

Integration of Low-Fidelity MDO and CFD-Based Redesign of Low-Boom Supersonic Transports

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A mixed-fidelity low-boom multidisciplinary optimization (MDO) problem is formulated for integration of low-fidelity MDO and computational fluid dynamics (CFD) based low-boom inverse design optimization. The mixed-fidelity low-boom MDO problem aims to enforce the weight consistency for CFD-based low-boom inverse design optimization: the optimum low-boom configuration is designed for the weight at the start of cruise of the same configuration for the low-boom overland mission. Moreover, it also seeks the optimum trades among the maximum takeoff gross weight (MTOGW), cruise Mach, and range for the low-boom overland mission while meeting the requirement of flying a transatlantic overwater mission. A block coordinate optimization (BCO) method is developed to find an approximate solution of the mixed-fidelity low-boom MDO problem. The BCO method is successfully applied to generate two CFD-based low-boom configurations that closely match two reversed equivalent area targets with ground noise levels below 70 PLdB, respectively. Moreover, these CFD-based low-boom configurations can be obtained with minor wing modifications from the solutions of the low-fidelity MDO problem. The low-fidelity MDO solutions are Pareto points for constrained multiobjective optimization of MTOGW, low-boom cruise Mach, and low-boom range. The best low-boom concept has a predetermined fuselage shape tailored for passengers and main gear storage, carries 40 passengers at seat pitch of 48 in, flies a low-boom overland mission with cruise Mach of 1.8 and range of 2,950 nm, cruises overwater at Mach 1.8 with range of 3,600 nm, satisfies the specified constraints for landing/cruise/takeoff, has MTOGW of 145,164 lb, trims the low-boom cruise flight with fuel redistributions instead of control surface deflections, and has a reversed equivalent area distribution closely matching a target with ground noise level below 70 PLdB.

Nomenclature

$\ \cdot\ $	= 2-norm of a function f on interval $[0, l_e]$ defined by $\ f\ = \left(\frac{1}{l_e} \int_0^{l_e} f(t) ^2 dt\right)^{\frac{1}{2}}$
$\ \cdot\ _{k,\lambda}$	= k -norm of a function f on interval $[0, \lambda]$ defined by $\ f\ _{k,\lambda} = \left(\frac{1}{\lambda} \int_0^\lambda f(t) ^k dt\right)^{\frac{1}{k}}$
A_e	= any equivalent area (ft ²) including $A_{e,m}$, $A_{e,LoFi}$, $A_{e,CFD}$, and $A_{e,r}$
$A_e(\mathbf{D})$	= A_e (ft ²) for $\mathbf{D}$
$A_e(x_e, \mathbf{D})$	= A_e value (ft ²) at x_e for $\mathbf{D}$
$A_{e,CFD}$	= $A_{e,m}$ (ft ²) calculated using computational fluid dynamics (CFD) lift distribution
$A_{e,LoFi}$	= $A_{e,m}$ (ft ²) calculated using low-fidelity aero lift distribution
$A_{e,m}$	= classical equivalent area (ft ²) defined by Mach angle cut method
$A_{e,r}$	= reversed equivalent area (ft ²) defined by using reverse propagation of CFD off-body pressure
$A_{e,CFD}^{lift}$	= lift part (ft ²) of $A_{e,CFD} = A_{e,CFD}^{lift} + A_e^{volume}$
$A_{e,LoFi}^{lift}$	= lift part (ft ²) of $A_{e,LoFi} = A_{e,LoFi}^{lift} + A_e^{volume}$
$A_{e,r}^{target}$	= $A_{e,r}$ target (ft ²) for $\mathbf{D}$
$A_{e,r,0}^{target}$	= $A_{e,r}$ target (ft ²) for $\mathbf{D}_0$
A_e^{volume}	= volume part (ft ²) of $A_{e,LoFi} = A_{e,LoFi}^{lift} + A_e^{volume}$ or $A_{e,CFD} = A_{e,CFD}^{lift} + A_e^{volume}$
$CG_{x,aft}$	= most aft CG_x (ft) at start of overland cruise using fuel redistribution for $\mathbf{D}$

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c_i	= chord length (ft) of airfoil at span location y_i
$CP_{x,CFD}$	= CFD longitudinal center (ft) of pressure for $\hat{\mathbf{D}}$
$CP_{x,LoFi}$	= low-fidelity longitudinal center (ft) of pressure at start of overland cruise for $\mathbf{D}$
$\mathbf{D}$	= vector of $\mathbf{d}_{lft}$, $\mathbf{d}_{wt}$, $\mathbf{d}_{aft}$, $\mathbf{d}_{LTO}$, $\mathbf{d}_{igt}$, F_s , H_{OL} , M_{OL} , and R_{OL}
$\hat{\mathbf{D}}$	= independent copy of $\mathbf{D}$ with $\mathbf{d}_{aft}$ and $\mathbf{d}_{wt}$ replaced by $\hat{\mathbf{d}}_{aft}$ and $\hat{\mathbf{d}}_{wt}$, respectively
$\mathbf{D}_0$	= $\mathbf{D}$ for baseline
$\mathbf{d}_{aft}$	= vector of design variables for pylon location and shape, horizontal tail location and shape, nacelle location, and vertical tail location
$\hat{\mathbf{d}}_{aft}$	= independent copy of $\mathbf{d}_{aft}$
$\mathbf{d}_{lft}$	= vector of widths and heights for fuselage cross sections
d_i	= width (ft) of fuselage cross section at x_i
$\mathbf{D}_{LoR}$	= $\mathbf{D}_{LoW}$ with $M_{OL} = 1.8$ and optimized R_{OL} , referred to as best trade solution
$\hat{\mathbf{D}}_{LoR}$	= $\hat{\mathbf{D}}$ corresponding to $\mathbf{D}_{LoR}$
$\mathbf{D}_{LoT}$	= $\mathbf{D}_{LoW}$ for cruise Mach 1.8 and better trim characteristics, referred to as trim solution
$\hat{\mathbf{D}}_{LoT}$	= $\hat{\mathbf{D}}$ corresponding to $\mathbf{D}_{LoT}$
$\mathbf{D}_{LoW}$	= $\mathbf{D}$ for the lowest weight solution on the Pareto frontier of multi-objective MDO, which also has an acceptable matching error for low-boom inverse design (i.e., $\ A_{e,LoFi}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\ _{k,\lambda} \leq \varepsilon$)
$\hat{\mathbf{D}}_{LoW}$	= $\hat{\mathbf{D}}$ corresponding to $\mathbf{D}_{LoW}$
$\mathbf{d}_{LTO}$	= vector of 6 auxiliary design variables for landing and takeoff: flap deflection angles, main gear length, and longitudinal center-of-gravity offsets for trim at landing and takeoff
$\mathbf{d}_{igt}$	= vector of 12 control-point coordinates of Bezier parametric curve for $A_{e,r,0}^{target}$ or $A_{e,r}^{target}$
$\mathbf{d}_{wt}$	= vector of 19 wing design variables and tail rotation angle θ_{tail}
$\hat{\mathbf{d}}_{wt}$	= independent copy of $\mathbf{d}_{wt}$
F_s	= sea-level static thrust (lb) for engine
$\hat{F}_s$	= independent copy of F_s (lb) as input for the Numerical Propulsion System Simulation (NPSS)
g_i	= constraint function for mission constraint (III.A.i)
H_{OL}	= cruise altitude (ft) for overland mission or for CFD analysis
k	= positive integer to define integral error for low-boom front shaping
$\mathbf{l}$	= lower bound vector for $\mathbf{D}$ or $\hat{\mathbf{D}}$
l_e	= effective length (ft) of configuration defined by $\mathbf{D}$ or $\hat{\mathbf{D}}$, which is the largest effective distance where Mach angle cut plane intersects the configuration
$l_{e,0}$	= l_e (ft) for $\mathbf{D}_0$
M_{OL}	= cruise Mach for overland mission or for CFD analysis
MTOGW	= maximum (lb) of $TOGW_{OL}$ and $TOGW_{OW}$
PLdB(A_e)	= perceived level in decibel (PLdB) of sonic boom ground signature for A_e
Q_{OL}	= dynamic pressure (psf) for cruise Mach and cruise altitude of overland mission
R_{OL}	= range (nm) of overland mission
t	= temporary variable
$TOGW_{OL}$	= takeoff gross weight (lb) for overland mission of $\mathbf{D}$
$TOGW_{OW}$	= takeoff gross weight (lb) for overwater mission of $\mathbf{D}$
$\mathbf{u}$	= upper bound vector for $\mathbf{D}$ or $\hat{\mathbf{D}}$
W_0	= weight (lb) at start of overland cruise for $\mathbf{D}_0$
W_{crs}	= weight (lb) at start of overland cruise for $\mathbf{D}$
x, y, z	= coordinates (ft) of point in space
x_e	= effective distance (ft) for A_e calculation
x_{lft}	= x coordinate (ft) of leading edge of wing root airfoil
x_i	= x coordinate (ft) of fuselage cross section location
x_{tip}	= x coordinate (ft) of trailing edge of tip airfoil of wing
y_i	= y coordinate (ft) of span location of wing
z_i	= height (ft) of fuselage cross section at x_i
α	= angle of attack (degree) at start of overland cruise for $\mathbf{D}$
$\hat{\alpha}$	= independent copy of α (degree) for calculation of $A_e^{volume}(\hat{\mathbf{D}})$ and $A_{e,r}(\hat{\mathbf{D}})$
δ	= lower limit for $A_{e,r,0}^{target}(l_{e,0}/2)$
ε	= tolerance of $0.007 \cdot A_{e,r}^{target}(l_e)$ for acceptable low-boom inverse design objective value

$\hat{\epsilon}$	= error tolerance for $\ A_e^{\text{volume}}(\hat{\mathbf{D}}) - A_e^{\text{volume}}(\mathbf{D})\ $
θ_{htail}	= deflection angle (degree) of an all-moving horizontal tail at start of overland cruise for $\mathbf{D}$
$\hat{\theta}_{\text{htail}}$	= independent copy of θ_{htail} for $\hat{\mathbf{D}}$
λ	= $l_e \cdot \lambda_0 / l_{e,0}$ (ft), upper limit (ft) for integration of A_e matching error for $\mathbf{D}$
λ_0	= x_e location (ft) of the highest point on $A_{e,\text{LoFi}}(\mathbf{D}_0)$
μ	= estimate of ratio $A_{e,r}(l_e)/A_{e,\text{CFD}}(l_e)$ for expected CFD-based low-boom design
η	= lower limit of low-fidelity trim margin for overland cruise
σ	= scaling parameter of $A_{e,r}^{\text{target}}(\mathbf{D})$
$\tau_{i,\text{max}}$	= design section lift coefficient for NACA 63-series airfoil at span location y_i
τ_{max}	= design section lift coefficient for NACA 63-series airfoil

I. Introduction

THE NASA X-59 low-boom flight demonstrator (LBFD) [1] brings low-boom supersonic overland flight closer to reality. The LBFD project will help determine future Federal Aviation Administration (FAA) regulations on the acceptable ground noise level for supersonic overland flights and validate sonic boom prediction tools. Despite the huge technical advances in design of low-boom supersonic aircraft, many barriers still exist for a low-boom supersonic transport to be economically viable. A set of design requirements for a supersonic business jet (SBJ) was articulated by Henne [2] based on multiple market research studies. Smith [3] provided a comprehensive review of design issues for low-boom SBJs and a historical summary of conceptual design efforts (including some hard-to-find configuration layouts such as the Lockheed Martin Quiet Supersonic Platform concept). An updated review about low-boom SBJ design issues can be found in Ref. [4]. Design requirements from NASA and used by Lockheed Martin and Boeing for future supersonic transports can be found in Refs. [5-6]. Alonso and Colonno [7] gave a review of the advances and challenges in design of low-boom supersonic aircraft from a multidisciplinary design optimization (MDO) perspective. For historical accounts on sonic boom research, see Refs. [8] and [9] for general reading and Ref. [10] for more in-depth reading.

Low boom has different meanings when different analysis metrics are used to define the low-boom design objective. Traditional sonic boom minimization treats a supersonic wing and body configuration as a body of revolution [11,12] and generates optimal F-functions [13,14] for shaping the corresponding equivalent area ($A_{e,m}$) distributions of low-boom configurations. For sonic boom minimization, low-boom design objectives evolved from reducing the initial pressure rise of the ground signature to matching $A_{e,m}$ with a low-boom target distribution. Analysis of lift distribution for $A_{e,m}$ calculation was also advanced from the modified linear theory to computational fluid dynamics (CFD). See Refs. [15] and [16] for low-boom design studies on minimizing the initial pressure rise of the ground signature and matching $A_{e,m}$ with a low-boom target, respectively, while using CFD lift for $A_{e,m}$ calculation. For supersonic aircraft in flight, CFD methods could accurately predict the off-body pressures obtained by in-flight flow-field probe measurements (see Chapter 4 of Ref. [10]). Further validation of CFD for low-boom design was performed on the Gulfstream Quiet Spike concept via wind tunnel [17] and flight [18] tests. The boom prediction workshops [19,20] highlight the advances of the state-of-the-art in using CFD analysis methods to predict off-body pressures of complex supersonic configurations. The propagation of a waveform through the atmosphere to get the sonic boom ground signature is improved from the original waveform parameter method [21] to the solution of the augmented Burgers equation [22,23]. Computational cost of solving the augmented Burgers equation is only slightly higher than the waveform parameter method, so the augmented Burgers equation becomes the preferred analysis model for sonic boom propagation through the atmosphere (see Table 2 in Ref. [24]).

