

# New Developments in the CompDam Fiber Kinking Model for the Interaction of Kinking and Splitting Cracks with Application to Open Hole Compression Specimens

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This paper describes two new additions to the CompDam fiber kinking model: 1) a failure index for fiber kinking to facilitate interaction between kinking and matrix cracking and 2) a strain-based criterion for fiber rupture to account for fiber breakage before or after fiber kinking. The model formulation is described with the aid of the graphical Considère construction, which is used to determine the conditions for fiber kinking. Verification studies are conducted with single element models to show the behavior of the model in light of the new developments and to evaluate the effect of numerical parameters of the model. The model is used to analyze open hole compression (OHC) specimens with soft, quasi-isotropic, and hard layups. Analysis results are shown for a conventional continuum damage mechanics model using a trilinear constitutive law for fiber damage and for the proposed model to highlight the differences between the two. Comparison with test data highlights the capabilities and remaining gaps in the quantitative prediction of the damage and failure response. While both models can predict many features of the OHC response, accurate prediction of kinking onset in the OHC specimens remains elusive. The results suggest that it is necessary to predict the combined effects of compression and shear loading as well as the structural effects (e.g., ply thickness, strain gradient effect) accurately to match the experimental observations of fiber kinking onset in the three OHC laminates.

## I. Introduction

ACCURATE mesoscale progressive damage analysis (PDA) simulation requires not only accurate physically-based models for each damage mode, but also careful consideration for damage mode interaction. Recent activities supported by NASA's Advanced Composites Project (ACP) have been focused on developing and validating accurate PDA methods for prediction of strength and life of post-buckled stiffened panels [1]. Studies conducted under the ACP have shown that capturing the kinematic interaction of delamination and matrix cracking is essential for predicting skin-stiffener separation [1–3]. In cases where longitudinal compression failure can occur, fiber kinking and fiber rupture must be considered in addition to matrix cracking and delamination. Mesoscale models that incorporate fiber kinking theory (FKT) [4–10] show promise for this purpose, since they include the kinematics of fiber kinking; namely the role of local fiber rotation and shear nonlinearity on the compressive load carrying capability.

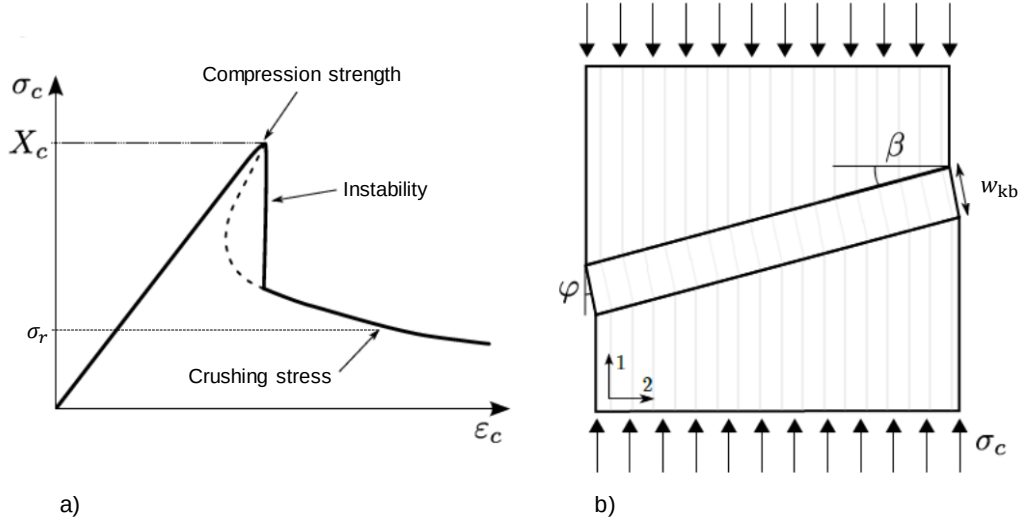
The FKT introduced by Budiansky offers a physically-based model for kink band initiation and propagation [11–13]. FKT considers the relevant mechanisms in kink band formation as a combination of an infinitely-wide band of initially misaligned fibers, nonlinear shear stress-strain behavior, and large fiber rotation. FKT produces the characteristic nonlinear constitutive law shown in Fig. 1a. The FKT stress-strain law shows a nearly linear response to the strength,  $X_c$ , followed by a rapid reduction in stress to a residual “crush stress” level. The kink band is idealized, as shown in Fig. 1b, with a fiber misalignment angle,  $\phi$ , kink band angle,  $\beta$ , and kink band width,  $w_{kb}$ . If the shear nonlinearity follows a Ramberg-Osgood [14] behavior,  $\gamma_{12} = (\tau_{12} + \alpha\tau_{12}^\eta)/G_{12}$ , FKT predicts the longitudinal strength is

