

Multi-point line focused laser differential interferometer for high-speed flow fluctuation measurements

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A multi-point focused laser differential interferometer (FLDI) has been developed to measure density fluctuations at 16 points along a line. A pair of cylindrical lenses on the transmitter side of a conventional single-point FLDI instrument form two closely spaced ($\leq 200 \mu\text{m}$), orthogonally polarized, parallel laser lines at the instrument's focus. On the receiver side of the instrument, the interference of the beams on a 16-element photodiode array results in a single line of measurements. The further addition of a Nomarski prism creates two separate measurement lines, and the addition of a second photodiode array to the instrument enables simultaneous measurements of density fluctuations along the two lines separated by several millimeters. These two lines of measurement can be conveniently oriented at any azimuthal angle relative to the instrument's optical axis on the measurement plane, coinciding with the instrument's focus. Two experiments were performed to demonstrate the capabilities of the instrument. In the first experiment, a laser-induced breakdown spark generated a traveling spherical shock wave, and measurements of the resulting density disturbance and wave velocity were obtained. These results were compared to high-speed schlieren images of the shock wave acquired at 400 kHz. In the second experiment, the multi-point FLDI instrument was used to measure density disturbances in the boundary layer of a flat plate in a Mach 6 freestream flow. The measurements were made along two lines, both approximately 6 mm in length, extending from the surface of the plate through the boundary layer. High-speed schlieren images were acquired at 100 kHz during separate wind tunnel runs at matching unit Reynolds numbers to visualize the unsteady boundary layer flow and compare to the FLDI measurements.

1. INTRODUCTION

The focused laser differential interferometer (FLDI) originally described by Smeets and George [1] is capable of providing time-resolved measurements of density fluctuations at a point. The instrument uses two closely spaced, orthogonally polarized, parallel beams that are transmitted from the transmitter side of the instrument and focused to two small spots at the measurement plane. These points are separated by a distance Δx_{WP} , as shown in Fig. 1(a), where orange and purple represent separate, orthogonally polarized laser light paths. A small change in density, $\Delta\rho$, in the flow results in a proportional change in refractive index, Δn , as described by the Gladstone–Dale relation:

$$\Delta n = K_{\text{air}} \Delta\rho, \quad (1)$$

where $K_{\text{air}} = 2.3 \times 10^{-4} \text{ m}^3 \cdot \text{kg}^{-1}$ is the Gladstone–Dale constant for air. If a difference in refractive index exists between the two focused spots persisting over some distance in the z direction, s_z , then a phase difference is imparted between the two orthogonally polarized beams according to

$$\Delta\phi = \frac{2\pi}{\lambda_0} s_z \Delta n, \quad (2)$$

where $\Delta\phi$ is the phase shift, and λ_0 is the wavelength of the laser light. After the orthogonally polarized laser light beams are combined on the receiver side of the instrument, it passes through a linear polarizing filter that is adjusted such that it transmits the polarization component corresponding to the middle of the interference fringe. The intensity of laser light incident on a high-speed photodiode (PD) will then change proportionally to $\Delta\rho$. Unlike path-averaged laser differential interferometers

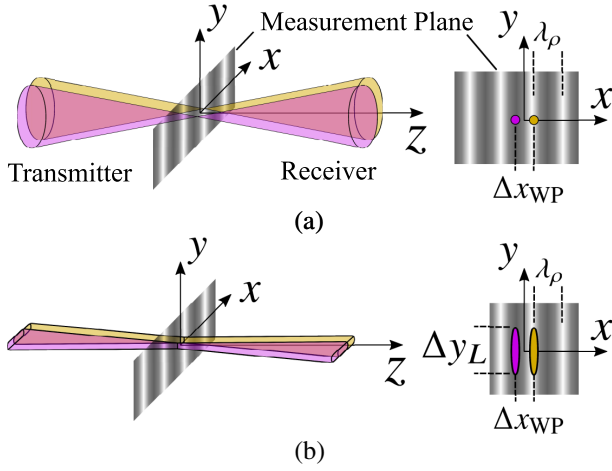


Fig. 1. Orthogonally polarized (purple/orange) laser light paths for (a) conventional single-point and (b) multi-point line FLDI instruments between transmitter ($-z$) and receiver ($+z$) sides of the instrument. Note that in (b), the beam is focusing in the x direction and is collimated in the y direction.

(LDI) that use collimated laser beams to sense density fluctuations [2–10], the FLDI has the benefit of attenuating the influence of density gradients away from the measurement plane. This attenuation of signal is the result of spatial filtering as the cross-sectional diameter of the laser light becomes large relative to the density disturbance wavelength [11–14], λ_ρ . Further discussions on the overall response of a single-point FLDI signal with the application of a series of transfer functions that are dependent upon disturbance wavenumber, $k_\rho = 2\pi\lambda_\rho^{-1}$, can be found in [11–16].

The single-point FLDI has been successfully applied to high-speed flow experiments in the last decade, as advancements in high-speed analog-to-digital conversion instrumentation have enabled acquisition rates in excess of tens of megahertz (MHz). Such acquisition rates are critical to the understanding of high-speed flow, such as hypersonic boundary layer transition-to-turbulence, where disturbances with frequencies in the hundreds of kilohertz (kHz) drive certain aspects of the flow physics. Azzazy *et al.* [17] were the first to demonstrate a variant of the single-point FLDI in an aerofoil boundary layer with a frequency response up to 1 MHz, analyzing the magnitude of the signal root mean square (RMS) to determine the transition-to-turbulence location. Parziale *et al.* [15,16,18] performed the first modern single-point FLDI measurements in a hypersonic wind tunnel, providing analyses of the spectral behavior of both the tunnel freestream and wind tunnel model boundary layer disturbances up to 10 MHz. Numerous other examples of the application of the single-point FLDI can be found in [9,11–13,19–25].

One drawback of the single-point FLDI is its inability to measure density disturbance velocity, U_ρ . Data obtained with the instrument are first acquired as a time series, and then a Fourier analysis is performed. The resulting spectra of the density disturbance field at a point in the measurement plane are given as a function of frequency, ν_ρ . However, in order to accurately determine the response of the instrument, the appropriate wavenumber-dependent transfer functions must be applied to

the raw spectra. This requires that the frequency-dependent spectra be recast as a function of wavenumber, $k_\rho = 2\pi\nu_\rho U_\rho^{-1}$, which requires U_ρ to be assumed or known. Smeets and George [1] were the first to propose a two-point FLDI measurement where a polarizing Koester prism was used to generate two measurement points that would enable measurement of U_ρ . More recent work using several different optics incorporated into a single-point FLDI instrument to generate two separate measurement points is described in [22,26–30]. Two-point FLDI velocity measurements have also been obtained using two separate, single-point FLDI instruments as in [18].

For high-speed boundary layer measurements, multiple measurement points distributed throughout the boundary layer are preferable in order to provide a complete understanding of its state (e.g., laminar or turbulent) and characterize the distribution of disturbance intensities and velocities. Smeets and George [1–3] and Smeets [4] were the first, to the best of our knowledge, to propose a multi-point LDI that used a series of Wollaston polarization prisms to generate up to eight measurement points for the study of boundary layer flow. For the eight-point measurements in [1,4], an eight-channel PD array was used to record the time-varying signal from each point. Wadhams *et al.* [31] used a five-point FLDI system for measurements above the surface of a cone in a hypersonic flowfield; however, it appears that these measurements were spaced sufficiently far apart such that correlated measurements could not be made. Recently, Hameed *et al.* [32] demonstrated a four-point FLDI instrument used to characterize boundary layer flow over a cone in a Mach 6 flow. These points, generated using two additional Wollaston polarizing prisms, enabled measurements of the density disturbance spectra and phase velocity using two beam pairs located at two wall-normal positions within the boundary layer.

For measurements in the boundary layer above a broad, flat surface, the use of conventional laser beam focusing methods in an FLDI instrument may not be appropriate, as portions of the beam can be truncated by the model itself. Recent work by Houpt *et al.* [20,23] replaced the spherical optics of a conventional FLDI instrument with cylindrical focusing optics. This approach allowed for single-point density disturbance spectra to be obtained above a flat wall boundary layer in a Mach 2 wind tunnel while maintaining a degree of spatial filtering for disturbances away from the measurement plane.

The purpose of this paper is to demonstrate the feasibility of a multi-point line FLDI instrument for use in high-speed wind tunnel experiments, where tunnel or model geometry would otherwise interfere with the spherically focused beams of a conventional FLDI instrument. In this work, a combination of spherical and cylindrical focusing optics forms a pair of closely spaced, orthogonally polarized lines (as opposed to a pair of spots) on the measurement plane, as shown in Fig. 1(b). This approach avoids the geometry-based beam-truncation limitations described in the preceding paragraph. Careful selection of the focal lengths of the cylindrical optics allows for control of the line pair length, Δy_L , at the measurement plane. Signals from density disturbances at 16 points along the length of this line pair are acquired using a 16-element linear array PD. The addition of a Nomarski polarizing prism, as used in [22,30], enables measurements from density disturbances between two closely spaced line pairs. A cross correlation, or

similar analysis, of the time-varying signals at the same relative locations along both line pairs provides for a measurement of the phase velocities, $U_\rho(y, t)$. The use of the Nomarski prism (NP) ensures that the propagation axes of all laser lines travel parallel to one another between the transmitter and receiver sides of the instrument, mitigating bias error in the velocity measurements. By rotating this prism about the optical axis, the azimuthal orientation of the focused laser lines at the measurement plane can be adjusted with relative ease. Two experiments are used to demonstrate the performance of this instrument. In the first experiment, measurements are obtained with the single- and two-line pair configurations oriented both parallel and normal to the surface of a traveling shock wave formed from a laser spark. The time-varying signals and velocities obtained with the multi-point FLDI instrument are then compared to results obtained simultaneously with a high-speed schlieren (HSS) system framing at 400 kHz. These results are used in part to show that the FLDI system measures accurate velocities of a simple, well-defined density wave. In the second experiment, single- and two-line measurements through the boundary layer of a flat plate model in a Mach 6 freestream flow are obtained in the wall-normal direction. HSS images from the flat plate flow are used to verify the boundary layer thickness estimate obtained with the multi-point FLDI instrument.

