

CFD Validation Study of a Hypersonic Cone-Slice-Flap Configuration

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Model validation is the process of determining the degree of accuracy between the real world and the model. The result of model validation can be used to either improve the model through calibration or quantify the model-form uncertainty. This work focuses on estimating the model-form uncertainty through an area metric for a hypersonic cone-slice-flap geometry configuration given the uncertainty in both the simulation and experimental data. A procedure that can give an accurate representation of the model-form uncertainty using a small number of high-fidelity runs is outlined. The work also assesses the impact of using different high-fidelity solvers and different turbulence models. The goal of performing this comparison is to provide a quantifiable measurement of the accuracy of each solver and turbulence model for this type of design. A low-fidelity analysis is also performed to get a model-form uncertainty for this type of analysis. The two high-fidelity CFD solvers used here are VULCAN-CFD and FUN3D, and the low-fidelity results comes from Cart3D. The experimental data comes from the 20-inch Mach 6 Tunnel located at NASA Langley Research Center. The work here shows that the models used tend to under-predict the drag and moment aerodynamic coefficients for this type of design leading to a model-form uncertainty estimate which can account for up to an 86% of the total uncertainty in the model predictions.

Nomenclature

AF	axial force (lbf)	f_i	quantity of interest for grid i
a_i	coefficient for i th basis function	GCI	grid convergence index
BMC	balance moment center	L	model length (in.)
C_D	drag coefficient	l_{ref}	model reference length (in.)
C_L	lift coefficient	M	Mach number
C_M	moment coefficient	MRC	moment reference center (in.)
$\mathbf{D}$	deterministic variables	M_∞	freestream Mach number
d	area metric	NF	normal force (lbf)
$E(x)$	empirical distribution function at x	N_t	number of samples needed
$F(x)$	cumulative distribution function at x	n	number of random variables
F_s	factor of safety	PM	pitching moment (lbf)

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p	order-of-accuracy	y^+	dimensionless wall distance
p_{nose}	model nose pressure (psia)	X_f	flap leading edge location (in.)
p_{T_∞}	facility stagnation pressure (psia)	X_s	slice leading edge location (in.)
q	order of polynomial		axial balance transfer distance
q_∞	freestream dynamic pressure (psia)	x_{tran}	from BMC to MRC (in.)
$U_{C_D}^g$	grid error uncertainty of C_D		vertical balance transfer distance
$U_{C_L}^g$	grid error uncertainty of C_L	z_{tran}	from BMC to MRC (in.)
$U_{C_M}^g$	grid error uncertainty of C_M	α	angle of attack (deg)
R_b	model base radius (in.)	$\varepsilon_{i+1,i}$	relative grid error of grid $i + 1$ to grid i
R_n	nose radius (in.)	γ	ratio of specific heats
SA	Spalart and Allmaras turbulence model	κ	Von Kármán's constant
S_{ref}	model reference area (in. ²)	Ψ_i	basis function representing i th mode
S_i	main effect Sobol index of parameter $\mathbf{X}_i$	σ, c_{w2}	SA model closure coefficients
ST_i	total-effect Sobol index of parameter $\mathbf{X}_i$	σ_{w1}, σ_{w2}	SST model closure coefficients
s_X	uncertainty range for small sample size	θ_f	flap deflection angle (deg)
SST	Menter Shear-Stress Transport model	ξ	standard random variable
r	grid refinement ratio		
Y	quantity of interest		

I. Introduction

With advancements in computers, engineers have increasingly relied on modeling and simulation for design and analysis of aerospace vehicles. Hence, there is a need to quantify how accurate or inaccurate these models are when compared with empirical data. Validation can be used to determine the adequacy of the mathematical model for an intended application. One approach to validation is to compare verified numerical solutions with experimental data for which uncertainty is estimated in both the computational model and experimental data. With the inclusion of uncertainty in both the experimental data and simulation, a statistical-based measurement of the model-form uncertainty can be determined.

The goal of the current work is to present a validation procedure for a cone-slice-flap configuration which has been studied since the early years of hypersonics to understand basic flow physics.^{1–5} by using multiple computational fluid dynamics (CFD) models. This procedure illustrates a process for performing model validation with limited computational resources and numerous sources of uncertainty in the experimental and simulation data. Previous research investigated a similar design with a validation study,^{6–9} but these studies do not include various turbulence models and CFD solvers with multiple sources of uncertainty. This work quantifies the uncertainties in the experimental and simulation data, formulates a quantifiable measurement for model-form uncertainty, and draws conclusions on modeling a cone-slice-flap hypersonic design with state-of-the-art CFD solvers.

The computational and experimental results presented herein are for a Mach 6, spherically blunted cone. This model was studied experimentally in the 20-inch Mach 6 Tunnel located at NASA Langley Research Center. The experimental database consists of force and moment data, two base pressure measurements, and a sting-cavity pressure measurement. The simulations are performed as three-dimensional, calorically perfect gas flows on two state of the art Reynolds Averaged Navier Stokes (RANS) CFD solvers and with two turbulence models. An inviscid solver is also used as a comparison point for this type of hypersonic design.

This paper is organized to first present the experimental test that was performed and the inherent uncertainty sources in Section II. Section III presents the computational models, grid convergence study, and the uncertainty sources in the computational models. Section IV presents the uncertainty quantification methodology and the validation approach for comparing the Mach 6 wind tunnel data and CFD uncertainty results. Section V discusses the results of the work, and Section VI describes conclusions.

II. Experimental Test

An experiment was run in the NASA Langley 20-Inch Mach 6 Tunnel to investigate the aerodynamic characteristics of a cone-slice-flap configuration. The stainless steel wind tunnel model was a 7° half-angle cone measuring 12 inches in length from the virtual sharp nose tip to the base and features a sliced surface

parallel to the centerline axis along with a flap mounted on the slice surface. An illustration of a model configuration on a sting support is shown in Figure 1 with in an inverted orientation to show the body slice and flap.

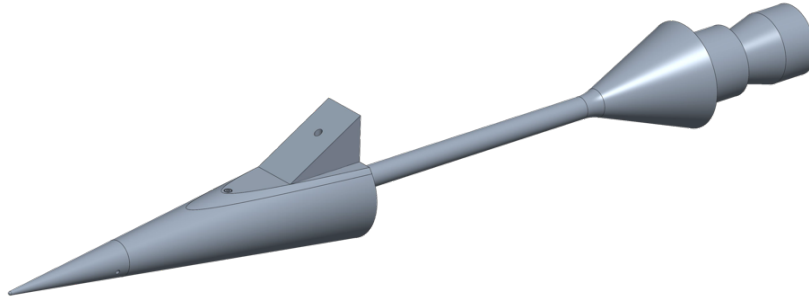


Figure 1: Cone-Slice-Flap wind tunnel model (shown inverted).

The model is designed with interchangeable parts that allows the investigation of the aerodynamic effects of nose bluntness, slice leading edge location, flap leading edge location, and flap deflection angle. The ranges of these geometric parameters are given in Table 1 with the slice and flap leading edge locations normalized by the model length, $L = 12$ inches, and the nose radius normalized by the model base radius, $R_b = 1.4734$ inches. An exploded view of the model parts is shown in Figure 2.

Table 1: Range of geometrical parameters.

Parameter	Variable	Minimum Value	Maximum Value
Nose Radius	R_n/R_b	0.04	0.30
Slice Leading Edge Location	X_s/L	0.65	0.75
Flap leading Edge Location	X_f/L	0.80	0.90
Flap Deflection Angle, deg	θ_f	0°	30°

The experimental model consists of 36 different configurations where 28 configurations were defined by design of experiments to fit a response surface, and eight are confirmation configurations used to verify a response surface for the aerodynamic analysis. The 28 configurations have three flap deflection angles that are located on the aft section of the model, three different flap leading edge locations, three different planar slice leading edge locations, and three different nose cone radii. The eight confirmation configurations have two flap deflection angles, two flap leading edge locations, one planar slice leading edge location, and three different nose radii. The work here focuses on a single configuration, labeled configuration 33, and future work may address the entire configuration space. This configuration was chosen because this will be the check standard wind tunnel model for the 20-inch Mach 6 wind tunnel. This configuration was also run at several Reynolds numbers and has multiple replicate runs within the test matrix which help quantify the random error uncertainty in the experiment. The geometrical parameters of configuration 33 can be seen in Table 2 and visualized in Figure 3.

Table 2: Parameters for configuration 33 model.

X_f/L	X_s/L	R_n/R_b	θ_f
0.85	0.70	0.15	15°

A minimum nose radius was specified such that a small pressure orifice could be installed on the stagnation point of the blunted nose. This orifice was used to obtain post-normal-shock stagnation pressure

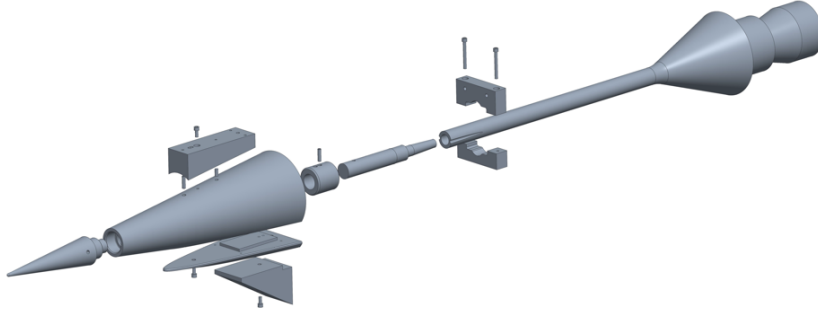


Figure 2: Exploded view of cone-slice-flap wind tunnel model.

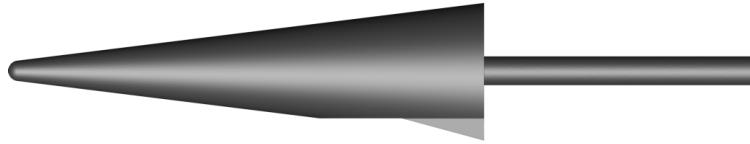


Figure 3: Geometry for configuration 33.

measurements at zero angle of attack so that the Mach number at the model location in the test section can be determined directly for subsequent calculations of the flow conditions.