$$X_c = \frac{G_{12}}{1 + \eta\alpha^{1/\eta} \left( \frac{G_{12}\phi}{\eta - 1} \right)^{\frac{\eta-1}{\eta}}} \quad (1)$$

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where  $G_{12}$  is the in-plane shear modulus,  $\alpha$  and  $\eta$  define the shape of the shear stress-strain curve, and  $\bar{\varphi}$  is an initial fiber misalignment [12]. Under a combined stress state, the peak stress from FKT varies with magnitudes of the shear and transverse stress components.



**Figure 1. The (a) characteristic constitutive response predicted by fiber kinking theory and (b) idealization of the kink band.**

While FKT has been used successfully to predict fiber kinking in various applications, it has not yet been incorporated into a general purpose mesoscale PDA method and validated for problems where consideration of damage mode interaction is essential. The author [4–6] and Costa, Gutkin, and Olsson [7–9, 15] have made contributions towards this long-term objective over the past few years, building on more than four decades of research into fiber kinking.

Recently, a constitutive model for fiber compression based on FKT was proposed and implemented in the continuum damage mechanics (CDM) code, CompDam [16]. CompDam is a research code developed to help improve understanding of material modeling requirements for CDM methods. Efforts to validate the CompDam FKT model through comparison with experimental test data and high fidelity micromechanical models have demonstrated that the model accurately predicts stiffness, strength, residual crush stress, and kinking location [4–6, 17]. However, relatively little attention has been given to the interaction between fiber kinking and matrix cracking. In addition, the model has not yet been applied to open hole compression (OHC) specimens, a common benchmark used in evaluation of PDA models that has proven to be difficult to predict accurately, especially for soft shear-dominated layups [18].

The CompDam code has two models for fiber compression failure: a conventional CDM approach using a trilinear stress-strain law, and the FKT model mentioned above. A validation study for OHC specimens using the conventional CDM approach for fiber damage has been reported in [19]. In general, the test vs. analysis correlation using the conventional CDM approach was found to be very good. OHC strength was within 3% error for two of the three layups. However, in the soft shear-dominated layup, the error in strength was around 10% and the quasi-brittle nature of the response was not predicted. Furthermore, despite qualitative agreement in pre-peak damage patterns in some cases, quantitatively accurate agreement was not achieved. One reason for the differences between test and analysis is that the conventional CDM approach for fiber compression ignores the kinematics of fiber kinking. Therefore, the focus of this study is on application of the CompDam FKT model to OHC specimens to evaluate its potential for improved predictions as compared with the conventional trilinear stress-strain law.

In this paper, the mesoscale fiber kinking model proposed by the author is extended with two main new additions: 1) a failure index for fiber kinking is developed to facilitate interaction between kinking and matrix cracking and 2) a criterion for fiber rupture is introduced to account for fiber breakage. In previous studies using the CompDam FKT model for structural analysis [4–6, 17], the finite element models were constructed such that whether an element was able to kink or matrix crack was defined *a priori* since no failure index was available for fiber kinking. In such an implementation, competition between kinking and splitting cannot occur. To overcome this limitation, a failure index for fiber kinking is implemented, which allows competition between the kinking and splitting failure modes up to the point of onset. When the criteria for one of the two modes is satisfied, propagation occurs by the critical mechanism. The second development builds on the work in [17] wherein a lock-up criterion based on fiber rotation was proposed

to model the post-kinking behavior. Herein, the fiber rotation is related to strain in the fibers in order to establish a physical basis for the criterion. To demonstrate the new developments, several verification studies are provided. Then, to evaluate the overall model performance in a laminate, the model is applied to predict OHC strength and damage evolution for three different layups. Particular attention is given to the predicted behavior under combined compression and shear stress.