2. MULTI-POINT FLDI INSTRUMENT

A. Optical Setup

A schematic of the two-line, multi-point FLDI system is shown in Fig. 2, with the transmitter/pitch side (P) to the left of measurement plane (section A-A), and the receiver/catch side (C) to the right. A single-mode, linearly polarized $\lambda_0 = 532$ nm laser (L) with maximum output power of 300 mW reflects off two turning mirrors (M^{1P} and M^{2P}) onto the optical axis of the system (dash-dot black line). A half-wave plate (HWP^{1P}), polarizing beamsplitter cube (PBS), and beam dump (BD) are used to adjust the transmitted laser power. A second HWP (HWP^{2P}) rotates the polarization angle by 45° relative to the y axis. A short focal length spherical lens (beam expander, BE) is then used to expand the beam in all directions. Next, a NP splits the vertical and horizontal components of polarization by a small angle about the z axis in the xz plane and then redirects them such that they cross the z axis in the xz plane approximately 267 mm beyond the prism's exit face. These orthogonally polarized beams are denoted in the schematic with either blue or red. After NP, a third HWP (HWP^{3P}) rotates the polarization angle

of each beam such that they are oriented at $\pm 45^\circ$ relative to the y axis. Up to this point, the instrument is essentially identical to the two-point FLDI system described in [22,30].

However, after HWP^{3P} , cylindrical lenses are placed at two positions along the instrument's optical axis so that lines of focused laser light are formed parallel to the y axis on the measurement plane. The first of these is a positive cylindrical lens (PCL^P) that collimates the beams with respect to the y axis while allowing them to continue to expand in the $\pm x$ directions. In Fig. 2, the purple color is used hereafter to describe the propagation of the laser beams when viewed orthogonal to the yz plane. For reference, the coordinate system orientations for all colors used in the schematic are shown in the top left of Fig. 2. A Wollaston prism (W^P) placed at the crossing point of the two beams split by NP further splits each beam by 2 arcmin relative to the z axis and parallel to the xz plane. This creates two additional orthogonally polarized beams. A negative cylindrical lens (NCL^P) is then placed one focal length from W^P to further expand the beam in the $\pm y$ directions. A field lens (FL^P) consisting of two back-to-back achromatic doublets (FL^{1P} and FL^{2P}) is used to focus the beams in the xz plane and collimate them in the yz plane. This is placed at one effective focal length (EFL) from W^P such that the central axis of each beam propagates parallel to one another and the z axis after exiting FL^P . Using this configuration, two line pairs (each referred to as a Wollaston pair) are formed at the measurement plane (section A-A), with each line oriented parallel to the y axis and separated in the x direction by Δx_{WP} . The two Wollaston pairs (referred to as a Nomarski pair) are separated by Δx_{NP} . The length of the lines Δy_L can be specified with careful selection of BE, PCL^P , and NCL^P focal lengths. While the FLDI beams are referred to as *lines* throughout this study, the beam is in fact an ellipse, where the major axis is much larger than the minor axis.

Beyond the measurement plane (section A-A) in Fig. 2, light from the laser lines diverges in the $\pm x$ directions while remaining collimated in the y direction. A second FL (FL^C) consisting of back-to-back achromatic doublets (FL^{1C} and FL^{2C}) is then placed at a distance from the measurement plane equal to that between FL^P and the measurement plane. For this work, both FL^P and FL^C are identical. After passing through FL^C , the beams are focused in both the x and y directions. Another 2 arcmin Wollaston prism (W^C) is placed at the EFL of FL^C to combine the orthogonally polarized beams from each Wollaston pair. The location of W^C also corresponds to the focus of the laser light in the y direction. After passing through W^C , the laser light again begins to diverge in the y direction. This light is then collimated in the y direction by placing a positive cylindrical

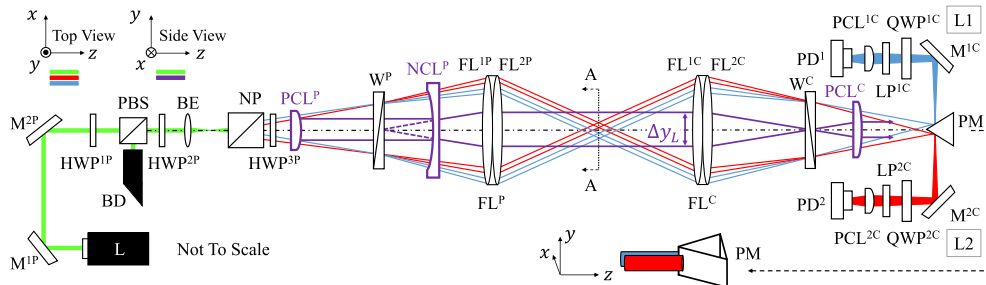


Fig. 2. Schematic of the two-line FLDI system, with the transmitter/pitch side on the left and the receiver/catch side on the right.

lens (PCL^C) one focal length from W^C . Beyond PCL^C , the laser light from the Nomarski pair continues to converge in the x direction until two focused laser lines are formed. These two lines, offset from one another by a small distance in the x direction, are then split using a right-angle prism M (PM) in two separate legs of the instrument (L1 and L2). A perspective view of this splitting of the two laser lines with PM can be seen at the bottom right of the schematic. After PM , both legs L1 and L2 contain the same optical components. First a turning M ($M^{#C}$) redirects the beam by 90° and can be adjusted for alignment onto the detector. The beam then passes through a quarter-wave plate ($QWP^{#C}$) that is adjusted to remove any ellipticity in the polarization of the beam. Next a linear polarizer ($LP^{#C}$) makes the two overlapping orthogonally polarized beams interfere with one another, and a short focal length positive cylindrical lens ($PCL^{#C}$) is used to adjust the thickness of the laser line incident on the silicon PIN PD linear array ($PD^\#$). Note that careful selection of the PCL^C focal length determines the length of the laser line incident on $PD^\#$. For the experiments described in this work, a 16-element linear array PD was used, with each element measuring 0.7 mm high and 2.0 mm wide (active area 1.4 mm^2). Therefore, the focal length of PCL^C was selected in order to slightly overfill $PD^\#$ in the $\pm y$ direction.

The orientation of the two laser lines can be changed to address the particular measurement needs for a flow. Figure 3 shows two laser line orientations applied to three configurations that were achieved with the instrument and are later described in Sections 3 and 4. Figure 3(a) shows a configuration (Configuration 1) that can be used if sequential measurements of an evolving density wave are desired. Here, W^P and W^C are rotated such that the laser lines within each Wollaston pair overlap one another, with an overlap offset of Δy_{WP} . The NP

is then rotated such that the overlapping laser lines in each Wollaston pair are separated by Δx_{NP} relative to their minor axes. Figures 3(b) and 3(c) show a configuration (Configuration 2) that can be used if measurements across a spatially varying disturbance field are desired, relative to the laser line pair's (Wollaston pair's) major axes, with the dominant phase velocity component oriented parallel to the Wollaston pair's minor axes. Here, W^P and W^C are rotated such that the laser lines within each Wollaston pair are separated by Δx_{WP} relative to their minor axes. The NP is then rotated such that the Wollaston pairs are separated by Δx_{NP} relative to the minor axes of the laser lines. To achieve the Wollaston pair and Nomarski pair orientations from Fig. 3, the appropriate optics were adjusted in their respective rotation mounts. Note that the PM and all subsequent optics in legs L1 and L2 were mounted together and connected with a rotation mount to the catch side of the instrument so that they could all be rotated about the instrument's optical axis (z) without the need for extensive realignment within each leg.

