



Absolute Characterization of Gravity Sag

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Self-Weight Deflection

Self-weight deflection consists of two components: mount and face-sheet deflection.

Mount Deflection or (or Gravity Sag) is low-order bending of the mirror substrate as it is accelerated by gravity against its opto-mechanical support mount.

Face-sheet Deflection (or Quilting) occurs when mirrors are light-weighted – leaving parts of the face-sheet without support. These parts deflect slightly due to the acceleration of gravity against the vertical core ribs – depending upon their unsupported span size – and deflect more due to fabrication tool forces.

For this discussion, we will ignore face-sheet deflection because it can be mitigated:

- Design the mirror with appropriately face-sheet and core pocket dimensions
- Pressurize mirror core cells to balance gravity during fabrication
- Correct via small mechanical/ion polishing process.



G-Release Error

G-release error is defined as the knowledge uncertainty of a mirror's gravity-sag or the residual error between the mirror's predicted and actual on-orbit shape.

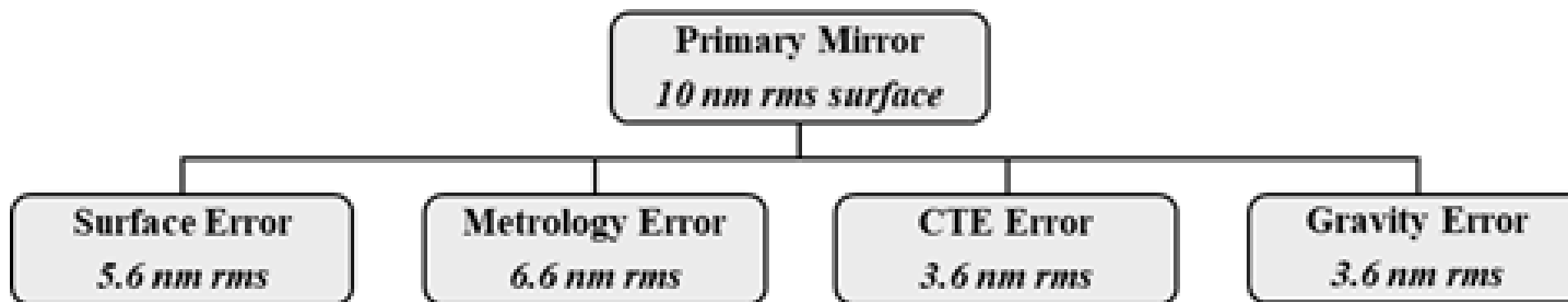
- G-release occurs because space mirrors are manufactured in a 1-G environment.
- In 1-G, mirrors have self-weight deflection.
- But in 0-G, there is no gravity and thus no self-weight deflection.
- The change in the mirror's shape from 1-G to 0-G is called G-release.
- G-release is most important for the primary because it is the largest, lowest stiffness mirror in the telescope.



G-Release Error

To achieve UV/Optical diffraction limited performance, the telescope's transmitted wavefront error needs to be better than 30 nm rms and the telescope's primary mirror surface figure error needs to be better than 10 nm rms.

