

# Uncertainty Quantification of Expected Time-of-Arrival in UAV Flight Trajectory

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One of the foremost requirements for accurate in-flight safety monitoring of autonomous unmanned aerial vehicles (UAVs) is tracking of their flight trajectory. Existing UAVs leverage autonomous flight functionalities based on trajectory generation algorithms developed in robotic applications such as polynomial or spline curves in order to facilitate kinematic smoothness, minimum vibrations and fuel efficiency. However in practice, the actual path may be subjected to unexpected local weather conditions, unexpected obstacles along the path or abrupt traffic changes in the low-altitude airspace resulting in large errors of the predicted time-of-arrival at way-points. In this study, an approach to quantify and propagate uncertainty in 4D trajectories is proposed. The paper presents a simple error interval propagation method based on the expected cruise speed of the UAV and its associated uncertainty. The uncertainty is then propagated in time to estimate reasonable confidence intervals on the times-of-arrival of the vehicle at each way-point as well as along the entire flight-path. The uncertainty propagation is demonstrated on a state-of-the-art trajectory generation algorithm based on non-uniform rational B-spline (NURBS) curves. Further, the effect of a stationary wind field is incorporated in the uncertainty propagation approach. The proposed method is implemented on synthetic and real data obtained from flight experiments with a small UAV.

## I. Introduction

The use of small unmanned aerial vehicles (sUAVs) in the low-altitude airspace is rapidly evolving, and the density of UAV operations is expected to grow at a fast pace in the next decade. A novel set of tasks like public safety monitoring and surveillance, delivery of goods and medical supplies, weather monitoring, and precision agriculture performed by small UAVs may drastically change the future of aerospace [1, 2]. New concepts for unmanned traffic management are currently being developed [1], but the future airspace and expected operation density will require holistic methods for in-time safety assessment of the airspace and health monitoring of the vehicles operating within.

Safety monitoring frameworks similar to the ones developed for manned aircraft [3–5] may enhance unmanned traffic management with in-time monitoring and prediction tools for low-altitude operating vehicles. However, the challenges

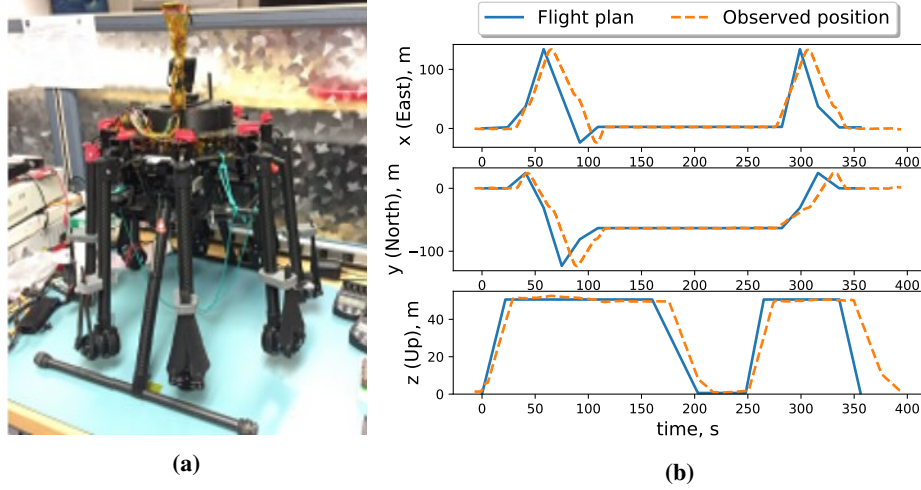
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involving monitoring and prediction capabilities of low-altitude airspace and autonomous sUAVs are substantially different when compared to that of traditional aviation. Failure behavior of sub-systems in UAVs and their effects during operations are different than what has been seen in traditional manned aviation [6]. Further, sUAVs have intrinsic size and cost limitations due to which they cannot carry heavy or expensive on-board sensors, computing systems or hardware for large-bandwidth communications with the ground station, thus limiting the ability to perform self-diagnosis and prognosis in real-time flight operations. For those same reasons, batteries installed in such vehicles are relatively small with limited redundancy and any discrepancy in the discharge rate during an autonomous flight can significantly affect the remaining state-of-charge in the battery.

One of the critical elements for UAV flight safety monitoring requires accurate in-time prediction of its trajectory as subsequent analyses of vehicle health and safety, such as prediction of the battery's end of discharge or the vehicle's proximity to obstacles are extensively dependent on the vehicles' predicted location. Trajectory prediction methods, including estimates of the time of arrival at each way-point and along the entire trajectory, will also benefit UAV flight schedule optimization, risk assessment [7, 8] as well as unmanned traffic management. Unlike manned aircraft, light-weight UAVs are exposed to strong external disturbances such as wind gusts and turbulence affecting their planned trajectory more frequently than larger aircrafts. Additionally, since sUAVs are expected to fly at lower altitudes (below 400*ft*), they are expected to be subjected to wind tunnel effects caused by buildings and other obstacles in urban environment. Modeling trajectories of flights in urban canyon is not only challenging but inherently associated with high uncertainties posed from external and internal sources. Uncertainty in external variables or environmental conditions (e.g., an un-modeled wind field) or un-modeled autopilot controller characteristics generates large fluctuations of the vehicle's cruise speed which eventually leads to deviated times-of-arrival (TA). Such deviations from the ETA may disrupt the optimal vehicle operation plan or the fleet operations and even require trajectory re-plans in high-density flight areas to minimize the chance of trajectory conflicts and the need to trigger mid-air collision avoidance systems. As an example, figure 1 (b) shows the flight plan executed with a *DJI S1000* Octocopter in autonomous mode. The vehicle, as depicted in figure 1 (a), was equipped with Pixhawk autopilot hardware (<http://pixhawk.org/>) and commanded with Ardupilot software (<http://ardupilot.org/>). The figure shows the vehicle location "unrolled in time," calculated by the autopilot using GPS locations and its internal measurement unit. The geospatial coordinates (latitude, longitude, and altitude) have been converted into a local, Earth-fixed reference frame with origin at the take-off location, x-axis pointing East, y-axis pointing North, and z-axis pointing upwards, forming a East-North-Up (ENU) reference frame. Even in this short flight (approximately 6 minutes), the times of arrival (TA) at the way-points is substantially different from the estimated times of arrival, (ETAs). The relative TA error at each way-point was between 1% and 50%, and such error is expected to grow on longer flights. Delay or discrepancy in TAs can disrupt the vehicle and airspace operations especially in case of high-density flight areas as expected in future urban environments. Besides, flight scheduling for UAVs involved in timely deliveries of packages or medical supplies may be strongly affected by uncertainty in



