

A Model Predictive Control Approach for In-Flight Acoustic Constraint Compliance

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Vehicle noise remains one of the major barriers to public acceptance of Urban Air Mobility-class aircraft. This work focuses on motion planning for aircraft in noise-sensitive areas. A nonlinear Model Predictive Path Integral (MPPI) control law is used to generate a finite-horizon trajectory that satisfies acoustic level constraints at a set of (three-dimensional) observer locations. The MPPI framework places no restrictions on the class of state-dependent cost functionals that can be employed, making it well-suited for use with sophisticated acoustic models and metrics, in addition to dynamic and mission-relevant constraints. The model predictive control architecture is also suitable for implementation in a real-time application. A simulation example demonstrates the ability of the controller to modify the flight trajectory in order to satisfy acoustic constraints at multiple measurement locations.

I. Nomenclature

x_i	=	Inertial position
v_i	=	Inertial velocity
ω_v	=	Angular velocity in velocity frame
q	=	Attitude quaternion representing rotation from inertial to velocity coordinates
R	=	Rotation matrix from velocity to inertial coordinates
V	=	Vehicle speed
a_v	=	Vehicle acceleration in velocity coordinates
g	=	Gravitational acceleration
m	=	Vehicle mass
t	=	Time
α	=	Angle of attack
p_s	=	Stability-axis roll rate
ρ	=	Air density
T	=	Net thrust of vehicle propellers
c_i	=	Thrust coefficient
C_N	=	Normal force coefficient
C_A	=	Axial force coefficient
S	=	Area of aerodynamic lifting surface
d_p	=	Propeller diameter
ω_p	=	Propeller angular speed
N_p	=	Number of propellers

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J	=	Propeller advance ratio
f	=	Propeller blade pass frequency
c	=	Speed of sound
r	=	Distance between acoustic source and observer
M_t	=	Rotor tip Mach number
M_{eff}	=	Effective rotor tip Mach number
$OASPL$	=	Overall Sound Pressure Level
$\mathcal{A}$	=	Set of acoustic observer locations

II. Introduction

NOISE management remains one of the major barriers to public acceptance of Urban Air Mobility (UAM) operations in urban areas [1]. Recent research into the noise mitigation problem for UAM-class aircraft spans a variety of approaches, from configuration design [2] to trajectory planning [3] and propeller phase synchronization for noise directivity control [4, 5]. Hierarchical control is a promising approach for noise mitigation that could be applied to existing vehicles with few (or no) modifications or configuration changes. Combining a trajectory planner and tracking controller with source noise directivity control provides a flexible architecture for noise mitigation, while ensuring separation from other air traffic, operation within the dynamic limitations of the vehicle, and accounting for other mission-relevant restrictions.

This work focuses on real-time trajectory planning for a single vehicle in a noise-constrained environment. It builds on prior work, which provided a framework for generating spatially or temporally deconflicted trajectories for multiple vehicles [6]. The framework provided guarantees on constraint satisfaction by parameterizing the trajectories as Bézier curves, but was limited to relatively simple dynamics and noise models that are differentially flat and that can be parameterized by (rational) polynomial functions. The present work focuses on motion planning for a single vehicle and uses a model predictive control (MPC) architecture to remove the constraints on the classes of dynamics and acoustic models that can be considered. A model predictive path integral (MPPI) [7] control law is used to generate an optimal control sequence that satisfies mission objectives as well as state and acoustic constraints. MPPI (see [7–9] for additional information) is a sample-based method for solving nonlinear finite-horizon optimal control problems, which utilizes a graphics processing unit (GPU) to propagate thousands of trajectories in parallel. The optimal control is obtained by a cost-weighted average of the sampled controls. The sample-based nature of MPPI imposes no restrictions (e.g., continuity, differentiability) on the class of cost functions allowed, making it an attractive option for use with sophisticated acoustic models and constraints. Acoustic constraints, models, and mission objectives can potentially be modified online.

The acoustic model and constraints, along with state constraints and mission requirements, are encoded in the cost function of the MPPI controller to use for trajectory planning. The control commands are formulated as pseudostates, where the MPPI control law treats the derivative of the commands as inputs. This ensures that, for the fixed-wing aircraft dynamics considered here, the MPPI control trajectory can be consistently initialized with a constant (zero) value anywhere in the state space, without a baseline control law to provide “trim” command. It also allows the pseudostates to be incorporated in the state-dependent portion of the cost function in order to place constraints on the commanded control variables. In the current formulation, the MPPI controller is used as a standalone planner, but it could also be used to navigate in the vicinity of a predefined trajectory, making online local adjustments to mitigate noise in the immediate area. Results from a simulation, based on vehicle properties and acoustic data from NASA’s GL-10 tilt-wing aircraft [10] in forward flight, are included to demonstrate the efficacy of the proposed control methodology.

The remainder of this document is organized as follows. Section III defines the vehicle dynamics and section IV describes the acoustic source model. An overview of the MPC algorithm used for trajectory planning is given in section V and results from a simulation example are provided in section VI. Section VII offers some concluding remarks.

