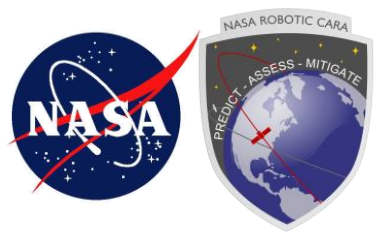


Conjunction Assessment Risk Analysis



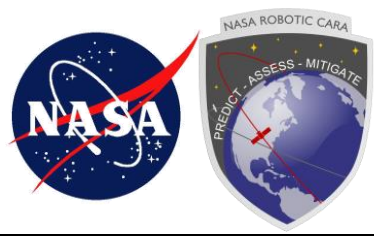
**2-D Pc, 3D-Nc, and
BFMC Pc
Calculations for
Single and Repeating
Conjunctions**

**M.D. Hejduk
Reporting on the work of D.T. Hall
September 2020**

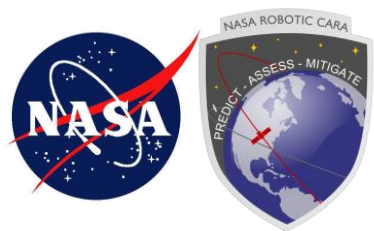


Agenda

- **3D-Nc description and treatment**
 - 3-D Pc basics and history
 - Algorithm flaw that caused errant behavior
 - Algorithm repair
- **2-D Pc and 3D-Nc performance against Monte Carlo for single events**
- **3D-Nc approximations for repeating conjunctions**
 - Approximation techniques
 - BFMC validation
- **Overall recommendations**



3D-Pc History, Description, and Repaired Methodology



Introduction and Motivation

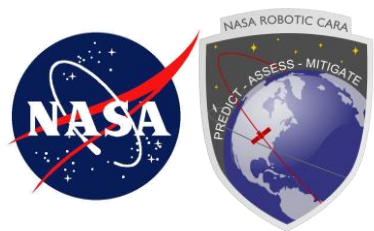
- **History of 3D-Pc approximation**

- Originally formulated by Coppola in 2012 (AAS 12-247 & AAS 12-248)
- Employed by DeMars & Jah in 2014 with OD solutions expressed using Gaussian Mixture Model PDFs (JGCD, vol.37, no.3, pp.1324-1335)
- CARA 3D-Pc numerical integration code implemented in 2017 by Hall *et al.* (AAS 17-271)

- **CARA analysis conducted during 2017 and 2018**

- Some conjunctions show very large differences between 2D-Pc and 3D-Pc
- Prompted implementing Brute Force Monte Carlo (BFMC) simulations to use as the “gold standard” adjudicator
- Conclusion: overall, the 2D-Pc method performs much better than the original 3D-Pc method
- 3D-Pc was operationally deprecated

- **Why did this happen?**



3-D Pc Calculation Methodology: Review

- **Methodology worked out by V.T. Coppola (2012)**

- Expanded by DeMars *et al.* (2014), who discuss the “probability rate,” dP_c/dt
- Probability rate is the instantaneous “rate of incursion” of uncertainty PDF into HBR sphere calculated by the surface area integral

$$\frac{dP_c}{dt} = F(t) = \oint_{4\pi} I(\hat{\mathbf{r}}, t) d^2\hat{\mathbf{r}}$$

- Approach greatly aided by extremely fast method of integrating over the unit sphere called Lebedev Quadrature (Lebedev 1999)

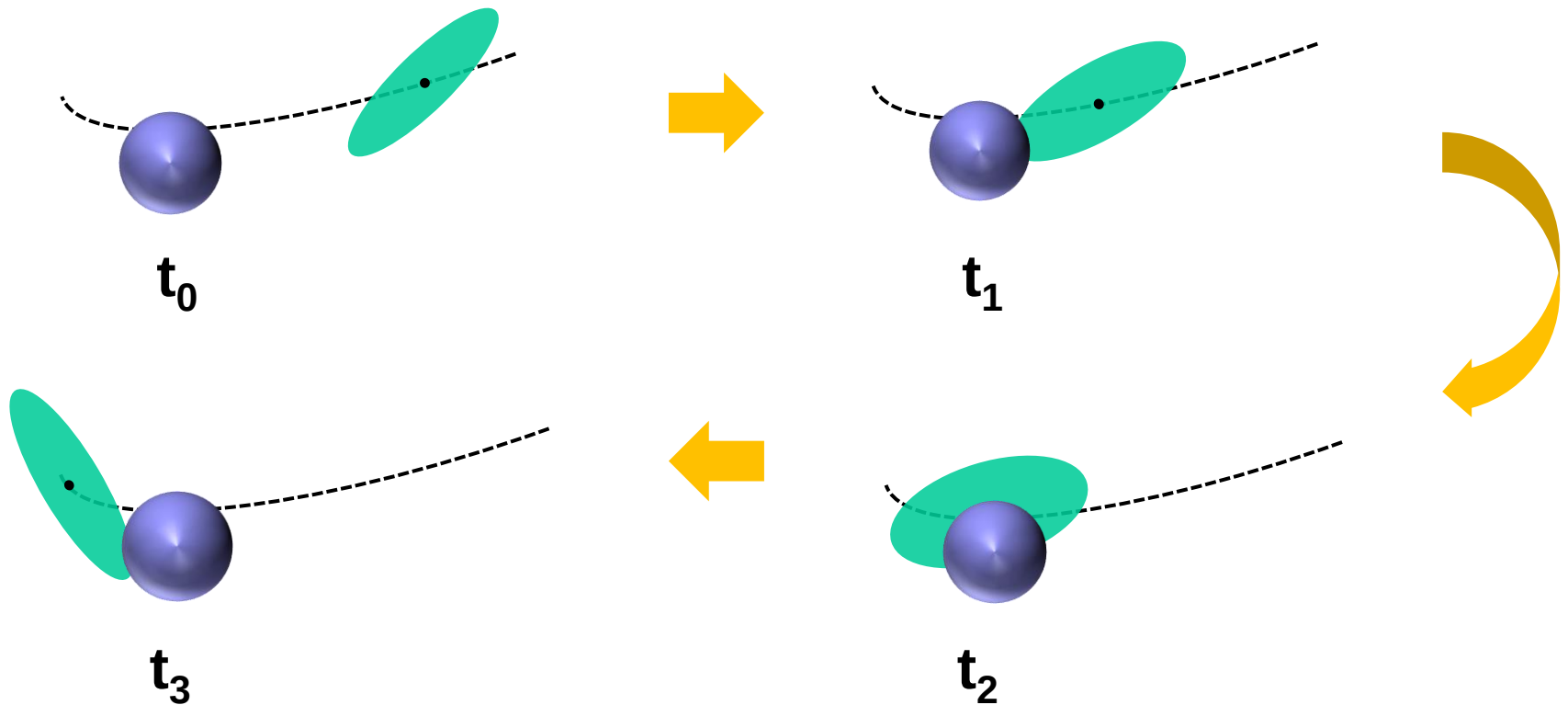
- **Pc for encounter a 1-D time integral of probability rate**

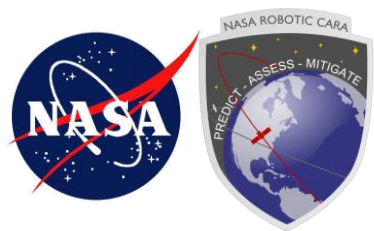
$$P_c = P_0 + \int_{t_0}^{t_0+T} \left(\frac{dP_c}{dt} \right) dt$$

- Integration bounds can usually be chosen to drive P_0 essentially to zero

3-D Pc Pictorial Progression

- Blue sphere is primary (as size of HBR); green ellipsoid is combined covariance ($1-\sigma$); black path is relative trajectory



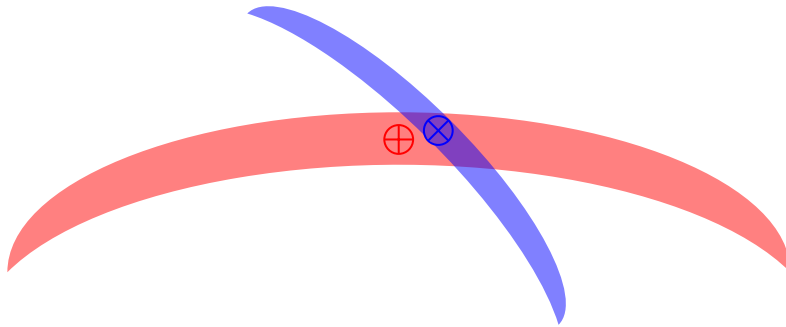


Schematic Illustrations for Two Conjunctions at Time of Maximum Pc Accumulation

CONJUNCTION A

Overlap occurs near the centers of both PDFs:

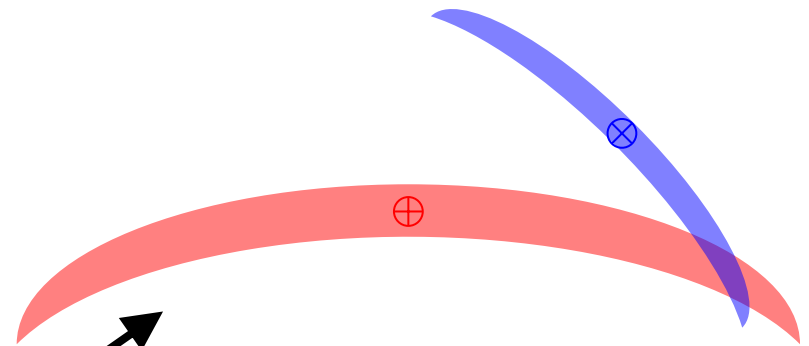
Monte Carlo-Pc $\approx$ 3D-Pc



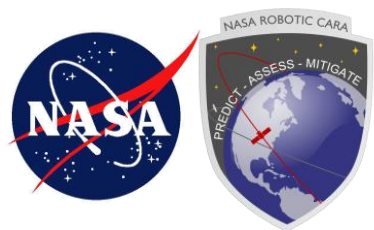
CONJUNCTION B

Overlap occurs near the end of either PDF, or both PDFs:

Monte Carlo-Pc $\neq$ 3D-Pc



Q1: Why is 3D-Pc inaccurate in these situations?
Q2: How does 3D-Nc address the issue?



