Handling singularities in meteoroid environment modeling

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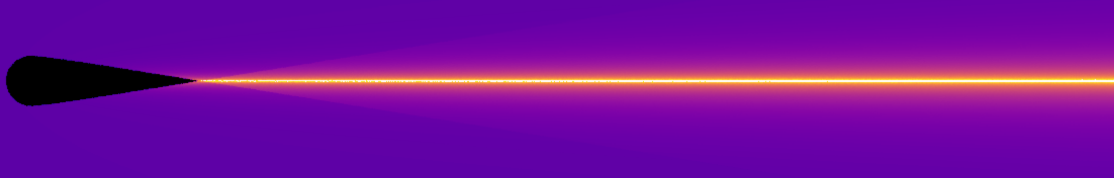
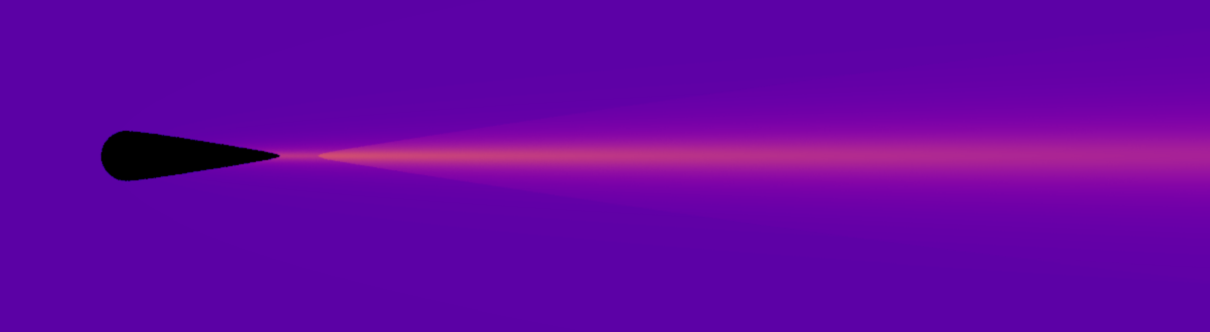
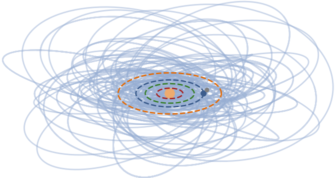


... or, “adding ϵ^2 to equations”



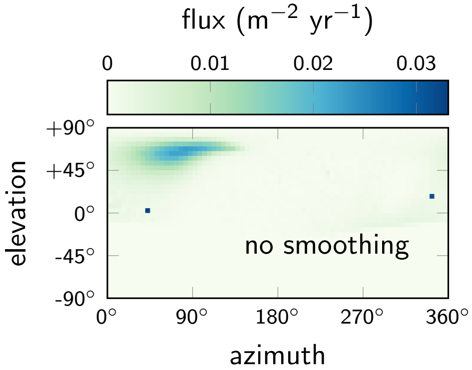
Sporadic meteoroids are $\sim$ evenly distributed in node and mean anomaly.



The spatial density along an orbit is inversely proportional to v_r:

$$\rho(r) = \frac{r}{\pi a} \frac{1}{\sqrt{(r-q)(Q-r)}}$$

When you happen to be near one orbit's perihelion ($r = q$), "hot pixels" can appear:

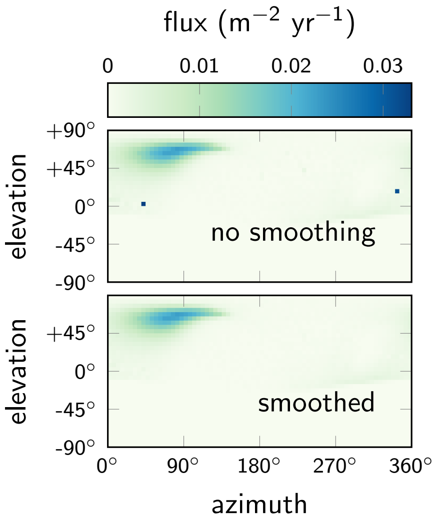


Adding a small smoothing term produces a finite density at q and Q.

