



Interpretation of Zerodur[®] Strength Data

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Abstract

The strength distribution of Zerodur[®] (Schott AG) prepared by grinding and etching is determined through statistical analysis to be best described as normal. Although a 3-parameter Weibull appearance is exhibited when multiple specimen batches are combined and analyzed together, this appearance is the result of mixing of components of variation. As a result of the particular type of flaw population, little effect of scale is exhibited.

Introduction

Recent, detailed fractographic analysis of Zerodur[®] (Schott AG) strength test specimens prepared by linear grinding and etching with a proprietary process indicated low-frequency damage to be the strength-limiting defects (Ref. 1). The 2-parameter Weibull distribution is usually assumed when working with ceramic and glasses such as Zerodur[®], although 3-parameter behavior is occasionally considered (Ref. 2). Detailed statistical modeling of the Zerodur[®] strength data with the initial perspective of a 3-parameter Weibull distribution gave unsatisfying results (Ref. 3). The subsequent fractographic investigation (Ref. 1) indicated that the usual assumption of many small, random, noninteracting flaws (Ref. 4) was not represented, but instead, less frequent, flaws aligned along etching ridges parallel to the grinding direction were present (see Figure 7 of Ref. 1). It was thus concluded that the strength of Zerodur[®] as prepared was not Weibull distributed. The flaws represent a sparse flaw population relative to typical grinding damage, but an extensive occurrence (high frequency) of handling damage. We examine the type of distribution present and explain the appearance of a 3-parameter Weibull distribution.

Distribution Analysis

The 2-parameter Weibull distribution is the usual assumption when working with ceramic and glasses, with less frequent observations and use of the 3-parameter Weibull distribution. If the Zerodur[®] is not Weibull distributed, then what distribution is representative? Fits to two of the datasets of Reference 3, Pop 4 and Pop 5, are shown in Figure 1 for the normal, 2-parameter Weibull, and 3-parameter Weibull distributions. The associated fit quality as represented by several goodness-of-fit statistics is given in Table I. Although reasonable 2-parameter fit statistics are attained, the 3-parameter fits are weaker. The normal distribution provides the best fits. The best representation of the data is a normal distribution with a large coefficient of variation (~23 percent), implying variation in the handling damage from specimen to specimen.

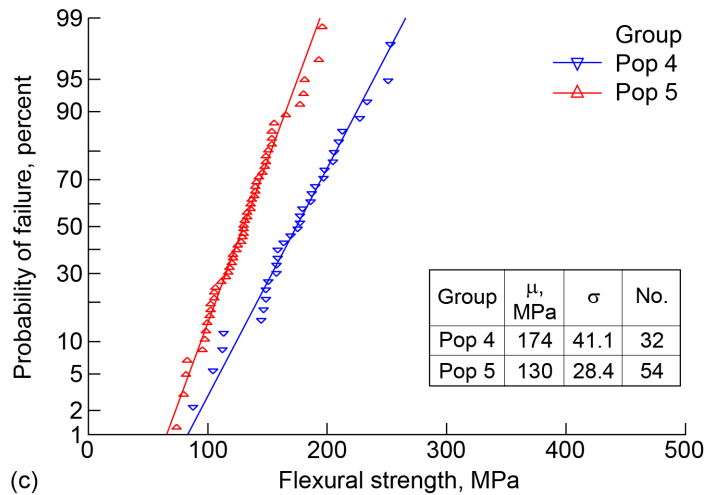
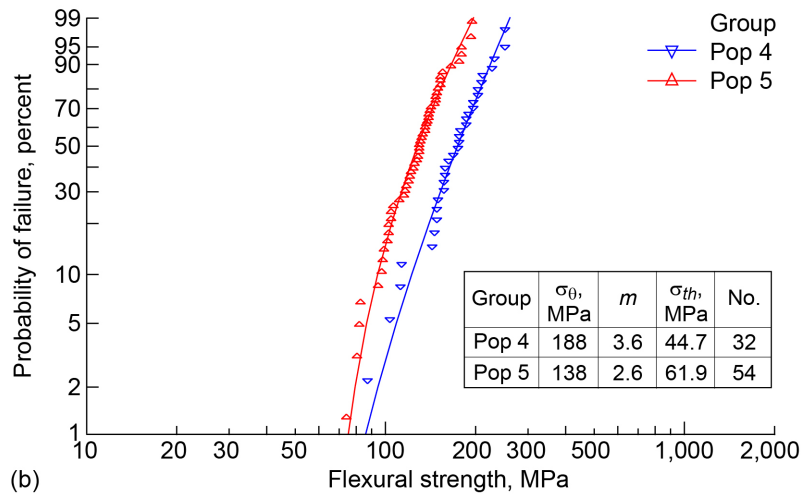
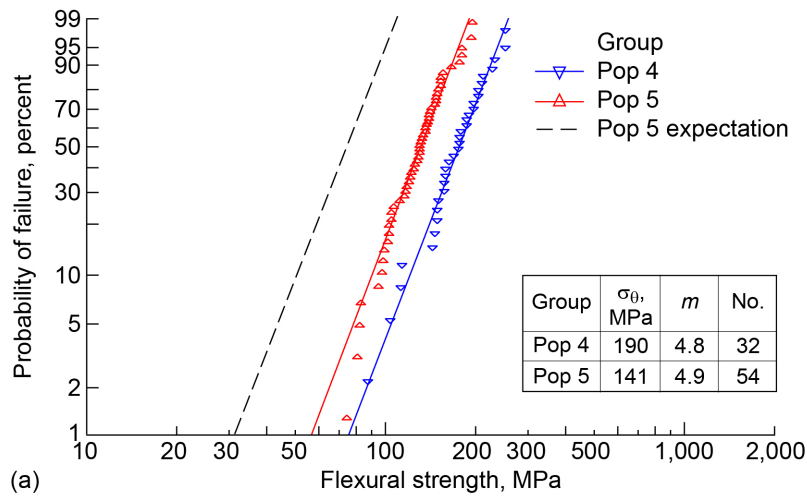