First-order Expansions of the Motion: the original 3D-Pc Formulation

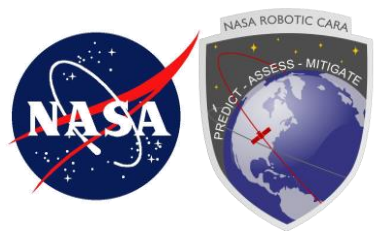
The original 3D-Pc formulation implicitly uses first-order Taylor series to approximate the orbital motion of both satellites, centering the expansions on their propagated mean states

3D-Pc centers the Taylor series approximating the motion of the secondary here

3D-Pc centers the Taylor series approximating the motion of the primary here

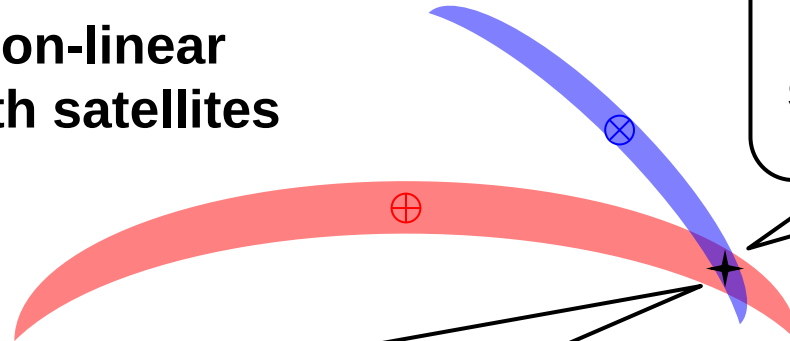
But this peak-overlap region is where both approximations need to be the most accurate....

Q1: Why is 3D-Pc inaccurate in these situations?
A1: Because the Taylor series are poorly centered.



First-order Expansions of the Motion: the 3D-Nc Formulation

The 3D-Nc formulation also uses first-order Taylor series to approximate the non-linear orbital motion of both satellites



3D-Nc centers the first-order Taylor series expansions for both satellites here

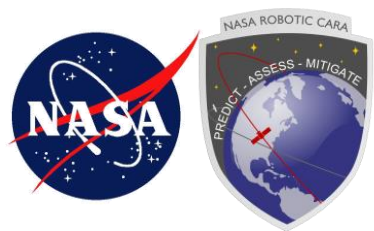
3D-Nc uses an iterative numerical algorithm to find the peak-overlap location of the two PDFs.

After the 1st iteration: $3D-Nc = 3D-Pc$

After iterative convergence: $3D-Nc \neq 3D-Pc$

Q2: How does 3D-Nc address the issue?

A2: It uses the same, optimized expansion-center location for both primary and secondary series expansions.



Optimally Centered 1st Order Series Corrects the Inaccuracies of the Original 3D-Pc Method

- The single-encounter probability rate of the original 3D-Pc method is given by a integral over the unit sphere¹

$$\dot{P}_c(t) = R^2 \oint d\hat{r} [\mathcal{F}(\hat{r}, R, \bar{\mathbf{X}}(t), \bar{\mathbf{P}}(t))]$$

- The 3D-Nc rate is also a unit-sphere integral, with an integrand of the same form, but using different arguments²

$$\dot{N}_c(t) = R^2 \oint d\hat{r} [\mathcal{F}(\hat{r}, R, \tilde{\mathbf{X}}(t), \tilde{\mathbf{P}}(t))]$$

- The different arguments arise because 3D-Nc uses optimal series expansion center points (rather than just using the mean orbital states)

27424_conj_31750_20171124_160046_20171118_071345

HBR=20 CA=6.97km RelV= 14.8km/s Angle=162°

PROP=6.47+12.1=18.6day CPU=1.99e6s

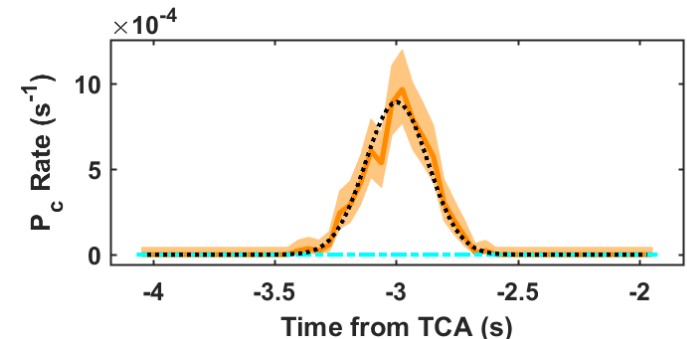
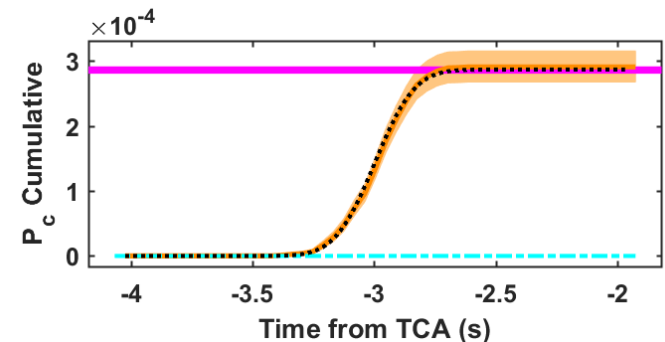
— Foster & Estes (1992) 2D-P_c = 2.86218e-4

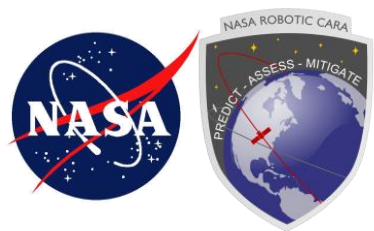
- - - Coppola (2012) 3D-P_c = 2.11075e-13

..... Hall (2020) 3D-N_c = 2.86932e-4

— BFMC (582 hits / 2e6 trials) P_c = 2.91e-4

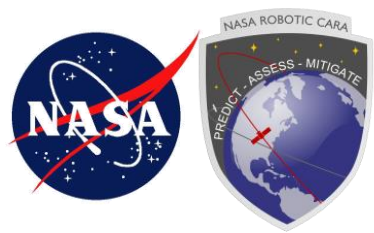
■ BFMC 95% confidence 2.68e-4 ≤ P_c ≤ 3.16e-4



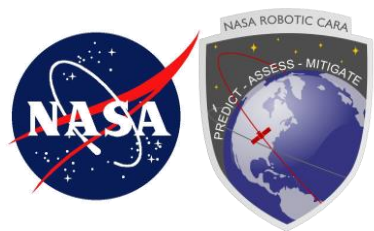


3D-Nc Formulation: Summary

- **The “3D-Nc” formulation addresses the 3D-Pc inaccuracy issue**
 - Estimates the statistically expected number of collisions (N_c)
 - For single encounters: $N_c = P_c$
 - For multi-encounter interactions: $N_c \geq P_c$
- **Formulated using an orbital state space based on Equinoctial elements along with time-invariant PDFs**
 - Employs first-order Taylor series approximations for the non-linear motion of both satellites, using optimized expansion-center states (rather than simply using mean states)
 - Yields single-encounter equations with the same functional form as 3D-Pc, but containing different specific mathematical terms
- **3D-Nc method can be used to decide when high-fidelity BFMC calculations are required**
 - 3D-Nc algorithm has been designed specifically to identify and process repeating/blended conjunctions, some of which will need BFMC

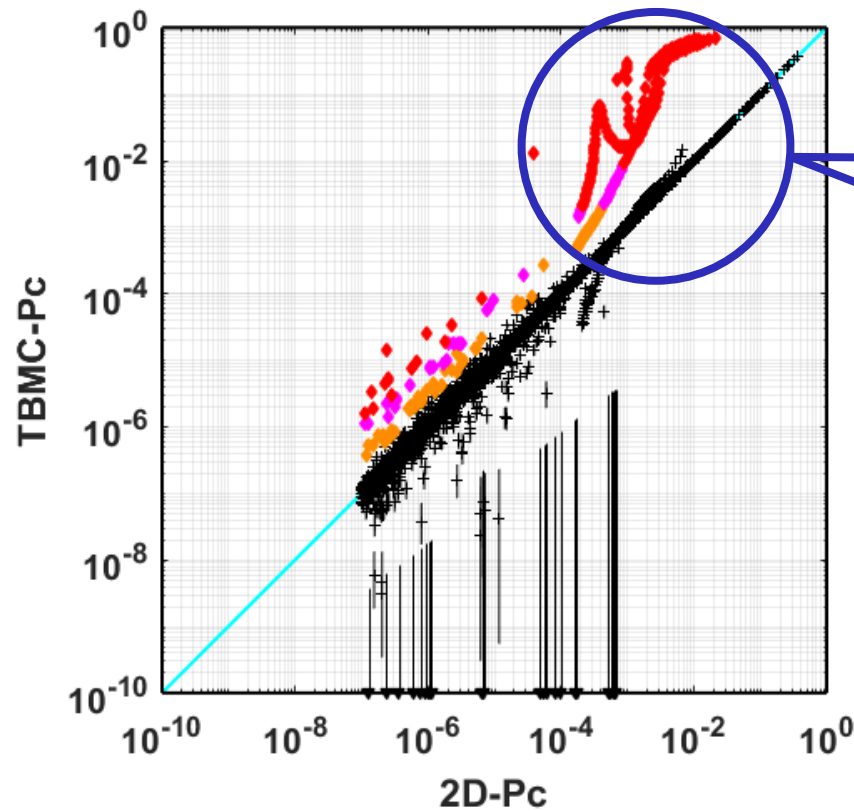


Comparison of Monte Carlo Pc Estimates to those from the 2D-Pc and 3D-Nc Methods for Single Encounters

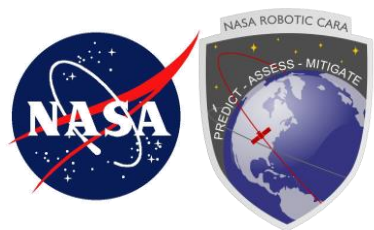


TBMC-Pc vs 2D-Pc for 64,376 CARA Conjunctions

- + TBMC-Pc/2D-Pc < 2.5 (64029 of 64376)
- ◆ $2.5 \leq \text{TBMC-Pc}/2\text{D-Pc} < 5$ (88 = 0.137%)
- ◆ $5 \leq \text{TBMC-Pc}/2\text{D-Pc} < 10$ (56 = 0.087%)
- ◆ $\text{TBMC-Pc}/2\text{D-Pc} \geq 10$ (203 = 0.315%)

