

Trajectory Optimization for the Virtual Telescope for X-Ray Observations

Kyle Rankin, Hyeonjun Park

New Mexico State University, Department of Mechanical and Aerospace Engineering
1040 S. Horseshoe Street, Las Cruces, NM 88003; +1 575-649-7675
krankii@nmsu.edu, hjpark@nmsu.edu

John Krizmanic

Center for Space Science & Technology -
NASA Goddard Space Flight Center, University of Maryland Baltimore County
8800 Greenbelt Road, Greenbelt, MD 20771; +1 301-286-6817
john.f.krizmanic@nasa.gov

Neerav Shah

NASA Goddard Space Flight Center
8800 Greenbelt Road, Greenbelt, MD 20771; +1 301-286-8174
neerav.shah-1@nasa.gov

Steven Stochaj

New Mexico State University Department of Electrical Engineering
1125 Frenger Mall, Las Cruces, NM 88003; +1 575-646-3115
sstochaj@nmsu.edu

Asal Naseri

Space Dynamics Laboratory
1695 Research Park Way, North Logan, UT 84341; +1 435-713-3400
asal.naseri@sdsl.usu.edu

ABSTRACT

The Virtual Telescope for X-Ray Observations (VTXO) is a long focal length telescope which promises to provide orders of magnitude improvement in angular resolution in the X-ray band. VTXO will include a Phased Fresnel Lens (PFL), which provides nearly diffraction-limited imaging, with a 1 km focal length. The PFL is carried by the Optics Spacecraft, which flies in a formation with the Detector Spacecraft, approximating a rigid telescope body. In order to maintain the formation requirements, while pointing the telescope axis at the desired astronomical targets, one spacecraft will be traveling on a non-natural trajectory, requiring the vehicle to maneuver regularly to maintain the telescope pointing. If care is not taken in the trajectory design, these paths result in large propellant consumption. However, there is an opportunity to optimize trajectories when re-arranging the formation between different astronomical targets. This paper presents an optimization scheme for re-pointing the telescope, utilizing a non-traditional path-based cost function to solve the propellant optimal trajectory. The resulting trajectories show a factor of four improvement in propellant consumption compared to the baseline. The optimization techniques developed for VTXO are applicable to orbits ranging from low-Earth orbit, to highly eccentric Earth orbits, and Lagrange point orbits.

INTRODUCTION

The Virtual Telescope for X-ray Observations (VTXO) mission is part of a new class of distributed component space telescopes. On the VTXO mission, the telescope uses a new type of optic known as a Phase Fresnel Lens (PFL) to perform high angular resolution imaging in the X-ray spectrum.¹ The PFL on VTXO provides an order of magnitude improve-

ment in angular resolution over Chandra, the current state of the art X-ray telescope. However, a major downside to the PFL is its long focal length, which is on the order of a kilometer for VTXO's optic. Since this is not reasonably achievable on a rigid spacecraft, the VTXO mission distributes the telescope components between two spacecraft. The PFL is carried on one spacecraft known as the Optics Space Craft

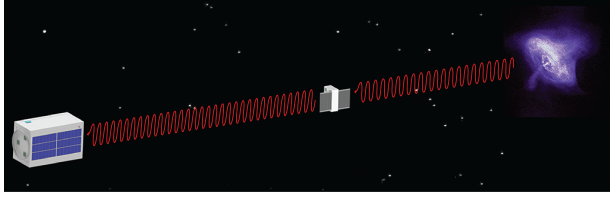


Fig. 1: Artist impression of the VTXO mission imaging the Crab Nebula

(OSC), and the X-ray camera is on a second spacecraft known as the Detector Space Craft (DSC). The OSC and DSC then fly in a precision formation approximating a rigid telescope structure.