The existing CFD-based low-boom design methods have a potential weight inconsistency gap issue: the specified weight for CFD-based low-boom optimization might be very different from the actual cruise weight of the low-boom design. The integration of low-fidelity MDO and CFD-based low-boom redesign aims to close this weight inconsistency gap. The resulting mixed-fidelity low-boom MDO problem uses low-fidelity aerodynamics for conceptual design of supersonic transports and CFD for low-boom inverse design. A block coordinate optimization (BCO) method is developed to find an approximate solution of the mixed-fidelity low-boom MDO problem. The BCO method is also capable of finding an optimum combination of the cruise speed, cruise altitude, range, and maximum takeoff gross weight (MTOGW) for the low-boom design. See Section II for details. In Section III, the BCO method is successfully applied to generate a low-boom supersonic transport that carries 40 passengers, flies a low-boom overland mission with cruise Mach of 1.8 and range of 2,930 nm, cruises overwater with Mach of 1.8 and range of 3,600 nm, and has the potential to attain a CFD-based ground signature below 70 PLdB. Concluding remarks are given in Section IV.

II. Block Coordinate Optimization for Mixed-Fidelity Low-Boom MDO

The ideal mathematical problem for integration of low-fidelity MDO and CFD-based low-boom redesign is formulated in Subsection II.A. The resulting mixed-fidelity low-boom MDO problem aims to ensure that the specified weight for CFD-based low-boom inverse design is the actual cruise weight of the low-boom design from the mission analysis with the optimum cruise Mach and range for the overland mission. Moreover, the CFD-based low-boom inverse design also has the lowest MTOGW possible and can be trimmed with fuel redistribution at the start of overland cruise. In Subsection II.B, a BCO method is proposed to find an approximate solution of the mixed-fidelity low-boom MDO problem. The BCO method uses $\mathbf{D}$ as the design vector for low-fidelity low-boom MDO and $\hat{\mathbf{D}}$ as the design vector for CFD-based low-boom inverse design, where $\hat{\mathbf{D}}$ is a copy of $\mathbf{D}$ with independent values for the wing design variables, tail rotation angle, and design variables for the aft components. The initial baseline $\mathbf{D}_0$ and design variables are defined in Subsection II.C. Implementation details for BCO are provided in Subsection II.D. The validity of the BCO method is demonstrated using numerical examples in Section III.

A. Mixed-Fidelity Low-Boom MDO

The integration of low-fidelity MDO and CFD-based low-boom optimization aims to ensure that the specified weight for CFD-based low-boom inverse design is the actual cruise weight of the low-boom design from the mission analysis. Moreover, all system-level design requirements are explicitly included in the optimization formulation. Unlike the conventional inverse design optimization of minimizing an inverse design objective, the low-boom inverse design objective is treated as a constraint to make the design requirement transparent: the optimum inverse design objective value must be close to its ideal value of zero. This leads to the following mixed-fidelity low-boom MDO problem with three objective functions.

$$\begin{aligned} \min_{\mathbf{D}} \{ & \text{MTOGW}, -M_{OL}, -R_{OL} \} \quad \text{subject to } \mathbf{l} \leq \mathbf{D} \leq \mathbf{u} \\ & g_i(\mathbf{D}) \geq 0 \quad (1 \leq i \leq 9), \quad \text{CG}_{x,\text{aft}} - \text{CP}_{x,\text{CFD}} \geq 0 \\ & \|A_{e,r}(\mathbf{D}) - A_{e,r}^{\text{target}}(\mathbf{D})\| \leq \varepsilon, \quad \text{PLdB}(A_{e,r}^{\text{target}}(\mathbf{D})) \leq 70 \end{aligned} \quad (1)$$

Here $g_i(\mathbf{D}) \geq 0$ ($1 \leq i \leq 9$) correspond to the following mission constraints.

- (III.A.1) CG margin to prevent tip over on the ground ≥ 4 ft (i.e., the longitudinal CG of the aircraft at empty weight is at least 4 ft before the main gear longitudinal location)
- (III.A.2) Static margin for overland cruise $\geq 2\%$
- (III.A.3) LTO static margins $\geq 2\%$
- (III.A.4) Tail rotation angles for trim at LTO $\geq -20^\circ$
- (III.A.5) LTO field lengths $\leq 8,300$ ft
- (III.A.6) Approach velocity ≤ 150 knots
- (III.A.7) $\mathbf{D}$ has storage space for main gear and carries 40 passengers for both overland and overwater missions.
- (III.A.8) $\mathbf{D}$ has an overwater mission with cruise Mach of 1.8, cruise ceiling of 60,000 ft, and range of 3,600 nm.
- (III.A.9) $\mathbf{D}$ has an overland mission with cruise Mach of M_{OL} , cruise altitude of H_{OL} , and range of R_{OL} .

The design vector $\mathbf{D}$ has $\mathbf{d}_{\text{fit}}$, $\mathbf{d}_{\text{wt}}$, $\mathbf{d}_{\text{aft}}$, $\mathbf{d}_{\text{LTO}}$, $\mathbf{d}_{\text{tgt}}$, F_s , H_{OL} , M_{OL} , and R_{OL} as its components. The partition of $\mathbf{D}$ into different weakly coupled blocks of design variables is an important step to solve Eq. (1). The subvector $\mathbf{d}_{\text{fit}}$ consists of the design variables for fuselage nose cone to define a low-boom front shape. The subvector $\mathbf{d}_{\text{wt}}$ has the wing design variables and the all-moving horizontal tail rotation angle θ_{ail} at the start of overland cruise. The wing design variables define the low-boom shape in the mid-range of equivalent distance and θ_{ail} determines the appropriate ratio between the wing lift and horizontal tail lift for a low-boom configuration. The subvector $\mathbf{d}_{\text{aft}}$ has the design variables for the aft components and determines the low-boom aft shape. The subvectors $\mathbf{d}_{\text{LTO}}$ and $\mathbf{d}_{\text{tgt}}$ contain the design variables for LTO performance and low-boom target $A_{e,r}^{\text{target}}$, respectively. The engine thrust F_s is an important design variable to find the most efficient engine to complete the dual missions. The overland mission parameters H_{OL} , M_{OL} , and R_{OL} are also used as the design variables for an optimum combination of cruise altitude, cruise Mach, and range for the overland mission. Such an optimum combination of H_{OL} , M_{OL} , and R_{OL} is critical for development of an economically viable low-boom concept.

Constraint (III.A.1) is a constraint for layout design. It is easily satisfied with a reasonable main gear location. Constraints (III.A.2) and (III.A.3) are typical longitudinal stability constraints for aircraft conceptual design. Because LTO trim analyses use linear aerodynamics, the limit of -20° on tail deflections for trim during LTO is imposed in (III.A.4). The constraints on LTO field lengths are determined by the relevant service airports and some margin for potential errors in the estimates of LTO field lengths. The maximum approach velocity of 150 knots is used to mitigate

the approach noise. For succinctness, $\mathbf{D}$ in (III.A.7)-(III.A.9) actually means the configuration defined by $\mathbf{D}$. Constraint (III.A.7) defines the payload weight and imposes volume constraints for 40 passengers and main gear storage. For the overwater mission constraint (III.A.8), the cruise Mach 1.8 is based on LTO noise, emissions, and technology availability for an advanced engine model; the cruise altitude for the overwater mission is limited to 60,000 ft for maximum performance benefit allowed by the engine; and the range of 3,600 nm is based on the origin-destination pairs for the transatlantic commercial flights. Constraint (III.A.9) defines M_{OL} , H_{OL} , and R_{OL} as the cruise Mach, cruise altitude, and range of the overland mission.

Each of LTO constraints includes two constraints for landing and takeoff, respectively; but two inequality constraints $t_1 \geq 0$ and $t_2 \geq 0$ can be written as $\min(t_1, t_2) \geq 0$ using one constraint function. Constraints (III.A.7)-(III.A.9) involve equality constraints, but an equality $t = 0$ can be written as an inequality $\min(t, -t) \geq 0$. So, each of (III.A.1)-(III.A.9) can be written as $g_i(\mathbf{D}) \geq 0$ with an appropriate definition of $g_i(\mathbf{D})$.

Equation (1) is called a mixed-fidelity low-boom MDO problem because of the different aerodynamics analysis fidelities for conceptual design of supersonic transports and CFD-based low-boom inverse design. In this paper, low-fidelity analyses are used to compute $CG_{x,aft}$ and the mission constraints (III.A.1)-(III.A.9). In the future, multi-fidelity computational methods could be used to reduce the modeling uncertainties in linear aerodynamics and empirical weights estimation for aircraft conceptual design.

In theory, Eq. (1) is the ideal constrained multiobjective optimization problem for finding a CFD-based low-boom design that can also perform the specified overland and overwater missions. In Eq. (1), $\mathbf{l} \leq \mathbf{D} \leq \mathbf{u}$ is the standard constraint to define the ranges of design variables; $g_i(\mathbf{D}) \geq 0$ ($1 \leq i \leq 9$) represent the mission constraints (III.A.1)-(III.A.9); $CG_{x,aft} - CP_{x,CFD} \geq 0$ is for trim at the start of overland cruise with fuel redistribution; $\varepsilon = 0.007 \cdot A_{e,r}^{target}(l_e, \mathbf{D})$ is a relatively small error tolerance for the $A_{e,r}$ [25] matching error and it can be replaced by another reasonable error tolerance; and the two constraints related to $A_{e,r}^{target}(\mathbf{D})$ ensure a close match of $A_{e,r}(\mathbf{D})$ with a low-boom target $A_{e,r}^{target}(\mathbf{D})$ below 70 PLdB. Here $\|A_{e,r}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\|$ is the integrated error of $|A_{e,r}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})|^2$ over the effective distance range $[0, l_e]$ and $PLdB(A_{e,r}^{target}(\mathbf{D}))$ is the noise level of the ground signature using $A_{e,r}^{target}(\mathbf{D})$, M_{OL} , and H_{OL} as inputs for sonic boom propagation analysis. The target PLdB limit of 70 is based on the NASA N+3 goal [5-6] for low-boom supersonic transports. The objectives in Eq. (1) are the system-level design goals for low-boom supersonic transports: the generated low-boom concept has the lowest MTOGW and the highest low-boom cruise Mach and range if possible.

In practice, the mixed-fidelity MDO Eq. (1) is extremely difficult to solve as a numerical optimization problem because of four reasons. The first is that the objective and constraint values could be undefined if the mission analysis fails. The mission analysis failure is mainly due to insufficient engine thrust to climb to the required cruise altitude. The second is that solutions for the $A_{e,r}$ matching error constraint $\|A_{e,r}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\| \leq \varepsilon$ are acceptable CFD-based low-boom inverse designs, which are rare (if they exist) in any design space $\mathbf{l} \leq \mathbf{D} \leq \mathbf{u}$. This is a difficult inverse design optimization problem by itself and requires many evaluations of CFD off-body pressure. Even for Euler CFD analysis, each off-body pressure evaluation takes several hours using a Linux cluster computer with parallel implementations and hundreds of compute nodes. So, it is time consuming to find a feasible solution of $\|A_{e,r}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\| \leq \varepsilon$. The third is that the existing solution method for $\|A_{e,r}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\| \leq \varepsilon$ [25,26] is based on minimization of $\|A_{e,r}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\|$ for a fixed cruise weight at fixed cruise Mach and altitude. A simultaneous search for an optimum low-boom target and the corresponding inverse design solution with changing cruise conditions is a huge leap forward. Finally, there is no guarantee that $\|A_{e,r}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\| \leq \varepsilon$ is always solvable.

In the next subsection, an extension of the BCO method in Ref. [27] will be used to find an approximate solution of Eq. (1).