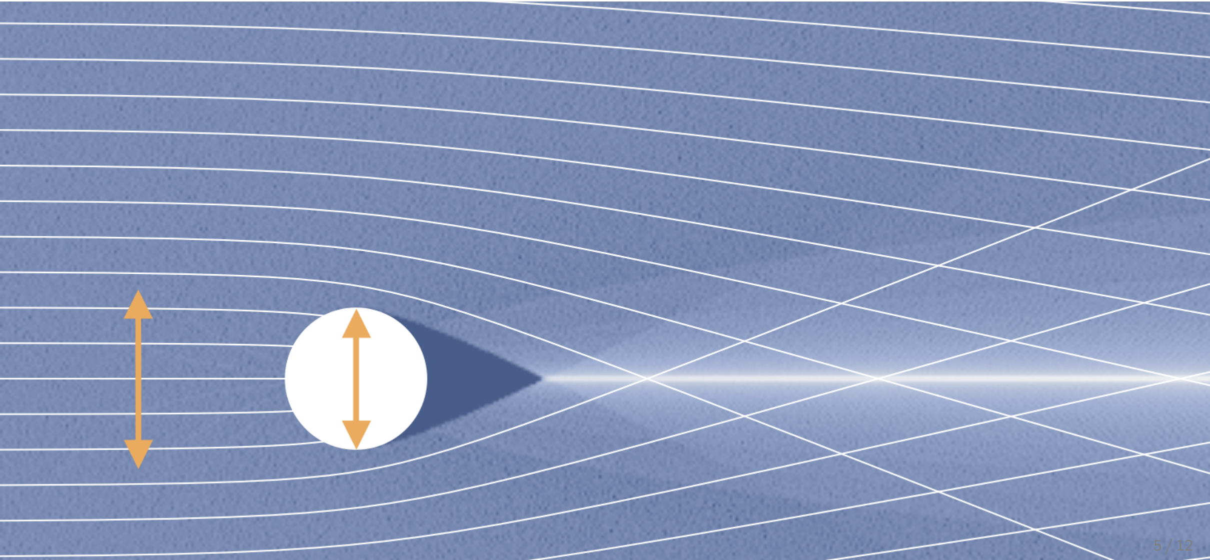
A smoothing term removes this singularity (pushes it out of range):

$$\rho(r) = \frac{r}{\pi a} \frac{1}{\sqrt{(r-q)(Q-r) + (0.01\ \mathrm{au})^2}}$$

Ideally, you'd model the orbit as a tube (future work).

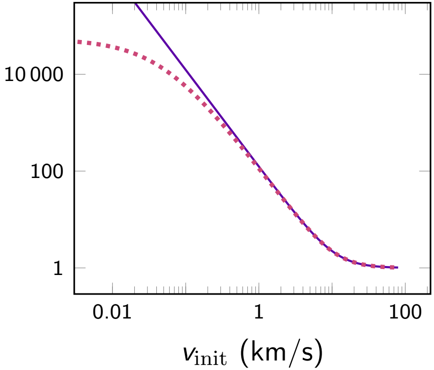


Gravity increases the effective cross-sectional area the Earth presents to a meteoroid stream.



As the initial speed drops, the degree of focusing increases.