**Fig. 1 Example of DJIS1000 octocopter (1a) test flight in autonomous mode: the observed position is compared against the flight plan in a East-North-Up reference frame (1b).**

predicted trajectory. This propels the need for generating reasonable uncertainty bounds within safety monitoring frameworks for both scheduling as well as in-time safety monitoring of UAV flights. Nonetheless, the quantification of precise uncertainty bounds affecting a 4D trajectory is extremely complex, and would require knowledge of a number of parameters making the task unfeasible. For example, precise wind conditions along the flight path are hardly available in most areas. Similarly, knowledge of the vehicle dynamic properties over the entire flight envelope including precise aerodynamic coefficients, inertia properties, type of controller and its parameters are information available only to the vehicle manufacturer.

Several previous research have focused on aircraft trajectory optimization [9, 10], tracking [11] and probabilistic prediction [12, 13] to enable conflict detection under sense-and-avoid and conformance monitoring in air traffic control [14]. Further in [15] the location of an aircraft in the airspace was defined using probability field functions in which speed variations were modeled by random walks around the average speed, therefore requiring sampling-based simulations to compute the confidence intervals of the aircraft position at each time step. A similar conflict-avoidance problem was addressed in [16], also using speed uncertainty models. A probabilistic model for aircraft flight time and fuel consumption under wind uncertainty was proposed in [17], where the authors used ensembles of weather forecasts to incorporate wind effects in the aircraft ground speed. An adaptive algorithm for prediction of aircraft climb trajectory under uncertainty was presented in [18].

However, all cited papers focus mainly on manned aircraft. A few studies pertaining to unmanned vehicles include [19] that presents a comparison of five path-following algorithm to control a UAV under different wind conditions and [20] in which the authors proposes a motion planning technique for an unmanned helicopter subjected to winds. Besides, Bayesian filtering techniques has been implemented to track UAV flight trajectory [21], yet estimation and

propagation of uncertainty in predicted trajectory caused due to speed variations has mostly remained unexplored for unmanned vehicles.

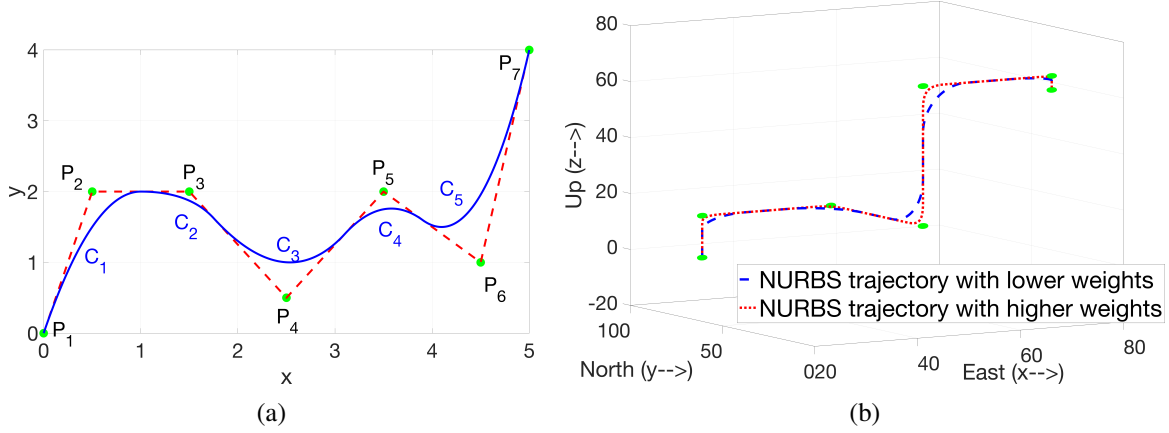
Given the need for in-flight monitoring capabilities, this paper proposes a simple method to estimate the uncertainty affecting flight-plan ETAs of small UAVs flying in autonomous mode. A state-of-the-art algorithm based on B-spline curves is utilized as base-line trajectory generation algorithm to present the case studies. The uncertainty quantification method enables the estimation of the confidence intervals of the position profile along the trajectory that the UAV is supposed to execute. The effects of uncertainty sources are condensed into random variations of the desired cruise speed, described using Gaussian independent random variables, and is propagated using error intervals. Contrary to other solutions proposed in literature, the method does not require simulations nor sampling-based techniques, thus being computationally efficient. Its simplicity enables drawing reasonable confidence intervals around the ETA of the flight plan based on the expected or measured uncertainty of the vehicle speed. It is vehicle agnostic and can be easily embedded in larger frameworks for UAV or UAM path planning, optimization and scheduling in presence of uncertainty. Its simplicity enables drawing reasonable confidence intervals around the ETA of the flight plan based on the expected, or measured, uncertainty of the vehicle speed. The trajectory generation algorithm is briefly presented in Section II, while the uncertainty quantification approach is presented in Section III. Section IV uses both simulated and real flight data to show the methodology, its advantages and drawbacks. Section V draws some conclusions on the presented work and discusses future research on the topic.

