

Stability Analysis of the Flow over a Swept Forward-Facing Step using PIV Base Flows

Koen J. Groot^{a,1} Jenna Eppink^{b,2} and Helen L. Reed^{c,1}

¹*Texas A&M University, College Station, TX 77843*

²*NASA Langley Research Center, Hampton, VA 23681*

I. Introduction

TURBULENT boundary layers contribute largely to the drag budget of commercial aircraft. Aircraft drag can therefore be significantly reduced by attenuating the disturbances responsible for laminar-turbulent transition. Surface imperfections are an important aspect that should be accounted for in accomplishing this. For one, such imperfections can strongly influence the mechanism by which disturbances are amplified. Second, they can create, rather than just amplify, disturbances.

The subject of this study is a specific kind of surface imperfection that is commonly encountered in aerospace applications: step excrescences. Due to accessibility requirements, the structure of wings and control surfaces usually consists of multiple panels. The effect on the boundary layer would be minimized if a perfectly flush intersection between panels could be produced. Due to finite manufacturing tolerances, however, these intersections are never perfectly seamless. Therefore, the boundary layer now has to traverse across an either forward- or backward-facing step geometry. In flowing across the step, the boundary layer experiences a sudden change in characteristics. If the step height is large enough, transition can be induced at the location of the step. At low speeds, it is known from the literature that, in the sense of the step height, ‘backward facing steps are roughly twice as dangerous as forward facing steps’ (Perraud et al., 2004; Holmes et al., 1985). This is mainly attributed to the fact that backward-facing steps result in a larger laminar separation length.

The context of the present study is furthermore focused on swept wings. Wings of commercial aircraft are swept to reduce wave drag. If the sweep angle is significant and the freestream turbulence intensity is low, stationary streamwise-oriented crossflow vortices emerge as the primary instability to the undisturbed laminar base flow (Saric et al., 2003). This perturbation mechanism is unstable from a location very close to the attachment line (Groot et al., 2020). Hence, crossflow vortices can attain a significant amplitude upstream of the step. Accordingly, the attention is focused on how the crossflow vortices interact with the step in order to understand step-induced transition on swept wings. By considering the forward-facing step from the point of view of stability analysis, Tufts et al. (2017) demonstrated the correlation between the critical step height and the vortex core of the crossflow vortices: the ‘Tufts-Reed criterion’. A synthesis of this criterion relevant for engineering purposes is that the critical step height can be determined through highly simplified means. It is proposed that the most amplified perturbation is determined with Linear Stability Theory (LST, Mack (1984)); the eigenfunction corresponding to this disturbance can be used to approximate the height of the vortex core.

Although the Tufts-Reed criterion indicates a culprit that drives forward-facing-step-induced transition on swept wings, many questions remain unanswered. It is unknown, for example, *how* the step-induced distortion of the crossflow vortices encourages transition in the portion of the flow downstream of the step if the step height is supercritical. The aim of the present study is to shed light on this aspect by analyzing the stability characteristics of the flow immediately downstream of a forward-facing step subject to a boundary

^aAssistant Research Scientist, Department of Aerospace Engineering, Member AIAA

^bResearch Aerospace Engineer, Flow Physics and Control Branch, M.S. 170, Member AIAA

^cUniversity Distinguished Professor, Department of Aerospace Engineering, Fellow AIAA

layer with crossflow vortices. In particular, this allows assessing whether the main effect of the step-induced distortion is to:

1. Impart an adaptation of the primary instability mechanism, *and/or*
2. Cause the emergence of secondary instability mechanisms.

Providing this information can help identifying the origin of the emergent unsteady character of the flow downstream of the step, as observed by Eppink (2018, figure 21).

A. Stability analysis of a measured base flow

To study the stability of a given flow field, a high-quality representation of that flow field is required. In this context, this flow field consists of the crossflow vortices in the boundary layer, subject to the influence of the forward-facing step. On the one hand, the crossflow vortices naturally introduce gradients in the flow that are approximately aligned with the spanwise direction. On the other hand, the step introduces a strong gradient orthogonal to it. The present study will focus on the experimental measurement data published independently by Eppink (2018). In this case, the step is oriented parallel to the leading edge. This means that the flow field is three-dimensional in the direct vicinity of the step; the gradient occupies all 3 spatial dimensions. Up to a reasonable approximation, the flow is steady immediately downstream of the step. This flow field will from now on be referred to as “the base flow”.