III. Vehicle Dynamics

The vehicle dynamics used by the model predictive controller are based on the differentially flat fixed-wing dynamics with coordinated flight constraint from [11]:

$$\dot{\mathbf{x}}_i = \mathbf{v}_i, \quad (1)$$

$$\dot{\mathbf{v}}_i = \mathbf{g} + \mathbf{R}\mathbf{a}_v, \quad (2)$$

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q} \otimes \begin{bmatrix} 0 \\ \boldsymbol{\omega}_v \end{bmatrix}, \quad (3)$$

$$\boldsymbol{\omega}_v = \begin{bmatrix} p_s & -\mathbf{e}_3^\top (\mathbf{a}_v + \mathbf{g})/V & \mathbf{e}_3^\top \mathbf{g}/V \end{bmatrix}^\top, \quad (4)$$

$$\begin{bmatrix} T & \alpha & p_s \end{bmatrix}^\top = \mathbf{u}, \quad (5)$$

where $V = \|\mathbf{v}\|$ is the total speed and $\otimes$ denotes the standard quaternion multiplication [12, §6.2]. The position, $\mathbf{x}_i$, and velocity, $\mathbf{v}_i$, are both relative to a North-East-Down inertial coordinate system, while the angular rates, $\boldsymbol{\omega}_v$, and acceleration, $\mathbf{a}_v$, are expressed in coordinates fixed to the vehicle in the velocity frame. The vector $\mathbf{e}_3$ is defined as $\mathbf{e}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top$. We note that the absence of the normal rotational dynamics is due to the coordinated flight constraint.

The acceleration of the vehicle (resolved in velocity coordinates) is a function of the applied thrust, T , angle of attack, α , and the aerodynamic forces:

$$\mathbf{a}_v = \begin{bmatrix} \frac{T}{m} \cos \alpha - \frac{\rho V^2 S}{2m} \left(\sin \alpha (C_{N_0} + C_{N_\alpha} \alpha) + \cos \alpha (C_{A_0} + C_{A_{\alpha^2}} \alpha^2) \right) \\ 0 \\ -\frac{T}{m} \sin \alpha - \frac{\rho V^2 S}{2m} \left(\cos \alpha (C_{N_0} + C_{N_\alpha} \alpha) - \sin \alpha (C_{A_0} + C_{A_{\alpha^2}} \alpha^2) \right) \end{bmatrix}, \quad (6)$$

where the aerodynamic parameters are components of a polynomial aerodynamic model developed from wind tunnel testing of NASA's GL-10 aircraft [13], shown in Figure 1.

The propeller/motor speed, ω_p , is required for use in the acoustic model and is computed from the quadratic relationship

$$T = c_2(J)\omega_p^2 + c_1(J)\omega_p + c_0(J), \quad (7)$$

where the c_i are dependent on the propeller advance ratio

$$J = \frac{V \cos \alpha}{n d_p} = \frac{2\pi V \cos \alpha}{\omega_p d_p}, \quad (8)$$

where any edge-wise component of airflow over the propeller is ignored in the calculation of the advance ratio.

Implicit in the formulation of the vehicle dynamics is, from the perspective of the MPC, the existence of a lower level control law capable of tracking axial acceleration, stability axis roll rate, angle of attack, and regulating angle of sideslip sufficiently well that the tracking errors are small enough to be considered as disturbances.

IV. Acoustic Model

In order to predict the noise level at the observer locations, the MPC uses the overall sound pressure level (OASPL) as a metric. The model assumes that each propeller radiates acoustic power omnidirectionally and that the propellers are incoherent sources. Under these assumptions the OASPL varies only with distance from the vehicle and has no directional dependence. The OASPL, based on the first N_f harmonics of propeller rotation, is then given as

$$OASPL = 10 \log_{10} \left(\frac{1}{\hat{p}^2} \sum_{k=0}^{N_f} \left[\hat{p}_{\text{rms},k}^2 \left(\frac{M_{\text{eff}}}{\hat{M}_{\text{eff}}} \right)^{\xi_k} \right] \left(\frac{\hat{r}}{r} \right)^{2N_p} \right), \quad (9)$$

where $\hat{p}$ is the acoustic reference pressure, $\hat{p}_{\text{rms},k}$ is the RMS pressure of the k -th harmonic at the reference condition, r is the distance between the vehicle and acoustic observer, $\hat{r}$ is the reference distance, and N_p is the number of propellers.



Fig. 1 NASA's GL-10 aircraft, from [10].

The effective tip Mach number, M_{eff} , is determined from the propeller tip Mach number, $M_t = \omega_p d_p / 2c$, where c is the speed of sound in the air and the propeller advance ratio, J , given by

$$M_{\text{eff}} = \frac{M_t}{1 + J(1 - M_t)}. \quad (10)$$

The values of the exponents, ξ_m , on the effective tip Mach number ratio were determined according to [14] from the Propeller Analysis System of the Aircraft Noise Prediction Program (PAS-ANOPP) [15]. The distance, r , between the vehicle and a particular acoustic observer is defined as $r = \|\mathbf{x}_i - \mathbf{x}_{obs}\|$, where, for an observer region or volume $\mathcal{A}$, $\mathbf{x}_{obs}$ is the point in the region or volume closest to the vehicle, i.e., $\mathbf{x}_{obs} = \arg \min_{\mathbf{a} \in \mathcal{A}} \|\mathbf{x}_i - \mathbf{a}\|$.

Optionally, A-weighting [16] can be added to the acoustic model to account for relative loudness of different frequencies as perceived by the human ear. The A-weighting is added to the acoustic model as

$$OASPL = 10 \log_{10} \left(\frac{1}{\hat{p}^2} \sum_{k=0}^{N_f} \left[\hat{p}_{\text{rms},k}^2 \left(\frac{M_{\text{eff}}}{\dot{M}_{\text{eff}}} \right)^{\xi_k} R_A(f_k) \right] \left(\frac{\hat{r}}{r} \right)^2 N_p \right), \quad (11)$$

where

$$R_A(f) = \frac{12194^2 f^4}{(f^2 + 20.6^2) [(f^2 + 107.7^2) (f^2 + 737.9^2)]^{1/2} (f^2 + 12194^2)} \quad (12)$$

and the harmonic frequencies are computed from the propeller angular rate and number blades, N_b , as

$$f_k = k f_0 = k \frac{\omega_p N_b}{2\pi}. \quad (13)$$

We note that, in the current version of the acoustic model, the delay in sound wave propagation from the vehicle to the observer locations is not accounted for. This does not qualitatively affect the results, as it would merely shift the acoustic levels temporally without changing the magnitude of the OASPL at the observer.

