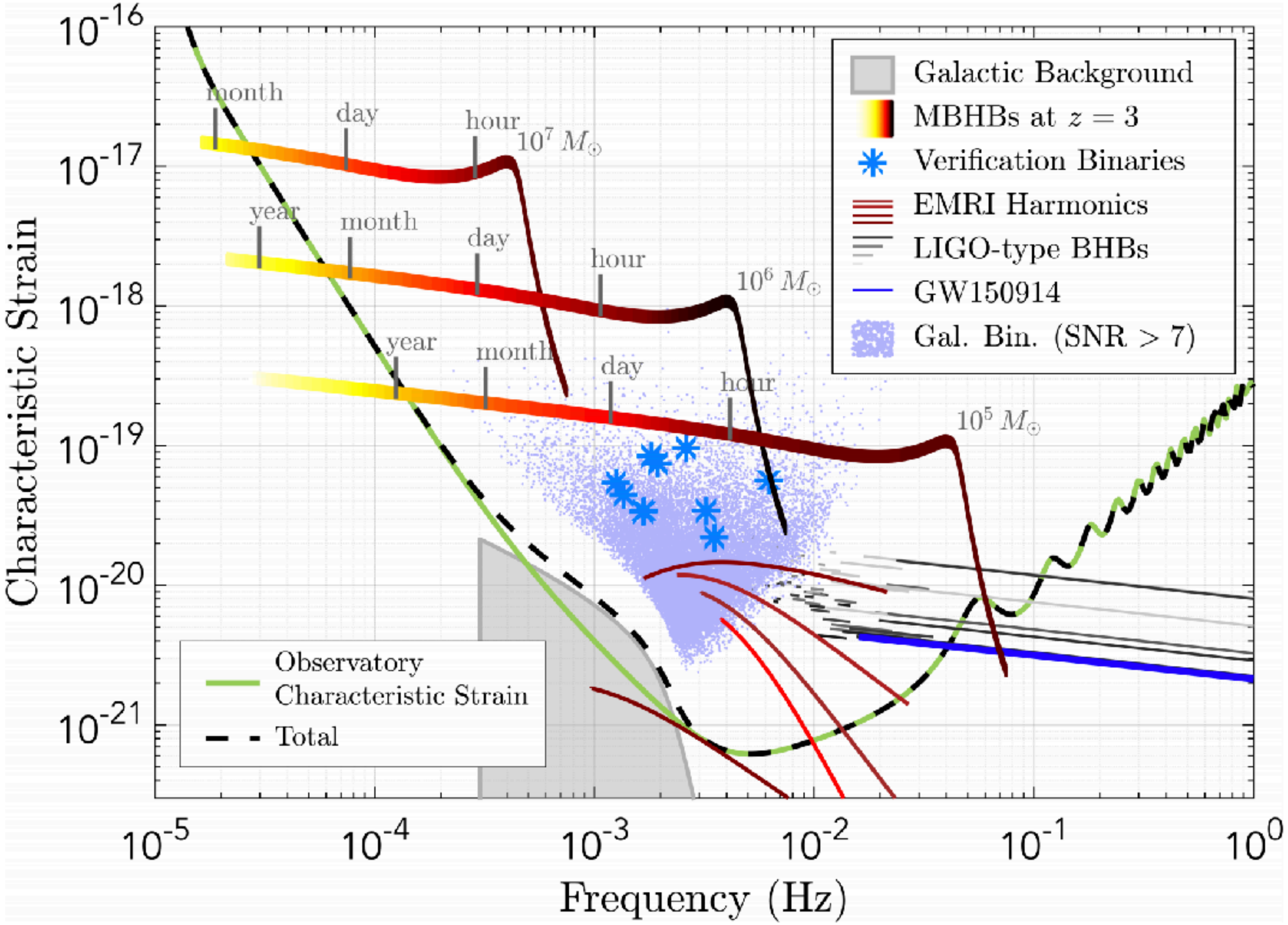
Designing a Global Fit Analysis For LISA

Tyson Littenberg @ NASA Marshall Space Flight Center
Neil Cornish @ Montana State University

The Global Fit problem

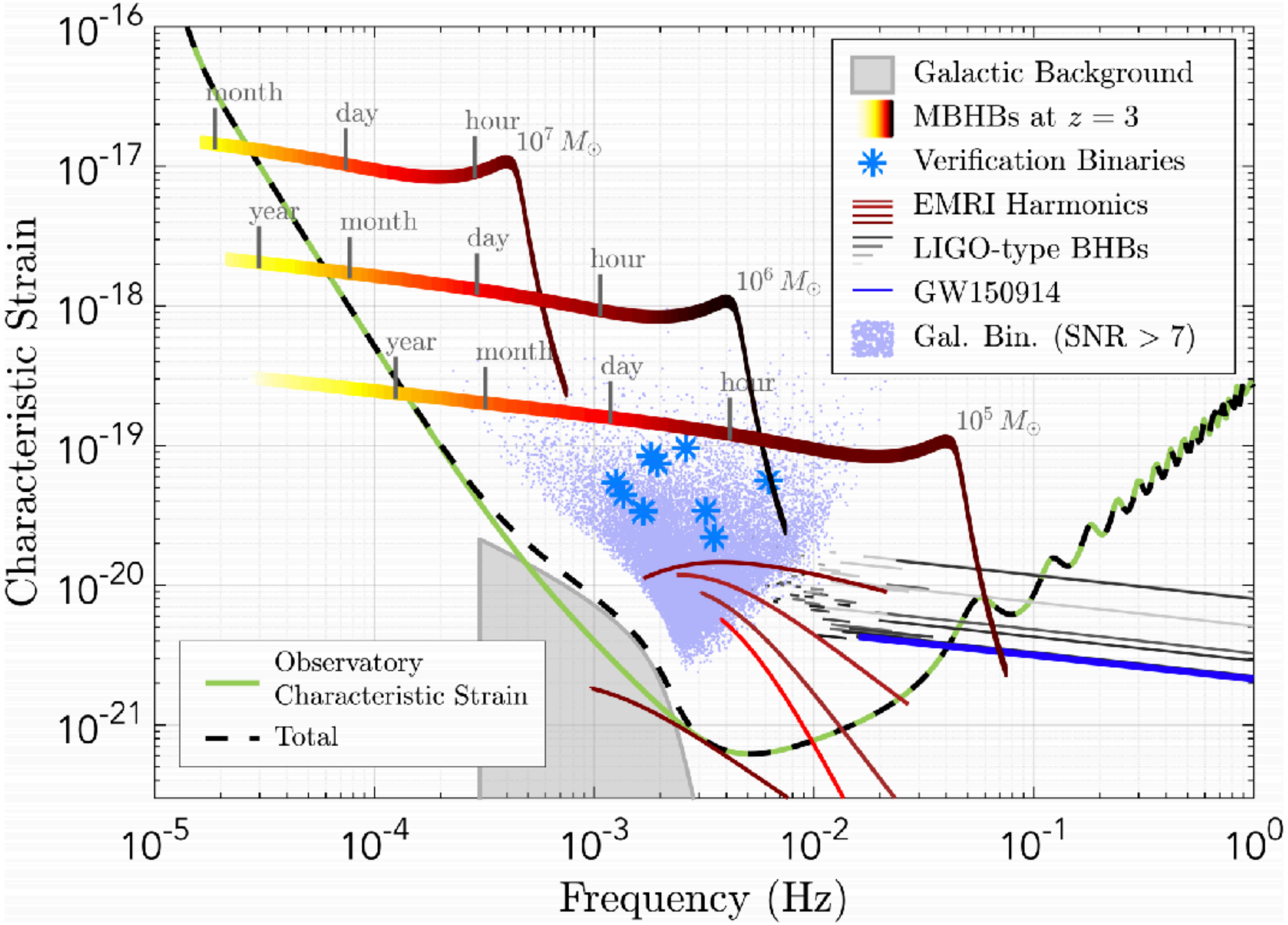


I. Number of detectable sources is unknown a priori—but it is **LARGE**.

II. Sources are overlapping in time and frequency.

III. Individual overlaps between pairs are small. But see I.

The Global Fit problem



IV. Advantages of the Global Fit:

I. Avoids source confusion

II. Properly marginalize over all model uncertainties

V. Disadvantage of the Global Fit:

I. It is **logistically** complicated.

MCMC

Why MCMC?

Given some **model** for the data: M

Why MCMC?

Given some **model** for the data: M

…with **parameters**: x

Why MCMC?

Given some **model** for the data: M

…with **parameters**: x

…the **posterior probability density** for the parameters is:

$$p(x \mid d, M) = \frac{p(d \mid x, M)\, p(x \mid M)}{p(d \mid M)}$$

Likelihood = “Goodness of Fit” for parameters

Prior = Previously known values for parameters

Marginalized Likelihood = “Goodness of Fit” for model

Why MCMC?

