Deriving Telescope Wavefront Error (WFE) Stability Error Budget from Coronagraph Performance

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Introduction

Deriving a Telescope's Wavefront Stability Error Budget from its Coronagraph Performance is an exercise in Science-Driven Systems Engineering.

Motivation:

- Directly imaging and characterizing Earth-like habitable-zone planets with a coronagraph requires an ultra-precise, ultra-stable optical telescope.
- Telescope wavefront errors (WFEs) can cause stellar light to pass around the occulting mask and introduce noise.
- Sources of these errors can be rigid-body misalignments between optical components, mounting error, or low-order and mid-spatial frequency figure errors.
- Because different coronagraphs perform differently, the telescope and coronagraph must be designed as an integrated system.

Science Driven Systems Engineering: Stability Error Budget

Recently, we published two papers in JATIS

- Nemati, et. al., "Method for deriving optical telescope performance specifications for Earth-detecting coronagraphs," doi: 10.1117/1.JATIS.6.3.039002.
- Stahl, et. al., "Habitable-Zone Exoplanet Observatory (HabEx) baseline 4-m telescope: systems-engineering design process and predicted structural thermal optical performance," doi: 10.1117/1.JATIS.6.3.034004.

that describes in detail a process for developing a stability error budget and applying it to a specific coronagraph/telescope pair:

- Determine the desired science measurement's signal-to-noise ratio (SNR) requirement
- Determine what are the sources of science limiting noise
- Determine coronagraph's contrast leakage sensitivity to each noise source mode.
- Predict via STOP modeling (structural thermal optical performance) the amplitude of each of these.
- Create WFE stability error budget by allocating a 'weighted' amount of contrast leakage to each mode.

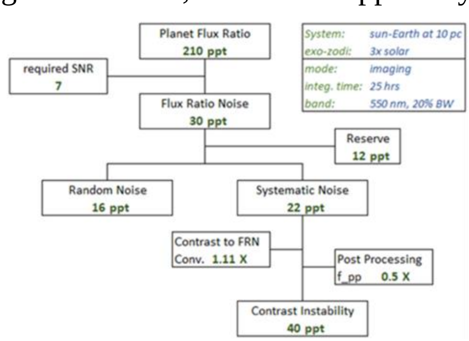
1. Science Measurement

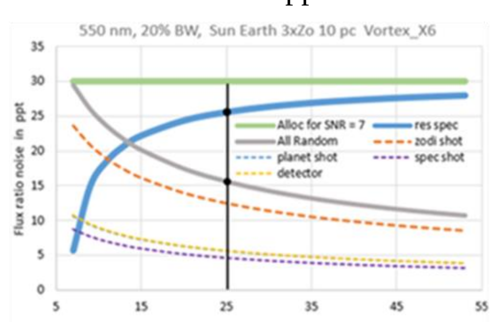
To characterize an exo-Earth at 10 parsec, its PSF needs to be detectable above starlight halo.

Flux ratio for an exo-Earth at 10 pc is 210 parts per trillion (ppt).

If we require a SNR= 7, then noise from all sources must contribute no more than ~30 "ppt".

Integrating for 25 hours, allocates 22 ppt to Systematic Noise == 40 ppt of Contrast Instability





This is different for planets that are larger or smaller, closer to or further from their star.

2. Noise Sources: Telescope WFE Instability Modes

Primary source of systematic contrast instability is telescope wavefront error (WFE).

WFE instability arise from mechanical and thermal disturbances

- LOS WFE instability occurs when LOS drift/jitter causes beam-shear on SM and TM.
- Inertial WFE instability occurs when PM is accelerated against its mounts.
- Thermal WFE instability occurs when telescope's temperature changes.
  - If mirror's CTE is homogeneous, then a temperature should only result is a defocus error.
  - But any CTE inhomogeneity results in a temperature dependent WFE.

Typically, mechanical WFE is low-spatial-frequency, while thermal WFE can have significant mid- and high-spatial-frequency content.

Also, mechanical WFE is measured in Hertz, while thermal WFE is measured in hours.

2. Noise Sources: Telescope WFE Instability Modes

There are many different ways to describe WFE instability modes. We choose to use modes produced by telescope-structure rigid-body motions or optical component deformations.

Please note that monolithic and segmented telescopes have different perturbation modes.

