begins. This wall geometry is the same as that of the strong pressure gradient (SPG) model investigated during the TAMU experiment as shown in Figure 1a. As shown in Table 1, Cases M5ZPG and M5FPG are designed to have the same freestream and wall-temperature conditions except for a different wall geometry to isolate the influence of

the curvature-driven pressure gradients on boundary-layer turbulence.

Table 1 Freestream and wall-temperature conditions for various cases.

Case	M_∞	$U_\infty (m/s)$	$\rho_\infty (kg/m^3)$	$T_\infty (K)$	$T_w (K)$	T_w/T_r	$(\beta_{inc})_{Location2}$
M5ZPG	4.9	794	0.265	65.4	317	1.0	≈ 0
M5FPG	4.9	794	0.265	65.4	317	1.0	-1.0

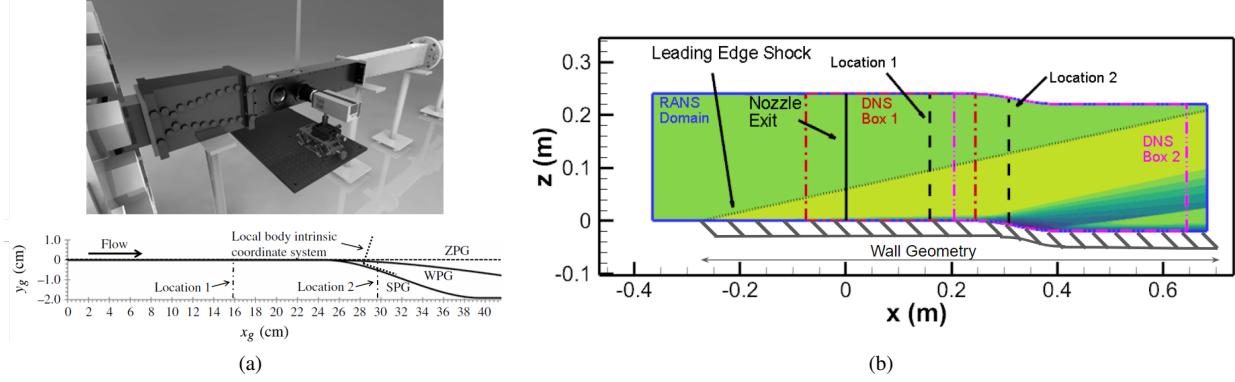


Fig. 1 (a) Schematic of experimental arrangement and measurement locations (adapted from Tichenor et al. [1]); (b) RANS and DNS domains for Case M5FPG. $x = 0$ indicates the location of the nozzle exit in the experiment, and locations 1 and 2 depict the two measurement locations at $x = 0.159$ m and $x = 0.298$ m, respectively. The curvature begins at $x = 0.2454$ m and ends at $x = 0.3964$ m.

A. DNS of Hypersonic Turbulent Boundary Layers

To perform the DNS of turbulent boundary layers, the compressible Navier-Stokes equations in conservation form are solved numerically. The inviscid fluxes of the governing equations in curvilinear coordinates are computed by using a seventh-order weighted essentially nonoscillatory (WENO) scheme. The viscous fluxes are discretized with a fourth-order central difference scheme and time integration is performed using a third-order low-storage Runge-Kutta scheme [45].

Prior to the DNS, we first carry out a precursor RANS computation with the freestream conditions and wall geometry matching those of the TAMU experiment. The RANS calculation is conducted by using the CFL3D code [46] and provides the initial condition for the DNS of Case M5FPG as well as the boundary conditions for both DNS (Figure 1b). The RANS calculation for the M5FPG case simulates the full curved wall geometry and encompasses the DNS domain. The DNS is conducted in two stages of streamwise overlapping boxes. An independent auxiliary DNS of a turbulent boundary layer developing spatially over a smooth flat plate (Box 1 DNS) is carried out first in order to provide the inflow boundary condition for the principal DNS over the curved-wall domain (Box 2 DNS). In the Box 1 DNS, a modified rescaling/recycling method [47] is used to generate inflow turbulence. The inflow location of the Box 1 DNS will be significantly upstream of the first measurement location in the experiment (labeled as Location 1 in Tichenor et al. [1]) in order to ensure that the DNS has achieved a fully developed equilibrium state of a zero pressure gradient turbulent boundary layer at the above measurement location. The single auxiliary DNS will be used to provide a common inflow condition for the simulations of the downstream region with and without wall curvature. The strategy of using a common inflow between both DNS computations is designed to isolate the influence of the curvature-driven pressure gradients on boundary-layer turbulence. The computational setup for the auxiliary (i.e., Box 1) and the principal (i.e., downstream) simulations is sketched in Figure 2. For each wall shape, no-slip conditions are applied for the three velocity components at the wall, and an isothermal condition is used for the temperature, with $T_w = 317$ K ($T_w/T_r \approx 1.0$). At the top and the outflow boundaries, unsteady non-reflecting boundary conditions based on Thompson [48] are imposed, and periodic boundary conditions are used in the spanwise direction.