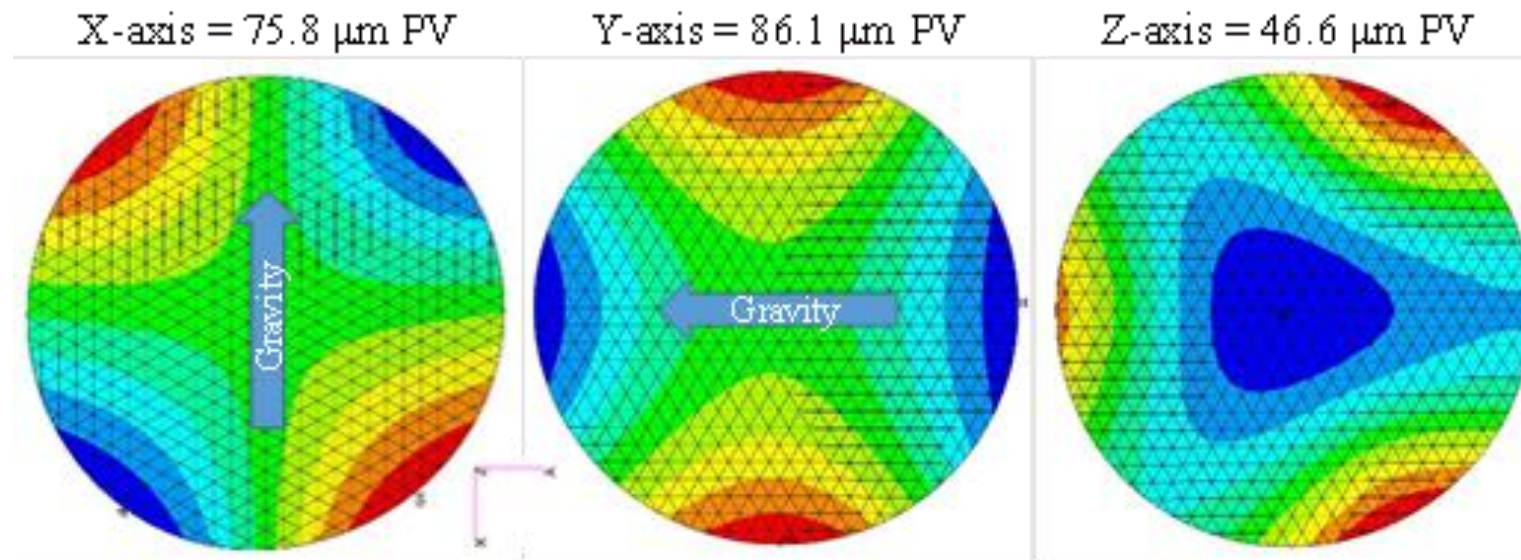
To achieve this specification, G-release needs to be less than 4 nm rms.





G-Release Error

The problem is that large mirror's have large gravity sags. For example, the 4-meter HabEx baseline primary mirror predicted gravity sag is:



The challenge is – **how to mitigate 100 micrometers PV of gravity sag with a precision of better than 4 nm rms.**



Gravity Sag State of Practice

Gravity-sag has been studied extensively since the 1960s.

There are four possible mitigation approaches:

- Make the mirror as stiff as possible
- Support the mirror during fabrication in a 0-G configuration
- Analytically remove G-sag from the mirror during the fabrication process
- Actively correct the mirror in space



Make the Mirror as Stiff as Possible

Self-weight deflection is proportional to geometry, mass density and mount interface

$$1G \text{ Gravity Sag} \sim C_{SP} \left(\frac{D^4}{t^3} \right) \rho_{AD} \sim 1/(2\pi f)^2$$

Where: C_{SP} = mount support constant, D = diameter, T = thickness, ρ_{AD} = Areal Density

Support Constant	C_{SP}	Factor of Reduced Deflection Compared to 3-Pt Support
Ring at 68% of Diameter	0.028	11
6 Points Equal Spaced at 68.1% of Diameter	0.041	8
Edge Clamped	0.187	1.5
3 Points, Equal Spaced at 64.5% of Diameter	0.316	-
3 Points, Equal Spaced at 66.7% of Diameter	0.323	~1
3 Points, Equal Spaced at 70.7% of Diameter	0.359	0.9
Edge Simply Supported	0.828	1/3
Continuous Support along the Diameter	0.943	1/3
“Central Support” (Mushroom or Stalk Mount; r = radius of stalk)	1.206	1/4
3 Points Equal Spaced at Edge	1.356	1/4

Yoder, Paul and Danial Vukobratovich, Opto-Mechanical Systems Design, Fourth Edition, Two Volume Set, CRC, 2015



Make the Mirror as Stiff as Possible

In practice, it is not possible to make a mirror sufficiently stiff that it will have a gravity sag of less than 4 nm rms.