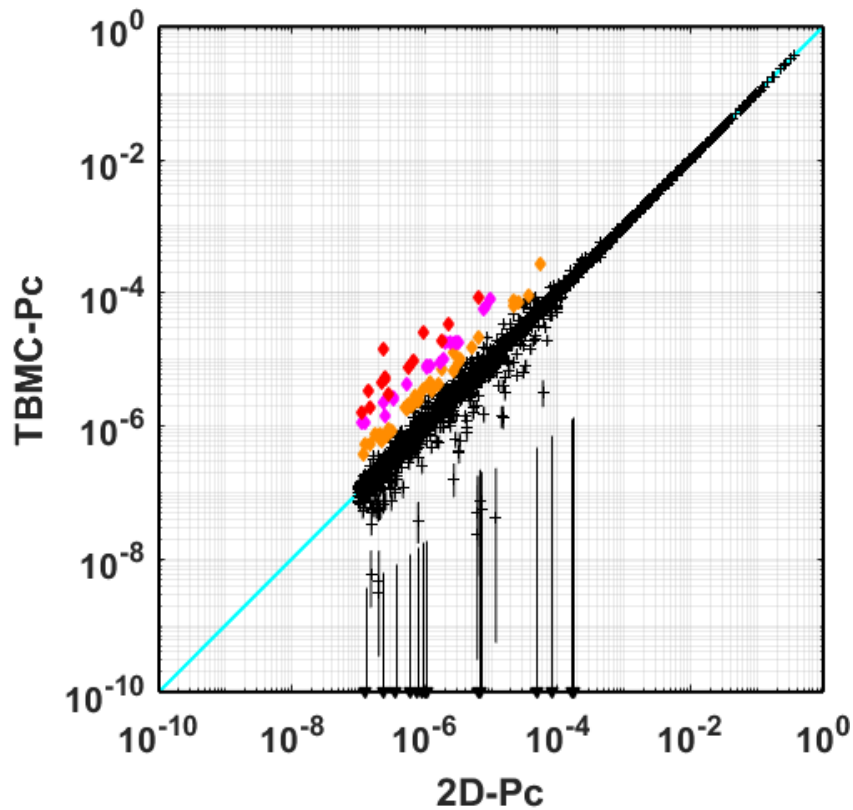


Repeating
conjunctions
involving ISS

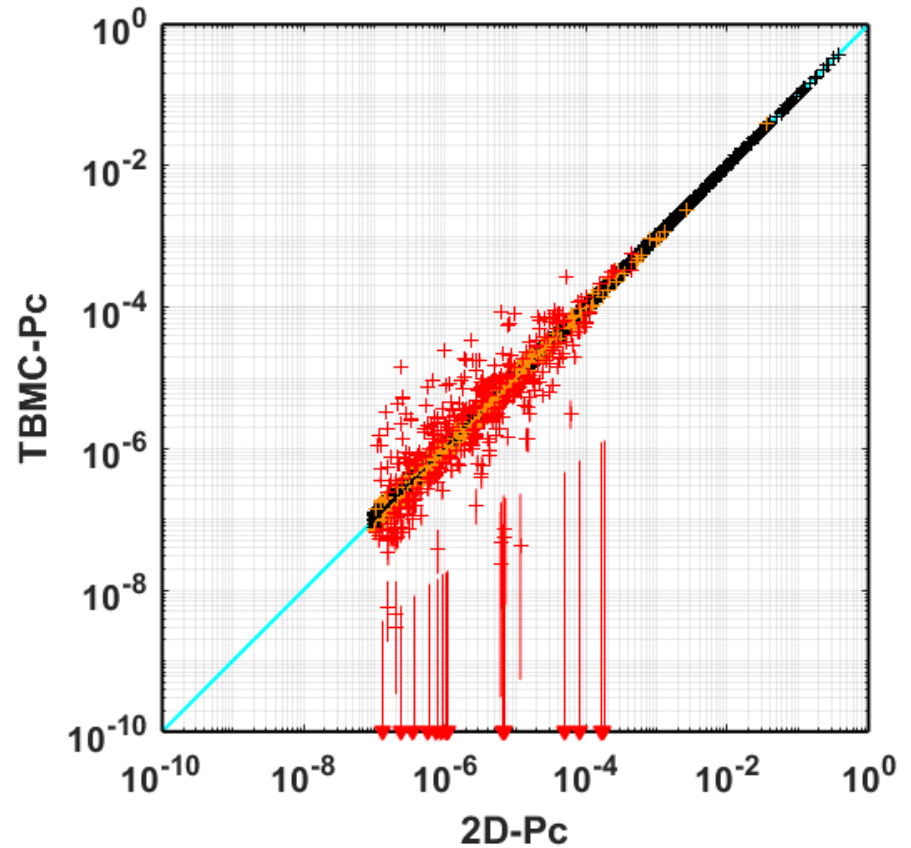


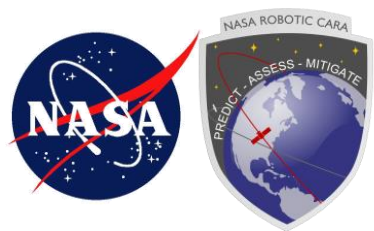
TBMC-Pc vs 2D-Pc for 63,603 CARA Conjunctions

- + TBMC-Pc/2D-Pc < 2.5 (63531 of 63603)
- ◆ $2.5 \leq \text{TBMC-Pc}/2\text{D-Pc} < 5$ (35 = 0.055%)
- ◆ $5 \leq \text{TBMC-Pc}/2\text{D-Pc} < 10$ (22 = 0.035%)
- ◆ $\text{TBMC-Pc}/2\text{D-Pc} \geq 10$ (15 = 0.024%)



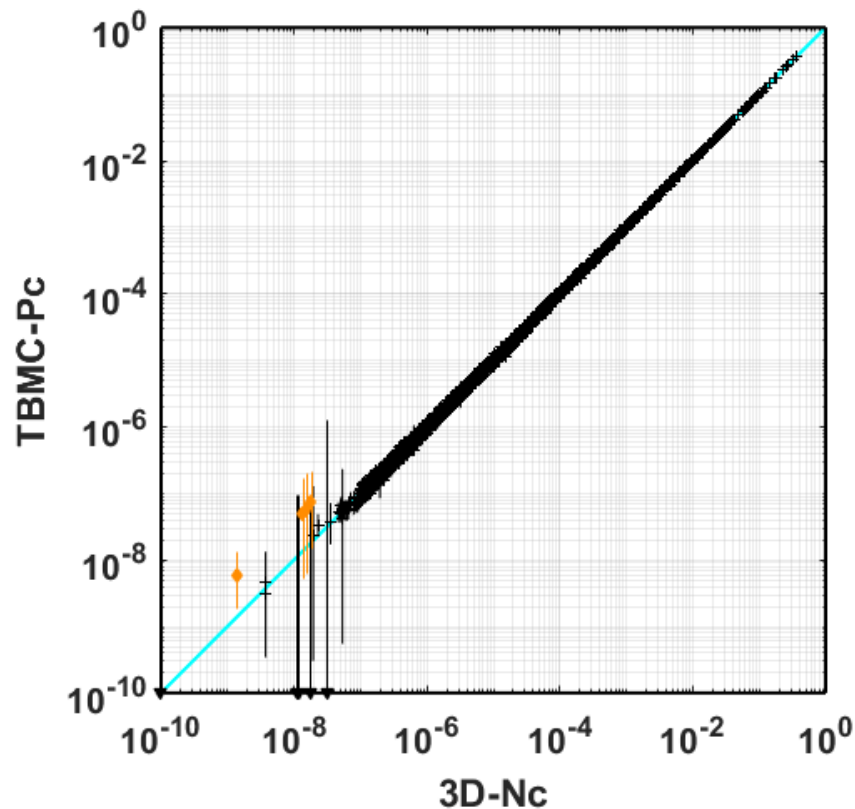
- + Pvalue > 1e-3 (62913 of 63603)
- + $1e-6 < \text{Pvalue} \leq 1e-3$ (254 = 0.399%)
- + $\text{Pvalue} \leq 1e-6$ (436 = 0.686%)



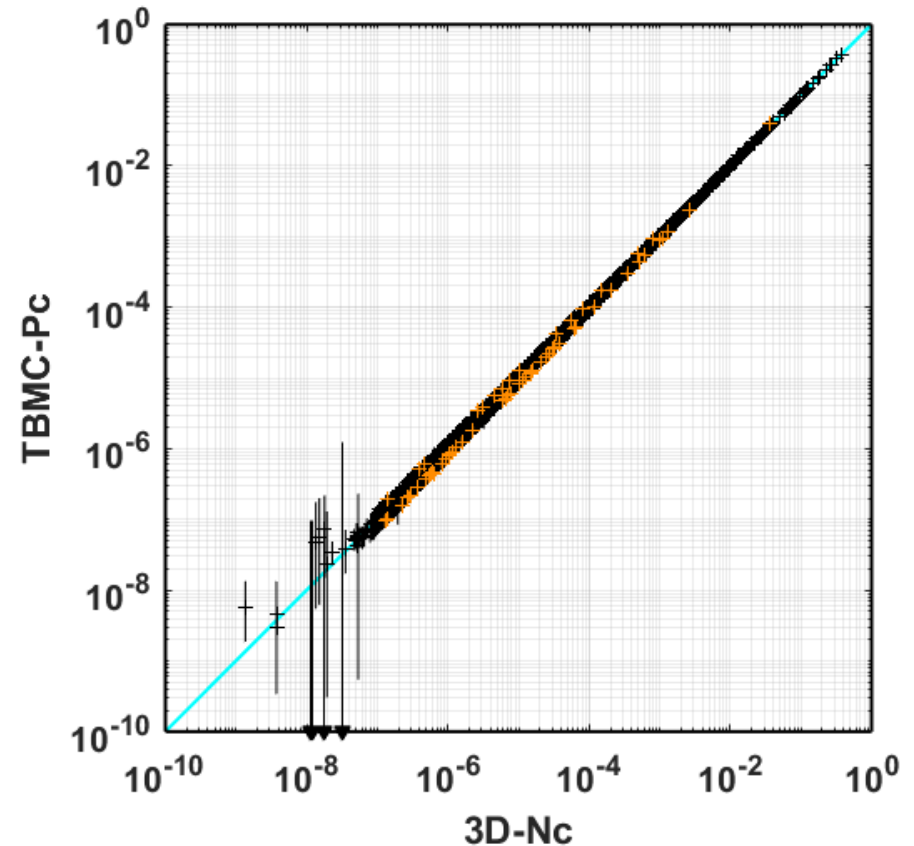


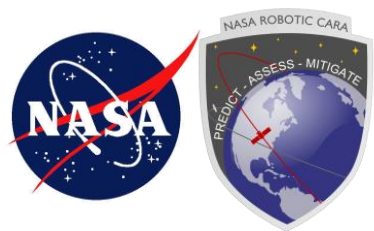
TBMC-Pc vs 3D-Nc for 63,603 CARA Conjunctions

- + TBMC-Pc/3D-Nc < 2.5 (63599 of 63603)
- ◆ $2.5 \leq \text{TBMC-Pc}/3\text{D-Nc} < 5$ (4 = 0.006%)
- ◆ $5 \leq \text{TBMC-Pc}/3\text{D-Nc} < 10$ (0 = 0.000%)
- ◆ TBMC-Pc/3D-Nc ≥ 10 (0 = 0.000%)