On the VTXO mission, propellant is likely to be the dominate factor in determining spacecraft lifetime, and in turn the total science return of the mission.² As such it is highly desirable to minimize propellant consumption. One of the keys to this is to place the formation as far from a gravitational body as is practical, this minimizes the difference in gravitational acceleration between the two spacecraft. Additionally, in selecting an orbit it is necessary for VTXO to conduct observations outside of Earth's radiation belt to reduce the noise floor in the X-ray detector, and the VTXO baseline assumes the two spacecraft will launch as a ride share payload. To meet these requirements, the VTXO mission has baselined a Super Synchronous geostationary Transfer Orbit (SSTO), with an apogee around 90,000km altitude, and a perigee between 400km and 800km altitude. By observing near apogee, this orbit keeps the spacecraft above Earth's radiation belts during observations, and places the observation window at a high enough altitude to minimize propellant consumption. Finally, this orbit is used periodically for the initial orbit of spacecraft going to a GEostationary Orbit (GEO) providing an opportunity for ride share.³

However, while Highly Elliptical Earth Orbits (HEO) such as an SSTO significantly reduce propellant consumption during the observation window, it is not feasible to maintain the observation formation for the entire orbit, as holding the formation through perigee would require unsustainably high ΔV .⁴ Hence, VTXO breaks the formation and then flies a propellant optimized trajectory between the end of one observation window and the start of the next. The current mission baseline takes a naive approach, where at the end of the observation window in the relative frame, the DSC flies a straight line trajectory at a constant speed to a point in a very close formation with the OSC. This minimum safe distance is then

held for a few hours past perigee, at which point the spacecraft will then begin moving back out to the observation formation, again flying a straight line constant velocity trajectory.

The baseline trajectory while simple to fly, is in no way optimal. As such an attempt is made to find a theoretically optimal trajectory, this is done by utilizing a non-traditional path-based cost function. This cost function calculates the total acceleration along the trajectory, and then subtracts the acceleration due to gravity and disturbance forces from the acceleration function, resulting in the acceleration due to the propulsion system. Then, by integrating over the propulsion curve, the ΔV required to fly the trajectory can be calculated. The trajectory is then run through an optimization algorithm to determine the propellant optimal path.

DISTRIBUTED TELESCOPES

Beyond VTXO, the technology being developed for VTXO is an enabling technology for numerous classes of future very large space telescope missions, ranging from distributed aperture telescopes, to extreme focal length telescopes like VTXO, and occulting missions.

Distributed Aperture Telescope

The creation of space based distributed aperture telescopes where the telescopes primary mirror is split amongst several spacecraft promises to solve limitations on telescope performance caused by the upper limit on spacecraft size, mass, and cost. The desire for more powerful astronomical instruments has driven the continual development of larger and more capable telescopes starting with instruments such as Hubble Space Telescope, and more recently with the James Web Space Telescope (JWST).^{5,6} The future looks towards even larger and more powerful telescopes such as the proposed Large UV / Optical / IR Surveyor (LUVOIR) enabled by the largest currently announced launch vehicles such as NASA's Space Launch System (SLS) or SpaceX's Big Falcon Rocket (BFR).^{7,8} However, even with these large launch vehicles it is difficult to imagine achieving the significantly larger apertures compared to LUVOIR, which are needed to advance many fields of astronomy within the constraints of a single spacecraft. By creating a distributed aperture telescope where the primary optic is split between a few spacecraft, it becomes possible to move past these constraints, to provide the orders of magnitude increase in angular resolution.