The paper is organized as follows. The constitutive formulation and implementation details are discussed in Section II. Section III describes simple single-element verification studies that illustrate the behavior of the model under various conditions. Finally, in Section IV, the mesoscale model is applied for prediction of the damage evolution behavior and ultimate strength of OHC specimens. The model predictions are assessed via comparison with results obtained using the conventional CDM model for fiber damage and with test data. The capabilities of the kinking model are discussed along with its limitations.

## II. Model Formulation

A mesoscale constitutive model based on Budiansky's fiber kinking theory [11–13] with  $\beta = 0$  is formulated in this section with the aim of representing the fiber kinking phenomenon using geometric nonlinearity and shear nonlinearity. Kinking is considered in two planes: the 1-2 plane for in-plane kinks and the 1-3 plane for out-of-plane kinks. The model includes the kinematics of the fiber kinking process by tracking fiber misalignment throughout loading. The characteristic constitutive response shown in Fig. 1a is not directly prescribed in the model. Instead, the response is a result of the shear nonlinearity and large rotation of the fiber direction. The model is formulated considering finite deformations and implemented as a VUMAT material subroutine for Abaqus/Explicit [16].

### A. Fiber kinking constitutive model

Consider a material point with a reference frame,  $\mathbf{X}_i$ . Initial fiber misalignment angles,  $\bar{\varphi}_{12}$  in the 1-2 plane and  $\bar{\varphi}_{13}$  in the 1-3 plane are assumed such that a fiber-aligned reference frame,  $\mathbf{X}'_i$ , is defined with rotation,  $\mathbf{R}$ , as

$$\mathbf{X}' = \mathbf{R}\mathbf{X} \quad (2)$$

where

$$\mathbf{R} = \mathbf{R}_{12}\mathbf{R}_{13} \quad (3)$$

$$\mathbf{R}_{12} = \begin{bmatrix} \cos \bar{\varphi}_{12} & -\sin \bar{\varphi}_{12} & 0 \\ \sin \bar{\varphi}_{12} & \cos \bar{\varphi}_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$\mathbf{R}_{13} = \begin{bmatrix} \cos \bar{\varphi}_{13} & 0 & -\sin \bar{\varphi}_{13} \\ 0 & 1 & \cos \bar{\varphi}_{13} \\ \sin \bar{\varphi}_{13} & 0 & 0 \end{bmatrix} \quad (5)$$

The initially misaligned fiber is shown in Fig. 2. The total initial misalignment,  $\bar{\varphi}$ , results from the components in the 1-2 plane  $\bar{\varphi}_{12}$ , and the 1-3 plane,  $\bar{\varphi}_{13}$ . In this way, a generic misalignment of the fiber from the nominal 1-direction can be represented in three dimensions.

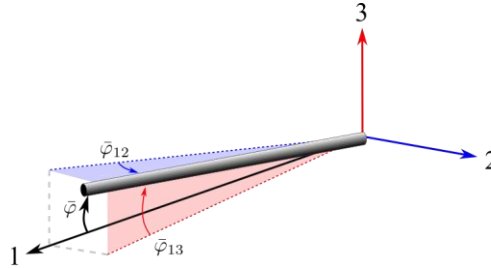


Figure 2. Initial fiber misalignment.