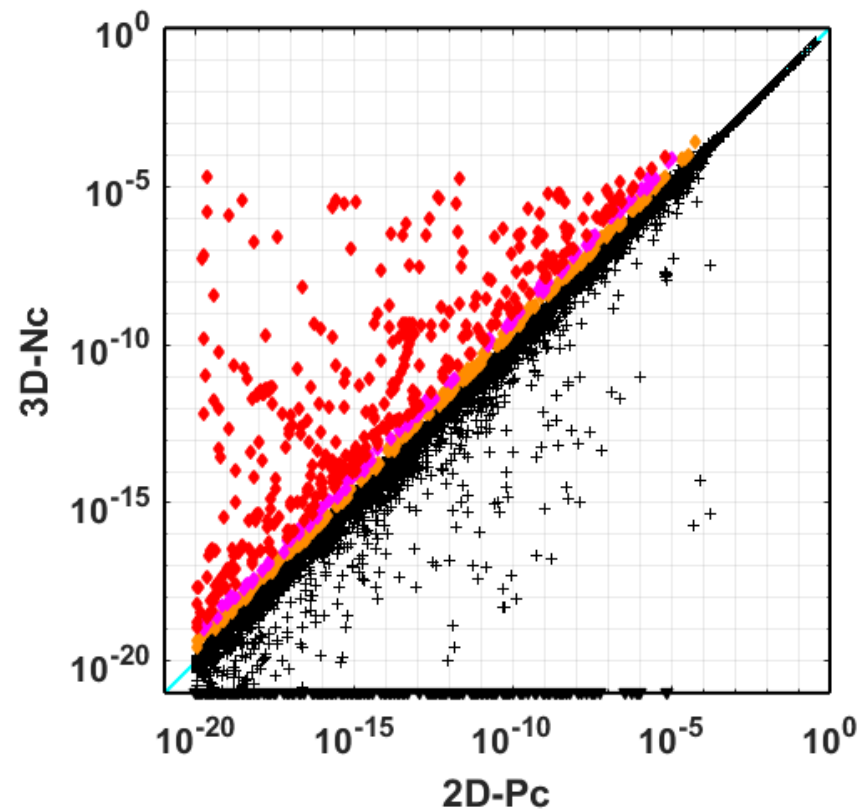
- + Pvalue > 1e-3 (63538 of 63603)
- + $1e-6 < \text{Pvalue} \leq 1e-3$ (65 = 0.102%)
- + Pvalue $\leq 1e-6$ (0 = 0.000%)





3D-Nc vs 2D-Pc for 164,601 CARA Conjunctions

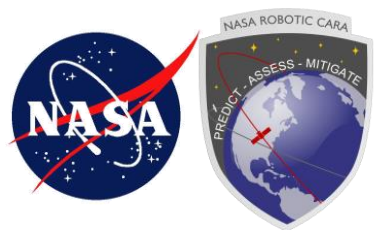
- + 3D-Nc/2D-Pc < 2.5 (163710 of 164601)
- ◆ 2.5 ≤ 3D-Nc/2D-Pc < 5 (891 = 0.541%)
- ◆ 5 ≤ 3D-Nc/2D-Pc < 10 (605 = 0.368%)
- ◆ 3D-Nc/2D-Pc ≥ 10 (470 = 0.286%)



3D-Nc analysis of a large number of archived conjunctions reveals the existence of some additional 2D-Pc failures

Several of these new cases have
 $2D-Pc < 10^{-10}$
and $3D-Nc > 10^{-7}$

No cases have been discovered with
 $2D-Pc < 10^{-10}$
and $3D-Nc > 10^{-4}$



Example 2D-Pc Failure:

29499_conj_31746_20170820_050235_20170813_221640

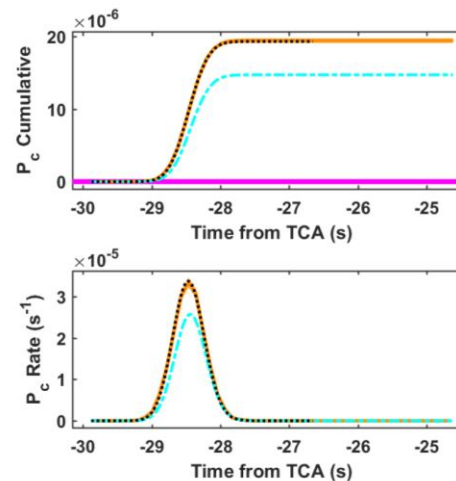
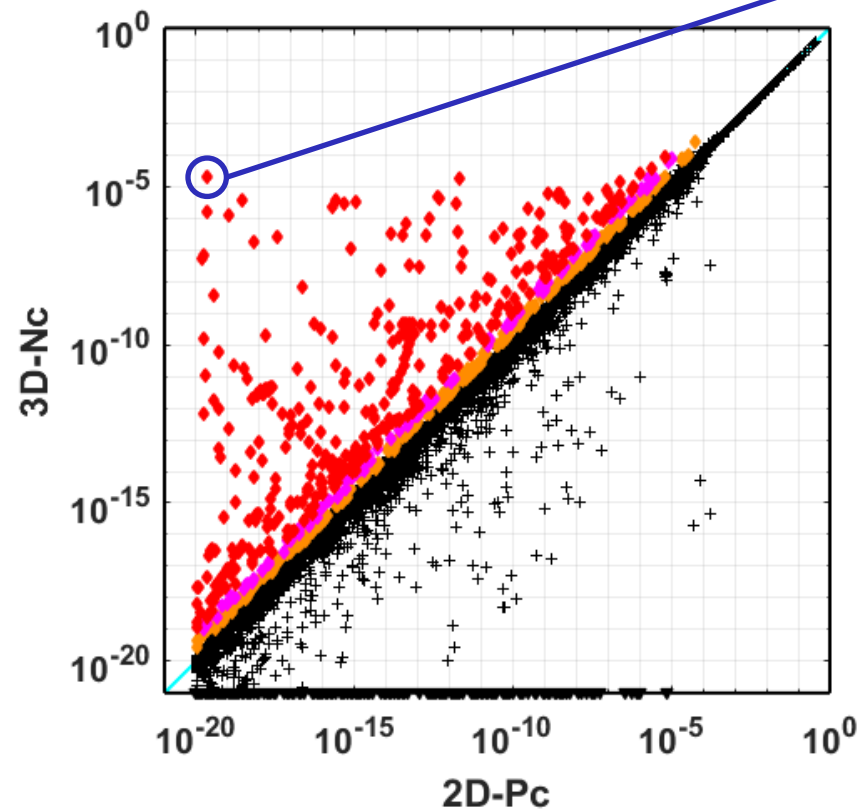
- + 3D-Nc/2D-Pc < 2.5 (163710 of 164601)
- ◆ 2.5 ≤ 3D-Nc/2D-Pc < 5 (891 = 0.541%)
- ◆ 5 ≤ 3D-Nc/2D-Pc < 10 (605 = 0.368%)
- ◆ 3D-Nc/2D-Pc ≥ 10 (470 = 0.286%)

2D-Pc = 2.27E-20

3D-Nc = 1.93E-05

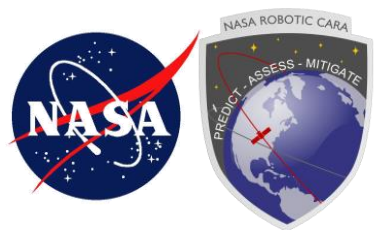
TBMC-Pc = 1.91E-05 to 1.97E-05 (95%)
(19,394 hits / 1e9 samples)

BFMC-Pc = 1.6E-05 to 3.7E-05 (95%)
(25 hits / 1e6 samples)



29499_conj_31746_20170820_050235_20170813_221640
HBR=20m MissDistance=7306.1m VelocityAngle=0.5°

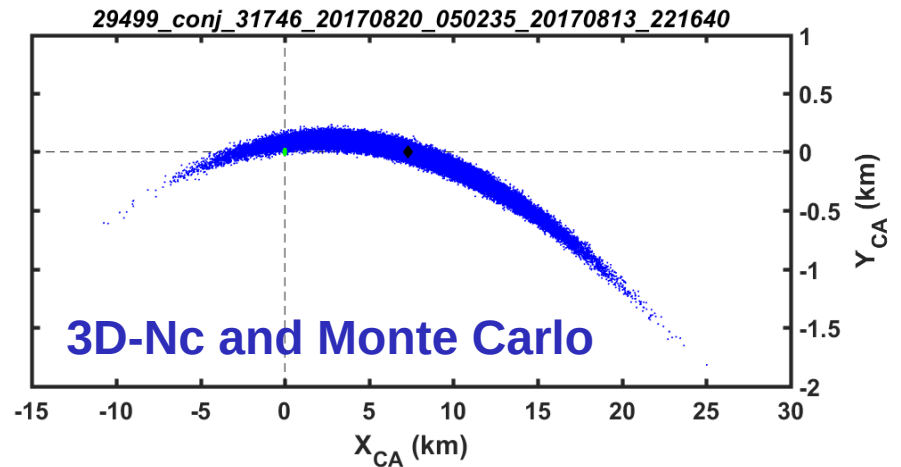
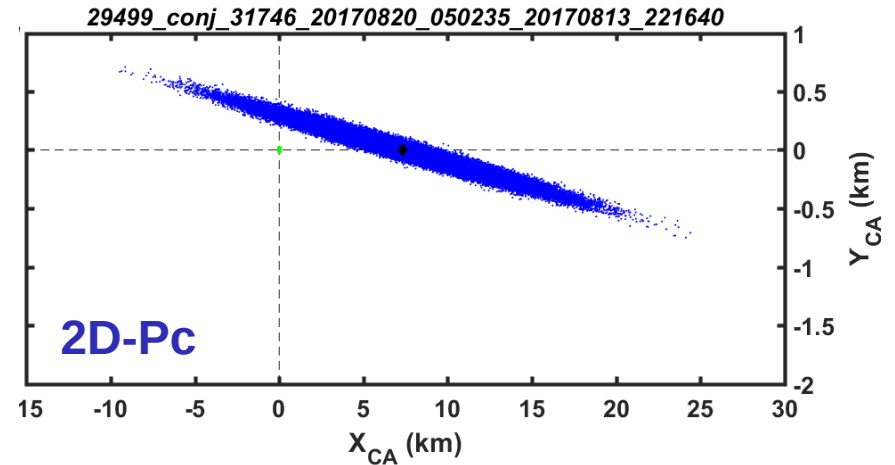
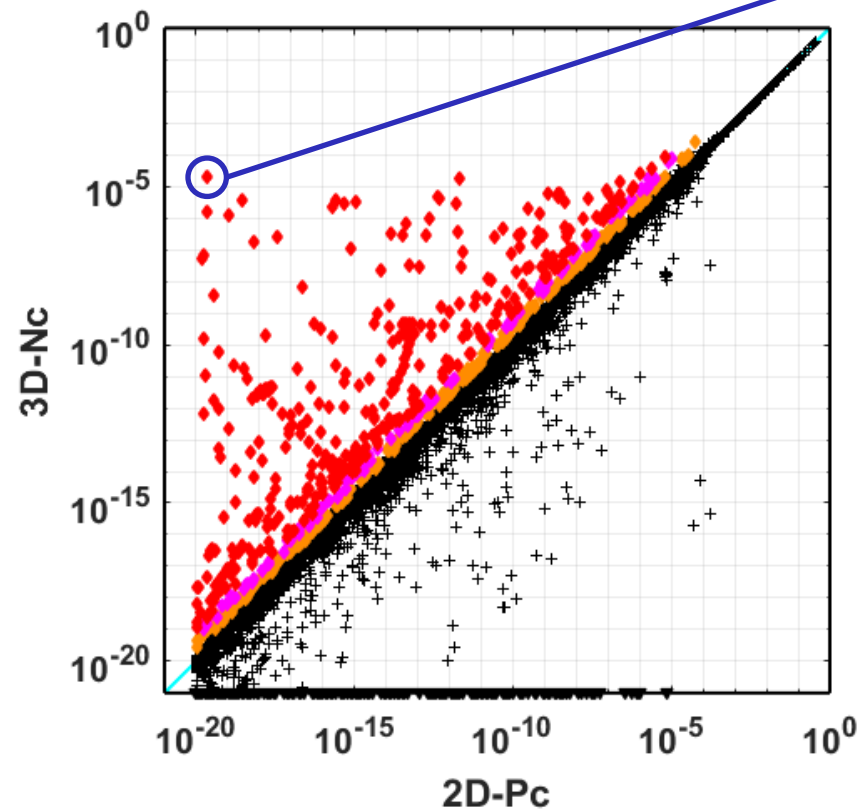
- Foster & Estes (1992) 2D- $P_c = 2.26597e-20$
- - - Coppola (2012) 3D- $P_c = 1.468e-05$
- Hall (2020) 3D- $N_c = 1.92724e-5$
- TBMC (1e9 trials) $P_c = 1.938e-5$
- TBMC 95% confidence $1.911e-5 \leq P_c \leq 1.966e-5$