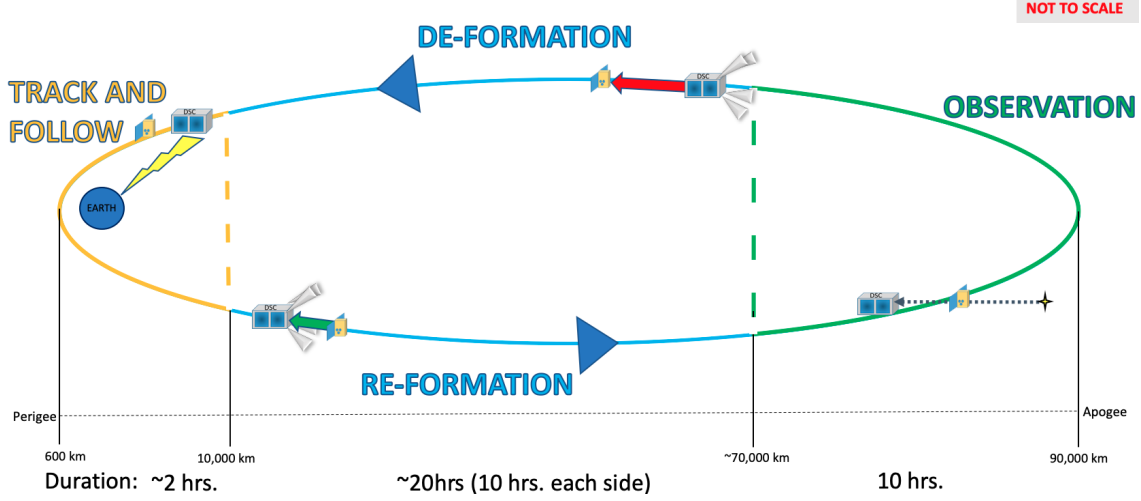


Fig. 2: VTXO Concept of Operations Showing a Typical Observation Orbit

Occulter Disk Telescopes

Occulting disks are traditionally used to obscure the sun's disk to enable viewing of its relatively dim corona. However, they can also be used to obscure a distant star in order to observe the exoplanets orbiting it.⁹ There are several formation flying occulter mission concepts being developed, including the Remote Occulter mission being developed at NASA Goddard, and the Jet Propulsion Laboratory's Star Shade which will fly formation with NASA Goddard's Wide Field Infrared Survey Telescope (WFIRST).^{10–12}

CONCEPT OF OPERATIONS

In the current VTXO baseline trajectory, the OSC is free flying while the DSC contains the propulsion system and flies formation with the OSC. As can be seen in Figure 2, at the end of the observation period the DSC breaks the observation formation and moves to a propellant efficient trajectory which is required to move the DSC back into position to re-establish the observation formation at the beginning of the observation window.

SYSTEM DYNAMICS

The equations of motion for VTXO are based on a derivation by Calhoun and Shah.¹³ The position vector $\vec{r}$ from the OSC to the DSC in a non-rotating frame can be expressed as

$$\vec{r} = \vec{R}_D - \vec{R}_O, \quad (1)$$

where $\vec{R}_D$ is the position vector from the Earth to DSC, and $\vec{R}_O$ is the position vector from the Earth to the OSC.

Equation (1) can be differentiated twice to obtain the acceleration vector $\ddot{\vec{r}}$ from the OSC to the DSC as

$$\ddot{\vec{r}} = \ddot{\vec{R}}_D - \ddot{\vec{R}}_O, \quad (2)$$

where the acceleration vectors $\ddot{\vec{R}}_D$ and $\ddot{\vec{R}}_O$ are given by

$$\ddot{\vec{R}}_D = -\frac{\mu}{\|\vec{R}_D\|^3} \vec{R}_D + \vec{a}_T, \quad (3)$$

$$\ddot{\vec{R}}_O = -\frac{\mu}{\|\vec{R}_O\|^3} \vec{R}_O. \quad (4)$$

Here, $\vec{a}_T$ is the thrust vector.

Inserting Equations (3) and (4) into Equation (2), we obtain

$$\ddot{\vec{r}} = -\frac{\mu}{\|\vec{R}_D\|^3} \vec{R}_D + \frac{\mu}{\|\vec{R}_O\|^3} \vec{R}_O + \vec{a}_T. \quad (5)$$

Equation (5) represents the full non-linear equation of motion for the system. However, it is desirable to use Equation (1) to re-write the system in terms of purely $\vec{R}_O$ and $\vec{r}$, using Equation (6).

$$\vec{R}_D = \vec{R}_O - \vec{r} \quad (6)$$

By rearranging, applying a binomial expansion, and removing higher order terms we can get Equation (7).

$$\frac{1}{\|\vec{R}_O\|^3} - \frac{1}{\|\vec{R}_D\|^3} = \frac{3 \left([\vec{r}]^T [\vec{r}] - 2[\vec{R}_O]^T \right)}{2 \|\vec{R}_O\|^5} \quad (7)$$

Similarly, we can show Equation (8) is also true.

$$\frac{\mu}{\|\vec{R}_D\|^3} = \frac{\mu}{\|\vec{R}_O\|^3} \left(1 - \frac{3}{2} \frac{[\vec{r}]^T [\vec{r}]}{\|\vec{R}_O\|^2} + 3 \frac{[\vec{R}_O]^T [\vec{r}]}{\|\vec{R}_O\|^2} \right) \quad (8)$$