Therefore, the only practical options are:

- Support the mirror during fabrication in a 0-G configuration
- Analytically remove G-sag from the mirror during the fabrication process
- Actively correct the mirror in space

However, it is still advisable to make the mirror as stiff as possible, because it makes the remaining three mitigation approaches easier to implement.



Gravity Off-Loading

Gravity off-loading is typically done via:

- Multipoint Mount
- Air Bag Support

For mid-quality mirrors, air bags are adequate.

But for high-precision mirrors, multi-point mounts are preferred.

The reason is that discrete mount points are more deterministic than a bag surface.

- Air bag exerts a force that is perpendicular to the mirror's back – which is not necessarily anti-parallel to gravity; and
- Stiction between bag and mirror can introduce shear forces.

Thus, air bags can introduce rotationally symmetric errors, i.e. power or spherical.



Gravity Off-Loading

Multi-point metrology mount technology was developed with NASA funding in the 1970s for the Large Space Telescope Program (Hubble).

Hubble primary mirror's mounted 7.6 micrometer PV self-weight deflection was characterized to an accuracy of 1.4 nm rms using a 135 point metrology mount.

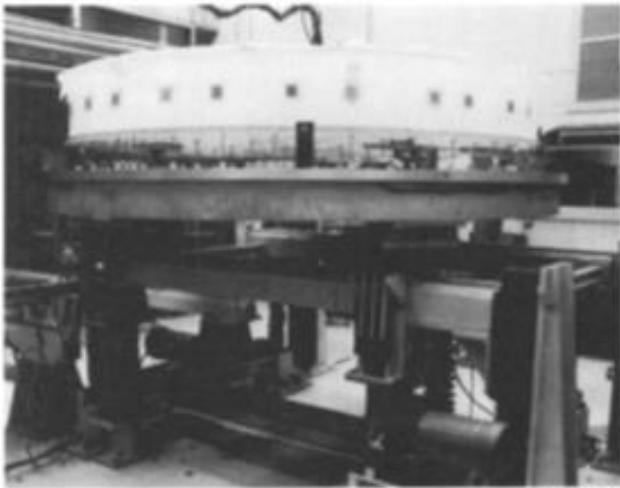


Figure 3. Primary mirror metrology mount.

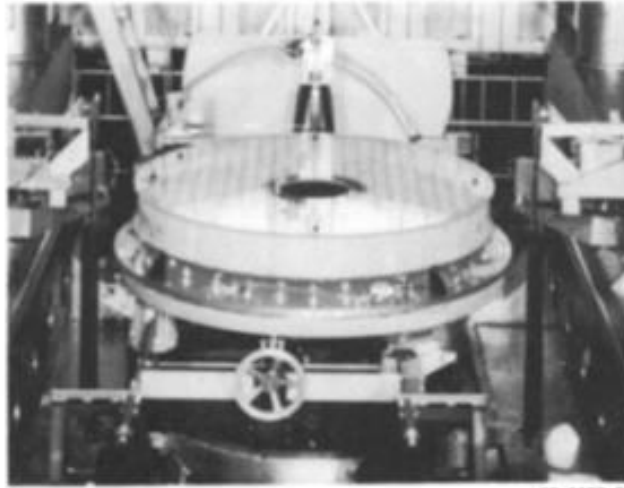
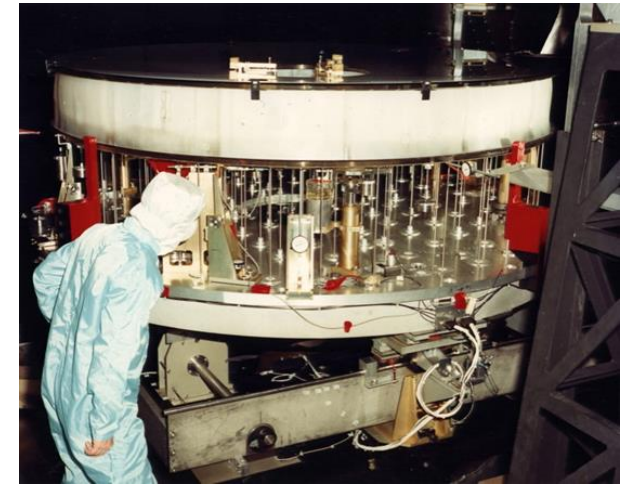


Figure 4. Primary mirror/mount assembly on six-degree-of-freedom table.