B. RANS Simulation of Hypersonic Turbulent Boundary Layers

RANS simulations were conducted for the ZPG and the FPG TBLs with the same flow conditions and thermodynamic equation of state as the DNS cases listed in Table 1. In RANS, NASA's CFL3D code [46] is used to solve the compressible Favre-averaged Navier-Stokes equations [49]. The Favre-averaged RANS equations can be written as:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j}{\partial x_j} = 0 \quad (2)$$

$$\frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}^T}{\partial x_j} \quad (3)$$

$$\frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i (\tilde{E} + \bar{p}/\bar{\rho})}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\bar{\sigma}_{ij} \tilde{u}_i + \bar{\sigma}_{ij} u_i'' \right) - \frac{\partial}{\partial x_j} \left(\bar{q}_j + q_j^T - \tilde{u}_i \tau_{ij}^T + \frac{1}{2} \overline{\rho u_i'' u_i'' u_j''} \right) \quad (4)$$

where $\tilde{E} = \tilde{e} + \frac{1}{2} \tilde{u}_i \tilde{u}_i + k^T$, $\bar{q}_j \approx \frac{C_p \bar{\mu}}{Pr} \frac{\partial \tilde{T}}{\partial x_j}$, and $\bar{\sigma}_{ij} \approx 2\bar{\mu} \left(\tilde{S}_{ij} - \frac{1}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right)$. Throughout the paper, standard Reynolds averages are denoted by an overbar, $\bar{f}$, whereas the density-weighted (i.e., Favre) averages are denoted by a tilde, $\tilde{f} = \overline{\rho f}/\bar{\rho}$; fluctuations around standard and Favre averages are denoted by single and double primes, as with $f' = f - \bar{f}$ and $f'' = f - \tilde{f}$, respectively.

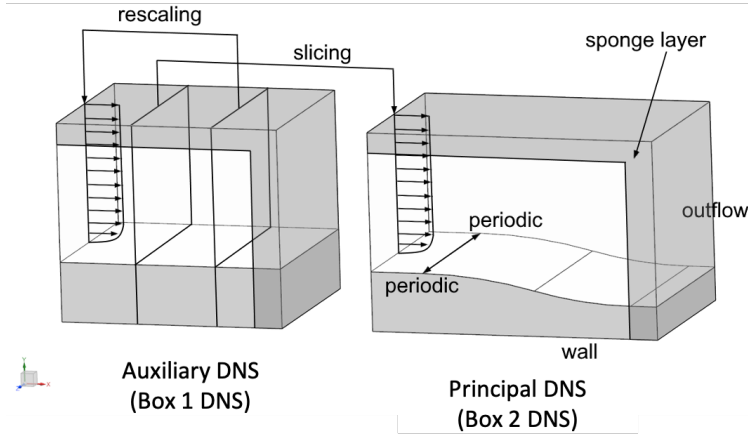


Fig. 2 Schematic of DNS setup for Case M5FPG along with the boundary conditions.

In the one- and two-equation turbulence models used herein, the Reynolds stress term $\tau_{ij}^T = -\overline{\rho u_i'' u_j''}$ is modeled by the Boussinesq approximation:

$$\tau_{ij}^T = -\overline{\rho u_i'' u_j''} = 2\mu_t \left(\tilde{S}_{ij} - \frac{1}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \rho k^T \delta_{ij} \quad (5)$$

where $\tilde{S}_{ij}$ is the mean velocity strain, and μ_t is the eddy viscosity obtained via either of the turbulence models, and k^T denotes the turbulent kinetic energy. In the current study, we consider two representative turbulence models that are in widespread use within the aerospace industry, namely, the one-equation model of Spalart–Allmaras [50] and the shear-stress transport $k-\omega$ model by Menter [51]. Unless otherwise mentioned, the "standard" form (i.e., the original published form) of the model equations was used, with the detailed information on the model formula and coefficients listed in Reference [52–54].

The turbulent heat flux term $q_i^T = \overline{\rho h'' u_i''}$ was modeled either by assuming a constant turbulent Prandtl number or via an algebraic model for the turbulent energy flux that was developed by Bowersox [42]. In the constant Prandtl number model, the turbulent heat flux term is modeled as:

$$q_i^T = \overline{\rho h'' u_i''} \approx -\frac{C_p \bar{\mu}_t}{Pr_t} \frac{\partial \tilde{T}}{\partial x_i} \quad (6)$$

where the turbulent Prandtl number Pr_t is assumed to be a constant, and is set equal to 0.9. The algebraic model of Bowersox [42, 43] is based on a simplified form of the transport equation for the energy flux $\theta_i^T = \overline{\rho e'' u_i''}$:

$$\begin{aligned} a_{ik} \theta_k^T &= b_i \\ a_{ik} &= \left[\frac{1}{\tau_\theta} + \frac{R}{C_v} \frac{\partial \widetilde{u_m}}{\partial x_m} \right] \delta_{ik} + \frac{\partial \widetilde{u_i}}{\partial x_k} \\ b_i &= \tau_{ik}^T \left(\frac{\partial \widetilde{h}}{\partial x_k} - \frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial x_k} \right) + \frac{1}{\bar{\rho}} \left(\tau_{kl}^T \frac{\partial \widetilde{u_l}}{\partial x_k} \right) \frac{\partial \bar{p}}{\partial x_i} \tau_e \end{aligned} \quad (7)$$

Here, $\tau_\theta = \sigma_\theta \tau_u$ and $\tau_e = \sigma_e \tau_u$ are time scales that are proportional to the turbulence time scale $\tau_u = k^T / \epsilon$ as in the $k - \epsilon$ model, and the Reynolds stress term τ_{ik}^T can be modeled by an aforementioned turbulence model. Once θ_i^T is calculated using Eqn. 7, the turbulent energy flux $q_i^T = \overline{\rho h'' u_i''}$ can be found as $q_i^T = \gamma \theta_i^T$.

