

Fast and Precise Trajectory Simulation for Entry, Descent, and Landing Using A Multi-Model Monte Carlo Approach

James E. Warner^{*}, Robert A. Williams[†], Soumyo Dutta[‡], Geoffrey F. Bomarito[§], Patrick E. Leser[§], and William P. Leser[§]

NASA Langley Research Center, Hampton, VA 23681, USA

Predicting landing radius and other quantities of interest (QoI) for entry, descent, and landing (EDL) applications requires a viable uncertainty propagation method for quantifying the impact of uncertainties in aerodynamics, atmosphere, mass properties, etc. While standard Monte Carlo (MC) simulation is the de facto standard for providing robust and unbiased predictions, it is often infeasible for expensive, high-fidelity models. Low-fidelity models are commonly constructed to replace the high-fidelity model in MC simulation for computational speedup, but at the expense of accuracy and unbiasedness. Emerging multi-model MC methods are bridging this gap by combining predictions from two or more models of varying fidelity and computational cost for efficient and unbiased uncertainty propagation. This work explores the use of multi-model MC for increasing the speed and precision of trajectory simulation for EDL. It is shown that combining a high-fidelity EDL model with low-fidelity models (e.g., data-driven, reduced physics) yields substantial computational speedup versus standard MC with high-fidelity models. Moreover, the unbiasedness of multi-model MC predictions is highlighted by showing increased accuracy versus an approach that leverages a low-fidelity surrogate model alone.

I. Introduction

NASA's aspirations for human class Mars missions are contingent upon significant breakthroughs in entry, descent, and landing (EDL) systems. To support a crew of four on the Martian surface, NASA's Evolvable Mars Campaign (EMC) has considered landing four separate 20 metric ton payloads that must be delivered within a 50 m radius of the desired target. In order to accomplish this feat, NASA needs an order-of-magnitude increase in payload capability and nearly three orders-of-magnitude reduction in landing radius over the current state-of-the-art [1]. This work begins to address the latter challenge by exploring new methods for precise trajectory simulation.

Trajectory simulation is complicated by uncertainties in a range of factors that impact a quantity of interest (QoI), including wind pattern variations, atmospheric uncertainties, and spacecraft sensor errors. Predicting a vehicle's landing radius or other QoIs requires simulating a huge number of potential trajectories by varying these uncertainties using a Monte Carlo (MC) dispersion analysis [1, 2]. Since time-consuming, high-fidelity EDL models are needed for accurate trajectory simulation, it can be challenging to deliver precise predictions for missions with strict time constraints.

Broadly speaking, the problem of estimating the statistics of model QoIs given uncertainties in input parameters is that of uncertainty propagation. While MC simulation can often be computationally prohibitive for expensive models, it is still the most well known and general-purpose approach for uncertainty propagation. MC estimators are known to be robust, unbiased, and have a convergence rate that is independent of dimensionality (the number of random inputs). A common approach to reduce the computational expense is to construct low-fidelity surrogate models to replace the high-fidelity model for MC simulation. While substantial speedup can be obtained, relying solely on the low-fidelity model for prediction will generally lead to biased and potentially inaccurate estimators.

Multi-model MC methods have recently emerged to bridge this gap, combining information from two or more models of varying fidelity and computational cost to produce efficient, accurate predictions. By retaining the high-fidelity model, unbiased estimators of a QoI can be constructed, while introducing an ensemble of low-fidelity models provides computational speedup. Existing multi-model methods are distinguished by the types of models used and the approach to allocate computational resources across the available models. Multilevel MC (MLMC) [3] restricts low-fidelity

^{*}Research Computer Scientist, Durability, Damage Tolerance, and Reliability Branch, M/S 188E.

[†]Research Computer Scientist, Atmospheric Flight and Entry Systems Branch, M/S 489.

[‡]Aerospace Engineer, Atmospheric Flight and Entry Systems Branch, M/S 489, AIAA Member.

[§]Materials Engineer, Durability, Damage Tolerance, and Reliability Branch, M/S 188E.

models to those arising from coarsened discretizations of the same governing equations in space/time, while multifidelity MC (MFMC) [4] generalizes the types of models permitted to have different forms (e.g., data-driven, reduced-order, analytical, etc.). Both MLMC and MFMC introduce simplifying assumptions that yield closed form expressions for resource allocation across models. A generalized approximate control variates (ACVs) [5] approach was later proposed that unified and improved upon MLMC-based and MFMC methods. It was shown that significant performance improvements could be obtained relative to MLMC and MFMC by optimizing for resource allocation with a more complex numerical approach.