The model was sting-mounted through the base on a six-component force balance equipped with a water-cooling jacket to regulate the balance temperatures in the high-temperature hypersonic flow. Base pressure was measured using two flush orifices on the model base. The model was tested over an angle of attack range from -4° to $+12^\circ$ and at unit Reynolds numbers from 1.0 to $4.0 \times 10^6/\text{ft}$.

A. Uncertainty Sources

This section outlines the uncertainty quantification approach that was used to gauge uncertainties within the wind tunnel experiment. For the current study, both epistemic and aleatory uncertainties are considered. The approach assumes a calorically perfect gas and isentropic flow. Under these assumptions, the isentropic flow relations are exploited, and a full Monte Carlo approach is used to estimate systematic load uncertainties. The first section will describe the systematic error approach, and the subsequent section will describe random error within the experiment.

1. Systematic Error

Systematic error occurs due to measurement error in the experiment. Within the wind tunnel, there are calibration uncertainties associated with axial force (AF), normal force (NF), and pitching moment (PM). These uncertainties are determined using a standard balance calibration where the measurement of each force was assumed to have a normal distribution. There is also a settling chamber uncertainty for the facility stagnation pressure, p_{T_∞} . The facility stagnation pressure is measured using a 500-psia Druck pressure transducer. The uncertainty comes from the manufacturer's specifications and was described as a normal distribution. There is a model nose pressure, p_{nose} , uncertainty which comes from the uncertainty of the ESP module that was used to measure the nose pressure and was found to be normally distributed. Other uncertainties include the axial balance transfer distance, x_{tran} , from balance moment center, BMC, to the moment reference center, MRC, vertical balance transfer distance, z_{tran} , from the balance moment center, BMC, to the moment reference center, MRC, and angle of attack (α). The x_{tran} and z_{tran} quantities are based off engineering judgment on the device used to measure the model coordinates. Given the lack of information for accuracy of the transfer distances, they are both considered epistemic variables. The angle of

attack uncertainty was determined by mounting an angle measurement system, AMS, accelerometer package to the model using the model level plate. The model is then run through a series of move-pause angles for a wind-off run. The residuals between the AMS package and pitch encoder help determine the uncertainty. Because this was only done once for the model at an angle of attack of 12° , this uncertainty is determined to be epistemic with interval bounds being the max residual between the AMS package and pitch encoder. Table 3 shows all the wind tunnel load input uncertainties for the systematic error. The uncertainty for the normal distribution is given as the standard deviation, sd, of the distribution, and the nominal for the distribution is given as the wind tunnel, WT, reading.

Table 3: Wind tunnel load input uncertainties.

Uncertainty Variable	Classification	Uncertainty Type	Nominal	Uncertainty
AF (lbf)	aleatory	normal	AF (lbf) WT reading	0.01 (lbf) sd
NF (lbf)	aleatory	normal	NF (lbf) WT reading	0.005 (lbf) sd
PM (lbf)	aleatory	normal	PM (lbf) WT reading	0.0165 (lbf) sd
p_{T_∞} (psia)	aleatory	normal	p_{T_∞} (psia) WT reading	0.1 (psia) sd
p_{nose} (psia)	aleatory	normal	p_{nose} (psia) WT reading	0.0075 (psia) sd
x_{tran} (in.)	epistemic	interval	0.2535 (in.)	+0.001 (in.)
z_{tran} (in.)	epistemic	interval	0 (in.)	± 0.001 (in.)
α (deg)	epistemic	interval	α (deg) WT reading	± 0.01 (deg)

An assumption of a perfect gas and isentropic flow are used to propagate the uncertainties for the systematic error. These assumptions allow for ease of implementation of the Monte Carlo analysis and give a relatively close result to what is expected in the wind tunnel due to the Mach 6 flow. Eqs. (1)-(3) are used to transform the wind tunnel measurements from body fixed axis to wind axis.

$$C_D = \frac{NF \sin(\alpha) + AF \cos(\alpha)}{q_\infty S_{ref}} \quad (1)$$

$$C_L = \frac{NF \cos(\alpha) - AF \sin(\alpha)}{q_\infty S_{ref}} \quad (2)$$

$$C_M = \frac{PM - NF x_{tran} - AF z_{tran}}{q_\infty S_{ref} l_{ref}} \quad (3)$$

The freestream dynamic pressure (q_∞) is found using the isentropic flow relation in Eq. (4) with the assumption that the spscific heat ratio (γ) is 1.4.

$$q_\infty = 0.7 p_{T_\infty} M_\infty^2 \left(1 + \frac{M_\infty^2}{5} \right)^{-7/2} \quad (4)$$

The freestream Mach number, M_∞ , from Eq. (4), is calculated using the Newton-Raphson root finding method by employing the normal shock relation with γ equal to 1.4 as shown in Eq. (5).

$$\frac{p_{nose}}{p_{T_\infty}} = \left(\frac{6M_\infty^2}{M_\infty^2 + 5} \right)^{7/2} \left(\frac{6}{7M_\infty^2 - 1} \right)^{5/2} \quad (5)$$

This work defines the reference area, S_{ref} , as the circular area of the base of the cone which is 6.8202 in.² and the reference length, l_{ref} , as the diameter of the circular portion of the cone which is 2.9468 in. The uncertainty distributions in Table 3 are propagated through the equations above to assess the distributions of the loads at angles of attack considered in this study.

Experimental uncertainty was quantified for angles of attack of -4° , 0° , 4° , and 12° to obtain a spectrum of the flow behavior over the full range of angles of attack. Figure 4 shows the load coefficient probability boxes of the cumulative distribution functions, CFDs, for the wind tunnel test run at a Reynolds number of $2.5 \times 10^6/\text{ft}$ and angle of attack of 12° . Table 4 gives the 95% probability output uncertainty range for each load at the angles of attack of interest. The 95% probability output range shown here represents that there

is at least a 95% chance the true wind tunnel value will lie between that range given the known aleatory and epistemic uncertainties. Because wind tunnel measurements are not obtained at the exact angle of attack of interest, the intermediate loads at a particular angle of attack were determined by linearly interpolating the wind tunnel load readings around the angle of attack of interest. Linear interpolation is used because the data were acquired at 20 samples/sec while continually pitching the model at $1^\circ/\text{sec}$.

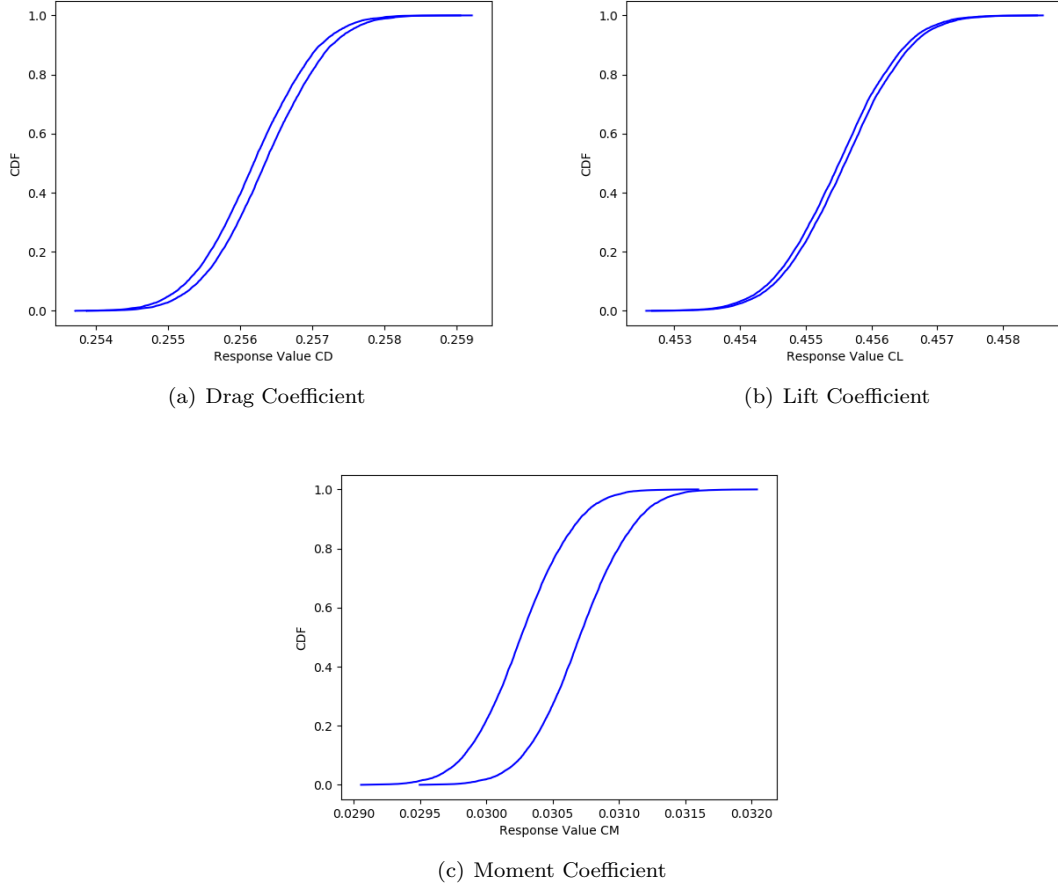


Figure 4: Probability boxes for WT loads at $\alpha = 12^\circ$ and $\text{Re} = 2.5 \times 10^6/\text{ft}$