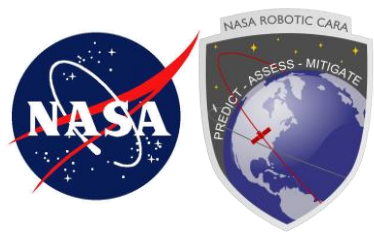


B-Plane Plots for 2D-Pc Failure:

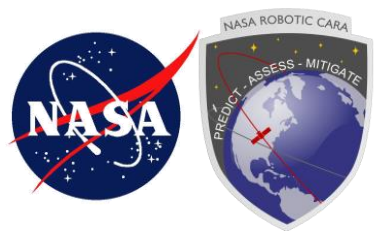
29499_conj_31746_20170820_050235_20170813_221640

- + 3D-Nc/2D-Pc < 2.5 (163710 of 164601)
- ◆ 2.5 ≤ 3D-Nc/2D-Pc < 5 (891 = 0.541%)
- ◆ 5 ≤ 3D-Nc/2D-Pc < 10 (605 = 0.368%)
- ◆ 3D-Nc/2D-Pc ≥ 10 (470 = 0.286%)





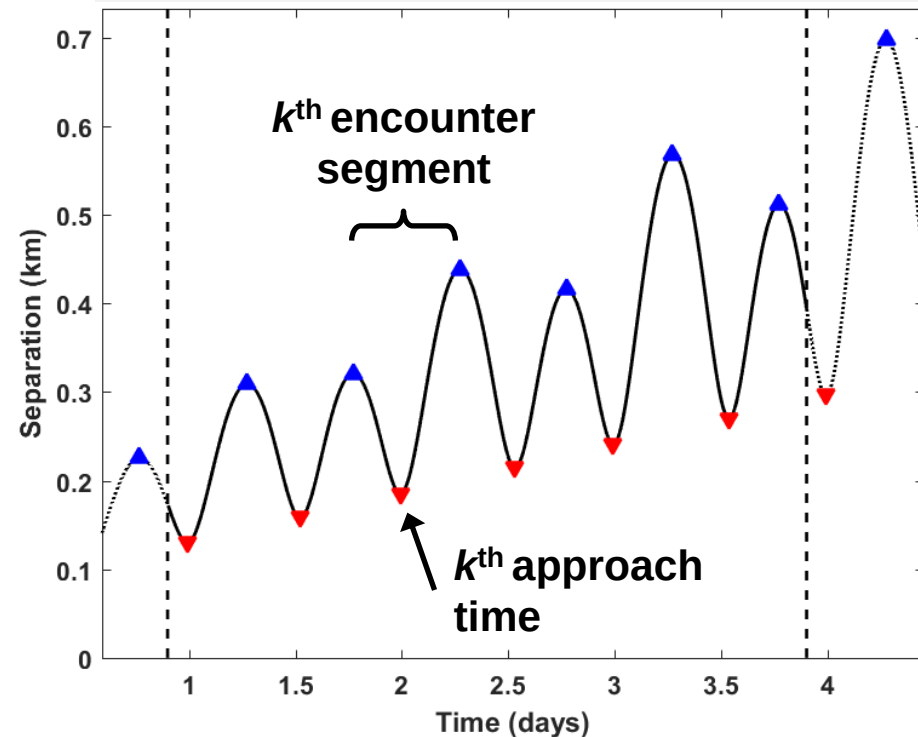
Comparison of Monte Carlo Pc Estimates to those from the 3D-Nc Methods for Repeating Conjunctions

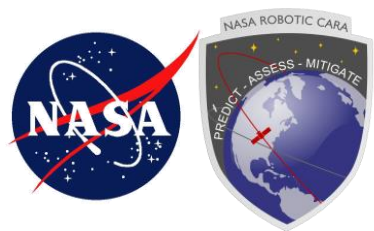


The 3D-Nc Method for Multi-Encounter Collision Risk Assessment

- Divide multi-encounter interactions into time segments, each containing a single approach
- Determine number of collisions (P_c , essentially) for each segment
- If individual segments are largely to entirely uncorrelated, then composite risk / number of collisions easy to calculate
- If not, then conjunctions are blended and BFMC is needed

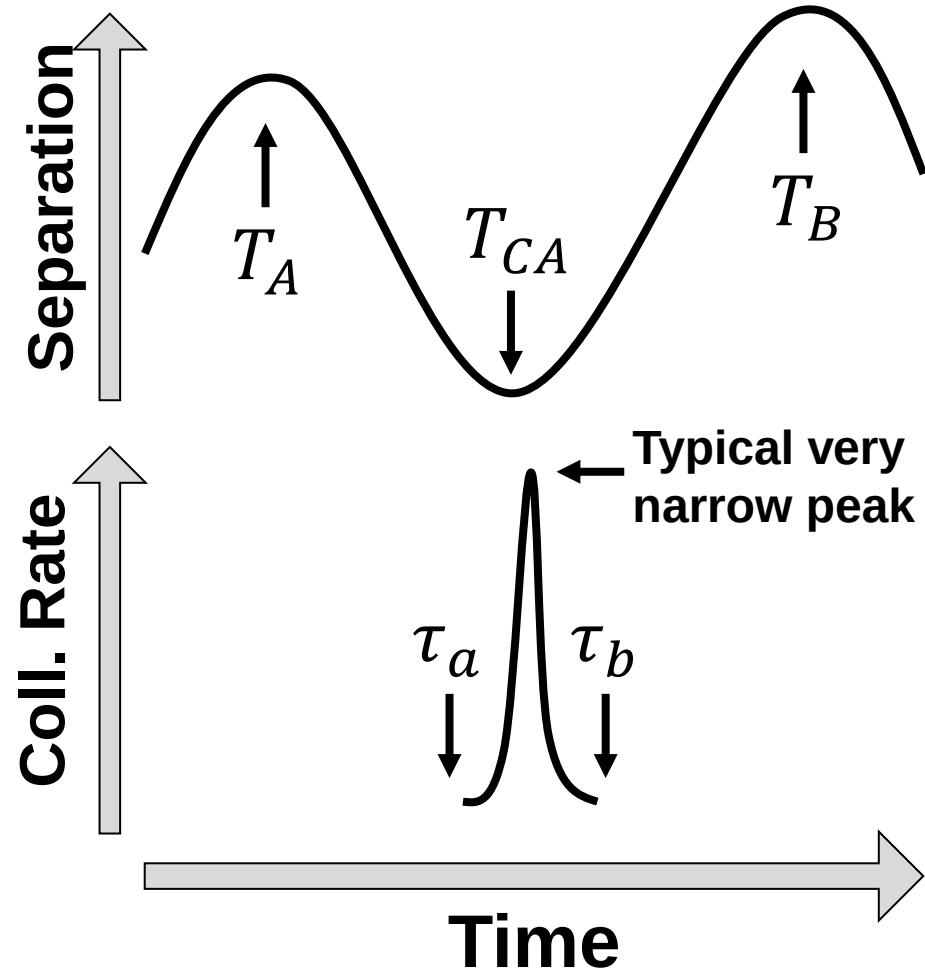
Nominal distance between two closely-orbiting GEO satellites