V. Model Predictive Path Integral Control

A. Overview

The information-theoretic formulation of MPPI (see [8, 9, 17] for details regarding the MPPI control theory) is used here for trajectory generation in the presence of acoustic constraints. A brief overview of the MPPI controller, following the description in [18], is provided in this section. The MPPI controller makes use of the discrete-time dynamics

$$\mathbf{z}_{t+1} = \mathbf{f}(\mathbf{z}_t, \mathbf{v}_t), \quad (14)$$

where $\mathbf{v}_t$ is a random control input vector with distribution $\mathbf{v}_t \sim \mathcal{N}(\mathbf{u}_t, \Sigma)$. The input control vector can be equivalently written as $\mathbf{v}_t = \mathbf{u}_t + \boldsymbol{\epsilon}_t$, where $\boldsymbol{\epsilon}_t$ is sampled from the zero-mean normal distribution, $\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \Sigma)$. The optimization objective at each time step is to determine the mean control sequence, $\mathbf{U}_t = \{\mathbf{u}_t, \mathbf{u}_{t+1}, \dots, \mathbf{u}_{t+T}\}$, which minimizes the cost functional

$$\mathbf{J}(\mathbf{U}) = \mathbb{E} \left[\phi(\mathbf{z}_{t+T}) + \sum_{k=t}^{t+T-1} q(\mathbf{z}_k) + \lambda \mathbf{u}_k^\top \Sigma^{-1} (\mathbf{v}_k - \mathbf{u}_k) \right]. \quad (15)$$

At each time step, t , given the current state $\mathbf{z}_t$ and previous control sequence $\mathbf{U}_{t-1}$, thousands of trajectories are sampled in parallel, each computed from a separate control input realization $\mathbf{V}_t^i = \{\mathbf{v}_t^i, \mathbf{v}_{t+1}^i, \dots, \mathbf{v}_{t+T}^i\}$, where $\mathbf{v}_t^i = \mathbf{u}_t + \boldsymbol{\epsilon}_t^i$. The state-dependent cost-to-go for the i -th trajectory is defined as

$$S(\mathbf{V}_t^i) = \phi(\mathbf{z}_{t+T}^i) + \sum_{k=t}^{t+T-1} q(\mathbf{z}_k^i). \quad (16)$$

Then, defining the cost for each trajectory as

$$C(\mathbf{V}_t^i) = S(\mathbf{V}_t^i) - \sum_{k=t}^{t+T-1} \mathbf{u}_k^\top \Sigma^{-1} \boldsymbol{\epsilon}_k^i \quad (17)$$

with minimum $\beta = \min_i C(\mathbf{V}_t^i)$, the cost is mapped to a trajectory weight via

$$w(\mathbf{V}_t^i) = \exp \left[-\frac{1}{\lambda} (C(\mathbf{V}_t^i) - \beta) \right]. \quad (18)$$

The mean of the optimal control distribution is then approximated by

$$\mathbf{u}_t^* = \mathbf{u}_t + \frac{1}{\sum_{i=1}^{N_s} w(\mathbf{V}_t^i)} \sum_{i=1}^{N_s} w(\mathbf{V}_t^i) \boldsymbol{\epsilon}_t^i, \quad (19)$$

where N_s is the number of sampled trajectories.

The sampling and trajectory propagation are parallelized on a GPU in order to obtain thousands of sampled trajectories in real-time. Because of the sample-based optimization, the state dependent cost terms on the right hand side of (16) can be arbitrarily complex, and are not required to be continuous or differentiable. This feature makes the MPPI architecture an attractive candidate for motion planning with acoustic constraints, since many acoustic models may be highly nonlinear and/or based on lookup tables or empirical data.

B. Application to Motion Planning with Acoustic Constraints

To use the MPPI controller for trajectory planning with acoustic constraints, we discretize the vehicle dynamics and define the state vector to be

$$\mathbf{z}_t = \begin{bmatrix} \mathbf{x}_{i,t}^\top & \mathbf{v}_{i,t}^\top & \mathbf{q}_t^\top & \mathbf{x}_{c,t}^\top \end{bmatrix}^\top, \quad (20)$$

where $\mathbf{x}_{c,t} = \begin{bmatrix} T_t & \alpha \end{bmatrix}^\top$ are control pseudostates, and the angular rates are again omitted since they are constrained by the coordinated flight assumption. The discretized dynamics are propagated as

$$\mathbf{z}_{t+1} = \mathbf{z}_t + \begin{bmatrix} \mathbf{v}_{i,t} & \mathbf{g} + \mathbf{R} \mathbf{a}_{v,t} & \frac{1}{2} \mathbf{q}_t \otimes \begin{bmatrix} 0 \\ \boldsymbol{\omega}_{v,t} \end{bmatrix} & \begin{bmatrix} \dot{T} & \dot{\alpha} \end{bmatrix} \end{bmatrix} \Delta t. \quad (21)$$