By assuming $\|\vec{R}_O\| \gg \|\vec{r}\|$ and substituting Equations (7) and (8) into (5), the resulting equation is of the form,

$$\ddot{\vec{r}} = [\Gamma_{GG}] \vec{r} + \vec{a}_T, \quad (9)$$

where

$$[\Gamma_{GG}] = -\frac{\mu}{\|\vec{R}_O\|^3} \left([I] - 3 [\hat{R}_O] [\hat{R}_O]^T \right). \quad (10)$$

BASLINE TRAJECTORY

The VTXO baseline trajectory can be seen in Figure 3, where the formation maintains a constant pointing direction during an orbit. It shows each of the components of the trajectory in the relative frame and two orbits starting at perigee. The DSC then moves linearly along that pointing direction between the 1 km separation during the observation window near

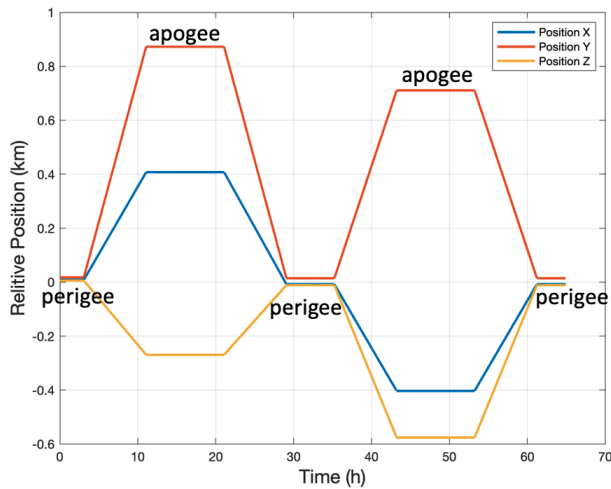


Fig. 3: VTXO baseline trajectory

apogee, and the 20 m separation that is held during perigee to minimize ΔV .¹⁴

Figure 4 shows a typical ΔV plot for the baseline mission. This also shows each of the components of the trajectory in the relative frame. The baseline trajectory consumes around 0.36 m/s of ΔV for a typical orbit.

OPTIMIZATION

In order to improve on the baseline, a trajectory optimization scheme is utilized. The optimization attempts to generate a theoretically propellant optimal trajectory in the relative frame. This then results in a function which gives a position as a function of time. This function can then be given to the spacecrafts on board control system to follow.

Cost Function

The non-traditional cost function utilized for VTXO is based on calculating the acceleration of the trajectory function $\ddot{\vec{r}}$, then subtracting the acceleration due to gravity, and the disturbance forces along that trajectory. The resulting difference will be the acceleration due to the propulsion system as can be seen in the below derivation.

First, we split the acceleration of the path into its components

$$\ddot{\vec{r}} = \ddot{\vec{r}}_g + \vec{a}_T + \ddot{\vec{r}}_d, \quad (11)$$

where $\ddot{\vec{r}}_g$ is the gravitational acceleration, $\vec{a}_T$ is the acceleration of the thruster, and $\ddot{\vec{r}}_d$ is the disturbance

