Fluid System Stability Analysis Techniques

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Motivation

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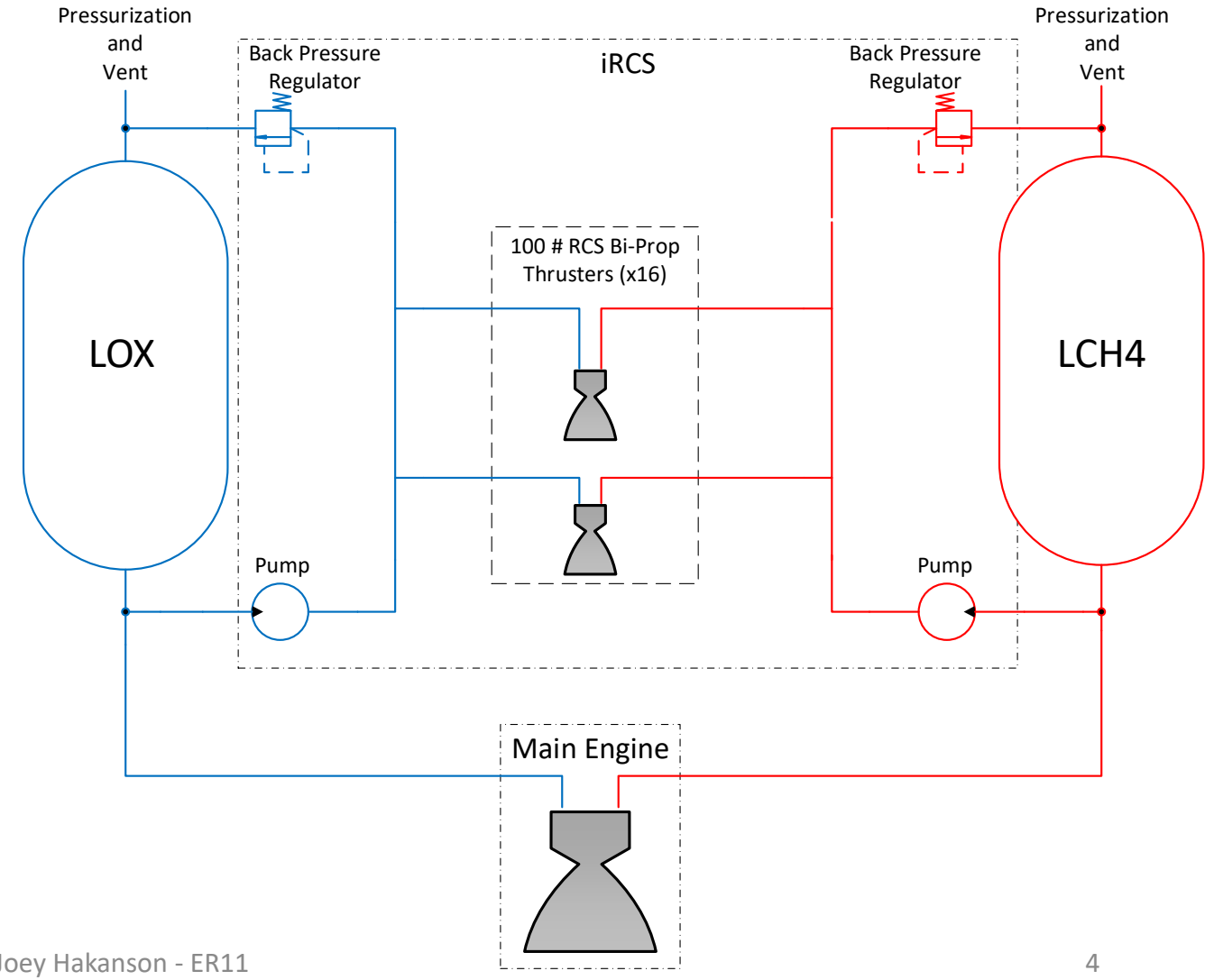
Inter-Component Dynamics

- Generically, a fluid system is a network of components that interact to resolve a thermo-fluid state solution $(P, \dot{m}, h, \dots)$ at desired design conditions, or "equilibrium points"
  - These components are often governed by nonlinear differential equations
  - Transient component-to-component interactions can result in network instability
  - Individual steady-state performance isn't sufficient to predict these phenomena
- Small perturbations around equilibrium points can drive instability, which can go undetected until issues arise during integrated system testing
  - **Dynamic Instability**: Pogo Oscillations
    - Combustion surge causes perturbation in chamber pressure and thrust
    - Dynamic coupling between pressure, mass flow, thrust, and structural elasticity cause instability
  - **Static Instability**: Aircraft Static Margin
    - Natural variations in the atmosphere can cause angle-of-attack to be perturbed from trim
    - An unstable aircraft (SM < 0) will produce moments that amplify this perturbation
- This presentation explores a method for designers to think of the fluid network as an assembly of components, each with their own thermo-fluid surfaces of partial stability, called nullclines
  - **Static Stability Example**: Perturbations about fluid system equilibrium points

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Integrated Reaction Control System (iRCS)

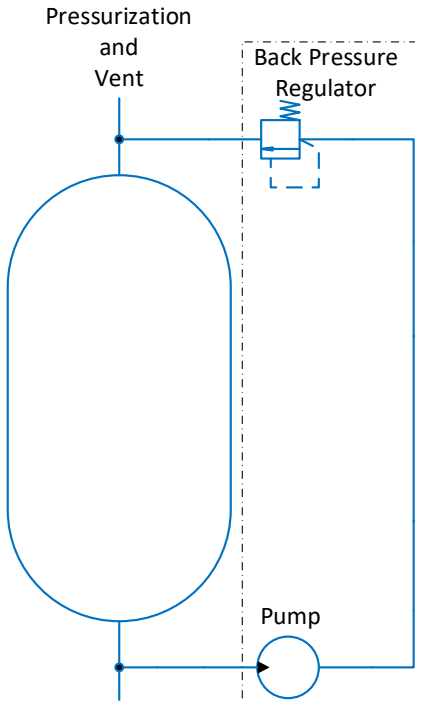
- Objectives:
  - MPS and RCS share Prop Tanks
  - Pulse-firing, pump-fed thrusters
  - Excess flow recirculates back to tanks via pressure-control device
- Benefits:
  - Potential mass savings from shared tanks and hardware
  - Cryogenic bi-prop RCS is more efficient than storable options
  - Using a common propellant can reduce system complexity
  - Possibly leveraging In-Situ Resource Utilization (ISRU)

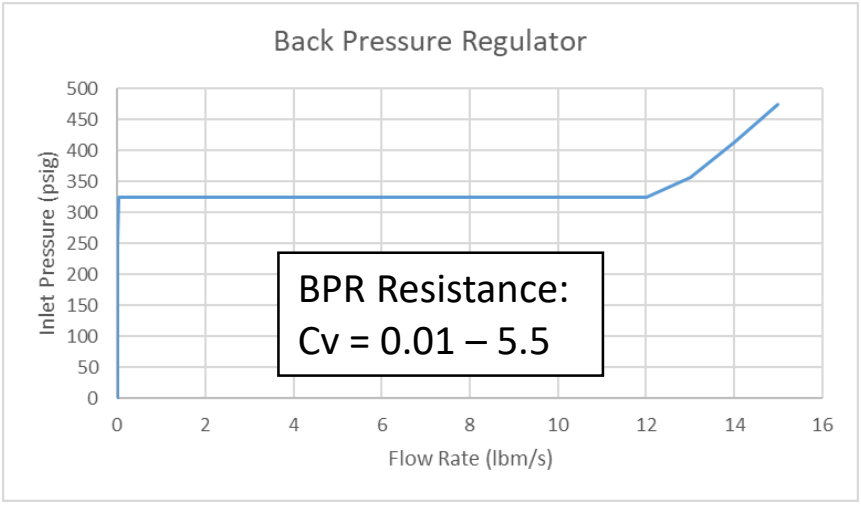


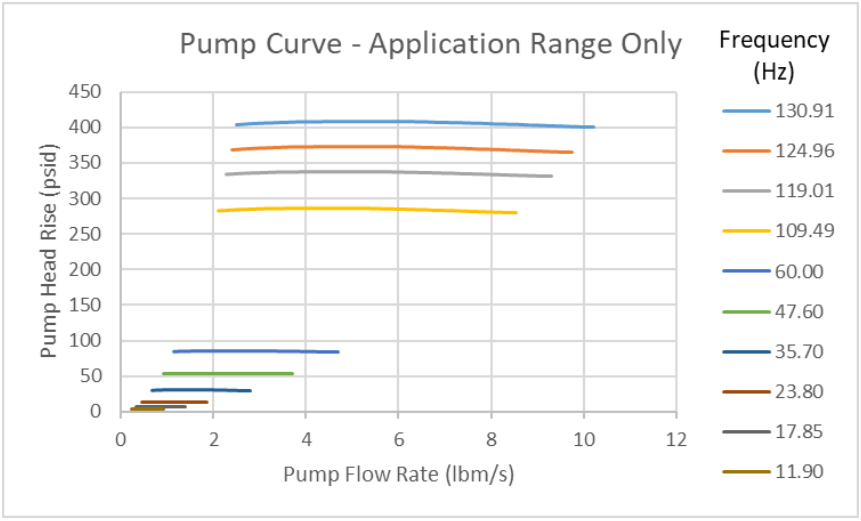
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Stability Question

- Proposed Procedure:
  1. Set BPR upstream pressure to 350 psig
  2. Adjust pump speed with VFD until the desired flow rate is achieved
     - 0.54 lbm/s to fire Two 100 lbf Thrusters
     - 2.16 lbm/s to fire Eight 100 lbf Thrusters
- Possible Stability Issue:
  - All desired operating points are on the Left-Hand Side of the pump curve (approximated as a quadratic function)
  - BPR will passively adjust system resistance over a wide range to achieve set-pressure
  - <u>Is it possible to hold 350 psig @ 0.54 lbm/s?</u>







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Phase Portrait Examples

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Linear Time Invariant (LTI) Example

Example 1:

Using LTI theory, draw a phase portrait of:

$\dot{x} = x + 2y$

$\dot{y} = -y$



$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}x + 2y\\ -y\end{bmatrix} \rightarrow \boldsymbol{\dot{X} = AX + B}$$

$$\boldsymbol{A} = \begin{bmatrix}\partial\dot{x}/\partial x & \partial\dot{x}/\partial y\\ \partial\dot{y}/\partial x & \partial\dot{y}/\partial y\end{bmatrix} = \begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix} \rightarrow \begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\text{Equilibrium Point : } \begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \rightarrow \begin{bmatrix}x^*\\ y^*\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\text{Eigen Values: } |\boldsymbol{A} - \lambda\boldsymbol{I}| = 0 \rightarrow \begin{matrix}\lambda_1 = -1\\ \lambda_2 = 1\end{matrix} \rightarrow \text{Saddle Point}$$

$$\text{Eigen Vectors: } (\boldsymbol{A} - \lambda\boldsymbol{I})v = 0 \rightarrow v_1 = \begin{bmatrix}1\\ -1\end{bmatrix}, \quad v_2 = \begin{bmatrix}1\\ 0\end{bmatrix}$$

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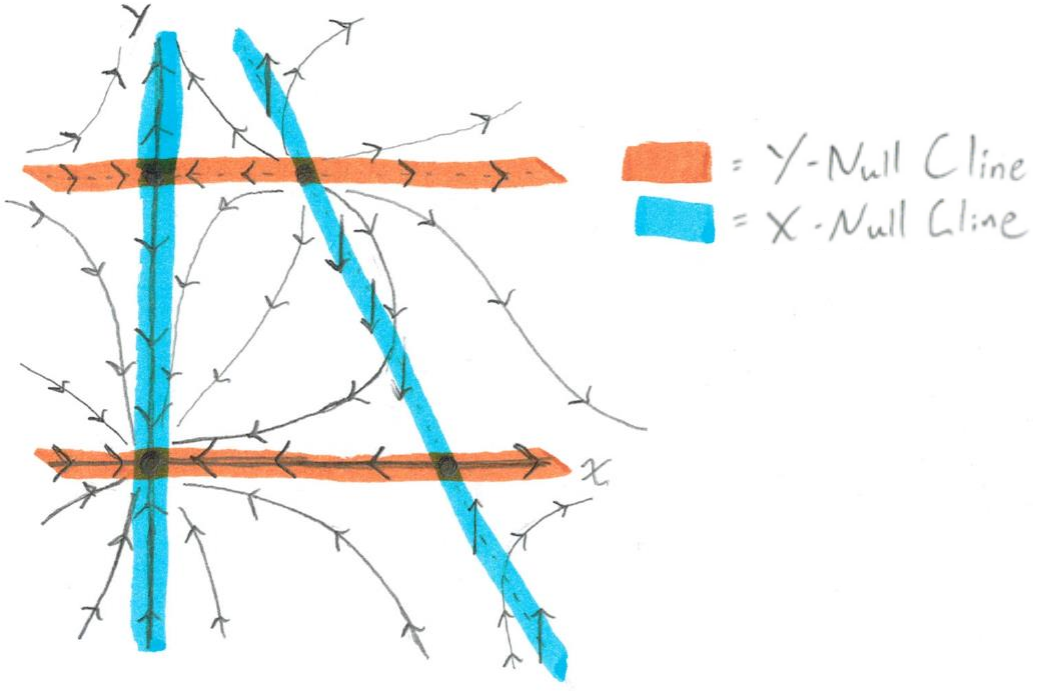
Non-Linear Time Invariant (TI) Example