An important assumption in stability approaches is that the base flow satisfies the steady Navier-Stokes equations. In secondary instability analyses of crossflow vortices, for example, the base flow is computed by performing Direct Numerical Simulations (DNS, Bonfigli and Kloker (2007)). A strong argument for representing the base flow through computational means can be made upon regarding the strong sensitivity of the stability results to details (derivatives of) the base flow, the “base-flow uncertainty”, see Arnal (1994), Reed et al. (1996) and Theofilis (2003). A computational framework allows for a fine-tuned control of the residuals, so that the satisfaction of the steady Navier-Stokes equations is ensured and so that the base-flow uncertainty can be measurably eliminated. A significant drawback of all computational techniques, however, is the requirement for a careful receptivity calibration in order to introduce the primary crossflow vortices. Also, obtaining such solutions comes at a relatively high computational cost. That is, besides requiring a large amount of memory and time, it is very expensive to perform detailed grid-convergence studies (pivoted toward the elimination of the base-flow uncertainty, for example) and receptivity-parameter sweeps.

The complication of modeling the primary instability through computational means is directly circumvented in the present work by using an experimental measurement for the representation of the base flow. The time-averaged flow is accessed using highly-spatially-resolved stereographic Particle Image Velocimetry (stereo-PIV). This technique resolves all velocity components of the base flow in a plane. The aforementioned base-flow-sensitivity argument can be used inversely; as an advantage for the use of a measured base flow. Such a representation incorporates all features that are specifically inherent to the experiment, per definition. This includes features that might be overlooked when modeling the primary vortices numerically or *anyway* require opportune calibration with experimental datasets. Furthermore, there is no uncertainty about whether the Navier-Stokes equations are satisfied. In short, working this closely with measurement data creates a potential for:

- a direct validation of the stability results with the experimental data, on the one hand, and explaining experimental observations from the perspective of stability theory, on the other;
- the prediction of the *unsteady* perturbation content based on the measurement of the *steady* flow field (note that the measurement of an instantaneous flow field is considerably more challenging than the mean field);
- the prediction of the (unsteady) *pressure* perturbation field, while measuring just the mean *velocity* field (this application is restricted to the near-incompressible speed regime);
- providing feedback for enhancements or extensions of the measurements (e.g. which frequencies to target and whether the expected pressure perturbation will be measurable at the wall).

It should be emphasized that the aforementioned attributes are solely implied from the perspective of linear stability methods. That is, it is assumed that the instantaneous field is closely approximated by a linear

mode superposed on a mean flow. Despite this restriction, obtaining the pertinent mode’s characteristics through this approach goes beyond what is currently possible in the experimental framework alone.

Besides the linear mode assumption, using a measurement for the representation of the base flow does also come at a cost, of course. In particular, the sensitivity of the stability results comes into play, but now in relation to the parameters inherent to the experimental technique and “experimental reality”. Instead of a purely steady, laminar base flow, access is limited to the mean of instantaneous vector fields. This is allowable under the hypothesis that the difference between the base and mean flow becomes negligible as the ensemble size for the mean is increased; the “mean = base flow”-hypothesis. It has been demonstrated before that this assumption can yield success, even if the amplitude of unsteady perturbations is larger than what can reasonably be taken as representative of linear dynamics. What is most important is that all relevant mean flow features are properly accounted for (Fischer and Dallmann, 1991; Jordan and Colonius, 2013; Groot et al., 2018). In the present case, the experimentally observed perturbation amplitude is large in the region of interest. The velocity component in the direction parallel to the step, w'_{rms} , attains the largest values. In the region of interest, w'_{rms} is bounded above by 10% of the edge velocity, $\bar{W}_e$. This circumstance is quite similar to that considered by Groot et al. (2018).

The remainder of this abstract is structured as follows. The treatment of the measured base flow will be treated in §II, the stability approach and preliminary results will be outlined in §§III and IV. The abstract will be concluded in §V, which lists important aspects that will be added in the final manuscript.

II. Measured base flow

As mentioned before, this study will focus on the experimental measurement data by Eppink (2018). For the purposes of this abstract, only the essential features of the base flow will be addressed. The considered model represents a flat plate at 30° sweep and consists of two pieces, which allows the precise variation of the step height. In this abstract, the analysis is focused on the case with a step height of 1.7 mm. The step is located 41 cm downstream of the leading edge. A three-dimensional pressure body was used to introduce a favorable pressure gradient in order to activate the crossflow instability mechanism. Discrete Roughness Elements (DREs) were applied with a spanwise spacing of 11 mm.