The material model computes the stress state given the current deformation, state variables, and material properties as follows. For a given deformation,  $\mathbf{F}$ , the Green-Lagrange strain,  $\mathbf{E}$ , is calculated as

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I}) \quad (6)$$

The strain is written in the misaligned (fiber) coordinates as

$$\mathbf{E}' = \mathbf{R}^T \mathbf{E} \mathbf{R} \quad (7)$$

Shear nonlinearity is accounted for in the  $\mathbf{X}'_1, \mathbf{X}'_2$  and  $\mathbf{X}'_1, \mathbf{X}'_3$  coordinates using a simple plasticity model based on the Ramberg-Osgood stress-strain law [14] as follows. The plastic strain tensor is

$$\mathbf{E}'_{PL} = \frac{\gamma_{12}^{PL}}{2} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{\gamma_{13}^{PL}}{2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (8)$$

where the plastic portion of the shear strain,  $\gamma_{1i}^{PL} = \gamma_{1i} - \gamma_{1i}^{EL}$ , is found iteratively using

$$\gamma_{1i} = \frac{1}{G_{1i}} [\tau_{1i} + \text{sign}(\tau_{1i}) \alpha_i |\tau_{1i}|^{\eta_i}] \quad (9)$$

where  $\tau_{1i}$  is the shear stress and  $G_{1i}$ ,  $\alpha_i$ , and  $\eta_i$  are the material properties that define the Ramberg-Osgood curve for  $i = 2, 3$ . Herein the shear nonlinearity is assumed to follow the same law in both planes, thus

$$\begin{aligned} \alpha_2 &= \alpha_3 = \alpha \\ \eta_2 &= \eta_3 = \eta \end{aligned} \quad (10)$$

Since it has been shown experimentally that the shear response is similar in the 1-2 and 1-3 planes for IM7/8552 [20], the assumption seems reasonable for traditional unidirectional materials. However, in general, the shear response in the two planes may differ.

The calculations are performed using the second Piola-Kirchoff stress (2PK) in the fiber-aligned reference frame,  $\mathbf{S}'$  and then the result is transformed back to Cauchy stress. The 2PK stress is calculated using the elastic stiffness tensor,  $\mathbf{C}$

$$\mathbf{S}' = \mathbf{C} : (\mathbf{E}' - \mathbf{E}'_{PL}) \quad (11)$$

with

$$\mathbf{C} = \mathbf{H}^{-1} \quad (12)$$

$$\mathbf{H} = \begin{bmatrix} \frac{1}{E_1^*} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1^*} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{13}}{E_1^*} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix} \quad (13)$$

using the typical elastic constants. Fiber nonlinearity is accounted for following Kowalski [21] as

$$E_1^* = E_1 (1 + c_l \varepsilon_{11}) \quad (14)$$

where  $c_l$  is the nonlinearity coefficient for the ply and is an additional material property to be obtained from test data. In the reference frame  $\mathbf{X}_i$ , the second Piola-Kirchoff stress is

$$\mathbf{S} = \mathbf{R} \mathbf{S}' \mathbf{R}^T \quad (15)$$

Finally, the Cauchy stress is calculated.

$$\boldsymbol{\sigma} = \mathbf{F} \mathbf{S} \mathbf{F}^T |\mathbf{F}|^{-1} \quad (16)$$

### B. Modified shear nonlinearity law

Application of the model described above has revealed that the shear strain can become quite large (>10%) in certain cases, at which point the shear stiffness is small, and therefore numeric difficulties can arise. To circumvent this problem, a minimum stiffness is used as other authors have proposed previously, e.g., reference [22].