Given some **model** for the data: M

…with **parameters**: x

…the **posterior probability density** for the parameters is:

$$p(x \mid d, M) = \frac{p(d \mid x, M)\, p(x \mid M)}{p(d \mid M)}$$

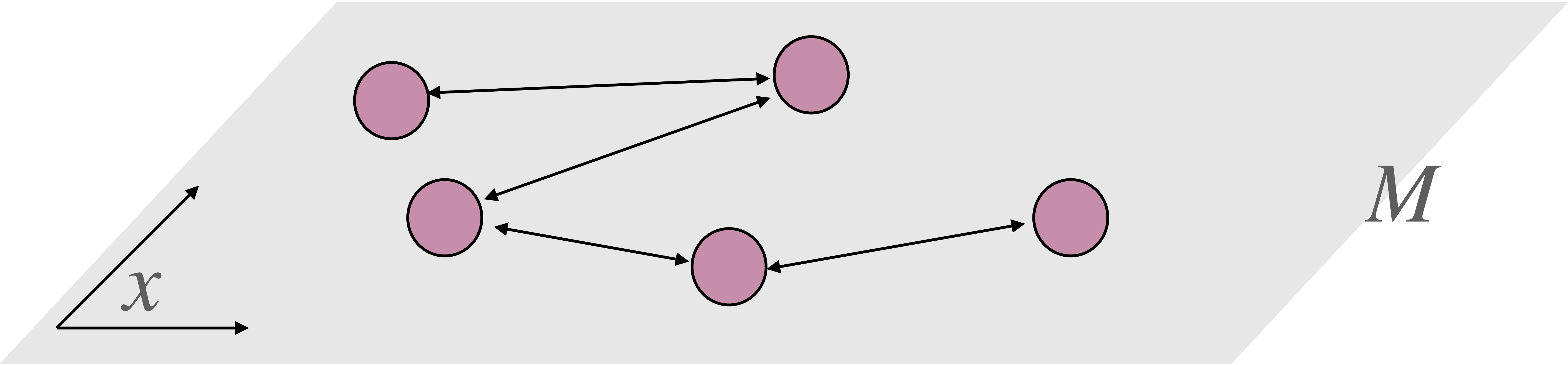
Likelihood = "Goodness of Fit" for parameters

Prior = Previously known values for parameters

Marginalized Likelihood = "Goodness of Fit" for model

MCMC produces independent samples from $p(x \mid d, M)$

Why MCMC?



$$p(x|d,M) = \frac{p(d|x,M)p(x|M)}{p(d|M)}$$

I. Stochastically sample large and complicated parameter spaces

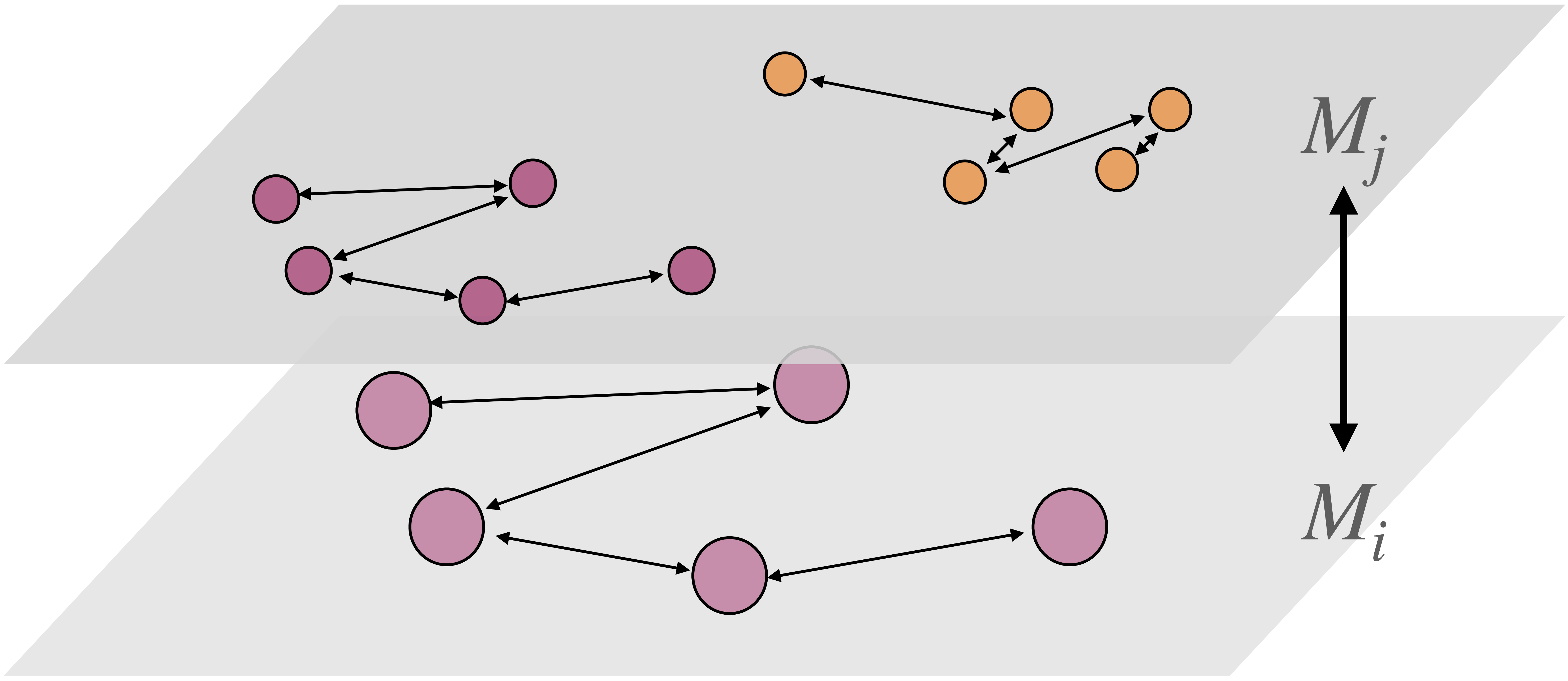
II. Always converges, usually faster than grid-based approaches when parameter space is **LARGE**.

III. One-stop shop for detection, characterization, and quantifying confidence

IV. **Stochastically sample between models with RJMCMC**

“But you are describing a number of stochastic samplers. Why ***MCMC***?”

Why MCMC?



$$p(x, M \mid d) = \frac{p(d, M \mid x)\, p(x, M)}{p(d, M)}$$

I. Stochastically sample large and complicated parameter spaces

II. Always converges, usually faster than grid-based approaches when parameter space is **LARGE**.

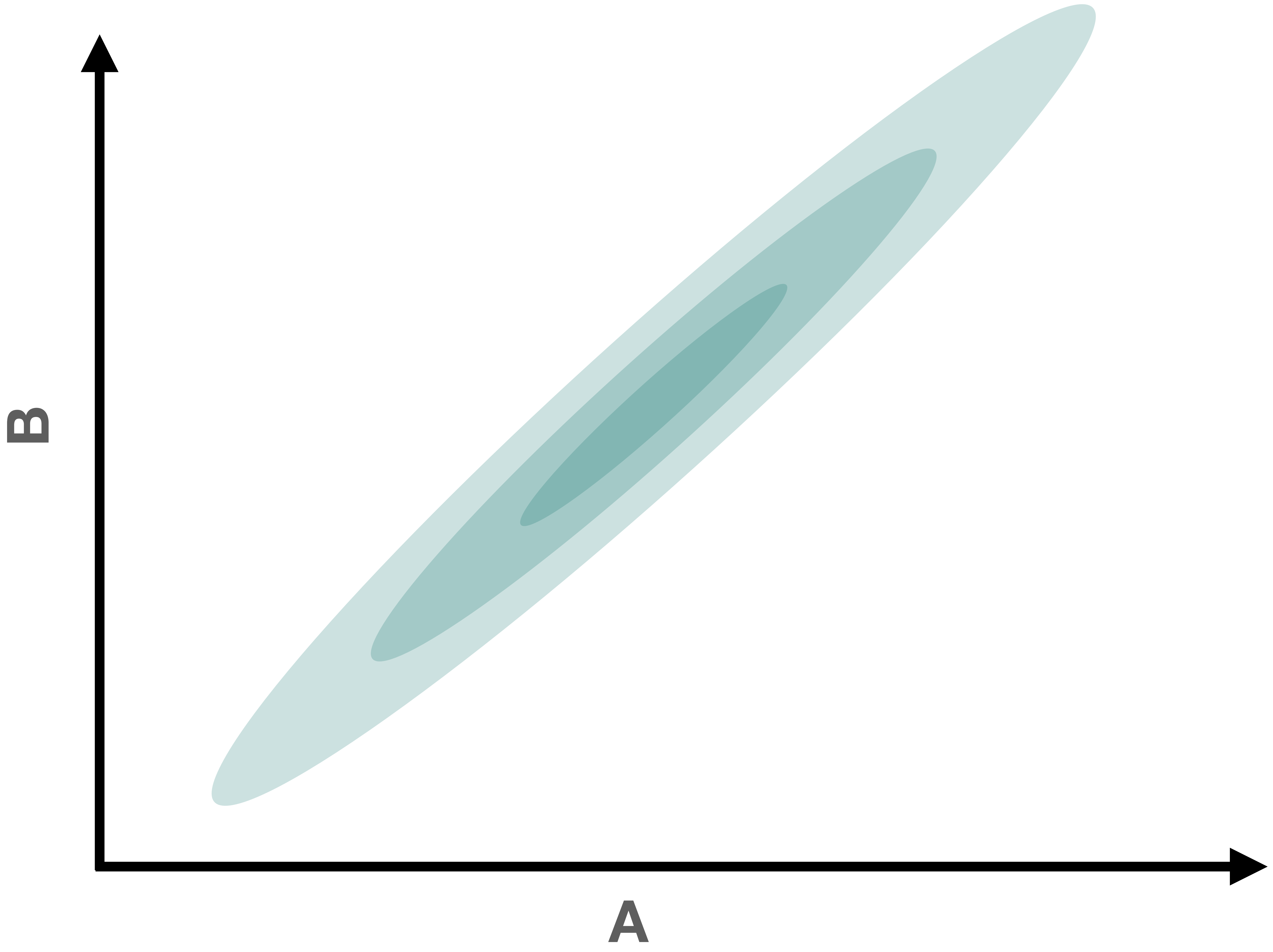
III. One-stop shop for detection, characterization, and quantifying confidence