The terms associated with molecular diffusion $\sigma_{ij} u_i'' u_j''$ and turbulent transport $-\frac{1}{2} \overline{\rho u_i'' u_i'' u_j''}$ within the Favre-averaged energy equation are neglected in the current study [55]. Similar to the DNS, ideal gas relations were used in the RANS simulations with air as the working fluid, and Sutherland's law was used for molecular viscosity. A constant molecular Prandtl number of 0.71 is used for calculating the molecular thermal conductivity.

III. Results

In this section, we discuss the results obtained from the RANS computations for Cases M5ZPG and M5FPG. Specifically, the performance of the selected turbulence models is assessed by comparing the RANS predictions against the experimental measurements by Tichenor et al. [1].

Figure 3 shows the schematics of the computational domains for the ZPG and the FPG geometries, respectively, along with the boundary conditions used in each case. The RANS domain in each case extends from $x = -0.366$ m to $x \approx 0.7$ m in the axial direction, encompassing the two measurement locations and the curved-wall region for the Case M5FPG. A uniform grid distribution with a grid spacing of $\Delta x / \delta_r = 0.176$ is used in the axial direction, where the reference length $\Delta_r = 7.3$ mm denotes the boundary-layer thickness at the first measurement location ($x = 0.159$). In the wall-normal (z) direction, a hyperbolic tangent distribution is used to cluster the computational mesh close to the wall. The grid resolution in the wall-normal direction corresponds to $\Delta z^+ = 0.186$ at the wall and $\Delta z^+ = 22.3$ near the boundary-layer edge, with the grid resolutions normalized by the viscous length $z_\tau = 8.74 \mu\text{m}$ at the first measurement location ($x = 0.159$ m). To monitor any sensitivity to grid resolution, a grid study was conducted for the $k - \omega$ SST model by comparing the results obtained with the baseline grid and those based on a refined grid with twice the baseline grid resolution in each direction. The two grids yielded very close results as shown in Figure 4. The difference in C_f between the baseline grid and the refined grid is less than 1.0 percent over the entire domain. Because the same inflow was used for the two cases with and without wall curvature, respectively, the values of C_f for the ZPG and FPG cases match within the upstream part of the computational domain, until the wall shapes deviate from each other at the start of the curved wall region at $x = 0.2454$ m.

Figure 5 shows a comparison between the skin-friction coefficients predicted by the various turbulence models and the values predicted by the van Driest II theory [56] for the ZPG and FPG cases. The turbulence models under investigation included the algebraic model of Baldwin-Lomax, the one-equation model of Spalart-Allmaras, and the $k - \omega$ SST model by Menter. For the ZPG case, both the $\kappa\omega$ and the SA models show good agreement with the Van Driest II theory, while the Baldwin Lomax model underpredicts c_f by approximately 10%. For the FPG case, all the RANS models predict a dip in C_f over the curved-wall region, and the Baldwin-Lomax model again yields lower predictions of C_f than the one- and two-equation models for the TBL across the entire domain including the curved-wall region. The van Driest II theory, which was developed mainly by using the experiments for zero-pressure-gradient, flat-plate TBLs, appears to provide a qualitatively incorrect trend in the region of the curved wall.

Next, Figure 6 shows a comparison with the van Driest transformed velocity, U_{VD}^+ , at measurement location 1 ($x = 0.159$ m), which is located upstream of the curved wall region. The van Driest transformed velocity is defined as follows:

$$U_{VD}^+ = \frac{1}{u_\tau} \int_0^{\bar{u}} (\rho / \rho_w)^{1/2} d\bar{u} \quad (8)$$

Following the van Driest transformation, the mean velocity predicted by all of the turbulence models conforms well to the incompressible law of the wall and shows a narrow logarithmic region. In addition, the predicted mean velocity profile

