Uncertainty Analysis of the Pitch Damping Coefficient of Blunt Bodies, Measured from Magnetic Suspension Wind Tunnel Tests

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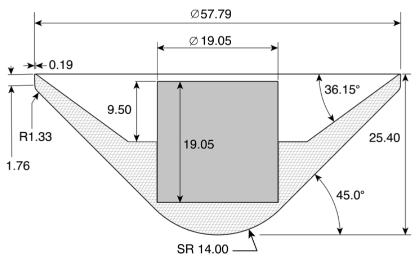
ABSTRACT

Blunt body aerodynamics in re-entry conditions are characterized by oscillatory behavior that is often encapsulated by the pitch damping coefficient (C_{mq}). Magnetic Suspension Wind Tunnel (MSWT) testing with a Magnetic Suspension Balance System (MSBS) aims to improve blunt body wind tunnel testing by removing the sometimes unknown aerodynamic effects of traditional sting mounts, as well as generate more constrained outputs than vertical spin tunnel testing. Efforts to calculate the uncertainty of the pitch damping coefficient with a proposed Earth Entry Vehicle (EEV) capsule model have been made by fitting an analytic prediction of the attitude time history to video data of a free-to-oscillate MSBS test.

1. Introduction

The oscillation of a blunt body constrained to one axis of rotation can be approximated with a simple harmonic oscillation (SHO) equation defined by Eq. 1. Determining uncertainty associated with C_{mq} must be made in conjunction with determining uncertainty associated with $C_{m\alpha}$, the pitch moment slope with respect to α. The weighted least squares estimate with a covariance matrix can be used to then determine both uncertainties (C_{mq} and $C_{m\alpha}$). This was performed with video data of a low subsonic EEV MSBS test at a dynamic pressure (q) of 130 Pa. The dimensions of the EEV model tested are in Figure 1.



Symbol	Value
A	scaling constant
$C_{L\alpha}$	lift slope coeff.
C_{MAG}	magnetic coeff.
d	diameter
I	moment of inertia
m	mass
S	surface area
t	time
V	velocity
ρ	fluid density
ϕ	phase shift

Fig. 1 EEV Geometry [mm] and SHO Nomenclature

2. Method

One degree of freedom constrained blunt body oscillations (pitch angle, α) can be approximated with the SHO model below, which is found by using the planar equations of motion and modifying exponential terms to account for magnetic force contributions[1].

$$\alpha = Ae^{\frac{\rho V S}{4m}\left(-C_{L\alpha}+C_{MAG}+\frac{md^2}{2I}C_{mq}\right)t} \cos\left(\sqrt{(-\frac{\rho V^2 Sd}{2I}C_{m\alpha})}t+\phi\right) \quad (1)$$

The covariance weighted least squares estimator solution allows for the simultaneous solving of parameters' uncertainty, which is defined by Eq. 2 [2]. The covariance parameters in P are all independent variables in Eq. 1 outside of C_{mq} and $C_{m\alpha}$, and H and G are found by linearizing about the fitted SHO expression at the estimated states and nominal parameter values. Further, M is the covariance matrix of the camera-measured α value, estimated as 0.2°. Inability to access the test setup prevented further analysis or a calibration test to fully define the uncertainty in the camera data collection.

$$cov\left(\begin{bmatrix} C_{mq} \\ C_{m\alpha} \end{bmatrix}\right) = \left(H^T M^{-1} H\right)^{-1} + \left(H^T M H\right)^{-1} H^T M^{-1} G P G^T M^{-1} H \left(H^T M^{-1} H\right)^{-1} \quad (2)$$

$$P = cov\left(\begin{bmatrix} V \\ C_{MAG} \\ \rho \\ \vdots \end{bmatrix}\right), \quad M = cov\left(\alpha\right) \quad (3)$$

3. Results and Discussion

The Taylor Series Uncertainty Method was used for all parameters' uncertainties that were not directly measured or quoted, which is defined according to Eq. 4 [3]. The error of each parameter is defined at a specific velocity and related dynamic pressure.

$$U_f = \sqrt{\sum_{i=1}^{J}\left(\frac{\partial f}{\partial X_i}\right)^2 U_i^2} \quad (4)$$

The velocity and density are calculated through the following equations during wind tunnel testing

$$V = \sqrt{\frac{2q_{test}}{\rho}}, \quad \rho = \frac{p}{RT} \quad (5)$$

where q_{test} is the test section dynamic pressure. δp=7.25 Pa (for single tap pressure recording runs using a pressure transducer, which is an upper magnitude bound) and δT=2.7°C including the rounding error with the thermocouple's listed uncertainty. δq=1.665 Pa according to Neill from systematic uncertainty analysis of inlet pressure tap readings[4]. For an ideal case

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at standard atmospheric conditions, p=101,325 Pa, T=288.15K, the uncertainty of fluid density, $\delta\rho$=0.0115 kg/m^3. Further, when ρ=1.225 kg/m^3, the uncertainty of fluid velocity can be calculated at each run, as shown in the Table 1.