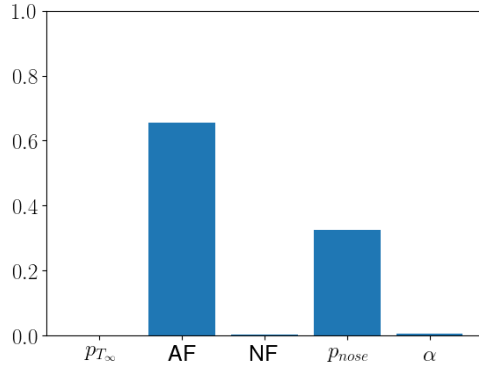
Additionally, a Sobol method was implemented to gauge the sensitivity of the input parameters on the loads. The Sobol sensitivity index is a variance-based method to perform a global sensitivity analysis which uses the variance as the basis to define a measure of input parameter influence on the overall output variation.¹⁰ This method was chosen here, because it is easy to use with Monte Carlo methods and gives a quantifiable measurement of the sensitivity input parameters to output parameters. Eq. (6) gives the first order sensitivity index or main effect Sobol index, S_i , which measures the effect of varying parameter X_i without any other parameters varying.

$$S_i = \frac{\mathbb{V}_i[\mathbb{E}_{\sim i}[\mathbf{Y}|\mathbf{X}_i]]}{\mathbb{V}[\mathbf{Y}]} \quad (6)$$

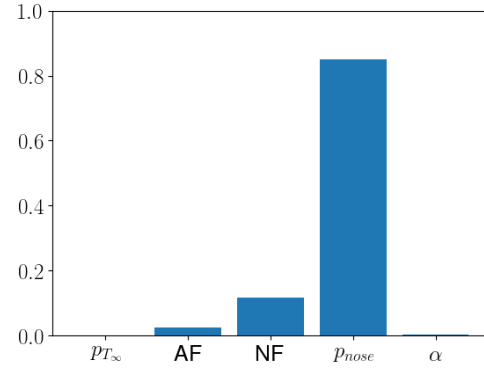
Here, $\mathbb{V}_i[\mathbb{E}_{\sim i}[\mathbf{Y}|\mathbf{X}_i]]$ is the variance of the conditional expectation of the output $\mathbf{Y}$ given parameter $\mathbf{X}_i$ is fixed, and $\mathbb{V}[\mathbf{Y}]$ represents the total variance of the output. Figure 5 shows the Sobol main effect index for the load coefficients at an angle of attack of 12° and a Reynolds number of $2.5 \times 10^6/\text{ft}$. At these conditions, the greatest contribution to the drag coefficient uncertainty is due to the measured balance axial force. For the lift coefficient, the greatest contribution was the nose pressure uncertainty. For the moment coefficient, the pitching moment calibration uncertainty has the most significant contribution.

Table 4: 95% probability range for the actual wind tunnel load.

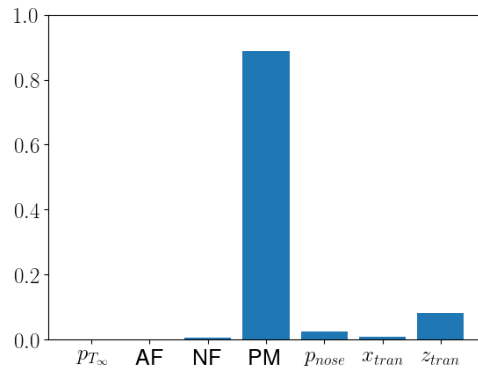
α	Uncertainty Variable	WT Reading	95% Uncertainty Range
-4°	C_D	0.1069	[0.1059,0.1084]
	C_L	-0.1118	[-0.1132,-0.1117]
	C_M	-0.0237	[-0.0250,-0.0235]
0°	C_D	0.0958	[0.0945,0.0970]
	C_L	0.0101	[0.0098,0.0110]
	C_M	-0.0083	[-0.0090,-0.0076]
4°	C_D	0.1172	[0.1160,0.1185]
	C_L	0.1379	[0.1374,0.1389]
	C_M	-0.0020	[-0.0030,-0.0015]
12°	C_D	0.2558	[0.2548,0.2578]
	C_L	0.4547	[0.4539,0.4572]
	C_M	0.0303	[0.0296,0.0314]



(a) Drag coefficient Sobol main effect index.



(b) Lift coefficient Sobol main effect index.



(c) Moment coefficient Sobol main effect index.

Figure 5: Sobol main effect indices for $\alpha = 12^\circ$ and $\text{Re} = 2.5 \times 10^6/\text{ft}$

2. Random Error

Random error in the wind tunnel is due to inherent changes in the wind tunnel experiment from one run to the next. The work here obtains the random error uncertainty by using the replicate runs for configuration

33 which were performed at a Reynolds number of $2.5 \times 10^6/\text{ft}$. Classically, random uncertainty is estimated by taking the mean and standard deviation of the replicate runs. However, due to the small sample size of the number of replicate runs, Eq. (7) can be used to convert the range into an unbiased estimator of the standard deviation by assuming a normal distribution.^{11,12}

$$s_X = \frac{X_{\max} - X_{\min}}{2.059} \quad (7)$$

Here $X_{\max}$ and $X_{\min}$ represent the maximum and minimum values of all the runs for the quantity of interest, respectively, and the value 2.059 is based on the number of replicate runs. Table 5 shows the wind tunnel readings of the four replicate runs at α equal to 12° and a Reynolds number of $2.5 \times 10^6/\text{ft}$ and the associated uncertainty.

Table 5: Replicated wind tunnel data for $\text{Re} = 2.5 \times 10^6/\text{ft}$ and $\alpha = 12^\circ$.

Run	C_D	C_L	C_M	Uncertainty (Eq. (7))
14	0.2558	0.4547	0.0303	$U_{C_D} = 0.0008, U_{C_L} = 0.0010,$ $U_{C_M} = 0.0003$
29	0.2554	0.4537	0.0296	
52	0.2561	0.4536	0.0297	
76	0.2570	0.4557	0.0298	

To combine the systematic and random error uncertainty, a new upper and lower distribution are formed assuming a normal distribution with the mean from the upper and lower probability bounds from the systematic error and standard deviation shown in Eq. (8). A normal distribution is assumed because historically the random error should follow a normal distribution given that it is formulated through repeated measurements. Also, the systematic error presented in Section II.1 can be shown to follow a normal distribution using the Anderson-Darling test.¹³ Hence, two combined normal distribution should create a new normal distribution.

$$\sigma_{\text{new}_k}^\pm = \sqrt{(\sigma_{\text{bounds}_k}^\pm)^2 + U_k^2}, \quad k \in \{C_D, C_L, C_M\} \quad (8)$$

For Eq. (8), $\sigma_{\text{bounds}_k}^\pm$ represents the standard deviation of lower and upper bound CDFs from the systematic error with σ^- being the lower bound standard deviation and σ^+ being the upper bound standard deviation. Also, U_k comes from the uncertainty column in Table 5. After new lower and upper probability distributions are formed, a 95% uncertainty interval is calculated the same way it was done in Section II.A.1, which can be seen in Table 6 for the relevant angles of attack. The repeatability or random error uncertainty given in Table 5 is much less significant to the total uncertainty compared to the systematic error that represents good repeatability for this design in the 20-inch Mach 6 wind tunnel. The largest total uncertainty for the experimental data exists in the moment coefficient at an angle of attack of 12° that is expected given the larger moment arm that is present at a higher angle of attack.

III. Computational Models

Two RANS CFD solvers, both developed at NASA Langley Research Center, were used in this study: Fully Unstructured Navier-Stokes (FUN3D)¹⁴ and Viscous Upwind Algorithm for Complex Flow Analysis (VULCAN-CFD).¹⁵ FUN3D is a unstructured, node-centered, finite volume solver while VULCAN-CFD is a structured, cell-centered, multi-block solver. Both solvers used the Edwards low-diffusion flux-splitting scheme,¹⁶ an inviscid flux scheme, and the Van Leer limiter. In this study, both codes assume steady-state, calorically perfect gas models with a ratio of specific heats equal to 1.4. Cold wall boundary conditions of 80°F were assumed as the model was allowed to cool to approximately 80°F between runs. Both solvers also employed two turbulence models: the one-equation Spalart-Allmaras (SA) model¹⁷ and the two-equation Menter Shear-Stress Transport (SST) model.¹⁸ Both models are frequently used in hypersonic flow applications making them of interest for studying their predictive capability. The work also leverages the inviscid solver, Cart3D,¹⁹ which was developed at NASA Ames Research Center. In the remainder of this section, geometry modeling assumptions and grid convergence studies with both RANS CFD codes are discussed with a listing of uncertainty sources considered in the modeling.

Table 6: 95% probability range for the actual wind tunnel load.

α	Uncertainty Variable	WT Reading	95% Uncertainty Range
-4°	C_D	0.1069	[0.1051,0.1092]
	C_L	-0.1118	[-0.1146,-0.1104]
	C_M	-0.0237	[-0.0252,-0.0233]
0°	C_D	0.0958	[0.0938,0.0977]
	C_L	0.0101	[0.0084,0.0125]
	C_M	-0.0083	[-0.0092,-0.0074]
4°	C_D	0.1172	[0.1153,0.1193]
	C_L	0.1379	[0.1361,0.1403]
	C_M	-0.0020	[-0.0032,-0.0013]
12°	C_D	0.2558	[0.2541,0.2585]
	C_L	0.4547	[0.4530,0.4581]
	C_M	0.0303	[0.0294,0.0316]

A. Geometry Modeling Assumptions

The computational models include an attached sting to the base of the cone-slice-flap geometry to account for sting effects within the wind tunnel data. A VULCAN-CFD analysis was run with the nominal flow field conditions to determine where the sting could be truncated to ensure it has less than a 0.1% change on the loads of interest on the forebody while minimizing the computational burden of modeling the entire sting hardware. This analysis showed a sting length of approximately 6 inches aft from the model base could be used in CFD calculations whereas the wind tunnel model sting length was, in total, 17 inches from the base of the model to the wind tunnel attachment point. Neither the wind tunnel walls, nor the model support system was modeled. Using the model geometry, tunnel geometry, and knowledge of the freestream Mach number, isentropic and oblique shock relations predicted that no reflected shocks would impinge on the model or sting. All of the CFD results presented have the same reference area of 6.8202 in.² and reference length of 2.9468 in.