Montagnino, Lucian A., "Test And Evaluation Of The Hubble Space Telescope 2.4-meter Primary Mirror," Proc. SPIE 0571, Large Optics Technology, (21 February 1986)

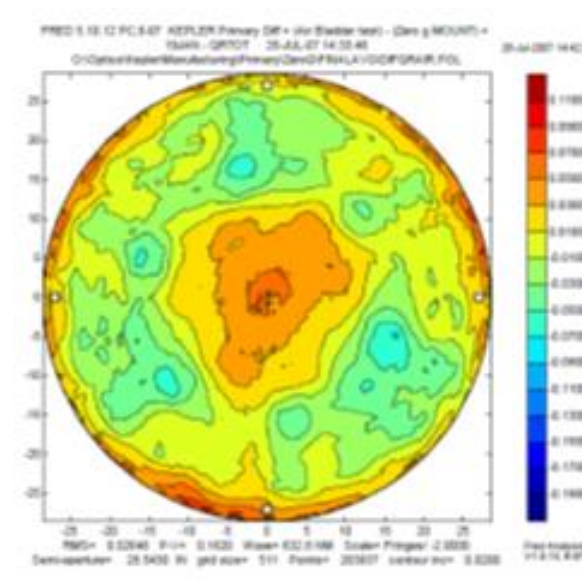
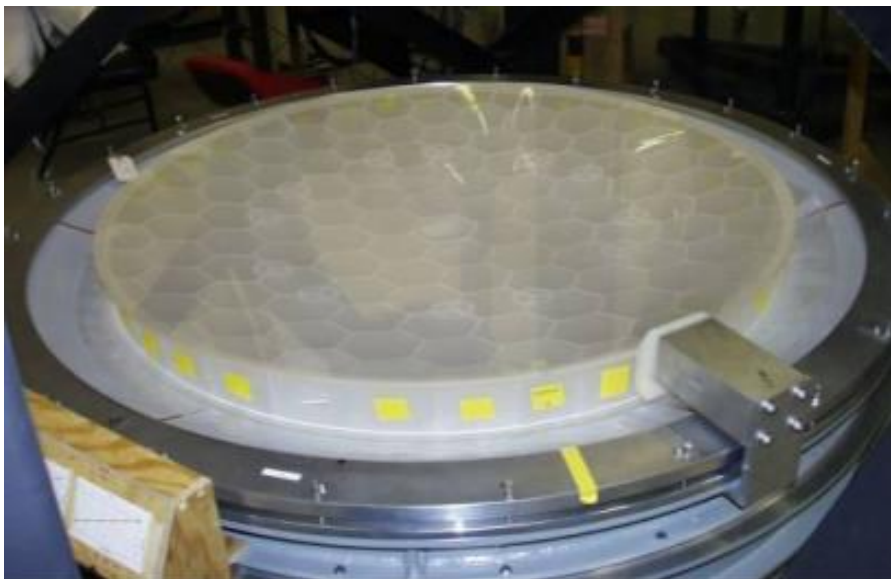
Yoder, Paul and Danial Vukobratovich, Opto-Mechanical Systems Design, Fourth Edition, Two Volume Set, CRC, 2015 ISBN-10: 1439839778

<https://www.hexagonkh9.com/blog/2019/1/19/hexagon-looked-at-the-earth-the-hubble-looked-at-the-stars>



Kepler Primary Mirror

Kepler 1.4-m primary mirror was tested using both an air bag and a 108 point mount. The air bag was estimated to off-load gravity sag with an uncertainty of 5.6 nm rms. Difference between air bag test and multi-point mount test was 16.4 nm rms (mostly spherical aberration).