## II. Generating kinematic profile using non-uniform rational B-spline algorithm

The first step of the UAV flight simulation framework is trajectory generation which denotes the process of computing the position, velocity and acceleration profiles of the UAV based on the planned way-points and associated ETAs. The trajectory generation method utilized in this paper is based on Non-uniform rational B-splines (NURBS), which are parametric composite curves used in previous robotic applications owing to their desirable properties of convex hull and maintaining continuity up to the  $k - 1$  derivative for a curve of degree  $k$  [22, 23]. The resultant NURBS is a curve that passes close to each way-point such that the corresponding velocity and acceleration profile remains continuous and therefore representative of a real flight path. Further, clamped B-spline curves [23] ensures that the trajectory passes exactly through the first and last way-points to ensure fixed take-off and landing positions.

Given a set of  $n + 1$  way-points, a NURBS curve of degree  $k$  is defined as a piecewise polynomial curve  $P(u)$  described by parameters  $u \in \mathbb{R}_{\geq 0} : 0 \leq u \leq n - k + 2$ , defined according to Eq. (1). The resultant curve  $P(u)$  is composed of  $n - k + 1$  sections where each section of the curve  $\{[0, 1], [1, 2], \dots, [(n - k + 1), (n - k + 2)]\}$  is of degree  $k$ . Figure 2 (a) depicts a typical NURBS curve generated from 7 ( $n = 6$ ) waypoints with desired degree of  $k = 3$ , thereby comprising  $7 - 3 + 1 = 5$  sections denoted by  $\{C_1, C_2, C_3, C_4, C_5\}$ .

A segment of the NURBS curve consists of a weighted contribution from each waypoint defined according to a



**Fig. 2** (a) Schematic of NURBS trajectory  $P(u)$  from example way-points  $P_1 - P_7$  (b) Example of two trajectories obtained by varying relative weights in NURBS algorithm.

basis function  $N_{i,k}(u)$  which is computed for  $i^{th}$  way-point and  $k^{th}$  degree. The NURBS basis function is described in equations (2a) and (2b).

$$P(u) = \frac{\sum_{i=0}^n h_i N_{i,k}(u) P_i}{\sum_{i=0}^n h_i N_{i,k}(u)}, \quad 0 \leq u \leq n - k + 2 \quad (1)$$

$$N_{i,k}(u) = \frac{(u - t_i)N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}} \quad (2a)$$

$$N_{i,1}(u) = \begin{cases} 1 & \text{if } t_i \leq u \leq t_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (2b)$$

The term  $t$ , known as the knot vector, is defined in Eq. (3).

$$t_i = \begin{cases} 0 & \text{if } i \leq k \\ i - k + 1 & \text{if } k \leq i \leq n \\ n - k + 2 & \text{if } i \geq n \end{cases} \quad (3)$$

It identifies the waypoints that contribute to each segment of the NURBS curve via the basis function. For example, in figure 2 (a), the degree  $k$  is set to 3, hence waypoints  $P_1$ ,  $P_2$ , and  $P_3$  define the curve segment  $C_1$ , waypoints  $P_2$ ,  $P_3$  and  $P_4$  define the curve segment  $C_2$  and the trend continues for the remaining segments  $C_3 - C_5$ . Finally, the weight  $h_i$  associated with each way-point controls the distance of the curve from  $i^{th}$  way-point.

One of the other advantages of using NURBS is that no in-flight parameters are required for generating the trajectory. Both the position and velocity profile of planned trajectory can be obtained using the global locations and expected time

of arrival (ETA) at the way-points, which makes this approach applicable for multiple UAVs and fleet operations.

The existing NURBS algorithm dictates that the distance of the resultant curve from each way-point depends not on its absolute weight ( $h_i$ ) but on its weight relative to the surrounding way-points in the flight path. Hence, pseudo-way-points associated with lower weights are added in between every two consecutive true way-points within the NURBS algorithm [24]. Assigning higher weights to the true waypoints generates a smooth trajectory which passes through the waypoints. The smoothness of the trajectory as well as distance of curve from each way-point can be adjusted by regulating the weights of the true way-points relative to that of the pseudo way-points. By varying the weights, the shape of the resultant curve can be changed, as shown in figure 2 (b).

### III. Uncertainty affecting times-of-arrival

Several factors affect a UAV flight trajectory, introducing uncertainty in its position and velocity along the flight path and TA at specific locations, as specified in Section I. The following assumptions were considered in this work:

- 1) External disturbances (wind gusts) or unmodeled controller characteristics generate variations in expected speed
- 2) Mission profile is characterized by cruise flight and minimal speed variations
- 3) Uncertainty in the cruise speed is "low" compared to the absolute value of the commanded cruise speed.

Firstly, external factors and unmodeled controller characteristics are assumed to generate variations on the vehicle expected speed which produces uncertainty in the TAs at all locations throughout the entire flight. Secondly, the vehicle acceleration along with the associated uncertainty has been assumed negligible, thus expecting a mission profile characterized by cruise flight and minimal speed variations. Finally, the third assumption is imperative for implementation of uncertainty propagation via error intervals.