B. BCO for Mixed-Fidelity Low-Boom MDO

The BCO method in Ref. [27] can be extended to a partition of $\mathbf{D}$ for finding an approximate solution of the mixed-fidelity MDO Eq. (1). To decouple the low-fidelity MDO and CFD-based low-boom inverse design optimization, independent copies $\hat{\mathbf{d}}_{wt}$ and $\hat{\mathbf{d}}_{aft}$ of $\mathbf{d}_{wt}$ and $\mathbf{d}_{aft}$ are used for CFD-based low-boom inverse design. The BCO method solves the following optimization problems sequentially to get an approximate solution of Eq. (1).

$$\min_{\mathbf{d}_{tgt}} PLdB(A_{e,r,0}^{target}) \text{ subject to } A_{e,r,0}^{target}\left(\frac{l_{e,0}}{2}\right) \geq \delta, A_{e,r,0}^{target}(l_{e,0}) = \mu \cdot A_{e,LoFi}(l_{e,0}, \mathbf{D}_0) \quad (2.1)$$

(Baseline low-boom target optimization)

$$\min_{F_s} MTOGW \text{ subject to } \mathbf{l} \leq \mathbf{D} \leq \mathbf{u}, g_8(\mathbf{D}) \geq 0, g_9(\mathbf{D}) \geq 0 \quad (2.2)$$

(Engine thrust optimization)

$$\begin{aligned} \min_{d_{\text{ftt}}, d_{\text{wt}}} \{ & \text{MTOGW}, \|A_{\text{e,LoFi}}(\mathbf{D}) - A_{\text{e,r}}^{\text{target}}(\mathbf{D})\|_{k,\lambda} \} \text{ with } \lambda = \frac{l_e}{l_{e,0}} \cdot \lambda_0 \\ \text{subject to } & \mathbf{l} \leq \mathbf{D} \leq \mathbf{u}, g_i(\mathbf{D}) \geq 0 \ (7 \leq i \leq 9), \text{ CG}_{x,\text{aft}} - \text{CP}_{x,\text{LoFi}} \geq \eta \geq 0 \\ & \text{(Low-fidelity low-boom MDO)} \end{aligned} \quad (2.3)$$

$$\begin{aligned} \text{Find a subvector } & \mathbf{d}_{\text{LTO}} \text{ of } \mathbf{D} \text{ in } \mathbf{l} \leq \mathbf{D} \leq \mathbf{u} \text{ for a feasible solution of } g_i(\mathbf{D}) \geq 0 \ (1 \leq i \leq 9) \\ & \text{(Feasibility optimization for mission constraints)} \end{aligned} \quad (2.4)$$

$$\begin{aligned} \max_{H_{\text{OL}}, M_{\text{OL}}} & M_{\text{OL}} \text{ subject to } g_8(\mathbf{D}) \geq 0, g_9(\mathbf{D}) \geq 0, \text{PLdB}(A_{\text{e,r}}^{\text{target}}(\mathbf{D})) \leq 70, Q_{\text{OL}} \text{ is fixed} \\ & \text{(Maximization of cruise Mach for overland mission)} \end{aligned} \quad (2.5)$$

$$\begin{aligned} \max_{H_{\text{OL}}, R_{\text{OL}}} & R_{\text{OL}} \text{ subject to } g_8(\mathbf{D}) \geq 0, g_9(\mathbf{D}) \geq 0, \text{PLdB}(A_{\text{e,r}}^{\text{target}}(\mathbf{D})) \leq 70, \text{TOGW}_{\text{OL}} \leq \text{TOGW}_{\text{OW}} \\ & \text{(Maximization of range for overland mission)} \end{aligned} \quad (2.6)$$

$$\begin{aligned} \min_{\hat{\mathbf{d}}_{\text{aft}}, \hat{\mathbf{d}}_{\text{wt}}, \hat{\alpha}, \sigma} & \|A_{\text{e,r}}(\hat{\mathbf{D}}) - \sigma \cdot A_{\text{e,r}}^{\text{target}}(\mathbf{D})\| \text{ subject to } \mathbf{l} \leq \hat{\mathbf{D}} \leq \mathbf{u}, \text{ (CFD lift for } \hat{\mathbf{D}}) = W_{\text{crs}} \\ & \text{(CFD-based low-boom inverse redesign of low-fidelity MDO solution)} \end{aligned} \quad (2.7)$$

The low-boom target $A_{\text{e,r}}^{\text{target}}(\mathbf{D})$ in the above formulations is defined by the following scaling formulas.

$$A_{\text{e,r}}^{\text{target}}(x_e) = \frac{W_{\text{crs}}}{W_0} \cdot A_{\text{e,r},0}^{\text{target}}\left(\frac{l_{e,0}}{l_e} \cdot x_e\right) \quad \text{for Eqs. (2.3) and (2.7)} \quad (3.1)$$

$$A_{\text{e,r}}^{\text{target}}(x_e) = \frac{\int_0^\lambda A_{\text{e,LoFi}}(t) \cdot A_{\text{e,r},0}^{\text{target}}(t) \cdot dt}{\int_0^\lambda A_{\text{e,r},0}^{\text{target}}(t) \cdot A_{\text{e,r},0}^{\text{target}}(t) \cdot dt} \cdot A_{\text{e,r},0}^{\text{target}}\left(\frac{l_{e,0}}{l_e} \cdot x_e\right) \quad \text{for Eqs. (2.5) and (2.6)} \quad (3.2)$$

The optimization problems Eqs. (2.1)-(2.7) are just iteration steps to get an approximate solution of Eq. (1) and they do not include some of the actual design requirements in Eq. (1). The following post-optimization constraints must be satisfied by a solution pair $(\mathbf{D}, \hat{\mathbf{D}})$ of Eqs. (2.1)-(2.7) to obtain an approximate solution of Eq. (1).

$$\|A_{\text{e,LoFi}}(\mathbf{D}) - A_{\text{e,r}}^{\text{target}}(\mathbf{D})\|_{k,\lambda} \leq \varepsilon, \text{PLdB}(A_{\text{e,r}}^{\text{target}}(\mathbf{D})) \leq 70 \quad (4.1)$$

$$\|A_{\text{e,r}}(\hat{\mathbf{D}}) - \sigma \cdot A_{\text{e,r}}^{\text{target}}(\mathbf{D})\| \leq \varepsilon, \text{PLdB}(\sigma \cdot A_{\text{e,r}}^{\text{target}}(\mathbf{D})) \leq 70 \quad (4.2)$$

$$\|A_{\text{e}}^{\text{volume}}(\hat{\mathbf{D}}) - A_{\text{e}}^{\text{volume}}(\mathbf{D})\| \leq \hat{\varepsilon} \quad (4.3)$$

$$\text{CG}_{x,\text{aft}} - \text{CP}_{x,\text{CFD}} \geq 0 \quad (4.4)$$

Equations (4.1) and (4.2) are the implicit requirement for inverse design optimization: the optimum inverse design objective value must be close to its ideal value of zero. So, an optimal solution of the low-boom MDO Eq. (2.3) is useless unless it also satisfies Eq. (4.1), which can only be checked after the optimization. Any design $\mathbf{D}$ satisfying Eqs. (4.1) is called a *plausible* low-boom design. The A_e matching error $\|A_{\text{e,LoFi}}(\mathbf{D}) - A_{\text{e,r}}^{\text{target}}(\mathbf{D})\|_{k,\lambda} \leq \varepsilon$ implies that $\mathbf{D}$ has a low-boom shape up to the location $x_e = \lambda$ in the aft region of the wing. Similarly, an acceptable solution of the low-boom inverse design problem Eq. (2.7) must satisfy Eq. (4.2). Any design $\hat{\mathbf{D}}$ satisfying Eq. (4.2) is called a *feasible* low-boom design. A design $\hat{\mathbf{D}}_{\text{opt}}$ satisfying $\text{PLdB}(A_{\text{e,r}}(\hat{\mathbf{D}}_{\text{opt}})) \leq 70$ is called an *ideal* low-boom design. These terminologies can be used to discuss low-boom configurations in different design phases from low-fidelity low-boom front shaping to high-fidelity ground signature shaping. An ideal low-boom design is the ultimate goal, but its existence is not guaranteed. In practice, a feasible low-boom design $\hat{\mathbf{D}}$ can be reshaped to obtain $\hat{\mathbf{D}}_1$ with $\text{PLdB}(A_{\text{e,r}}(\hat{\mathbf{D}}_1)) < 79$ using an inverse design method for $A_{\text{e,r}}$ [25,26] or $\hat{\mathbf{D}}_2$ with $\text{PLdB}(A_{\text{e,r}}(\hat{\mathbf{D}}_2)) \approx 76$ using an adjoint-based inverse design method [28-30].

In Section III, two examples show that a feasible low-boom design is not too far away from a plausible low-boom design generated by the BCO method. In theory, a feasible low-boom design $\hat{\mathbf{D}}$ for Eq. (4.2) indicates that an ideal low-boom design (if exists) is not too far away. So, the plausible, feasible, and ideal low-boom designs can be similar configurations. This validates a progressive design method with increasing fidelities from a plausible low-boom design to an ideal low-boom design in development of low-boom supersonic transports. It is also the theoretical foundation for the validity of using low-fidelity analyses for system-level trades between aircraft conceptual design goals and

low-boom requirement without violating high-fidelity physics. Obviously, the error tolerance ε and the upper limit of 70 PLdB can be replaced by any reasonable small number and another low-boom loudness requirement (e.g., an upper limit of 75 PLdB) in the above discussion. For example, $\hat{\mathbf{D}}_1$ and $\hat{\mathbf{D}}_2$ mentioned above can be called ideal low-boom designs for 79 PLdB and 76 PLdB, respectively.

The approximation error between $\hat{\mathbf{D}}$ and $\mathbf{D}$ is defined by the difference of volume equivalent areas in Eq. (4.3). The volume equivalent area depends not only on the OML but also on the angle of attack and the tail rotation angle at the start of overland cruise. Here $\|A_e^{\text{volume}}(\hat{\mathbf{D}}) - A_e^{\text{volume}}(\mathbf{D})\|$ is the integrated error of $|A_e^{\text{volume}}(\hat{\mathbf{D}}) - A_e^{\text{volume}}(\mathbf{D})|^2$ over the effective distance range $[0, l_e]$. In this paper, an acceptable approximation error is based on a visually close match between the volume equivalent areas, which is attainable by the BCO method as shown in Section III. No direct solution method is available to solve a low-boom inverse design optimization problem with the CFD trim constraint Eq. (4.4). So, Eq. (4.4) can only be checked after the solution pair $(\mathbf{D}, \hat{\mathbf{D}})$ is generated by the BCO method. However, the BCO method has the capability to implicitly enforce Eq. (4.4).

The BCO method is represented by the flowchart in Fig. 1. Each optimization problem in Eqs. (2.1)-(2.7) is one iteration step of the BCO method with Eqs. (4.1)-(4.3) as termination criteria for BCO iterations. The BCO method starts with an initial design exploration for a baseline $\mathbf{D}_0$ that depends heavily on empirical knowledge about low-boom supersonic transports. The design vector $\mathbf{D}$ and its approximate copy $\hat{\mathbf{D}}$ are partitioned into seven blocks of design variables: d_{tgt} , F_s , $(d_{\text{ftr}}, d_{\text{wt}})$, d_{LTO} , $(H_{\text{OL}}, M_{\text{OL}})$, $(H_{\text{OL}}, R_{\text{OL}})$, and $(\hat{a}_{\text{wt}}, \hat{a}_{\text{aft}})$. Each of Eqs. (2.1)-(2.7) is an optimization problem using one block of design variables. In the BCO iterations, each optimum solution is used as the initial design for the next optimization problem. The BCO method has three nested iteration loops.

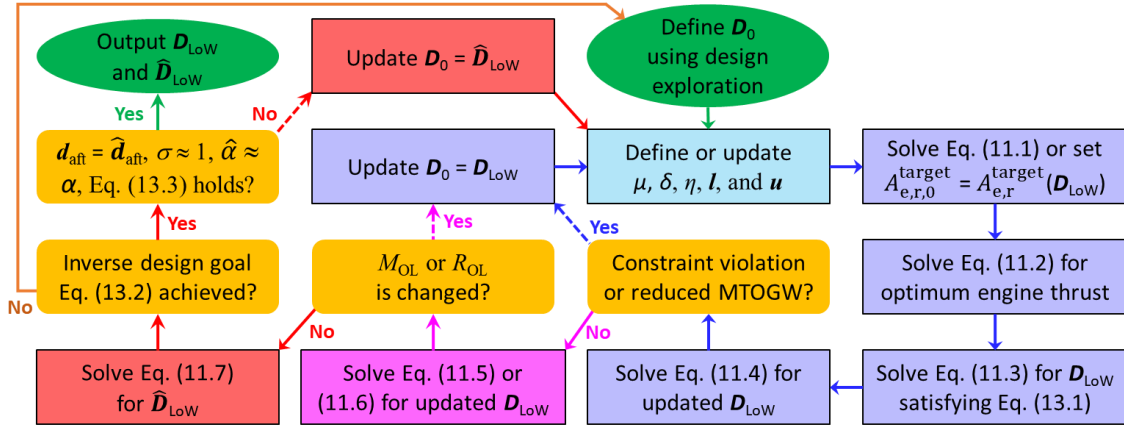


Fig. 1 BCO for approximate solution of mixed-fidelity low-boom MDO problem.