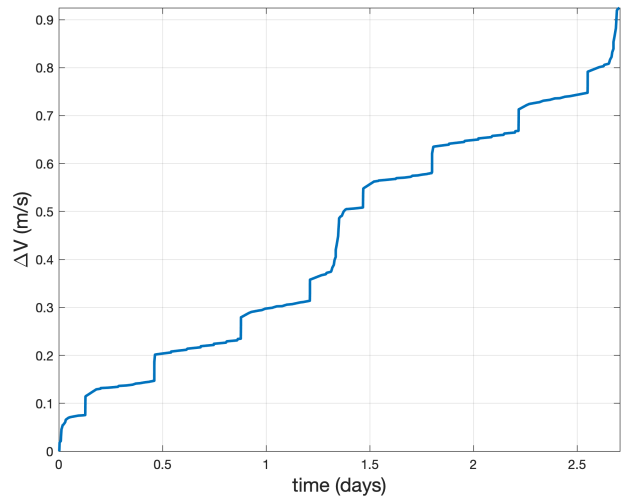


Fig. 4: VTXO Baseline ΔV

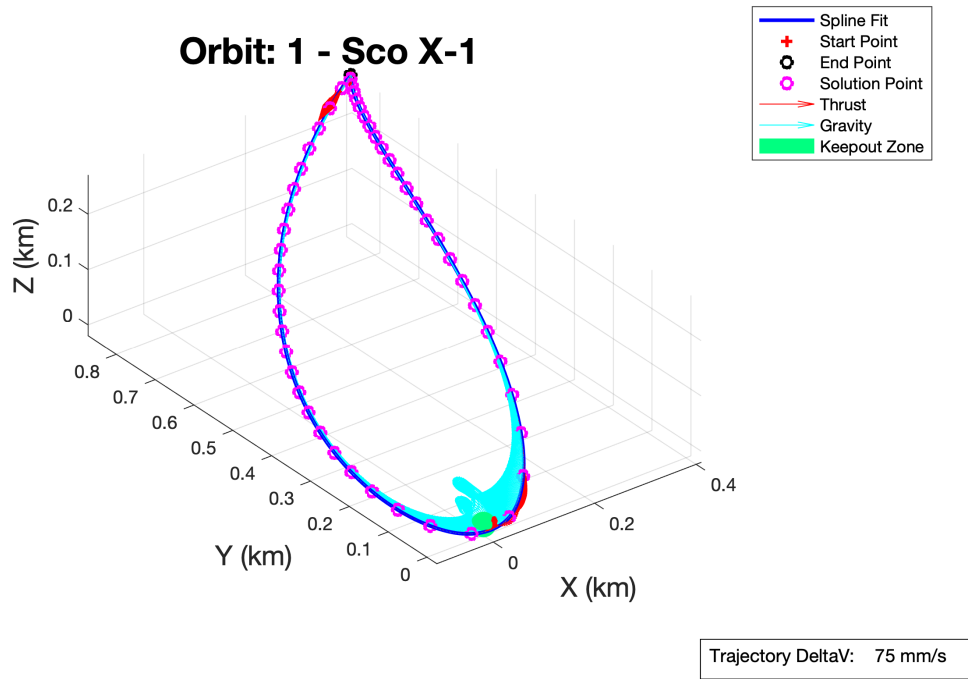


Fig. 5: VTXO Optimized Trajectory between the 1st and 2nd Observation Windows

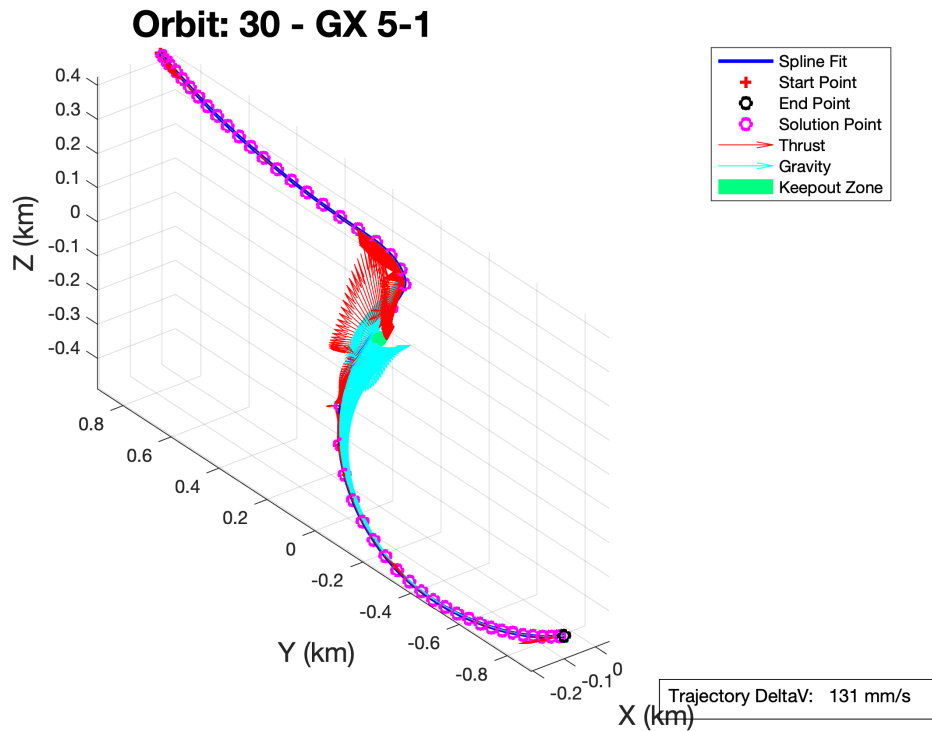


Fig. 6: VTXO Optimized Trajectory between the 30th and 31st Observation Windows

forces, respectively. ΔV can then be calculated by solving Equation (11) for $\vec{a}_T$, and integrating over the path, substituting Equation (10) for the gravitational acceleration given by

$$\Delta V = \int_{t_0}^t \left(\ddot{\vec{r}}(t) - [\Gamma_{GG}] \vec{r}(t) - \ddot{\vec{r}}_d(t) \right) dt \quad (12)$$

The propellant optimal trajectory can be found by calculating the global minimum of Equation (12) by varying $\vec{r}(t)$, subject to zero velocity end point constraints. Additional constraints are implemented to impose a minimum spacecraft separation distance $\|\vec{r}\|$ for collision avoidance between two spacecraft and a maximum acceleration due to the propulsion $\vec{a}_T$. These ensure that the vehicles maintain a safe separation and that the trajectory can be followed utilizing a finite thrust level.

Optimization Results

In this section, we present a sample of optimized trajectories produced by the optimization algorithm. Figures 5 and 6 show the trajectories from the beginning of one observation window to the beginning of the next. In these results, the disturbance forces are neglected as being approximately zero. In Figure 5, the orbit is moving between two observation windows of Scorpius X-1. It shows the thrust and gravitational acceleration vectors. ΔV for the trajectory is 0.075 m/s. Figure 6 presents this orbit moving between an observation window viewing Cygnus X-3 to one viewing GX 5-1. ΔV for this trajectory is 0.131 m/s.