This section uses the following notation. Lower-case letters  $x$  define scalar quantities, bold lower-case letters  $\mathbf{x}$  define vectors and bold upper-case letters  $\mathbf{X}$  define matrices. The symbol  $\bar{x}$  indicates expectation of the variable  $x$  and  $\delta x$  indicates the error interval of  $x$ . Standard deviation and variance of normal distributions are described by  $\sigma_x$  and  $\sigma_x^2$ , respectively, while the covariance matrix is defined by  $\Sigma_x$  (the bold symbol is omitted when defining indices instead of random variables). The differential operator is defined as  $\Delta x_i = x_i - x_{i-1}$ . With an abuse of notation, random variables and their realizations are described by lower-case letters.

#### A. Uncertainty representation and quantification

Without loss of generality, uncertainty is represented in a local East-North-Up (ENU) reference frame consisting of  $x$ ,  $y$  and  $z$  dimensions. For example, the position of way-point  $k$ ,  $k = \{0, 1, \dots, K\}$ , is defined as  $\mathbf{p}_k = [p_{x,k}, p_{y,k}, p_{z,k}]^T \in \mathbb{R}^{3 \times 1}$ , with the take off location being  $\mathbf{p}_0 = [p_{x,0}, p_{y,0}, p_{z,0}]^T = [0, 0, 0]^T$ . For more information about ENU coordinate systems and conversion from geo spatial coordinates (i.e., latitude, longitude, altitude) to ENU, the reader is referred to [24]. With the assumption of negligible acceleration by the vehicle, the kinematic equations of the UAV in between

way-points can be described by:

$$\Delta \mathbf{p}_k \approx \mathbf{v}_k \Delta t_{a,k} \quad , \quad (4)$$

where  $\Delta \mathbf{p}_k = \mathbf{p}_k - \mathbf{p}_{k-1} \in \mathbb{R}^{3 \times 1}$  is the vector distance between way-point  $k$  and way-point  $k-1$ ,  $\Delta t_{a,k} = t_{a,k} - t_{a,k-1} \in \mathbb{R}_{>0}$  is the difference between the TA at way-point  $k$  and way-point  $k-1$ , and  $\mathbf{v}_k = [v_{x,k}, v_{y,k}, v_{z,k}]^T \in \mathbb{R}^{3 \times 1}$  is the corresponding cruise speed in between way-point  $k$  and way-point  $k-1$ . Equation (4) is an approximate relationship, which is valid by assuming piece-wise constant velocities in between two consecutive way-points. The differential TA at each way-point can thus be calculated by inverting Eq. (4) and computing the norm of the velocity vector  $\|\mathbf{v}_k\|_2$ .

$$\Delta t_{a,k} \approx \frac{\|\Delta \mathbf{p}_k\|_2}{\|\mathbf{v}_k\|_2} \quad (5)$$

The velocity profile of the UAV is modeled by either uniform or normal distribution with intervals  $\delta \mathbf{v}_k = [\delta v_{x,k}, \delta v_{y,k}, \delta v_{z,k}]^T \in \mathbb{R}_{\geq 0}^{3 \times 1}$  or covariance  $\Sigma_v = \text{diag}([\sigma_{v,x}^2, \sigma_{v,y}^2, \sigma_{v,z}^2]) \in \mathbb{R}^{3 \times 3}$ . The errors affecting the cruise speed along the three directions are assumed to be independent and identically distributed.

## B. Uncertainty propagation via error intervals

The error propagation rule utilized in this work grounds on the algebra of random variables, and can be defined as follows. Given the quantity  $q$  defined by the fraction:

$$q = \frac{a b c \dots}{x y z \dots} \quad , \quad (6)$$

and the error of the independent variables  $a, b, \dots, z$  equal to  $\delta a, \delta b, \dots, \delta z$ , then:

$$\frac{\delta q}{|q|} \approx \left[ \left( \frac{\delta a}{a} \right)^2 + \left( \frac{\delta b}{b} \right)^2 + \left( \frac{\delta c}{c} \right)^2 + \dots + \left( \frac{\delta x}{x} \right)^2 + \left( \frac{\delta y}{y} \right)^2 + \left( \frac{\delta z}{z} \right)^2 + \dots \right]^{1/2} \quad , \quad (7)$$

showing that fractional uncertainties add in quadrature, as described in [25]. Similarly, the relationship holds for Gaussian, independent random variables:

$$\frac{\sigma_q^2}{q^2} \approx \left( \frac{\sigma_a}{a} \right)^2 + \left( \frac{\sigma_b}{b} \right)^2 + \left( \frac{\sigma_c}{c} \right)^2 + \dots + \left( \frac{\sigma_x}{x} \right)^2 + \left( \frac{\sigma_y}{y} \right)^2 + \left( \frac{\sigma_z}{z} \right)^2 + \dots \quad . \quad (8)$$

Equations (7) and (8) come from the approximation of power laws:  $\sigma_{\ln x} \approx \sigma_x / \bar{x}$ , which is valid for uncorrelated input variables and small variances or errors (i.e.,  $\sigma_{XY} = 0 \forall X, Y$ ;  $\sigma_X^2 \ll \bar{x}$ , and  $\delta x \ll \bar{x}$ ). In this work, the quantity of interest is TA, therefore  $q \rightarrow t_a$ .