The blue iteration loop solves Eqs. (2.1)-(2.4) sequentially and checks the post-optimization constraint (4.1) for an acceptable inverse design $\mathbf{D}_{\text{LoW}}$ with the minimum MTOGW. It starts with the choices of five parameters μ , δ , η , l , and u that are not design variables, but their values have a significant influence on the solution pair of Eqs. (2.1)-(2.7). The parameter μ is *a priori* estimate of $A_{e,r}(l_e, \hat{\mathbf{D}})/A_{e,\text{CFD}}(l_e, \hat{\mathbf{D}})$ for an expected solution $\hat{\mathbf{D}}$ of Eqs. (2.7) and (4.2), which is critical to a successful termination of the BCO method. Initially, the choice of $\mu > 1$ could be an empirical estimate such as 1.25. The value of μ determines the accuracy of $|A_{e,r}(l_e, \hat{\mathbf{D}}) - A_{e,r}^{\text{target}}(l_e, \mathbf{D})|$ for a solution $\hat{\mathbf{D}}$ of Eq. (2.7). A small $|A_{e,r}(l_e, \hat{\mathbf{D}}) - A_{e,r}^{\text{target}}(l_e, \mathbf{D})|$ is necessary for Eq. (4.2) with $\sigma \approx 1$. So, once an acceptable low-boom inverse design $\hat{\mathbf{D}}$ for Eqs. (2.7) and (4.2) is obtained, μ must equal $A_{e,r}(l_e, \hat{\mathbf{D}})/A_{e,\text{CFD}}(l_e, \hat{\mathbf{D}})$ to ensure that Eq. (4.2) holds with $\sigma \approx 1$ for the next CFD-based inverse design. The parameter δ implicitly enforces the CFD trim constraint. The initial choice of δ is always zero. Once a solution of Eq. (4.2) violates the CFD trim constraint Eq. (4.4), δ can be increased to implicitly enforce Eq. (4.4) (see Subsection III.C for an example). The choice of $\eta \geq 0$ is empirical for the low-fidelity trim constraint and its effect on trim for overland cruise is much less effective than δ . The design vector bounds l and u were extensively discussed in Ref. [27]. The key idea is to avoid having an optimized geometry design variable close to one of its bounds, while using small design ranges so that Eq. (2.3) can be solved effectively. In practice, a trial and error approach might be required to get a set of appropriate values for δ , l , and u in the process of solving Eqs. (2.1)-(2.7). For example, if the takeoff field length constraint in Eq. (2.4) cannot be satisfied with any LTO design subvector d_{LTO} , a change of the engine thrust F_s might be needed to solve Eq. (2.4). The resulting engine thrust can then be used as a lower bound on F_s for the BCO iterations. Similarly, if Eq. (4.2) has no solution with the optimized M_{OL} , R_{OL} , and H_{OL} , but it has a solution for a lower cruise Mach, then the lower cruise Mach can be used as an upper bound on M_{OL} .

to ensure that the BCO method can be terminated successfully. In other words, δ , $\mathbf{l}$, and $\mathbf{u}$ are actually knowledge-based parameters for the BCO method, which cannot be determined by any numerical optimization method at the moment.

The optimum baseline low-boom target $A_{e,r,0}^{\text{target}}$ from Eq. (2.1) and the scaling formula Eq. (3.1) allow the decoupling of low-boom target optimization and inverse design optimization. Here W_0 and W_{crs} are the weights at the start of overland cruise for $\mathbf{D}_0$ and $\mathbf{D}$, respectively, while $l_{e,0}$ and l_e represent the equivalent lengths of $\mathbf{D}_0$ and $\mathbf{D}$, respectively. The weight scaling formula Eq. (3.1) was first used in Ref. [27]. Its purpose is to align $A_{e,r}^{\text{target}}(\mathbf{D})$ with $A_{e,\text{LoFi}}(\mathbf{D})$ for the weight and effective length changes during optimization. Equation (2.1) has two very important shape control parameters μ and δ for $A_{e,r,0}^{\text{target}}$ that depend on the data from CFD-based inverse design solutions of Eq. (2.7) as discussed above. The option of using $A_{e,r,0}^{\text{target}} = A_{e,r}^{\text{target}}$ instead of solving Eq. (2.1) in the blue iteration loop is for convenience to make $A_{e,r,0}^{\text{target}}$ correspond to the low-boom target of the updated $\mathbf{D}_0$, when μ is not changed and there is no need to change the low-boom target for better trim characteristics.

Equation (2.2) is a simple engine thrust optimization problem with MTOGW as the objective. The constraints $g_8(\mathbf{D}) \geq 0$ and $g_9(\mathbf{D}) \geq 0$ in Eqs. (2.2)-(2.6) are included to indicate that the MTOGW is generated by the fixed-range dual-mission analysis for (III.A.8) and (III.A.9). They are not explicitly used by any numerical optimization algorithm.

Equation (2.3) is the key low-fidelity MDO problem that searches for a plausible low-boom design with the lowest MTOGW. The low-fidelity trim constraint is strongly related to the low-boom inverse design objective function. This leads to a potential conflict between the cruise trim with fuel redistribution and low-boom shaping, a well-known practical design challenge for low-boom concepts. The inclusion of the low-fidelity trim constraint in Eq. (2.3) is to ensure that such a conflict can be resolved optimally by a numerical optimizer at the conceptual design level. The desired solution $\mathbf{D}_{\text{LoW}}$ of Eq. (2.3) is the lowest weight Pareto point that is a plausible low-boom design for Eq. (4.1). The inverse design objective function $\|A_{e,\text{LoFi}}(\mathbf{D}) - A_{e,r}^{\text{target}}(\mathbf{D})\|_{k,\lambda}$ represents the integrated average error of $|A_{e,\text{LoFi}}(\mathbf{D}) - A_{e,r}^{\text{target}}(\mathbf{D})|^k$ over the effective distance interval $[0, \lambda]$. The definition of λ in Eq. (2.3) using a scaling of λ_0 ensures that the inverse design error calculation accounts for the effective length change during optimization, where λ_0 is the x_e location of the highest point on $A_{e,\text{LoFi}}(\mathbf{D}_0)$. The positive integer k determines the influence of the maximum error between $A_{e,\text{LoFi}}$ and $A_{e,r}^{\text{target}}$ on the inverse design objective function. An increase of k forces a numerical optimization algorithm for Eq. (2.3) to focus more on minimizing the maximum matching error. The two cases of Eq. (2.3) with $k = 2$ and $k \rightarrow \infty$ were used in Ref. [27] for minimizing the average and maximum matching errors, respectively. The constraint $g_7(\mathbf{D}) \geq 0$ is included to indicate that the configuration has enough volume for 40 passengers and main gear storage, which was enforced by the fuselage width constraints in Ref. [27] and is always satisfied in this paper by a parametric fuselage shape with variable nose cone. Only derivative-free multi-objective optimization methods are applicable to finding $\mathbf{D}_{\text{LoW}}$. The most sensible strategy is to use enough design variables that can generate a desired low-boom OML of $\mathbf{D}_{\text{LoW}}$, while Eq. (2.3) can still be solved effectively by a derivative-free multi-objective optimization method. Reference [27] shows that Eq. (2.3) for 35 shape design variables can be solved by a multi-objective genetic optimization algorithm within a wall time of 72 hours using a Windows server with 48 cores at 2.1 GHz and a shared RAM of 1024 GB.

Equation (2.4) is the feasibility optimization that aims to satisfy the LTO constraints (III.A.3)-(III.A.6) using $\mathbf{d}_{\text{LTO}}$. As mentioned before, (III.A.1) is easily satisfied with a reasonable main gear location. Constraint (III.A.2) follows from the cruise trim constraint in Eq. (2.3). With proper fuselage shape and payload weight, (III.A.7) can be satisfied. The constraints (III.A.8) and (III.A.9) are the consequences of a fixed-range dual-mission analysis. It should be noted that the horizontal tail size and its longitudinal location have a significant influence on the LTO performance. Here it is implicitly assumed that the horizontal tail is already properly designed for the LTO performance requirements during the initial design exploration. After the initial design exploration, the LTO performance constraints can be easily satisfied by modifications of $\mathbf{d}_{\text{LTO}}$ for all design cases in this paper. The output of Eq. (2.4) is an updated $\mathbf{D}_{\text{LoW}}$ with $\mathbf{d}_{\text{LTO}}$ from a solution of Eq. (2.4).

Once the MTOGW of $\mathbf{D}_{\text{LoW}}$ cannot be further reduced and $\mathbf{D}_{\text{LoW}}$ satisfies the specified mission constraints, the blue iteration loop terminates. The blue iteration loop is a simplified version of the BCO method in Ref. [27]. It is based on an implicit assumption used in this paper: Eqs. (2.3) and (4.1) can be solved simultaneously. This assumption is true if $M_{\text{OL}} = 1.6$, $R_{\text{OL}} = 2,500$ nm, and $H_{\text{OL}} = 45,000$ ft (see Ref. [27]). However, if the mission requirements are significantly different from (III.A.5)-(III.A.8), it might be difficult to know *a priori* what values of M_{OL} , R_{OL} , and H_{OL} can ensure a plausible low-boom design for Eq. (4.1) that also satisfies the mission constraints. In such a case, it might be necessary to add a feasibility exploration phase [27] in the blue iteration loop.

The pink iteration loop solves either Eq. (2.5) or Eq. (2.6) for trading any margin in the target PLdB for a low-boom design that flies faster or further. For Eqs. (2.5) and (2.6), a different scaling formula Eq. (3.2) is used for $A_{e,r}^{\text{target}}(\mathbf{D})$. Equation (3.2) defines the scaling coefficient using the least-squares solution of minimizing

$\|A_{e,LoFi}(\mathbf{D}) - A_{e,r}^{target}(\mathbf{D})\|_{2,\lambda}$. The least-squares scaling formula Eq. (3.2) gives a good prediction on how the changes of (M_{OL}, R_{OL}, H_{OL}) would affect the target PLdB for $\mathbf{D}$ that already has a low-boom shape up to $x_e = \lambda$.

The trades among the target PLdB, M_{OL} , R_{OL} , and H_{OL} are extremely important for low-boom concept development, because intuitions about a sensible combination of (M_{OL}, R_{OL}, H_{OL}) for the overland mission could be misleading and result in a poor concept. When H_{OL} is fixed, it is obvious that an increase of M_{OL} or R_{OL} leads to an increase of $TOGW_{OL}$, which results in an increase of $PLdB(A_{e,r}^{target})$. So, $PLdB(A_{e,r}^{target})$ can be directly traded with M_{OL} or R_{OL} . A trade between $PLdB(A_{e,r}^{target})$ and H_{OL} is more complicated. If H_{OL} is increased and the cruise weight remains the same, then $A_{e,LoFi}(l_e)$ at the higher altitude is higher than that at the original altitude due to a reduced air density, which requires an increase of $A_{e,r}^{target}(l_e)$ and results in a higher target PLdB. On the other hand, a higher altitude tends to increase supersonic cruise efficiency and to reduce W_{crs} (due to fuel weight reduction), which might yield a lower value for $A_{e,r}^{target}(l_e)$ and a lower target PLdB. No theory is available to predict whether an altitude change is beneficial for $PLdB(A_{e,r}^{target})$ or not. For a trade between M_{OL} and R_{OL} , some economic analysis is required to determine whether the trade is beneficial or not, which is beyond the scope of this paper. As a result, Eqs. (2.5) and (2.6) are used for the optimum trades among the target PLdB, M_{OL} , and R_{OL} , and H_{OL} .

In theory, Eq. (2.5) or (2.6) leads to an increased M_{OL} or R_{OL} , respectively, when $PLdB(A_{e,r}^{target}(\mathbf{D}_{LoW})) < 70$. The constraint for dynamic pressure Q_{OL} in Eq. (2.5) is to ensure that the same lift coefficient corresponds to the same cruise weight at different cruise conditions. This tends to minimize the angle-of-attack change at the start of cruise for different cruise conditions and maintain the low-boom shape of $A_{e,LoFi}$ during the optimization iterations. With fixed Q_{OL} , M_{OL} is determined by H_{OL} , which reduces Eq. (2.5) to a simple optimization problem with one design variable H_{OL} . The weight constraint in Eq. (2.6) ensures that $MTOGW$ is not changed for range optimization. With the weight constraint, the maximum range R_{OL} for H_{OL} in Eq. (2.6) is actually achieved by $TOGW_{OL} = TOGW_{OW}$. With fixed M_{OL} and $TOGW_{OL}$, R_{OL} is determined by H_{OL} . As a result, Eq. (2.6) is also a simple optimization problem with one design variable H_{OL} . So, both Eqs. (2.5) and (2.6) can be solved by finding H_{OL} such that $PLdB(A_{e,r}^{target}(\mathbf{D})) \approx 70$. If M_{OL} or R_{OL} is changed by Eq. (2.5) or (2.6), the output is $\mathbf{D}_{LoW}$ with the updated values for M_{OL} , R_{OL} , and H_{OL} . Then the baseline $\mathbf{D}_0$ is replaced by the updated $\mathbf{D}_{LoW}$ and the blue iteration loop is restarted. In this case, the baseline low-boom target optimization is optional and the corresponding low-boom target for the updated $\mathbf{D}_{LoW}$ from Eq. (2.5) or (2.6) could be used as $A_{e,r,0}^{target}$. Once the cruise speed and range for the overland mission cannot be further improved by Eq. (2.5) or (2.6), the pink iteration loop terminates.