Zinn, John W., George W. Jones, "Kepler primary mirror assembly: FEA surface figure analyses and comparison to metrology," Proc. SPIE 6671, Optical Manufacturing and Testing VII, 667105 (14 September 2007)



Analytical Removal

Gravity Sag can be estimated via an orientation test:

- Face-up/Face-down Test
- Horizontal Rotation Test

Face-Up / Face-Down Test

Mirror is tested on its flight mount.

G-Sag should be identical in the two orientations except for its polarity.

$$S(x, y)_{face-up} = S(x, y)_{G(0,0,0)} - S(x, y)_{G(0,0,-1)}$$

$$S(x, y)_{face-down} = S(x, y)_{G(0,0,0)} + S(x, y)_{G(0,0,1)}$$

$$0G \text{ Shape} = S(x, y)_{G(0,0,0)} = \{S(x, y)_{face-up} + S(x, y)_{face-down}\}/2$$

$$1G \text{ Sag} = S(x, y)_{G(0,0,-1)} = \{S(x, y)_{face-up} - S(x, y)_{face-down}\}/2$$

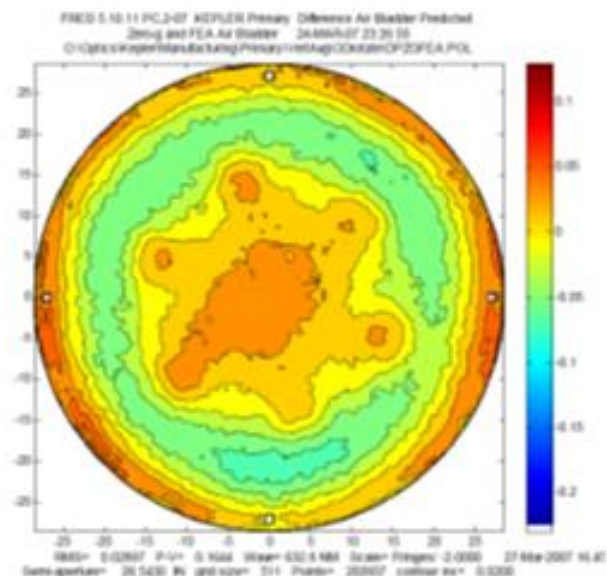
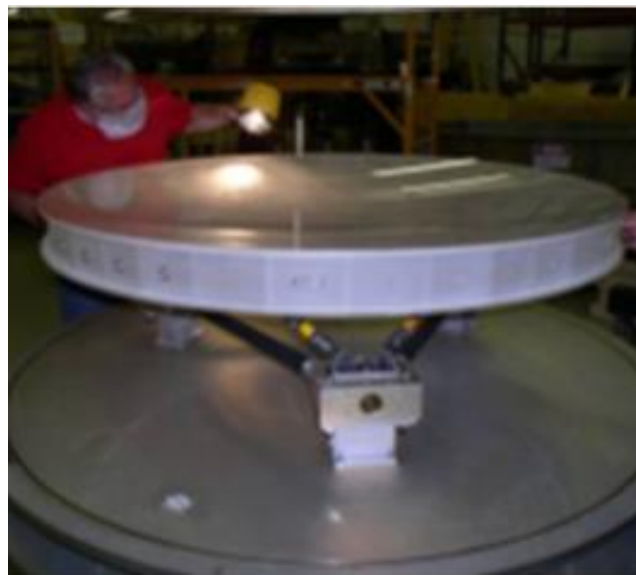


Kepler Primary Mirror

Kepler primary mirror was assembled with a low thermal expansion flight mount and characterized in a face-up/face-down gravity orientation test.

Unpredicted difference between the air bag test and the up/down test was 18.4 nm rms.