Quantities presented in this abstract will be scaled with the length $\lambda_r = 10$ mm, i.e. a slightly different quantity than the roughness spacing.^a As the velocity scale, the value 24.3 m/s is used. This value approximates the local flow velocity component in the direction of the inviscid streamline, $\bar{U}_{s,e}$, in the vicinity of the step. In that vicinity, the inviscid streamline is removed by an angle of approximately $\psi_s = 35.3^\circ$ from the leading-edge orthogonal direction. The Reynolds number based on the aforementioned scales, $\bar{U}_{s,e}\lambda_r/\nu$, equals 1.66×10^4 , which is comparable to the case considered by Groot et al. (2018).

The base flow quantities need to be preprocessed before they can be fed to the stability problem. The handling of this is identical as that presented in Groot et al. (2018), see that reference for more details. First, the no-slip condition is enforced at the location of the wall ($y = 0$). The first off-the-wall datapoint is overwritten by the value of a spline interpolant based on the no-slip condition and the rest of the data. This is a precautionary measure in relation to the fact that it is challenging to perform PIV measurements close to the wall.

Stability solutions usually require a significant extent of the domain into the inviscid portion of the flow. For this dataset, this aspect is not much of a concern, because the PIV domain reaches up to $0.6\lambda_r \approx 3\delta_{99}$, where δ_{99} is the (distorted) boundary-layer thickness. Nevertheless, an overlap region between the largest available y -coordinate of the PIV data, and $0.5\lambda_r$ above that largest coordinate, is defined in order to test the influence of the domain height for the stability problem. The flow quantities are forced to tend to constant inviscid values based on the aforementioned values of $\bar{U}_{s,e}$ and ψ_s . Besides the wall-normal direction, the PIV plane is oriented in the direction parallel to the step (and the leading-edge). This coordinate will be indicated with z . In this direction, the PIV covers a length more than $5\lambda_r$, meaning that 5 crossflow vortices are captured in the spanwise direction.

The stability problem requires the first derivatives in two dimensions (the choice of z and y will be justified) of all velocity components representing the base flow field.^b From the first off-the-wall coordinate onward, the wall-normal derivatives were obtained through fourth-order finite differences (central in the

^aThe mismatch of these length scales will be resolved in the final manuscript.

^bFor the compressible problem, this requirement would be extended to second and cross derivatives of the velocity components as well as the temperature. Furthermore, the pressure and its first derivatives are required.

interior and forward and backward at the boundaries). A first-order difference was used to determine the wall-normal derivative at the wall. Derivatives in the step-parallel direction were determined with fourth-order finite difference throughout the domain. The resolution in the wall-normal direction equals $\Delta y = 7.83\lambda_r \times 10^{-3}$. This value is almost trice as small as the value used in Groot et al. (2018, $0.15/6.89 = 21.8\lambda_r \times 10^{-3}$). The same resolution is used in the z -direction.