The red iteration loop uses the updated solution $\mathbf{D}_{LoW}$ as the initial design to find a low-boom inverse design solution $\hat{\mathbf{D}}_{LoW}$ of Eq. (2.7). Two extra design variables $\hat{\alpha}$ and σ are used in Eq. (2.7) to mitigate any adverse effect on the inverse design problem caused by the prediction error $|\mu - A_{e,r}(l_e, \hat{\mathbf{D}})/A_{e,CFD}(l_e, \hat{\mathbf{D}})|$ of μ for a solution $\hat{\mathbf{D}}$ of Eqs. (2.7) and (4.2). Here $\hat{\alpha}$ is the angle of attack for CFD analysis of $\hat{\mathbf{D}}$ and σ is a scaling parameter for $A_{e,r}^{target}(\mathbf{D})$. The inverse design problem Eq. (2.7) shall be solved in three steps: (i) minimize $\|A_{e,r}(\hat{\mathbf{D}}) - A_{e,r}^{target}(\mathbf{D})\|_{2,\lambda}$ with $\hat{\alpha} = \alpha$ and adjust $\hat{\theta}_{tail}$ to match the CFD lift with W_{crs} ; (ii) shape the aft components to minimize $\|A_{e,r}(\hat{\mathbf{D}}) - A_{e,r}^{target}(\mathbf{D})\|$; and (iii) adjust $\hat{\alpha}$ to match the CFD lift with W_{crs} (if needed) and minimize the inverse design error $\|A_{e,r}(\hat{\mathbf{D}}) - \sigma \cdot A_{e,r}^{target}(\mathbf{D})\|$ for the best scaling parameter σ . Here α denotes the angle of attack at the start of cruise for $\mathbf{D}$. This is very different from using $\hat{\alpha}$ to match the CFD lift with W_{crs} during optimization iterations for low-boom inverse design of $A_{e,r}$ [25,26]. In theory, the adjustments of $\hat{\alpha}$ and σ are not needed if $\mu = A_{e,r}(l_e, \hat{\mathbf{D}})/A_{e,CFD}(l_e, \hat{\mathbf{D}})$ for the last iteration solution $\hat{\mathbf{D}}$ of Eqs. (2.7) and (4.2) along the red loop. If the generated solution $\hat{\mathbf{D}}_{LoW}$ of Eq. (2.7) satisfies Eq. (4.2) and the difference between $\mathbf{D}_{LoW}$ and $\hat{\mathbf{D}}_{LoW}$ is significant, then update $\mathbf{D}_0$ with $\hat{\mathbf{D}}_{LoW}$ and continue along the red iteration loop. Otherwise, Eq. (4.3) holds for a small error tolerance $\hat{\epsilon}$ and the red iteration loop terminates. The other termination conditions for the red iteration loop in Fig. 1 are listed to emphasize that $\mathbf{D}_{LoW}$ and $\hat{\mathbf{D}}_{LoW}$ have the same aft components, the same lift with approximately the same angle of attack, and approximately the same low-boom target for inverse design optimization. The equality $\mathbf{d}_{aft} = \hat{\mathbf{d}}_{aft}$ is the result of the updating rule $\mathbf{D}_0 = \hat{\mathbf{D}}_{LoW}$ in Fig. 1, which forces another iteration cycle along the red loop if $\hat{\mathbf{d}}_{aft}$ is changed when solving Eq. (2.7). The termination conditions $\hat{\alpha} \approx \alpha$ and $\sigma \approx 1$ are consequences of Eqs. (4.1)-(4.3). Because of the weak couplings among the seven blocks of coordinates, the BCO method in Fig. 1 does terminate after a few iterations along the red loop. However, if one cannot find a solution of Eq. (2.7) that also satisfies Eq. (4.2), then it means a restart of the low-boom concept development. For all the numerical examples in this paper, Eqs. (2.7) and (4.2) can always be solved simultaneously.

The most important contribution of this paper is to demonstrate that a feasible low-boom design for Eq. (4.2) can be obtained from a plausible low-boom design for Eq. (4.1) with a tail rotation angle change and minor wing modifications. That is, Eqs. (4.1)-(4.3) can be solved *simultaneously*. The BCO method in Fig. 1 will be applied in Section III to generate two solution pairs of Eqs. (2.1)-(2.7) and (4.1)-(4.3) with $|\sigma - 1| \leq 0.02$, $|\hat{\alpha} - \alpha| \leq 0.2^\circ$, and relatively small values of $\hat{\epsilon}$.

C. Baseline and Design Variables for BCO

The baseline $\mathbf{D}_0$ is extremely important for the BCO method in Fig. 1. The initial choice of $\mathbf{D}_0$ is difficult because it is based on empirical knowledge and experiences from low-boom supersonic concept development. The fuselage and wing design variables for the BCO method in this paper are significantly different from those used in Ref. [27].

The fuselage is completely redesigned based on the cabin layout requirement and the knowledge about the supersonic cruise performance benefits of a slender body (see, *e.g.*, Ref. [31]). The redesigned fuselage in Fig. 2 has the minimum cross section areas for passengers and main gear storage. The passenger cabin has a height of 7.3 ft, a width of 5.5 ft, and a length of 80 ft, with a seat pitch of 48 in. The fuselage width for main gear storage is 6 ft. Only the widths and heights at three cross sections for the nose cone (see the blue variables in Fig. 2) are used as design variables ($\mathbf{d}_{\text{frt}}$) for BCO.

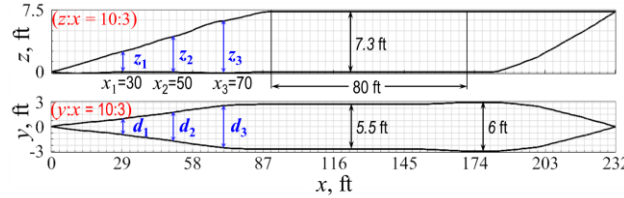


Fig. 2 Fuselage side/top views and design variables.

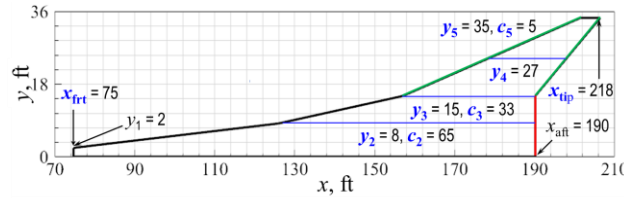


Fig. 3 Baseline wing planform and design variables.

The wing has the same parametric planform as in Ref. [27] with 9 design variables (see the blue variables in Fig. 3). A design lift parameter τ_{max} (*i.e.*, the parameter CLI in the computer code to generate NACA 63-series airfoils [32]) is used to parameterize each NACA 63-series airfoil. Four parameters $\tau_{i,\text{max}}$ for airfoils at the span locations y_i ($i=1,2,3,4$) are used as design variables. Moreover, the twist angles for airfoils at the span locations y_4 and y_5 are used as design variables. The inboard wing twist variables are fixed to ensure that the main gear strut can be stored inside the wing. The dihedral angles of the four outboard wing sections are also used as the design variables. A total of 10 design variables are available to modify the camber surface of a given wing planform. The design vector $\mathbf{d}_{\text{wt}}$ for the low-fidelity low-boom MDO Eq. (2.3) consists of the 19 wing design variables and the tail rotation angle θ_{htail} . The dihedral angles were not used as design variables in Ref. [27].

With the given fuselage and wing parametric shapes, (III.A.7) is always satisfied with a proper payload for 40 passengers.

The baseline is derived from the re-optimization solution [27] shown in Fig. 4 for $M_{\text{OL}} = 1.6$, $R_{\text{OL}} = 2,500$ nm, and $H_{\text{OL}} = 45,000$ ft. It provides the initial values for $(M_{\text{OL}}, R_{\text{OL}}, H_{\text{OL}})$ that has a plausible low-boom design. As mentioned before, finding such a plausible low-boom baseline is a nontrivial task if any of the mission requirements (III.A.5)-(III.A.8) are changed. The fuselage and wing of the configuration in Fig. 4 are remodeled as in Figs. 2 and 3 to get the initial baseline $\mathbf{D}_0$ for the BCO method.

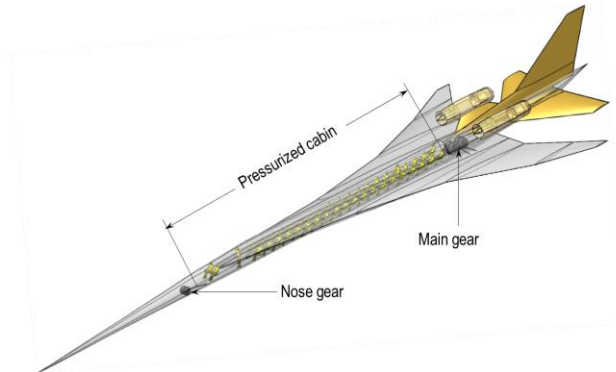


Fig. 4 Low-boom configuration for $M_{\text{OL}} = 1.6$, $R_{\text{OL}} = 2,500$ nm, and $H_{\text{OL}} = 45,000$ ft.

The design vector $\mathbf{d}_{\text{aft}}$ for aft components is based on empirical knowledge on how to shape the aft $A_{e,r}$, which will be discussed with examples later. A parametric Bezier curve with 8 control points is used to define $A_{e,r,0}^{\text{target}}$ in Eq. (2.1): two control points are used to fix the start and end points of $A_{e,r,0}^{\text{target}}$, and the remaining 12 control-point coordinates are used as components of $\mathbf{d}_{\text{tgt}}$ for the baseline low-boom target optimization Eq. (2.1). The same six auxiliary design variables as in Ref. [27] are used as components of $\mathbf{d}_{\text{LTO}}$ in the feasibility optimization Eq. (2.4) for mission constraints: trailing edge flap deflection angles for LTO, longitudinal CG offsets for LTO, leading edge flap deflection angle for landing, and main gear length. Flap sizes also affect LTO performance, but they are not used as design variables and change proportionally with wing chords in this paper. The horizontal tail size and its longitudinal location affect LTO performance too, but they are determined in the initial design exploration stage and fixed in this paper. Two other important parameters related to LTO performance are engine thrust and main gear longitudinal location. The engine thrust affects the mission analysis, engine weight, nacelle size, and LTO field lengths. The baseline has an engine thrust of 36,700 lb. The main gear location affects the aircraft CG, LTO static margins, and LTO field lengths. Main gear longitudinal location was a design variable during the initial layout design, but its value is fixed at 176 ft for all configurations in this paper.

D. Implementation of BCO

The initial choices of μ and δ are 1.25 and 0, respectively. The low-fidelity trim margin $\eta = 4$ is empirical. The positive integer $k = 8$ for the inverse design objective function in Eq. (2.3) provides a good balance for minimizing both maximum and average matching errors. The low-boom cruise Mach or range optimization, Eq. (2.5) or (2.6), is performed only when $\text{PLdB}(A_{e,r}^{\text{target}}(\mathbf{D}_{\text{LOW}})) < 69.4$. As pointed out in Subsection III.B, Eqs. (2.5) and (2.6) are trivial optimization problems with one design variable H_{OL} , which are solved for $\text{PLdB}(A_{e,r}^{\text{target}}(\mathbf{D})) \approx 69.4$ using a parametric study method on H_{OL} . In this paper, Eq. (2.5) is used for $M_{\text{OL}} < 1.8$ and Eq. (2.6) is used for $M_{\text{OL}} = 1.8$.

All functions of $\mathbf{D}$ used in Eqs. (2.1)-(2.6) are computed using the multidisciplinary feasible (MDF) architecture in Fig. 5 (see Ref. [33] for classifications of MDO architectures). The MDF architecture in Fig. 5 is based on the MDO frameworks documented in Refs. [34] and [35].

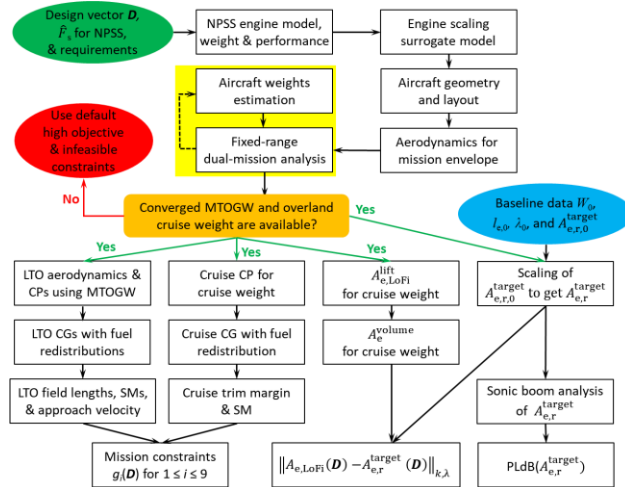


Fig. 5 MDF architecture for low-fidelity analyses of supersonic aircraft concept.

An advanced engine model was developed using the Numerical Propulsion System Simulation (NPSS) [36]. This engine model is only scaled by the engine thrust F_s in this paper. Equation (2.2) is a trivial optimization problem using one design variable F_s . To avoid expensive NPSS runs, an independent copy $\hat{F}_s$ of F_s for NPSS is fixed at the initial value of F_s for a baseline configuration and a surrogate model is used during the optimization iterations. The engine size, weight, and performance data are linearly scaled by the surrogate model for any F_s . The optimized engine thrust F_s is reanalyzed using NPSS to validate the scaling after the optimization. This can be done simply by setting $\hat{F}_s = F_s$ (of $\mathbf{D}$) in Fig. 5, which forces NPSS to regenerate the size, weight, and performance data when the value of $\hat{F}_s$ is changed. The engine thrust optimization problem, Eq. (2.2), can be easily solved using a parametric study on F_s . The optimum engine thrust mainly changes with the overland cruise condition, wing, and MTOGW. It also has a significant effect on the takeoff field length. So, a lower bound could be imposed on F_s to help maintain the takeoff field length constraint if it cannot be satisfied using $\mathbf{d}_{\text{LTO}}$.