Total flux enhancement: $1 + v_{\mathrm{esc}}^2 / v_{\mathrm{init}}^2$



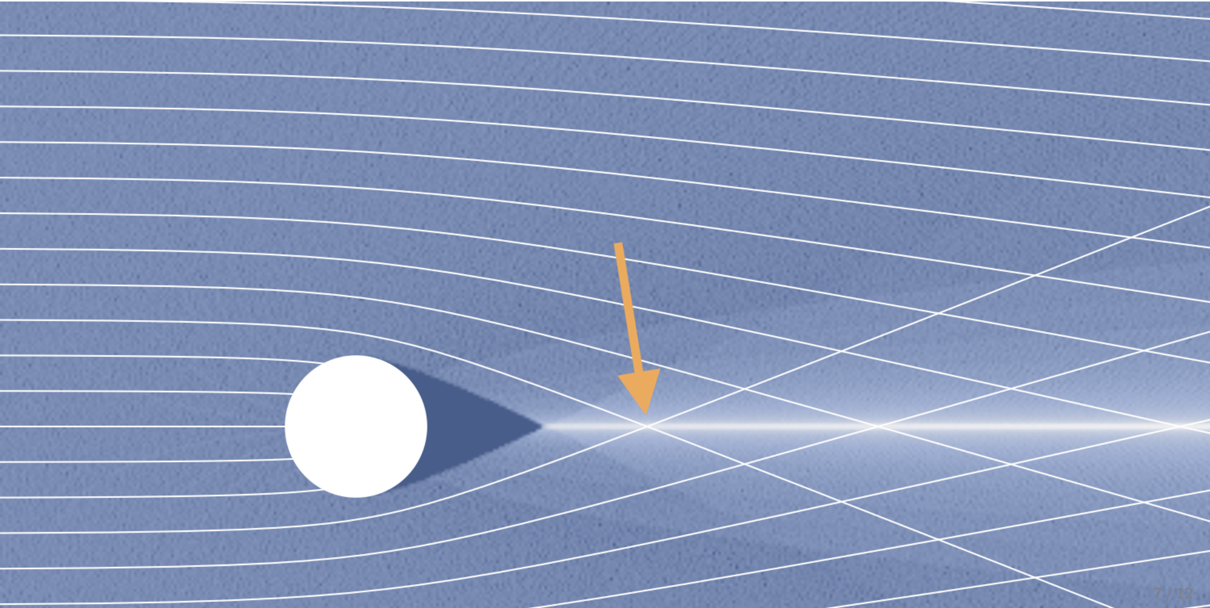
The largest impact factor that allows a particle to hit the Earth is

$$b_{\max} = \sqrt{R_\oplus^2 + 2GM_\oplus R_\oplus / v_{\mathrm{init}}^2}$$

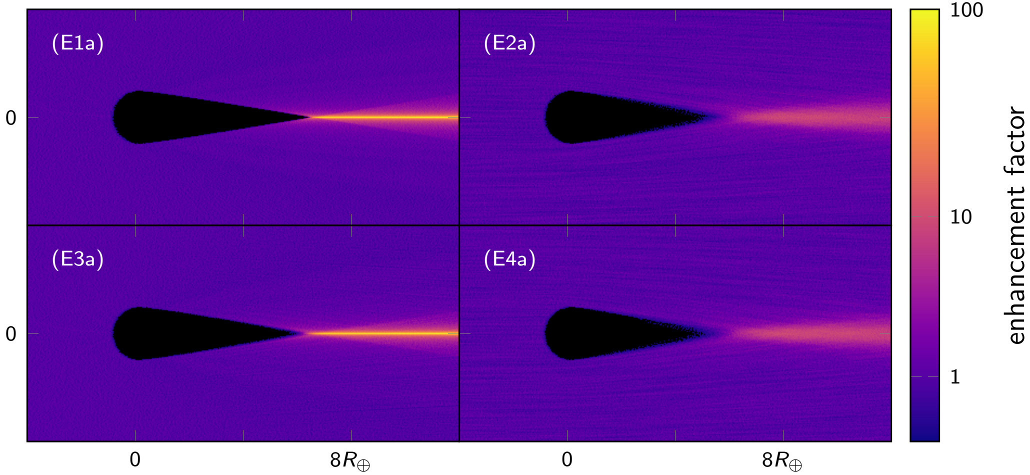
If we set $b_{\max} = r_{\mathrm{Hill}}$, we can derive an equivalent minimum speed $v_{\min} = 0.048$ km/s.

$$1 + v_{\mathrm{esc}}^2 / v_{\mathrm{init}}^2 \to 1 + v_{\mathrm{esc}}^2 / (v_{\mathrm{init}}^2 + v_{\min}^2)$$

Along the anti-radiant line, the enhancement is unbound.



Dispersions in radiant (direction) and speed both modify the pattern, but a radiant dispersion *eliminates* the singularity.