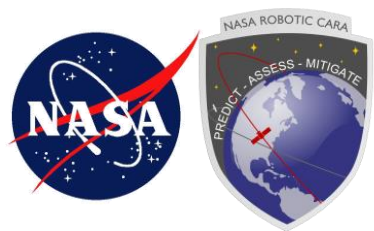




The Expected Collision Rate Evaluated for Each Encounter Segment

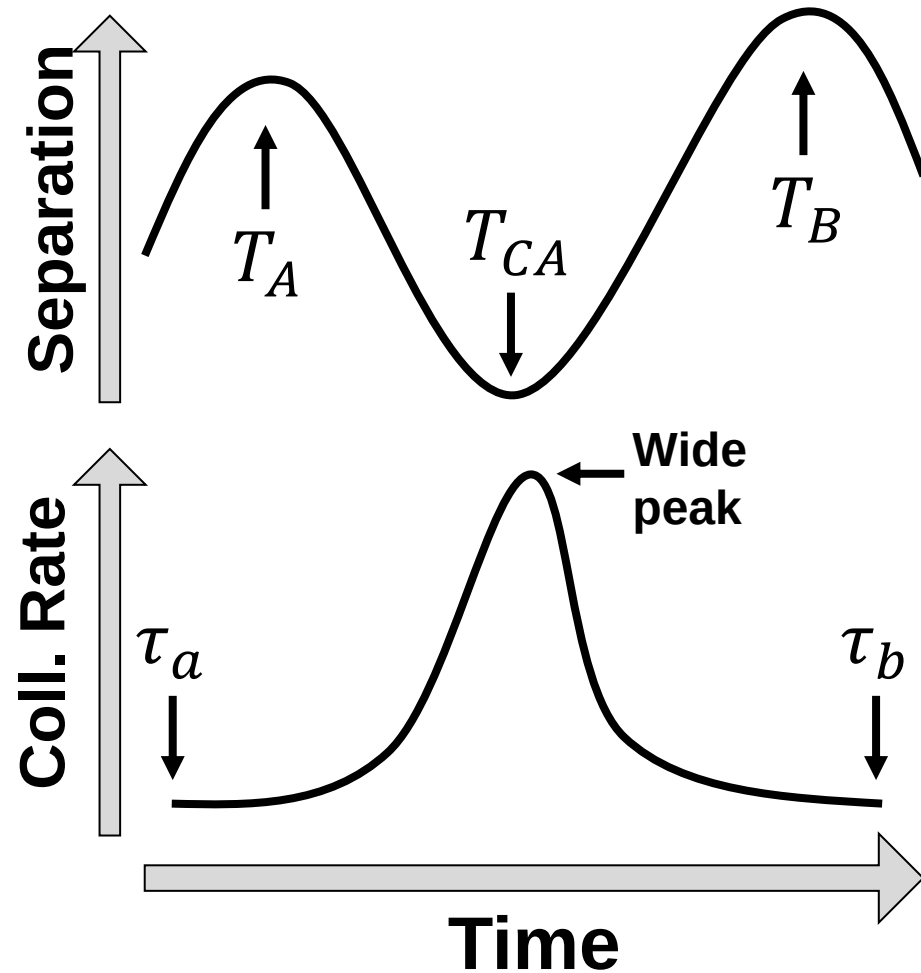
- **Most satellite conjunctions have very high relative velocities**
 - Typical for most CIRA events
 - Very short conjunction durations
- **Expected collision rates tend to peak sharply at or near the nominal close approach time**
 - Orbital perturbations can be neglected over such short time scales
 - Justifies approximating trajectories using two-body equations of motion
 - Trajectories relative to the close-approach position are approximated as segments of Keplerian elliptical orbits

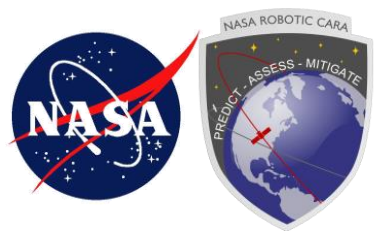




Low-Velocity Interactions: Extended and Blended Encounters

- For low-velocity encounters, expected collision rate curves can have considerably wider peaks
- The resulting extended conjunction durations can degrade the accuracy of the two-body motion approximation
- Low velocities can also shift the peak time, increasing the likelihood that the collision rate curves become blended with those of adjacent conjunctions
- **BFMC follow-up analysis should be performed for such extended or blended encounters**





3D-Nc Method Usage Violations Evaluated for Each Encounter Segment

- Extended duration violations:

$$\tau_b - \tau_a \geq F_E(T_B - T_A)$$

- Blended duration violations:

$$\tau_a - T_A \leq F_B(T_{CA} - T_A)$$

or

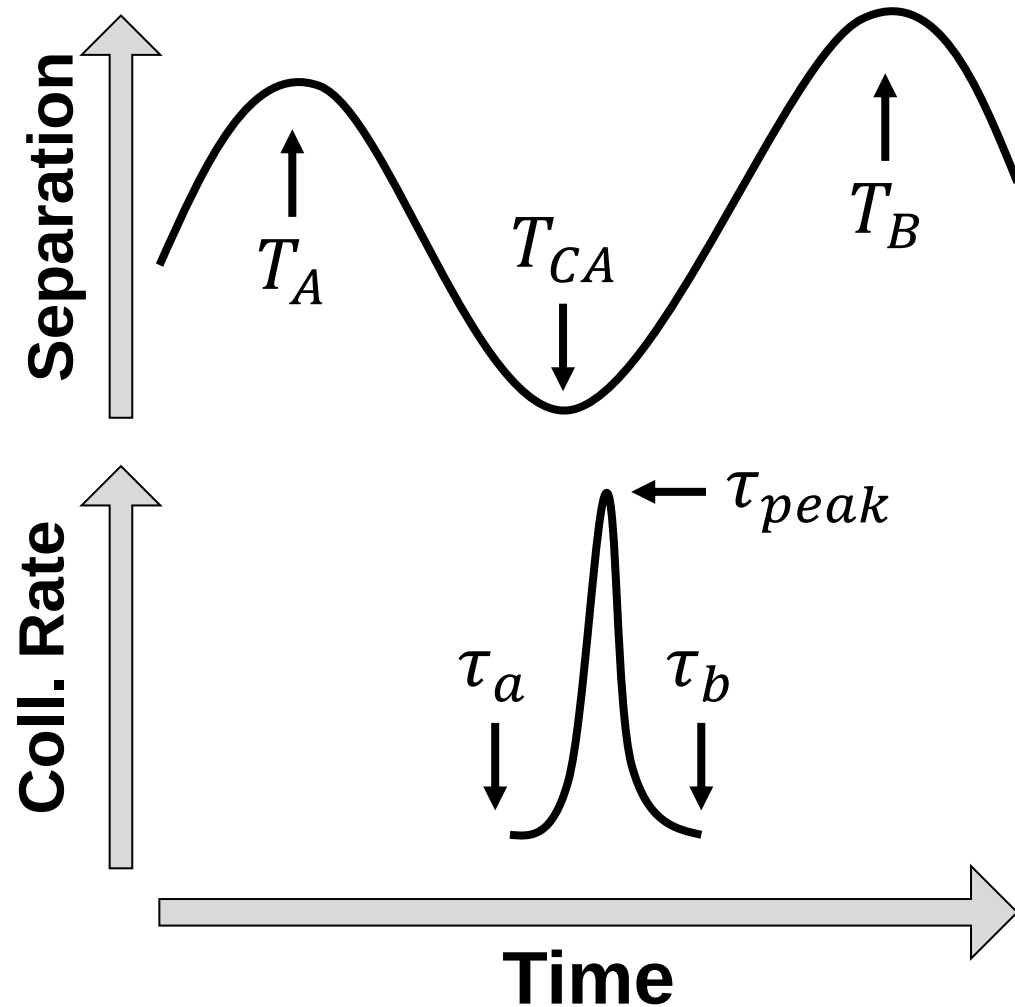
$$T_B - \tau_b \leq F_B(T_B - T_{CA})$$

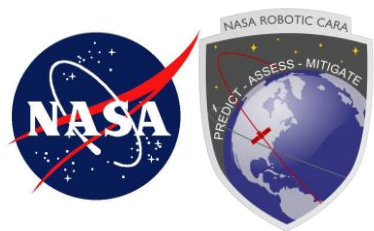
- NPD remediation violations:

$$|N_{c,k} - N_{c,k}^{rem}| \geq F_N N_{c,k}$$

- Empirical threshold factors:

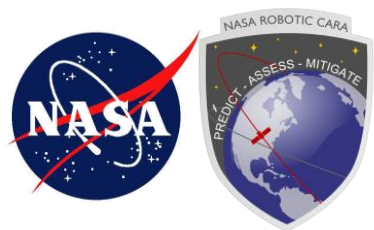
$$F_E = F_B = 0.5 \quad F_N = 0.05$$





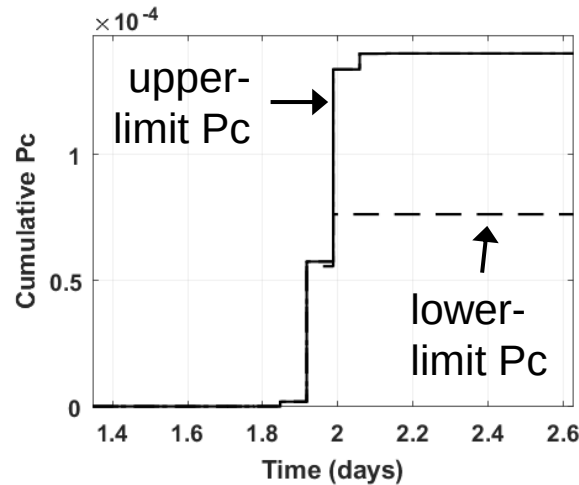
BFMC Follow-up Analysis Recommendations and Priorities

- **BFMC follow-up analysis estimates the cumulative P_c value for a specified primary/secondary pair of satellites**
 - Typically over a multi-day risk assessment interval (e.g., 7 or 10 days)
- **BFMC recommendation priorities should include the following considerations:**
 - 3D-Nc usage violations: extended, blended, and/or NPD-affected
 - The potential for BFMC to change best-estimate P_c (multi-encounter)
- **These can be combined into a single priority**
 - Constructed to vary from 0% to 100%, least to most important
 - Most CARA screenings produce no BFMC follow-up recommendations



Modest Repeating Conjunction: Analysis of Screening CDMs (i.e., CNJs)

3D-Nc method
cumulative $P_c \rightarrow$
upper & lower limits



--- OCMDB FILE CONJUNCTION ANALYSIS ---
OCMDB_20170501_to_20190815_Nc1255043.mat
Screening creation time: 2017-09-30 06:37:16

Primary (Last Obs. = 2 hours)
Secondary (Last Obs. = 2.5 days)
Combined hard-body radius = 6 m

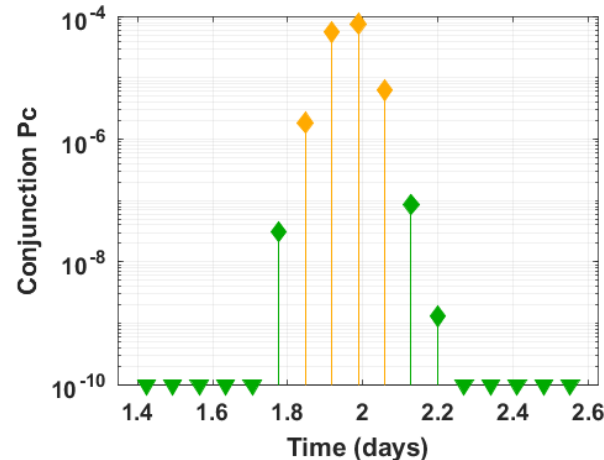
Number of conjunctions: 17
(7 with $P_c \geq 1e-10$)
Min conj. CA = 293.2 m @ 2017-10-02 04:37:38.331
Max conj. $P_c = 7.62e-5$ @ 2017-10-02 06:19:08.536

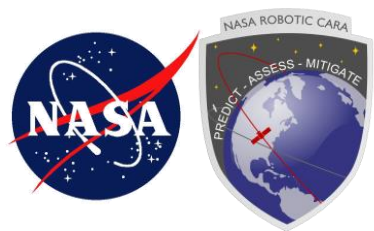
Collision risks estimated using 3D-Nc method:
Expected number of collisions: $N_c = 1.4e-4$
Accumulated collision probability: $P_c \leq 1.4e-4$