Example 2:

Find the Equilibrium Points, linearize to find their type, then use Null Clines to help draw the non-linear phase portrait of:

$\dot{x} = x(y + 2x - 2)$

$\dot{y} = y(y - 1)$



$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}x(y+2x-2)\\ y(y-1)\end{bmatrix} \rightarrow \boldsymbol{\dot{X} = AX + B}$$

$$\boldsymbol{A} = \begin{bmatrix}\partial\dot{x}/\partial x & \partial\dot{x}/\partial y\\ \partial\dot{y}/\partial x & \partial\dot{y}/\partial y\end{bmatrix} = \begin{bmatrix}y+4x-2 & x\\ 0 & 2y-1\end{bmatrix}$$

Equilibrium Point : $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \rightarrow \begin{bmatrix}x^*\\ y^*\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}, \begin{bmatrix}1\\ 0\end{bmatrix}, \begin{bmatrix}0\\ 1\end{bmatrix}, \begin{bmatrix}1/2\\ 1\end{bmatrix}$

Linearized Eigen Values: $|\boldsymbol{A}(\boldsymbol{X}^*) - \lambda\boldsymbol{I}| = 0$

$\begin{bmatrix}x^*\\ y^*\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \rightarrow \lambda = -2, -1 \rightarrow$ **Sink**

$\begin{bmatrix}x^*\\ y^*\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} \rightarrow \lambda = -1, 2 \rightarrow$ **Saddle**

$\begin{bmatrix}x^*\\ y^*\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix} \rightarrow \lambda = -1, 1 \rightarrow$ **Saddle**

$\begin{bmatrix}x^*\\ y^*\end{bmatrix} = \begin{bmatrix}1/2\\ 1\end{bmatrix} \rightarrow \lambda = 1, 1 \rightarrow$ **Source**

x Null Cline: $\dot{x} = 0 \rightarrow x = 0, \quad y = 2 - 2x$

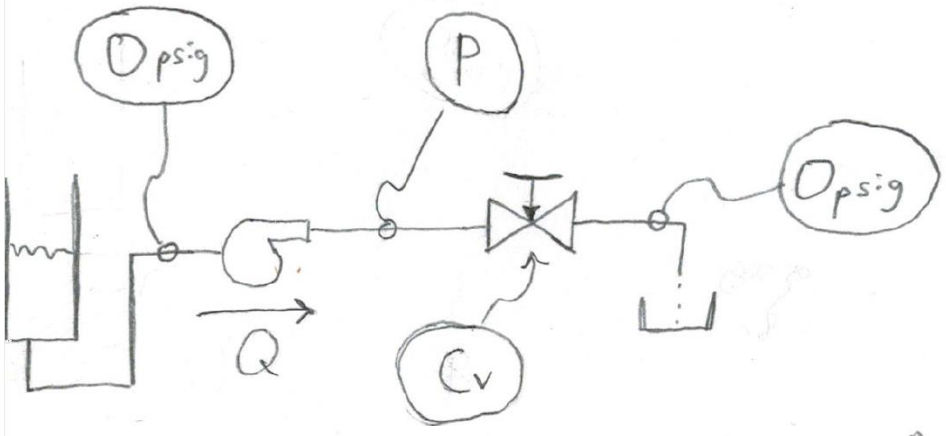
y Null Cline: $\dot{y} = 0 \rightarrow y = 0, \quad y = 1$

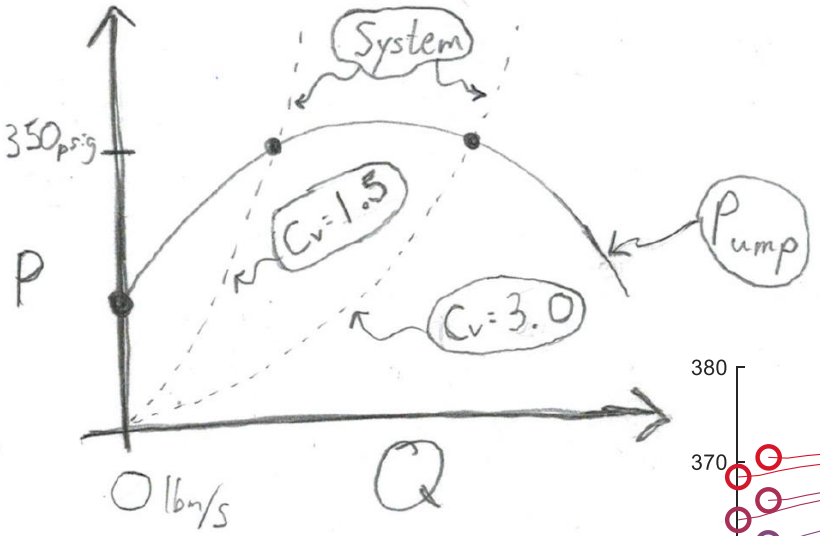
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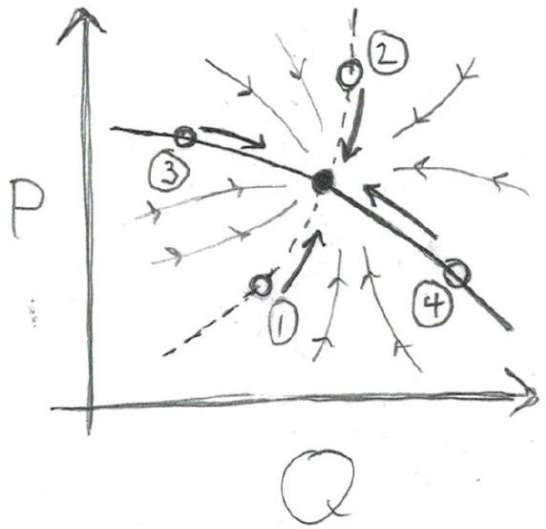
Qualitative iRCS Phase Portrait

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Centrifugal Pump + Needle Valve







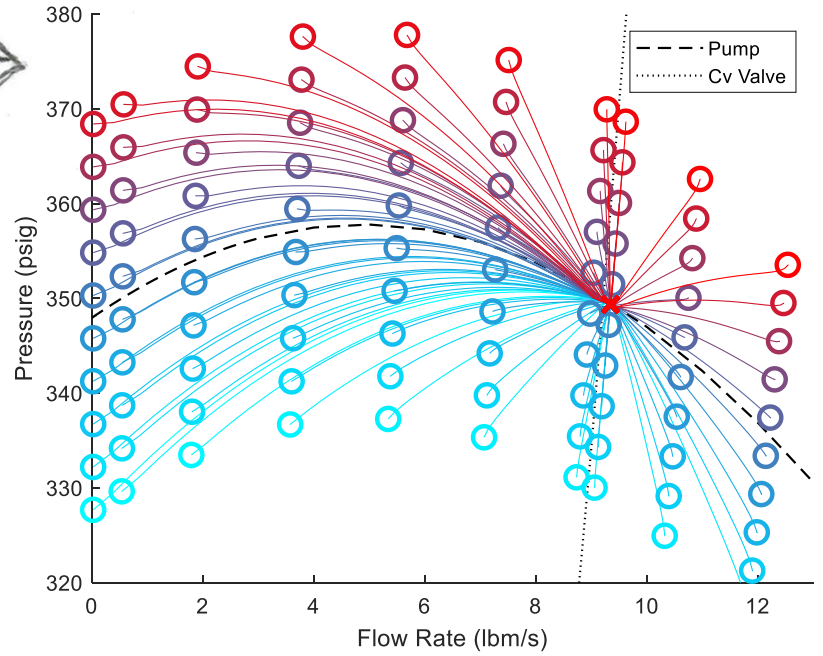
① Under Pump Curve

② Over Pump Curve

③ Left of System Curve

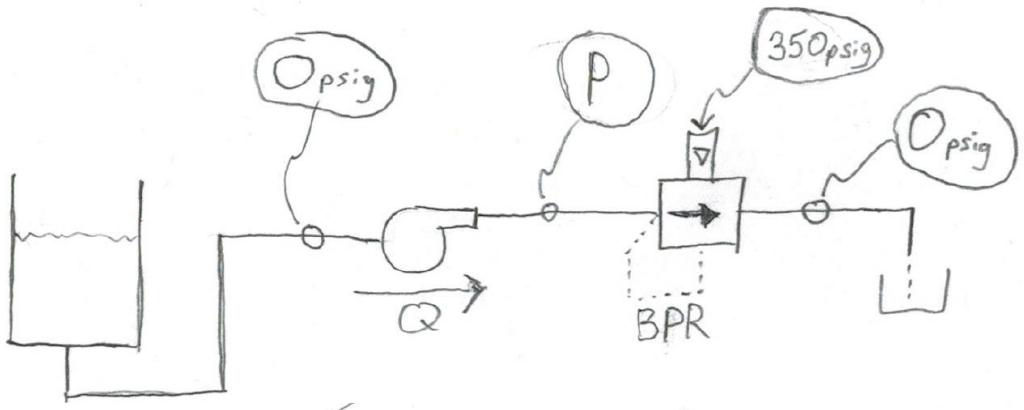
④ Right of System Curve

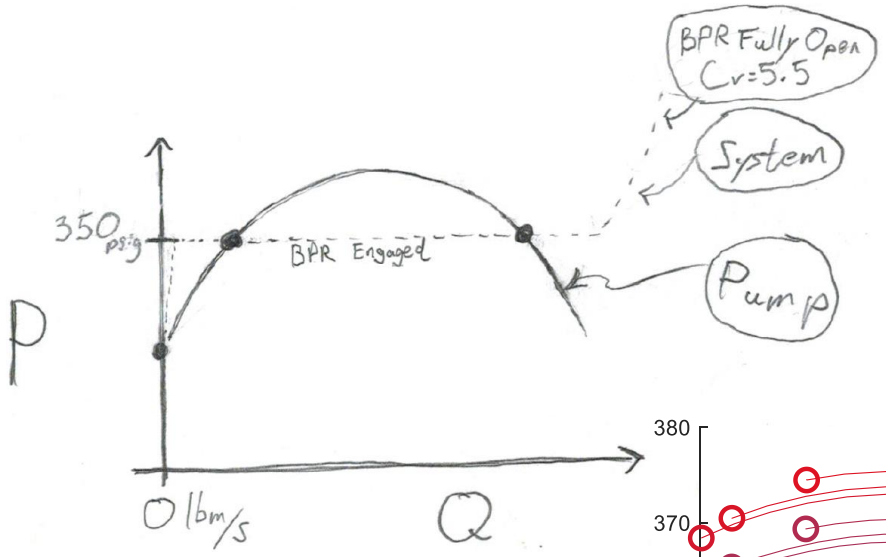
Test Hypothesis in Simulation

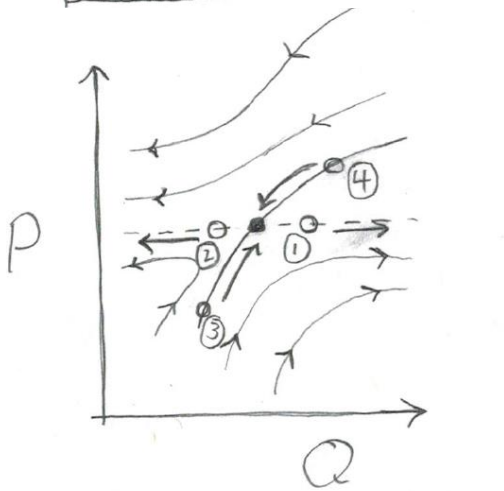


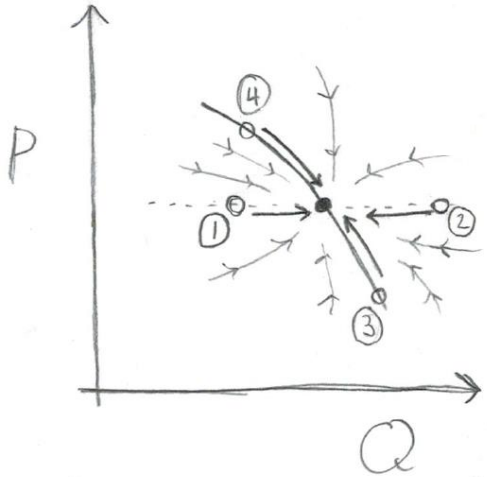
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Centrifugal Pump + BPR