### C. Confidence intervals of times of arrival at way-points

Assuming that the cruise speed is uniformly distributed with intervals  $\delta \mathbf{v}$  or normally-distributed with variances  $\delta \mathbf{v}$ , and that the position of the way-point  $k$  is a deterministic variable, the uncertainty in the times of arrival at way-point  $k$  can be described as Eq. (9).

$$\delta t_k \triangleq \Delta t_{a,k} \frac{\delta \|\mathbf{v}_k\|_2}{\|\mathbf{v}_k\|_2} \quad (9)$$

It should be noticed that if Eq. (9) is applied to all way-points, singularities may result from the computation in specific segments of the flight, where the vehicle is not moving in a particular direction  $j \in \{x, y, z\}$ . Those same singularities can still cause numerical problems when the uncertainty of the cruise velocity is not much smaller than the cruise velocity itself, since the error interval propagation method works under the assumption that  $\sigma_x \ll x$ . In those segments, the expected velocity in between the way-points, i.e.,  $\bar{v}_x$ ,  $\bar{v}_y$  or  $\bar{v}_z$ , would be zero or close to zero, therefore generating  $\delta t \rightarrow \infty$ . We therefore apply some conditions to Eq. (9), to ensure the error on TA does not diverge, by enforcing that direction  $j$ , in which the UAV is not moving, does not generate uncertainty in the TA. Equation (9) is therefore enhanced by Eqs. (10) to avoid singularity in its solution.

$$\delta t_k \triangleq \begin{cases} \Delta t_{a,k} \frac{\delta \|\mathbf{v}_k\|_2}{\|\mathbf{v}_k\|_2} & \forall \quad \delta \mathbf{v}_k, \mathbf{v}_k : \delta \|\mathbf{v}_k\|_2 / \|\mathbf{v}_k\|_2 < \delta \tilde{v}_0 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Equation (10) highlights that the ratio between the norm of the speed uncertainty along the three direction  $\delta \|\mathbf{v}\|_2$  should be larger than a minimum threshold  $\delta \tilde{v}_0$  to contribute to uncertainty. Once Eq. (10) is applied to all way-points, the flight plan information is intrinsically enhanced by adding the expected error on the TA at every way-point,  $\delta t_a$ , by computing the cumulative sum of all  $\delta t_i, \forall i \leq k$ ,

$$\delta t_{a,k} = \sum_{i=1}^k \delta t_i \quad , \quad (11)$$

By estimating  $\delta t_{a,k}$  using Eq. (11), it is possible to compute the lower and upper limit of the ETA of all way-points. However, the output distribution of the ETA, that is the probability distribution of  $t_{a,k}$ , will not follow any known distribution, since it is the sum of Uniformly-distributed random variables  $\delta t_i$ .

On the other hand, by assuming Normally-distributed error, it is possible to model the arrival time at each way-point as a normal random variable as stated in Eq.(8), since the sum of uncorrelated Gaussian random variables yields a Gaussian random variable.

Hence, Eqs. (10) can be substituted by Eqs. (12) and the variance of the time of arrival at each way-point  $k$  is

computed by cumulative sum of variances up to  $\sigma_{t_k}^2$ , as in Eq. (13).

$$\sigma_{t_k}^2 \triangleq \begin{cases} \Delta t_{a,k} \frac{\|\sigma_{\mathbf{v}_k}^2\|_2}{\|\mathbf{v}_k\|_2} & \forall \sigma_{\mathbf{v}_k}^2, \mathbf{v}_k : \|\sigma_{\mathbf{v}_k}^2\|_2 / \|\mathbf{v}_k\|_2 < \delta \tilde{v}_0 \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

$$\sigma_{t_{a,k}}^2 = \sum_{i=1}^k \sigma_{t_i}^2 \quad . \quad (13)$$

Eventually, the time of arrival at way-point  $k$  can be described by a normal probability density function (pdf):

$$t_{a,k} \sim \mathcal{N}(\bar{t}_{a,k}, \sigma_{t_{a,k}}^2) \quad . \quad (14)$$

For simplicity, all results presented afterwards have been obtained by setting the minimum threshold to introduce uncertainty to 10%, which means that  $\delta \tilde{v}_0 = 0.1$ .

#### D. Correction for wind effects

The methodology can be easily adjusted to deal with airspeed and wind speed uncertainty, under the assumption of stationary wind field. Thus, the equations presented here hold for  $\partial w(x, y, z, t) / \partial t \approx 0$ . The error interval on the arrival time defined by (10) and (12) can be modified by simply substituting  $\delta v_{a,j,k} + \delta w_{j,k}$ ,  $\forall j = \{x, y, z\}$  to  $\delta v_{j,k}$  for the Uniform pdf case and  $\sigma_{v_{a,j,k}}^2 + \sigma_{w_{j,k}}^2$ ,  $\forall j = \{x, y, z\}$  to  $\sigma_{v_{j,k}}^2$  for the Gaussian pdf case. Similarly to previous equations, the subscripts  $j, k$  indicate direction and way-point number, respectively, while the subscript  $a$  indicates the airspeed. It should be noticed that some UAVs are controlled in ground speed, like small rotorcraft using the *Arducopter* control systems, while fixed-wing UAVs may be controlled in indicated air speed, for example when they use the *Ardupilot* control system. Therefore the correction of the speed uncertainty should be performed accordingly.