Table 1: Telescope Wavefront Error Instability Modes

'Global' Monolithic Primary Mirror Telescope • Rigid-Body Motion between PM/SM (beam shear) • Inertial WFE (bending modes of PM as it reacts against mount) • Thermal WFE (CTE inhomogeneity shape changes)	Tip/Tilt
	Power
	Astigmatism
	Coma
	Spherical
	Terfoil
	Secondary Trefoil
	Higher-Order Zernikes Coefficients
'Global' Segmented Primary Mirror Telescope • Rigid-Body Motion between PM/SM (beam shear) • Bending Modes of the Back Plane Structure	Power
	Astigmatism
	Coma
	Spherical
'Segment' Level Modes • Rigid-Body Motion of Segment • Inertial WFE (bending modes of Segment as it reacts against mount) • Thermal WFE (CTE inhomogeneity Segment shape changes)	Piston
	Tip/Tilt
	Power
	Astigmatism
	Trefoil
	Secondary Trefoil
	Higher-Order Zernike Coefficients

3. Coronagraph Contrast Sensitivity per Telescope Mode

WFE causes stellar light to pass around the occulting mask and introduces noise.

Magnitude of this noise depends on the coronagraph's sensitivity to a given WFE mode

Noise appears in dark hole based on WFE mode's spatial frequency.

- WFEs with low spatial frequency contribute noise closer to the inner working angle
- Mid-spatial frequency WFEs contribute noise in the middle of the dark hole.

To quantify this noise, we follow a method defined by Shaklan et. al. (2011)

- Calculate residual contrast for each mode,
- Divide it into annular rings in working angle, and
- Compute standard deviation $\sigma\_\Delta C$

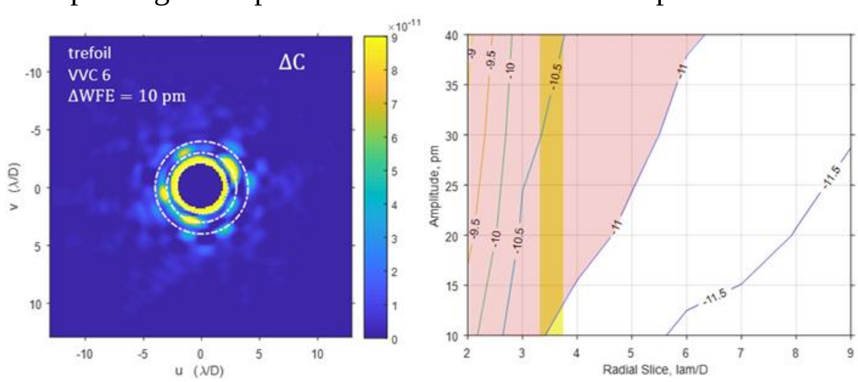
Example: Residual Spackle Noise for Trefoil

Left graphic shows residual speckle map (labeled Δ"C") for trefoil and VVC-6.

Right graphic shows contour plot of residual noise vs perturbation amplitude.

Red shaded region is angular separation where residual noise would exceed 1e-11

Vertical yellow strip is angular separation for an exo-Earth at 10 pc



Case Studies

Contrast sensitivity was calculated for five cases:

- Unobscured off-axis 4-meter monolithic aperture telescope VVC-4
- Unobscured off-axis 4-meter monolithic aperture telescope VVC-6
- Unobscured off-axis 4-meter monolithic telescope with Hybrid Lyot coronagraph (HLC)
- Off-axis 6-m 37-segmented aperture telescope with a VVC-6
- On-axis 6-meter 36-segment telescope with apodized pupil Lyot coronagraph (APCL)

Case Studies

The following graphs plot residual speckle error ($\sigma_{\Delta C}$) caused by 5 pm rms of selected perturbation modes computed over a 1 λ/D-wide annular zone near 100 mas (location of a 1 AU exoplanet at 10 pc) as a function of initial raw contrast.

- Because residual speckle error is a cross-term, as raw contrast of the planet increases, so too does the sensitivity of the contrast to wavefront change.
- The central wavelength in all cases is assumed to be 550 nm.