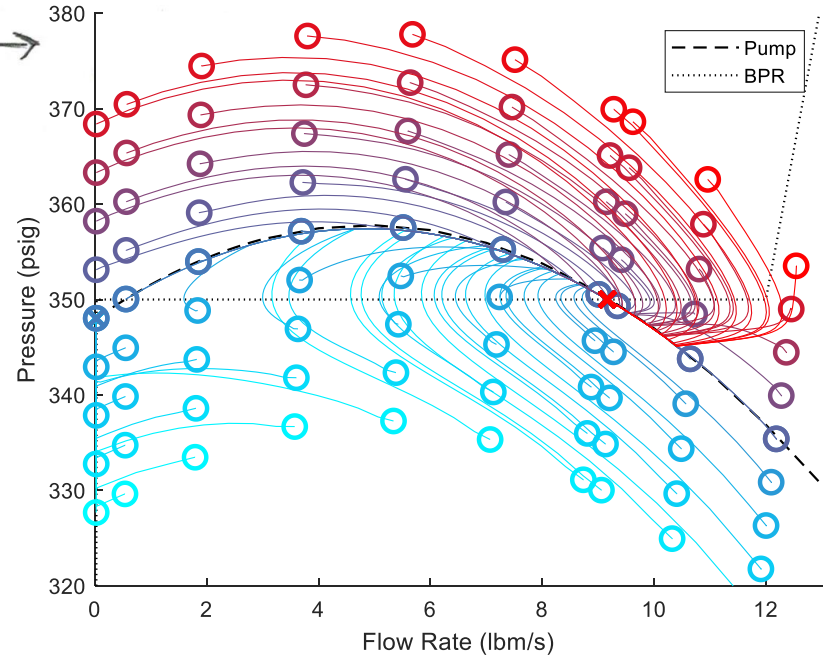






① Under Pump Curve
② Over Pump Curve
③ Under System Curve
④ Over System Curve

Test Hypothesis in Simulation



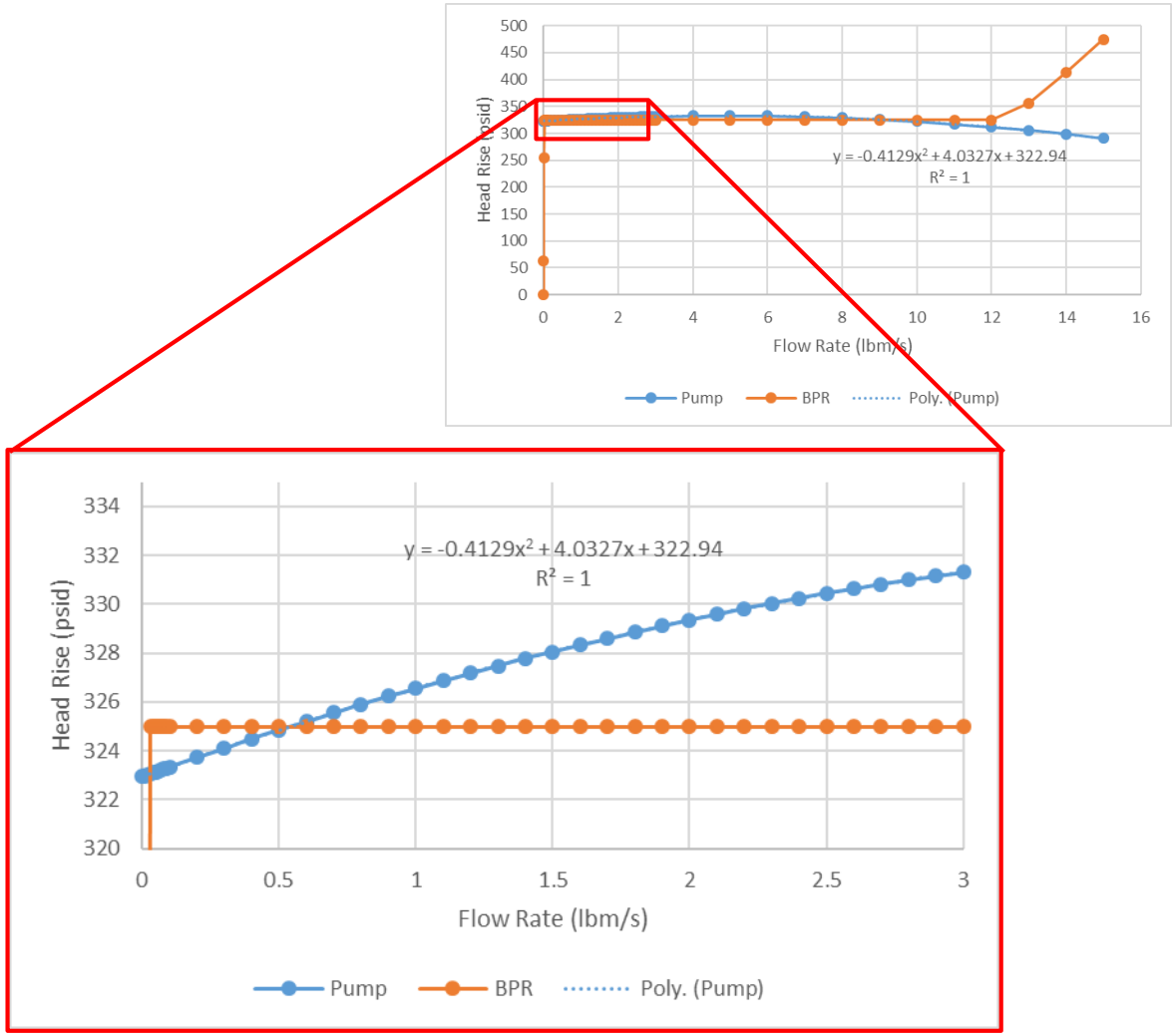
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Generalized Fluid System Simulation Program (GFSSP) Stability Simulation

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iRCS Component Inputs

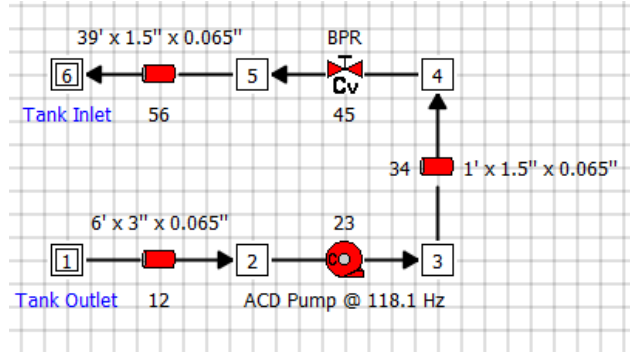
- Desired Operating Point
  - Tank Pressure = 25 psig
  - Manifold Pressure = 350 psig
  - Pump Flow Rate = 0.54 lb_m/s
- Equilibar BPR (GSDM6) [1]
  - CV = 0.01 – 5.5
- ACD Pump (AC/TC-30) [2]
  - Pump Speed = 118.1 Hz

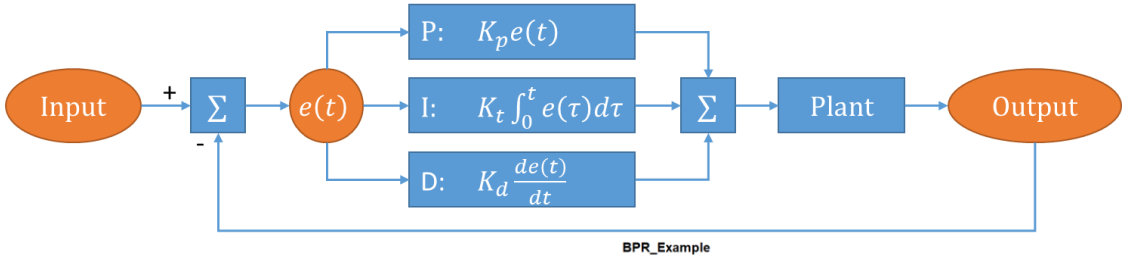


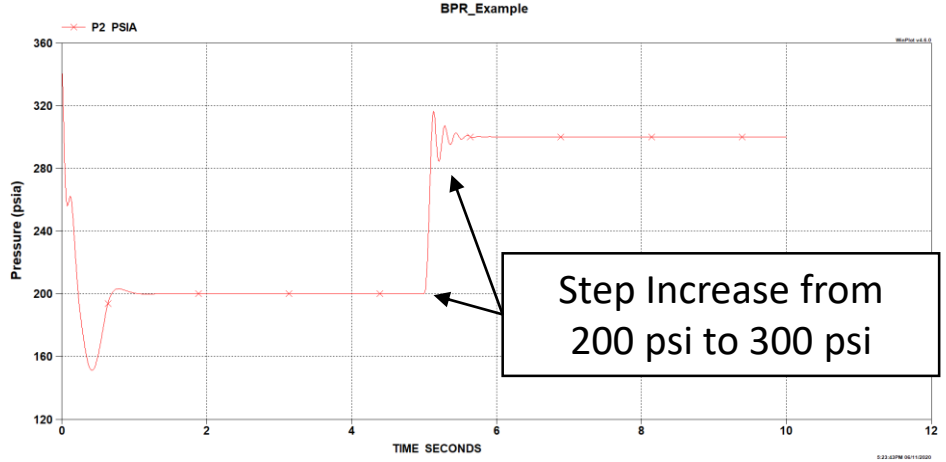
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GFSSP Model Structure

- Simplified Recirculation Loop
  - Tube Diameters = Minor Pressure Loss & Capacitance
  - Pump Characteristics = Quadratic Function
  - BPR = Cv Valve with PID Control
- User Subroutines
  - BPR Using PID Logic
    - Input = Target Pressure (psia)
    - Output = Measured Pressure (psia)
    - Control Variable = Valve Cv (-)
    - Plant/Process = GFSSP Simulation
  - Set “Initial, Perturbed” Pressure and Flow Rates
    - Momentum source in Branch 34, then ramp to 0 lbf
    - Override BPR Cv to fixed value, then turn on PID



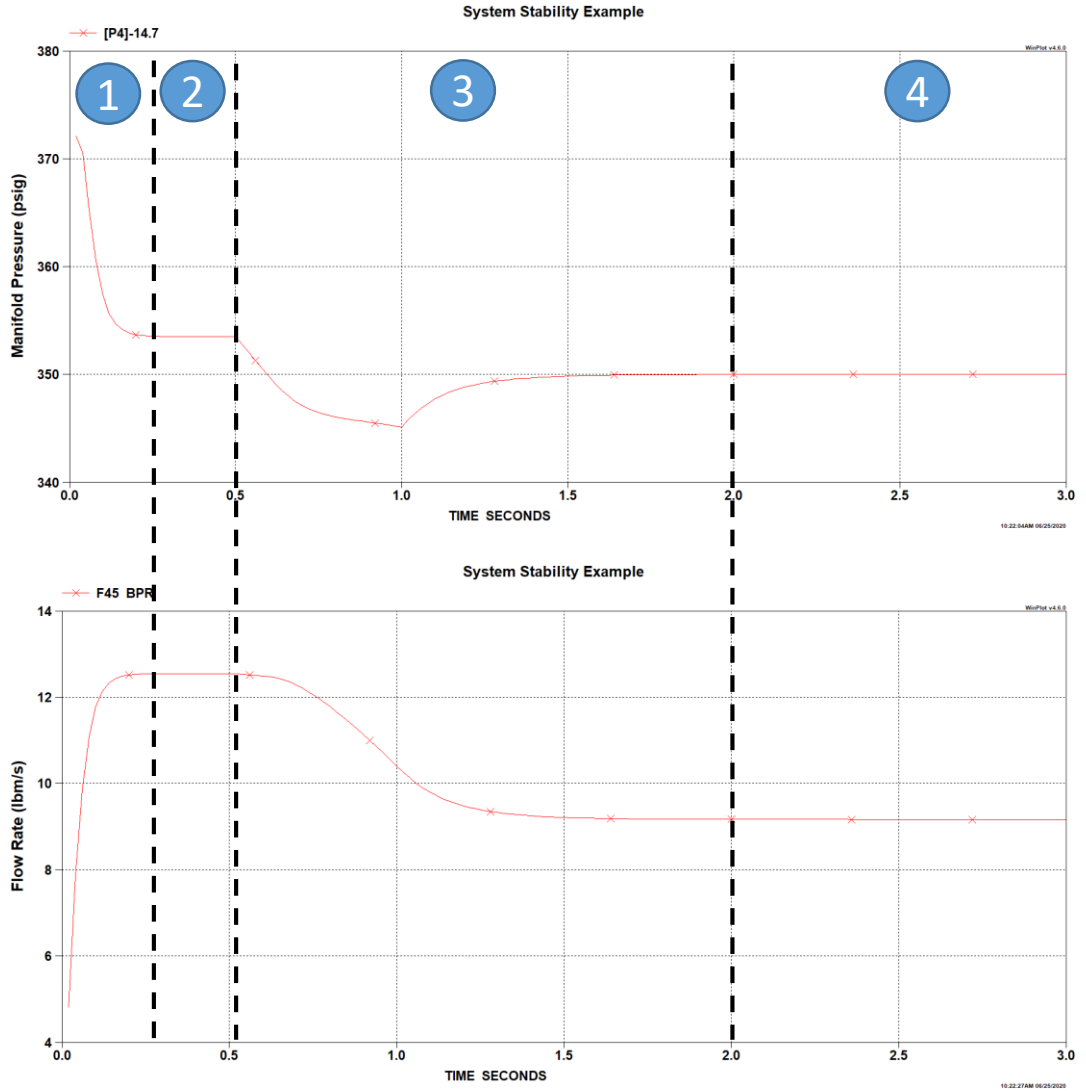




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Typical Simulation Results

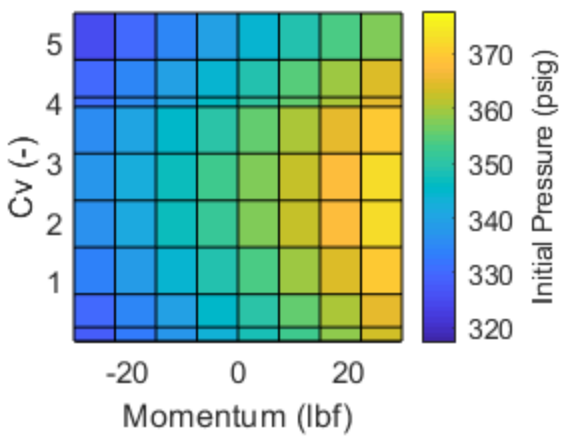
1. 0.0 - 0.25 seconds
   - GFSSP numerically resolving the “Guessed” flow conditions into an “Initial, Perturbed” flow state
   - Filtered out of final results
2. 0.25 – 0.5 seconds
   - “Initial, Perturbed” flow conditions
   - User defined Pressure and Flow Rate via Momentum Source and Initial BPR Cv
3. 0.5 – 2.0 seconds
   - Momentum Source ramped down to 0 lbf and BPR PID control turned “on”
4. 2.0 – 10.0 seconds
   - Model has reached a stable Steady State solution