IV. **Stochastically sample between models with RJMCMC**

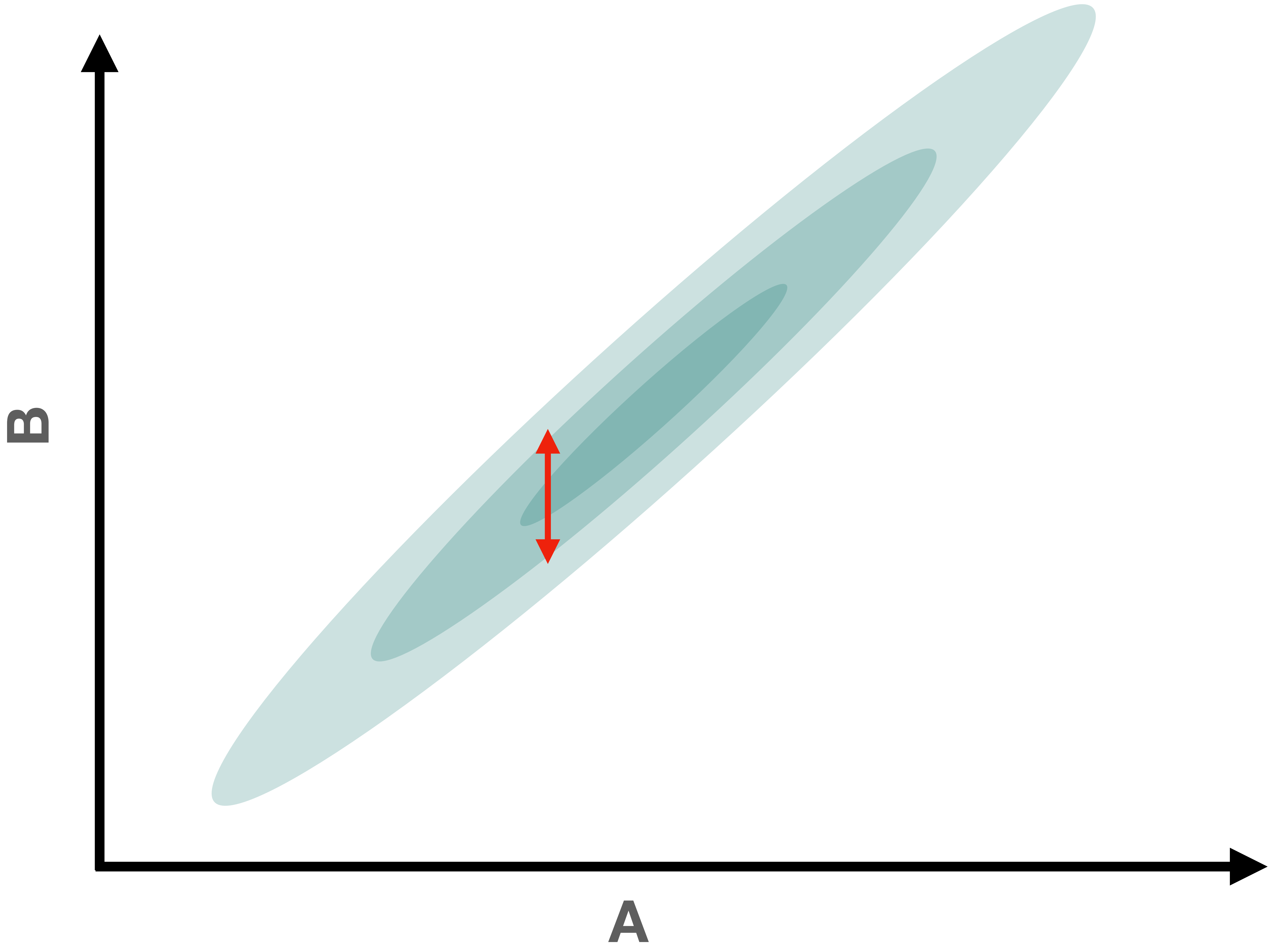
“How can you possibly plan to efficiently sample over that **LARGE** parameter space?”

BLOCKED GIBBS SAMPLING

What is Gibbs Sampling?

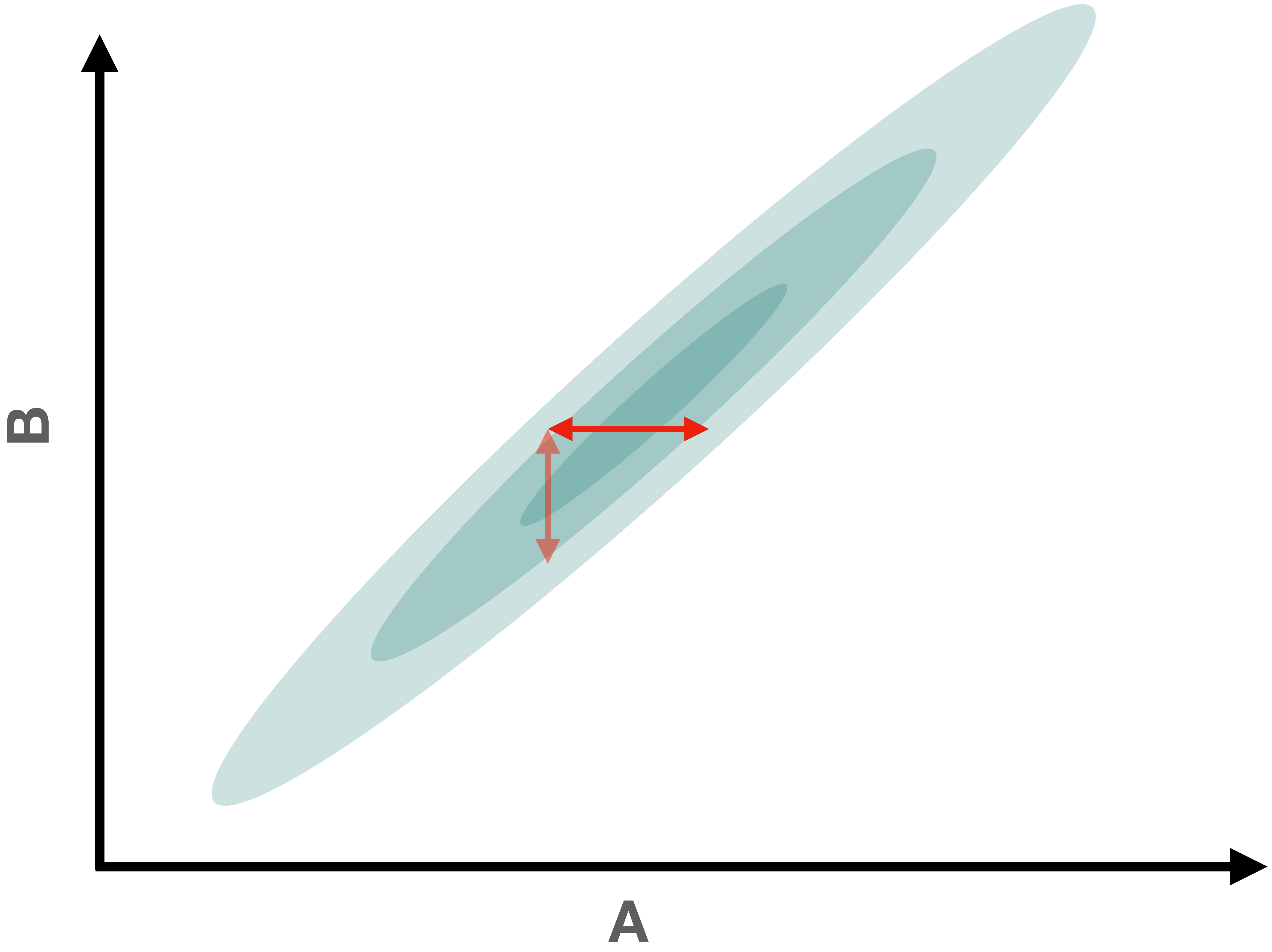


What is Gibbs Sampling?