The current implementation is compatible with most strain-hardening shear nonlinearity models and therefore is stated in a general way. Assume the constitutive law for shear behavior is defined as  $\tau(\gamma, s_i)$  where  $\gamma$  is the shear strain and  $s_i$  is the set of state variables. The minimum shear stiffness is defined as a parameter,  $\tilde{G}$ . It is assumed that the corresponding strain  $\tilde{\gamma}$ , beyond which the stiffness is held constant, can be calculated for the particular nonlinear constitutive law. The constitutive behavior with the minimum stiffness constraint imposed is thus

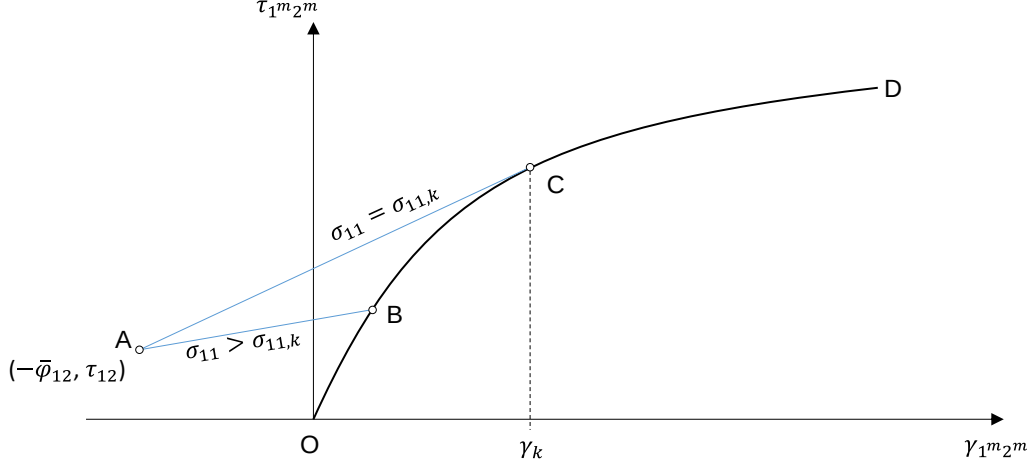
$$\tau_{ms} = \begin{cases} \tau(\gamma, s_i) & \text{for } \gamma \leq \tilde{\gamma} \\ \tilde{G}\gamma + b & \text{for } \gamma > \tilde{\gamma} \end{cases} \quad (17)$$

and referred to as  $\tau_{ms}$  for minimum stiffness. In Eq. (17),  $\tilde{G} = \tau'(\tilde{\gamma})$ , which is to say that the minimum stiffness is equivalent to the slope of the constitutive law  $\tau' = \partial\tau/\partial\gamma$  calculated at  $\tilde{\gamma}$ . The constant  $b$  in Eq. (17) is  $b = \tau(\tilde{\gamma}) - \tau'(\tilde{\gamma})\tilde{\gamma}$ , which is obtained knowing the two curves are continuous in value and slope at  $\tilde{\gamma}$ . This modified shear law, is used in place of Eq. (9). Although Eq. (17) is written for  $\gamma > 0$ , the same approach applies for  $\gamma < 0$ .

### C. Fiber kinking criterion

The fiber kinking model as described in section A predicts fiber kinking behavior in terms of the nonlinear constitutive behavior (Fig. 1a) but it does not provide a direct indication of kinking onset under a combined stress-state. In other words, while the constitutive model reaches a stress peak at kinking onset, there is no parameter in the model that maps to this event. To exert control over how the fiber kinking model interacts with models for other damage modes in a CDM framework, it is necessary to know when fiber kinking will occur. For this purpose, a fiber kinking failure index is defined based on the instability condition in the FKT. The resulting criterion is essentially identical to the LaRC04 criterion for fiber kinking [23], with some additional numerical considerations introduced for solution stability. This section develops the fiber kinking criterion on which the failure index is based.