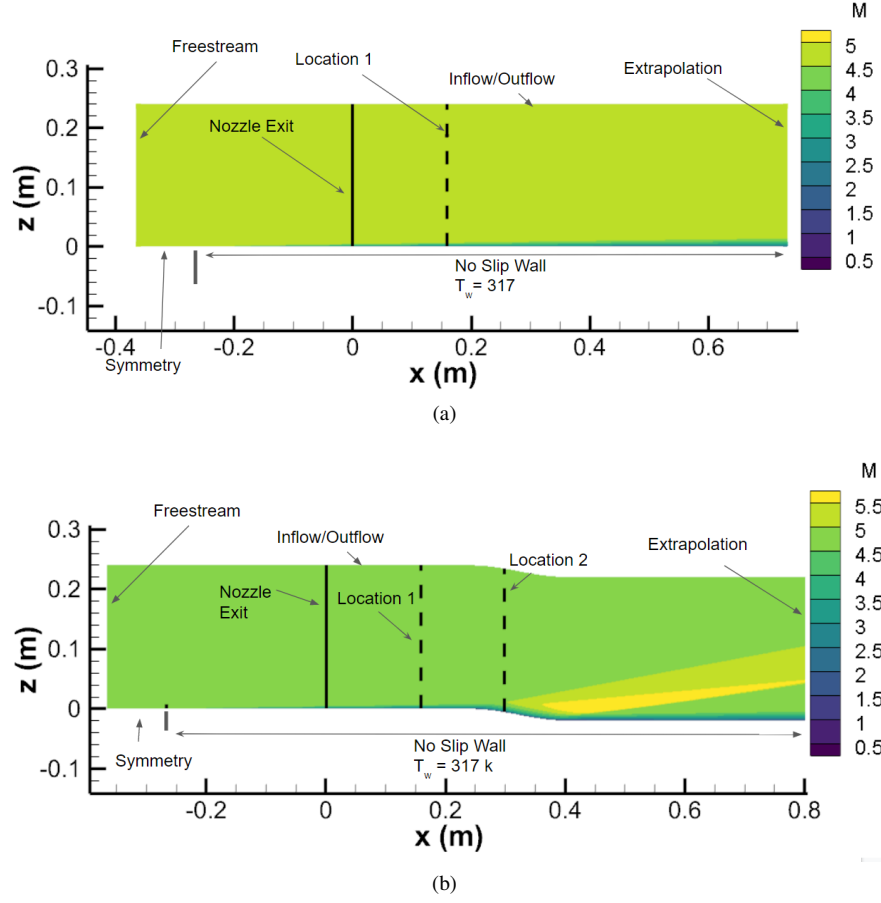


Fig. 3 Schematics of RANS domains for Cases M5ZPG and M5FPG, respectively, along with the boundary conditions.

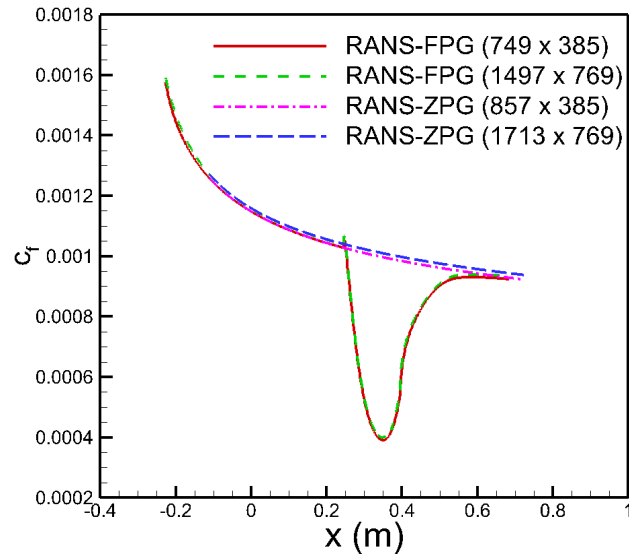


Fig. 4 Grid sensitivity study for the skin friction coefficients for the ZPG and FPG cases, respectively, as obtained with the $k - \omega$ SST model.

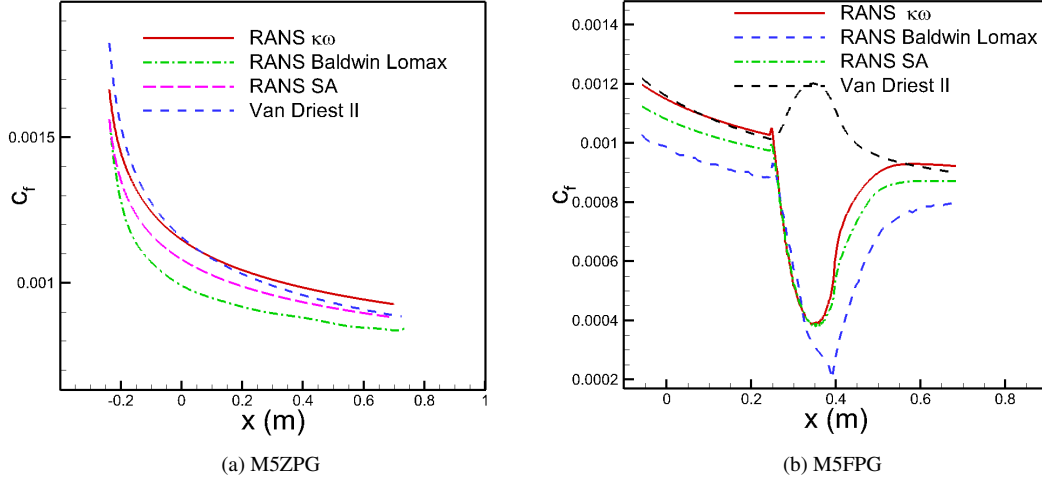


Fig. 5 Comparison between the skin friction coefficients predicted by various turbulence models and the van Driest II theory for the ZPG and FPG cases.

compares well with the experimental measurements by Tichenor et al. [1]. Compared with the one- and two-equation models, the Baldwin-Lomax model predicts approximately 9% higher values of U_{VD}^+ near the boundary layer edge, which is largely due to the lower values of the wall shear stress predicted by this model. Figure 7a shows a comparison of the RANS-based mean velocity profile normalized with the inner scaling with the similarly normalized profile based on the experimental measurement at $x = 0.298$ m, where the TBL is subjected to both a convex wall curvature and a strong favorable pressure gradient. An analogous comparison between the mean velocity profiles normalized via the defect law scaling is indicated in Figure 7b. The one- and two-equation models are seen to predict the mean velocity reasonably well, whereas the algebraic Baldwin-Lomax model significantly underpredicts the velocities measured in the experiment.