Figure 3(d) shows an image of the flat plate model used in the wind tunnel testing of Section 4, viewed from the pitch side of the tunnel (with model coordinate system indicated). Note that the leading edge of the model is not visible in the image. Flow is traveling in the positive $\tilde{x}$ direction, and the full width of the model can be seen (178 mm). Laser scatter from both Wollaston pairs is also shown, where the narrower beams are located at the tunnel centerline ($\tilde{z} = 0 \text{ mm}$) at the focus of the FLDI system ($z = 0 \text{ mm}$), while the wider beams show the FLDI lines some distance away from the focus. The center of the Nomarski pair is located 341.3 mm from the leading edge of the model (in the positive $\tilde{x}$ direction). The location of the model surface with respect to the laser lines is indicated with a solid red line for both

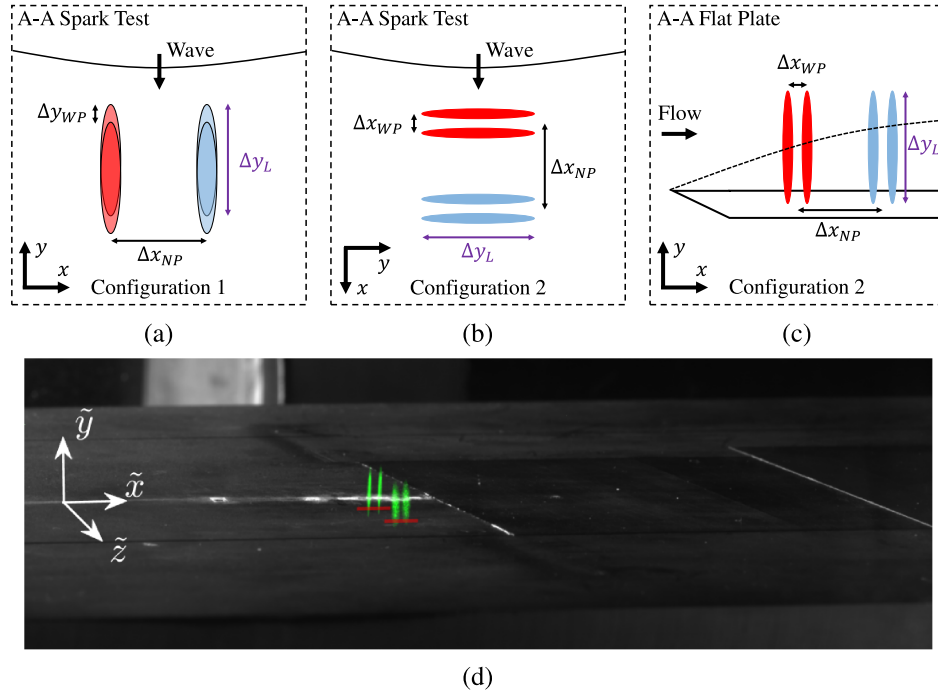


Fig. 3. Laser line configurations (not to scale) for (a), (b) the spark experiment and (c) the wind tunnel test (with instrument coordinate systems), and (d) flat plate model in the wind tunnel with FLDI measurement lines made visible using laser scatter at two lateral ($\tilde{z}$) positions (with model coordinate system). The model surface intersection with the laser lines is shown in red in (d).

laser beam frames, and the method for determining the exact cutoff of the laser lines by the model is discussed in Section 2.C. The FLDI beam laser scatter images were acquired without the model in the tunnel test section, which is why the beams are visible below the model surface. Figure 3(d) shows the need for a system that focuses only along a single direction in order to probe a region at the model surface. If a conventional FLDI instrument was used, the beams would be partially truncated by the flat plate model as they focused down in the y direction (along the model $\tilde{z}$ axis).

B. Photodiodes and Data Acquisition

As briefly mentioned in Section 2.A, two 16-element linear array silicon PIN PDs, PD¹ and PD², were used to measure intensity variations from each Wollaston pair. The active area of each element on PD¹ and PD² measured 0.7 mm high by 2.0 mm wide, with the center of each element spaced 0.8 mm apart, giving a total sensor height of 12.8 mm and total sensor width of 2.0 mm. The -3 dB cutoff frequency is 25 MHz when illuminated with 830 nm light, reverse biased at -10 V, and terminated at $50\ \Omega$. Although the laser wavelength in this work was $\lambda_0 = 532$ nm, the bandwidth of the array was sufficient at this wavelength to resolve the unsteady high-speed flow phenomena discussed in Section 4, and the spectral response is approximately 0.33 A/W.

Each PD array was surface-mounted to a printed circuit board (PCB) designed such that the 16 elements were supplied a reverse bias voltage of -10 V from a common ground plane powered by an external DC source. The soldered connections for the even and odd elements on each array were attached to ribbon cable connector receptacles mounted to either side of the array via copper leads. A 10 cm $\times$ 10 cm $\times$ 5 cm aluminum enclosure was used to house the PCB and limit interference from stray light. Insulated ribbon cables were attached to each receptacle on the PCB with a removable connector, while at the opposite ends of the cables, bayonet Neill–Concelman (BNC)-type connectors were used to couple to in-line low-pass electrical filters with a -3 dB cutoff frequency greater than 6 MHz.

The time-varying current signals from the PD elements were terminated at $50\ \Omega$ and digitized at 14 bit resolution with two eight-channel high-speed oscilloscope modules. While each channel had a bandwidth of 250 MHz, data were acquired at 10 MHz on each channel. Each oscilloscope module had 1.5 GB of on-board memory that was transferred to a separate computer for permanent data storage. Custom oscilloscope control software written in the LabVIEW programming language allowed the user to control data acquisition remotely from a computer.

Tests were performed to ensure there was no cross talk between channels on the PD. A turbulent jet, supplied with 100 psi compressed air through a 1.8 mm diameter nozzle, was used to provide a signal with broad frequency spectrum fluctuation content, and channels were sequentially blocked as data were acquired. Power spectral density (PSD) plots for all channels at every blocking position were compared, and there was no visible change in any of the channels' signals throughout the test, indicating that cross talk was negligible.

C. Calibration Method

Calibrations were performed before each of the three tests [Figs. 3(a)–3(c)] to determine the spacing of the PD array elements (channels) projected onto the measurement plane, Δy_{CH} . For each calibration, a razor blade was affixed to a motorized linear translation stage that was then mounted such that the blade would traverse the major axis of the laser lines at the measurement plane. With the laser light initially unblocked, the blade was advanced incrementally in steps of 100 μ m until the laser light was completely blocked. The calibration voltage signal ($V_{\text{calibration}}$) recorded from the PD array elements at each incremental razor blade position was then scaled by the voltages recorded with the blade fully removed from (V_{removed}) and inserted into (V_{inserted}) each laser line's path in the measurement plane according to

$$S_{\text{calibration}} = \frac{V_{\text{calibration}} - V_{\text{inserted}}}{V_{\text{removed}} - V_{\text{inserted}}}, \quad (3)$$

where $S_{\text{calibration}}$ is the scaled calibration signal for each element on the PD[#] array recorded at each blade position. A cubic spline was fit to values of $S_{\text{calibration}}$ between 0.25 and 0.95 for each element, and the razor blade position at $S_{\text{calibration}} = 0.5$ was computed, resulting in the relative position of the elements within a single PD array.

Figure 4(a) shows the result of this calibration procedure performed for the configuration shown in Fig. 3(a) with a single 16-element PD array, while Fig. 4(b) shows the result of this calibration procedure for a two-line configuration, such as that shown in Figs. 3(b) or 3(c). Note that for the two-line calibration procedure results shown in Fig. 4(b), only signals from the even-numbered channels from each 16-element PD array are presented, since the number of channels available on the oscilloscope was limited to 16. Additionally, while great care was taken to align the laser lines of each Wollaston pair such that they were equally blocked by the razor blade at each position, a small level of misalignment ($\approx 250\ \mu$ m) was observed. This, however, does not change the resulting calculated channel spacing. The mean channel spacing values determined from Figs. 4(a) and 4(b) are shown in Table 1.

Complementary metal–oxide–semiconductor (CMOS) camera images taken at the beam focus (measurement plane) were used to determine the spacing between the Wollaston pairs and between the Nomarski pair, with a characteristic image shown in Fig. 5(a) (pixel pitch of 3.45 μ m/pixel). The sensor of the camera was placed at the beam focus, and no imaging lenses were used. The laser lines in this image are aligned according to Configuration 2 [Fig. 3(c)], where the left two lines in red false color are the upstream (US) Wollaston pair, the right two lines in blue false color are the downstream (DS) Wollaston pair, and the Nomarski pair comprises both Wollaston pairs. A profile normal to the major axis of the lines was extracted, and the peaks of each line were fit with a low-order polynomial, where the distances between the peaks were used to determine the spacing of the beam pairs. Figure 5(b) shows the corresponding beam profile along the major axis of the lines for both the US and DS Wollaston pair, denoted with solid colored lines. The image for this plot was acquired with a separate scientific CMOS (sCMOS) camera mounted outside the tunnel that was used to

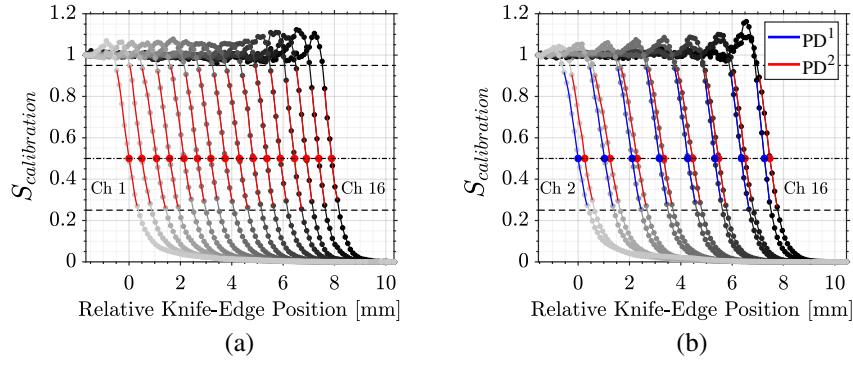


Fig. 4. Knife-edge calibration result for the (a) single-line (Configuration 1) and (b) two-line (Configuration 2) multi-point FLDI instrument.