The constraints $g_8(\mathbf{D}) \geq 0$ and $g_9(\mathbf{D}) \geq 0$ are automatically satisfied with a coupled weight and fixed-range dual-mission analysis method (see the highlight iteration loop in Fig. 5 and more details in Ref. [27]) using the Flight Optimization System (FLOPS) [37,38]. The coupled weight and mission analysis outputs MTOGW and the weight at the start of overland cruise, along with the component weight and CG data. Linear aerodynamics codes WINGDES/AERO2S [39] are used to generate aero data for mission, cruise, and LTO analyses. In addition to drag due to lift from WINGDES, skin friction and wave drag coefficients are estimated using the methods in Refs. [40] and [41], respectively. The trim margin ($CG_{x,aft} - CP_{x,LoFi}$) for overland cruise and static margins are based on the WINGDES/AERO2S centers of pressure and the most aft CG_x values for cruise and LTO weights. For approach velocity and LTO field lengths, the landing weight is 70% of MTOGW and all relevant FAR Part 25 requirements are met [42].

The equivalent areas A_e^{volume} and $A_e^{lift_{e,LoFi}}$ are computed using the methods in Refs. [41] and [43], respectively. The solver for the augmented Burgers equation [23] is used to compute the ground signature for A_e in the standard atmosphere [44]. The method in Ref. [45] is used to compute PLdB of the ground signature for A_e .

All CFD results for CFD-based low-boom optimization Eq. (2.7) are generated using the Cart3D solver [46] based on three-dimensional Euler equations. The engine shown in Fig. 4 is modeled using a flow-through nacelle for CFD analysis. A volume mesh of about 32M cells is used for calculation of CFD off-body pressure at 3 body lengths below the aircraft (see Ref. [47]), as well as $CP_{x,CFD}$. The Mach, altitude, angle of attack for CFD analysis are M_{OL} , H_{OL} , and $\hat{\alpha}$. The CFD off-body pressure is used for calculation of $A_{e,r}$ [25].

III. Low-Boom Supersonic Transports at Mach 1.8 for 40 Passengers

Two solution pairs will be generated using the BCO method in this section. Subsection III.A shows how the configuration in Fig. 4 can be further improved to get $\mathbf{D}_{LoR}$ for reduced MTOGW and increased M_{OL} and R_{OL} . In Subsection III.B, the best trade solution $\mathbf{D}_{LoR}$ is redesigned with CFD-based low-boom inverse design optimization to obtain a solution pair $(\mathbf{D}_{LoR}, \hat{\mathbf{D}}_{LoR})$ satisfying Eqs. (4.1)-(4.3). Subsection III.C shows how to enforce the CFD trim constraint Eq. (4.4) using δ in Eq. (2.1). Only the optimization results for the last iteration cycle of the red loop in Fig. 1 are included.

A. Optimum Trades of Target PLdB with Mach and Range

Equations (2.5) and (2.6) are used to increase the cruise Mach and range for the low-boom overland mission. Initially, the baseline is optimized using the blue iteration loop in Fig. 1 to get a plausible low-boom design $\mathbf{D}_{LoW}$ with the lowest MTOGW. Then, Eq. (2.5) is solved for $PLdB(A_{e,r}^{target}(\mathbf{D})) \approx 69.4$ for the maximum cruise Mach and the corresponding cruise altitude. The resulting solution is updated by the optimum solution from the blue iteration loop in Fig. 1 for the new cruise Mach and altitude. The pink iteration loop in Fig. 1 continues until the cruise Mach cannot be further increased. For the current design study, the cruise Mach is limited by the maximum cruise Mach of 1.8 allowed by the engine. This limit is primarily driven by takeoff noise and high-altitude emission considerations. So, any excess target PLdB margin at cruise Mach 1.8 is traded for increased range. Equation (2.6) is used for the range increase and $\mathbf{D}_{LoW}$ is updated by the optimum solution from the blue iteration loop in Fig. 1 for the new cruise altitude and range. This range optimization continues along the pink iteration loop in Fig. 1 until no range increase is possible.

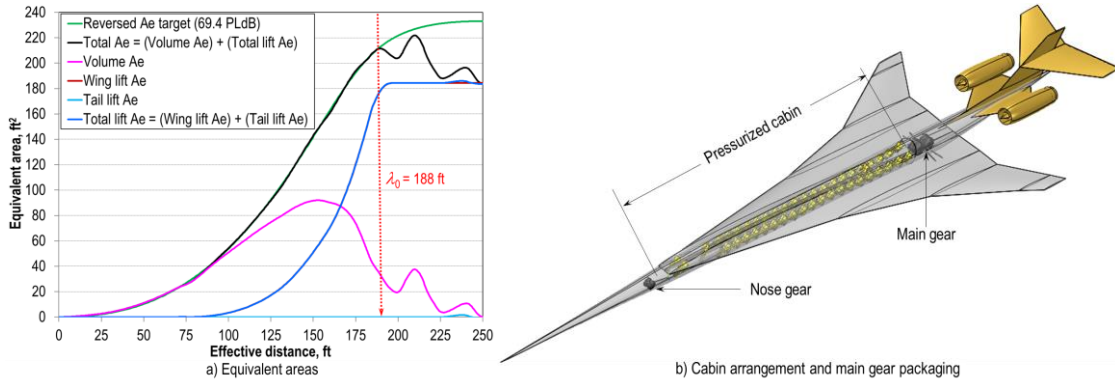


Fig. 6 Equivalent areas and cabin arrangement of $\mathbf{D}_{LoR}$ for cruise Mach 1.8.

The trades described above lead to the best trade solution $\mathbf{D}_{LoR}$ for $M_{OL} = 1.8$, $R_{OL} = 2,900$ nm, and $H_{OL} = 50,000$ ft. The equivalent areas, cabin arrangement, and main gear packaging for $\mathbf{D}_{LoR}$ are shown in Fig. 6. The low-fidelity

trim constraint is satisfied: $CG_{x,aft} - CP_{x,LoFi} = 170.6 - 164.0 = 6.6 \text{ ft} \geq \eta = 4 \text{ ft}$. The mission constraints (III.A.1)-(III.A.6) are satisfied by $\mathbf{D}_{LoR}$ with the following constraint values.

- (IV.B.1) CG margin to prevent tip over on the ground = 11.2 ft
- (IV.B.2) Static margin for overland cruise = 15.0%
- (IV.B.3) LTO static margins are 3.3% and 3.0%, respectively
- (IV.B.4) Tail rotation angles for trim at LTO are -8.4° and -12.4° , respectively
- (IV.B.5) LTO field lengths are 6,916 ft and 8,216 ft, respectively
- (IV.B.6) Approach velocity = 137 knots

The best trade solution $\mathbf{D}_{LoR}$ has a much lower MTOGW of 145,625 lb than MTOGW of 154,474 lb for the low-boom concept in Fig. 4. This shows the importance of the system-level trades (the pink iteration loop in Fig. 1) among $PLdB(A_{e,r}^{target})$, MOL , and R_{OL} using the optimum cruise altitude H_{OL} for the overland mission. The values of θ_{htail} and α for $\mathbf{D}_{LoR}$ at the start of overland cruise are -1.0° and 1.8° , respectively. In comparison, θ_{htail} and α for the low-boom concept in Fig. 4 at the start of cruise are -1.6° and 2.5° , respectively. More details about $\mathbf{D}_{LoR}$ and the low-boom concept in Fig. 4 will be given later in Table 1 of Subsection IV.C. The changes for aft components of $\mathbf{D}_{LoR}$ in Fig. 6 are from an intermediate CFD-based low-boom inverse design.

B. CFD-Based Low-Boom Inverse Redesign

The low-boom inverse design method using $A_{e,r}^{target}$ described in Subsection III.B is used to reshape $A_{e,r}(\mathbf{D}_{LoR})$. The inverse design goal is to show that $\mathbf{D}_{LoR}$ can be redesigned with minor wing modifications to obtain a feasible low-boom design $\hat{\mathbf{D}}_{LoR}$ that satisfies Eqs. (2.7) and (4.2).

Multiple iterations along the red loop in Fig. 1 were performed to obtain $\hat{\mathbf{D}}_{LoR}$. Two major CFD-based corrections were required to achieve the termination of the BCO method: (i) a major redesign of aft components for a low-boom shape of the aft $A_{e,r}$ and (ii) updating of μ in Eq. (2.1): $\mu = A_{e,r}(l_e, \hat{\mathbf{D}})/A_{e,CFD}(l_e, \hat{\mathbf{D}})$ for intermediate solutions $\hat{\mathbf{D}}$ of Eq. (11.7). Figures 4 and 6b show how the aft components were redesigned by an intermediate CFD-based low-boom shaping. The changes include the pylon camber modifications, aft shifts of the pylons and nacelles, and the shift of the horizontal tail from the side of the fuselage to the vertical tail.

The reversed equivalent area curve of $\hat{\mathbf{D}}_{LoR}$ matches the low-boom target closely in Fig. 7a. The CFD-based ground signature of $\hat{\mathbf{D}}_{LoR}$ oscillates along the target signature with the same magnitude (see Fig. 7b). After matching $A_{e,r}(\hat{\mathbf{D}}_{LoR})$ with $A_{e,r}^{target}(\mathbf{D}_{LoR})$ for $\hat{\alpha} = \alpha = 1.8^\circ$, $A_{e,r}(l_e, \hat{\mathbf{D}}_{LoR})$ is slightly lower than $A_{e,r}^{target}(l_e, \mathbf{D}_{LoR})$. This is due to the minor difference of $A_{e,r}(l_e, \hat{\mathbf{D}}_{LoR})/A_{e,CFD}(l_e, \hat{\mathbf{D}}_{LoR})$ and μ value for $A_{e,r,0}^{target}$. The identical values of $A_{e,CFD}(l_e, \hat{\mathbf{D}}_{LoR})$ and $A_{e,LoFi}(l_e, \mathbf{D}_{LoR})$ mean that the cruise weights for $\hat{\mathbf{D}}_{LoR}$ and $\mathbf{D}_{LoR}$ are identical (see Eq. (6) in Ref. [16]). So, $\sigma = 0.98$ is used to minimize $\|A_{e,r}(\hat{\mathbf{D}}_{LoR}) - \sigma \cdot A_{e,r}^{target}(\mathbf{D}_{LoR})\|$. The low-boom target $\sigma \cdot A_{e,r}^{target}(\mathbf{D}_{LoR})$ in Fig. 7a has a lower PLdB value of 68.8 than the target PLdB of 69.4 for $A_{e,r}^{target}(\mathbf{D}_{LoR})$ in Fig. 6a. The value of $\hat{\theta}_{htail}$ for $\hat{\mathbf{D}}_{LoR}$ is 5.2° . So, $|\sigma - 1| = 0.02$, $|\hat{\alpha} - \alpha| = 0^\circ$, and $|\hat{\theta}_{htail} - \theta_{htail}| = 3.6^\circ$.

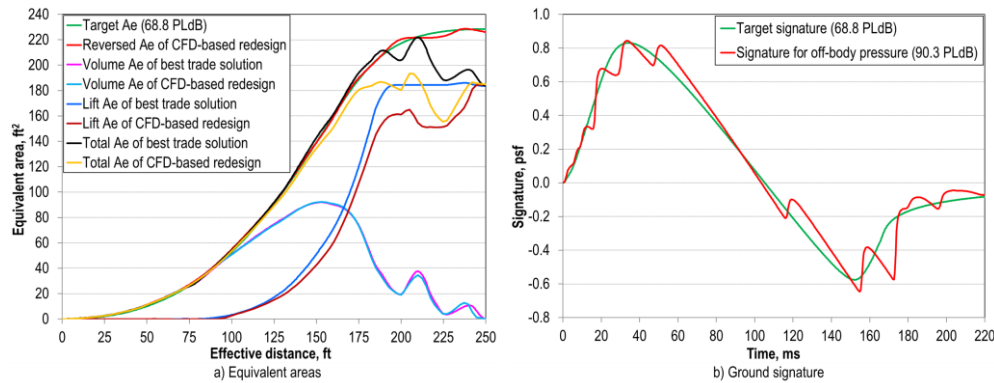


Fig. 7 Equivalent areas of $\mathbf{D}_{LoR}$ and $\hat{\mathbf{D}}_{LoR}$, and ground signature of $\hat{\mathbf{D}}_{LoR}$.