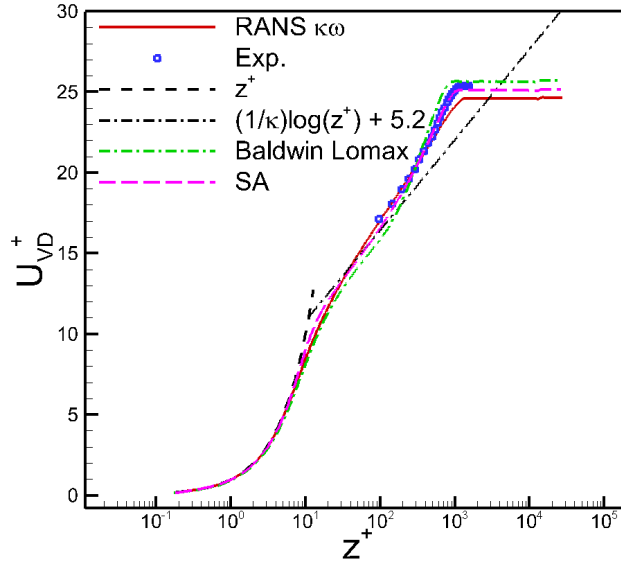


Fig. 6 Comparison of van Driest transformed velocity profiles U_{VD}^+ obtained via RANS and the experimental measurements by Tichenor et al. [1] at Location 1 ($x = 0.159$ m). $\kappa = 0.41$ is the von Karman constant.

Finally, Figure 8 shows a comparison of the RANS-based Reynolds-stress profile with that inferred from the

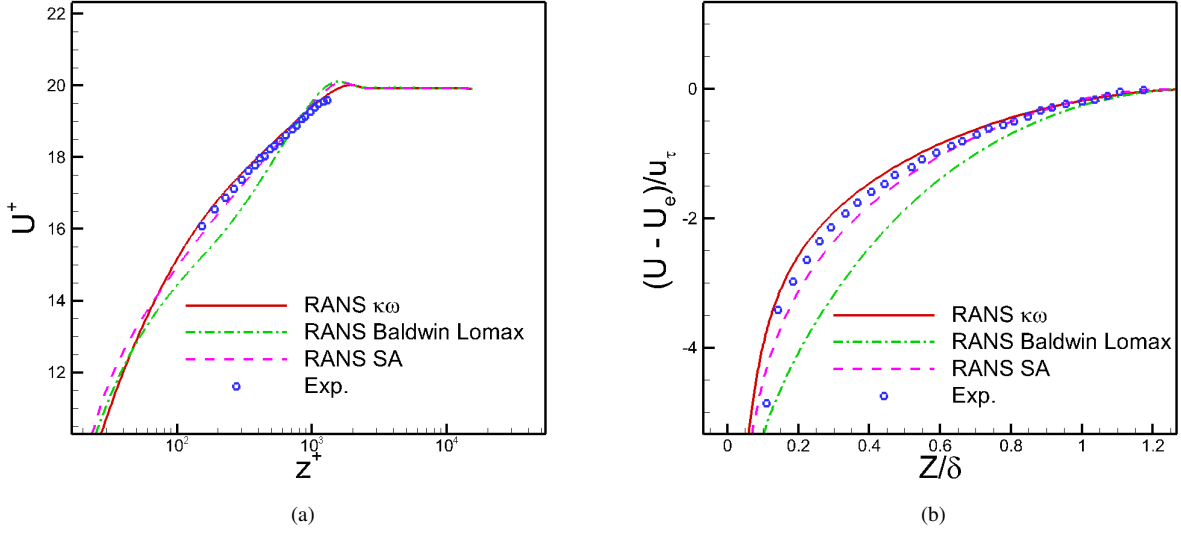


Fig. 7 Comparison of mean velocity profiles obtained from RANS and the measurements by Tichenor et al. [1] at Location 2 ($x = 0.298$ m). The profiles are normalized using (a) the inner scaling and (b) the defect-law scaling.

experiments at each of the two measurement locations ($x = 0.159$ m and $x = 0.298$ m). While the standard one- and two-equation models provide reasonable predictions for the Reynolds shear stress at $x = 0.159$ m, where the TBL is in mechanical equilibrium (i.e., without pressure gradient and wall curvature), these turbulence models fail to predict the Reynolds shear stress in the regions of strong wall curvature and a favorable pressure gradient.