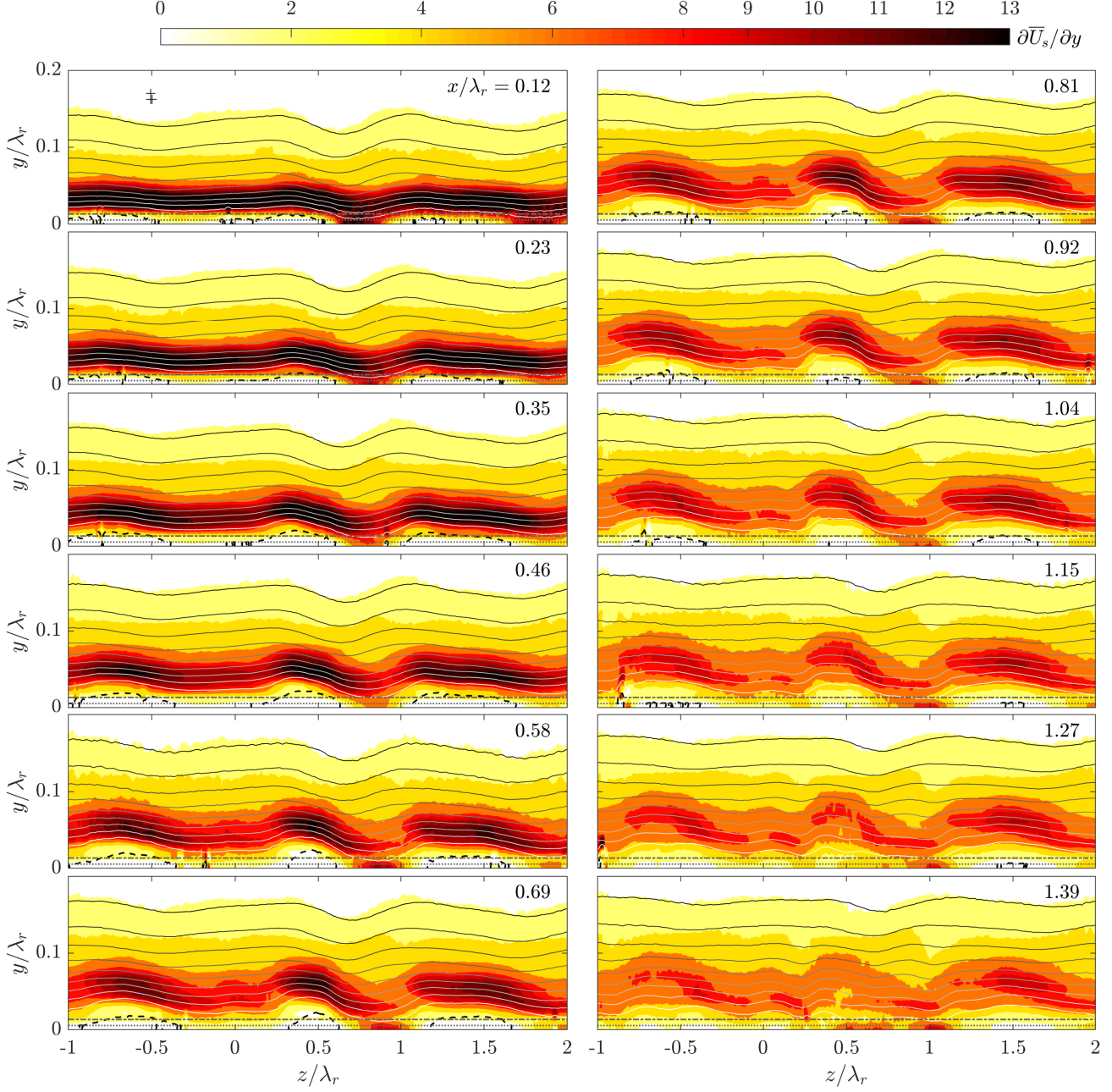


Figure 1: Streamwise evolution of properties related to the velocity component, $\bar{U}_s$, aligned with the local inviscid streamline direction. Value (gray solid contour lines, 0.1, 0.2, ..., 0.9 of $\bar{U}_{s,e}$; $\bar{U}_s = 0$ is indicated by the black dashed contour) and wall-normal derivative (color contours). Near-wall region (dash-dotted line) and first off-the-wall datapoint (dotted line).

The wall-normal derivative of the velocity component in the local direction of the inviscid streamline, $\partial\bar{U}_s/\partial y$ is illustrated in figure 1. The different panels represent the flow for the streamwise stations that will be considered for the stability analysis. The coordinate orthogonal to the step (and the leading edge) will be denoted by x . The gradients in the spanwise direction are so small that the figure of $\sqrt{(\partial\bar{U}_s/\partial y)^2 + (\partial\bar{U}_s/\partial z)^2}$

is practically indistinguishable from figure 1. Besides the shear magnitude, also contours of $\bar{U}_s$ are shown.

It is important to note that these contours do not resemble the typical shape corresponding to crossflow vortices. Usually, those vortices are recognized in the shape of the 80% to 90% contours with respect to $\bar{U}_{s,e}$. Here, instead, the contours close to the wall are curved the most and follow the outline of localized regions of reverse flow ($\bar{U}_s < 0$, shown as the dashed black contours). The primary features, the largest shear magnitude and the shape of the shear layer, are confined to the region below δ_{50} . Crossflow vortices themselves typically reach to about $2\delta_{99} > 4\delta_{50}$. As the localized regions of reverse flow are traced downstream, the shear layer evolves into localized pockets above the reverse flow regions. Heading downstream, the maximum shear magnitude decreases by a factor 2. For $x/\lambda_r > 1.15$, the coherency of the shear layers diminishes. This could be related to the perturbation amplitude becoming very significant at that point. In fact, $w'_{rms}/\bar{W}_e > 10\%$ at that point, where w'_{rms} denotes the maximum root-mean-square amplitude of the perturbation in the velocity component in the z -direction and $\bar{W}_e$ the inviscid edge velocity component attained by the base flow in that same direction.