I. Hold Block A fixed and sample over B

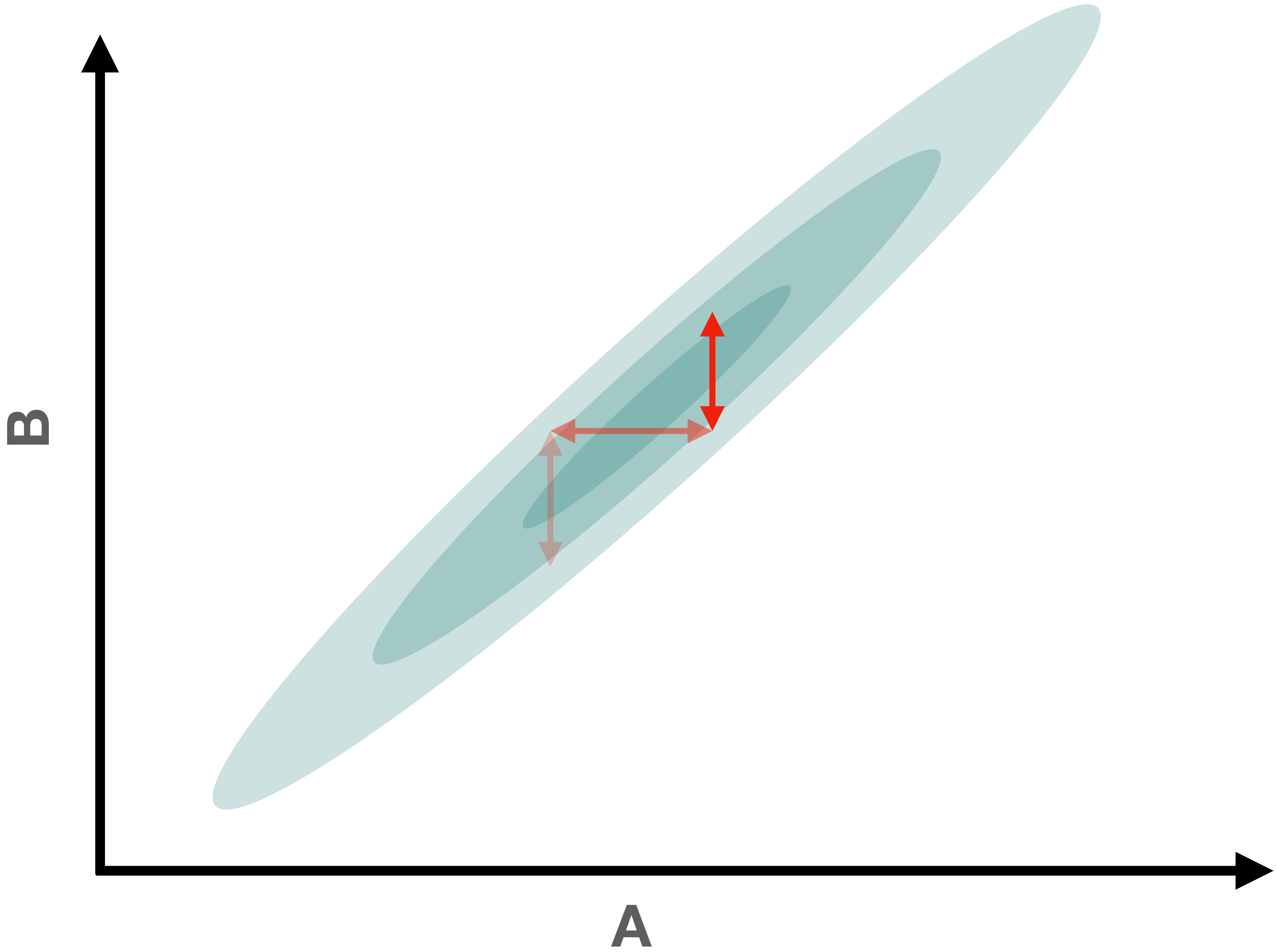
What is Gibbs Sampling?



I. Hold Block A fixed and sample over B

II. Hold Block B fixed and sample over A.

What is Gibbs Sampling?

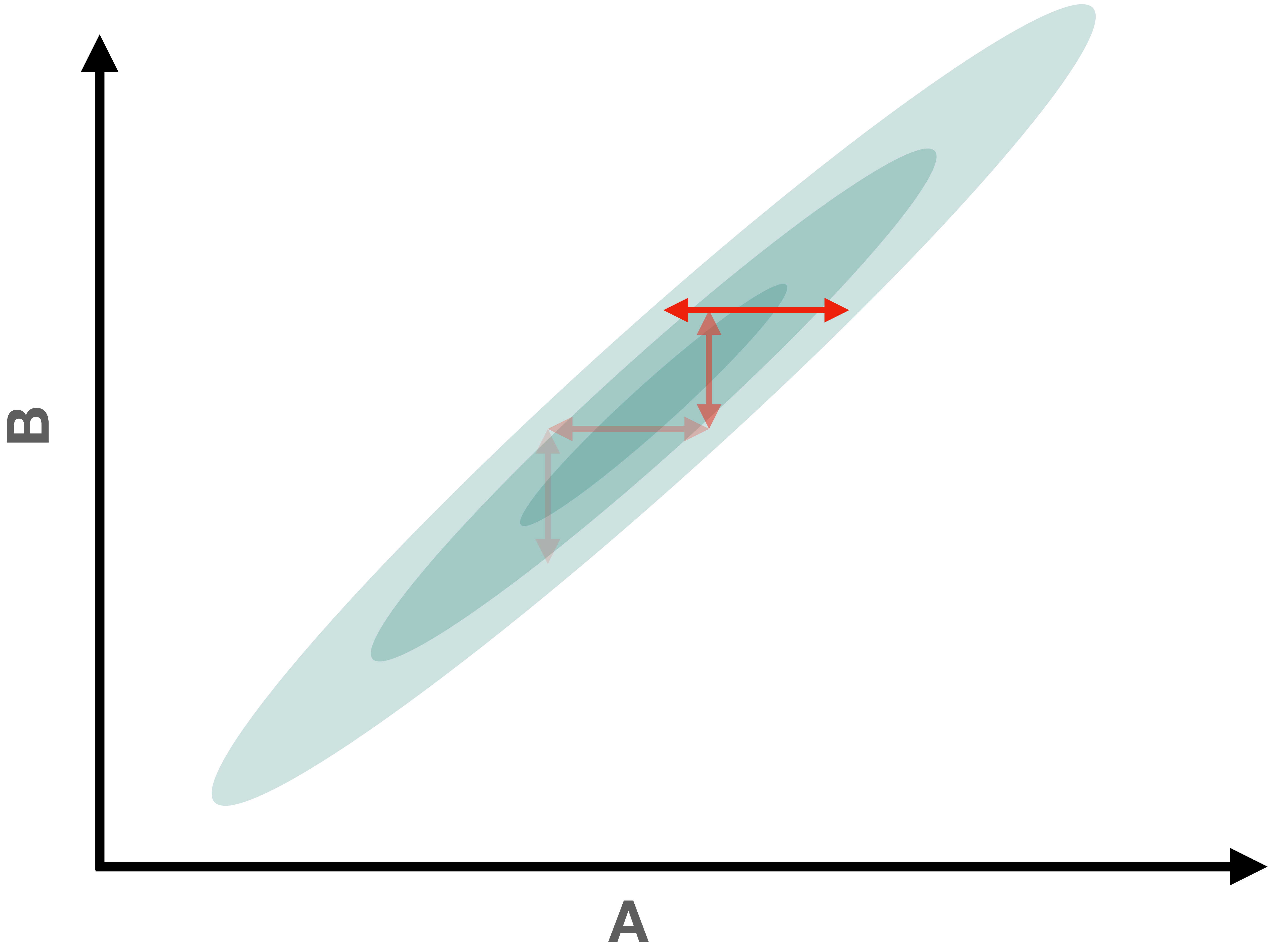


I. Hold Block A fixed and sample over B

II. Hold Block B fixed and sample over A.

III. Repeat

What is Gibbs Sampling?