Table 1. Calibration Results from CMOS Images and Knife-Edge Measurements, Where an Over-Bar Indicates an Average Value Over All Channels

Value	Configuration 1	Configuration 2	
	Spark Test (μm)	Spark Test (μm)	Wind Tunnel (μm)
Δy_{WP1}	172.7 ± 2.6	-	-
Δy_{WP2}	172.5 ± 1.9	-	-
Δy_{CH1}	526.1 ± 66 [Fig. 4(a)]	-	-
Δx_{WP1}	-	172.6 ± 7.2	172.2 ± 3.3
Δx_{WP2}	-	172.8 ± 4.0	172.4 ± 4.8
Δx_{NP}	-	1841.9 ± 14.4	1837.5 ± 10.1
Δy_{CH1}	-	1036.4 ± 160.6 [Fig. 4(b)]	502.1 ± 18.2
Δy_{CH2}	-	1029.5 ± 123.6 [Fig. 4(b)]	502.5 ± 46.6

image scattered light from the laser lines. The Gaussian shape of the beam is visible with the Gaussian fits denoted with the corresponding dashed line. The approximate location of the model surface is indicated with a dashed black line, and positive y -axis values denote the region above the surface. The profile shown in this figure can be compared to the schematic in Fig. 3(c) and the image in Fig. 3(d).

Calibrations (knife-edge data and CMOS images) were performed prior to data acquisition for each measurement

(spark test, wind tunnel) and configuration (1, 2), with the results reported in Table 1. Note that the variables in the first column correspond to the variables in Figs. 3(a)–3(c), and that the channel spacing is listed as an average (with an over-bar) over all PD channels. The CMOS image shown in Fig. 5(a) includes text indicating the appropriate measurement from the table as an example. The uncertainties reported for values derived from CMOS images take into account the Gaussian fit to the beam profiles and are reported at a 95% confidence interval.

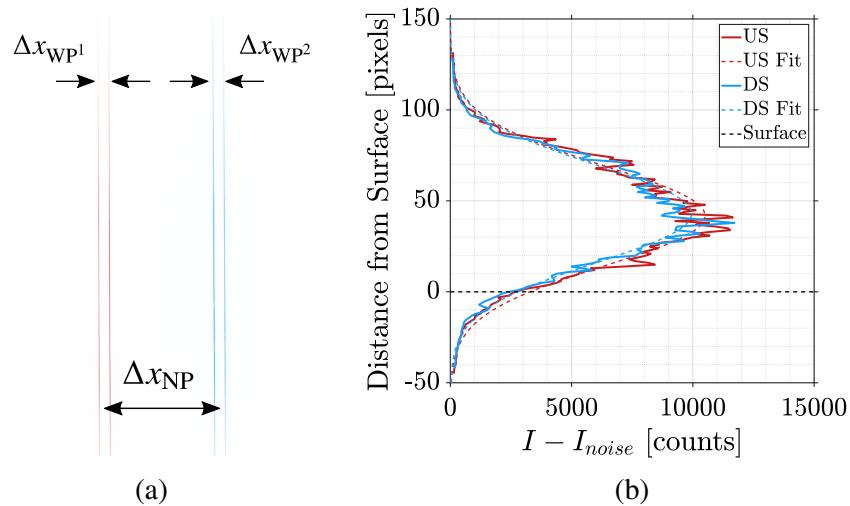


Fig. 5. (a) CMOS image of the Nomarski pair with two Wollaston pairs at the 20-Inch Mach 6 Air Tunnel test section focus and (b) laser line major axis beam profiles with Gaussian fits.

Uncertainties reported for values obtained using the knife-edge data take into account data fitting uncertainties and influences from the motorized linear translation stage (bi-directional repeatability, backlash error, resolution). Unless otherwise stated in the following text, the uncertainty is reported as a 95% confidence interval, which includes the appropriate Student's t distribution multiplier based on the degrees of freedom of the measurement.

D. High-Speed Schlieren Setup

Schlieren imaging is used for comparison to FLDI data in this study for three main reasons: both techniques are sensitive to density gradients, are non-intrusive, and are capable of providing time-resolved measurements (although FLDI has better time resolution than schlieren). Both measurements can be acquired simultaneously, allowing for comparison of the same flowfield. A lens-based HSS system acquired images of the laser-induced breakdown (LIB) shock wave simultaneously with the FLDI system measurements. The HSS optical axis was aligned orthogonal to the optical axis of the FLDI instrument (z axis). Multiple images of the shock wave passing through the FLDI measurement location were acquired at a frame rate of 400 kHz (using a pulsed LED light source) and a field-of-view of 256×80 pixels, with the knife edge placed parallel to the front surface of the shock wave to be sensitive to the gradient in the index of refraction across the shock wave front. Laser scatter from the mist of a fogging machine was used to image the location of the FLDI beams in the HSS field-of-view. The HSS velocity measurements were calculated by fitting a seventh-order polynomial to 11 data points centered around the peak pixel intensity and using a minimization algorithm to find the sub-pixel peak locations [30,33]. A synchronization signal from the HSS camera was recorded along with the FLDI signal to be able to align the FLDI signal with the HSS images. The HSS system was also used during the wind tunnel tests (frame rate of 100 kHz) and was aligned along the same optical axis as the FLDI system (orthogonal to the flow direction), but was acquired after completion of the FLDI tests.

Uncertainty in the HSS velocities (for the spark test data) was computed based on influences from the camera spatial

resolution, blur caused by finite width of the LED pulse that illuminated the shock wave, variations in time at which the pulse occurred during the acquired image, and the fitting of the intensity peaks used for computing the frame-to-frame wave position.

3. LASER-INDUCED BREAKDOWN SPARK EXPERIMENT

Characterization of the FLDI system was performed using a weak shock wave generated by a LIB spark, where the system was the same as that used in [22,30]. Both the FLDI and HSS systems were centered approximately 37 mm directly below the spark. Results from the two configurations shown schematically in Figs. 3(a) and 3(b) are presented below in Sections 3.A and 3.B, respectively.

A. Configuration 1

A single-spark experiment consisted of a single instance of the LIB-generated shock wave, with data acquisition performed using both FLDI and HSS systems. For this configuration, 50 instances of the spark experiment were run using the beam orientation shown in Fig. 3(a). As the wave expanded from the LIB spark location, it was first registered on channel 1, with each successive channel registering the signal from the wave at subsequent times, with the last registered signal occurring on channel 16. To visualize the LIB shock wave passing through the FLDI measurement location, the false-color image in Fig. 6 shows six selected HSS frames, with the red false color showing the shock wave propagation, and the green false color showing scattered light from the FLDI measurement line about its focus (obtained with a fogging machine). The shock front is clearly apparent with the expansion region trailing after. The plot to the far right shows the Gaussian beam profile of the scattered-light FLDI line.

The FLDI measurement raw voltage from a single-spark experiment can be seen in Fig. 7(a), where the DC offset was removed from the data by subtracting each channel's mean voltage prior to the arrival of the shock wave. This was done solely for visualization purposes, since the mean voltage for each

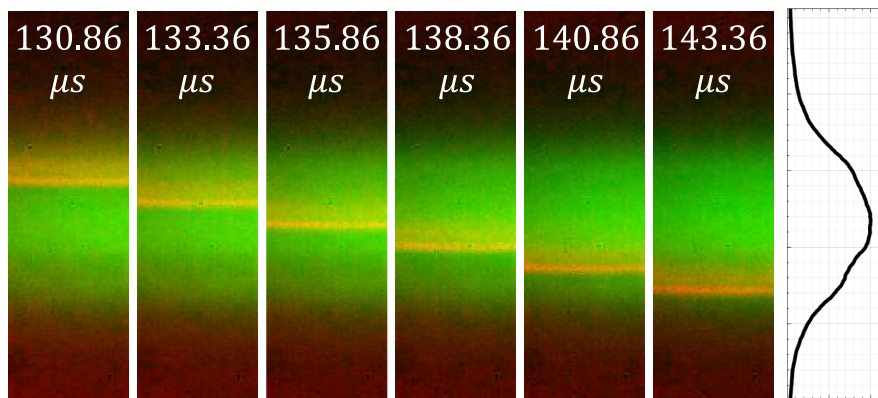


Fig. 6. Six selected frames of the HSS shock wave propagation (red false color) with the FLDI line measurement profile (green false color) for Configuration 1 and Gaussian beam profile of the FLDI line at the far right. Values indicated on the plot are time durations since the LIB spark was generated.

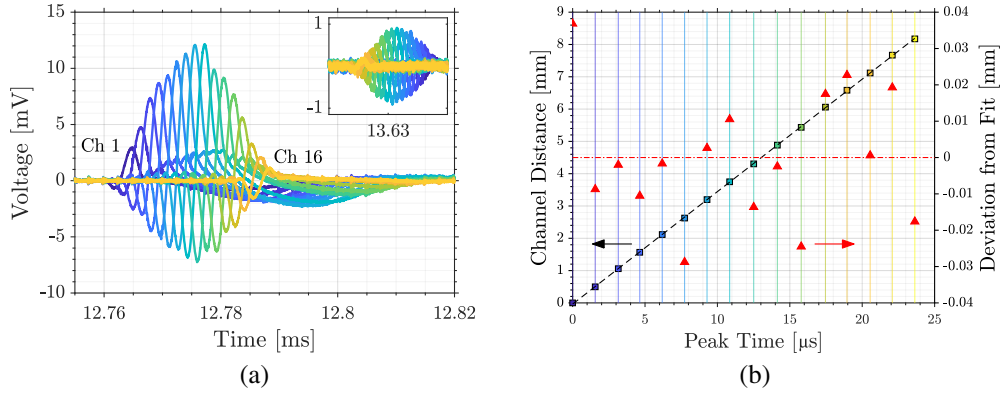


Fig. 7. Configuration 1: (a) DC-offset raw voltage for all channels on a single PD and (b) channel position versus peak voltage time (left axis, squares) and channel distance deviation from the linear fit (right axis, triangles).

channel follows the typical Gaussian distribution of the beam intensity along the line; this can still be seen in the maximum measured voltage for each channel, with the outer channels (e.g., 1 and 16) reading lower amplitude voltages than the inner channels. The low-amplitude repetitive pulses seen in the voltage signal are due to the pulsing of the high-power LED used for the simultaneous HSS. While these disturbances do not affect the computed wave velocities, data sets were also taken without the HSS, and this coupled disturbance in the FLDI data disappears. The signal is first registered on the upper-most channel (Ch 1) and continues down along the length of the PD to the lower-most channel (Ch 16). The inset of the figure shows the signal from the same PD a short time later, after the shock wave has reflected off a flat plate below the measurement region and passes through the channels in reverse order. The amplitude of the reflected wave signal has decreased significantly due to the continuous expansion of the spherical shock wave front.