The PIV technique makes use of a cross-correlation algorithm, which has a smoothing effect on the flow field. This is important, because a too harsh smoothing of the shear layer could compromise the validity of the stability results. A crude estimation of this model error can be made based on Groot (2018, figure 4.21), which relies on the following two parameters. First, the vorticity thickness, δ_v , can be derived from the shear magnitude, see Groot et al. (2018, figure 32) for more details. In the present case, it is assumed that this length is very closely aligned with the wall-normal direction. It is approximated to be: $\delta_v \approx 0.07\lambda_r$. Second, note that the aforementioned Δy mesh size does not indicate the height of the interrogation window over which cross-correlation is performed. A 75% overlap was used, which implies that the height of the interrogation window, $\Delta y_{vol} = 4\Delta y = 31.3\lambda_r \times 10^{-3}$. This implies that the shear layer is covered by approximately $\delta_v/\Delta y_{vol} \approx 2.2$ interrogation window height if stacked on top of each other. Using the $\delta_v/\Delta y_{vol}$ -parameter, the estimation of the expected error due to the smoothing effect follows from figure 4.21 by Groot (2018). It should be noted that the analysis presented by Groot (2018) considers the temporal problem, while here the spatial formulation of the stability problem will be considered. The results can be directly converted from one formulation to the other when assuming that the error in the perturbation's phase and group speed is negligible. Extracting the values from the figure, a 10% change on the maximum growth rate is expected and a 10% to 15% change in the frequency of the most unstable mode and a 10% to 20% change of the largest neutral frequency. Although these expected model errors are significant, they are argued to be sufficiently small to provide useful information for the current application.

III. Stability Approach

Given the structure of the base flow and the measurement data, it is elected to use the spanwise BiGlobal stability approach. This approach allows capturing all linear perturbation dynamics in a plane, while omitting the variation of the base flow and perturbation shape, wavenumber and growth rate in a third dimension, see Theofilis (2003); Groot (2018) for more details. Here, it is chosen to align this plane with the zy -plane, i.e. parallel to the step (and leading-edge) and the wall-normal direction. There are 2 reasons for this election.

First, as mentioned, the flow in the vicinity of the step is three-dimensional. It is argued, however, that, besides the wall-normal direction, the spanwise variation of the flow is the most constructive to integrate in the analysis. In the limit of a negligible spanwise variation imposed by the incident crossflow vortices on the step, an analysis performed in the zy -plane would yield the primary instability, the crossflow vortices themselves. So all aperiodicity can be assigned to a distortion of the perturbation mechanism by the crossflow-vortex-induced spanwise variation. The variation in the streamwise direction is definitely not negligible, but the flow does not necessarily change shape in that direction. It moreover changes shape in the spanwise direction, therefore that change is argued to be more important from a qualitative perspective.

Second, the stereo-PIV set-up is aligned with the zy -plane. While different downstream position were considered, providing data in a volume, the x -direction was sampled at a lower resolution (approximately $1\text{ mm} \approx 100\lambda \times 10^{-3}$ as indicated with the spacing in figure 1). By aligning the BiGlobal plane with the z - and y -coordinates, the maximal potential of the currently available experimental data is exhausted.

This assumption leads to the following ansatz for the perturbation quantities, q' :

$$q'(x, y, z, t) = \tilde{q}(z, y) e^{i(\alpha x - \omega t)} + c.c., \quad (1)$$

where α is the streamwise wavenumber of the perturbation, ω the (radial) frequency, $\tilde{q}$ represents the

eigenfunction equivalent of the quantity q' (for which boundary conditions are necessary) and *c.c.* represent the complex conjugate rendering the left-hand side real. The radial frequency ω is related to the cycle frequency as $\omega = 2\pi f$. Despite the different orientation of x and z , this approach is identical to that presented by Groot et al. (2018).

Perturbations are assumed to be convective, i.e. they are amplified while the flow carries them downstream. The way to represent this is by allowing α to be complex, while restricting ω to be real. The variable $-\alpha_i$ will denote the exponential growth rate in the x -direction. Perturbations can be stationary, in that case $\omega = 0$.

It is necessary to prescribe boundary conditions for the eigenfunctions $\tilde{q}$. At the top boundary, placed at $1.5\lambda_r \approx 7.5\delta_{99}$, all perturbation quantities are zeroed. At the wall, the no-slip condition is applied to the velocity component and the y -momentum equation is used as a compatibility condition for the pressure. In the absence of more reasonable conditions, periodic boundary conditions are applied in the z -direction. For the preliminary study presented here, the domain in the z -direction is limited to a unit of λ_r . In the final manuscript, the influence of the domain width on the stability results will be quantified. As demonstrated by Groot et al. (2018), this influence is small as long as the eigenfunctions are localized in the shear layer in the z -direction (even if they protrude through the boundary).