B. Grid Convergence Study

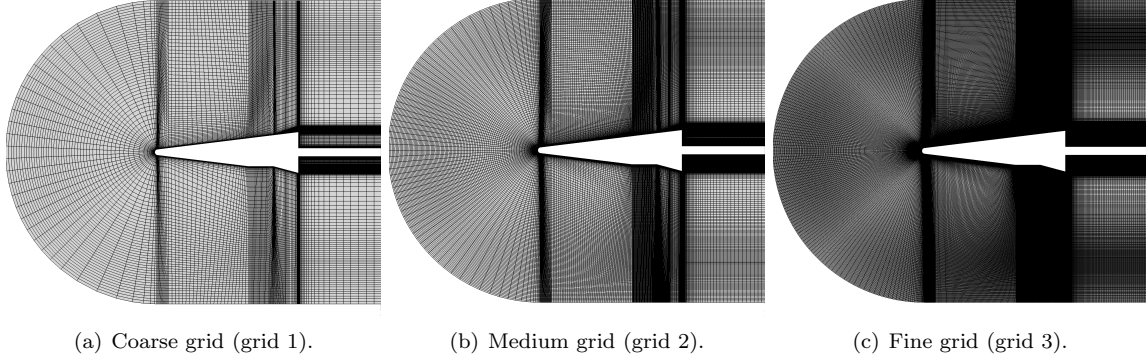
A grid convergence study was performed to quantify the spatial resolution component of solution verification error. The configuration was run at nominal wind tunnel conditions with a total temperature of 410° F and a total pressure of 143 psia. The Reynolds number was 2.5×10^6 /ft, the angle of attack was 12° , and the Mach number was 6. The angle of attack value was chosen because it represents the maximum angle of attack the model was tested and potentially the most extreme conditions the model will experience in terms of bow shock strength and separation.

1. Viscous Grid Convergence Study

VULCAN-CFD with the SST turbulence model was used to perform the grid convergence study. Once a suitable grid, with quantified error was determined, both solvers used the same grid for all subsequent analyses. Different grid levels considered in the study consisted of a coarse, medium, and fine grid. The number of cells, number of processors, and approximate time to convergence on that number of processors are shown in Table 7. The medium grid is built by coarsening the fine grid by a factor of two in each spatial dimension. Similarly, the coarse grid is built by coarsening the medium grid by a factor of two in each direction. The three computational grids can be seen in Figure 6. Grid wall spacing was selected such that the nominal y^+ was 0.5. Near separation regions and sharp edges, the maximum y^+ was 3.8. Solutions were run until no noticeable changes occurred over 5,000 iterations in the L^2 norm, mass flow rate error, and loads, thereby ensuring iterative convergence of the quantities of interest were effectively zero.

Table 7: Convergence level densities and computational cost.

Grid	Processors	Convergence Time	Number of Cells
Coarse (Grid 1)	280	16 minutes	786,959
Medium (Grid 2)	560	64 minutes	6,295,672
Fine (Grid 3)	560	512 minutes	50,365,376

**Figure 6: Three grids used for grid convergence study.**

Richardson extrapolation²⁰ was used to obtain error estimates for the medium grid and to calculate higher-order estimates of the flow fields from a series of low-order discrete values.²¹ Using this technique, the value estimated from the Richardson extrapolation is the value, f_{exact} , that would result if the cell size tended to zero. The extrapolated value is made from two grid sizes, but for a grid refinement study, at least three grids are used.

Roache generalized Richardson extrapolation method to p th order methods which use Eq. (9).²⁰

$$f_{\text{exact}} \approx f_1 + \frac{f_1 - f_2}{r^p - 1} \quad (9)$$

Here, f_1 and f_2 are the quantity of interest for grids 1 and 2, respectively. In this study, the grid refinement ratio, r , a constant value of two was used. From Eq. (9), the extrapolated value is varied by the order p , and Stern et al. determined that order-of-accuracy can be estimated by Eq. (10).²²

$$p = \frac{\ln(\varepsilon_{32}/\varepsilon_{21})}{\ln(r)} \quad (10)$$

Here, the relative grid error from grid $i + 1$ to i , $\varepsilon_{i+1,i}$, is defined as

$$\varepsilon_{i+1,i} = f_{i+1} - f_i \quad (11)$$

for $i = 1, 2, 3$. For the present work, grids 1, 2, and 3 are defined as the coarse, medium, and fine grids, respectively.

For the computational grids used here, the observed order of accuracy was found to be much less than the theoretical order of accuracy of two. This indicates that the current level of grid refinement is not in the asymptotic range, and a Richardson extrapolated value is not a valid estimation of the grid error. To obtain a measure of the discretization error, the order of convergence is taken equal to the theoretical order of convergence (the order of the flow solver numerical scheme), and the grid convergence index (GCI) approach was applied.

The GCI provides a uniform measure of convergence for grid refinement studies²⁰ and is a percentage measurement of the difference between the computed value and the asymptotic numerical value. It indicates how much the solution would change with further refinement of the grid. A small value for the GCI indicates that the computation is within the asymptotic range. The GCI for the fine and medium grid can be computed

by Eq. (12).

$$GCI_{fine} = F_s \frac{|f_3 - f_2|}{f_2(r^p - 1)} \quad GCI_{med} = r^p GCI_{fine} \quad (12)$$

Here, F_s is a factor of safety. Because the solutions were found to perhaps not be in the asymptotic range using a Richardson extrapolation, a factor of safety of three is applied to all error bounds.²³

GCI values for the medium and fine grids are shown in Table 8. The medium grid was chosen for subsequent computations due to a small change in aerodynamic coefficients of interest from the medium to the fine grid and is significantly cheaper than the fine grid in terms of computational time. Note that the moment GCI values appear large, but the actual moment values about the reference location were relatively small numbers for which even small changes can exacerbate an error measure.

Table 8: Grid convergence index results.

	GCI_{C_D}	GCI_{C_L}	GCI_{C_M}
GCI_{med}	$\pm 0.74\%$	$\pm 2.24\%$	$\pm 17.70\%$
GCI_{fine}	$\pm 0.19\%$	$\pm 0.56\%$	$\pm 4.42\%$

Values in Table 8 were applied to each of the force predictions as uncertainty as the true value is unknown. In addition, this grid convergence study was only completed at an angle of attack of 12° . An assumption here was that the relative uncertainty values above apply uniformly to the other angles of attack considered. As mentioned above, the 12° angle of attack was expected to be the most extreme condition in terms of shock strength and separation. Therefore, the computed grid convergence error was expected to cover the potential grid resolution uncertainty at the less significant angles of attack.

2. Cart3D

Cart3D is a scalable multi-level, linearly-exact upwind solver that uses domain decomposition to achieve scalability for both steady and time-dependent flows.¹⁹ The solver uses a cell-centered finite-volume upwind differencing scheme for Cartesian meshes. For this work, Cart3D adjoint-based mesh refinement was used to minimize the discretization errors. The analysis was done on a half-body with Riemann boundary conditions, and nine adaption cycles were performed. The mesh for the model can be seen in Figure 7. The model was run on 16 processors and required 120 minutes to complete.

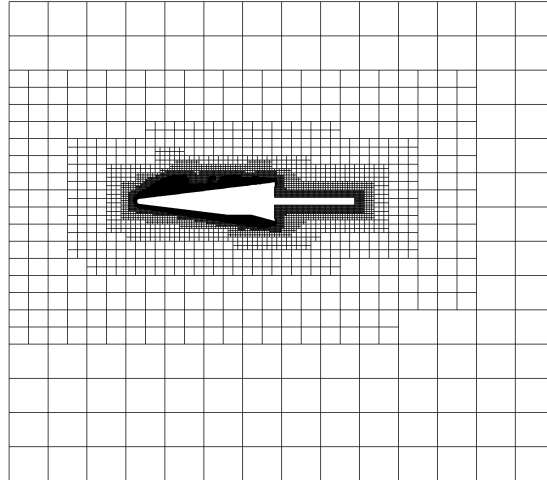


Figure 7: Cart3D adapted grid.

The grid error uncertainty for adaptive Cart3D was found by using the estimated discretization error term that is provided by the adaptive Cart3D analysis and is shown in Eq. (37) of Nemec et al.²⁴ The discretization error term is derived to be the difference between an adjoint correction term and the adjoint solution on the working mesh. The Cart3D grid error percent uncertainty (U^g) are summarized in Table 9.

Table 9: Overall comparisons of the grid convergences with estimated uncertainties.

Flow Solver	$U_{C_D}^g$	$U_{C_L}^g$	$U_{C_M}^g$
Cart3D	$\pm 0.14\%$	$\pm 0.08\%$	$\pm 13.2\%$

C. Uncertainty Sources

Uncertainty sources identified in this work arose from three sources: grid convergence errors, turbulence model closure coefficient uncertainty, and model-form uncertainty. The closure coefficients for the SA and SST models have been studied throughout literature to assess simulation uncertainty.^{25–28} Uncertainty in the closure coefficients is inherently epistemic due to the lack of knowledge of the best coefficient to use for this study. In this study, key uncertainty sources from the results given by Erb et al., which assessed the sensitivities of the various closure coefficients in SA and SST models.²⁵ The analysis conducted by Erb et al. was used due to its similarity in geometry and Mach number to the current study.

For the SA model, the closure coefficients σ , κ , and c_{w2} are the parameters that are varied; for more details on the model, refer to Spalart and Allmaras.¹⁷ For the SST model, the closure coefficients σ_{w1} , σ_{w2} , and κ are the varied uncertainty parameters; for more detail on the model, refer to Menter.¹⁸ Descriptions of each closure coefficient can be seen in Table 10. The closure coefficients σ_{w1} and σ_{w2} for the SST model behave as diffusion constants for the ω -equation which dictates the specific rate of turbulence dissipation from the kinetic energy into thermal energy. For the ω -equation, σ_{w1} dictates the extent in which the inner equation of the SST model influences the solution, and σ_{w2} does the same for the outer $k - \epsilon$ equation. The three uncertainty parameters were chosen such that they contribute at least 90% of the overall variance in the loads from the study done by Erb et al.²⁵ Given the similarity in this work and the Erb et al. study, it is assumed these closure coefficients will have a similar contribution to the overall uncertainty for this work. Previous work by Schafer et al.²⁷ gives lower and upper bounds for each of these uncertain variables in each model which can be seen in Table 10.