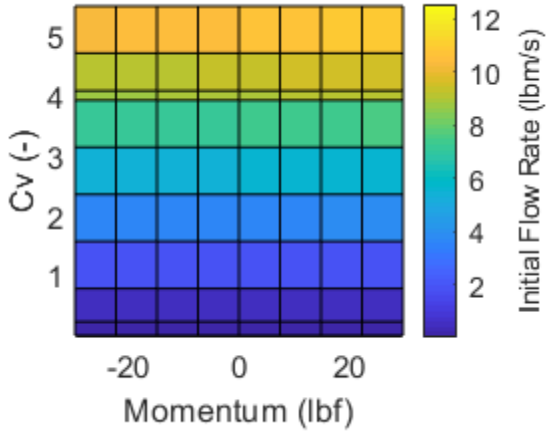


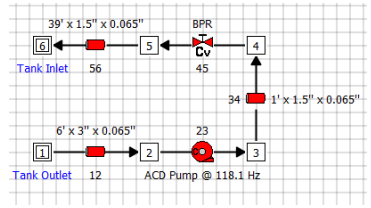
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Simulation Automation

1. Generate a “Typical Simulation” GFSSP Input File (*.dat)
2. Use MATLAB to modify Momentum Source and BPR Cv in the *.dat file
   - Momentum = -30 to +30 lbf
   - BPR Cv = 0.01 to 5.5
   - Added off-grid CV at 0.2393 and 4.0880 to test near Equilibrium Points
3. Use MATLAB to call a batch of 100 simulations (via CMD terminal) and compile results

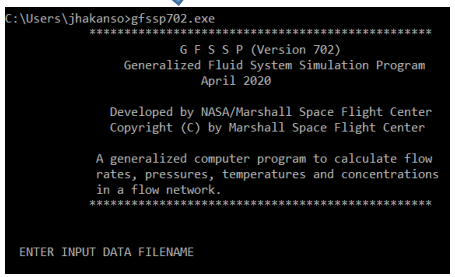






MIG





*.dat

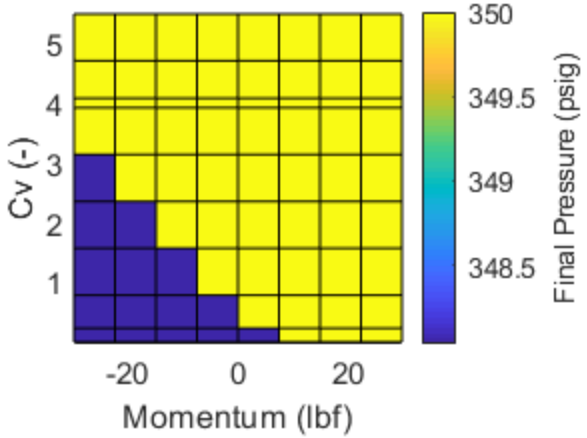
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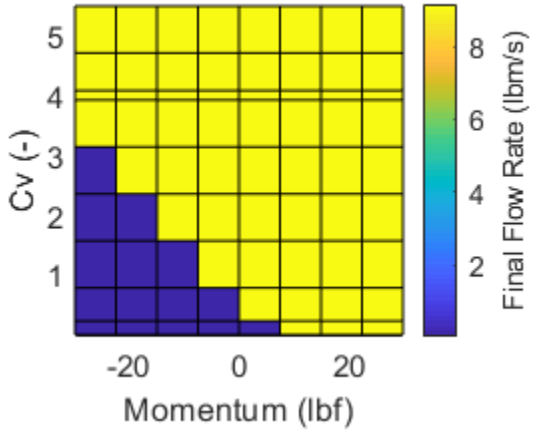
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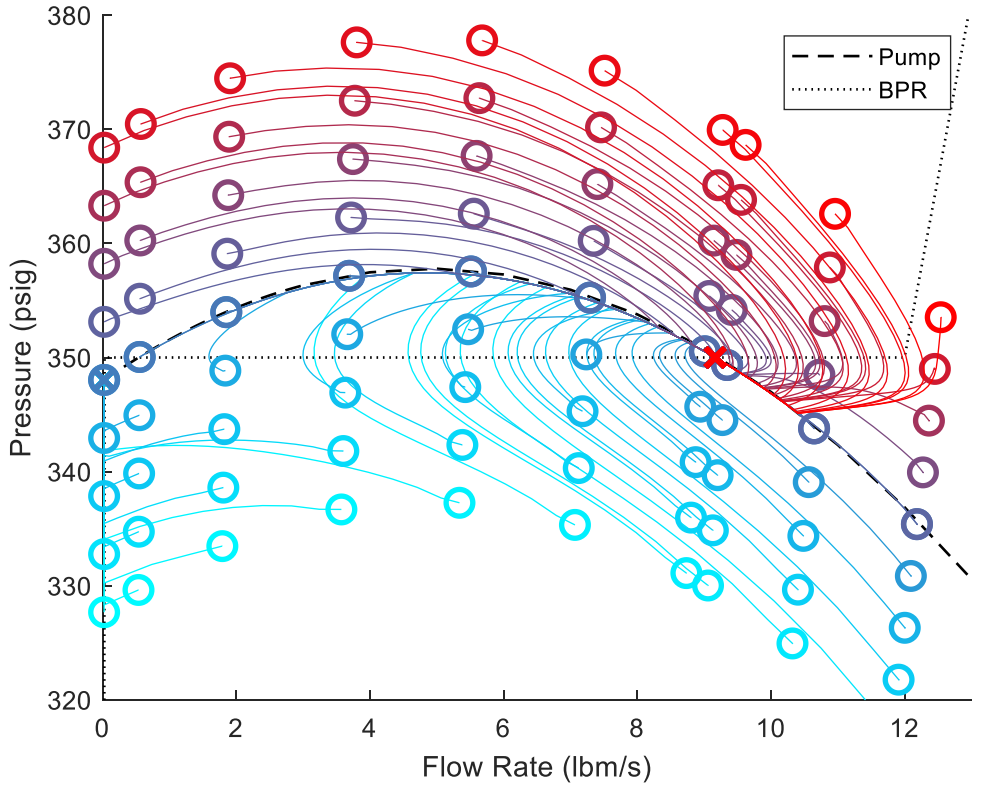
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Batch Results

- All simulations were attracted to the two stable fixed points at about 9 lbm/s and 0 lbm/s
- Even the point placed very close to the saddle point (0.54 lbm/s) dynamically diverged



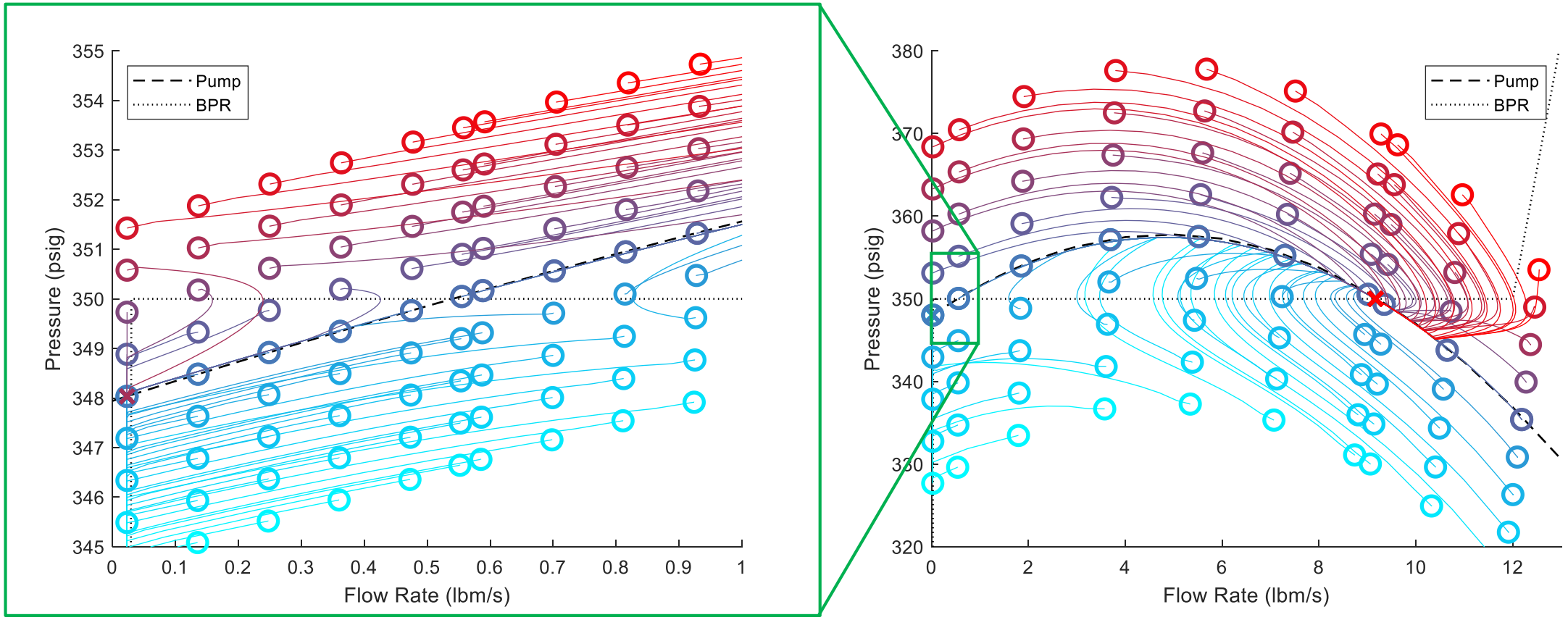




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Detailed View of Saddle Point

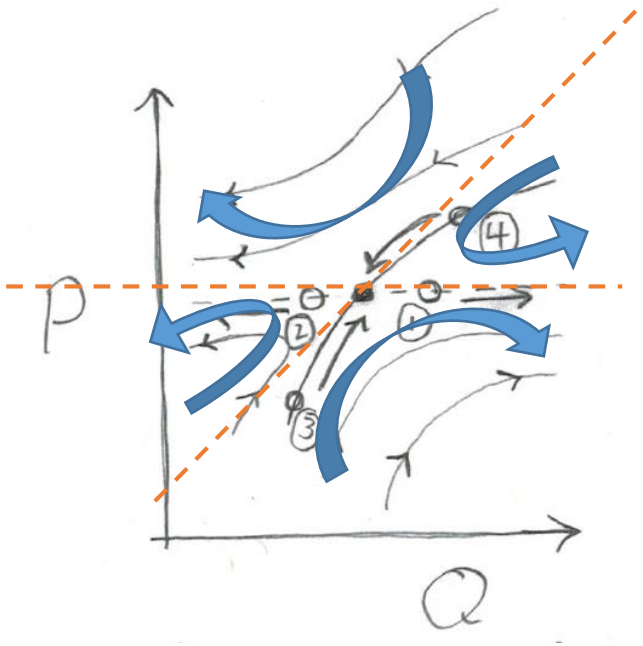
- Even though this system is nonlinear, linearization still applies to a localized region around each fixed point