#### E. Confidence intervals for entire trajectory

Once the probability distributions of TAs at every way-point are calculated, the remaining step is the computation of the uncertainty bounds for the trajectory in between way-points. The introduction of the NURBS algorithm helps defining such uncertainty bounds by computing 3 different trajectories: the expected, i.e., "average" trajectory, computed with the ETAs, and two trajectories computed using the upper and lower bounds of the ETA,

$$\begin{aligned} \mathcal{P}^+(t) &= \text{NURBS}(\mathbf{P}, t_a^+) \quad , \\ \mathcal{P}^-(t) &= \text{NURBS}(\mathbf{P}, t_a^-) \quad , \end{aligned} \quad (15)$$

where, in case of normally-distributed errors,

$$\begin{aligned} \mathbf{t}_a^+ &= [\bar{t}_{a,0}, \bar{t}_{a,1} + \gamma\sigma_{t_{a,1}}, \dots, \bar{t}_{a,K} + \gamma\sigma_{t_{a,K}}] \quad , \\ \mathbf{t}_a^- &= [\bar{t}_{a,0}, \bar{t}_{a,1} - \gamma\sigma_{t_{a,1}}, \dots, \bar{t}_{a,K} - \gamma\sigma_{t_{a,K}}] \quad , \end{aligned}$$

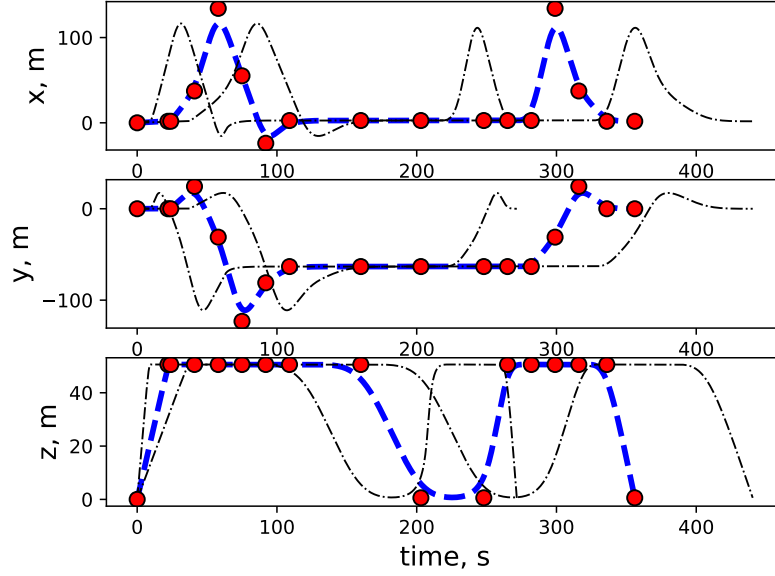
where  $\gamma$  is a real positive value. The matrix  $\mathbf{P} \in \mathbb{R}^{K \times 3}$  collects the  $K$  way-points in x, y, and z directions column-wise, and NURBS( $\cdot$ ) has been used to define the NURBS algorithm, which returns the 4D trajectory profile, with corresponding time derivatives, from the input way-points  $\mathbf{P}$  and TAs  $\mathbf{t}_a$ . The resulting profile is  $\mathcal{P}(t)$ , which contains the entire kinematic profile over the entire trajectory, and where the superscript  $+$  and  $-$  indicate the upper and lower interval of the trajectory.

As mentioned before, the uncertainty quantification approach proposed in the paper is independent of the trajectory generation technique. Although the NURBS algorithms has been implemented here as a possible flight trajectory for UAVs, the quantification of uncertainty in cruise velocity and its propagation to the ETA at each waypoint remains valid for other trajectory generation methods as well.

#### IV. Application to synthetic and real flight data

The error propagation method is applied to the flight path shown in figure 1b. The trajectories generated by introducing variation in expected cruise speed in the form of Gaussian distributions are shown in figure 3, using  $\pm 2\sigma$ . Uncertainty grows as time passes by, as expected. The confidence intervals reported here would be identical to the confidence intervals that can be computed with the uniformly-distributed velocity error, given the similarity of Eq. (7) and (8). In that case, the intervals computed from the uniformly-distributed cruise speed can be considered the upper and lower bound of the expected ETAs but the probability distribution of the TA at a specific position will not follow the uniform distribution, since those intervals are computed as the sum of uniformly-distributed random variables.

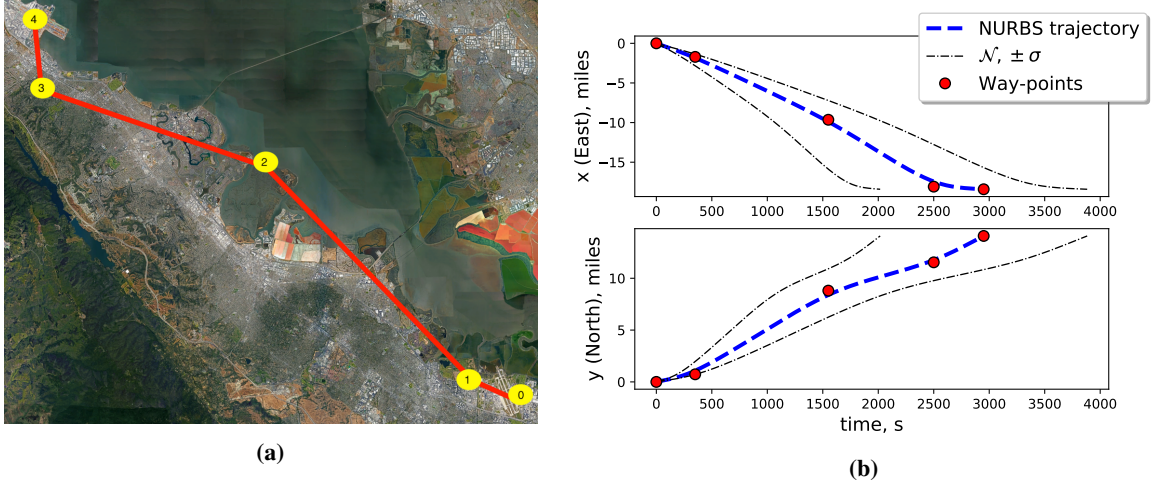
Further, to demonstrate the robustness of the proposed methodology, the uncertainty quantification technique is also applied to a set of synthetic way-points describing the flight plan of an unmanned fixed-wing aircraft on a much longer route. The unmanned vehicle takes off at Moffett Federal Airfield (NUQ), CA, and lands at San Francisco International Airport (SFO), CA. The 50 minute flight plan and trajectory generated with the NURBS algorithm are described by figure 4. During cruise, the altitude remains at 100 m from the ground. Way-point 3 on the flight route is necessary to approach the airport runway with the appropriate course angle. The uncertainty on the aircraft expected speed is propagated to compute the confidence intervals on the TAs at each way-point. Subsequently, the NURBS-based trajectory is computed for nominal, lower and upper limits of the interval (either from uniform or Gaussian distribution), and the resulting profiles are depicted in figure 4b. Both standard deviation and error interval of the cruise speed are kept constant to  $0.5m/s$  in horizontal direction.