The sampled controls are then defined as $\mathbf{v}_t = \begin{bmatrix} \dot{T}_t & \dot{\alpha} & p_{s,t} \end{bmatrix}^\top$. Defining the sampled input as the roll rate and the derivative of thrust and angle of attack in this manner is advantageous for two reasons. (i) The control can be initialized to zero anywhere in the state space. After the optimal control is computed, the first control input(s) in $\mathbf{U}_t$ is sent to the

vehicle. At the next time step, the control is shifted such that the first unused control input is the first entry of U_{t-1} , and a new control input is appended to the end of U_{t-1} such that $U_{t-1} = \{u_t, \dots, u_{t+T} = u_{\text{init}}\}$. If a nonzero trim control input is required, but the value varies throughout the state space, the exploration provided by the control variance Σ may not be sufficient to recover the trim control. It is therefore helpful to have an initialization that is consistent throughout the state space, especially if no trim command look-up is available. Similarly, if the vehicle is operating in a continuous highly dynamic maneuver, initializing with the last control applied (zero derivative) may improve performance. (ii) Control (position) constraints for angle of attack can be treated as state constraints. In this formulation of information theoretic MPPI, there are limited options for directly constraining the control inputs [9]. By treating the angle of attack as a pseudostate, the constraint can be incorporated into the state cost (16), either as an indicator function or as a “soft” constraint. However, the approach here may not be ideal for every application, as adding integrators between the sampled controls and the vehicle control states may limit the exploration of the state space in the sampled trajectories.

The acoustic model and constraint are incorporated directly into the running cost, q , as are any applicable state constraints. Depending on the relative weighting of performance and constraint costs, as well as the type of constraint penalty (linear, quadratic, indicator function, etc.), constraints may not be satisfied exactly.

VI. Simulation Results

In this section, we present the results of a simulation example to demonstrate the proposed trajectory generation method. The simulation is conducted in a C++ environment, with the vehicle dynamics running at 200 Hz and the controller running at 20 Hz. The vehicle dynamics, without the control pseudostates, are identical to the dynamics used in the MPPI controller. Integration of the pseudostates is handled by a simple integrator control law external to the MPPI controller. In this example, the vehicle is tasked with flying straight and level along a specific (linear) spatial path. The vehicle starts offset to the desired path, and there are two acoustic observers on the ground plane in the vicinity of the flight path. The first is a point observer located at the position $x_{obs,1} = [225 \ 15 \ 0]^T$ and the second observer is a volume enclosed by an octodecahedron centered at $x_{obs,2} = [375 \ -15 \ -5]^T$. The running cost for this example is chosen as a linear-quadratic form:

$$q(z_t) = \frac{1}{N_s} (\xi_k - \xi_d)^T Q_2 (\xi_k - \xi_d) + \|\max\{\mathbf{0}, Q_1 (\xi_k - \xi_d)\}\|_1, \quad (22)$$

where

$$\xi = \begin{bmatrix} x_i^T & V & OASPL^T \end{bmatrix}^T, \quad (23)$$

$$Q_2 = \text{diag} \left(\begin{bmatrix} 0 & 100 & 100 & 10e3 & 0 & 0 \end{bmatrix} \right), \quad (24)$$

$$Q_1 = \text{diag} \left(\begin{bmatrix} 0 & 0 & 0 & 0 & 200e3 & 200e3 \end{bmatrix} \right), \quad (25)$$

$$\xi_d = \begin{bmatrix} 0 & 10 & -10 & 27.5 & 65 \end{bmatrix}^T, \quad (26)$$

and the terminal cost is the same as the running cost, given by $\phi(z_{t+T}) = q(z_{t+T})$. Table 1 provides other relevant simulation parameters, including initial conditions, control parameters, and vehicle properties.

Figures 2, 3, and 4 show the flight path and acoustic footprint of the vehicle. In the left half of the figures, the term containing Q_1 in (22) is omitted and no acoustic constraint is enforced. As a result, the vehicle quickly moves to the desired flight path and, in the process, the acoustic constraint of 60 dB is exceeded. The 60 dB acoustic limit is also exceeded at observer 2 to an even greater degree. In the right half of Figures 2, 3, and 4 the full cost function (22) is used, enforcing the acoustic constraint. The flight path deviates from the nominal trajectory in order to reduce the OASPL at both observers. The OASPL at each observer, with and without the acoustic constraint, is shown in Figure 5. From Figure 5, it is clear that the trajectory deviates from the desired flight path only enough to meet the acoustic constraint. The aircraft speed, target speed and flight path angle are displayed in Figure 6. A larger deviation from the desired flight speed can be observed with the acoustic constraint enforced. Figure 7 shows roll and heading attitude of the velocity frame. The control states are visible in Figure 8, where we note the significant drops in thrust coincident with times that the vehicle passes closest to the acoustic observers to reduce the OASPL.