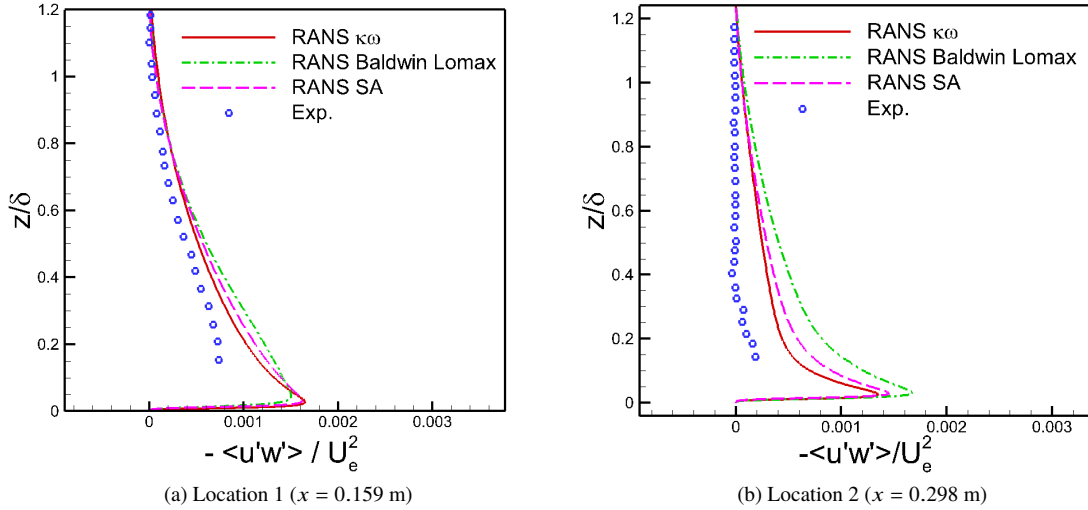


Fig. 8 Comparison of profiles of Reynolds shear stress predicted by RANS with the experimental results of Tichenor et al. [1].

IV. Summary

This abstract has outlined precursor RANS computations of ZPG and FPG TBLs, with the freestream and wall boundary conditions matching those of TAMU's experiments at Mach 4.9. These computations were conducted using NASA's CFL3D code and will be used to provide both initial and boundary conditions for the DNS of these flows, which will be described in the final paper. The sensitivity of the RANS results to the choice of turbulence model was also examined for several standard, eddy-viscosity-based turbulence models, including the algebraic model by Baldwin and Lomax, the one-equation model of Spalart and Allmaras, and the $k - \omega$ SST model by Menter for Reynolds stress, and the turbulent heat flux was modeled with a constant turbulent Prandtl number of $Pr_t = 0.9$. The RANS results showed that the Spalart–Allmaras and $k - \omega$ SST models perform better than the algebraic Baldwin-Lomax model for the current flow. While the standard one- and two-equation models provide reasonable predictions for skin friction and mean velocity, these turbulence models cannot accurately predict the Reynolds shear stress in regions of strong wall curvature and favorable pressure gradient.

In the full paper, we will include the results of the DNS at representative conditions from the TAMU experiment. The DNS is being initiated and, based on our extensive previous experience in performing such simulations, no significant technical risk is anticipated in timely completion of the DNS. Furthermore, the full paper will include RANS results based on Bowersox's algebraic energy flux model and their comparison with the DNS and the experiments, as well as an assessment of the model's potential to improve upon the shortcomings of the standard constant Pr_t models in predicting the Reynolds stresses and other turbulence statistics in the region of strong favorable pressure gradient.

Acknowledgments

Financial support for the work is being provided by the Office of Naval Research (under grant N00014-20-1-2194), managed by Dr. Eric Marineau. Author M. Choudhari was supported by the NASA Hypersonic Technology Project. Computational resources were provided by the DoD High Performance Computing Modernization Program and the Ohio Supercomputer Center. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the funding agencies or the U.S. Government.

References

- [1] Tichenor, N. R., Humble, R. A., and Bowersox, R. D. W., "Response of a Hypersonic Turbulent Boundary Layer to Favourable Pressure Gradients," *Journal of Fluid Mechanics*, Vol. 722, 2013, pp. 187–213.
- [2] Bradshaw, P., "The analogy between streamline curvature and buoyancy in turbulent shear flow," *Journal of Fluid Mechanics*, Vol. 36, 1969, pp. 177–191.
- [3] Bradshaw, P., "The effect of mean compression or dilatation on the turbulence structure of supersonic boundary layers," *Journal of Fluid Mechanics*, Vol. 63, 1974, pp. 449–464.
- [4] Lee, J.-H., and Sung, H. J., "Structures in Turbulent Boundary Layers Subjected to Adverse Pressure Gradients," *Journal of Fluid Mechanics*, Vol. 639, 2009, pp. 101–131.
- [5] Schlatter, P., and Örlü, R., "Assessment of Direct Numerical Simulation Data of Turbulent Boundary Layers," *Journal of Fluid Mechanics*, Vol. 659, 2010, pp. 116–126. doi:10.1017/S0022112010003113.
- [6] Vila, C. S., Örlü, R., Vinuesa, R., Schlatter, P., Ianiro, A., and Discetti, S., "Adverse-pressure-gradient effects on turbulent boundary layers: statistics and flow-field organization," *Flow, Turbulence and Combustion*, Vol. 99, No. 3-4, 2017, pp. 589–612.
- [7] Bobke, A., Vinuesa, R., Örlü, R., and Schlatter, P., "Large-eddy simulations of adverse pressure gradient turbulent boundary layers," *Journal of Physics: Conference Series*, Vol. 708, IOP Publishing, 2016, p. 012012.
- [8] Bobke, A., Vinuesa, R., Örlü, R., and Schlatter, P., "History effects and near equilibrium in adverse-pressure-gradient turbulent boundary layers," *Journal of Fluid Mechanics*, Vol. 820, 2017, pp. 667–692.
- [9] Jayaram, M., Taylor, M. W., and Smits, A. J., "The response of a compressible turbulent boundary layer to short regions of concave surface curvature," *Journal of Fluid Mechanics*, Vol. 175, 1987, pp. 343–362.
- [10] Donovan, J. F., Spina, E. F., and Smits, A. J., "The structure of a supersonic turbulent boundary layer subjected to concave surface curvature," *Journal of Fluid Mechanics*, Vol. 259, 1994, pp. 1–24.