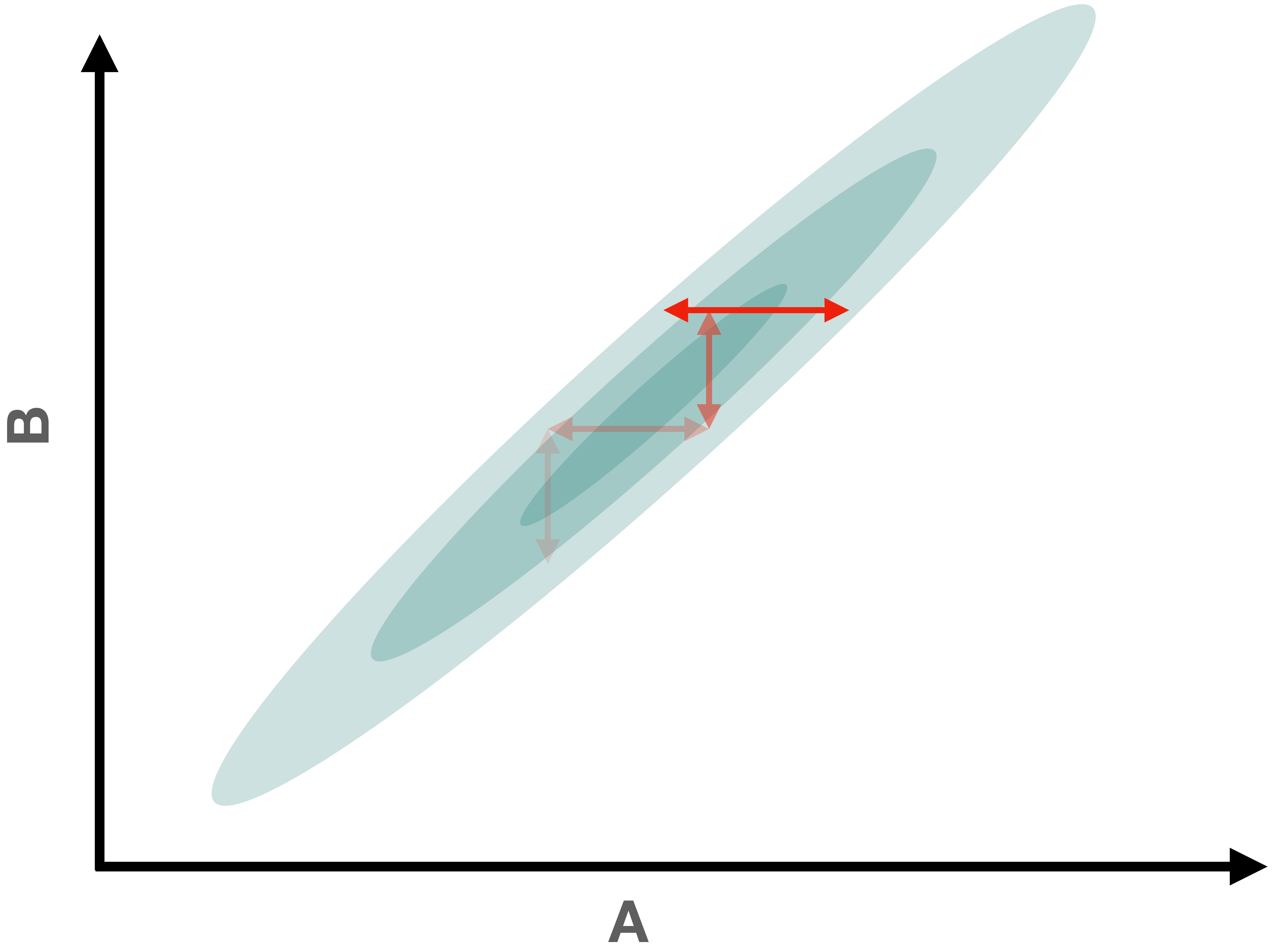
I. Hold Block A fixed and sample over B

II. Hold Block B fixed and sample over A.

III. Repeat

IV. Keep repeating…

What is Gibbs Sampling?



I. Hold Block A fixed and sample over B

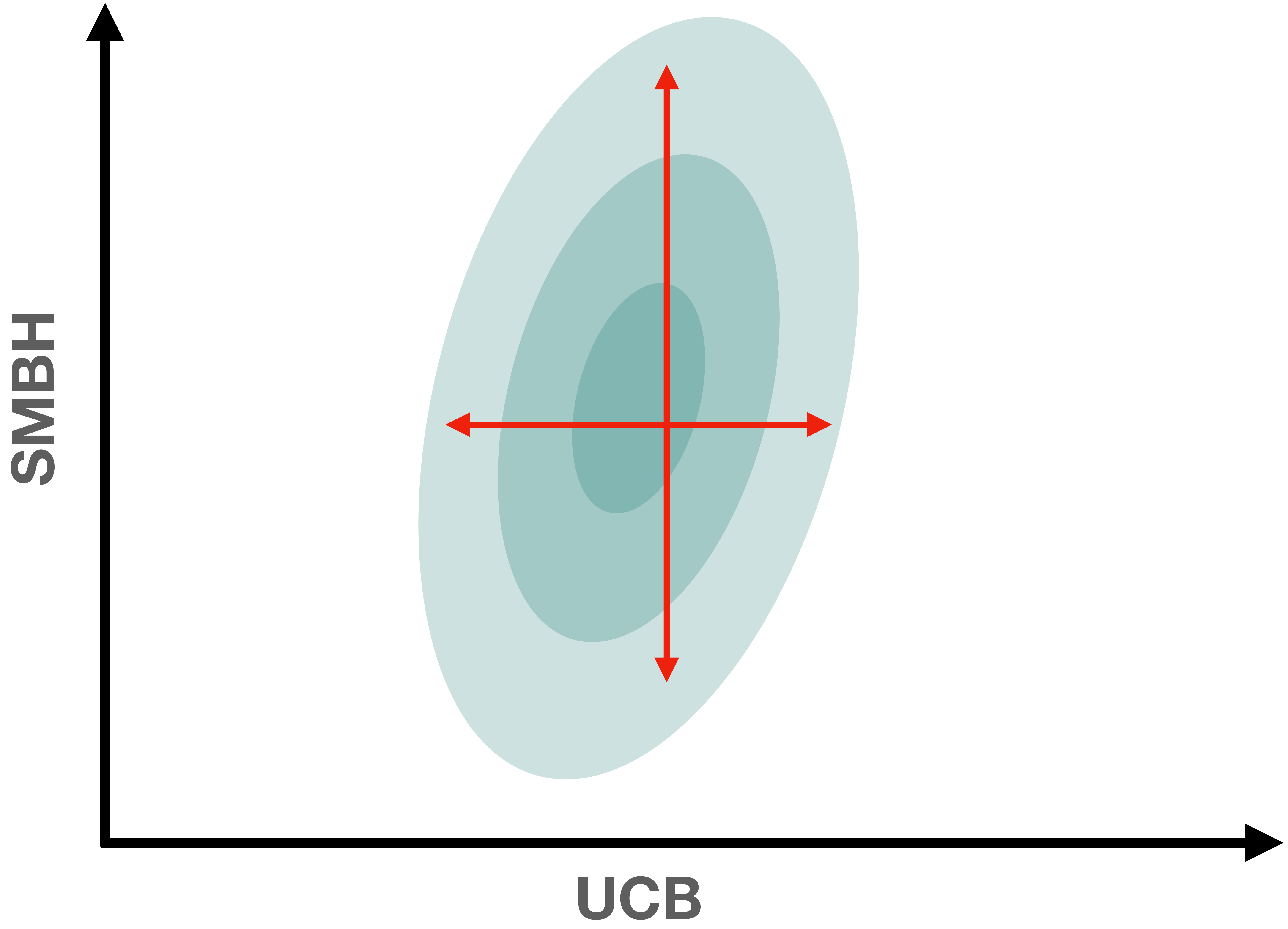
II. Hold Block B fixed and sample over A.

III. Repeat

IV. Keep repeating…

V. Converges miserably for correlated blocks

Why Gibbs Sampling?



I. Correlations between different source types are small.

II. “Blocks” are developed independently w/ common API.

III. With clever scheduling is nicely parallelized.

IV. Blocks can be inserted/removed without disrupting the workflow.

Global Fit Design

Current State of Development

“I’m convinced. So what’s new?”

Efficiently Adding New Data

Efficiently Adding New Data

Start with data d_1: $p(x \mid d_1) = \frac{p(d_1 \mid x)p(x)}{p(d_1)}$

Efficiently Adding New Data

Start with data d_1 $$p(x \mid d_1) = \frac{p(d_1 \mid x)p(x)}{p(d_1)}$$

…now add new data d_2: $$p(x \mid d_1, d_2) = \frac{p(d_1, d_2 \mid x)p(x)}{p(d_1, d_2)}$$

Efficiently Adding New Data

Start with data d_1 $$p(x \mid d_1) = \frac{p(d_1 \mid x)p(x)}{p(d_1)}$$

…now add new data d_2 $$p(x \mid d_1, d_2) = \frac{p(d_1, d_2 \mid x)p(x)}{p(d_1, d_2)}$$

…d_1 and d_2 are independent $$p(x \mid d_1, d_2) = \frac{p(d_1 \mid x)p(d_2 \mid x)p(x)}{p(d_1)p(d_2)}$$

Efficiently Adding New Data

Start with data d_1 $$p(x|d_1) = \frac{p(d_1|x)p(x)}{p(d_1)}$$

…now add new data d_2 $$p(x|d_1, d_2) = \frac{p(d_1, d_2|x)p(x)}{p(d_1, d_2)}$$

…d_1 and d_2 are independent $$p(x|d_1, d_2) = \frac{p(d_1|x)p(d_2|x)p(x)}{p(d_1)p(d_2)}$$