Minor wing modifications are quantified by A_e^{volume} differences between $\mathbf{D}_{LoR}$ and $\hat{\mathbf{D}}_{LoR}$, which are very small (see Fig. 7a). The aft A_e^{volume} differences are mainly due to approximation errors of the trimmed pylon volume for the low-fidelity A_e^{volume} and the tail rotation angle difference of 3.6° between $\mathbf{D}_{LoR}$ and $\hat{\mathbf{D}}_{LoR}$. Only the three outboard airfoils on the wing of $\mathbf{D}_{LoR}$ are reshaped to obtain $\hat{\mathbf{D}}_{LoR}$. Figures 8 and 9 compare the three outboard airfoils and planforms of $\mathbf{D}_{LoR}$ and $\hat{\mathbf{D}}_{LoR}$.

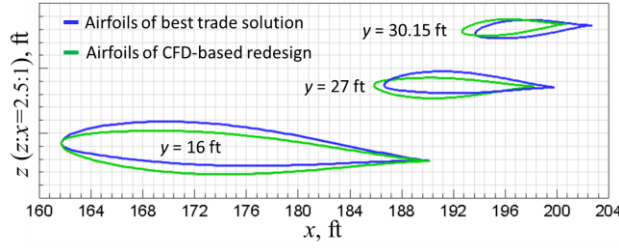


Fig. 8 Airfoil comparison of D_{LoR} and $\hat{D}_{LoR}$

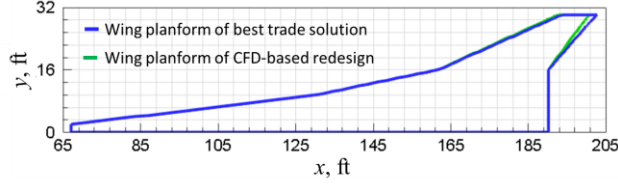


Fig. 9 Wing planform comparison of D_{LoR} and $\hat{D}_{LoR}$

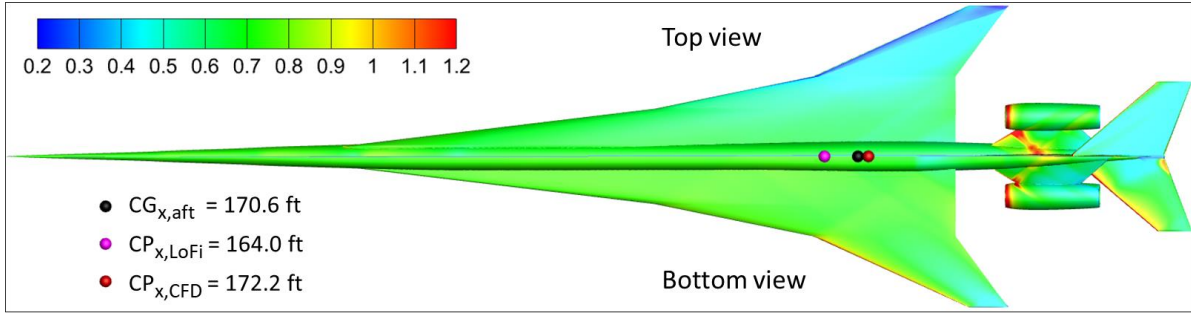


Fig. 10 Pressure contour on OML of $\hat{D}_{LoR}$ and CP_x locations at start of cruise for low-boom mission

It is expected that $\hat{D}_{LoR}$ satisfies the specified LTO performance constraints. The cruise performance of $\hat{D}_{LoR}$ is not expected to be significantly degraded by minor changes of the wing surface (shown in Figs. 8 and 9). The CFD lift-over-drag ratio (not including viscous drag) for the cruise condition of $\hat{D}_{LoR}$ is 8.8, while the low-fidelity lift-over-drag ratio (including an empirical viscous drag estimate) at the start of cruise for D_{LoR} is 7.0. In comparison, the Concorde cruise lift-over-drag ratio is about 7.5 [48]. The complex flow interactions in the aft region of $\hat{D}_{LoR}$ (see Fig. 10) should be carefully redesigned with powered engine simulation for a better propulsion and airframe integration.

Because intermediate CFD-based low-boom redesign results violated the trim constraint with $CG_{x,aft} - CP_{x,CFD} < -2$ ft, the trim margin for D_{LoR} was increased to 6.6 ft in an attempt to satisfy the trim constraint for CFD-based low-boom redesign. Unfortunately, the significant differences between $A_{e,LoFi}^{lift}$ and $A_{e,CFD}^{lift}$ (see Fig. 7a) cause a difference of 8.2 ft between $CP_{x,CFD}$ of $\hat{D}_{LoR}$ and $CP_{x,LoFi}$ of D_{LoR} (see Fig. 10). The trim constraint $CG_{x,aft} - CP_{x,CFD} \geq 0$ does not hold for $\hat{D}_{LoR}$ because $CG_{x,aft} - CP_{x,CFD} = 170.6 - 172.2 = -1.6$ ft (see Fig. 10). The next subsection shows how to move $CP_{x,CFD}$ forward for the CFD-based low-boom redesign using an increased δ in Eq. (2.1).

C. Redesign for Low-Boom Cruise Trim Constraint

In this subsection, D_{LoR} is used as the initial design to show how to increase $(CG_{x,aft} - CP_{x,CFD})$ by increasing δ in Eq. (2.1). The baseline target for D_{LoR} was generated using Eq. (2.1) with $\delta = 0$, which is a scaled version of Target 1 in Fig. 11. The baseline target $A_{e,r,0}^{target}$ for the trim solution D_{LoT} is Target 2 in Fig. 11. When Target 2 is generated using Eq. (2.1), $\mu = A_{e,r}(l_e, \hat{D}_{LoR}) / A_{e,CFD}(l_e, \hat{D}_{LoR}) = 1.22$, $\delta = 99$, $M_{OL} = 1.8$, $H_{OL} = 50,700$ ft, $l_{e,0} = 250$ ft, and $A_{e,r,0}^{target}(l_{e,0}) = 229$ ft².

The widths and heights (d_{fit}) for the fuselage nose cone are increased slightly to match the new low-boom target. A forward move of 2 ft for x_{fit} of D_{LoR} (i.e., a forward extension of the wing root airfoil by 2 ft) is used to shift $A_{e,LoFi}$ forward to better match the new low-boom target. This aims to increase the lift tailoring capability to fill the increased gap between the new low-boom target and $A_{e,LoFi}(x_e, D_{LoR})$ after $x_e = 75$ ft, where the wing of D_{LoR} has no lift (see Fig. 7a). The modified D_{LoR} is used as the initial baseline D_0 . Then Eqs. (2.2)-(2.4) and (2.6) are solved to get D_{LoT} .

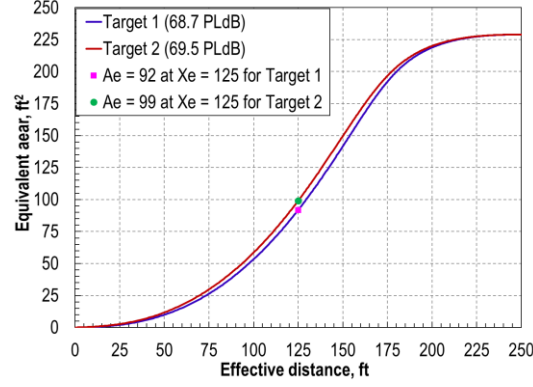


Fig. 11 Low-boom targets for different area constraints.

The CFD-based low-boom redesign $\hat{\mathbf{D}}_{\text{LoT}}$ of $\mathbf{D}_{\text{LoT}}$ requires a significant change of the wing to match $A_{e,r}^{\text{target}}(x_e, \mathbf{D}_{\text{LoT}})$ and to keep $\text{CP}_{x,\text{CFD}}$ ahead of $\text{CG}_{x,\text{aft}}$. As a result, the red iteration loop in Fig. 1 continues and $\hat{\mathbf{D}}_{\text{LoT}}$ is used as the new baseline $\mathbf{D}_0$ to get an updated $\mathbf{D}_{\text{LoT}}$, which is significantly different from the original $\mathbf{D}_{\text{LoT}}$ and has a more forward $\text{CG}_{x,\text{aft}}$ at $x = 170.0$ ft than $\text{CG}_{x,\text{aft}}$ of 170.5 ft for the original $\mathbf{D}_{\text{LoT}}$. This shows the importance of keeping $\hat{\mathbf{D}}_{\text{LoT}}$ close to $\mathbf{D}_{\text{LoT}}$. Otherwise, the actual $\text{CG}_{x,\text{aft}}$ of $\hat{\mathbf{D}}_{\text{LoT}}$ might be significantly different from $\text{CG}_{x,\text{aft}}$ of $\mathbf{D}_{\text{LoT}}$. The values of α and θ_{htail} for the updated $\mathbf{D}_{\text{LoT}}$ are 1.9° and -0.4° , respectively. The equivalent areas, cabin arrangement, and main gear packaging for the updated $\mathbf{D}_{\text{LoT}}$ are shown in Fig. 12. The updated $\mathbf{D}_{\text{LoT}}$ has a low-fidelity trim margin of $\text{CG}_{x,\text{aft}} - \text{CP}_{x,\text{LoFi}} = 170.0 - 165.0 = 5.0$ ft. The mission constraints (III.A.1)-(III.A.6) are also satisfied by $\mathbf{D}_{\text{LoT}}$ with the following constraint values, while the Mach, altitude, and range for the overland mission are listed in Table 1.

- (IV.C.1) CG margin to prevent tip over on the ground = 11.8 ft
- (IV.C.2) Cruise static margin for the low-boom mission = 10.8%
- (IV.C.3) LTO static margins are 3.5% and 2.6%, respectively
- (IV.C.4) Tail rotation angles for trim at LTO are -11.9° and -15.7° , respectively
- (IV.C.5) LTO field lengths are 7,432 ft and 8,296 ft, respectively
- (IV.C.6) Approach velocity = 145 knots

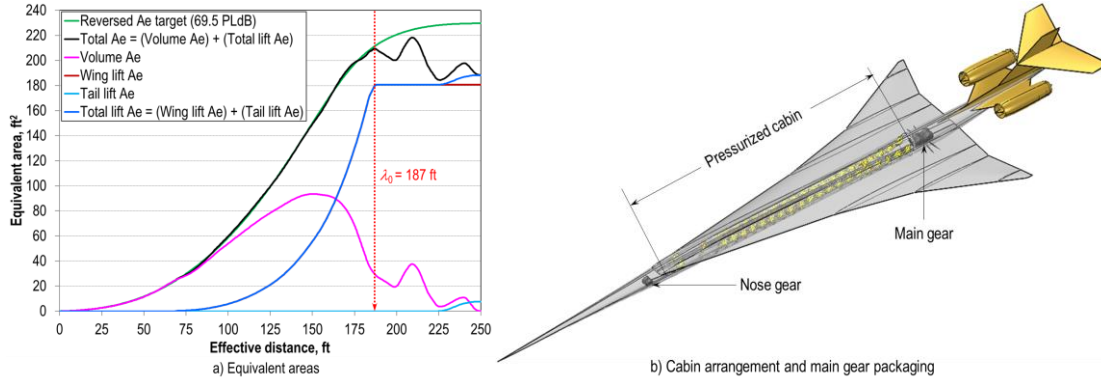
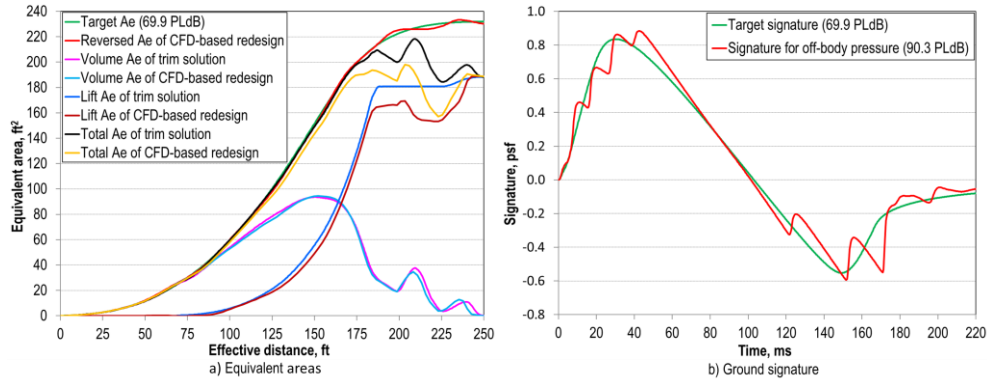
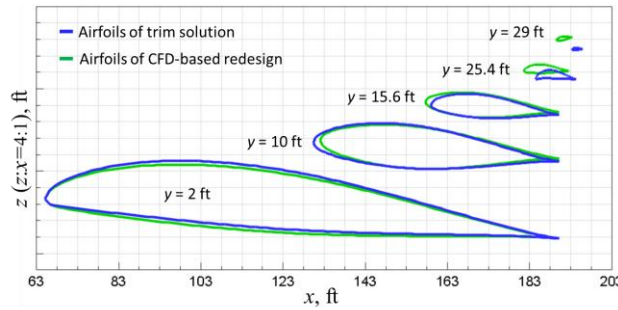


Fig. 12 Equivalent areas and cabin arrangement of $\mathbf{D}_{\text{LoT}}$ for cruise Mach 1.8.