Of this, 16.5 nm rms was symmetric (as expected for air bag test) and 6 nm rms was a 6-point bond pad pattern.



Zinn, John W., George W. Jones, "Kepler primary mirror assembly: FEA surface figure analyses and comparison to metrology," Proc. SPIE 6671, Optical Manufacturing and Testing VII, 667105 (14 September 2007)



Absolute Metrology

Because interferometry is a relative measurement, the ability to manufacture precision surfaces is limited by knowledge of the reference wavefront.

Absolute tests were needed to 'calibrated' the reference wavefront – allowing its error to be removed from the surface measurement.

Absolute calibration tests falls into four basic categories: Rotation Tests, Shear Test, Rotation-Shear Test, and Composite Averages.

An excellent summary of absolute calibration techniques is given by

- Schultz, G., and J. Schwider, “Interferometric Testing of Smooth Surfaces”, Progress in Optics XIII, pp.119-26, Wolf, North-Holland Publishing Co., 1976
- Bruning, J. H., “Fringe Scanning Interferometers”, Optical Shop Testing, Malacara, Wiley and Son's, 1978.



Rotation Test Removes Gravity Sag

Because horizontal gravity sag is asymmetric, rotation tests can absolutely quantify gravity sag – thus allowing its removal from a surface measurement.

$$S(x, y, \theta_i)_{Horizontal} = \left[S(x, y)_{G(0,0,0)_{symmetric}} + S(x, y, \theta_i)_{G(0,0,0)_{asymmetric}} \right] + S(x, y, \theta = 0)_{G(0,-1,0)}$$

Rotating a mirror in angles around its surface normal that are symmetric with its flight mount, only rotates the mirror's asymmetric component. The gravity sag does not rotate, nor does its rotationally symmetric component.

Averaging these N-rotation measurements results in a measurement consisting of only the mirror's symmetric surface shape and its gravity-sag.

Back rotating each data set to $\theta = 0$ before averaging, removes the gravity sag term, leaving only the mirror's symmetric and asymmetric terms.

$$\begin{aligned} \frac{1}{N} \sum_{i=0}^{N-1} S(x, y, \{\theta_i = 0\})_{Horizontal} \\ = \left[S(x, y)_{G(0,0,0)_{symmetric}} + S(x, y)_{G(0,0,0)_{asymmetric}} \right] \end{aligned}$$



Three Flat Test

The rotation test and its extension the rotation-shear test are based upon the work of Schulz and Schwider (1967).

By inter-comparing three surfaces (A-B, A-C, and B¹⁸⁰-C), it is possible to calculate an exact (no interpolation) absolute calibration of a single diameter.

This diameter is the line of symmetry about which surface B is inverted when it is moved from the bottom to the top, and the corresponding A, C surface diameters.

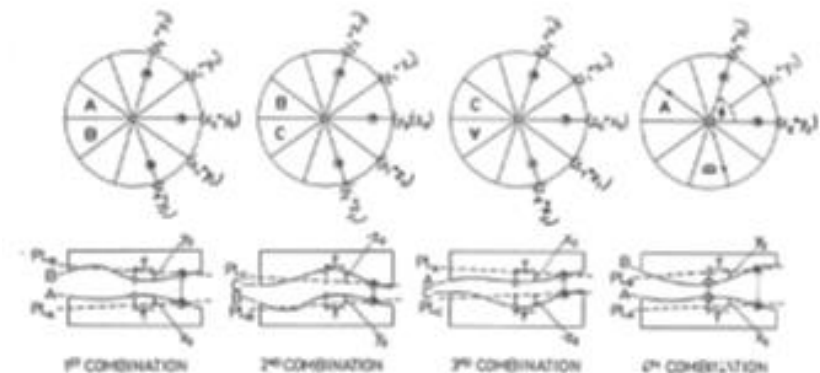


Fig. 3.2. Four combinations of positions of the surfaces A, B and C of Fig. 3.1. They enable one to test along N diameters (central sections). In the 4th combination the surface B is rotated through the angle $\Phi = 2\pi \times M/N$, compared to the 1st combination (M and N are natural numbers which are prime to each other, the example shows $M = 1$, $N = 5$).