Efficiently Adding New Data

Start with data d_1

$$p(x \mid d_1) = \frac{p(d_1 \mid x)p(x)}{p(d_1)}$$

…now add new data d_2

$$p(x \mid d_1, d_2) = \frac{p(d_1, d_2 \mid x)p(x)}{p(d_1, d_2)}$$

…d_1 and d_2 are independent

$$p(x \mid d_1, d_2) = \frac{p(d_1 \mid x)p(d_2 \mid x)p(x)}{p(d_1)p(d_2)}$$

$$p(x \mid d_1, d_2) = \frac{p(d_2 \mid x)p(x \mid d_1)}{p(d_2)}$$

Efficiently Adding New Data

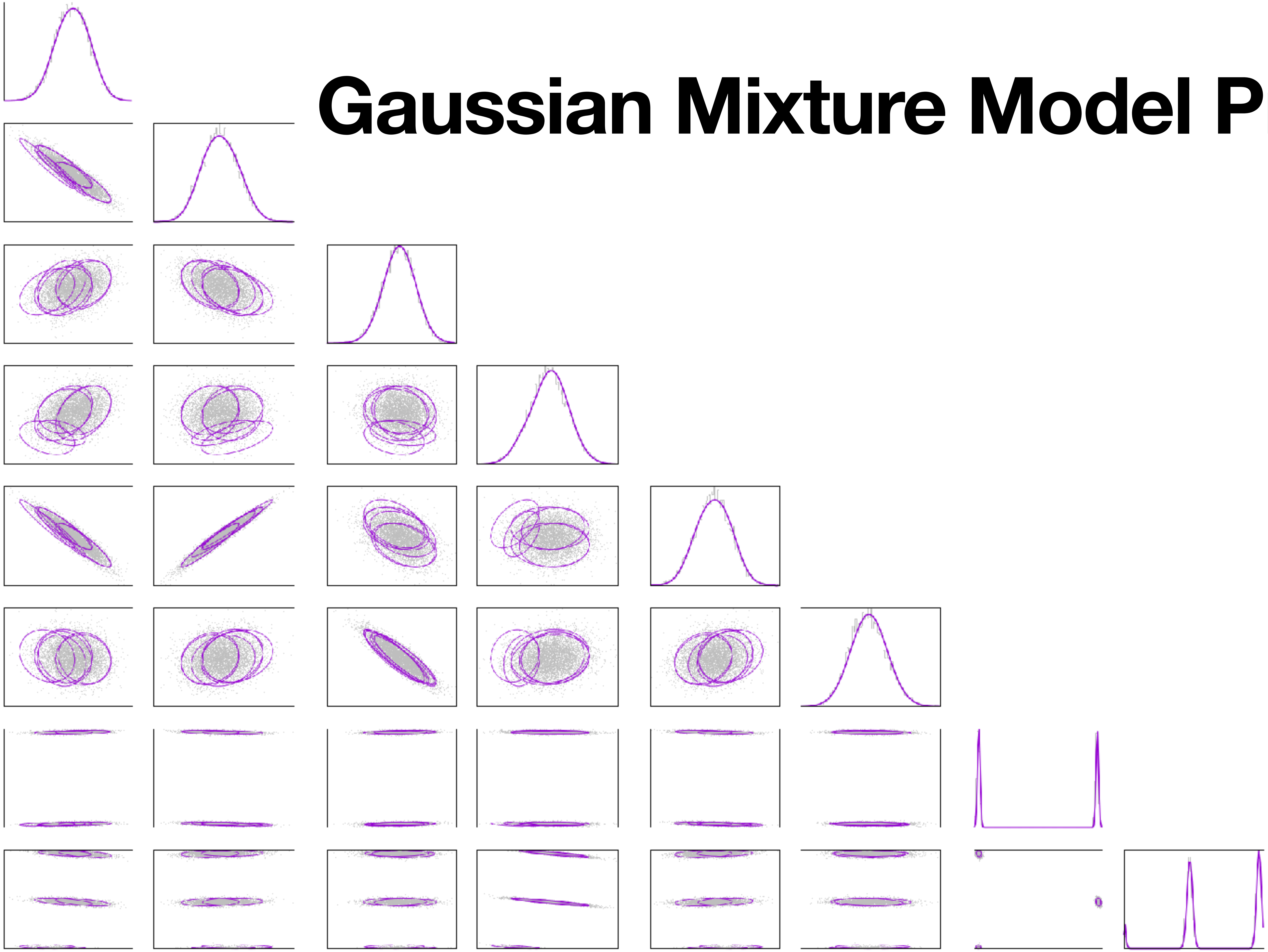
Start with data d_1 $$p(x|d_1) = \frac{p(d_1|x)p(x)}{p(d_1)}$$

…now add new data d_2 $$p(x|d_1, d_2) = \frac{p(d_1, d_2|x)p(x)}{p(d_1, d_2)}$$

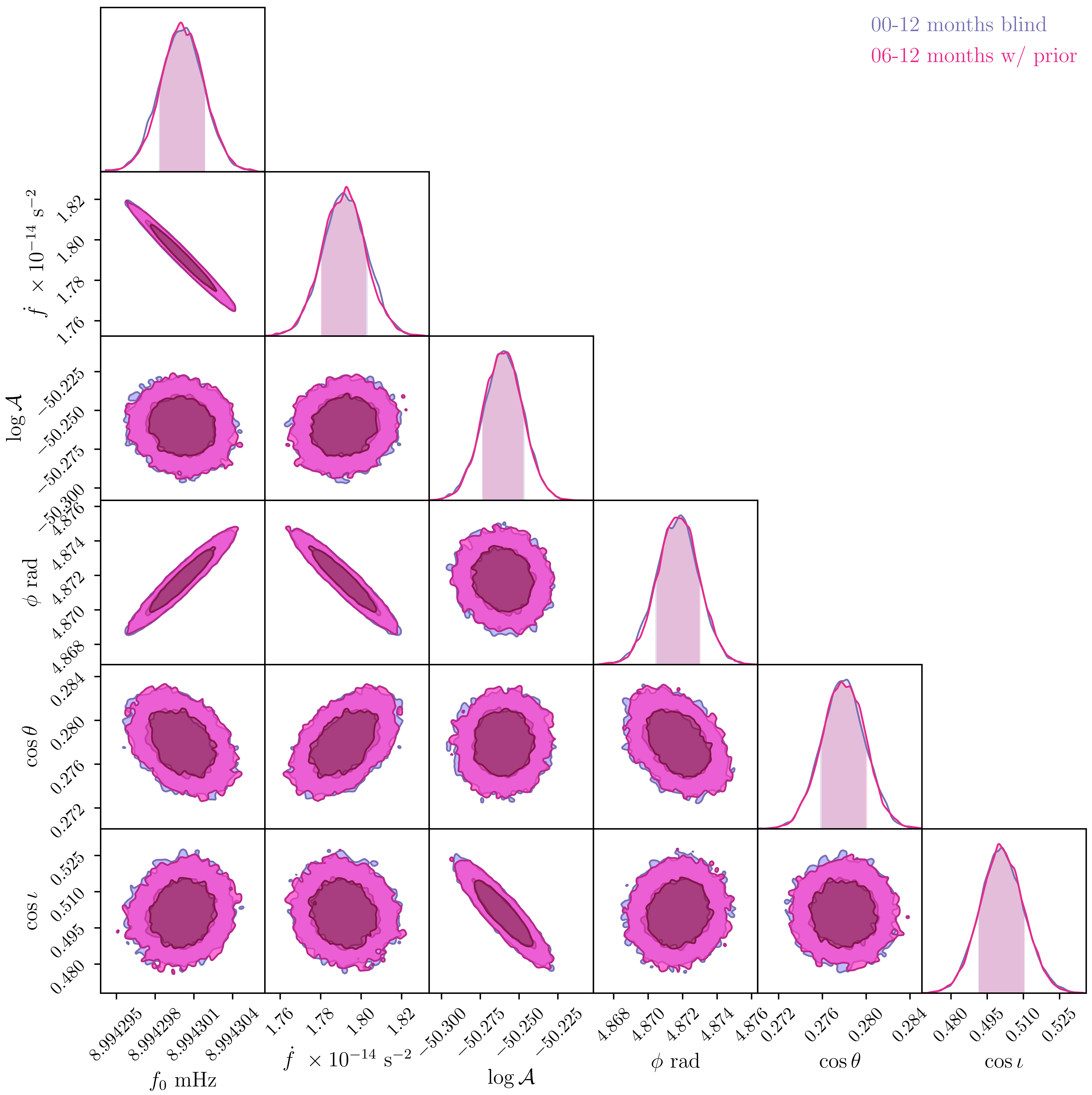
…d_1 and d_2 are independent $$p(x|d_1, d_2) = \frac{p(d_1|x)p(d_2|x)p(x)}{p(d_1)p(d_2)}$$