As can be seen in Figure 7, the optimization is averaging a ΔV of around 0.091 m/s. This is approximately 1/4 of the ΔV for the baseline solution.

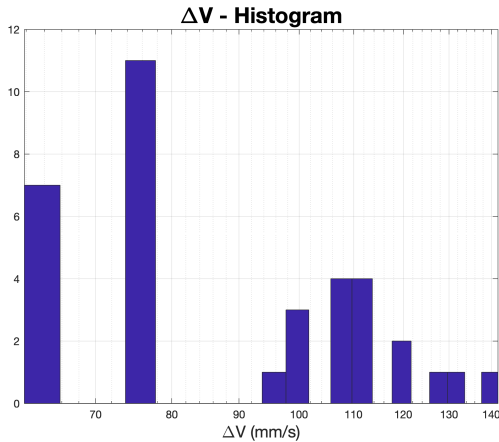


Fig. 7: Histogram of the Drift Segment ΔV

CONCLUSION

A novel optimization approach has been studied for re-pointing the VTXO telescope, based on a non-traditional path-based cost function. The preliminary results show an improvement in propellant consumption over the baseline by around a factor of 4, which is directly related to a corresponding proportional increase in the overall mission lifetime.

Most of the areas of future work revolve around improving the optimizer. Different optimization algorithms have potential to improve the solution. There is also a potential for improved optimization solution point spacing, possibly involving iterative solutions to find an ideal step size. The software also needs efficiency improvements in order to reduce run times to make the tool viable for operational use.

Additionally, this cost function technique has potential applications to other mission trajectories where the mission has a fixed start and end point, simply by changing the gravity model. In these cases, this method provides the possibility of finding a theoretically optimal trajectory, without being limited by propulsion profiles.

Finally, comparisons need to be run between the theoretical propellant consumption of the optimizer and the actual propellant consumed with realistic thrust, controlled by a flight like control system, in a fully non-linear dynamics environment including disturbance forces.

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