Table 10: Uncertainty sources for the CFD models.

Uncertainty Variable	Description	Classification	Lower Bound	Upper Bound
SA Model				
σ	Turbulent Prandtl number	epistemic	0.6	1.0
c_{w2}	Part of g function	epistemic	0.055	0.3525
κ	Von Kármán's constant	epistemic	0.38	0.42
SST Model				
σ_{w1}	coefficient in ω -equation	epistemic	0.3	0.7
σ_{w2}	coefficient in ω -equation	epistemic	0.7	1.0
κ	Von Kármán's constant	epistemic	0.38	0.42

IV. Uncertainty Quantification and Validation Approach

This section outlines the uncertainty quantification and validation approach used in this study. For the current study, both epistemic and aleatory uncertainty are considered. In order to accommodate mixed uncertainties, this work uses a second-order probability analysis approach which was developed by Eldred and Swiler.²⁹ To handle the computational burden of this type of approach, non-intrusive polynomial chaos will be used as an efficient and accurate means of propagating the uncertainty through the computational model. The first section will describe the surrogate-based approach, and the subsequent section will describe the validation methodology that will be used to assess the model discrepancy between the experimental and computational results.

A. Non-intrusive Polynomial Chaos

Polynomial chaos expansion (PCE) is a way of representing a quantity of interest as a function of random variables using a polynomial expansion.³⁰ The polynomial expansion gives a spectral representation of the uncertainty which can represent the response value as a linear combination of orthogonal stochastic or basis functions, Ψ_i , and suitable coefficients, a_i , as seen in Eq. (13).

$$Y(\mathbf{D}, \xi) = \sum_{i=0}^{\infty} a_i(\mathbf{D}) \Psi_i(\xi) \quad (13)$$

Here, $\mathbf{D}$ represents the deterministic variables, and ξ are n independent, standard normal variables. The basis functions, Ψ_i , of each random variable is represented by an orthogonal polynomial of order q which is determined using the Askey key³⁰ and is dependent on the distribution of each variable. For practical applications Eq. (13) must be truncated to a limited number of basis functions, N_t , which leads to Eq. (14).

$$Y(\mathbf{D}, \xi) \approx \sum_{i=0}^{N_t-1} a_i(\mathbf{D}) \Psi_i(\xi) \quad (14)$$

To form a complete basis, the number of terms, N_t , can be determined by Eq (15).

$$N_t = \frac{(n+q)!}{n!q!} \quad (15)$$

Further details on polynomial chaos theory can be seen throughout literature.^{31–34}

In order to obtain a PCE model of the form Eq. (14), the coefficients a_i must be computed. Here, a point-collocation non-intrusive polynomial chaos (NIPC) is used due to the ease of implementation with black-box functions. This method has also been used extensively in aerospace applications for improved computational efficiency over other spectral based approaches.^{35–40} The point-collocation NIPC method starts by computing coefficients using a least squares problem. Given N_t samples of the random variable ξ_k for $k = 1, \dots, N_t$, the corresponding basis functions, $\Psi_i(\xi_k)$, and the stochastic process, $Y(\mathbf{D}, \xi_k)$, Eq. (14) is used to find a suitable linear system which is written in Eq. (16).

$$\begin{pmatrix} Y(\mathbf{D}, \xi_0) \\ \vdots \\ Y(\mathbf{D}, \xi_{N_t}) \end{pmatrix} = \begin{pmatrix} \Psi_0(\xi_0) & \cdots & \Psi_{N_t}(\xi_0) \\ \vdots & \ddots & \vdots \\ \Psi_0(\xi_{N_t}) & \cdots & \Psi_{N_t}(\xi_{N_t}) \end{pmatrix} \begin{pmatrix} a_0 \\ \vdots \\ a_{N_t} \end{pmatrix} \quad (16)$$

For this system, N_t is the minimum number of required samples. If there are more samples, the system in Eq. (16) is over-determined and can be solved as a least squares problem.⁴¹ The number of samples required over the minimum is known as the over sampling ratio (OSR). The number of collocation points are determined by multiplying Eq. (15) by the OSR. For the work here, an OSR of 2 was used to compute a robust solution and to prevent overfitting as shown in Hosder et al.³⁹

B. Validation Approach

There are many forms of how to assess a model for validation as stated by Oberkampf et al.^{42–46} One approach to quantify the difference between the model prediction and experimental prediction is the area metric proposed by Ferson et al.⁴⁷ The area metric, d , is defined by Eq. (17), which is the integral of the absolute value of the difference between the prediction CDF at x , $F(x)$, and the experimental empirical distribution function, EDF, at x , $E(x)$.

$$d(F, E) = \int_{-\infty}^{\infty} |F(x) - E(x)| dx \quad (17)$$

This metric defines a value that is used to represent the discrepancy between theory and empirical evidence. The proposed metric also represents the model-form uncertainty that exists within the model. The formulation in Eq. (17) is only used to handle aleatory uncertainty and cannot account for when both epistemic and aleatory uncertainty exist within the system.

Oberkampff and Ferson⁴⁸ later extended Eq. (17) to include both epistemic and aleatory uncertainty. The mathematical formulation is given in Eq. (18).

$$d(F, E) = \int_{-\infty}^{\infty} \Delta([F_R(x), F_L(x)], [E_R(x), E_L(x)]) dx \quad (18)$$

where

$$\Delta(A, B) = \min_{\substack{a \in A \\ b \in B}} |a - b|$$

Here, L and R subscripts denotes left and right probability box boundaries. This metric results in the non-overlap between the probability boxes and can be visualized in Figure 8. While this metric allows both epistemic and aleatory uncertainty to exist in both the prediction and experiment, it falls short in resolving the true differences between the possible probability distributions contained within the prediction and experiment data. With this approach, the metric d could potentially be independent of the amount of epistemic uncertainty in the measurements, meaning that there are some instances where an increase or decrease in the epistemic uncertainty in the left or right bounds would have no impact on d . For example, in Figure 8, if the measurement had a larger epistemic uncertainty due to a decrease in the left bound, the metric d would not change. Therefore, with this metric, two separate problems can result in the same model-form uncertainty even though one exhibits a larger amount of epistemic uncertainty. To adjust for this, Voyles and Roy introduced a modification that separately tracks where the experimental values are larger than the simulation and those where the experimental values are smaller than the simulation values.⁴⁹ However, this method includes a factor of safety when it describes the model-form uncertainty that can lead to more conservatism than necessary.

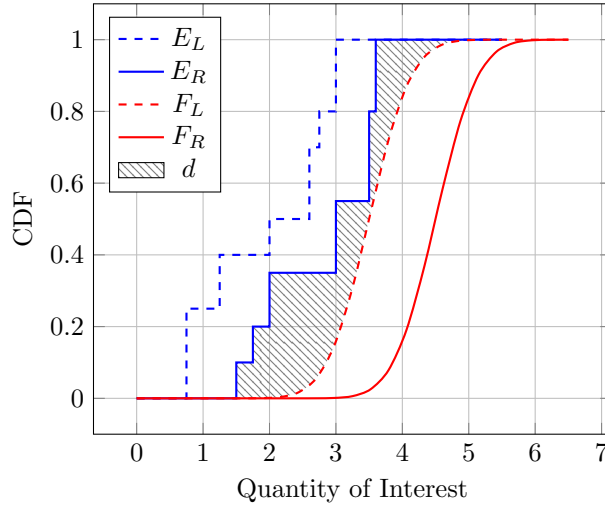


Figure 8: Area metric given by Eq. (18)

To resolve the issues with existing area metrics, the current work uses a new area metric that was proposed by Brune et al.⁴⁰ Keeping in line with previous approaches, the area metrics in Eqs. (17) and (18) can be extended to measure the disagreement between both the upper and lower bounding CDFs of the prediction and measurement without using a factor of safety. This results in two metrics, d_L and d_R , for left and right bounding CDFs, respectively, shown in Eq. (19).

$$d_L = \int_{-\infty}^{\infty} g_L(x) dx, \quad d_R = \int_{-\infty}^{\infty} g_R(x) dx \quad (19)$$

where,

$$g_L(x) = \begin{cases} 0 & \text{for } x \text{ such that } F_L(x) \geq E_L(x) \\ E_L(x) - F_L(x) & \text{for } x \text{ such that } F_L(x) < E_L(x) \end{cases}$$

and

$$g_R(x) = \begin{cases} 0 & \text{for } x \text{ such that } F_R(x) \leq E_R(x) \\ F_R(x) - E_R(x) & \text{for } x \text{ such that } F_R(x) > E_R(x) \end{cases}$$

The resulting metrics d_L and d_R quantify the evidence for disagreement between both the left and right bounding probability distributions while preserving the range of possible distributions from both the prediction and measurements. With this metric, the amount of epistemic uncertainty in the measurements is now accounted for in the metric values. There is still a possibility that one or both values could be zero. A zero value does not imply the two agree, but rather suggests that there is no evidence for disagreement. Stated another way, the validation metric cannot represent a reduction in epistemic uncertainty of the simulation. The model-form uncertainty is independent of the input epistemic uncertainty and, therefore, is not reducible without calibrating the simulation inputs to the same known measurement inputs. An example of Eq. (19) can be seen in Figure 9. For more information regarding how this metric is defined please see Brune et al.⁵⁰

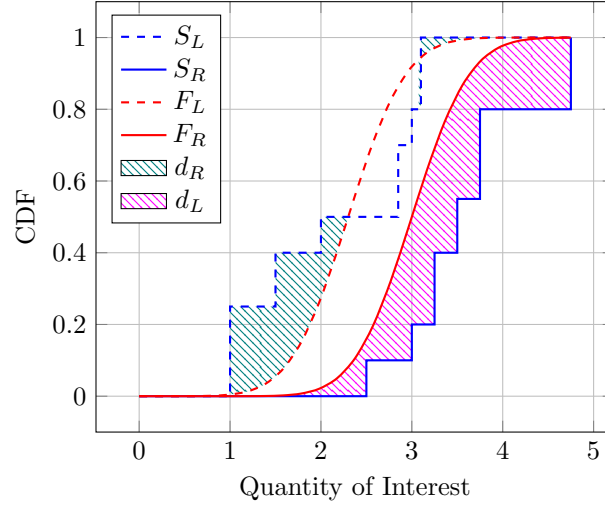


Figure 9: Example of Eq. (19) where predictions have partial overlap with measurements.