This work explores the use of multi-model MC for increasing the speed and precision of trajectory simulation for EDL. By leveraging recently-released NASA software [6] that implements MLMC, MFMC, ACV, and other multi-model methods, the relative performance of each is assessed with respect to standard MC simulation. Program to Optimize Simulated Trajectories II (POST2), a legacy code developed at NASA Langley Research Center, is used for high-fidelity trajectory simulation. Several low-fidelity models are constructed for use within the multi-model framework, including data-driven machine learning models trained from POST2 input/output data as well as lower-fidelity POST2 simulations that use time steps that are orders of magnitude larger than the high-fidelity model. An analysis of the Adaptable Deployable Entry and Placement Technology (ADEPT) test flight [2] is revisited to quantify accuracy and computational cost for predicting several relevant EDL QoIs. It is shown that a multi-model MC approach substantially outperforms a standard MC simulation that relies on either POST2 or a low-fidelity surrogate model alone.

II. Approach

Predicting landing location and other QoIs for EDL under uncertainties in parameters such as wind pattern variations and atmospheric conditions defines an uncertainty propagation problem. Let $Y \in \mathbb{R}^1$ represent a scalar QoI and $\mathcal{M}$ define a computational model that maps some input parameters, $\mathbf{X} \in \mathbb{R}^d$, to Y :

$$\mathcal{M} : \mathbf{X} \rightarrow Y. \quad (1)$$

Then, given a description of uncertainty in $\mathbf{X}$ (i.e., its probability density function, $p(\mathbf{x})$), uncertainty propagation methods seek to estimate the statistics of the QoI Y using $\mathcal{M}$.

In particular, this work considers the problem of constructing accurate estimators for the expected value of a QoI $\mathbb{E}[Y]$,

$$\mathbb{E}[Y] = \int Y(\mathbf{x})p(\mathbf{x})d\mathbf{x} \quad (2)$$

when $\mathcal{M}$ is an expensive, high-fidelity model. Using standard MC simulation, an estimator, $\hat{Y}$, for $\mathbb{E}[Y]$ is obtained as follows

$$\hat{Y} = \frac{1}{N} \sum_{i=1}^N Y(\mathbf{x}^{(i)}), \quad (3)$$

where $\{\mathbf{x}^{(i)}\}_{i=1}^N$ is a collection of independent and identically distributed (i.i.d.) samples from $p(\mathbf{x})$. It is easy to show that $\hat{Y}$ is an unbiased estimator and as such, the mean squared error (MSE) associated with it is given as follows

$$e(\hat{Y}) = \mathbb{E}[(Y - \hat{Y})^2] = \text{Var}[\hat{Y}] = \frac{\text{Var}[Y]}{N}, \quad (4)$$

where $\text{Var}[Y]$ is the variance of the QoI Y . Equation (4) shows that the convergence rate of standard MC estimators is relatively slow ($O(N^{-1})$) and so a huge number of samples (and model evaluations) can be required to obtain accurate estimators, especially when $\text{Var}[Y]$ is large. Thus, using MC simulation when it is time consuming to evaluate $\mathcal{M}$ can be impractical.

The approach adopted in this work is to use multi-model MC methods to construct estimators, $\tilde{Y}$, for $\mathbb{E}[Y]$ that have superior accuracy compared to standard MC ($e(\tilde{Y}) \ll e(\hat{Y})$) for the same amount of computation time.*. Here, it is assumed that there exists a collection of $M \geq 1$ low-fidelity models

$$\mathcal{M}_i : \mathbf{X} \rightarrow Y_i, \quad i = 1, \dots, M, \quad (5)$$

Alternatively, it could be said that multi-model MC methods can reach a prescribed accuracy e^ with substantially less computational effort than standard MC.

that provide faster, but potentially less accurate, approximations of Y predicted by the high-fidelity model in Equation (1). Note that $\mathcal{M}_i$ can represent a simple analytical model, a data-driven machine learning model (i.e. response surface, metamodel, etc.), or a coarser discretization of the same physics when $\mathcal{M}$ represents a model derived from discretized differential equations. As long as a low-fidelity model exhibits faster run time than the high-fidelity model and displays some degree of correlation with respect to Y , it can potentially increase the performance of a multi-model MC approach.