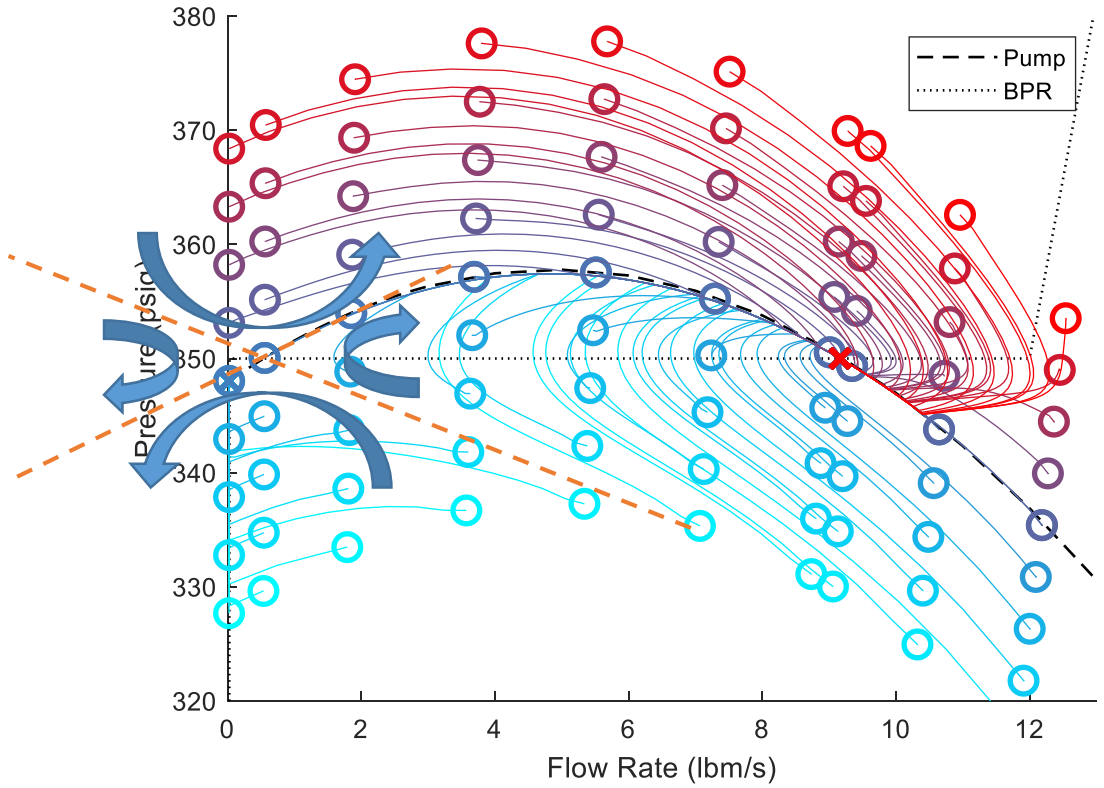


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Conclusion

- Qualitative analysis wasn't perfect, but gave quick insight into behavior around fixed points
- Solution: To achieve design points, either a different pump or a different pressure control device is needed





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References

[1] Equilibar Precision Pressure Control, “Equilibar GS Series General Service Back Pressure Regulators”. September 2019.

[2] ACD, “Model AC/TC-30 Close Coupled Pump”. February 2017.

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Backup

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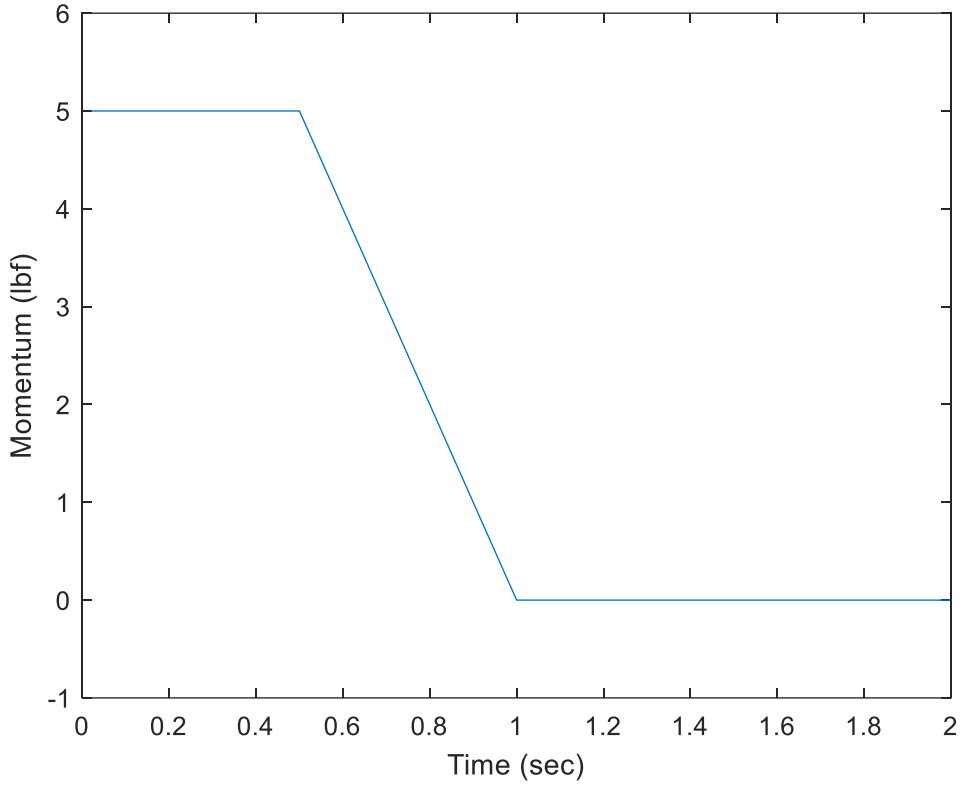
Abstract

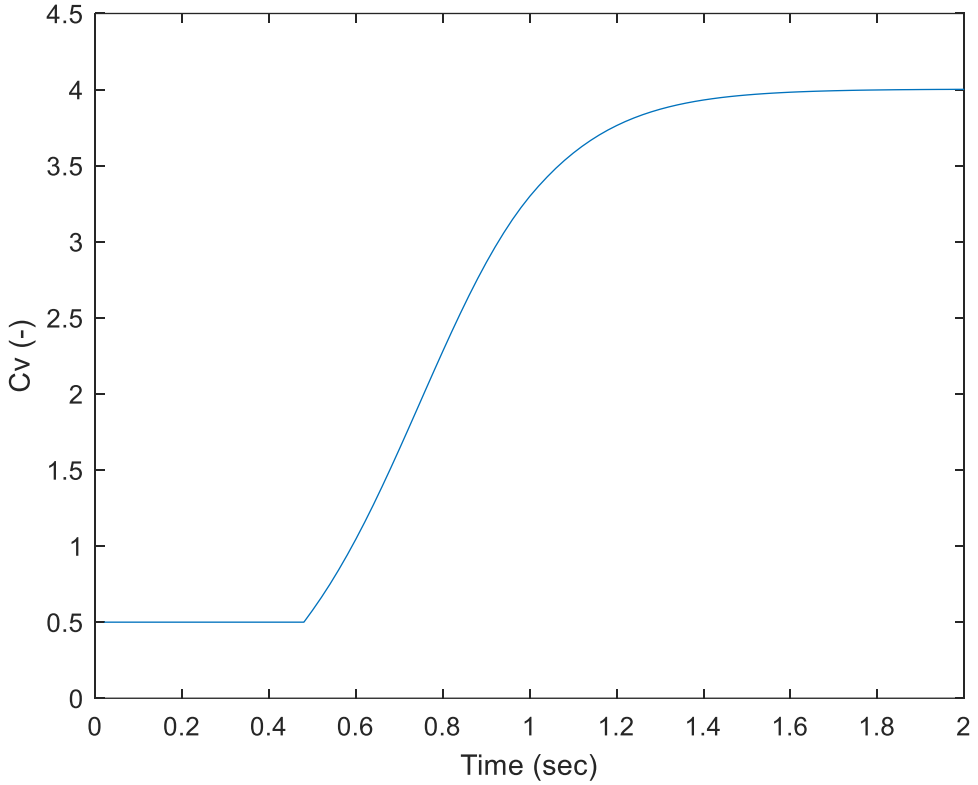
Fluid systems, or networks, consist of multiple components that work together to achieve some desired thermofluid state. For any generic application, this fluid state can be a combination of the fluid pressure, flow rate, enthalpy, or species concentration. Fluid system components, such as pumps and valves, are often governed by nonlinear differential equations, resulting in complex component-to-component interactions. System-level fluid network stability occurs when the flow through the system can maintain a steady-state solution in the presence of small perturbations, which depends on these component interactions. System instability, however, can go undetected until issues arise during integrated system testing. This presentation explores a method for system designers to think of the fluid network as an assembly of components, each with their own thermofluid surfaces of partial stability, called nullclines. The intersections of all nullclines yields system-level solutions, called equilibrium points. When designers define operating points, they are tuning system parameters so that these equilibrium points move to the desired location in the thermofluid state plane. However, linearization theory shows us that the dynamic behavior around these points can be unstable. The local stability of these equilibrium points can be assessed analytically with eigen-analysis, or numerically by propagating state-plane samples to construct a phase portrait. Investigating a phase portrait can help designers gain a qualitative understanding of a system's dynamic performance. This understanding can then help inform requirement definitions, component selection, and operational procedures. This presentation includes an example of the phase portrait technique on a system featuring a centrifugal pump and a back-pressure regulator (BPR). Numerical modeling of this system suggests that equilibrium points on the left-hand side of the pump curve are dynamically unstable.

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User-Defined Parameter Behavior (1/3)

- Momentum Ramp Time = 0.5 sec
- PID Coefficients: K_P = 1.0E-4 , K_I = 0.0 , K_D = 0.0

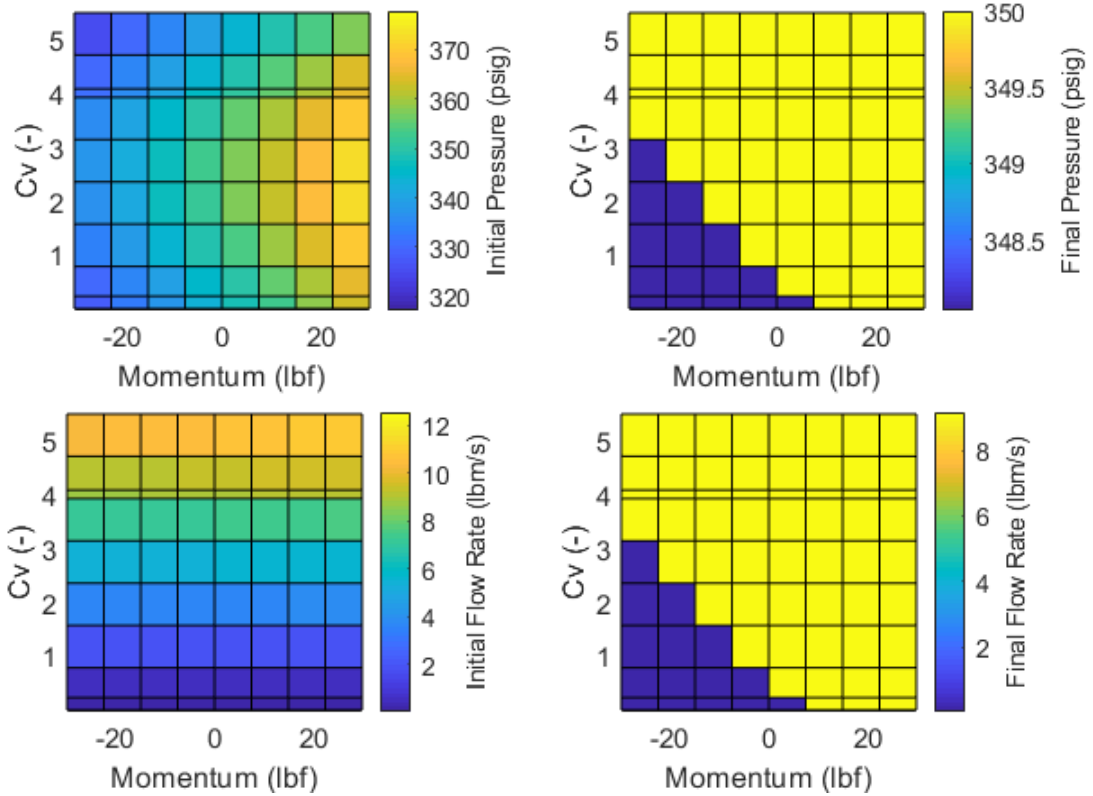


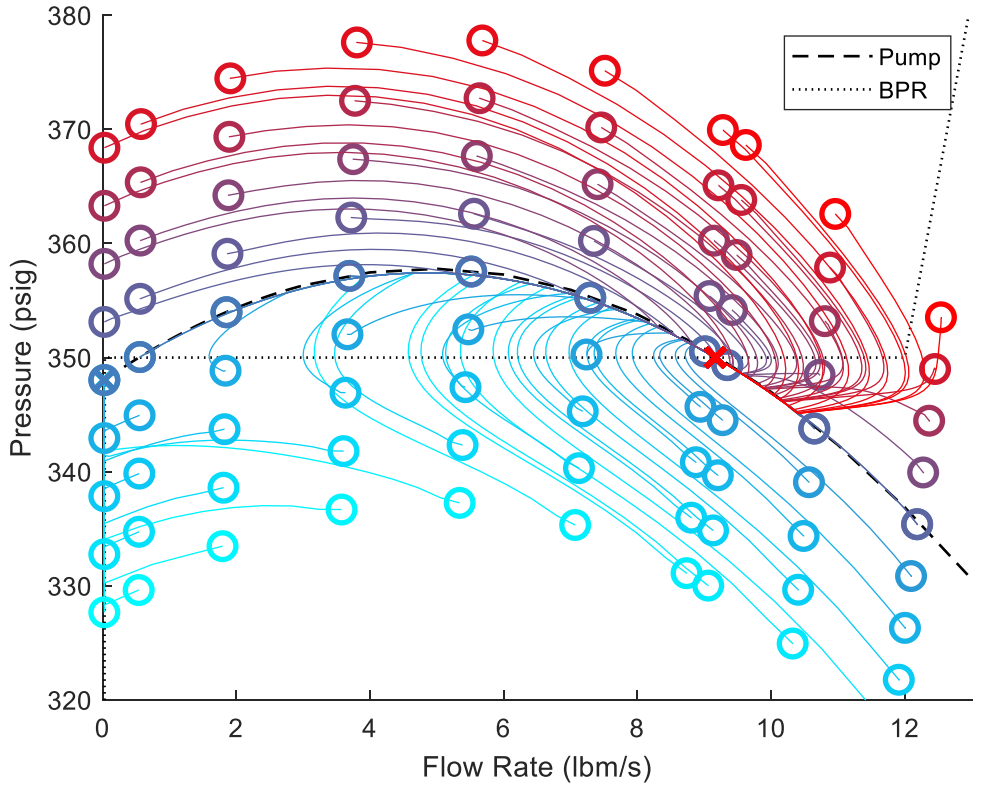


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Phase Portrait (1/3)