V. Results and Discussion

This section shows comparisons between wind tunnel data and CFD data. Then the statistical variations from the three uncertain parameters that were considered in Section III using non-intrusive PCE are presented. Finally, the validation metric developed in Section IV.B is used to quantify the model-form uncertainty of each solver based on its respective statistical variation. Given the generated statistical variation, the validation metric is then used to quantify the model-form uncertainty of each solver.

A. Analysis and Results

A CFD study was conducted at several angles of attack across the wind tunnel angle of attack range to address the different flow behaviors the wind tunnel model experiences in one run. The boundary conditions are based off the nominal wind tunnel test conditions. Given the nominal Reynolds number test condition, the calculated total pressure was used as a boundary condition for the flow solvers. Each viscous solver uses a cold wall boundary condition of 80 °F and assumes a calorically perfect gas model with a ratio of specific heats of 1.4.

For this study, VULCAN-CFD and FUN3D used the same numerical schemes that were described in the grid convergence study in Section III.B.1. Solutions were obtained on the medium grid where it was previously shown to have acceptably small grid error. Cart3D used an adjoint solver for each angle of attack as previously mentioned in Section III.B.2. Because Cart3D is an inviscid solver, a viscous correction term was used for the drag by anchoring Cart3D predicted drag, $C_D^{Cart3D_0}$, to the average of each viscous solver

and model, $C_D^{visc.avg.}$, as shown in Eq. (20).

$$C_D^{Cart3D} = C_D^{Cart3D_0} + |C_D^{Cart3D_0} - C_D^{visc.avg.}| \quad (20)$$

Pressure contours from each solver at an angle of attack of 12° can be viewed in Figure 10. With the viscous correction term included in the drag for Cart3D, all the loads for each angle of attack and solver can be viewed in Table 12 and Figure 11. The trends in Figure 11 show that the solvers consistently under predict the experimental data in the moment coefficient and over predict the drag coefficient. For the lift coefficient the solvers both over and under predict depending on the angle of attack. For an angle of attack of 4° the moment coefficient does not follow the same trend as the experimental data. Future work will investigate what may cause this, as there is currently no clear and definitive explanation.

Table 11: Aerodynamic coefficient results.

α	Model	Flow Solver	C_D	C_L	C_M
-4°	Wind Tunnel		0.1069	-0.1118	-0.0237
	SA Model	FUN3D	0.1173	-0.1197	-0.0205
		VULCAN-CFD	0.1180	-0.1187	-0.0264
	SST Model	FUN3D	0.1187	-0.1177	-0.0258
		VULCAN-CFD	0.1197	-0.1167	-0.0308
	Inviscid Model	Cart3D	0.0675	-0.1150	-0.0314
0°	Wind Tunnel		0.0958	0.0101	-0.0083
	SA Model	FUN3D	0.1003	0.0141	-0.0184
		VULCAN-CFD	0.1086	0.0146	-0.0204
	SST Model	FUN3D	0.1091	0.0156	-0.0205
		VULCAN-CFD	0.1110	0.0145	-0.0201
	Inviscid Model	Cart3D	0.0609	0.0200	-0.0247
4°	Wind Tunnel		0.1172	0.1379	-0.0020
	SA Model	FUN3D	0.1322	0.1558	-0.0265
		VULCAN-CFD	0.1342	0.1543	-0.0232
	SST Model	FUN3D	0.1352	0.1548	-0.0229
		VULCAN-CFD	0.1345	0.1522	-0.0190
	Inviscid Model	Cart3D	0.0827	0.1584	-0.0224
12°	Wind Tunnel		0.2558	0.4547	0.0303
	SA Model	FUN3D	0.2753	0.4705	0.0251
		VULCAN-CFD	0.2701	0.4649	0.0205
	SST Model	FUN3D	0.2706	0.4619	0.0248
		VULCAN-CFD	0.2693	0.4623	0.0229
	Inviscid Model	Cart3D	0.2158	0.4824	0.0151

B. Uncertainty Propagation and Statistical Variation

The second-order expansion surrogate models at an angle of attack of 4° were used to quantify the impact of turbulence model closure coefficient uncertainty in the lift, drag, and moment coefficient. For a second-order surrogate model, with three uncertain parameters in the turbulence model, 30 CFD runs with each solver and turbulence model combination were performed. The uncertainty was then propagated through the surrogate models for each coefficient using Monte Carlo sampling with 10,000 samples that were found to be sufficient to converge the epistemic bounds of the uncertainty.

The uncertainty range for the CFD is epistemic since all the closure coefficients are also epistemic this leads to a min-max interval for each aerodynamic coefficient. The wind tunnel uncertainty interval is the 95% total uncertainty interval discussed in Section II.A.2. The uncertainty range captured from an angle of attack of 4° was used to apply a solver uncertainty range to each angle of attack which was calculated as a percentage of the nominal. Each nominal run was conducted at the wind tunnel test conditions. Figure 12 shows the uncertainty intervals for each data set and includes the grid uncertainties reported in Table 8 for the viscous solvers and Table 9 for Cart3D. Figure 12 indicates agreement between the solvers and experiment for lift coefficient at an angle of attack of 12° for each viscous model, but none of the other angles of attack or solvers have agreement. For the moment coefficient, there is agreement, given the respective uncertainties, between viscous solvers and wind tunnel data at angles of attack of -4° and 12° .

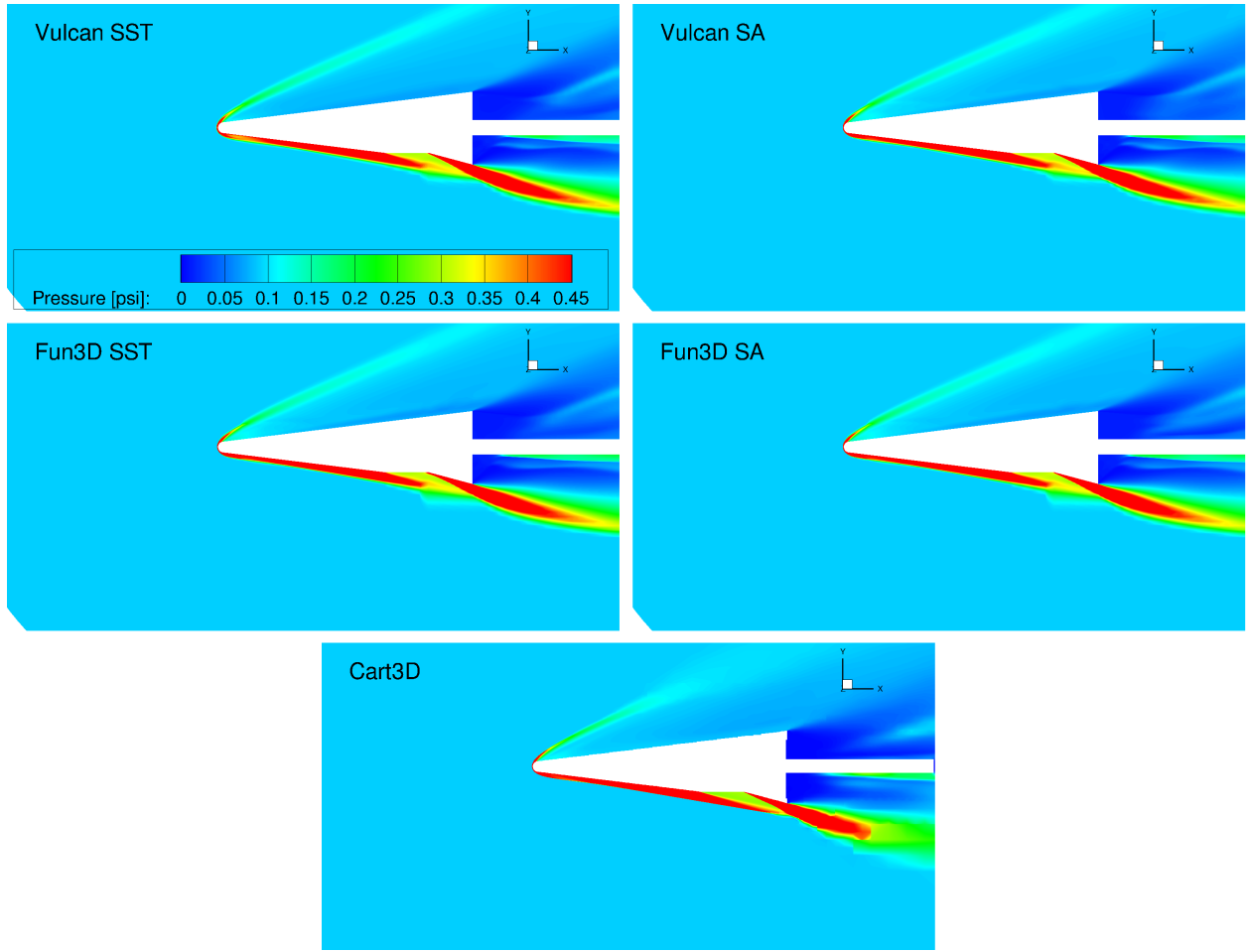


Figure 10: Pressure contours of each solver at $\alpha = 12^\circ$.