Table 1 Simulation input parameters

Description	Units	Value	Description	Units	Value
Initial Position, $\mathbf{x}_i$	m	$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$	# Samples, N_s	—	4096
Initial Velocity, $\mathbf{v}_i$	m/s	$\begin{bmatrix} 25 & 0 & 0 \end{bmatrix}^T$	Time Horizon, T	s	12.8
Initial AoA, α	deg	4.84	Sim Δt	s	0.005
Initial Thrust, T	N	199.45	Control Δt	s	0.05
Initial Attitude, $\mathbf{q}$	—	$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$	Mass	kg	28.31
Initial Control, $\begin{bmatrix} \dot{T} & \dot{\alpha} & p_s \end{bmatrix}^T$	$N/s, \text{deg}/s$	$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$	Wing Area, S	m^2	0.737
Control Covariance, Σ	$(N/s), (\text{deg}/s)$	$\text{diag} \left(\begin{bmatrix} 2 & 1 & 2 \end{bmatrix} \right)$	C_{N_0}	—	0.5
λ	—	1.0	C_{N_α}	rad^{-1}	5.7
Ref. Prop. Tip Mach#, $M_{t,ref}$	—	0.4	C_{A_0}	—	0.6
Ref. Distance, r_{ref}	m	30	$C_{A_{\alpha^2}}$	rad^{-2}	3.4
Prop. Diameter, d	m	0.4064	Ref. OASPL	dB	53.2

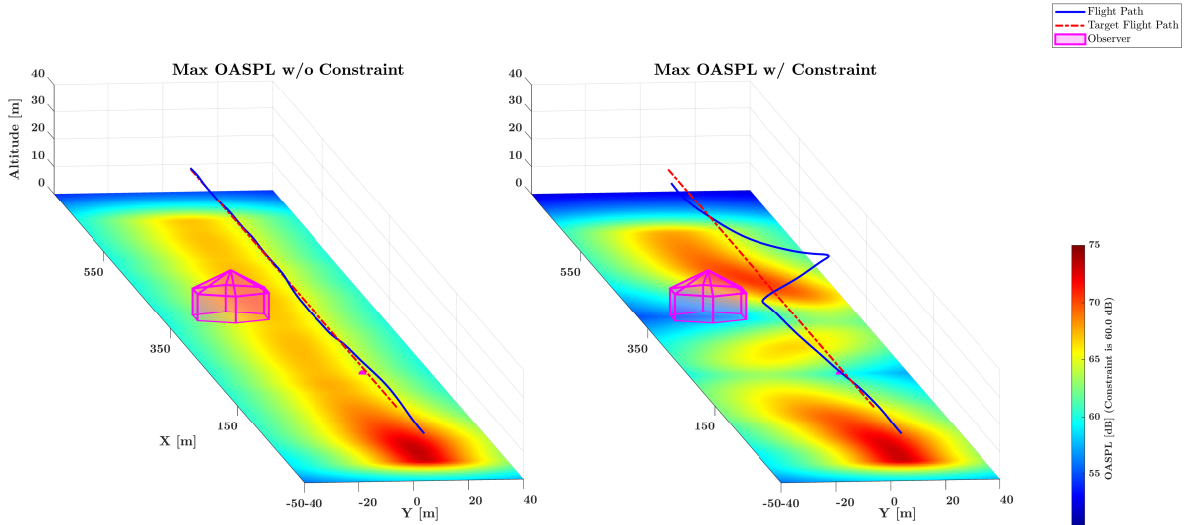


Fig. 2 Acoustic footprint on the ground plane.

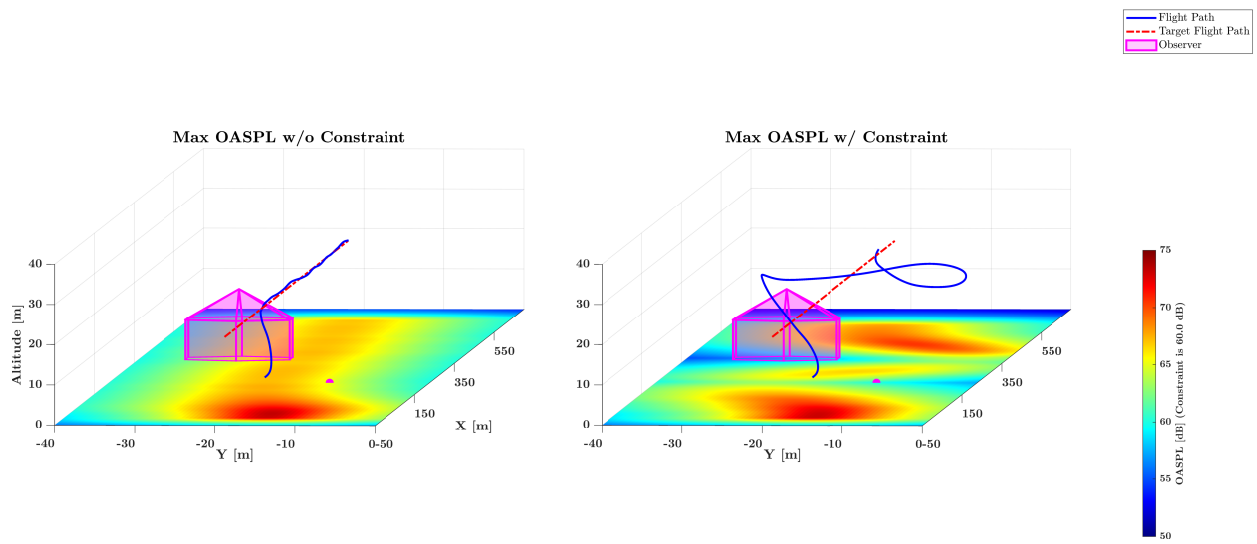


Fig. 3 Acoustic footprint on the ground plane, alternative view.