- Momentum Ramp Time = 0.5 sec
- PID Coefficients: K_P = 1.0E-4 , K_I = 0.0 , K_D = 0.0

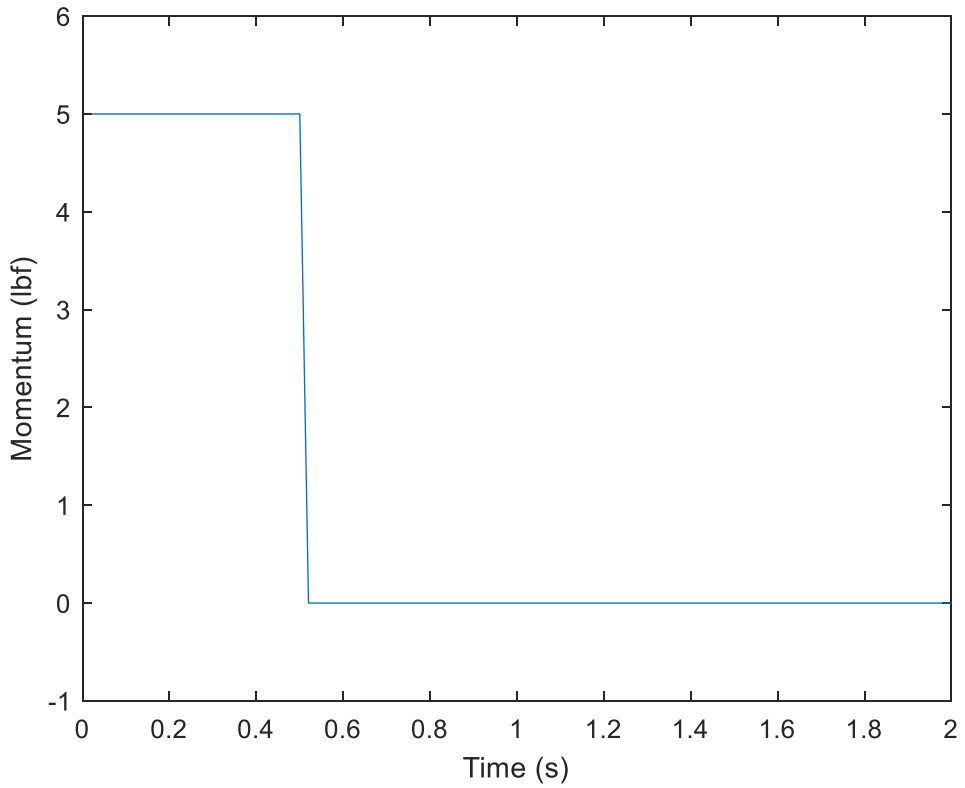


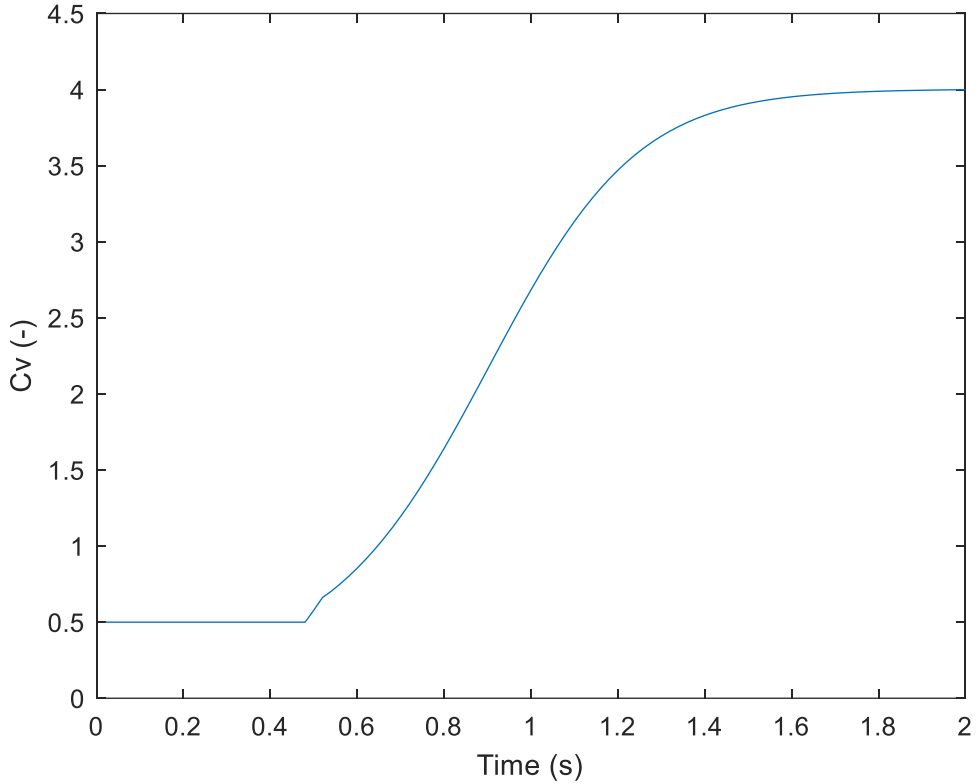


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User-Defined Parameter Behavior (2/3)

- Momentum Ramp Time = 0.0 sec
- PID Coefficients: K_P = 1.0E-4 , K_I = 0.0 , K_D = 0.0

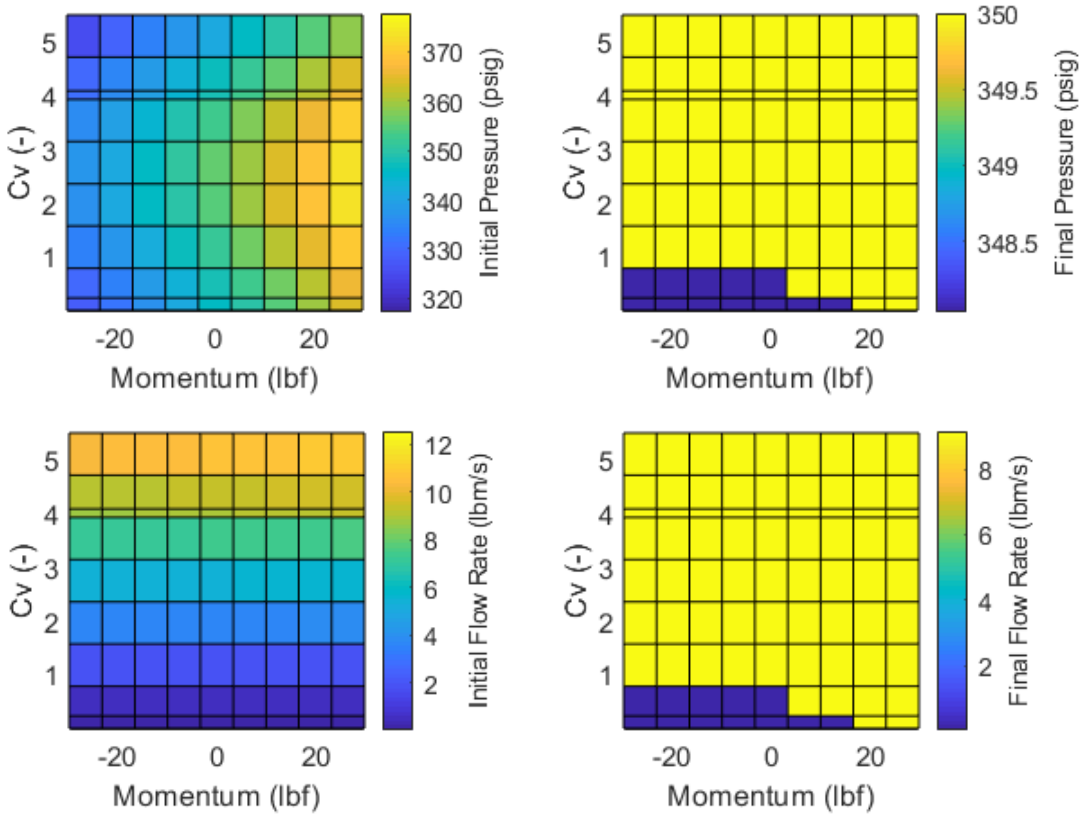


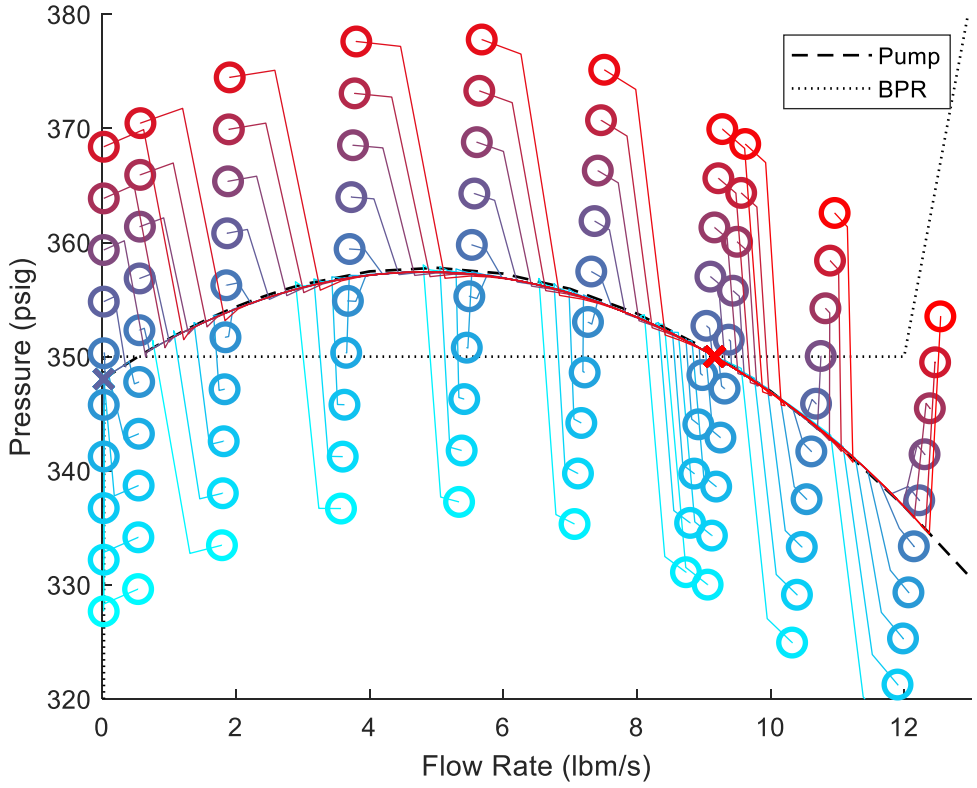


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Phase Portrait (2/3)

- Momentum Ramp Time = 0.0 sec
- PID Coefficients: K_P = 1.0E-4 , K_I = 0.0 , K_D = 0.0