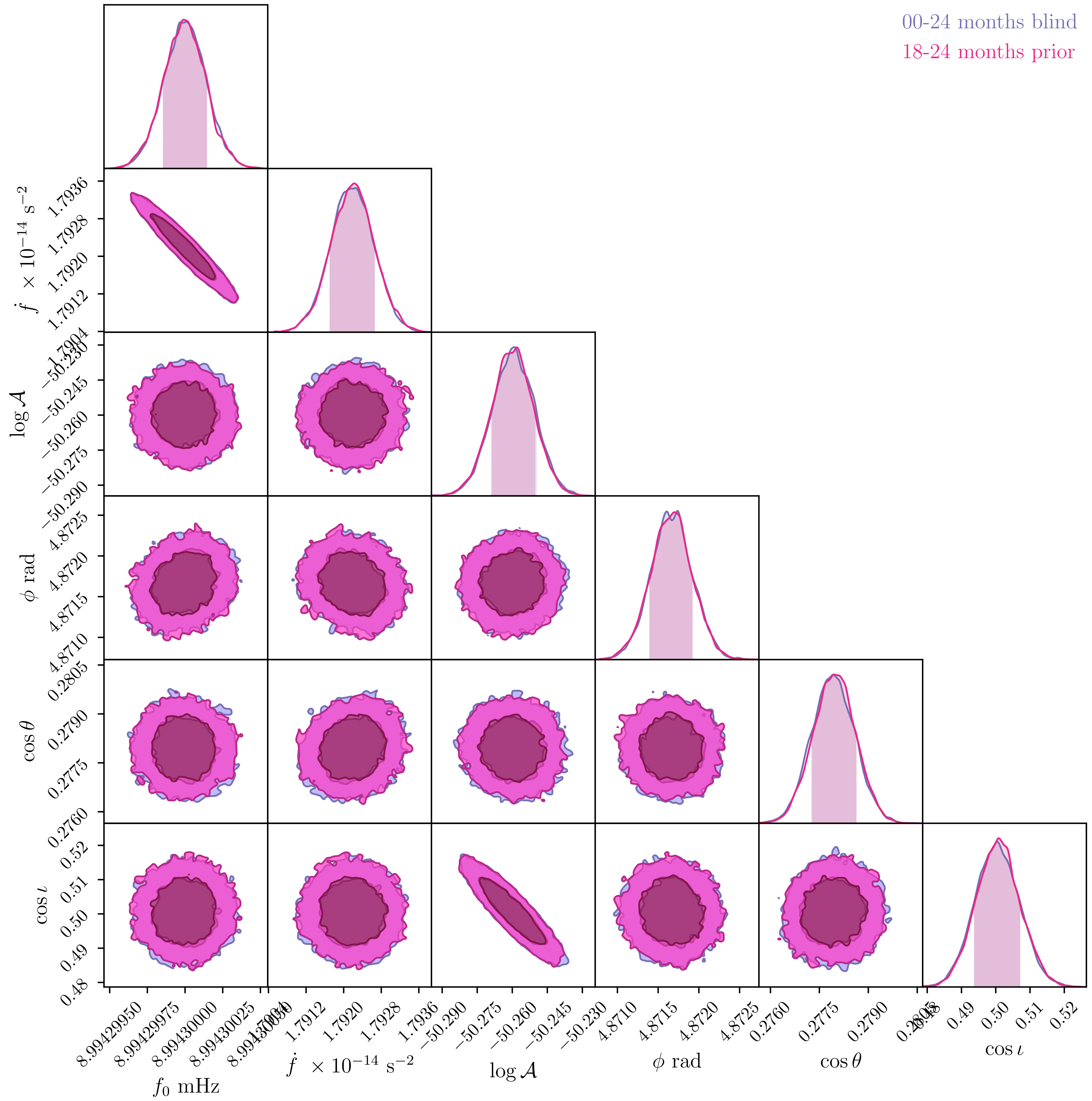
$$p(x|d_1, d_2) = \frac{p(d_2|x)p(x|d_1)}{p(d_2)}$$