The results presented in Figure 13 pictorially shows the validation metric given in Eq. (19) for VULCAN-CFD SA model. Note that in Figure 13, the CFD bounds are vertical lines that indicates the epistemic min-max interval of each aerodynamic coefficient. The nonzero d_L for the VULCAN-CFD SA model for both the drag coefficient and lift coefficient and nonzero d_R for the moment coefficient is represented by the shaded blue shaded area in Figure 13. This model-form uncertainty estimate in the shaded blue region represents that 0.013 and 0.014, respectively should pad the left side of the uncertainty interval of the lift and drag, while 0.01 should pad the right side of the moment uncertainty interval. Table 12 shows the left and right model-form uncertainty for each solver and turbulence model with an angle of attack of 4° solver uncertainty and grid uncertainty included for each angle of attack.

Table 12 reveals that for each solver and turbulence model, the drag and lift coefficients have a right model-form uncertainty, d_R , of zero except at an angle of attack of 4° and for the moment coefficient the left model-form uncertainty, d_L , is zero. Other results from Table 12 show that the largest discrepancy between the models and experiment occur at an angle of attack of 4° . One possible explanation for this angle of attack having greater uncertainty is that the current models are not accurately capturing the shock wave boundary layer interaction around the slice and wedge for this case. Further investigation into potential reasons should be conducted in the future. In general, the largest model-form uncertainty for the lift coefficients accounts for up to 47% variation in the CFD model. The largest model-form uncertainty for the moment coefficient accounts for up to 86% variation in the CFD model. For the drag coefficient, the largest model-form uncertainty accounts for up to 11% variation in the CFD model. With the model-form uncertainty added to each angle of attack, the error bars from Figure 12 can be adjusted and are shown in Figure 14. Given the added model-form uncertainty to the CFD data, each one of the angles of attack nearly

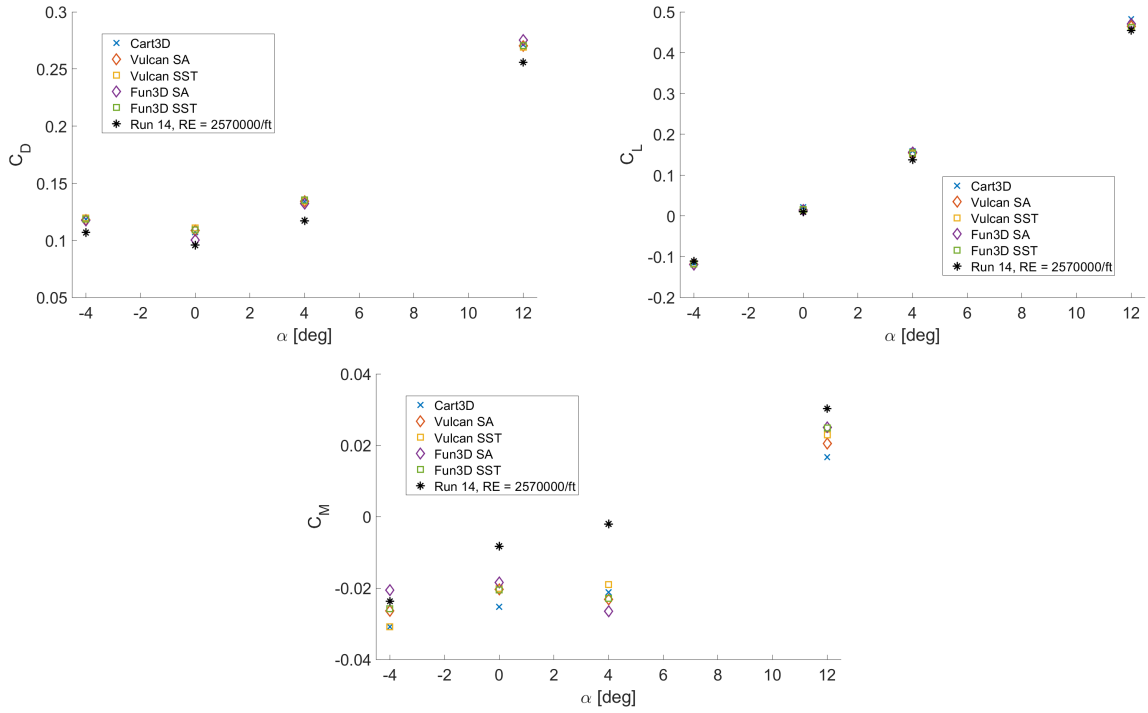


Figure 11: Overall comparison of each data sets.

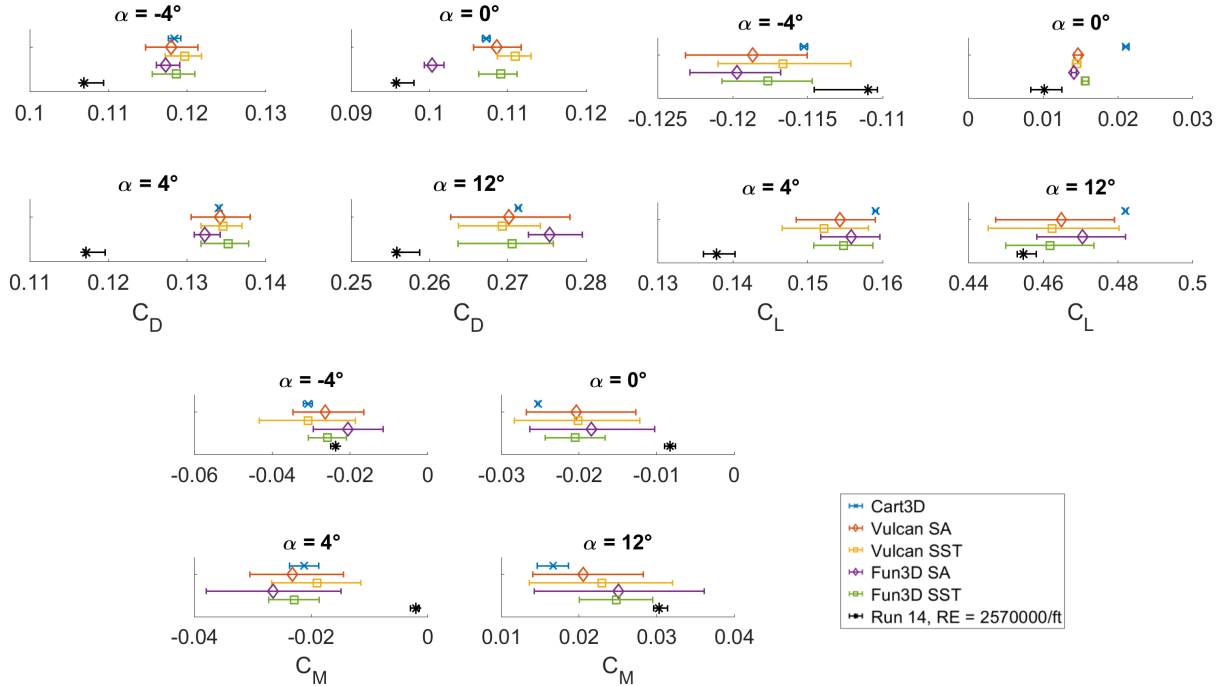


Figure 12: Comparison of solvers with total uncertainty included in all data sets.

captures the experimental data. Therefore, there is now agreement between the solvers and experiments for each angle of attack when all uncertainties are included in the models.

This work shows that there are deficiencies in the aerodynamic modeling capability. To improve the aerodynamic predictions for this hypersonic design, one direction would be to run simulations using a higher fidelity model, such as a detached eddy simulation or DES. This type of model uses a hybrid approach

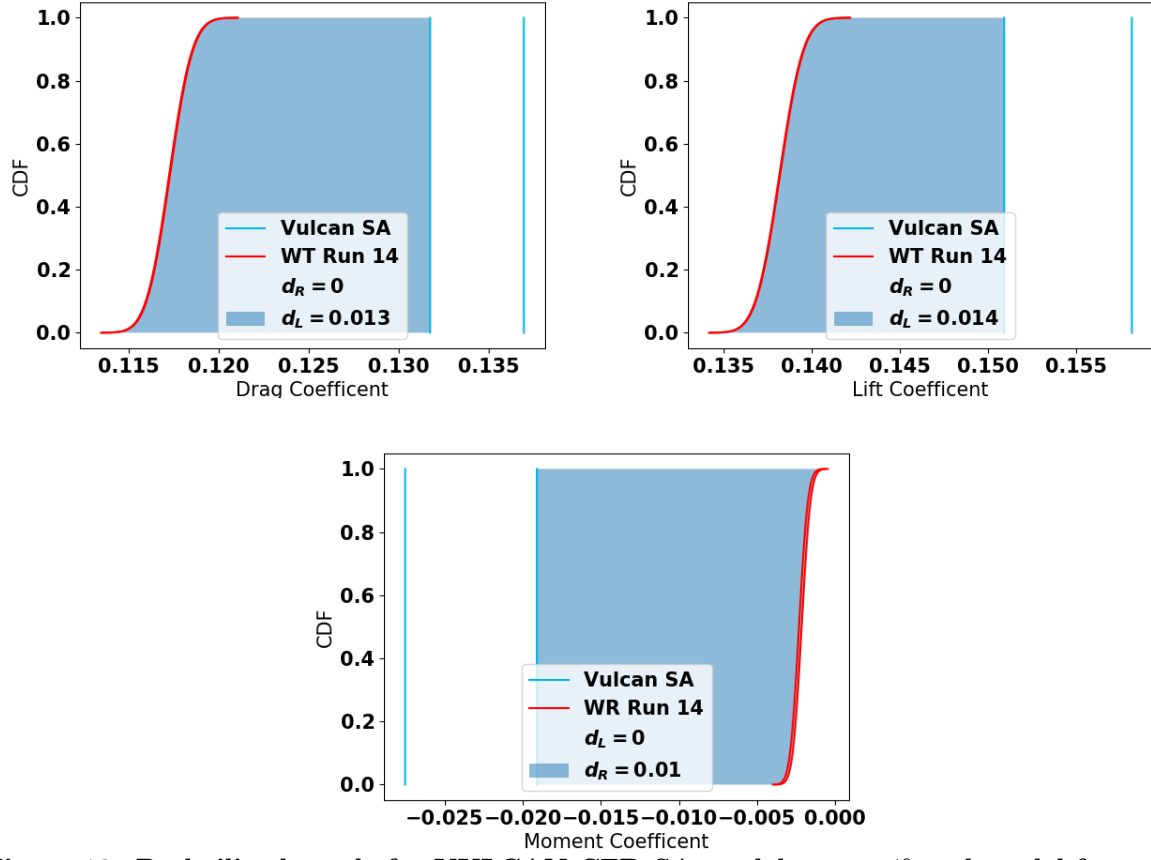


Figure 13: Probaility bounds for VULCAN-CFD SA model at $\alpha = 4^\circ$ and model-form uncertainty using Eq. (19)