The domain is discretized with the Chebyshev collocation approach, see Groot (2018). The BiQuadratic mapping, introduced in Groot et al. (2018), is used with $(z_{i1}, z_{i2}) = (1, 2)\lambda_r/3$ in order to yield an approximately uniform spacing in the interior of the domain. The same mapping is used in the wall-normal direction with the parameters $(y_{i1}, y_{i2}) = (0.045, 0.15)\lambda_r$.

IV. Results

In figure 2, the growth rates are presented corresponding to a frequency sweep for $x/\lambda_r = 0.46$. Interestingly, the $\tilde{w}$ -eigenfunctions, representing the perturbation velocity component in the z -direction, attain the largest magnitudes. Usually this role is taken by the component in the direction orthogonal to the step, $\tilde{u}$. For this reason, the $\tilde{w}$ -eigenfunctions are shown for the frequencies for which the mode family attains the maximum growth rate (1 – 5). Also two stationary perturbations are shown, (0a) and (0b).

A low resolution was used in order to scan the parameter space rapidly. In particular, $N_z \times N_y = 50 \times 50$ nodes were used in the $z \times y$ space. In order to check the validity of the results, a convergence study was executed at the frequencies for which a particular mode family attained the maximum growth rate. The results are indicated with the symbols. The red box corresponds to $N_z \times N_y = 90 \times 90$ and the green cross to $N_z \times N_y = 85 \times 85$. These results indicate that a slightly higher resolution is required for mode families (2), (3) and (5). Nevertheless, as a preliminary result, the $N_z \times N_y = 50 \times 50$ -node resolution is relatively close. Even though there is a noticeable change in the growth rates, the $\tilde{w}$ -eigenfunctions appear to be invariant to using a higher resolution; the $N_z \times N_y = 90 \times 90$ equivalent is superposed as the dashed white contours on top of the contours using $N_z \times N_y = 50 \times 50$. Clearly, the $N_z \times N_y = 50 \times 50$ -node resolution is good enough for an preliminary study. For this reason, this resolution was considered for the remainder of the data presented in this abstract. In the final paper, fully converged results will be presented.

From figure 2, it becomes clear that the flow immediately downstream of the step is unstable to perturbations with an estimated maximal frequency equal to 2700 Hz. It appears that a similar frequency range is observed by Eppink (2018, figure 21). It is possible, therefore, that a further investigation can lead to a conclusive explanation for the observation of the observed unsteady content. This further investigation will be presented in the final manuscript.

In order to get a feel for the dependency of the results on the step-orthogonal x -coordinate, a frequency sweep was executed at all considered location (again at the lower resolution). Figure 3 shows the evolution of the growth rates and the $\tilde{w}$ -eigenfunction corresponding to the most unstable frequency. Clearly, the unstable frequency band diminishes as more downstream locations are analyzed. The width of the frequency band appears to be related to how localized the $\tilde{w}$ -eigenfunction is in the z -direction. For $x/\lambda_r = 0.35$ and 0.46, the largest growth rates are attained while the eigenfunction barely protrudes through the boundary at $z = \lambda_r$. Furthermore, note that the eigenfunctions appear to avoid the separation region.

The integration of the spatial growth rates is avoided in this abstract, because it is highly complicated to ensure one matches up the correct mode family from one x -station to the next. A crude estimate can still be obtained from the growth rates, because of a scaling trick. The growth rates are non-dimensionalized by the length scale λ_r . Upon assuming that the growth rate $-\alpha_i$ is constant in x , the value $-\alpha_i\lambda_r$ indicates