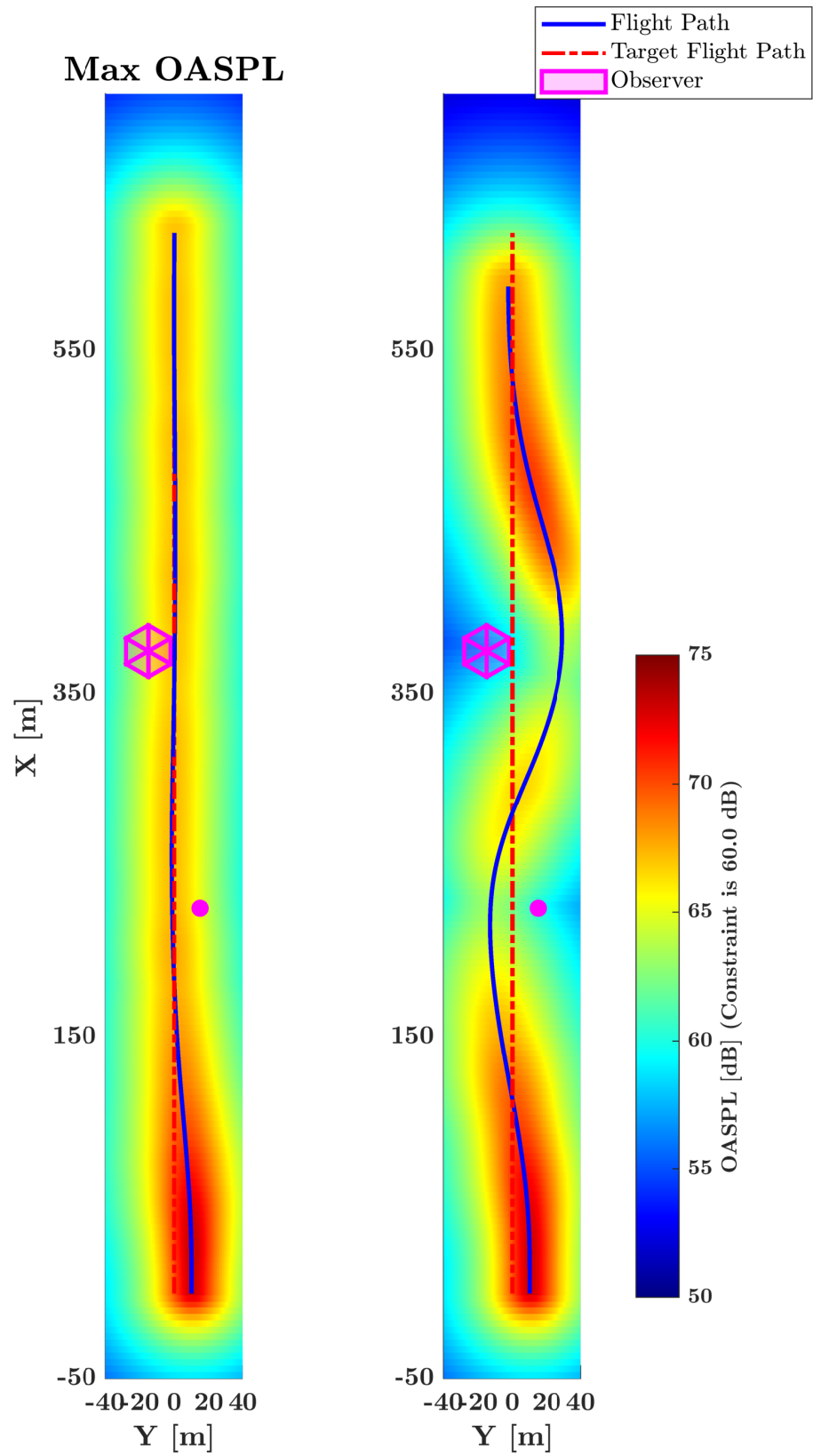


Fig. 4 Acoustic footprint on the ground plane, alternative view.

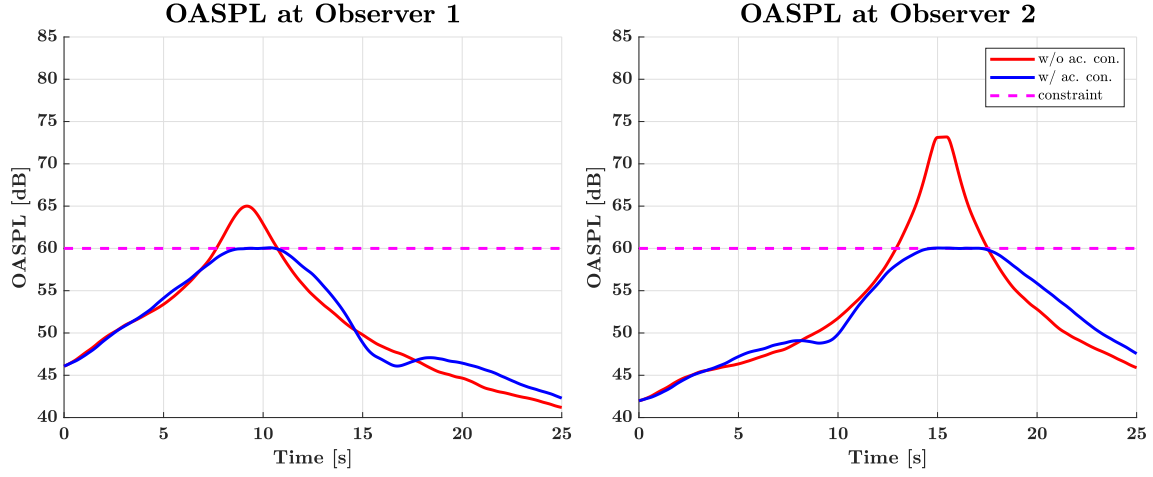


Fig. 5 OASPL value at each observer. Note that propagation delay is not accounted for in these plots.