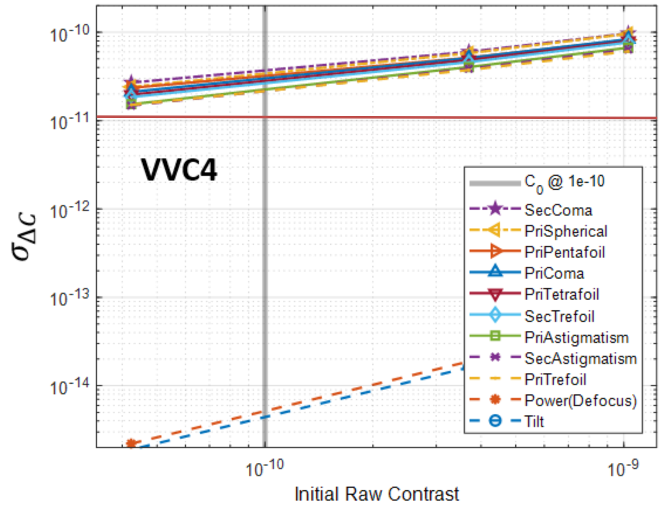
The vertical line is 10^{-10} raw contrast needed to observe an exo-Earth.

Table lists contrast sensitivity taken from the crossing points at C_0 of 10^{-10}.

Horizontal line at 10^{-11} is delta-contrast needed to detect an exo-Earth.

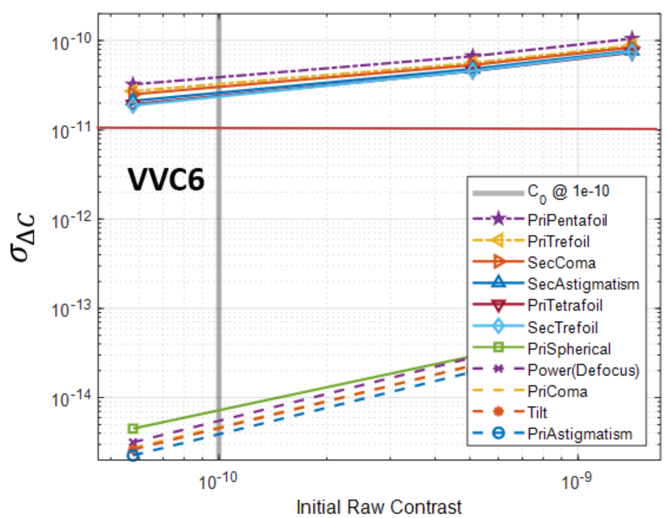
If residual speckle line for mode is above '**red**' line, then WFE must be < 5 pm rms.

4-m Off-Axis Monolithic Telescope with a VVC-4



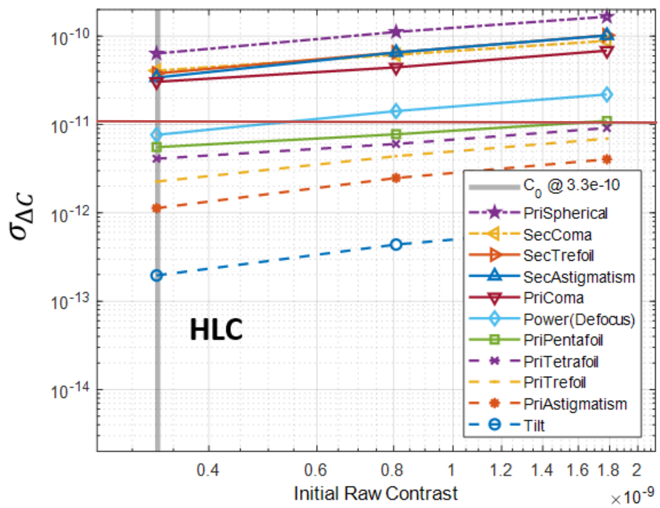
Perturbation Mode	Sensitivity
	ppt/pm
Tip/Tilt	0.00088
Defocus	0.00103
Astigmatism	4.08
Coma	5.44
Trefoil	3.92
Spherical	6.17
Secondary Atigmatism	3.93
Tetrafoil	5.09
Secondary Coma	6.72
Secondary Trefoil	4.80
Pentafoil	5.83

4-m Off-Axis Monolithic Telescope with a VVC-6



Perturbation Mode	Sensitivity
	ppt/pm
Tip/Tilt	0.00091
Defocus	0.0011
Astigmatism	0.00078
Coma	0.00092
Trefoil	6.06
Spherical	0.0014
Secondary Atigmatism	4.82
Tetrafoil	4.44
Secondary Coma	5.62
Secondary Trefoil	4.34
Pentafoil	7.21