**Fig. 3** Confidence intervals of the time of arrival along the trajectory computed by propagating the error on the expected cruise speed.

The method further allows to compute the uncertainty of the UAV position at a particular time instant based on the planned trajectory and expected wind conditions. Owing to the linear relationship approximation between position and time as stated in Eq. (4), we assume that the pdf of the vehicle position at time  $t = t^*$  follows the pdf form of  $t^*$ , i.e., if  $t^* \sim \mathcal{N}$ , then  $p(t = t^*) \sim \mathcal{N}$ . The pdfs of the aircraft position along East and North directions at time  $t^* = 2200.0$  s are shown in figure 5 (top and center), whereas the pdfs of the TA at a specific location, defined by  $\mathbf{p}^* = [-19.308\text{e}3, 16.090\text{e}3, 100]$  m, (corresponding to -12 East, +10 North) miles from the take off location, are available in figure 5 (bottom).

The plots in figure 5 highlights that the ETA computed from Gaussian distributions coincides with the TA of the mean of NURBS trajectory since the mode of the Gaussian distribution coincides with its mean (bottom plot). However, the mean position along  $x$  and  $y$  direction at time  $t^* = 2200.0$  s differs from the mean position computed from the NURBS trajectory in Eq. 15 (top and center plots). This is due to nonlinear transformation introduced by the NURBS algorithm, and stresses the fact that the computed pdfs are approximated. Refined distributions of the quantities in figure 5 could be computed using, e.g., the probabilistic transformation method used in [17], sampling-based techniques, or other methods for nonlinear transformations. Besides, since Gaussian pdfs are not bounded, the ETA pdf in the bottom panel of Figure 5 falls into the negative side of  $\mathbb{R}$ , but it can be easily corrected by transforming the pdf into, e.g., a truncated Gaussian or log-Normal distribution, as typically done to correct altitude pdf. The proposed methodology for uncertainty propagation is further applied to the experimental flight described in Fig. 1b. A flight executed from the

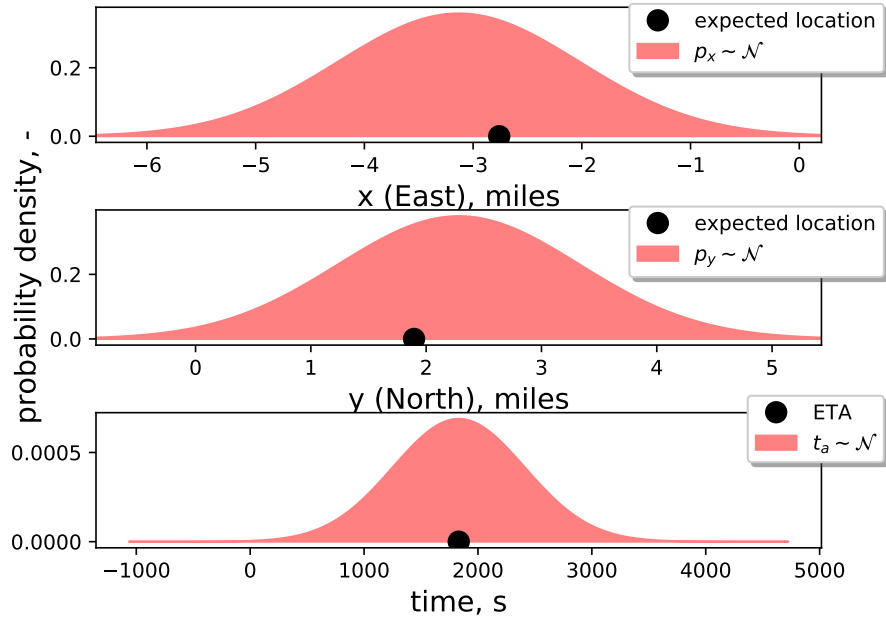


**Fig. 4** Synthetic flight plan of a fixed-wing aircraft along the San Francisco Bay Area (Google Maps) (a) and NURBS-based 4D trajectory with corresponding confidence intervals (b).