Multiple diameters for each surface can be obtained by taking a fourth measurement

The number of diameters (N) desired determines the rotation angle

$$\theta = 2\pi \left[\frac{M}{N} \right] \quad \text{where } M \text{ and } N \text{ are Prime to each other}$$



Three Sphere Test

A similar technique for spherical surfaces, first suggested by Jensen (1973) was demonstrated by Bruning (1974).

As with Shultz and Schwider's three surface test, the Jensen test is blind to harmonics of its rotation angle.

A 180-degree rotation test only removes errors such as coma.

To remove astigmatism requires four rotations of 90-degrees.

To trefoil requires six 60-degree rotations.

An important feature is the cat's eye retro-reflection. This measurement allows the method to eliminate test setup symmetric and asymmetric errors from the part under test. Without this measurement, a rotation test can only calibrate the rotationally asymmetric errors.

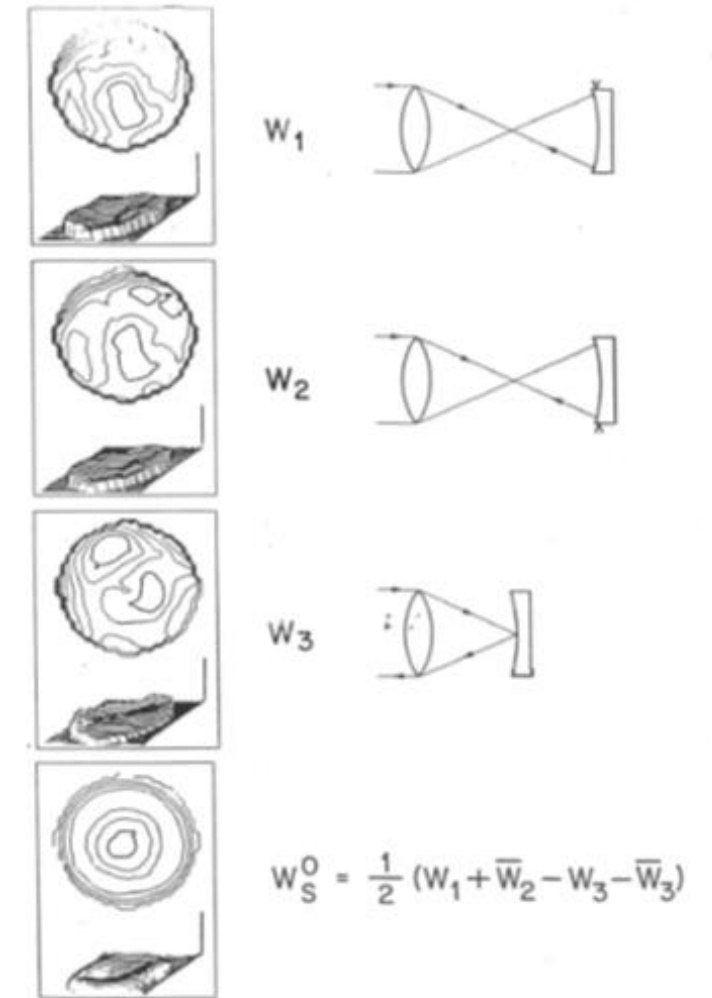


Figure 13.6. Interferometer calibration wavefronts. Wavefront contours are $\lambda/20$. (From Bruning et al. 1974)



N-Rotation Test

Building on work by Bruning and Parks (1978)

In 1996, Evans and Kestner published the 'N-rotation' algorithm where the interferometer measures a surface at N-rotation angles, i.e. $N=3$ (0° , 120° , 240°) or $N=6$ (0° , 60° , 120° , 180° , 240° , 300°).

Averaging removes all test errors with angular dependence up to $(N-1)$ Theta; and, all higher angular order errors that are not harmonics of N.