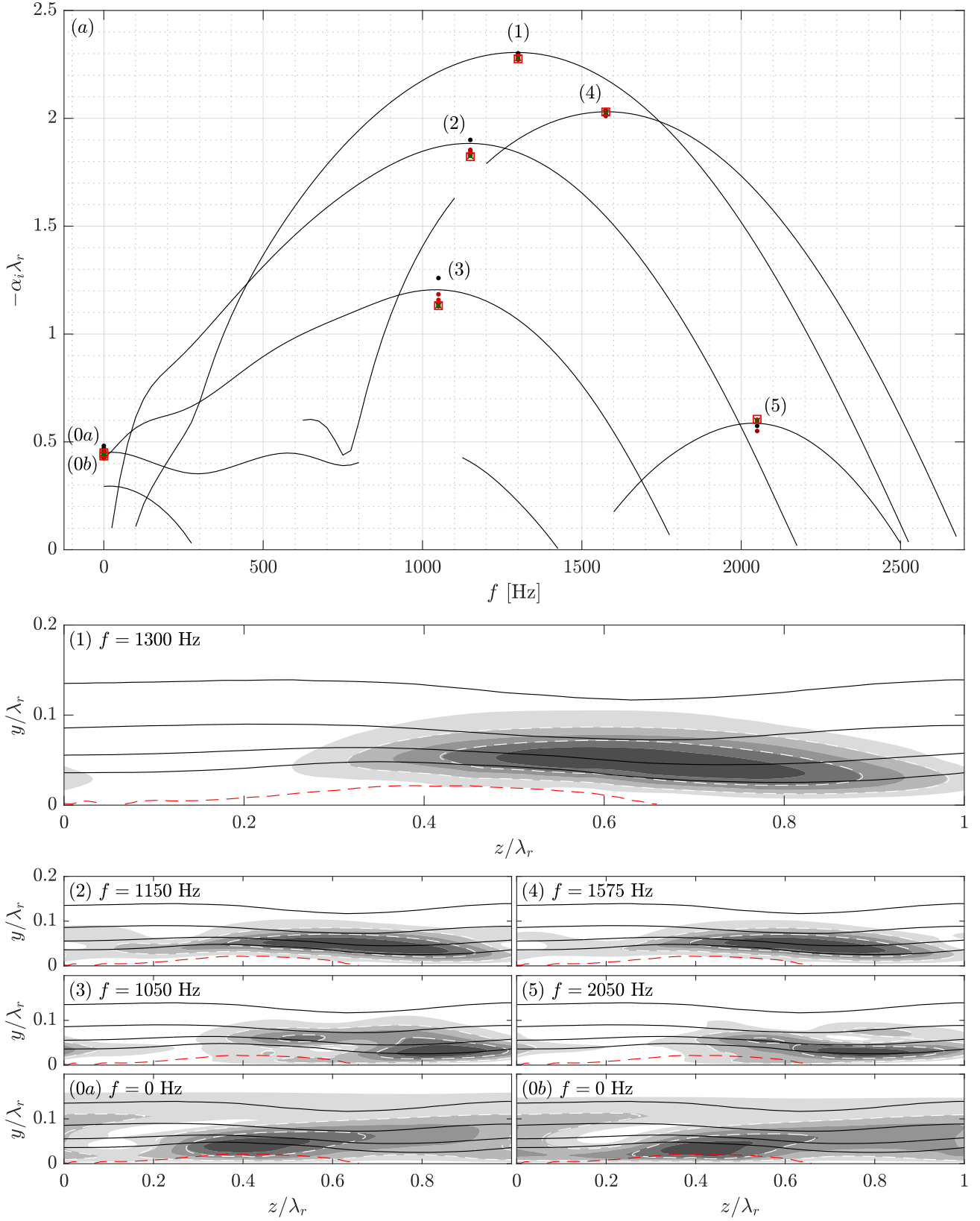


Figure 2: (a) Spatial growth rates versus frequency for all found mode families at $x/\lambda_r = 0.46$ with $N_z \times N_y = N \times N = 50 \times 50$. Symbols represent a stability-grid-resolution study (green $\times$: $N = 80$, red $\square$: $N = 90$). (1 – 5, 0a, 0b) Magnitude of the $\tilde{w}$ -eigenfunction (filled contours: $N = 50$, levels relative to $\max \tilde{w}$ and span $[1, 5]/6$ with $\Delta = 1/6$; white dashed contours: $N = 90$, levels: $1/3$ and $2/3$) corresponding to the modes labelled in (a). Contours of $\bar{U}_s$ (black solid lines: 0.2, 0.4, 0.6, and 0.8; red dashed line: 0.0).