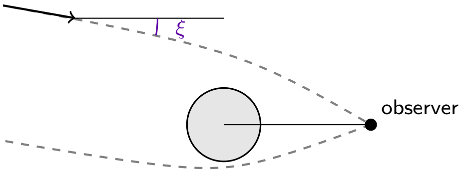


When we rearrange the Staubach et al. equations, it becomes clear how to apply a smoothing term.

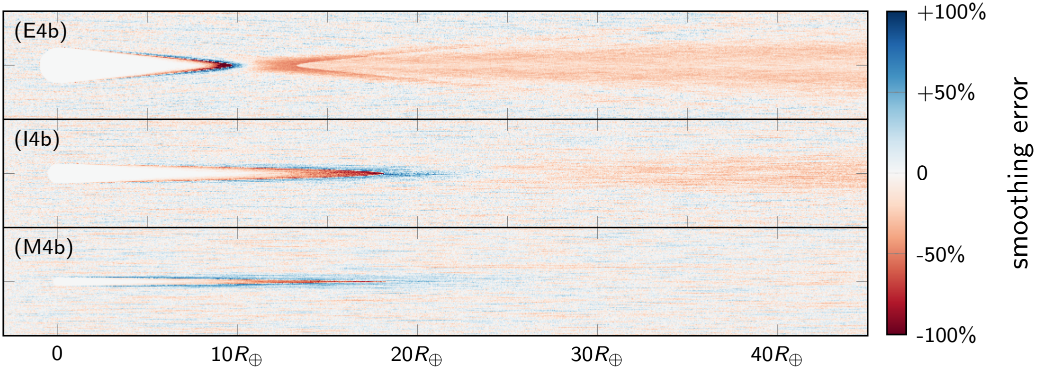
$$\eta = \frac{1}{2}\left| 1 \pm \frac{\sin^2\frac{\xi}{2} + f}{\sqrt{\sin^2\frac{\xi}{2}\left(\sin^2\frac{\xi}{2} + 2f\right)}} \right|$$

$$\downarrow$$

$$\eta' = \frac{1}{2}\left| 1 \pm \frac{\sin^2\frac{\xi}{2} + \sin^2\frac{\xi_{\min}}{2} + f}{\sqrt{\left(\sin^2\frac{\xi}{2} + \sin^2\frac{\xi_{\min}}{2}\right)\left(\sin^2\frac{\xi}{2} + \sin^2\frac{\xi_{\min}}{2} + 2f\right)}} \right|$$

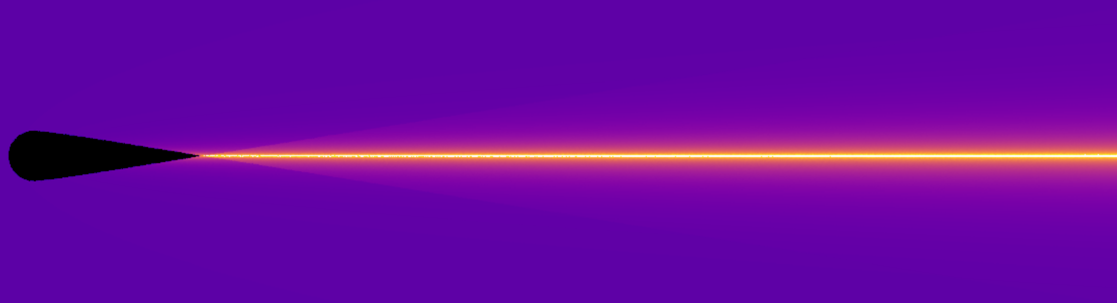


How well does the analytic approximation work?
To within $\sim$ 50%



Several equations used in meteoroid environment modeling contain singularities:

- Meteoroid number density: singularities near $r = q$ and $r = Q$
- Flux onto the Earth: singularity near $v_{\mathrm{init}} = 0$
- Flux near Earth: singularity along anti-radiant line



In each case, we obtain more realistic behavior by introducing a small smoothing term:

- Meteoroid number density: $r_{\mathrm{min}} = 0.01$ au
- Flux onto the Earth: v_{min} derived from Hill radius
- Flux near Earth: ξ_{min} is mode of radiant dispersion