3D-Nc analysis method usage violations
(for events with $P_c \geq 1e-10$):
Affected by NPD covariances: 0 of 7 <no violation>
Extended or blended in time: 0 of 7 <no violation>

--- ANALYSIS RECOMMENDATION ---
Confirm with BFMC method, if possible (priority = 4.4%)
(estimated potential P_c reduction from BFMC $\leq 45.6\%$)

3D-Nc method
individual
conjunction $P_c \rightarrow$
estimates

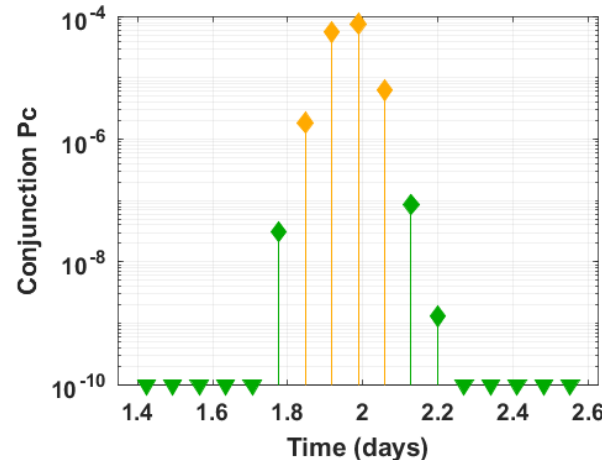
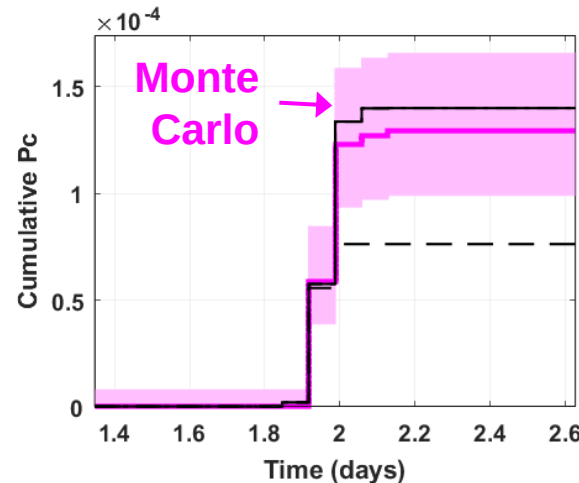




Modest Repeating Conjunction: Operationally Useful BFMC Analysis

BFMC + 3D-Nc
cumulative Pc →
estimates

3D-Nc method
individual
conjunction Pc →
estimates



--- OCMDB FILE CONJUNCTION ANALYSIS ---
OCMDB_20170501_to_20190815_Nc1255043.mat
Screening creation time: 2017-09-30 06:37:16

Primary (Last Obs. = 2 hours)
Secondary (Last Obs. = 2.5 days)
Combined hard-body radius = 6 m

Number of conjunctions: 17
(7 with $P_c \geq 1e-10$)
Min conj. CA = 293.2 m @ 2017-10-02 04:37:38.331
Max conj. $P_c = 7.62e-5$ @ 2017-10-02 06:19:08.536

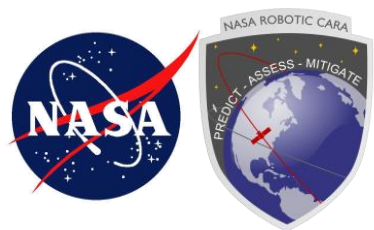
Collision risks estimated using 3D-Nc method:
Expected number of collisions: $N_c = 1.4e-4$
Accumulated collision probability: $P_c \leq 1.4e-4$

3D-Nc analysis method usage violations
(for events with $P_c \geq 1e-10$):
Affected by NPD covariances: 0 of 7 <no violation>
Extended or blended in time: 0 of 7 <no violation>

--- ANALYSIS RECOMMENDATION ---
Confirm with BFMC method, if possible (priority = 4.4%)

BFMC $P_c = 1.29e-4 = 62/4.8e5$ (CPU 3.84e5s)
95% conf. $9.9e-5$ to $1.66e-4$ (check-out passed)

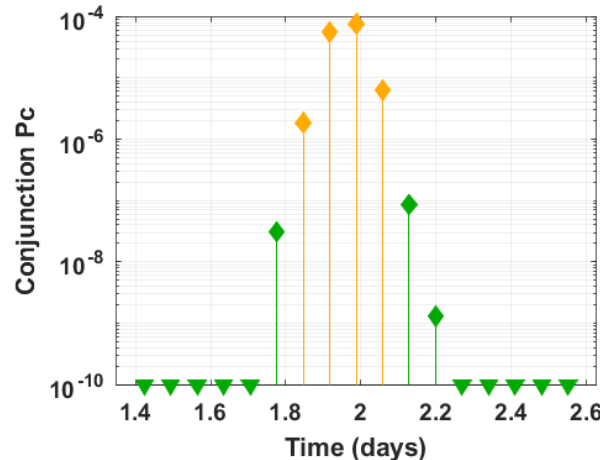
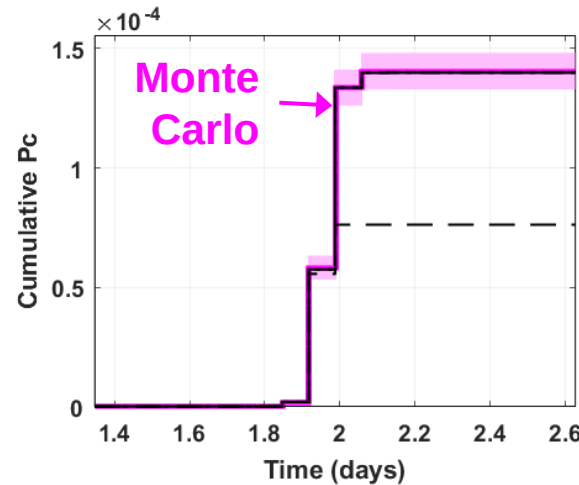
BFMC analysis results
(3.3-hours/32-processors)



Modest Repeating Conjunction: High Accuracy BFMC Analysis

**BFMC + 3D-Nc
cumulative Pc →
estimates**

**3D-Nc method
individual
conjunction Pc →
estimates**



--- OCMDB FILE CONJUNCTION ANALYSIS ---
OCMDB_20170501_to_20190815_Nc1255043.mat
Screening creation time: 2017-09-30 06:37:16

Primary (Last Obs. = 2 hours)
Secondary (Last Obs. = 2.5 days)
Combined hard-body radius = 6 m

Number of conjunctions: 17
(7 with $P_c \geq 1e-10$)
Min conj. CA = 293.2 m @ 2017-10-02 04:37:38.331
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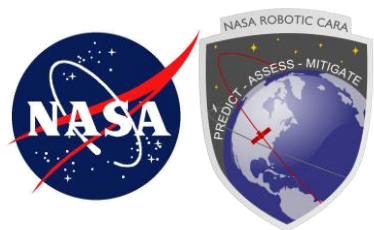
Collision risks estimated using 3D-Nc method:
Expected number of collisions: $N_c = 1.4e-4$
Accumulated collision probability: $P_c \leq 1.4e-4$

3D-Nc analysis method usage violations
(for events with $P_c \geq 1e-10$):
Affected by NPD covariances: 0 of 7 <no violation>
Extended or blended in time: 0 of 7 <no violation>

--- ANALYSIS RECOMMENDATION ---
Confirm with BFMC method, if possible (priority = 4.4%)

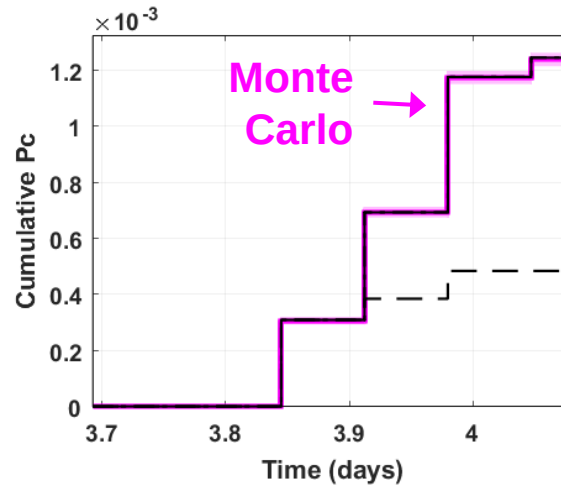
BFMC $P_c = 1.404e-4 = 1404/1e7$ (CPU 8e6s)
95% conf. $1.332e-4$ to $1.479e-4$ (check-out passed)

**BFMC analysis results
(2.9-days/32-processors)**

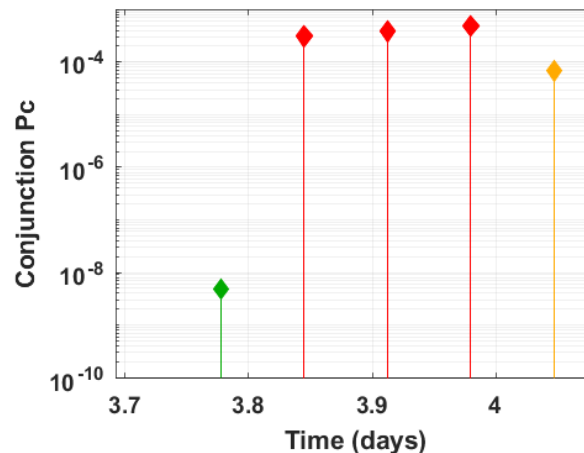


Repeating Conjunction: Very High Accuracy BFMC Analysis

**BFMC + 3D-Nc
cumulative Pc →
estimates**



**3D-Nc method
individual
conjunction Pc →
estimates**



--- OCMDB FILE CONJUNCTION ANALYSIS ---
OCMDB_20170501_to_20190815_Nc1255043.mat
Screening creation time: 2018-04-15 06:12:30

Primary (Last Obs. = 1.8 hours)
Secondary (Last Obs. = 7.8 hours)
Combined hard-body radius = 20 m

Number of conjunctions: 5
(5 with $P_c \geq 1e-10$)
Min conj. CA = 1.458 km @ 2018-04-19 04:06:06.628
Max conj. $P_c = 4.84e-4$ @ 2018-04-19 05:43:08.326

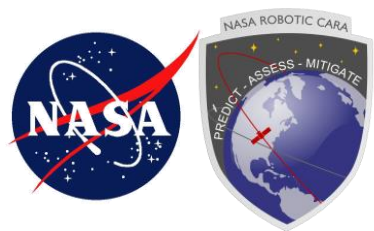
Collision risks estimated using 3D-Nc method:
Expected number of collisions: $N_c = 1.25e-3$
Accumulated collision probability: $P_c \leq 1.25e-3$

3D-Nc analysis method usage violations
(for events with $P_c \geq 1e-10$):
Affected by NPD covariances: 0 of 5 <no violation>
Extended or blended in time: 0 of 5 <no violation>