Figure 1.—Distribution fits to Zerodur® data. (a) 2-parameter Weibull distribution. (b) 3-parameter Weibull distribution. (c) Normal distribution. Weibull modulus (m). Mean (μ). Threshold stress (σ_{th}). Characteristic strength (σ_θ) Standard deviation (σ).

TABLE I.—GOODNESS-OF-FIT QUALITY STATISTICS
[Larger values imply better model fits.]

Distribution	P-value, percent		Anderson-Daring, percent	
	Pop 4	Pop 5	Pop 4	Pop 5
Normal	81	55	68	54
2-parameter Weibull	75	32	62	12
3-parameter Weibull	50	43	55	46

Size Effect

In the initial study (Ref. 3), testing of several sets of standard-sized beams, a set of oversized beams, and a set of plates was performed to check for the usual effect of scale exhibited with the 2-parameter Weibull distribution. A clear trend was not identified. For the small and large beams (Pop 4 and Pop 5, respectively), Figure 1(a) shows that the effect was about 45 percent of that based on the common formulas:

$$\frac{S_{f1}}{S_{f2}} = \left(\frac{Ae_2}{Ae_1} \right)^{1/m} \quad (1)$$

and

$$Ae = \left[\frac{1 + m \left(\frac{S_i}{S_o} \right)}{(m+1)^2} \right] \left(1 + \frac{mB}{B+W} \right) (B+W) S_o \quad (2)$$

where A_e is the effective area, S_f is the strength at a probability, m is the Weibull modulus, S_i and S_o are the spans, and B and W are the width and thickness. Further, other populations from the same study exhibited a smaller scale effect. This implies no effect of scale, but variation from one set to another, likely due to process handling and preparation variations from set to set.

Plates (Pop 6) tested in the same study (Ref. 3) exhibited greater characteristic strength than all but one set of beams, an unusual occurrence in the experience of the author unless bevel damage dominates failure. Plates usually contain multiaxial stresses, thus sampling all flaw orientations and measuring the worst. The strength of Zerodur® as machined via the particular grinding and etching process is normally distributed with a large variance, implying sufficiency of a simpler design process than usually used. Although the results imply that handling damage involving many flaws is normally distributed, a case-by-case basis supported by fractography is required.

Components of Variation

Why the 3-parameter appearance for pooled data? When significantly different components of variation, such as batch-to-batch, operator-to-operator, orientation, test environment, etc., are combined, the distribution can be distorted with weaker groups being stronger than expected for the associated ranking, and vice versa for stronger groups. An example of this is given for AD995 alumina tested by various laboratories as part of a round robin exercise (Ref. 5). The individual populations are reasonably described by the 2-parameter Weibull fit as shown in Figure 2(a). However, when pooled, a 3-parameter shape emerges, as shown in Figure 2(b). When additional components of variation are added, the shape is more convincingly 3-parameter, as shown in Figure 2(c).