The channel separation distance in physical space at the measurement plane (section A-A in Fig. 2) was determined using the knife-edge calibration method described in Section 2.C. To compute the velocity of the shock wave registered on the channels of the linear array, the time shift between adjacent-channel data sets was used. These time shifts were computed using a second-order polynomial fit for the peak locations of the raw voltage data from Fig. 7(a). A plot of the channel position versus the channel peak time is shown in Fig. 7(b) with corresponding colored squares (left axis), where the velocity between two adjacent channels is the slope of the line connecting the two data points. The mean wave velocity as it passes through all channels at the focus of the laser line is calculated to be 348.5 ± 1.5 m/s

using the slope of a linear fit of all the data points (denoted with a dashed black line). The mean HSS velocity computed using 33 data sets (each with 16 images of the shock wave traversal) was 351.2 ± 37.9 m/s. The uncertainties include influences from the fitting process and the standard deviation (STD) over all the utilized data sets.

Small changes in the peak time or the channel separation distance result in non-negligible changes in the computed channel-to-channel velocity. The right axis of Fig. 7(b) shows the deviation of each channel distance from the linear fit. While these offsets are small (on the order of tens of microns), the corresponding velocity offset from the mean fit is relatively large (on the order of tens of meters per second). This can be seen more clearly by taking the forward difference between adjacent channels to compute the slope and, thus, velocity of the wave as it passes through the laser line. These velocity results are reported for each adjacent channel pair in Table 2 and range between approximately 319 and 376 m/s. The uncertainties in the velocity take into account uncertainty in the average channel spacing from the knife-edge calibration, timing resolution, and STD of the mean time lag between sequential channels. The uncertainties reported in parentheses of Table 2 are solely due to the STD of the mean time lag between sequential channels and are typically approximately forty times smaller than the overall uncertainty. Since each channel-pair separation distance is a single value determined from the knife-edge calibration, the parenthetical uncertainty results indicate that the relative peak times are consistent over all the spark experiments, and that it is the uncertainty in the channel spacing that results in the absolute velocity deviations from the mean. Future efforts

Table 2. FLDI Channel-to-Channel Velocity Measurements, Calculated Using a Forward Difference Between Adjacent Channels^a

Ch	$V_{\text{FLDI}}[\text{m/s}]$	Ch	$V_{\text{FLDI}}[\text{m/s}]$	Ch	$V_{\text{FLDI}}[\text{m/s}]$
1/2	$319.43 \pm 42.57(1.82)$	6/7	$370.43 \pm 42.96(1.13)$	11/12	$375.62 \pm 39.80(1.17)$
2/3	$351.02 \pm 41.43(1.35)$	7/8	$354.36 \pm 42.46(0.92)$	12/13	$349.71 \pm 44.25(1.20)$
3/4	$343.71 \pm 44.77(1.07)$	8/9	$333.95 \pm 39.86(0.87)$	13/14	$335.91 \pm 41.60(1.52)$
4/5	$355.47 \pm 42.85(1.17)$	9/10	$355.35 \pm 40.54(0.85)$	14/15	$357.31 \pm 43.41(3.54)$
5/6	$327.77 \pm 42.46(1.03)$	10/11	$335.62 \pm 40.26(0.93)$	15/16	$324.90 \pm 43.19(3.58)$

^aUncertainties reported in parentheses include only contributions from time lag standard deviations over 50 data sets.

will require a more accurate method for determination of the channel-to-channel spacing along the FLDI line at its focus. Nonetheless, the raw voltage data and computed mean velocity indicate that this beam orientation is promising for disturbance propagation measurements.

B. Configuration 2

The orientation of the beams for the second configuration of the spark experiment is shown schematically in Fig. 3(b). Only the even channels of both PDs were used (2, 4, 6, 8, 10, 12, 14, 16) due to the limitation on available acquisition channels of the oscilloscope. False-color images of four HSS images (red false color) and laser scatter from the FLDI beams (green false color) are shown in Fig. 8. Due to the lower resolution of the laser scatter images compared to the CMOS images, the two Nomarski pair beams are visible, but the separate Wollaston pair beams are not. The four HSS frames show the shock wave traveling down through the FLDI measurement location, first passing through the upper Wollaston pair and then a short time later through the lower Wollaston pair.

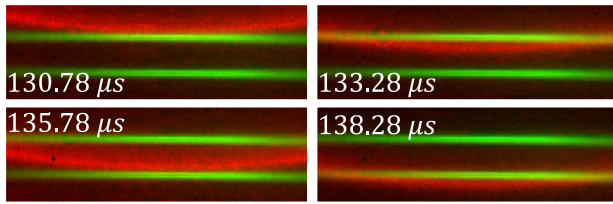


Fig. 8. Four selected frames of the HSS shock wave propagation (red false color) with the FLDI laser scatter line measurement location (green false color) for Configuration 2. Values indicated on the plot are time durations since the LIB spark was generated.

The raw voltage data of the FLDI measurement from a single spark test for both the top PD (PD¹, red) and the bottom PD (PD², blue) are shown in Fig. 9(a), where the DC offset has been removed using the data prior to the shock wave arrival. The top PD channels register the shock wave first and the bottom PD channels a short time later. The signal from channels on both PDs exhibit the same characteristic abrupt spike as the shock wave arrives, followed by an expansion and return to ambient conditions. This is the same structure observed in the two-point FLDI system of [22,30] and in computational fluid dynamics (CFD) simulations such as in [34]. A zoom view near the initial spike in the FLDI signal as the shock front passes through each channel is shown in Fig. 9(b), where the data are now colored based on channels in each PD, as shown in Fig. 9(c). From Fig. 9(b), the different amplitudes of the registered signal peak can be seen and are due to the Gaussian beam intensity of the laser line along the major axis, as shown in Fig. 5(a); channels 2 and 16 have the smallest signal, while the middle channels have the largest signal. The curvature of the shock wave front is visible in the HSS images of Fig. 8 (red false color). The result of this curvature over the FLDI line length can also be seen in Fig. 9(b), where the middle channels register signals before the outer channels. Further, these results show that the FLDI lines were centered directly below the spark, since corresponding channels on opposite sides of the detector have very nearly the same time delay (i.e., channels 2/16, 4/14, etc.).

The velocity of the shock wave from the FLDI data is calculated using both the time delay of signal registration between PD¹ and PD² for identical channels and the Nomarski pair separation distance from Table 1. For all channels on each PD, data points bounding the initial shock wave front peak are fit with a low-order polynomial, and the peak time of the fit is determined using a minimization algorithm. Eight wave velocities are calculated, one for each PD¹/PD² channel pair.

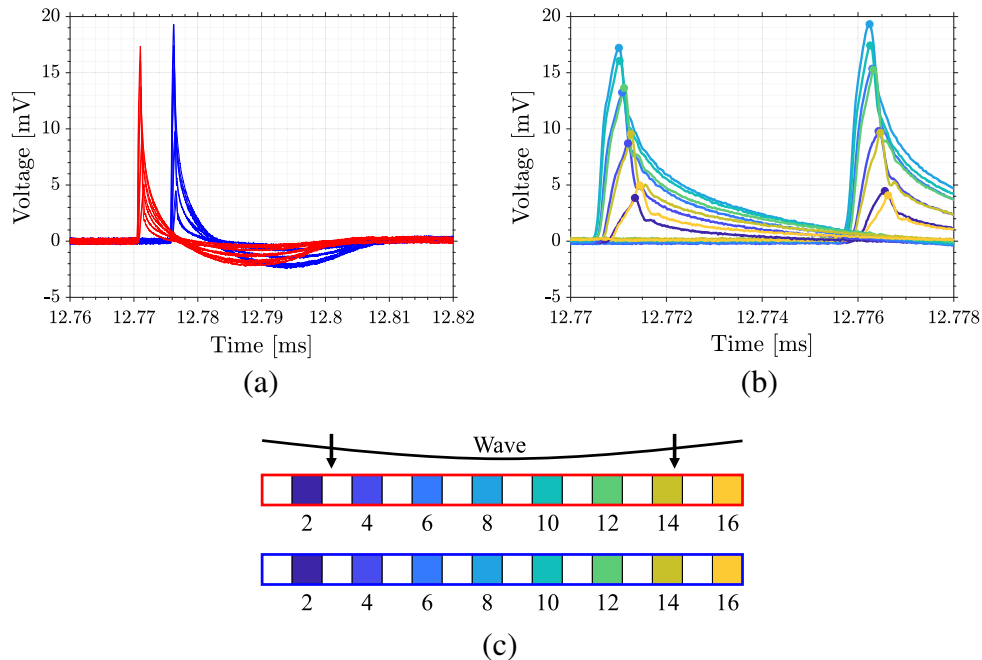


Fig. 9. Configuration 2: (a) DC-offset raw voltage for both PDs, (b) zoom view of voltage data colored by corresponding channels on both PDs, and (c) PD and channel positions colored corresponding to (a) and (b).