So, an $N=3$ rotation test will remove 2-Theta errors (i.e. coma, secondary coma, tertiary coma, etc.; and, astigmatism, secondary astigmatism, tertiary astigmatism, etc.).

An $N=6$ rotation test is required to remove error terms up to 5-Theta (i.e. trefoil, tetrafoil, and pentafoil).



HDOS Algorithm

In the mid-1990s, Hughes Danbury Optical Systems (HDOS) developed a proprietary three flat test to absolutely characterize the LIGO Pathfinder optic. \

A fundamental constraint of rotation tests is that their algorithms require the input data in polar coordinates; while interferometers use cartesian coordinates (because their detectors are square arrays). Thus, data interpolation is required.

The HDOS approach sought to overcome this limitation by implementing a method similar to Primak (1988) and process the data in Fourier space using circular harmonic transformation.

Finally, as with Evans and Kestner, HDOS performed N rotations. But, to overcome the (N-1) harmonic problem, extra rotations are added which are prime to each other.

Using the Shultz and Schwider equation: $\theta = 2\pi \left[\frac{M}{N} \right]$ where M and N are Prime to each other

If one wants 1009 diameters, then two good rotation angles would be 71.00 degrees (N=199) and 100.97 (N=283).



Rotation Tests

Rotation Tests are proven.

A 6-rotation tests was used to quantify the 0-G figure for each of the 18 JWST primary mirror segment assemblies (PSMAs) to an uncertainty of less than 10 nm rms.

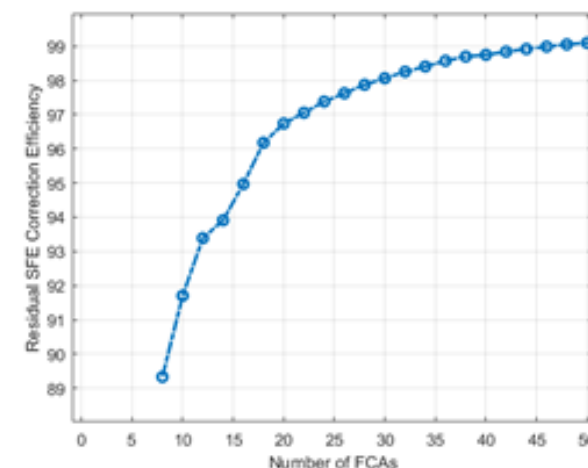
The HDOS rotation test was used to test two 200 mm diameter LIGO Pathfinder Flat with a full-aperture accuracy of better than 0.6 nm rms.



Correction

While it is desirable to manufacture the primary mirror with as small of a G-release uncertainty as possible (TRL-9 capability is < 10 nm rms), an active primary mirror can easily correct 50 to 200 nm rms of error.

Analysis of an alternative ULE® HabEx primary mirror indicates that 15 actuators can reduce low-order (i.e. astigmatic) G-release error by 20X, 25 actuators can reduce G-release error by 40X, and 50 actuators can reduce G-release error by 100X



It is interesting to note that, while they were never used, the Hubble PM (Primary Mirror) had 24 actuators to mitigate the risk of astigmatic G-release error.

And, JWST PM has 18 actuators to mitigate the risk of segment radius matching error.



Conclusions

G-release is a critical error budget element for space telescopes.

For a UV/Optical telescope, G-release needs to be less than 4 nm rms.

Because it is not possible to design a large mirror with a sufficient stiffness to insure a small enough G-release for a UVO diffraction limited telescope, it is necessary to fabricate a 0-G mirror, i.e. a mirror that will have its required shape in space.

This can be done by either supporting the mirror in a 0-G configuration or calculating and compensating for its G-sag during fabrication.

If necessary for further mitigation, one can design the mirror to be correctable to achieve its required shape in space.

Absolute Metrology techniques developed for high-accuracy optical surface testing should be able to characterize gravity-sag of space optics and remove it during fabrication to the required level of precision.