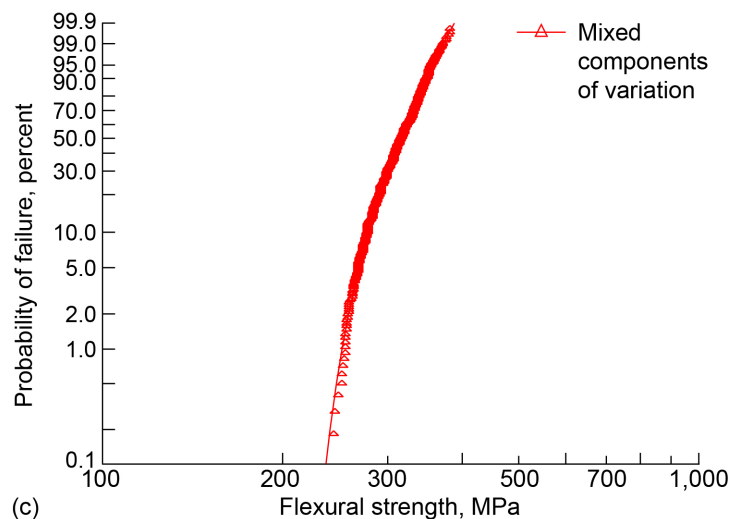
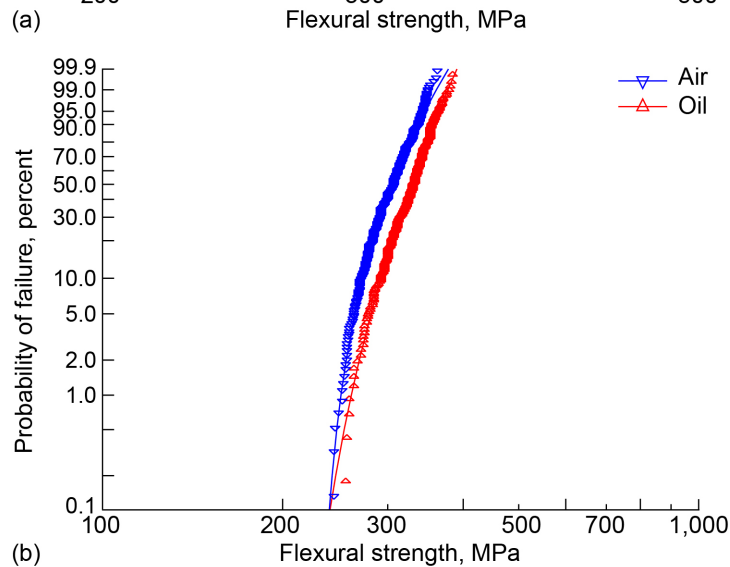
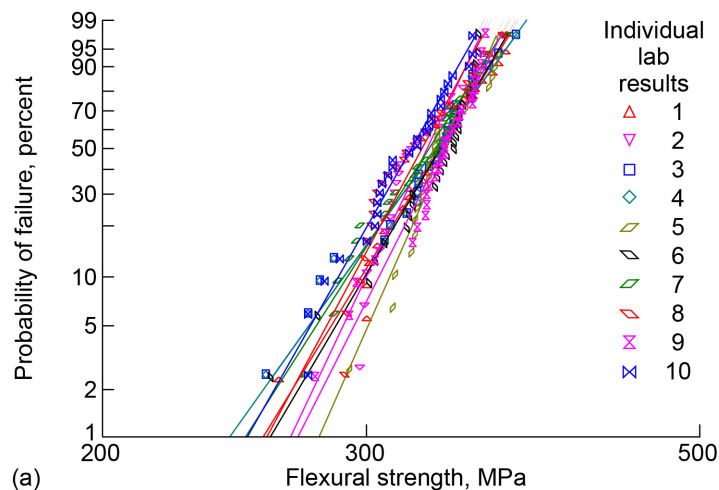


Figure 2.—Distributions for AD995 alumina. (a) Individual laboratory datasets fit to 2-parameter Weibull distribution for oil environment. (b) Pooled datasets for air and oil environments fit to 3-parameter Weibull distribution. (c) Pooled data for mixed components of variation fit to 3-parameter Weibull distribution.

Conclusions

Zerodur[®], as machined and etched (Refs. 1 and 3), is best represented by a normal distribution with large variance but without scale effects. Thus, a classical design approach might be used for structural design with this flaw population. The appearance of a 3-parameter Weibull distribution results from mixing of components of variation. Although a 3-parameter Weibull distribution might be reasonably used, the size effect is missing, and the fixity of the threshold is questionable.

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