Table 12: Model-form uncertainty results from Eq. (19)

α	Model	Flow Solver	C_D	C_L	C_M
-4°	SA Model	FUN3D	$d_L = 0.009, d_R = 0$	$d_L = 0, d_R = 0.005$	$d_L = 0, d_R = 0$
		VULCAN-CFD	$d_L = 0.008, d_R = 0$	$d_L = 0, d_R = 0.001$	$d_L = 0, d_R = 0$
	SST Model	FUN3D	$d_L = 0.008, d_R = 0$	$d_L = 0, d_R = 0.002$	$d_L = 0, d_R = 0$
		VULCAN-CFD	$d_L = 0.010, d_R = 0$	$d_L = 0, d_R = 0.004$	$d_L = 0, d_R = 0$
	Inviscid Model	Cart3D	$d_L = 0.011, d_R = 0$	$d_L = 0.004, d_R = 0.002$	$d_L = 0, d_R = 0.003$
0°	SA Model	FUN3D	$d_L = 0.001, d_R = 0$	$d_L = 0.005, d_R = 0$	$d_L = 0, d_R = 0.002$
		VULCAN-CFD	$d_L = 0.010, d_R = 0$	$d_L = 0.002, d_R = 0$	$d_L = 0, d_R = 0.004$
	SST Model	FUN3D	$d_L = 0.01, d_R = 0$	$d_L = 0.005, d_R = 0$	$d_L = 0, d_R = 0.008$
		VULCAN-CFD	$d_L = 0.013, d_R = 0$	$d_L = 0.004, d_R = 0$	$d_L = 0, d_R = 0.004$
	Inviscid Model	Cart3D	$d_L = 0.011, d_R = 0$	$d_L = 0.010, d_R = 0$	$d_L = 0, d_R = 0.013$
4°	SA Model	FUN3D	$d_L = 0.014, d_R = 0$	$d_L = 0.014, d_R = 0$	$d_L = 0, d_R = 0.013$
		VULCAN-CFD	$d_R = 0.013, d_L = 0$	$d_R = 0.010, d_L = 0$	$d_R = 0, d_L = 0.012$
	SST Model	FUN3D	$d_L = 0.014, d_R = 0$	$d_L = 0.013, d_R = 0$	$d_L = 0, d_R = 0.016$
		VULCAN-CFD	$d_L = 0.014, d_R = 0$	$d_L = 0.008, d_R = 0$	$d_L = 0, d_R = 0.009$
	Inviscid Model	Cart3D	$d_L = 0.015, d_R = 0$	$d_L = 0.020, d_R = 0$	$d_L = 0, d_R = 0.017$
12°	SA Model	FUN3D	$d_L = 0.016, d_R = 0$	$d_L = 0.001, d_R = 0$	$d_L = 0, d_R = 0$
		VULCAN-CFD	$d_L = 0.006, d_R = 0$	$d_L = 0, d_R = 0$	$d_L = 0, d_R = 0.002$
	SST Model	FUN3D	$d_L = 0.007, d_R = 0$	$d_L = 0, d_R = 0$	$d_L = 0, d_R = 0.009$
		VULCAN-CFD	$d_L = 0.007, d_R = 0$	$d_L = 0, d_R = 0$	$d_L = 0, d_R = 0$
	Inviscid Model	Cart3D	$d_L = 0.014, d_R = 0$	$d_L = 0.026, d_R = 0$	$d_L = 0, d_R = 0.013$

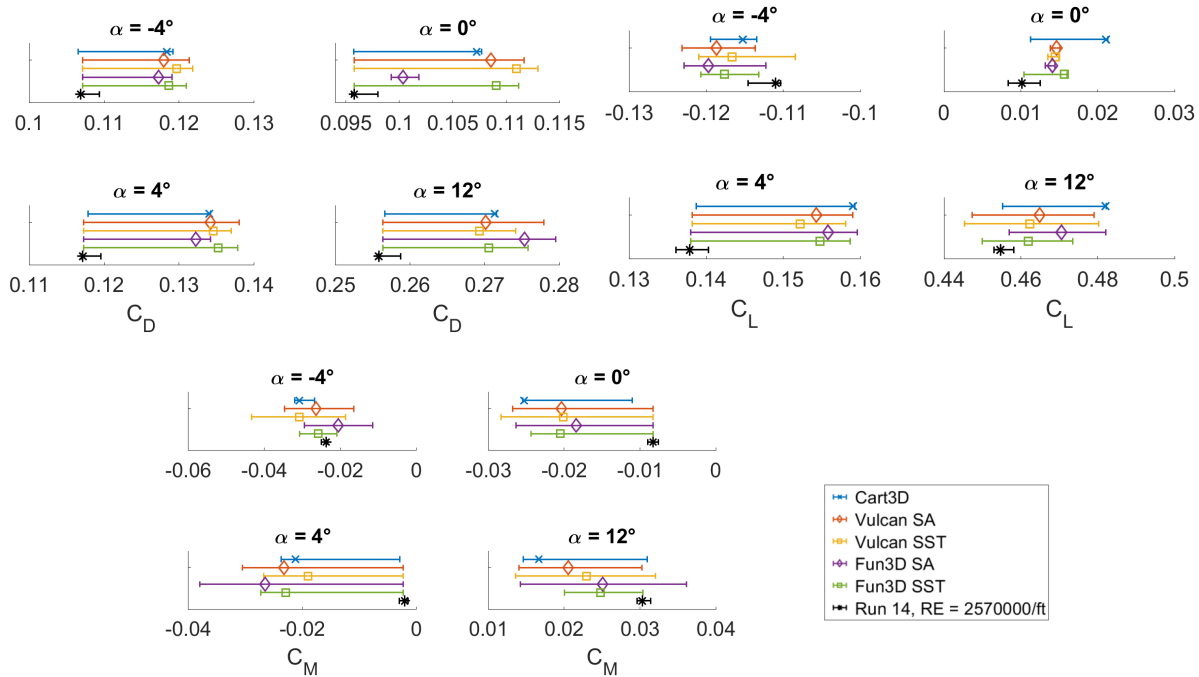


Figure 14: Load comparisons with model-form uncertainties included in the CFD data.

between RANS and a large eddy simulation or LES. The hybrid approach may be able to capture more of the turbulent effects that are occurring in the slice-flap attachment region of the model, as well as provide better resolution of the wake. However, this approach requires substantially more computational time and the improvement for integrated forces may be small. Experimental improvements could also be made which may lower the model-form uncertainty currently being estimated. For the experimental work, a more rigorous uncertainty study, which uses the test's facilities data reduction for the coefficients, would more adequately capture the experimental uncertainty. Also, having more repeat runs to assess the AMS package uncertainty range since currently only one run occurred to quantify that uncertainty. More accurately capturing the experimental uncertainty gives a better idea of the true uncertainty in the experimental data.

VI. Conclusions

An uncertainty analysis in the lift, drag, and moment coefficients for a cone-slice-flap hypersonic geometry was presented for wind tunnel conditions of a test performed in the 20-inch Mach 6 wind tunnel at NASA Langley Research Center. The analysis of this study accounts for systematic and random error uncertainty in the measurement and epistemic uncertainties in the computational fluid dynamic model used to predict the aerodynamic coefficients of the cone-slice-flap geometry. The experimental measurement data for a set of four angles of attack were compared to the prediction data to demonstrate a need to assess uncertainties and determine the statistical agreement in the predictions and experimental data. Uncertainty sources were identified and discussed for both the test and prediction data based on available references. Uncertainty classification and ranges were identified and used to propagate statistical uncertainty in the aerodynamic coefficients.

A full study was done to assess what the systematic and random error was in the measurement data. The uncertainty parameters were identified given the known ranges in the manufacturing tolerances on the test equipment for each uncertain parameter. For the measurement data, the perfect gas and isentropic flow relations are used to propagate the uncertainties through a Monte Carlo analysis of 10,000 samples. A sensitivity study was also conducted to gain an understanding of which uncertain parameter have the most influence over the statistical information of the aerodynamic coefficients. The random error uncertainty was captured by using an unbiased estimator of the standard deviation to handle the low sample size in repeated

runs. Finally, a methodology was presented on how the random and systematic errors are combined to create the overall uncertainty for the test data.

To limit the expense of computational fluid dynamic models used in this study, a surrogate-based approach using non-intrusive polynomial chaos was applied to construct a response surface for the aerodynamic coefficients at one angle of attack with three uncertain parameters. This uncertainty was used as a solver uncertainty for all the angles of attack considered in this study. The solver uncertainty was combined with the grid uncertainty from a grid convergence study to get an overall uncertainty estimate for each aerodynamic coefficient at four angles of attack.

A validation metric was introduced to describe the model-form uncertainty of each solver. The model-form uncertainty results reveal that each aerodynamic coefficient has discrepancy from the experimental data on the magnitude of at most 0.01 or larger which equate to approximately an 86% error for the aerodynamic coefficients. The comparisons between the multiple solvers and turbulence models show how these models differ when analyzing a cone-slice-flap geometry at these conditions.

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