The FKT instability condition was developed as graphical construction by Batdorf and Ko [24] and then in the LaRC04 criteria [23], among others. The graphical construction compares the external driving force to the intrinsic material response and is referred to as a Considère construction. A notional Considère construction for fiber kinking is shown in Fig. 3. The shear stress-strain state is analyzed in the misaligned reference frame where the subscript  $1m_2m$  indicates the quantity is in the misaligned coordinate system. Two curves are considered: the material constitutive response and the driving force for fiber kinking. The curve OBCD is the strain-hardening shear constitutive law (a Ramberg-Osgood law herein). The line AB defines a driving force for kinking where the point A is located at the coordinate  $(-\bar{\varphi}_{12}, \tau_{12})$  with  $\tau_{12}$  being the shear stress in the original coordinate system. Point B is the current rotated shear strain and corresponding shear stress from the constitutive law  $(\gamma_{1m_2m}, \tau_{1m_2m})$ . For small values of  $\gamma_{1m_2m}$ , the slope of AB is  $\propto \sigma_{11}$ . In general, the curve AB is nonlinear as will be described below. The current misalignment is  $\varphi_{12} = \bar{\varphi}_{12} + \gamma_{1m_2m}$  where  $\gamma_{1m_2m}$  is the current rotated shear strain. As  $\sigma_{11}$  decreases (becomes more negative), the point B moves along the shear constitutive law toward point D. FKT states that fiber kinking occurs when line AB is tangent to the shear constitutive law. The longitudinal stress at fiber kinking is denoted  $\sigma_{11,k}$ , the corresponding shear strain in the misaligned frame is  $\gamma_k$ , and the current misalignment is  $\varphi_k = \bar{\varphi}_{12} + \gamma_k$ . When  $\bar{\varphi}_{12} = 0$ , the instability condition is satisfied when  $\sigma_{11} = G_{12}$ , recovering Rosen's [25] solution.



**Figure 3. Notional Considère construction for fiber kinking with  $\bar{\varphi}_{12}, \tau_{12} > 0$ .**

Pinho et al. formalized the mathematical condition for kinking onset for a 3D stress state as

$$\tau(\gamma, s_i) = -\frac{\sigma_{11} - \sigma_{22}}{2} \sin(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) + |\tau_{12}| \cos(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) \quad (18)$$

$$\tau'(\gamma, s_i) = -(\sigma_{11} - \sigma_{22}) \cos(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) - 2|\tau_{12}| \sin(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) \quad (19)$$

in LaRC04 [23], where, as written above, the kinking plane is assumed to be the 1-2 plane. The left hand sides (LHS) of Eqs. (18)–(19) are the shear constitutive law and its slope, respectively, and the right hand sides (RHS) are the driving force for kinking, which is the shear stress in the misaligned reference frame, and its slope. While Fig. 3 and Eqs. (18)–(19) are given here for the 1-2 plane, the corresponding equations are equally valid for the 1-3 plane replacing the subscript 2 with 3. Based on reference [6], the assumption is made here that fiber kinking occurs in either the 1-2 plane (in-plane) or the 1-3 plane (out-of-plane). It follows that given a transverse and shear stress state  $(\sigma_{22}, \tau_{12})$ , Eqs. (18)–(19) can be solved to find  $\sigma_{11} = \sigma_{11,k}$  and  $\gamma_{1^m 2^m} = \gamma_k$  when kinking occurs. As noted in LaRC04, finding a solution to these two equations can be challenging.

Two minor modifications to Eqs. (18)–(19) are introduced to facilitate a robust numerical solution. The first is to replace the shear constitutive law  $\tau$  with the modified shear constitutive law having a minimum stiffness  $\tau_{ms}$ . By enforcing a minimum stiffness in the shear constitutive law, the numerical solution to Eqs. (18)–(19) is more stable. As long as  $\tilde{\gamma}$  is sufficiently large, the results for practical values of stress and strain are unaffected. The second modification is to consider cases where  $\tau_{12}$  and  $\bar{\varphi}$  have opposite signs (removing the absolute value operator). Thus, Eqs. (18)–(19) are rewritten as

$$\tau_{ms}(\gamma, s_i) = -\frac{\sigma_{11} - \sigma_{22}}{2} \sin(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) + \tau_{12} \cos(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) \quad (20)$$

$$\tau'_{ms}(\gamma, s_i) = -(\sigma_{11} - \sigma_{22}) \cos(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) - 2\tau_{12} \sin(2(\bar{\varphi}_{12} + \gamma_{1^m 2^m})) \quad (21)$$

and will be used in this form throughout the rest of the paper. It is noted that linearizing the RHS of Eqs. (20)–(21) by assuming small angles has little effect on the solution for practical cases and is a convenient simplification for most applications. Nonetheless, the full nonlinear solution is pursued here for generality.