- [11] Spina, E. F., Smits, A. J., and Robinson, S. K., "The Physics of Supersonic Turbulent Boundary Layers," *Annual Review of Fluid Mechanics*, Vol. 26, 1994, pp. 287–319.
- [12] Smith, D. R., and Smits, A. J., "A Study of the effects of curvature and compression on the behavior of a supersonic turbulent boundary layer," *Experiments in Fluids*, Vol. 18, 1995, pp. 363–369.
- [13] Arnette, S., Samimy, M., and Elliott, G., "Structure of supersonic turbulent boundary layer after expansion regions," *AIAA Journal*, Vol. 33, No. 3, 1995, pp. 430–438.
- [14] Arnette, S., Samimy, M., and Elliott, G., "The effects of expansion on the turbulence structure of compressible boundary layers," *Journal of Fluid Mechanics*, Vol. 367, 1998, pp. 67–105.
- [15] Smits, A. J., and Dussauge, J. P., *Turbulent Shear Layers in Supersonic Flow*, 2nd ed., Springer-Verlag New York, 2006. doi:10.1007/b137383.
- [16] Luker, J., Bowersox, R. D. W., and Buter, T., "Influence of curvature-driven favourable pressure gradient on supersonic turbulent boundary layer," *AIAA Journal*, Vol. 38, No. 8, 2010, pp. 1351–1359.
- [17] Ekoto, I. W., Bowersox, R. D. W., Beutner, T., and Goss, L., "Response of Supersonic Turbulent Boundary Layers to Local and Global Mechanical Distortions," *Journal of Fluid Mechanics*, Vol. 630, 2009, pp. 225–265.
- [18] Humble, R. A., Peltier, S. J., and Bowersox, R. D. W., "Visualization of the structural response of a hypersonic turbulent boundary layer to convex curvature," *Physics of Fluids*, Vol. 24, 2012, p. 106103.
- [19] Neel, I., Leidy, A. N., Bowersox, R. D. W., and Tichenor, N. R., "Influence of Favorable Pressure Gradients on a Mach 5.0 Turbulent Boundary Layer," *AIAA Paper 2016-4248*, 2016. <https://doi.org/10.2514/6.2016-4248>.
- [20] Tichenor, N. R., Neel, I., Leidy, A., and Bowersox, R. D., "Influence of Streamline Adverse Pressure Gradients on the Structure of a Mach 5 Turbulent Boundary Layer," *AIAA Paper 2017-1697*, 2017. <https://doi.org/10.2514/6.2017-1697>.
- [21] Carlson, J. D., Bowersox, R. D. W., and Tichenor, N. R., "Proper Orthogonal Decomposition Analysis of Hypersonic Turbulent Boundary Layers Subjected to Streamline Wall Curvature," *AIAA Paper 2020-2060*, 2020.
- [22] Wang, Q., Wang, Z., and Zhao, Y., "An experimental investigation of the supersonic turbulent boundary layer subjected to concave curvature," *Phys. Fluids*, Vol. 28, 2016, p. 096104.
- [23] Wang, Q., and Wang, Z., "Structural characteristics of the supersonic turbulent boundary layer subjected to concave curvature," *Appl. Phys. Lett.*, Vol. 108, 2016, p. 114102.
- [24] Wang, Q.-C., Wang, Z.-G., and Zhao, Y.-X., "On the impact of adverse pressure gradient on the supersonic turbulent boundary layer," *Physics of Fluids*, Vol. 28, No. 11, 2016, p. 116101.
- [25] Tichenor, N. R., Humble, R. A., and Bowersox, R. D. W., "Influence of Favorable Pressure Gradients on a Mach 5.0 Turbulent Boundary Layer," *AIAA Paper 2011-0748*, 2011. <https://doi.org/10.2514/6.2011-748>.
- [26] Wang, X., Wang, Z., Sun, M., Wang, Q., and Hu, Z., "Direct numerical simulation of a supersonic turbulent boundary layer subject to adverse pressure gradient induced by external successive compression waves," *AIP Advances*, Vol. 9, No. 8, 2019, p. 085215.
- [27] Sun, M., Sandham, N. D., and Hu, Z., "Turbulence structures and statistics of a supersonic turbulent boundary layer subjected to concave surface curvature," *Journal of Fluid Mechanics*, Vol. 865, 2019, pp. 60–99.
- [28] Wang, X., Wang, Z., Sun, M., Wang, Q., and Hu, Z., "Effects of favorable pressure gradient on turbulence structures and statistics of a flat-plate supersonic turbulent boundary layer," *Physics of Fluids*, Vol. 32, No. 2, 2020, p. 025107.
- [29] Wenzel, C., Peter, J. M., Selent, B., Weinschenk, M. B., Rist, U., and Kloker, M. J., "DNS of compressible turbulent boundary layers with adverse pressure gradients," *High Performance Computing in Science and Engineering'18*, Springer, 2019, pp. 229–242.
- [30] Wenzel, C., Gibis, T., Kloker, M., and Rist, U., "Self-similar compressible turbulent boundary layers with pressure gradients. Part 1. Direct numerical simulation and assessment of Morkovin's hypothesis," *Journal of Fluid Mechanics*, Vol. 880, 2019, pp. 239–283.
- [31] Gibis, T., Wenzel, C., Kloker, M., and Rist, U., "Self-similar compressible turbulent boundary layers with pressure gradients. Part 2. Self-similarity analysis of the outer layer," *Journal of Fluid Mechanics*, Vol. 880, 2019, pp. 284–325.