--- ANALYSIS RECOMMENDATION ---
Use these 3D-Nc method collision risk estimates

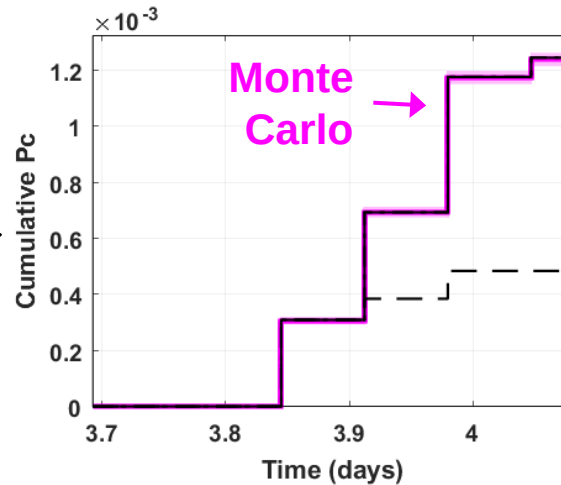
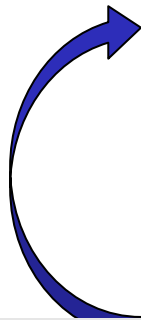
BFMC $P_c = 1.239e-3 = 12386/1e7$ (CPU 9.66e6s)
95% conf. $1.217e-3$ to $1.261e-3$ (check-out passed)

**BFMC analysis results
(3.5-days/32-processors)**



Repeating Conjunction: Very High Accuracy BFMC Analysis

BFMC + 3D-Nc
cumulative Pc →
estimates



The upper-limit cumulative Pc
estimate from the 3D-Nc method
matches the BFMC Pc estimate

Seems to apply to most CARA LEO
repeating conjunctions (but not all)

--- OCMDB FILE CONJUNCTION ANALYSIS ---
OCMDB_20170501_to_20190815_Nc1255043.mat
Screening creation time: 2018-04-15 06:12:30

Primary (Last Obs. = 1.8 hours)
Secondary (Last Obs. = 7.8 hours)
Combined hard-body radius = 20 m

Number of conjunctions: 5
(5 with Pc ≥ 1e-10)
Min conj. CA = 1.458 km @ 2018-04-19 04:06:06.628
Max conj. Pc = 4.84e-4 @ 2018-04-19 05:43:08.326

Collision risks estimated using 3D-Nc method:
Expected number of collisions: Nc = 1.25e-3
Accumulated collision probability: Pc ≤ 1.25e-3

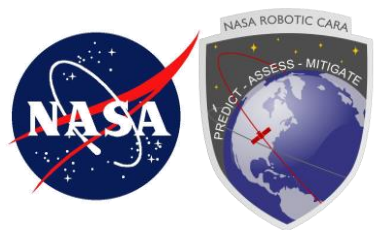
3D-Nc analysis method usage violations
(for events with Pc ≥ 1e-10):
Affected by NPD covariances: 0 of 5 <no violation>
Extended or blended in time: 0 of 5 <no violation>

--- ANALYSIS RECOMMENDATION ---
Use these 3D-Nc method collision risk estimates

BFMC Pc = 1.239e-3 = 12386/1e7 (CPU 9.66e6s)
95% conf. 1.217e-3 to 1.261e-3 (check-out passed)

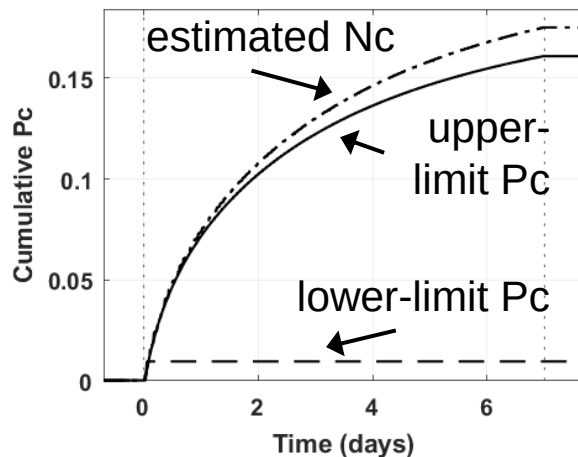


BFMC analysis results
(3.5-days/32-processors)

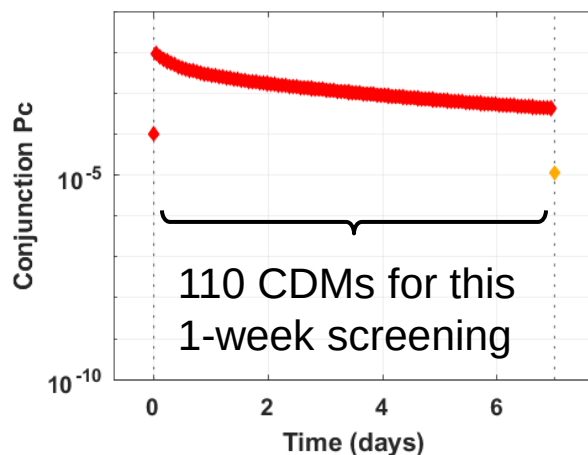


ISS and Soyuz Docking Failure Scenario: Very High Priority BFMC Recommendation

3D-Nc method
cumulative $P_c \rightarrow$
upper & lower limits



3D-Nc method
individual
conjunction $P_c \rightarrow$
estimates



--- OCMDB FILE CONJUNCTION ANALYSIS ---
OCMDB_20170501_to_20190815_Nc1255043.mat
Screening creation time: 2019-07-27 16:05:07

Primary 25544 (Last Obs. = 6.5 hours)
Secondary 44437 (Last Obs. = 9.6 hours)
Combined hard-body radius = 70 m

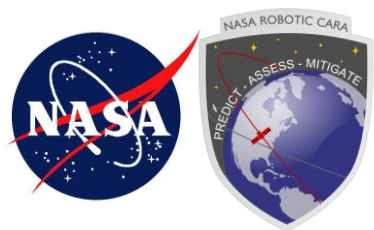
Number of conjunctions: 110
(110 with $P_c \geq 1e-10$)
Min conj. CA = 510.2 m @ 2019-07-27 17:14:06.758
Max conj. $P_c = 9.4e-3$ @ 2019-07-27 17:04:38.439

Collision risks estimated using 3D-Nc method:
Expected number of collisions: $N_c = 0.175$
Accumulated collision probability: $P_c \leq 0.161$

3D-Nc analysis method usage violations
(for events with $P_c \geq 1e-10$):
Affected by NPD covariances: 0 of 110 <no violation>
Extended or blended in time: 110 of 110 <violation>

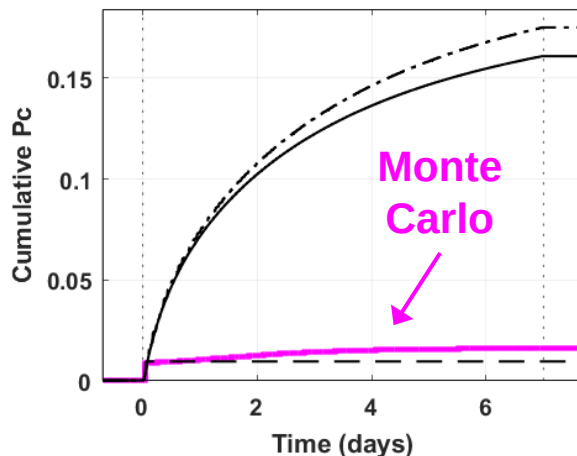
--- ANALYSIS RECOMMENDATION ---
Confirm with BFMC method, if possible (priority = 99.9%)

↑
BFMC follow-up analysis
recommended

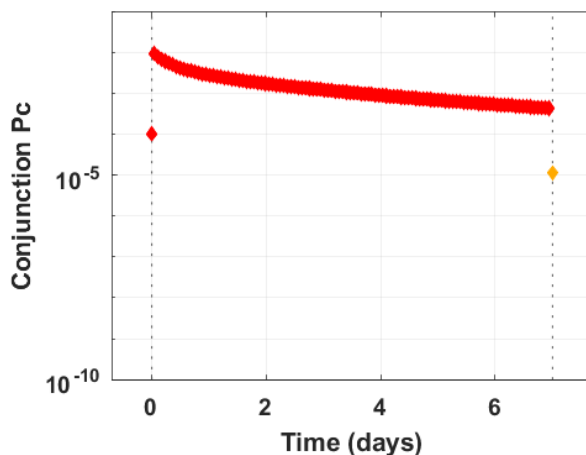


ISS and Soyuz Docking Failure Scenario: Very High Accuracy BFMC Follow-up Analysis

**BFMC + 3D-Nc
cumulative Pc →
estimates**



**3D-Nc method
individual
conjunction Pc →
estimates**



--- OCMDB FILE CONJUNCTION ANALYSIS ---
OCMDB_20170501_to_20190815_Nc1255043.mat
Screening creation time: 2019-07-27 16:05:07

Primary 25544 (Last Obs. = 6.5 hours)
Secondary 44437 (Last Obs. = 9.6 hours)
Combined hard-body radius = 70 m

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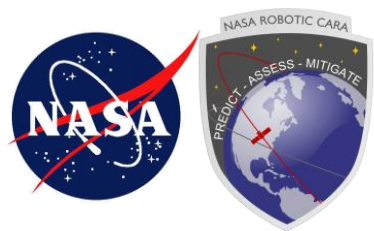
3D-Nc analysis method usage violations
(for events with $P_c \geq 1e-10$):
Affected by NPD covariances: 0 of 110 <no violation>
Extended or blended in time: 110 of 110 <violation>

--- ANALYSIS RECOMMENDATION ---
Confirm with BFMC method, if possible (priority = 99.9%)

BFMC $P_c = 0.01599 = 1599/1e5$ (CPU 1.34e5s)
95% conf. 0.01522 to 0.01679 (check-out failed)



**BFMC analysis results
(1.2-hours/32-processors)**



Repeating Conjunctions: Summary

- **3D-Nc outperforms 2D-Pc for single conjunctions**
 - Verified against ~65,000 actual conjunctions
 - Substantially fewer deviations from Monte Carlo, and no operationally significant underreporting of Pc
- **3D-Nc processes repeating conjunctions well**
 - Determines whether conjunctions are discrete (and therefore essentially uncorrelated)
 - DeMorgan's Law of Complements shown to work well for sets of discrete conjunctions in high-velocity cases
 - BFMC follow-up runs not necessary
 - Low-velocity “blended” cases do need BFMC to obtain accurate Pc
 - But 3D-Nc estimate does establish reliable upper-bound, so BFMC necessary only if 3D-Nc result above threshold
- **CARA moving to 3D-Nc as principal Pc calculation approach**
- **Software (source code) will be put on CARA github as soon as approved for public release**