Table 3. Velocity Comparisons Between Configuration 2 FLDI and HSS Measurements

Channel	$V_{\text{FLDI}}[\text{m/s}]$	$V_{\text{HSS}}[\text{m/s}]$	Channel	$V_{\text{FLDI}}[\text{m/s}]$	$V_{\text{HSS}}[\text{m/s}]$
2	352.42 ± 2.85	356.76 ± 52.63	10	351.28 ± 2.83	352.54 ± 52.59
4	351.22 ± 2.83	355.06 ± 52.05	12	352.14 ± 2.83	356.34 ± 52.23
6	351.25 ± 2.83	352.07 ± 52.82	14	352.94 ± 2.85	354.12 ± 51.80
8	351.37 ± 2.83	353.55 ± 51.70	16	354.40 ± 2.87	356.36 ± 52.31

The HSS velocity calculations are performed as described in Section 2.D. Using the false-color HSS/FLDI scatter images of Fig. 8, sub-regions of the HSS images corresponding to the FLDI channel positions were used to compute the HSS shock wave velocity. This allowed for comparisons of the two measurement techniques for each channel, but it is important to note that these positions are not coincident, since the FLDI and HSS measurement locations/field-of-view were orthogonal to each other. Assuming that the LIB shock wave is nearly spherical at the measurement location, the FLDI and HSS measurements can be reasonably compared.

Velocities for each channel from both FLDI and HSS measurements are presented in Table 3, where the uncertainty for both measurements is reported at the 95% confidence interval of the measurements over 50 spark tests. The precision of the FLDI velocity is high, and both the FLDI and HSS velocity agree within their uncertainty bounds. The larger separation distance between the PDs (and thus channels) in this configuration resulted in an improvement in the precision, as compared to the channel-to-channel velocity data in Section 3.A.

4. WIND TUNNEL EXPERIMENT

The FLDI system was installed at the NASA Langley 20-Inch Mach 6 Air Tunnel to probe the flow in the freestream and through a flat plate boundary layer. Details pertaining to the facility can be found in [35,36]. The system was placed on vibration-isolated mounting cages to avoid vibration interference from the facility. The separate pitch and catch sides of the FLDI instrument were mounted on breadboards that were themselves mounted onto high-load scissor jacks to provide adequate movement/adjustability for the alignment of the beams relative to the flat plate surface. The breadboards were also able to be moved laterally and longitudinally with respect to the tunnel flow axis for precise alignment relative to the tunnel centerline (so that z and $\tilde{z}$ were aligned) and along the length of the flat plate (so that x and $\tilde{x}$ were aligned), respectively. The

orientation of the beams for the flat plate experiment at the 20-Inch Mach 6 Air Tunnel is shown schematically in Fig. 3(c). A CMOS camera was positioned at the tunnel centerline to aid in the adjustment of the focus of the FLDI lines, estimate the minor axis waist radius of each laser line, and estimate the Wollaston and Nomarski pair separation distances. Knife-edge measurements similar to Section 2.C were used to obtain the precise channel separation and distance above the model surface. Since the knife-edge data needed to be taken with the model injected in the tunnel test section at its run position, the knife edge was initially placed on the model surface blocking all light from passing through to the catch side. The knife edge was then lifted off the surface in 250 μm increments, and the channel spacing was computed from the voltage versus knife-edge position data.

This paper contains results from two model configurations and two run conditions, encompassing a subset of all data acquired during testing. The length of the model is 711 mm, and the FLDI measurements were made at 341.3 mm DS from the sharp leading edge (measured to the center of the Nomarski pair). The model was equipped with removable leading edge tripping elements. These square elements had a side length of 2.2 mm (3.1 mm diagonal), a height of 3.1 mm, and a center-to-center spacing of 6.6 mm. Their diagonal axis was aligned with the flow direction, and the array of these elements extended from 13.8 to 42.7 mm off the centerline on the starboard side of the model. The trips were located 88.9 mm downstream of the leading edge of the model. Two run conditions with unit Reynolds numbers of $6.6 \times 10^6 \text{ m}^{-1}$ ($2 \times 10^6 \text{ ft}^{-1}$) and $13.1 \times 10^6 \text{ m}^{-1}$ ($4 \times 10^6 \text{ ft}^{-1}$) were tested. All FLDI data presented were acquired with a sampling rate of 10 MHz over a record length of 1 s. The model configurations and run conditions for the results discussed in the following sections are summarized in Table 4, where the unit Reynolds number, Re_x , is the Reynolds number per unit length.

Table 4. Run Information for Acquired FLDI and HSS Data Sets at the 20-Inch Mach 6 Air Tunnel, Where Re_x is the Unit Reynolds Number

Section	$\text{Re}_x[\text{m}^{-1}]$	Trips	Channel Configuration	Type
4.A	6.6×10^6	Off	PD ¹ , all channels	FLDI
	13.1×10^6	Off	PD ¹ /PD ² , even channels	FLDI
4.B	6.6×10^6	Off	PD ¹ , all channels	FLDI/HSS
	13.1×10^6	Off	PD ¹ /PD ² , even channels	FLDI/HSS
4.C	13.1×10^6	Off	PD ¹ /PD ² , even channels	FLDI
	13.1×10^6	On	PD ¹ /PD ² , even channels	FLDI
4.D	13.1×10^6	Off	PD ¹ /PD ² , even channels	FLDI

A. Time Series Fluctuation Correlation

Time series data from each channel on both PDs are used to compare the distribution and evolution of disturbances in the flowfield. Both freestream data and flat plate boundary layer data are discussed in this section. Unfortunately, a triggering error between the two eight-channel oscilloscope modules was discovered after the completion of the wind tunnel test, which prevented the measurement of absolute velocities for this study. This error delayed the triggering of the channels connected to the second scope module by approximately 100 ms, although this value could change by up to 10 ms from run to run. The data from each individual scope module were acquired simultaneously, so quantitative comparisons between channels on each individual module can be made, while qualitative discussions can be made with respect to all acquired data sets.

The magnitude of the raw voltage signal for each channel of the FLDI measurement is different due to the Gaussian beam profile of the laser line [see Fig. 5(b)]. The phase shift, $\Delta\phi$, is computed in Eq. (4) to allow for comparisons between all of the normalized channel data sets [15], where V is the raw measured voltage, and V_0 is the mean voltage of the 1 s data acquisition. Density fluctuations can be computed using the phase shift when other parameters of the flow and measurement system are known (integration length, laser wavelength, Gladstone–Dale constant, and mean flow density):

$$\Delta\phi = \sin^{-1} \left(\frac{V}{V_0} - 1 \right). \quad (4)$$

Freestream data are plotted in Fig. 10 for (a) $Re_x = 6.6 \times 10^6 \text{ m}^{-1}$ and (b) $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$. In both sub-figures, the phase shift is plotted as a function of time, and channels are shifted vertically for clarity, with channel numbers indicated to the right of each plot. In Fig. 10(a), all 16 channels on PD¹ are plotted, and similar disturbance features can be seen on adjacent channels, though the features are not spatially broad enough to be seen coherently on all channels. To account for the triggering error during acquisition, the data from the second oscilloscope module (channels 9 through 16) are shifted in time to align to the first oscilloscope module (channels 1 through 8) using the results from a cross correlation. This time shift is verified by comparing the channel 8 and 9 signals near 613.3 ms for instance, where similar fluctuation structures can be observed on both channels. In Fig. 10(b), both the US (PD¹)

and DS (PD²) even channel phase shifts are plotted versus time. The PD² data are again shifted in time for each channel based on a cross correlation of the signals, and the disturbance features are similar between the two PDs for each channel. Inter-channel disturbances are more difficult to identify due to the spacing being twice as far apart as that in Fig. 10(a), but some features can be identified between channels. In both sub-figures, the influence of the Gaussian beam distribution on the PDs can be seen as a lower signal-to-noise ratio on the outer channels (e.g., 2 and 16).

Figure 11 shows phase shift data for the even channels on the US (PD¹) and DS (PD²) PDs for the $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$ run, where the data have been high-pass filtered with a cutoff frequency of 90 kHz to remove lower-frequency undulations in order to highlight the higher-frequency disturbances. The unfiltered US data are plotted in light gray for comparison. The signals for each channel have again been shifted vertically for clarity, with channel numbers indicated to the right of the plot. Note that the signals from channels 14 and 16 have reduced color saturation because they are located below the model surface. This is apparent when comparing their intensities to the channels in the flow, as well as from the absence of lower-frequency fluctuations associated with passing flow structures. As with the data plotted in Fig. 10(b), the DS signals have been shifted in time based on a cross correlation between the US and DS data for each channel, which accounts for both the triggering time offset as well as the time delay between the Nomarski pair due to the true flow velocity. While the entire acquisition record length is 1 s, the narrower time bounds used for this plot show the presence of a second-mode disturbance (outlined roughly with a dashed black line in the figure). The intensity of this disturbance is highest on channels 4 and 6, with the signal still visible on channels 2 and 8, albeit with smaller intensities. Very little signal due to the disturbance is seen on channel 12, which is closer to the surface. This highlights a benefit of this non-intrusive off-body measurement; disturbances may be stronger away from the surface and can be measured adequately by the multi-point FLDI system, whereas surface probe measurements (pressure transducer, thin film, etc.) may be limited to measuring the lower amplitude disturbance signals at the surface.