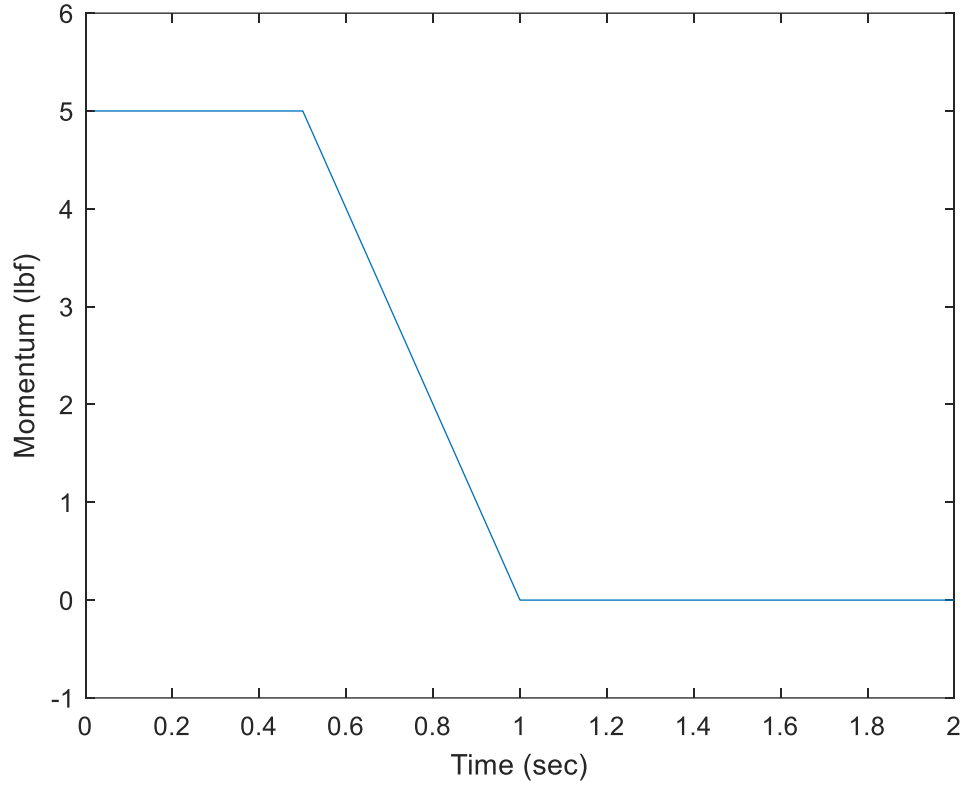


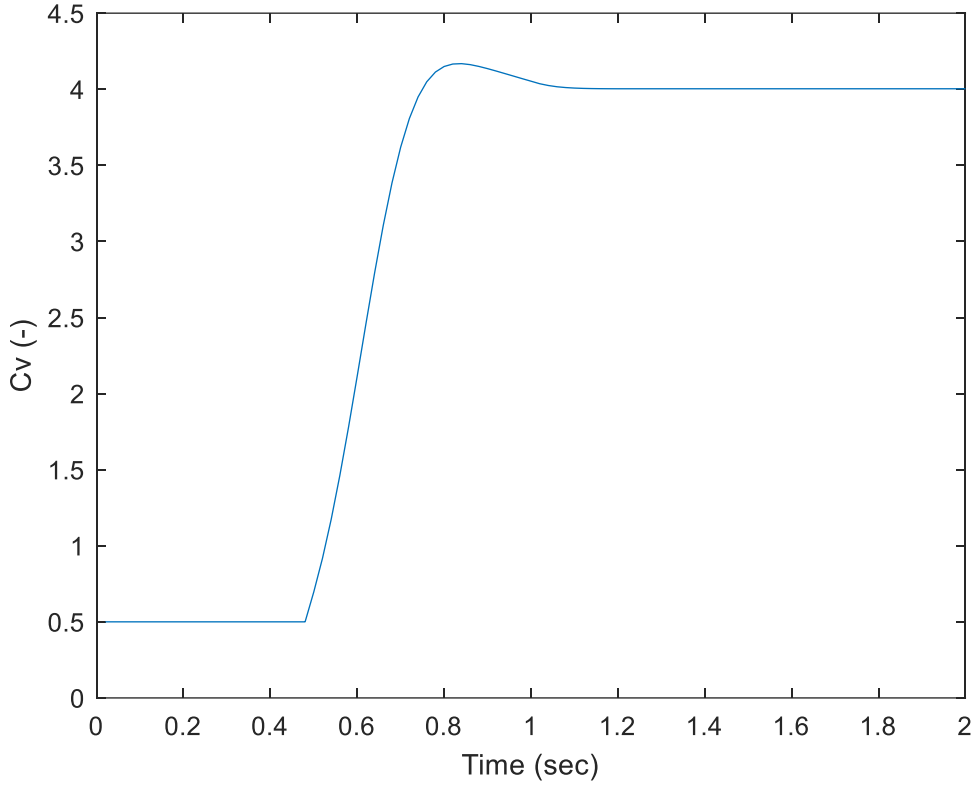


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User-Defined Parameter Behavior (3/3)

- Momentum Ramp Time = 0.5 sec
- PID Coefficients: K_P = 2.5E-4 , K_I = 0.0 , K_D = 0.0

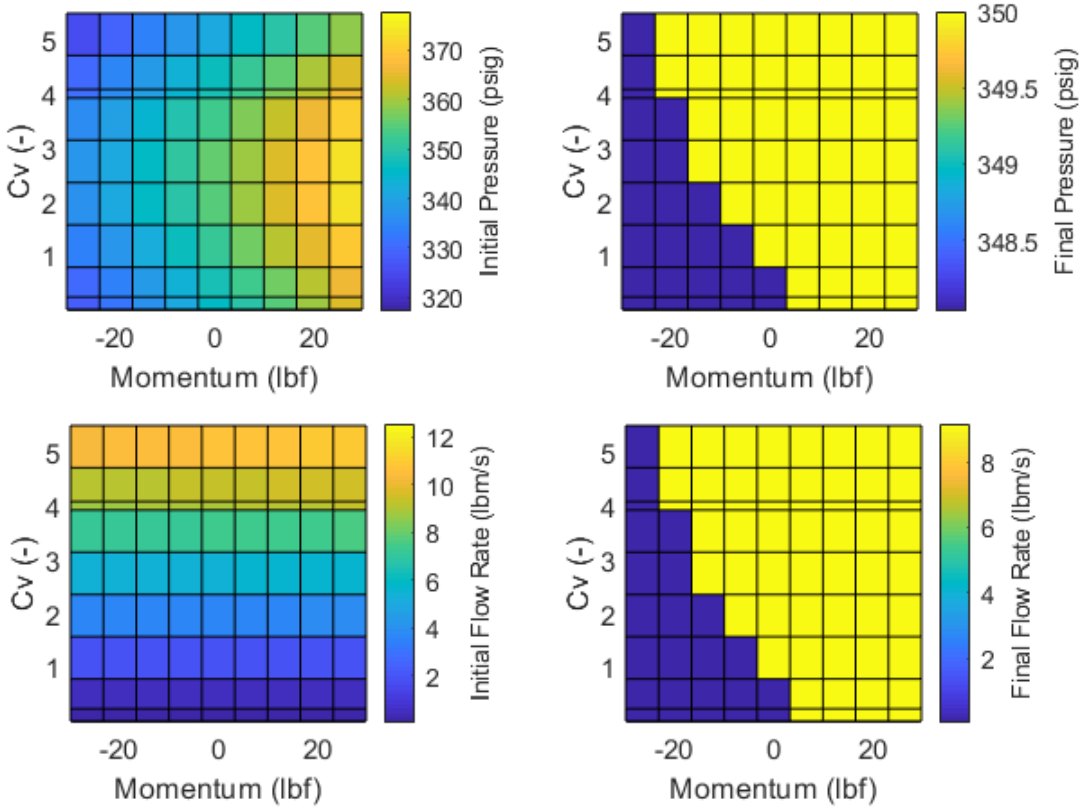


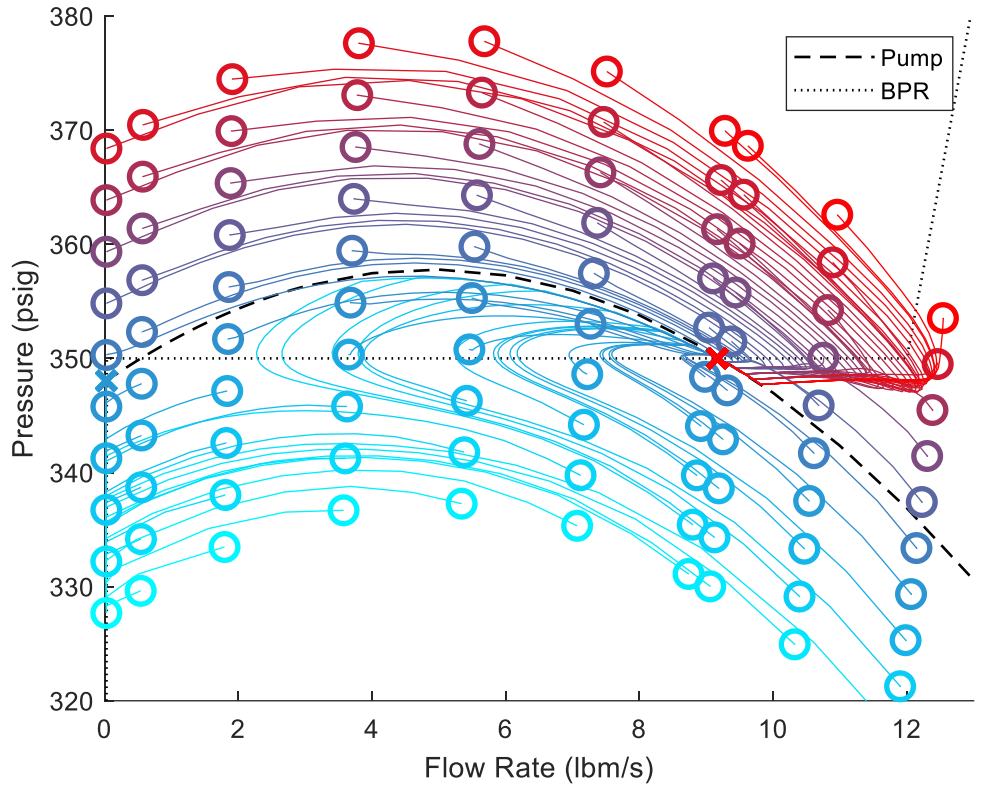


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Phase Portrait (3/3)

- Momentum Ramp Time = 0.5 sec
- PID Coefficients: K_P = 2.5E-4 , K_I = 0.0 , K_D = 0.0