A multi-model MC estimator for $\mathbb{E}[Y]$ can be defined as follows

$$\tilde{Y}(\boldsymbol{\alpha}, \{\mathbf{x}\}) = \hat{Y}(\mathbf{x}) + \sum_{i=1}^M \alpha_i \left(\hat{Y}_i(\mathbf{x}_{i+}) - \hat{Y}_i(\mathbf{x}_{i-}) \right), \quad (6)$$

where $\boldsymbol{\alpha}$ is a set of weighting coefficients and $\{\mathbf{x}\} \equiv \{\mathbf{x}, \mathbf{x}_{1+}, \mathbf{x}_{1-}, \dots, \mathbf{x}_{M+}, \mathbf{x}_{M-}\}$ is a collection of potentially overlapping sets of random input samples [5]. Since $\hat{Y}$ (Equation (3)) is an unbiased estimator and the bias of $\hat{Y}_i$ terms cancel out, it follows that $\tilde{Y}$ is also unbiased.

The key to multi-model MC approaches is the optimal allocation of computational resources across the available models, where *optimal* is defined as yielding an estimator in Equation (6) with the lowest MSE for a given computational budget. Since $\tilde{Y}$ is unbiased, the MSE, $e(\tilde{Y}) = \text{Var}[\tilde{Y}]$, and the optimal estimator (and resource allocation) can be obtained through the following optimization problem

$$\tilde{Y}^{opt} = \min_{\boldsymbol{\alpha}, \{\mathbf{x}\}} \text{Var}[\tilde{Y}(\boldsymbol{\alpha}, \mathbf{x})], \quad (7)$$

where this optimization is constrained by the total computation time that a user is willing to invest in generating the estimator. Note that $\{\mathbf{x}\}$ represents the resource (or sample) allocation, fully specifying how many times to execute each model for the best prediction. Practically speaking, the solution of Equation (7) typically leads to relatively few high fidelity model evaluations and many evaluations of the low-fidelity models, especially the models that are highly correlated and efficient with respect to the high-fidelity model. The existing variations of the multi-model MC approach differ in the assumptions made and strategy used to solve this resource allocation optimization, where the best performing method is generally problem dependent.

In the context of the EDL application studied in this work, multi-model MC estimators are constructed for various trajectory states (altitude, dynamic pressure, mach number, etc.) at several points during the ADEPT test flight [2]. The high-fidelity model, $\mathcal{M}$, is a POST2 simulation identical to the one used in the original ADEPT analysis. The random inputs, $\mathbf{X}$, represent parameters such as those governing atmosphere, aerodynamics, and mass properties and have probability distributions $p(\mathbf{x})$ matching those from [2]. A handful of low-fidelity models, $\mathcal{M}_i$, are constructed and considered within the multi-model MC framework. First, the user-defined time step within POST2 are increased by 1-2 orders of magnitude versus the high-fidelity model to define faster, but potentially less accurate, low-fidelity models. A *reduced physics* POST2 is instantiated by using simplified atmospheric models that are cheaper to evaluate through the simulation. Finally, a data-driven low-fidelity model is trained from POST2 input/output data using machine learning. With the collection of models defined, Equations (6) and (7) are leveraged to construct optimal multi-model MC estimators for the ADEPT QoIs.

References

- [1] Cianciolo, A. D., and Powell, R. W., “Entry, Descent, and Landing Guidance and Control Approaches to Satisfy Mars Human Mission Landing Criteria,” *27th AAS/AIAA Space Flight Mechanics Meeting*, AIAA, San Antonio, TX, 2017.
- [2] Dutta, S., and Green, J. S., “Flight Mechanics Modeling and Post-Flight Analysis of ADEPT SR-1,” *AIAA Aviation 2019 Forum*, AIAA, Dallas, TX, 2019. doi:10.2514/6.2019-2900.
- [3] Giles, M. B., “Multi-level Monte Carlo path simulation,” *Operations Research*, Vol. 56, No. 3, 2008, pp. 607–617.
- [4] Peherstorfer, B., Willcox, K., and Gunzburger, M., “Optimal Model Management for Multifidelity Monte Carlo Estimation,” *SIAM Journal on Scientific Computing*, Vol. 38, 2016, pp. A3163–A3194. doi:10.1137/15M1046472.
- [5] Gorodetsky, A., Geraci, G., Eldred, M., and Jakeman, J. D., “A generalized approximate control variate framework for multifidelity uncertainty quantification,” *Journal of Computational Physics*, 2020, p. 109257. doi:https://doi.org/10.1016/j.jcp.2020.109257, URL <http://www.sciencedirect.com/science/article/pii/S0021999120300310>.
- [6] Bomarito, G. F., Warner, J. E., Leser, P. E., Leser, W. P., and Morrill, L., “Multi Model Monte Carlo with Python (MXMCPy),” *NASA/TM-2020-220585*, 2020.