V [m/s]	**q [Pa]**	ρ **[kg/m^3]**	δV **[m/s]**
10	61.25	1.225	0.1438
15	137.81	1.225	0.1147
20	245	1.225	0.1157
25	382.81	1.225	0.1291
30	551.25	1.225	0.1477
35	730.31	1.225	0.1685

Table 1 Velocity Uncertainty

A balance that represents those used to generate Neill's mass values states an uncertainty of $\delta m = \pm\, 0.01$ grams[4]. Using *Space Electronics* services to measure moment of inertia of the model, and assuming the models are within tight manufacturing tolerances, moment of inertia errors can be expected within 0.5%. For EEV, δI= 1.9325e-08kg-m^2. The uncertainty of the calipers, and thus diameter, was found to be 0.1016 mm.

Strain gauge balance tests in the Langley Vertical Spin Tunnel provided lift versus angle of attack data for EEV. The lift curve slope, $C_{L\alpha}$ was calculated as -0.223 rad^{-1} using a linear fit to 10°. The uncertainty associated with $C_{L\alpha}$ for EEV is $\pm$0.0414 rad^{-1} considering nonlinearities in $C_{L\alpha}$ beyond the 10° fitted data.

The C_{MAG} term represents the averaged effect of the MSBS countering the model lift force as it oscillates. C_{MAG} was determined using the MATLAB Flight Simulation Environment (MAFSE) 6 degree of freedom simulation. The MSBS control law was integrated into MAFSE, and the simulation was shown to match experimental data. Fitting Eq. 1 to simulated data identified C_{MAG}, and partial derivatives of C_{MAG} with respect to other test parameters ($C_{L\alpha}$ and V being significant) were calculated for Taylor series uncertainty estimation as found in Eq. 4.

Mass and moment of inertia are correlated, thus an off-diagonal term in the parameter variance-covariance matrix (P) is needed to accurately represent the system. Using mass and moment of inertia measurements of the MSBS models, a linear relationship can be fitted as shown in Figure 2. Using the built-in linear fit function in Microsoft Excel R^2=0.9783, which gives an R value (correlation coefficient) of 0.989.

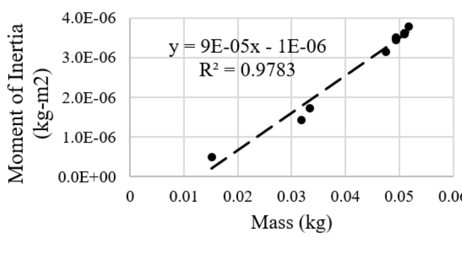

Fig. 2 MSBS models I and mass relationship

With MSBS video data, Eq. 1 can be fit to the output pitch data, extracted with MATLAB video tracking scripts. Setting C_{MAG} to its MAFSE estimated value at q=130 Pa, C_{mq}=-0.2183 as fit per Figure 3 with Eq. 1. The uncertainty of each parameter is defined according to Table 2, and using Eq. 2, δC_{mq} is calculated as **0.0038** for the EEV MSBS data at q=130 Pa.

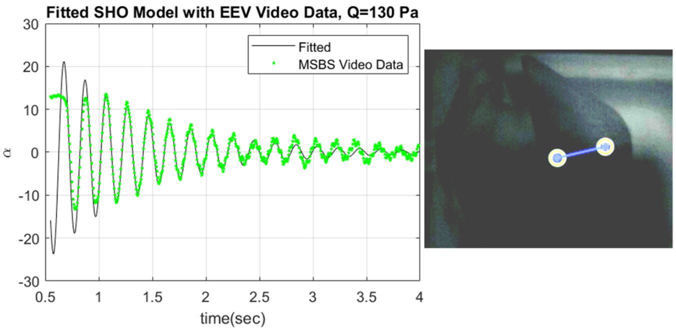

Fig. 3 EEV Video Data with Fitted Expression

Parameter	**Value**	δ	**Units**
ρ	1.225	0.0115	kg/m^3
V	14.57	0.115	m/s
m	48.90	0.01	g
I	3.865e-6	1.9325e-08	kg-m^2
d	57.8	0.102	mm
$C_{L\alpha}$	-0.223	0.0414	-
C_{MAG}	-0.07604	0.0122	-
α	varies	0.2	degrees

Table 2 Parameter Uncertainty EEV, q=130 Pa

4. Concluding Remarks

MSWT is a propitious technology for use in determining blunt body aerodynamics. The Langley MSBS accurately measured the dynamic stability of the EEV model, and its associated uncertainty was calculated by identifying and determining the main contributors to this uncertainty ($\delta C_{L\alpha}$ and δC_{MAG}). Uncertainty quantification will improve further with detailed calibration tests of the MSBS and camera system.

References

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