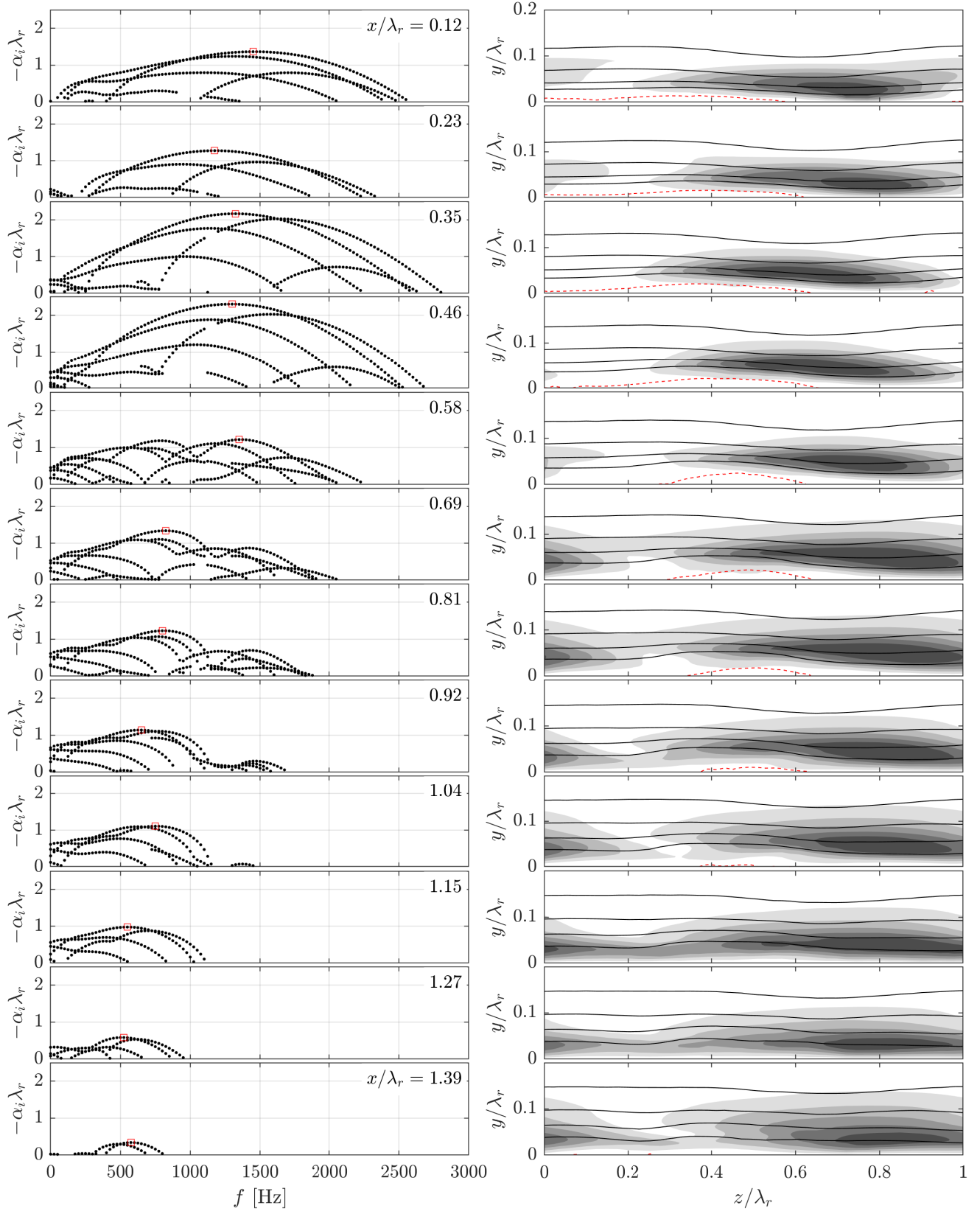


Figure 3: Growth rates versus frequency (left) and the most unstable $\tilde{w}$ -eigenfunction per step-orthogonal location (right, filled contours). The most unstable mode (left, red square), contours of $\tilde{U}_s$ (right, black solid lines: 0.2, 0.4, 0.6, and 0.8; red dashed line: 0.0).

the increase in the N -factor attained after one unit of λ_r . When considering $f = 500$ Hz, this product yields approximately $N \approx 1$ when arriving at $x/\lambda_r = 1.15$. It is expected therefore, that N -factors larger than unity will be attained.

V. Critical aspects to appear in the final manuscript

As illustrated already, there are many aspects upon which the current abstract will be extended. The final manuscript will contain:

- converged stability results for all mode families;
- a convergence study of the stability results with respect to the ensemble size, denoted by N_f , used to construct the mean flow, see (Groot et al., 2018, §4.2) for an outline of the intended approach;
- the domain will be truncated at different (larger) widths to assess the validity of the prescription of periodic boundary conditions;
- analysis attempting to perform validation with the experimental dataset (comparing growth rates, the shape of the perturbations, the pressure perturbation will be considered, for example, in order to identify whether the modes can be determined by high-frequency measurements at the wall, etc.);
- the instability mechanisms will be further analyzed in order to shed light on the path to transition.

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