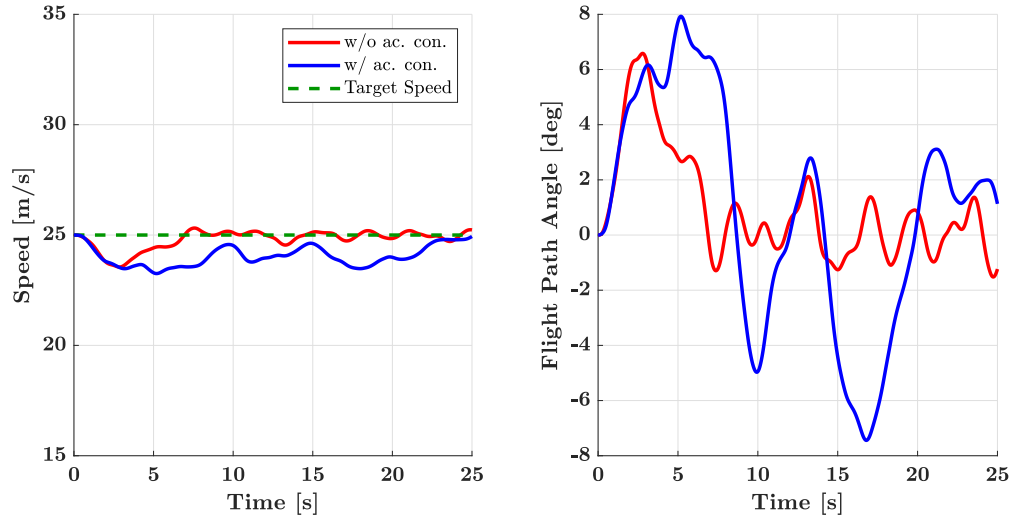


Fig. 6 Total speed and flight path angle. The desired speed of 25 m/s is shown in the left plot.

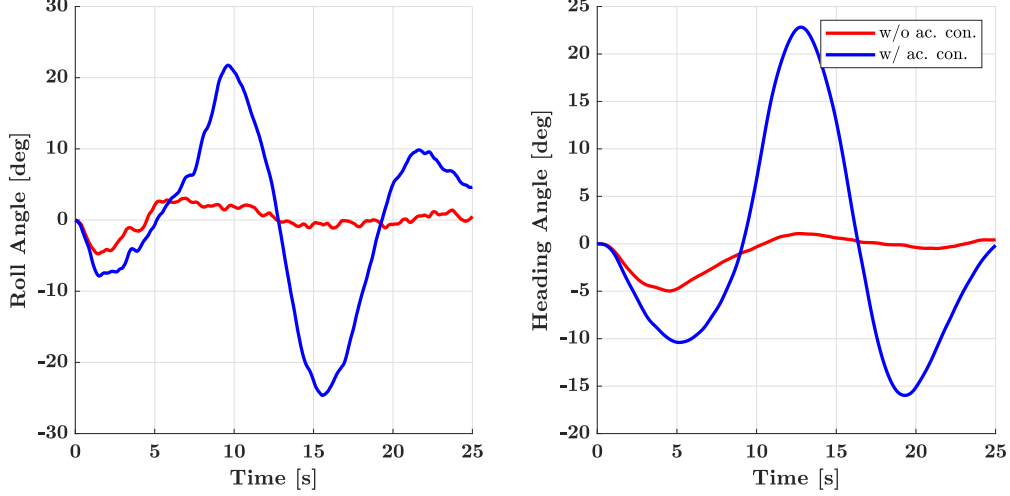


Fig. 7 Roll and heading of the velocity frame. The pitch attitude of the velocity frame is the same as the flight path angle.

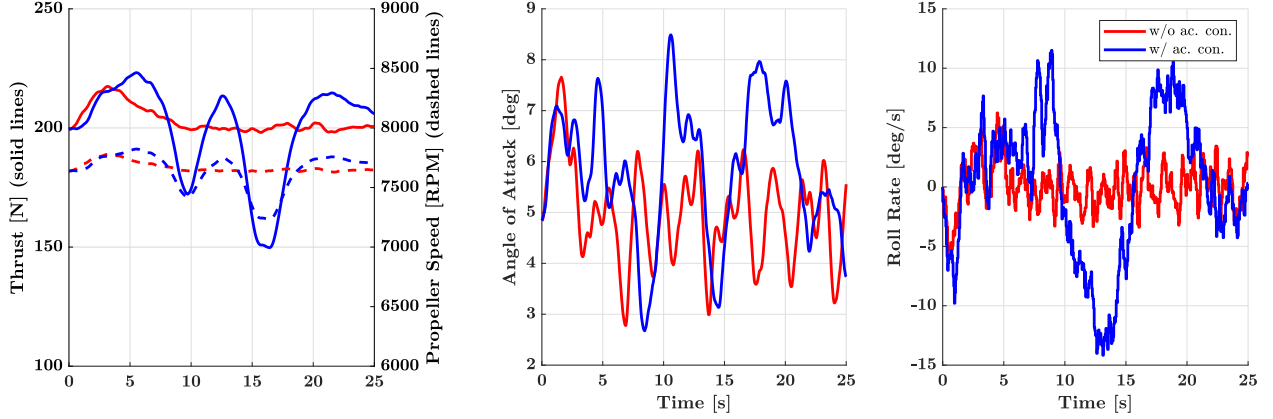


Fig. 8 Control states thrust, angle of attack, and stability-axis roll rate. The propeller speed, related to thrust by (7), is also shown in the leftmost plot.