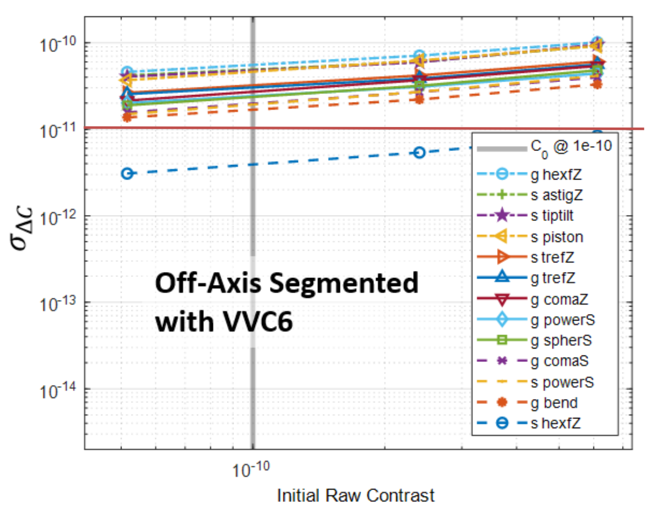
4-m Off-Axis Monolithic Telescope with a HLC



Perturbation Mode	Sensitivity
	ppt/pm
Tip/Tilt	0.039
Defocus	1.53
Astigmatism	0.23
Coma	6.11
Trefoil	0.45
Spherical	12.8
Secondary Atigmatism	6.83
Tetrafoil	0.82
Secondary Coma	8.18
Secondary Trefoil	7.60
Pentafoil	1.11

HLC cannot reach 1e-10 raw contrast, but has good sensitivities to various modes.

6-m Off-Axis Segmented Aperture Telescope with VVC-6



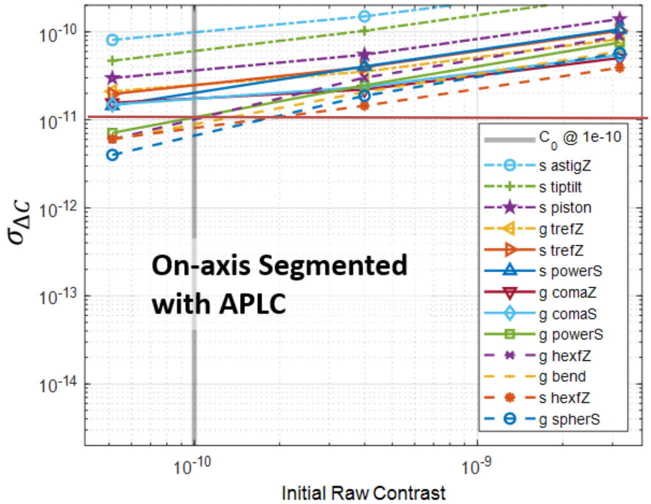
Perturbation Mode	Sensitivity
Global	ppt/pm
Bend	3.21
Defocus	4.64
Spherical	4.51
Coma (Seidel)	3.78
Coma (Zernike)	5.19
Trefoil	5.82
Hexafoil	10.61
Segmented	
Piston	8.84
Tip/Tilt	9.09
Power	3.68
Astigmatism	9.33
Trefoil	6.16
Hexafoil	0.745

'g' = global aberration

's' = segment level error

Segmented Case's larger diameter sees exo-Earth at 5.3 λ/D.

6-m On-Axis Segmented Aperture Telescope with APLC



Perturbation Mode	Sensitivity
Global	**ppt/pm**
Bend	1.65
Defocus	1.96
Spherical	1.26
Coma (Seidel)	3.26
Coma (Zernike)	3.31
Trefoil	4.66
Hexafoil	1.95
Segmented	
Piston	6.74
Tip/Tilt	11.12
Power	3.71
Astigmatism	18.33
Trefoil	4.55
Hexafoil	1.48

‘g’ = global aberration

‘s’ = segment level error

Segmented Case’s larger diameter sees exo-Earth at 5.3 λ/D.

Contrast Sensitivity Observations

The most important observation is that exoEarth science requires a telescope with picometer mid-spatial frequency wavefront stability.

For a segmented aperture telescope, the most important error sources are segment level rigid body motions of piston and tip/tilt.

For a monolithic aperture telescope with a VVC-6 the most important errors are trefoil, secondary trefoil and primary pentafoil.

For a monolithic telescope with a VVC-4, astigmatism is also an important error.

Regarding the HLC and segmented APLC designs, the models studied were relatively older and do not necessarily represent the best performance possible by these approaches.

4. Predicted Amplitude of Telescope WFE Modes

Knowing a corongraph's contrast sensitivity to specific WFE modes is only the first step.