Table 1 compares some specifications of the low-boom concept in Fig. 4, $\mathbf{D}_{\text{LoR}}$ in Fig. 6, and $\mathbf{D}_{\text{LoT}}$ in Fig. 12. The area constraint to improve trim in the low-boom target for $\mathbf{D}_{\text{LoT}}$ does not have a negative impact on the MTOGW, which benefits from the shortened wingspan and a smaller wing area. However, the trim penalty does show in the target PLdB for $\hat{\mathbf{D}}_{\text{LoT}}$ (see Figs. 7b and 13b). The wingspan of 58 ft for the updated $\mathbf{D}_{\text{LoT}}$ is not determined by optimization and is fixed to match that of $\hat{\mathbf{D}}_{\text{LoT}}$. When searching for the original $\mathbf{D}_{\text{LoT}}$, the wingspan was a design variable with its range between 60 ft and 63 ft. The optimal wingspan was 60.3 ft. Because the lower bound of 60 ft was not active, no attempt was made earlier to search for low-boom designs with wingspan below 60 ft. This shows that different choices of design variables and their design ranges yield different solutions of Eqs. (2.1)-(2.6) and (4.1). The implication is that two independent solutions of Eqs. (2.1)-(2.6) and (4.1) by two individuals could be very different. This is a common problem for any complex engineering design optimization.

Table 1 Comparison of initial low-boom concept, best trade solution, and trim solution

	Initial low-boom concept	Best trade solution ($\mathbf{D}_{LoR}$)	Trim solution ($\mathbf{D}_{LoT}$)
Number of passengers	40	40	40
Aircraft length	232 ft	232 ft	232 ft
Wingspan	69 ft	60.3 ft	58 ft
Wing area	3,009 ft ²	2,974 ft ²	2,930 ft ²
Overwater cruise Mach	1.8	1.8	1.8
Overwater cruise altitude	60,000 ft	60,000 ft	60,000 ft
Overwater range	3,600 nm	3,600 nm	3,600 nm
Overwater MTOGW	153,212 lb	145,615 lb	145,164 lb
Low-boom cruise Mach	1.6	1.8	1.8
Low-boom cruise altitude	45,000 ft	50,000 ft	50,700 ft
Low-boom cruise weight	145,766 lb	135,523 lb	134,377 lb
Low-boom range	2,500 nm	2,900 nm	2,950 nm
Low-boom MTOGW	154,474 lb	145,484 lb	144,655 lb
Zero fuel weight	76,249 lb	72,625 lb	71,834 lb
Engine thrust	36,700 lb	36,000 lb	36,000 lb
Target for inverse design	68.4 PLdB	69.4 PLdB	69.5 PLdB

**Fig. 13 Equivalent areas of $\mathbf{D}_{LoT}$ and $\hat{\mathbf{D}}_{LoT}$, and ground signature of $\hat{\mathbf{D}}_{LoT}$.****Fig. 14 Airfoil comparison of $\mathbf{D}_{LoT}$ and $\hat{\mathbf{D}}_{LoT}$**

The CFD-based low-boom redesign $\hat{\mathbf{D}}_{LoT}$ is obtained from $\mathbf{D}_{LoT}$ with wing surface modifications. The results are shown in Fig. 13. After matching $A_{e,r}(\hat{\mathbf{D}}_{LoT})$ with $A_{e,r}^{target}(\mathbf{D}_{LoT})$ for $\hat{\alpha} = \alpha = 1.9^\circ$, the CFD lift is slightly lower than W_{crs} of $\mathbf{D}_{LoT}$. This is also due to the minor difference of $A_{e,r}(l_e, \hat{\mathbf{D}}_{LoT})/A_{e,CFD}(l_e, \hat{\mathbf{D}}_{LoT})$ and μ value for $A_{e,r}^{target}$. So, $\hat{\alpha}$ is increased to 2.1° to match the CFD lift with W_{crs} . The identical values of $A_{e,CFD}(l_e, \hat{\mathbf{D}}_{LoT})$ and $A_{e,LoFi}(l_e, \mathbf{D}_{LoT})$ in Fig. 13a mean that the cruise weights for $\hat{\mathbf{D}}_{LoT}$ and $\mathbf{D}_{LoT}$ are identical (see Eq. (6) in Ref. [16]). The low-boom target of 69.9 PLdB in Fig. 13a is $\sigma A_{e,r}^{target}(x_e, \mathbf{D}_{LoT})$ with $\sigma = 1.01$, which minimizes $\|A_{e,r}(\hat{\mathbf{D}}_{LoT}) - \sigma A_{e,r}^{target}(\mathbf{D}_{LoT})\|$ after the adjustment of $\hat{\alpha}$. The value of $\hat{\theta}_{htail}$ for $\hat{\mathbf{D}}_{LoT}$ is 4.9° . So, $|\sigma - 1| = 0.01$, $|\hat{\alpha} - \alpha| = 0.2^\circ$, and $|\hat{\theta}_{htail} - \theta_{htail}| = 5.3^\circ$. From Fig. 13a, the A_e^{volume}

differences between $\mathbf{D}_{LoT}$ and $\hat{\mathbf{D}}_{LoT}$ are small. The aft A_e^{volume} differences are more significant than those shown in Fig. 7a because of a difference of 0.2° between angles of attack for $\mathbf{D}_{LoT}$ and $\hat{\mathbf{D}}_{LoT}$. The wing shape differences between $\mathbf{D}_{LoT}$ and $\hat{\mathbf{D}}_{LoT}$ (shown in Figs. 14 and 15) are small but more significant than those between $\mathbf{D}_{LoR}$ and $\hat{\mathbf{D}}_{LoR}$.

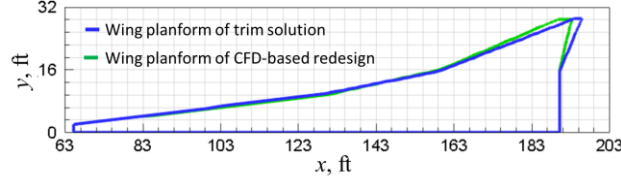


Fig. 15 Wing planform comparison of $\mathbf{D}_{LoT}$ and $\hat{\mathbf{D}}_{LoT}$

The volume constraint to improve trim for the low-boom target does not yield an improved low-fidelity trim margin for $\mathbf{D}_{LoT}$ when compared with $\mathbf{D}_{LoR}$. The trim benefit of $\mathbf{D}_{LoT}$ is in its noticeable tail lift (see Fig. 12a), which captures some of the tail lift of $\hat{\mathbf{D}}_{LoT}$ (see Fig. 13a). In comparison, the near zero lift on the tail of $\mathbf{D}_{LoR}$ is a gross underestimate of the tail lift of $\hat{\mathbf{D}}_{LoR}$ (see Figs. 6a and 7a), which results in a much larger aft shift of 8.2 ft for $CP_{x,CFD}$ of $\hat{\mathbf{D}}_{LoR}$ than 5.3 ft for $CP_{x,CFD}$ of $\hat{\mathbf{D}}_{LoT}$ when compared with $CP_{x,LoFi}$. The significant differences of $CP_{x,LoFi}$ and $CP_{x,CFD}$ in both cases are mainly due to the negative CFD lift on the pylon to reduce the volume increase of the nacelle for low-boom shaping, which requires a significant increase of the tail lift (see $A_{e,r}^{lift}$ curves in Figs. 7a and 13a) to align the aft shape of $A_{e,r}$ with the low-boom target. The trim constraint value $CG_{x,aft} - CP_{x,CFD} = 170.0 - 170.3 = -0.3$ ft is not feasible for $\hat{\mathbf{D}}_{LoT}$, but it is acceptable at the conceptual design phase because the constraint violation is only 0.1% of the aircraft length of 232 ft and is significantly less than the potential prediction error in $CG_{x,aft}$. However, the disconnection between $CP_{x,LoFi}$ and $CP_{x,CFD}$ makes it difficult to tell whether a specified trim margin for $CP_{x,LoFi}$ is enough for $CP_{x,CFD}$. More research is needed to develop an effective method to enforce the trim constraint while keeping the CFD-based low-boom redesign as close to the low-fidelity MDO solution as possible. A potential solution is to calibrate the aft lift distribution for the low-fidelity MDO solution so that the low-fidelity trim margin constraint in Eq. (2.3) is more closely related to the CFD trim constraint Eq. (4.4).

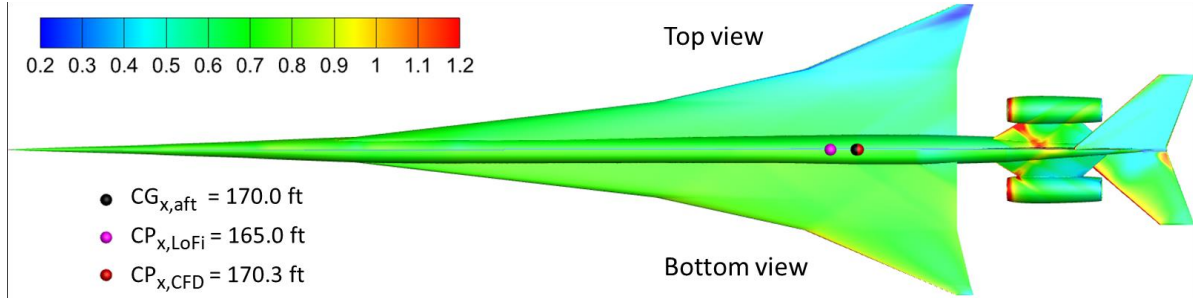


Fig. 16 Pressure contour on OML of $\hat{\mathbf{D}}_{LoT}$ and CP_x locations at start of cruise for low-boom mission

The cruise performance of $\hat{\mathbf{D}}_{LoT}$ is not expected to degrade significantly from that of $\mathbf{D}_{LoT}$. The CFD lift-over-drag ratio (not including viscous drag) for the cruise condition of $\hat{\mathbf{D}}_{LoT}$ is 8.9, while the low-fidelity lift-over-drag ratio (including an empirical viscous drag estimate) at the start of cruise condition for $\mathbf{D}_{LoT}$ is 7.2.

Except the wings, tail rotation angles, and small variations of the fuselage nose cone, the OMLs of $\mathbf{D}_{LoR}$, $\hat{\mathbf{D}}_{LoR}$, $\mathbf{D}_{LoT}$, and $\hat{\mathbf{D}}_{LoT}$ are identical. The pressure contours on the aft components of $\hat{\mathbf{D}}_{LoR}$ and $\hat{\mathbf{D}}_{LoT}$ are approximately the same (see Figs. 10 and 16).

IV. Concluding Remarks

A mixed-fidelity low-boom MDO problem is formulated for the integration of low-fidelity low-boom MDO and CFD-based low-boom inverse design. A block coordinate optimization (BCO) method is proposed for an approximate solution of the mixed-fidelity low-boom MDO problem. The BCO method has been applied to generate a low-boom supersonic transport that carries 40 passengers, flies a low-boom overland mission with cruise Mach of 1.8 and range of 2,950 nm, cruises overwater at Mach 1.8 with range of 3,600 nm, satisfies the specified landing and takeoff performance constraints, has a CFD lift-over-drag ratio (not including viscous drag) of 8.9 for low-boom cruise flight,

and approximately satisfies the low-boom trim constraint for CFD center of pressure. The reversed equivalent area of the generated low-boom supersonic transport closely matches a low-boom target below 70 PLdB.

One important discovery is that a low-boom supersonic transport could be designed with a predetermined shape of fuselage tailored for passengers and main gear storage. The generated low-boom configuration has a constant cross section height of 7.3 ft for the passenger cabin and main gear storage, a constant cross section width of 5.5 ft for the passenger cabin, and a constant cross section width of 6 ft for main gear storage. In contrast, significant fuselage cross section variations were usually required for low-boom supersonic configurations found in literature.

The trim constraint for low-boom cruise is difficult to enforce for CFD-based low-boom redesign with minor wing modifications of the low-boom low-weight solution generated by the low-fidelity MDO. There is a need to improve the low-fidelity prediction of aft lift distribution to make the trim constraints for low-fidelity and CFD centers of pressure more correlated.

The significance of this work is to demonstrate that (i) system-level trades among the target PLdB, cruise Mach, cruise altitude, and range are critical for development of economically viable low-boom supersonic transports, and (ii) minor wing modifications of a low-boom low-weight design from low-fidelity MDO can result in a CFD-based low-boom design with a reversed equivalent area close to a low-boom target below 70 PLdB. The latter provides a numerical method to obtain a CFD-based low-boom design that also approximately satisfies the specified mission performance constraints. The existing gap in CFD-based low-boom design methods—CFD-based low-boom shaping for a fixed low-fidelity cruise weight estimated from a baseline configuration that could be very different from the resulting CFD-based low-boom design—is significantly narrowed by the BCO method. The next step is to calibrate all low-fidelity aero analysis results by CFD analysis for low-fidelity low-boom MDO, which might be extremely difficult to accomplish.

Acknowledgments

This work is funded by the NASA Commercial Supersonic Technology Project and Transformational Tools & Technologies Project. The authors would like to thank James Fenbert at Analytical Mechanics Associates for many constructive comments that greatly improved the paper.

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