Gaussian Mixture Model Priors

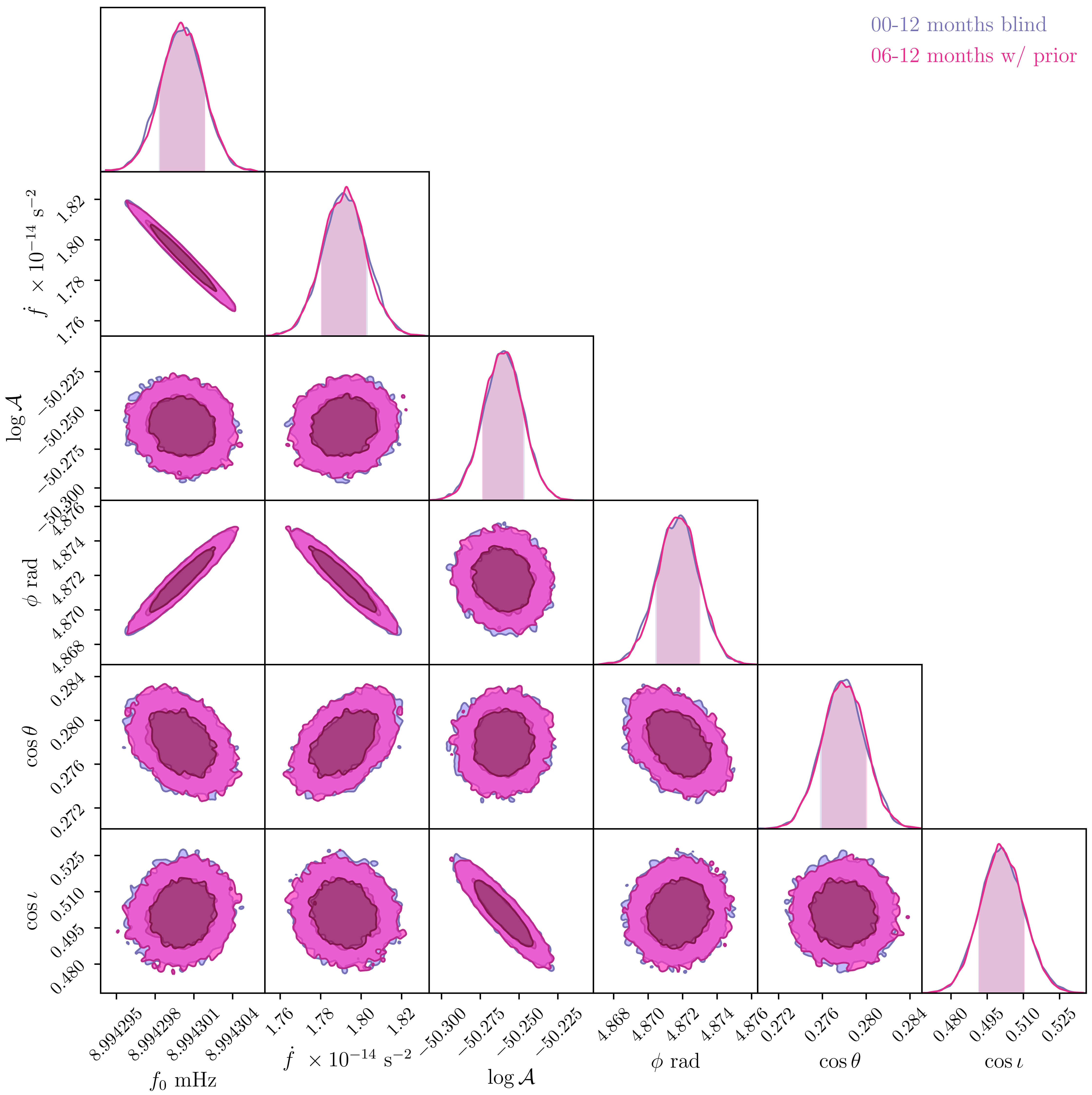


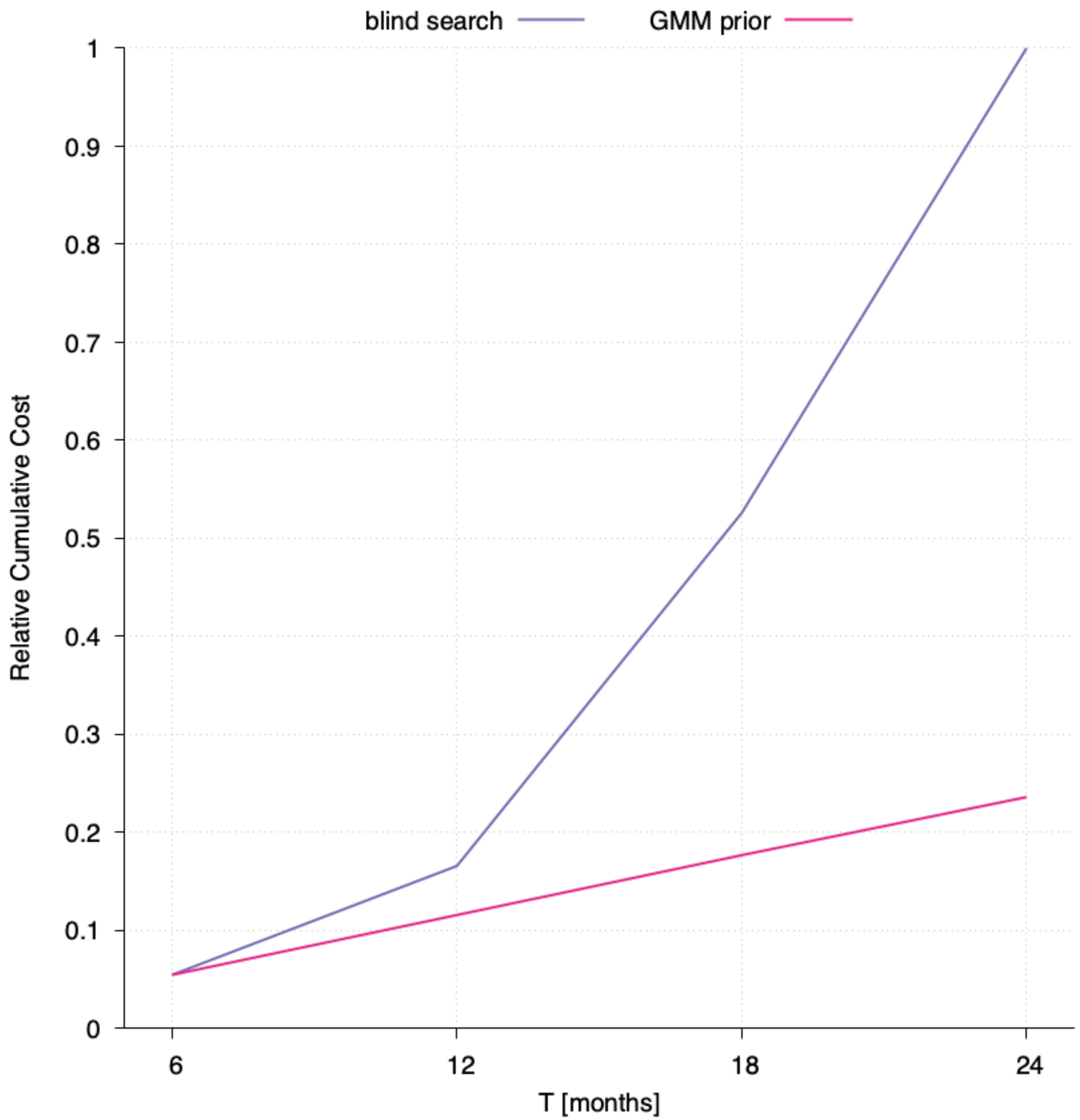
Gaussian Mixture Model Priors



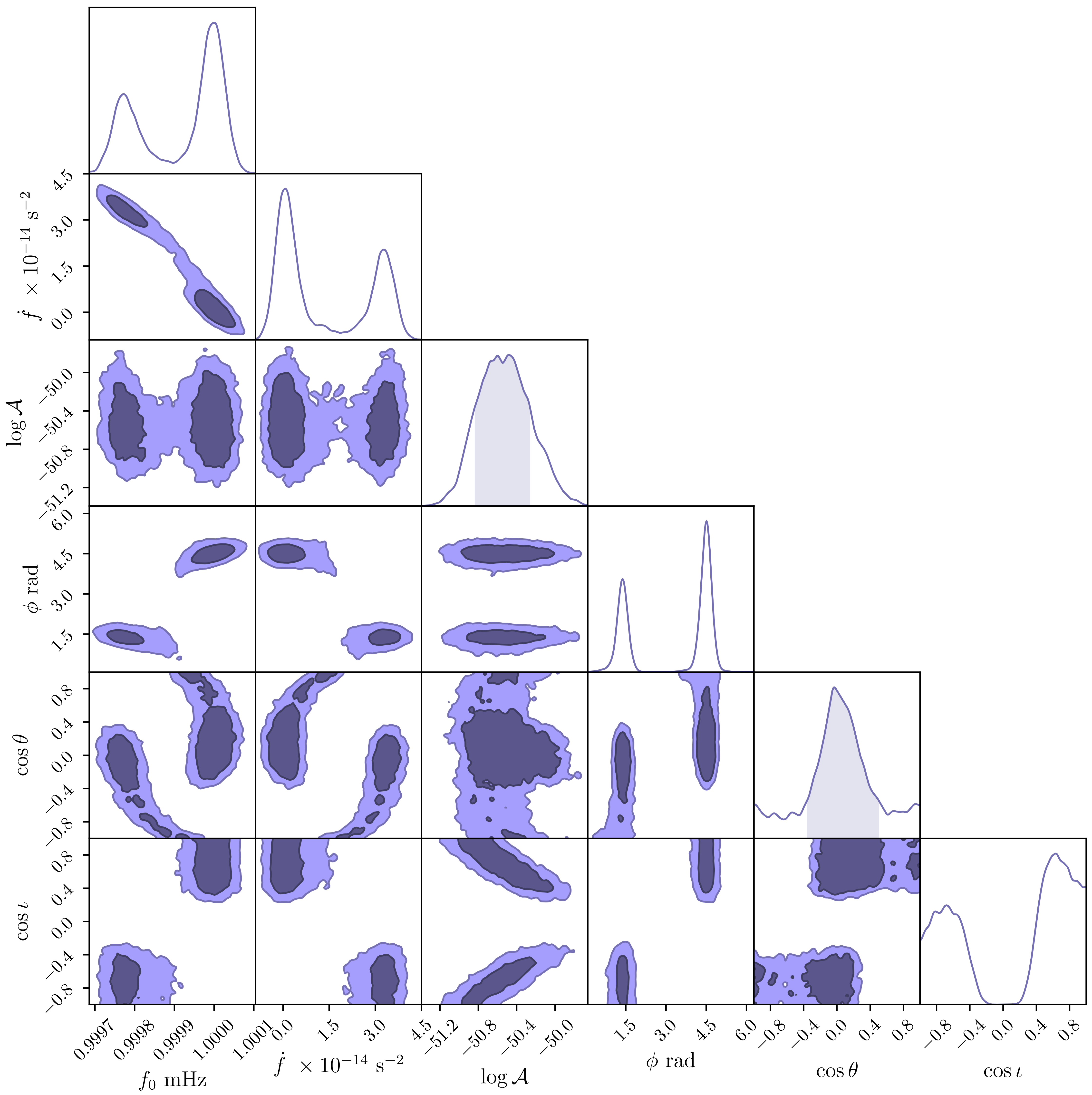


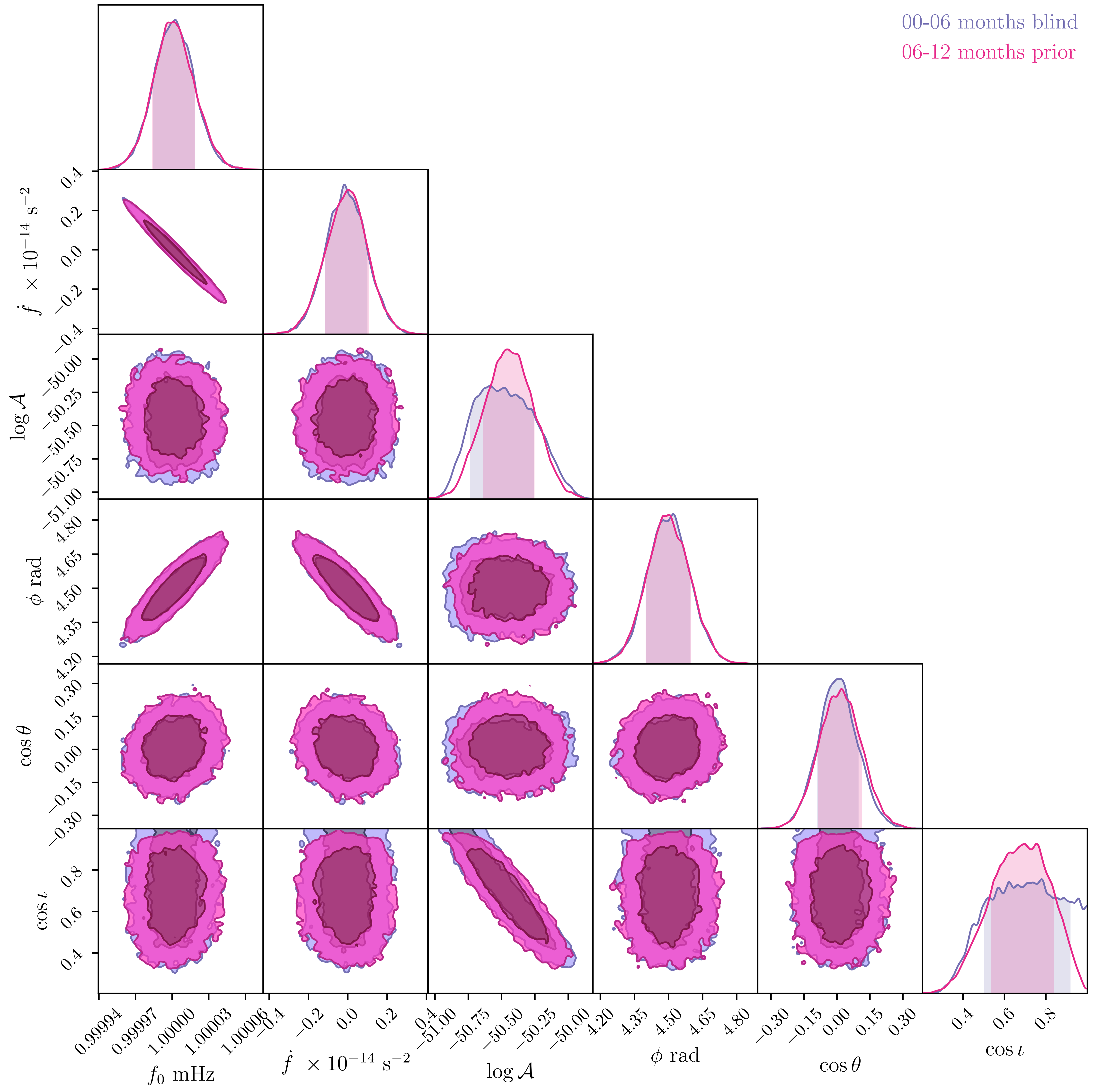
Gaussian Mixture Model Priors





Gaussian Mixture Model Priors





THE END