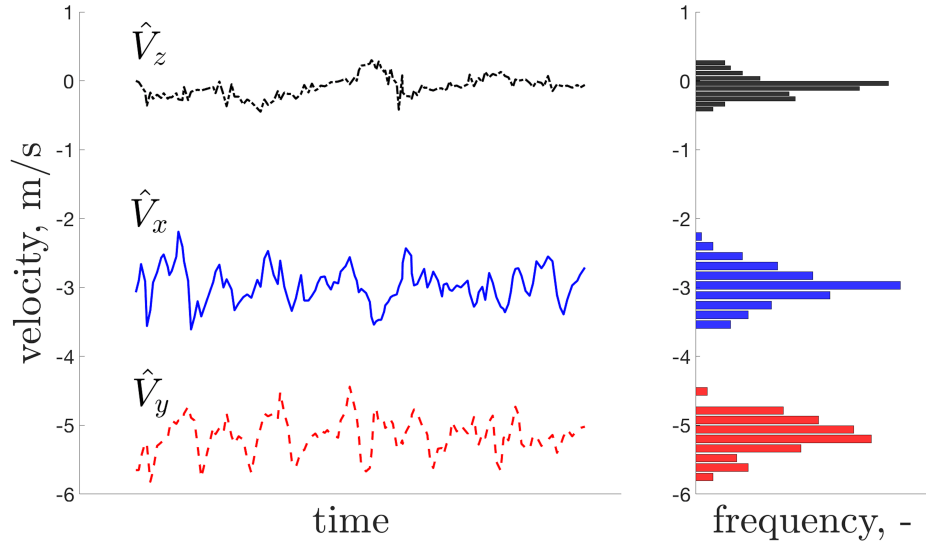
same vehicle, on another day, and on another route, has been utilized to compute the values  $\delta v$  and  $\Sigma_v$ . A segment corresponding to horizontal, steady flight, depicted in figure 6, has been selected to compute the variance of the different velocities in the local ENU reference frame. The resulting variances were  $\Sigma_v \approx \text{diag}([0.0625, 0.0625, 0.0289]^2)$ , (m/s)<sup>2</sup>. Figure 7 shows the corresponding confidence intervals against the observed flight path. The solid blue line defines the flight plan, the dot-dashed blue lines define the upper and lower bounds of the confidence interval, while the thick dashed red line represent the observed position of the UAV during flight. The figure shows that the actual path obtained from GPS measurements for the entire flight falls within the computed confidence intervals for the majority of the flight route.

## V. Conclusions

This work proposed a method to estimate and embed uncertainty in 4D trajectory profiles using error interval propagation applied to the cruise speed. The error interval propagation generates prediction intervals of the expected times of arrival (ETAs) at each way-point. When combined with the non-uniform rational B-spline (NURBS) utilized here, or any other trajectory generation algorithm it can map the relationship between velocity errors and the corresponding uncertainty along the flight path. The tests conducted during this work shows that the NURBS algorithm consumes most of the computing power, while the propagation of uncertainty is computationally efficient. Given its simplicity, the method could be embedded in larger systems for unmanned aerial vehicle (UAV) path planning, optimization, scheduling as well as for safety monitoring. An obvious drawback of the proposed approach is the necessity to quantify the uncertainty affecting the vehicle speed based on vehicle and environmental conditions in both no-wind and wind conditions. In absence of quantitative estimates of  $\sigma_v^2$ , the methodology may work by assuming conservative uncertainty bounds. The assumption of stationary wind profile is an additional but necessary limitation to enable analytical interval

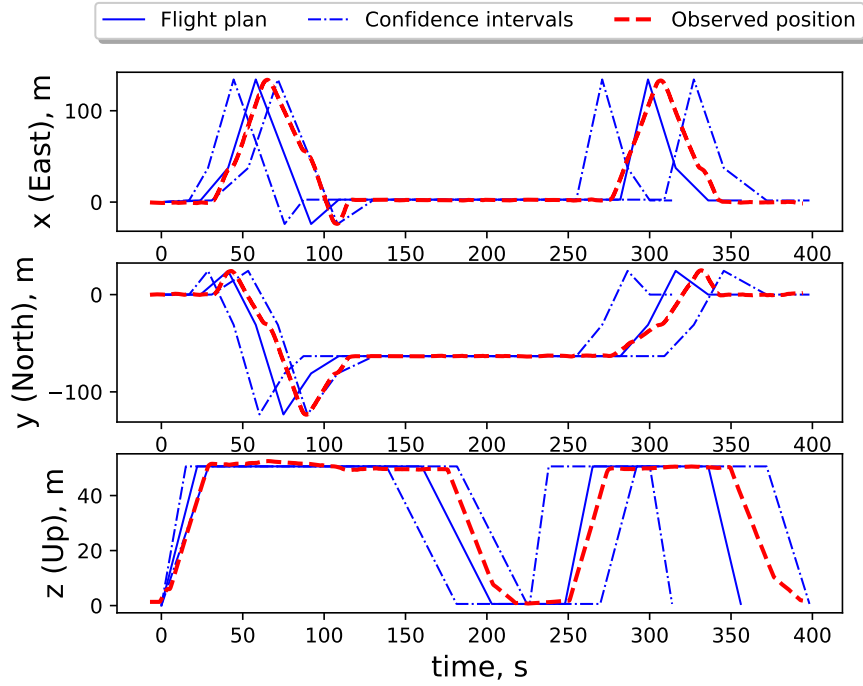


**Fig. 5** Position distributions at time  $t^*$  along East (top) and North (center) direction, respectively, and time of arrival distributions at position  $p^*$  (bottom).



**Fig. 6** Example of velocity recorded during a short segment of forward flight, and corresponding histograms of occurrence.

propagation. Other assumptions introduced in this work, e.g., negligible acceleration during cruise flight can be relaxed in future research. Potential extensions of this work include techniques to update such confidence intervals in real-time. If vehicle model(s) and fault detection methods are embedded into the framework, then ETA predictions could be further refined based on the actual condition of the vehicle.



**Fig. 7** Flight plan with  $\pm\delta t$  confidence intervals on the TA compared against the observed UAV flight.

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