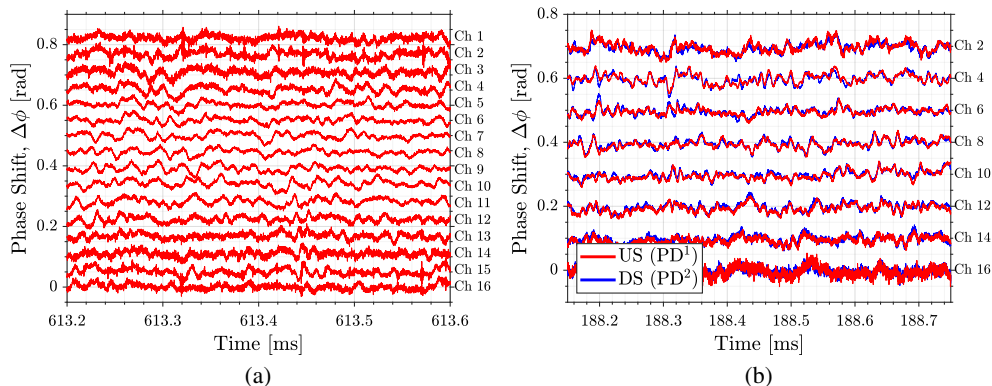


Fig. 10. Time series freestream data for (a) Run 65 and (b) Run 66, where the phase shifts are offset vertically for each channel for clarity.

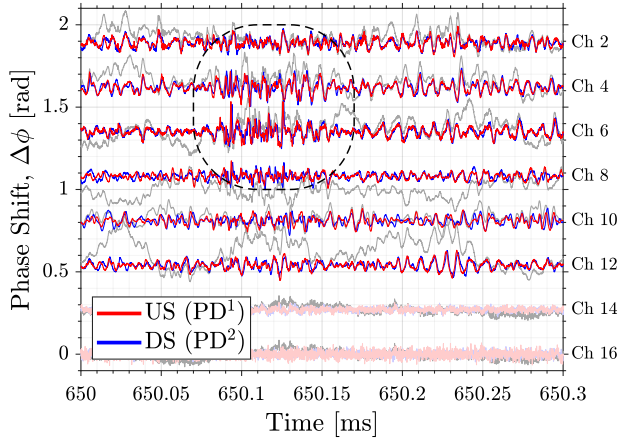


Fig. 11. Time series flat plate model data for $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$, where the phase shifts are offset for each channel for clarity, and the second-mode disturbance general location is outlined in dashed black. Channels 14 and 16 are pale in color because they are located below the model surface.

B. Wall-Normal Disturbance Fluctuation Profiles Through Boundary Layer

Higher-resolution single-line FLDI measurements at $Re_x = 6.6 \times 10^6 \text{ m}^{-1}$ and lower-resolution two-line FLDI measurements at $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$ are compared in this section, with corresponding Reynolds number HSS images (separate runs) to evaluate the structure of the flow through the boundary layer. For the HSS data, 50,000 images were acquired at a frame rate of 100 kHz. These images were first background-subtracted with images taken prior to the start of the run. To enhance visibility of the weaker flow features, each HSS frame was then subtracted by the average of all frames during the run, where a single frame (1674) is shown in the top image of Fig. 12. A proper orthogonal decomposition analysis (POD, a method originally developed by Lumley [37,38] to identify coherent flow structures [39]) of all HSS frames was performed, with the resulting second-highest energy mode plotted in the bottom image of Fig. 12. The numerical estimation of the compressible

boundary layer edge (δ_{99}) is plotted in the bottom image with a dashed gray line [40] and roughly corresponds to the region of highest density gradients in both images. The strongest gradients in the top image coincide with the numerically computed velocity boundary layer thickness in the bottom image.

To compare the FLDI and HSS data, the RMS of the FLDI phase shift, $\Delta\phi_{\text{RMS}}$, was plotted with the STD of the entire HSS acquisition. When transitioning from the FLDI measurement to the HSS measurement (which were acquired during separate runs), care was taken to ensure the HSS field-of-view was centered on the FLDI measurement location. A calibration target was used to compute the HSS field-of-view scale as 9.57 pixels/mm for the $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$ case and 10.24 pixels/mm for the $Re_x = 6.6 \times 10^6 \text{ m}^{-1}$ case; these values are different due to a realignment of the optics between runs. The FLDI channel separation and distance above the model surface from the knife-edge calibration allow for plotting and comparison of the HSS and FLDI data.

The FLDI and HSS measurements for the $Re_x = 6.6 \times 10^6 \text{ m}^{-1}$ case are shown in Fig. 13(a) and for $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$ are shown in Fig. 13(b), where the solid black region indicates the model surface (the model surface is located at 0 mm on the x axis). The data from both FLDI and HSS measurements in each plot are normalized by their peak value in order to compare the shapes of the profiles, since the magnitude relation between the two data sets is relative. Note that the HSS was sensitive to the vertical gradients in density, while the FLDI was sensitive to horizontal gradients in density. The vertical sensitivity for the HSS was chosen in order to better visualize the boundary layer edge.

In Fig. 13(a), the profile shape of the FLDI data generally tracks well with the HSS results, except for a pronounced amplitude signature in the HSS data located just below the global peak ($x = 2.5 \text{ mm}$). The location of the main peak for both measurements is the same and can be compared to the computed velocity boundary layer edge (indicated on the plot with a vertical dashed gray line), which is the same estimation used in Fig. 12. This value is 3.43 mm for the lower Reynolds number case and 2.46 mm for the higher Reynolds number case. Similar results can be seen in Fig. 13(b) for the two-line measurement, where the general trends of the FLDI and HSS

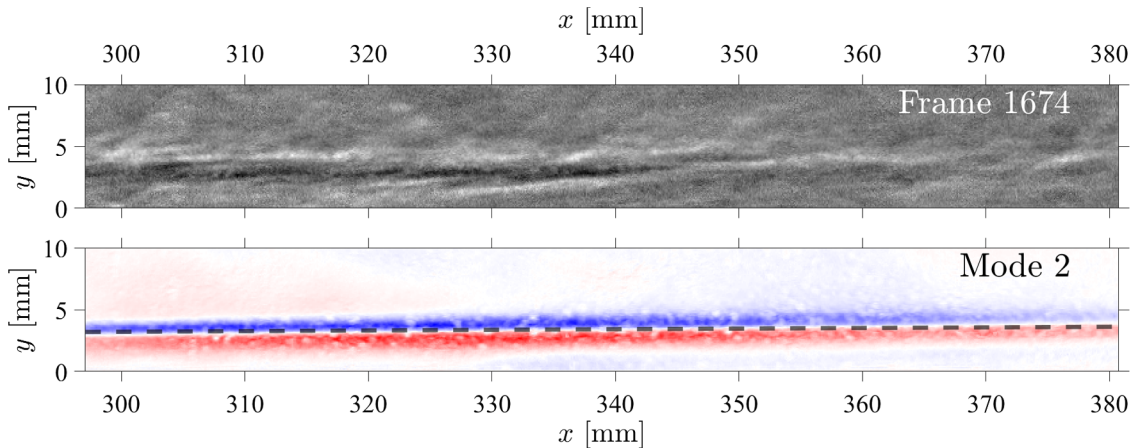


Fig. 12. (Top) Background- and average-subtracted HSS frame and (bottom) second highest energy mode from a POD analysis of the HSS data set, with computed boundary layer edge plotted in dashed gray.

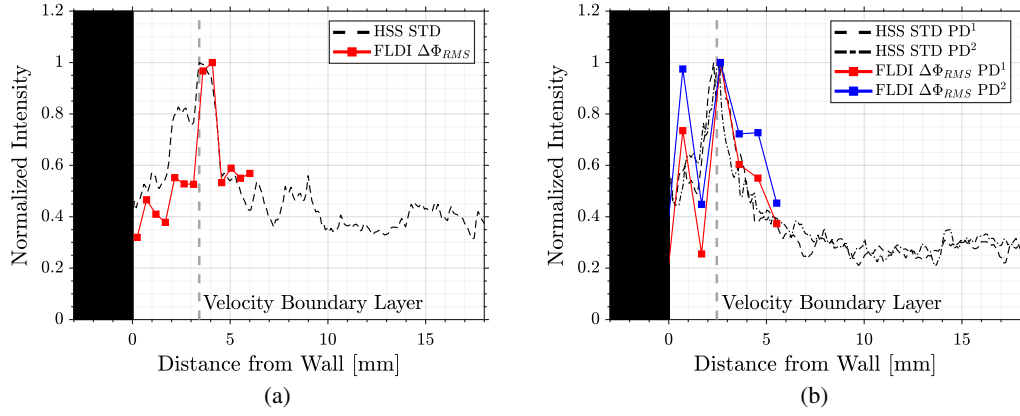


Fig. 13. Comparison of FLDI and HSS boundary layer profiles for unit Reynolds numbers of (a) $6.6 \times 10^6 \text{ m}^{-1}$ (single-line FLDI) and (b) $13.1 \times 10^6 \text{ m}^{-1}$ (two-line FLDI).

measurements are similar, except for a secondary peak in the FLDI near the surface at approximately 0.5 mm. The computed velocity boundary layer edge for this run condition is again denoted by a vertical dashed gray line, and matches the location of the largest fluctuation intensity for both the FLDI and HSS measurements. FLDI measurements beyond the edge of the boundary layer would be helpful for further comparison to the HSS, but were not available for these runs based on the selection of optics for the system. While the HSS and FLDI were sensitive to different orientations of the density gradients, the wall-normal fluctuation profile results in this section show good general agreement between the measurement techniques and can predict regions of high-intensity fluctuations reasonably well.