- [32] van Driest, E. R., "The Problem of Aerodynamic Heating," *Aeronautical Engineering Review*, Vol. 15, No. 10, 1956, pp. 26–41.
- [33] Walz, A., *Boundary Layers of Flow and Temperature*, MIT Press, Cambridge, MA, 1969.
- [34] Morkovin, M. V., "Effects of Compressibility on Turbulent Flows," In *Mécanique de la Turbulence* (ed. A. J. Favre), CNRS, 1962, pp. 367–380.
- [35] Gaviglio, J., "Reynolds Analogies and Experimental Study of Heat Transfer in the Supersonic Boundary Layer," *International Journal of Heat and Mass Transfer*, Vol. 30, No. 5, 1987, pp. 911–926. doi:10.1016/0017-9310(87)90010-X.
- [36] Huang, P. G., Coleman, G. N., and Bradshaw, P., "Compressible Turbulent Channel Flows: DNS Results and Modelling," *Journal of Fluid Mechanics*, Vol. 305, 1995, pp. 185–218.
- [37] Zhang, C., Duan, L., and Choudhari, M. M., "Direct Numerical Simulation Database for Supersonic and Hypersonic Turbulent Boundary Layers," *AIAA Journal*, 2018. <https://arc.aiaa.org/doi/pdf/10.2514/1.J057296>.
- [38] Huang, J., Nicholson, G. L., Duan, L., Choudhari, M. M., and Bowersox, R. D. W., "Simulation and Modeling of Cold-Wall Hypersonic Turbulent Boundary Layers on Flat Plate," *AIAA Paper 2020-0571*, 2020.
- [39] Rumsey, C. L., "Compressibility Considerations for k-omega Turbulence Models in Hypersonic Boundary-Layer Applications," *Journal of Spacecraft and Rockets*, Vol. 47, No. 1, 2010, pp. 11–20.
- [40] Gnoffo, P. A., Berry, S. A., and Van Norman, J. W., "Uncertainty assessments of hypersonic shock wave-turbulent boundary-layer interactions at compression corners," *Journal of Spacecraft and Rockets*, Vol. 50, No. 1, 2013, pp. 69–95.
- [41] Holden, M. S., Wadhams, T. P., and MacLean, M., "Measurements in Regions of Shock Wave/Turbulent Boundary Layer Interaction from Mach 4 to 10 at Flight Duplicated Velocities to Evaluate and Improve the Models of Turbulence in CFD Codes," 22nd AIAA Computational Fluid Dynamics Conference, Atlanta, GA, June, 2014. https://www.cubrc.org/_iassets/docs/Turbulent-SWTBL-paper.pdf.
- [42] Bowersox, R. D. W., "Extension of equilibrium turbulent heat flux models to high-speed shear flows," *Journal of Fluid Mechanics*, Vol. 633, 2009, pp. 61–70.
- [43] Bowersox, R. D. W., and North, S. W., "Algebraic Turbulent Energy Flux Models for Hypersonic Shear Flows," *Progress in Aerospace Sciences*, Vol. 46, 2010, pp. 49–61.
- [44] Poggie, J., "Compressible Turbulent Boundary Layer Simulations: Resolution Effects and Turbulence Modeling," *AIAA Paper 2015-1983*, 2015.
- [45] Williamson, J., "Low-Storage Runge-Kutta Schemes," *Journal of Computational Physics*, Vol. 35, No. 1, 1980, pp. 48–56.
- [46] Bartels, R. E., Rumsey, C. L., and Biedron, R. T., "CFL3D Version 6.4 - General Usage and Aeroelastic Analysis," Tech. rep., NASA/TM-2006-214301, 2006.
- [47] Duan, L., Choudhari, M. M., and Wu, M., "Numerical Study of Acoustic Radiation due to a Supersonic Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 746, 2014, pp. 165–192.
- [48] Thompson, K. W., "Time Dependent Boundary Conditions for Hyperbolic Systems," *Journal of Computational Physics*, Vol. 68, No. 1, 1987, pp. 1–24.
- [49] "Implementing Turbulence Models into the Compressible RANS Equations," <https://turbmodels.larc.nasa.gov/implementrans.html>, 2019.
- [50] Spalart, P. R., and Allmaras, S. R., "A One-Equation Turbulence Model for Aerodynamic Flows," *Recherche Aérospatiale*, Vol. 1, No. 5, 1994, pp. 5–21.
- [51] Menter, F. R., "Two-equation eddy-viscosity turbulence models for engineering applications," *AIAA Journal*, Vol. 32, No. 8, 1994, pp. 1598–1605.
- [52] "The Spalart-Allmaras Turbulence Model," <https://turbmodels.larc.nasa.gov/spalart.html#sa>, 2019.
- [53] "The Menter Baseline Turbulence Model," <https://turbmodels.larc.nasa.gov/bsl.html>, 2019.
- [54] "The Menter Shear Stress Transport Turbulence Model," <https://turbmodels.larc.nasa.gov/sst.html>, 2019.
- [55] "ANSYS® Fluent, Release 18.0, Help System, Theory Guide," *ANSYS, Inc.*, 2018.
- [56] Driest, E. V., "On turbulent flow near a wall," *Journal of the aeronautical sciences*, Vol. 23, No. 11, 1956, pp. 1007–1011.