VII. Conclusion

This paper presents a method for real-time motion planning to satisfy acoustic constraints at multiple (three-dimensional) observer locations. An acoustic model and metric are incorporated into the cost function of a Model Predictive Path Integral control law, which is applied to a fixed-wing, distributed electric propulsion aircraft dynamics with a coordinated flight constraint. The proposed control law places no restrictions on the form of the state-dependent portion of the cost functional and is suitable for use with sophisticated acoustic source model and metrics. A simulation example demonstrates the ability of the control law to modify trajectories as required to satisfy the noise constraints at multiple measurement locations. Future efforts are focused on incorporation of hemisphere-based acoustic models, noise directivity control, and higher-fidelity vehicle dynamics.

Acknowledgments

The authors would like to thank Dr. Kyle Pascioni of the Aeroacoustics Branch at NASA Langley Research Center for providing the acoustic model used in this work and Javier Puig Navarro of the Autonomous Integrated Systems Research Branch at NASA Langley Research Center for providing a distance query algorithm used to compute the minimum distance to polytopic acoustic observers.

References

- [1] Vascik, P. D., and Hansman, R. J., “Scaling Constraints for Urban Air Mobility Operations: Air Traffic Control, Ground Infrastructure, and Noise,” *2018 Aviation Technology, Integration, and Operations Conference*, 2018. AIAA 2018-3849.
- [2] Whiteside, S. K. S., Zawodny, N. S., Fei, X., Pettingill, N. A., Patterson, M. D., and Rothhaar, P. M., “An Exploration of the Performance and Acoustic Characteristics of UAV-Scale Stacked Rotor Configurations,” *AIAA Scitech Forum*, 2019. AIAA 2019-1071.
- [3] Greenwood, E., “Helicopter Flight Procedures for Community Noise Reduction,” *73rd American Helicopter Society Annual Forum*, 2017.
- [4] Pascioni, K. A., Rizzi, S. A., and Schiller, N., “Noise Reduction Potential of Phase Control for Distributed Propulsion Vehicles,” *AIAA Scitech Forum*, 2019. AIAA 2019-1069.
- [5] Patterson, A., Ackerman, K., Gahlawat, A., Hovakimyan, N., Schiller, N. H., and Gregory, I. M., “Controller Design for Propeller Phase Synchronization with Aeroacoustic Performance Metrics,” *AIAA Scitech Forum*, 2020. AIAA 2020-1494.
- [6] Ackerman, K. A., and Gregory, I. M., “Trajectory Generation for Noise-Constrained Autonomous Flight Operations,” *AIAA SciTech Forum*, 2020. AIAA 2020-0978.
- [7] Williams, G., Drews, P., Goldfain, B., Rehg, J. M., and Theodorou, E. A., “Aggressive Driving with Model Predictive Path Integral Control,” *International Conference on Robotics and Automation*, IEEE, 2016, pp. 1433–1440.
- [8] Williams, G., Aldrich, A., and Theodorou, E. A., “Model Predictive Path Integral Control: From Theory to Parallel Computation,” *Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 2, 2017, pp. 344–357.
- [9] Williams, G., Drews, P., Goldfain, B., Rehg, J. M., and Theodorou, E. A., “Information-Theoretic Model Predictive Control: Theory and Applications to Autonomous Driving,” *Transactions on Robotics*, Vol. 34, No. 6, 2018, pp. 1603–1622.
- [10] Rothhaar, P. M., Murphy, P. C., Bacon, B. J., Gregory, I. M., Grauer, J. A., Busan, R. C., and Croom, M. A., “NASA Langley Distributed Propulsion VTOL Tilt-Wing Aircraft Testing, Modeling, Simulation, Control, and Flight Test Development,” *14th AIAA Aviation Technology, Integration, and Operations Conference*, 2014. AIAA 2014-2999.
- [11] Hauser, J., and Hindman, R., “Aggressive Flight Maneuvers,” *IEEE Conference on Decision and Control*, Vol. 5, 1997, pp. 4186–4191.
- [12] Diebel, J., “Representing Attitude: Euler Angles, Unit Quaternions, and Rotation Vectors,” Tech. rep., Stanford University, 2006.
- [13] Murphy, P. C., and Landman, D., “Experiment Design for Complex VTOL Aircraft with Distributed Propulsion and Tilt Wing,” *AIAA Atmospheric Flight Mechanics Conference*, 2015. AIAA 2015-0017.
- [14] Pascioni, K., and Rizzi, S. A., “Tonal Noise Prediction of a Distributed Propulsion Unmanned Aerial Vehicle,” *2018 AIAA/CEAS Aeroacoustics Conference*, 2018. AIAA 2018-2951.
- [15] Nguyen, L. C., and Kelly, J. J., “A Users Guide for the NASA ANOPP Propeller Analysis System,” Tech. Rep. NASA-CR-4768, NASA, 1997.
- [16] McCurdy, R. G., “Tentative standards for sound level meters,” *Electrical Engineering*, Vol. 55, No. 3, 1936, pp. 260–263.
- [17] Williams, G., Wagener, N., Goldfain, B., Drews, P., Rehg, J. M., Boots, B., and Theodorou, E. A., “Information theoretic MPC for model-based reinforcement learning,” *2017 IEEE International Conference on Robotics and Automation (ICRA)*, IEEE, 2017, pp. 1714–1721.
- [18] Pravitra, J., Ackerman, K. A., Cao, C., Hovakimyan, N., and Theodorou, E. A., “L1-Adaptive MPPI Architecture for Robust and Agile Control of Multirotors,” *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2020.