It is also necessary to know the likely amplitude of each mode.

WFE instability is caused by telescope's response to mechanical and thermal stimuli:

- LOS WFE - Rigid Body Motion between Primary and Secondary mirrors.
  - Mostly Low-Order Aberration caused by Beam Shear
  - Mechanical Disturbances produce LOS Vibration – measured in Hz
  - Thermal Disturbances process LOS Drift – measured in Hours
- Inertial WFE – Reaction of mirror against its mount due to vibration acceleration.
  - Typically Low- and Mid-Spatial Frequency Error – looks like gravity sag
- Thermal WFE – Changes in Mirrors due to CTE inhomogeneity.
  - Typically Mid- and High-Spatial

WFE amplitudes are predicted via structural thermal optical performance (STOP) analysis.

5. WFE Stability Error Budget Allocation

Final step in creating a WFE stability error budget is to distribute contrast allocation to each WFE mode.

To produce an error budget where every mode has the same margin, distribute contrast allocation based on the ratio of the mode's predicted error to the telescope's total error.

Error Budget for 4-m Off-Axis Monolithic Telescope with VVC6

STOP modeling of the HabEx Baseline Telescope predicts Trefoil to be the most significant error, so it gets the most Residual Contrast (ppt) yielding an error budget with 4.1X margin.

Index			Predicted Performance Amplitude [pm rms]			Total WFE	VVC-6 Sensitivity	Raw Contrast	Allocation	WFE Tolerance	Margin
N	M	Aberration	LOS	Inertial	Thermal	[pm rms]	[ppt/pm PV]	[ppt]	[ppt]	[pm RMS]	
		TOTAL RMS	5.715	3.994	5.565	8.921		7.289	30.000	36.715	
1	1	Tilt	3.025	0.123	0.026	3.027	0.0002	0.001	0.005	12.459	4.1
2	0	Power (Defocus)	0.728	1.430	3.759	4.087	0.0003	0.002	0.010	16.821	4.1
2	2	Astigmatism	4.674	3.559	3.463	6.819	0.0002	0.003	0.013	28.066	4.1
3	1	Coma	1.064	0.099	0.345	1.123	0.0002	0.001	0.002	4.620	4.1
4	0	Spherical	0.005	0.213	0.405	0.458	0.0003	0.000	0.001	1.883	4.1
3	3	Trefoil	0.050	1.039	2.098	2.342	1.0016	6.634	27.303	9.638	4.1
4	2	Sec Astigmatism	0.019	0.178	0.108	0.209	1.6495	1.091	4.489	0.861	4.1
5	1	Sec Coma	0.003	0.026	0.105	0.108	1.6645	0.624	2.568	0.445	4.1
6	0	Sec Spherical	0.000	0.028	0.000	0.028	2.8902	0.214	0.881	0.115	4.1
4	4	Tetrafoil	0.001	0.198	0.189	0.274	0.9312	0.806	3.317	1.127	4.1
5	3	Sec Trefoil	0.000	0.112	0.233	0.259	1.8200	1.630	6.708	1.064	4.1
6	2	Ter Astigmatism	0.000	0.021	0.000	0.021	2.7219	0.214	0.880	0.086	4.1
7	1	Ter Coma	0.000	0.033	0.000	0.033	3.0608	0.404	1.663	0.136	4.1
5	5	Pentafoil	0.000	0.074	0.217	0.229	2.4409	1.939	7.979	0.944	4.1
6	4	Sec Tetrafoil	0.000	0.029	0.000	0.029	2.2050	0.239	0.985	0.119	4.1
7	3	Ter Trefoil	0.000	0.015	0.000	0.015	2.7946	0.168	0.690	0.062	4.1
6	6	Hexafoil	0.000	0.026	0.000	0.026	3.1667	0.308	1.268	0.107	4.1
7	5	Sec Pentafoil	0.000	0.015	0.000	0.015	3.0694	0.184	0.758	0.062	4.1
7	7	Septafoil	0.000	0.010	0.000	0.010	2.6510	0.106	0.436	0.041	4.1

Conclusion

Performing exoplanet science with an internal coronagraph requires an ultra-precise ultra-stable optical telescope.

The telescope and coronagraph must be specified and designed as an integrated system.

This paper summarized a science-driven systems engineering approach for:

- Calculating a coronagraph's contrast instability sensitivities to telescope wavefront error modes (Nemati, et. al.), and
- Creating a stability error budget based on predicted telescope performance (Stahl, et. al.)