C. Influence of Tripping Elements

The influence of leading edge boundary layer tripping elements is explored using two data sets at $\text{Re}_x = 13.1 \times 10^6 \text{ m}^{-1}$, where even channel data were acquired on each PD (PD^1 and PD^2). PSD estimates are computed using Welch's method with a window size of 2^{16} and an overlap of 50% and are plotted in Fig. 14 for both PDs and on/off trip configurations, but only for channels above the model surface. The spectra for both US

(PD^1) and DS (PD^2) PDs for either the trips-on case or the trips-off case track together, indicating minimal evolution of the flow between the two streamwise measurement locations, which are separated by 1.84 mm. The main differences between the two tripping cases occur at frequencies above 100 kHz, where the amplitude of the PSD estimate is larger with the trips on, indicating that the tripping elements incite higher-frequency fluctuations in the flow. This high-frequency trend is seen on all channels above the model surface and can be more clearly seen in the channel 4 data of Fig. 14(b). There are also differences at lower frequencies between the on/off configurations, but these seem to be limited to the channels closer to the surface, whereas channels farther from the surface have generally the same amplitudes out to 100 kHz.

D. Second-Mode Instability Wave Spectral Characteristics

The Configuration 2 spark test (Section 3.B) demonstrated the capability of the system to make measurements of disturbance propagation velocity between corresponding channels on two PDs (PD^1 , PD^2) spaced a short distance apart (Δx_{NP}), with the propagation direction of the disturbance traveling orthogonal to the major axis of the laser lines. Data were taken

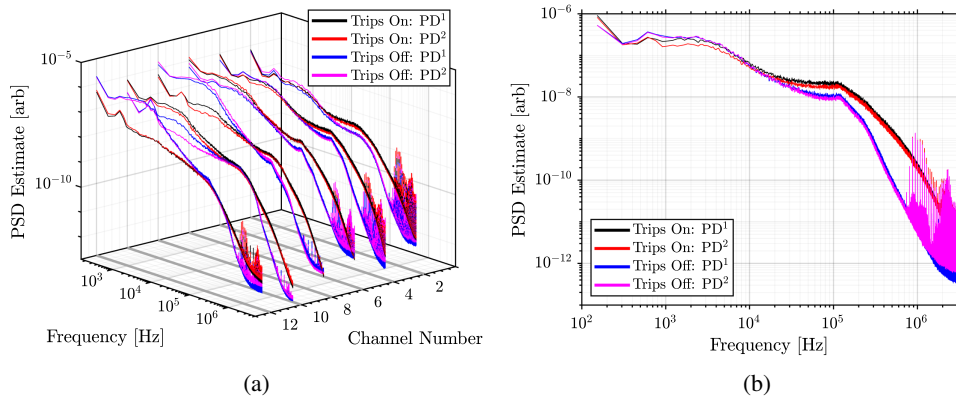


Fig. 14. PSD estimate plots for (a) even channels on US (PD^1) and DS (PD^2) PDs for cases with tripping elements attached and not attached ($\text{Re}_x = 13.1 \times 10^6 \text{ m}^{-1}$) and (b) of only the channel 4 data from (a).

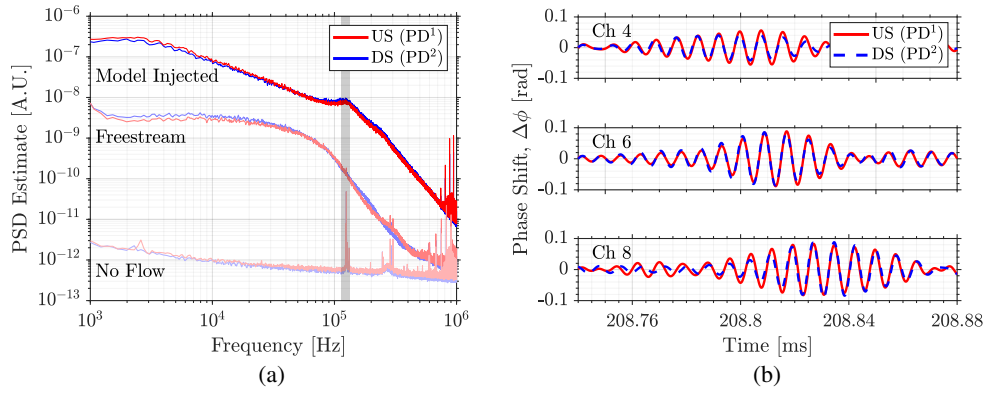


Fig. 15. (a) PSD plots for three periods of acquisition during a run at $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$ for channel 6, and (b) filtered phase shift centered at 123 kHz for a second-mode packet observed in both US and DS PDs on channels 4 (top), 6 (middle), and 8 (bottom).

at $Re_x = 13.1 \times 10^6 \text{ m}^{-1}$ using the even channels on both US and DS PDs, and a PSD plot of the data for channel 6 on both PDs is shown in Fig. 15(a). The data labeled *No Flow* were acquired when the tunnel was under vacuum before the start of the run, the data labeled *Freestream* were acquired during the run without the model in the test section, and the data labeled *Model Injected* were acquired during the run with the flat plate model in the test section.

For each of the three conditions at which data were acquired (*No Flow*, *Freestream*, *Model Injected*), the US and DS PD spectra are nearly identical. At and beyond 1 MHz, interference from RF noise dominates the signal, and it was truncated for clarity. The *Freestream* data exhibit the typical flat response out to $\approx 20 \text{ kHz}$ and between approximately 100 to 400 kHz has a constant log-log roll-off, where the slope can be expressed as f^{-C} , where C is a constant. The *Model Injected* data exhibit larger amplitudes over the entire frequency range, and a distinct peak in the spectra at approximately 123 kHz corresponds to the second-mode disturbance packets that occur sporadically throughout the entire 1 s acquisition. The frequency of this disturbance peak was also seen by Kegerise *et al.* on a similar geometry in a Mach 6 flow using atomic layer thermopiles [41].

The *Model Injected* data were then filtered using a 10 kHz wide bandpass filter centered at 123 kHz [gray region in Fig. 15(a)], and the filtered phase shifts for both US and DS PDs for three channels (4, 6, 8) are plotted in Fig. 15(b). As mentioned in Section 4.A, the triggering delay between scope modules precluded quantitative velocity measurements between corresponding channel signals on the two PDs, although the timing between signals on the same scope module (all red data or all blue data) is correct. To compare the US and DS signals, a cross correlation was computed between the signals for channel 6, and all three DS channel data sets were shifted using this time offset. Had the scope modules been triggered at the same time, a small time offset would be seen between the US and DS data sets for each channel due to the time it takes the disturbance to propagate the approximately 1.84 mm between the Nomarski pair measurement positions. The data shown in this plot are aligned however, because the cross correlation used to shift the DS data accounts for both the triggering delay and the disturbance propagation delay. Although no reliable velocity measurement can be computed from these data, it is

highly likely that these measurements can be made in future testing. Because the timing between channels on the same scope modules was not affected by the triggering issue, it can be seen that the second-mode disturbance is first registered on channel 4, then channel 6, and finally channel 8. Since the channels are oriented normal to the wall, this order of registration on the PD indicates a forward-angled wave and, due to the similar shape between the US and DS data, little evolution of the packet in the short distance between measurement locations. These wall-normal measurements have the potential to be used to compare to CFD flowfield disturbance models. Careful alignment of the two laser lines may also allow for disturbance amplification measurements and evolution of disturbances normal to the wall (through the boundary layer) and along the wall.

5. CONCLUSION

A two-point FLDI system has been modified to measure density gradient fluctuations along two lines. The modification to the two-point system includes the addition of cylindrical lenses and two linear array PDs each capable of measuring signals on 16 channels. Since the laser beam is shaped into a line (ellipse with a large major/minor axis length ratio), any number of channels can be theoretically measured with an appropriate PD and data acquisition system. Practically, the number of channels is limited by the amount of light available on each channel of the PD. A LIB spark experiment was used to evaluate the ability of the system to measure density gradient fluctuations and to compute the velocity of the spherical shock wave. Using simultaneous HSS measurements, the wave velocity determined by the FLDI system was validated, with velocities between the two techniques in good agreement. Two configurations of the laser line orientation were used to provide two different measurements of velocity: the first was between adjacent channels on the same PD, and the second was between the corresponding channels on two different PDs. An important parameter to quantify for accurate velocity measurements along a single FLDI line is the channel separation distance, as it was determined that small uncertainties in the channel spacing within a single linear array PD resulted in non-negligible uncertainties in the measured velocity when using the first configuration approach.

The line FLDI system was then used to measure density gradient fluctuations along a single line as well as along a pair of closely-spaced lines extending normal to the surface of a flat plate model in the 20-Inch Mach 6 Air Tunnel at the NASA Langley Research Center. The system was shown to be sensitive to both freestream and boundary layer fluctuations, with disturbance features tracking similarly between two separate line measurements separated by 1.84 mm. Density fluctuations of the FLDI system were compared with HSS measurements through the flat plate boundary layer, with good agreement for two unit Reynolds numbers. Results from the FLDI system further showed that the device was sensitive to second-mode disturbances, and that tracking the evolution of disturbances in the flow is possible. Unfortunately, during only the wind tunnel portion of this work, an oscilloscope triggering problem precluded measurements of disturbance velocities. The line FLDI system shows promise as a multi-point, non-intrusive, off-body measurement capable of high bandwidth measurements of small amplitude flow disturbances.

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