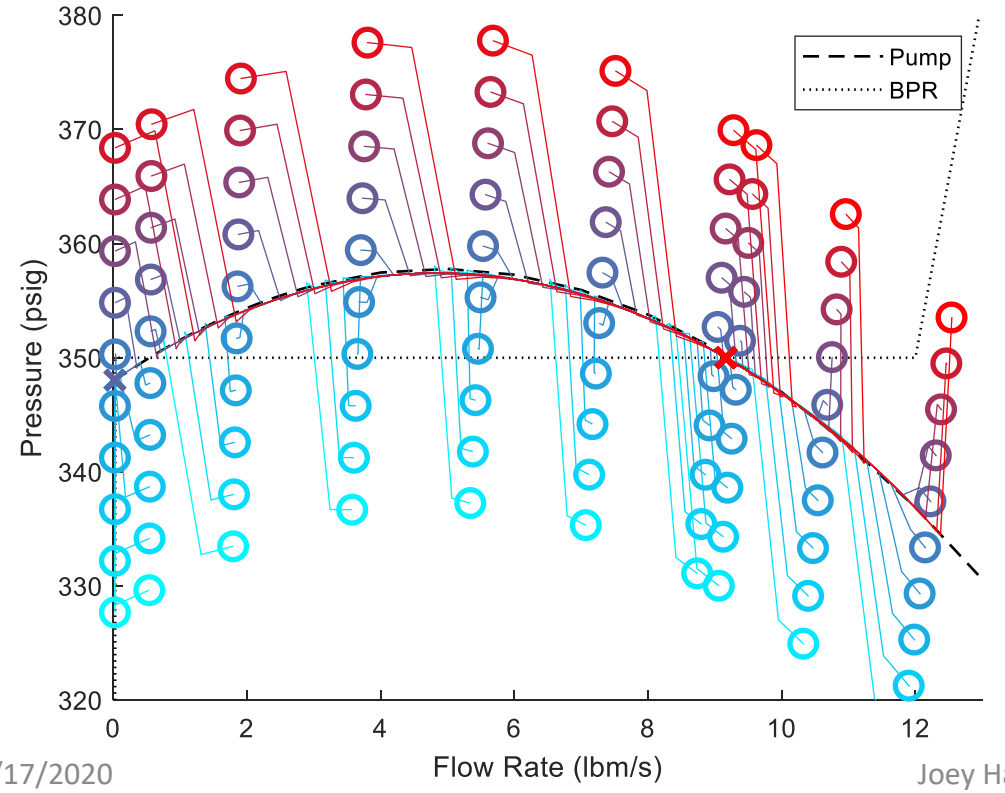


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Comparing the Extremes

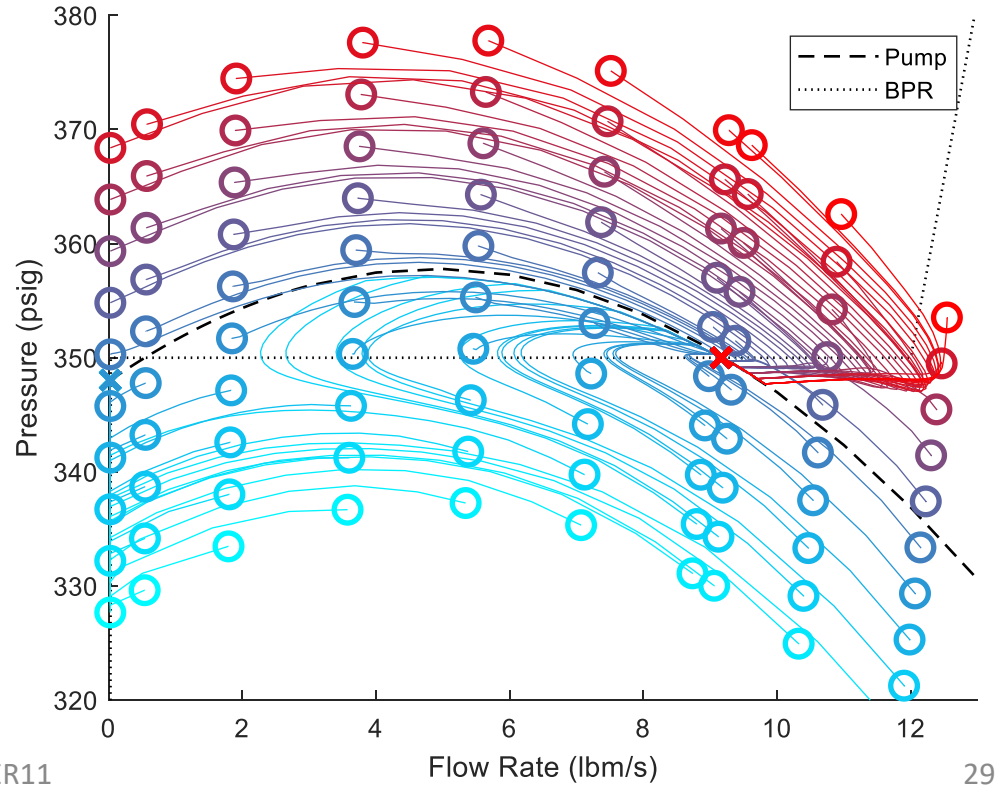
Pump is Boss:

- Fluid state is first forced to the Pump Curve
- Then BPR adjusts resistance to move state along pump curve towards a stable point



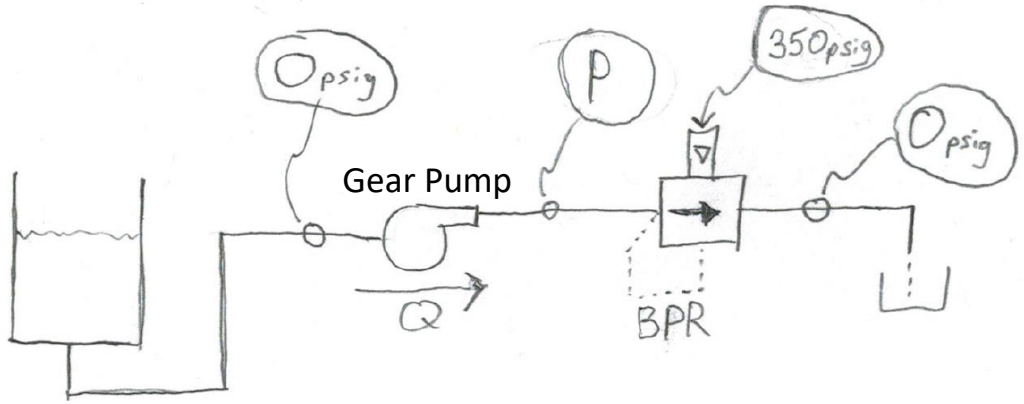
BPR is Boss:

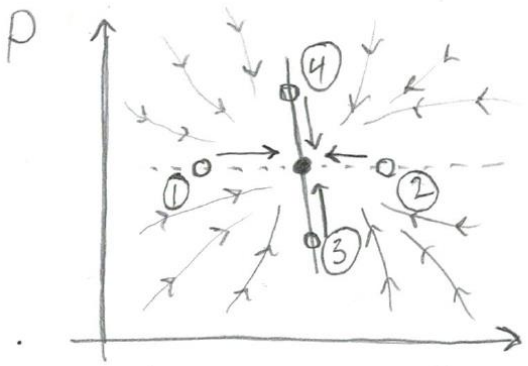
- Fluid momentum carries the fluid state along a parallel path to the pump curve as BPR adjusts resistance
- When fluid approaches set-pressure, pump takes over



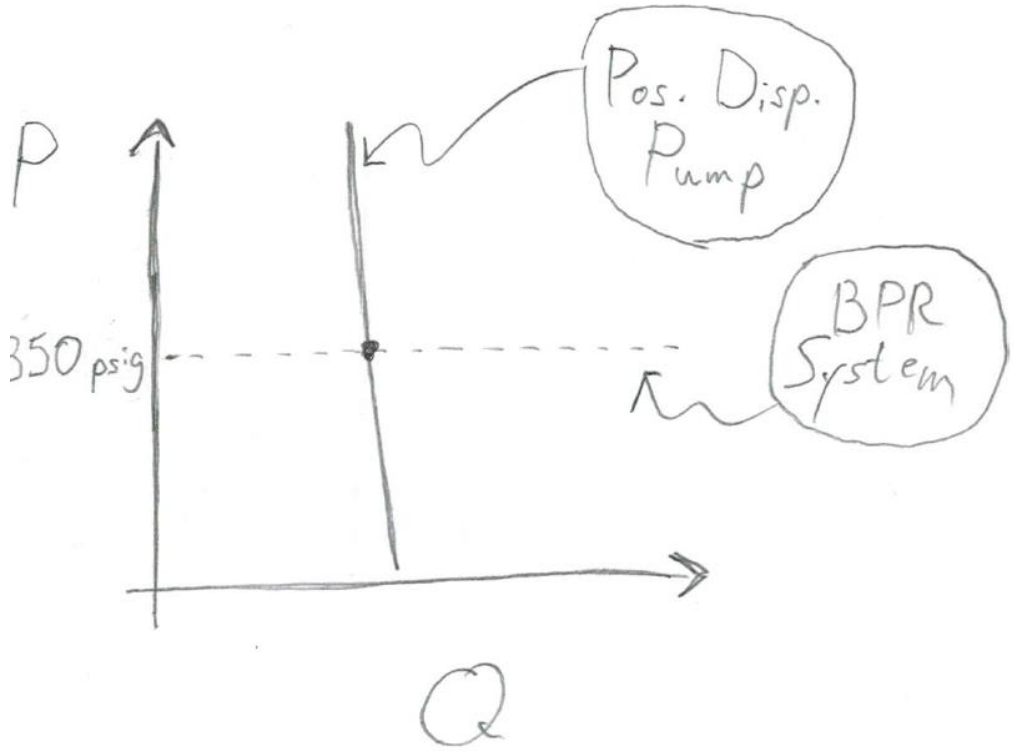
Distribution Statement A: Approved for public release; distribution is unlimited.

Positive Displacement Pump + BPR



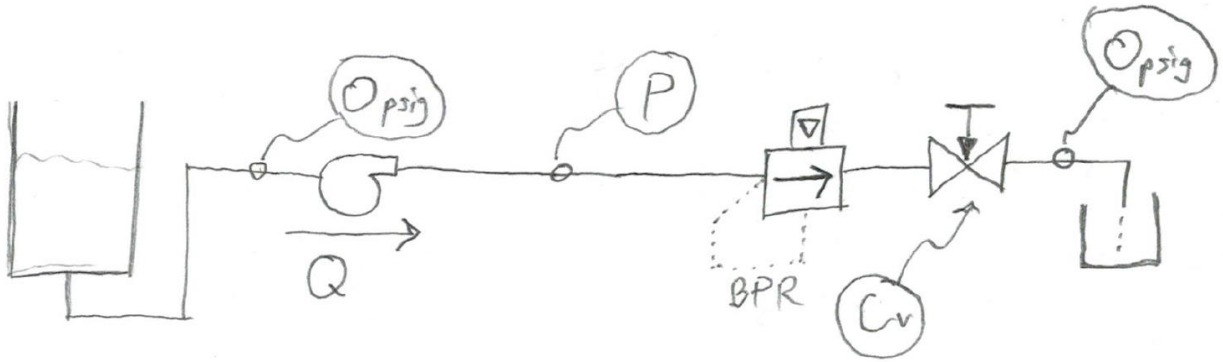


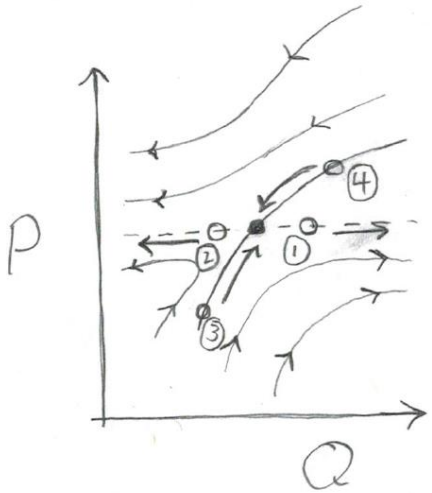
1. Left of Pump Curve
2. Right of Pump Curve
3. Under System Curve
4. Over System Curve



Distribution Statement A: Approved for public release; distribution is unlimited.

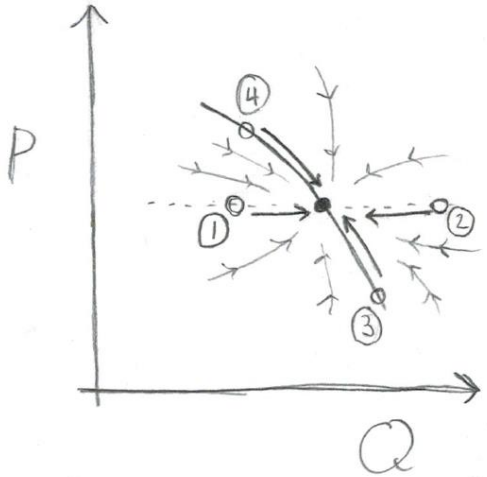
Centrifugal Pump + BPR + Needle Valve





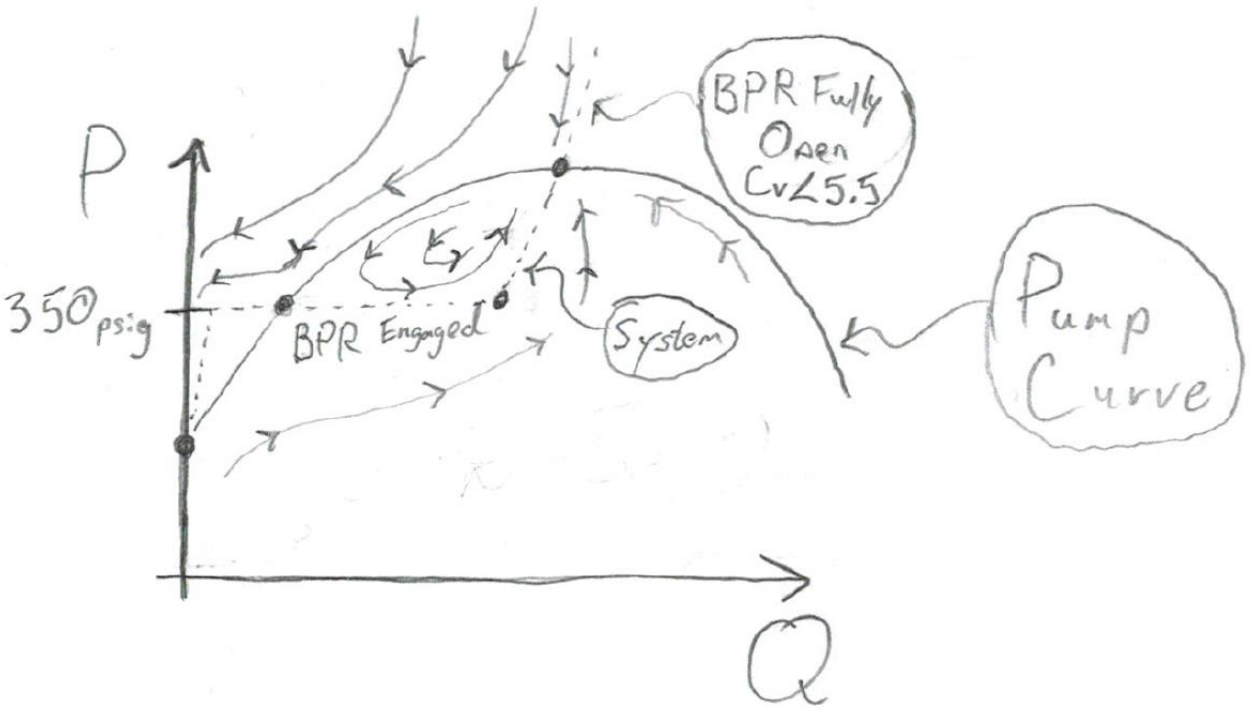
① Under Pump Curve

② Over Pump Curve



③ Under System Curve

④ Over System Curve



Distribution Statement A: Approved for public release; distribution is unlimited.