The solution to Eqs. (20)–(21) is found through Newton-Raphson iteration. As is typical, the robustness of the Newton-Raphson solution to Eqs. (20)–(21) is strongly dependent on the choice of the starting point. In most cases, an assumed starting point of  $\sigma_{11} = -X_c$  and  $\gamma_{1^m 2^m} = 0.02$  is quickly iterated to a solution. However, in some cases, nonphysical solutions are found or no solution may be found. For a solution to be physically relevant, it must satisfy

$$\begin{aligned} \sigma_{11,k} &< 0 \\ |\gamma_k| &\leq \tilde{\gamma} \end{aligned} \quad (22)$$

The first condition requires the material to be in compression, longitudinally. The second condition states that the shear strain at kinking onset must be no greater than the shear strain at which the constant stiffness is activated in the shear constitutive law. In the spirit of obtaining a robust numerical scheme, a strategy is implemented as follows to use an alternate starting point when the solution found using the default starting point does not satisfy Eq. (22). A set of candidate starting points are considered. The candidate starting points  $(\sigma_{11}^*, \gamma_{1^m 2^m}^*)$  are obtained with  $\sigma_{11, \min} \leq \sigma_{11}^* \leq -G_{12}$  where  $\sigma_{11, \min}$  is obtained by solving Eq. (21) for  $\sigma_{11}$  by setting  $\tau'_{ms} = \tilde{G}$  and  $\gamma_{1^m 2^m} = 0$ . The sampling strategy and number of values selected for  $\sigma_{11}^*$  are arbitrary; in this work, ten linearly spaced  $\sigma_{11}^*$  are used. For each  $\sigma_{11}^*$ , the corresponding value for  $\gamma_{1^m 2^m}^*$  is found by solving Eq. (21). Among these candidate starting points, the starting point with lowest error in Eqs. (20)–(21) is selected. Anecdotally, the revised starting point has always led to an admissible solution.

#### D. Binary condition specifying when kinking can occur under combined stresses

Based on the stress state, kinking may or may not be possible. Consider the four stress states A, B, C, and D shown in the Considère construction in Fig. 4. In the figure, the modified shear constitutive law with a minimum stiffness Eq. (17) is shown with the solid black line and the original Ramberg-Osgood law Eq. (9) is shown with the dashed black line. The two curves diverge at the point X when  $\gamma_{1^m 2^m} = \tilde{\gamma}$ . The four stress states have varying levels of  $\tau_{12}$ , with  $\sigma_{22} = 0$  (for simplicity). Stress state B is pure longitudinal compression and the line BB' is drawn such that B' is the point where kinking occurs. For stress state C, with  $(\tau_{12,C} > 0)$  the longitudinal stress for kinking is reduced compared with the pure compression case (evident by the lower slope of CC' compared with BB') and the shear strain at kinking (point C') is higher. As  $\tau_{12}$  is increased, eventually the shear strain at kinking onset is equal to  $\tilde{\gamma}$  (at point X). At yet higher  $\tau_{12}$ , kinking is not possible since there is no solution to Eq. (21). Consider stress state D, where no driving force curve exists that satisfies Eq. (21). By comparing the slope of the driving force curve DX to the minimum shear stiffness  $\tilde{G}$ , it is clear that kinking is not possible. Thus, it can be determined if kinking can occur for a given stress state in two steps. First, knowing  $\tau_{12}$ ,  $\sigma_{22}$ , and  $\bar{\varphi}_{12}$ , determine  $\tilde{\sigma}_{11}$  by solving Eq. (20) with  $\gamma_{1^m 2^m} = \tilde{\gamma}$

$$\tilde{\sigma}_{11} = \sigma_{22} - \frac{2\tau_{12} \cos(2(\bar{\varphi}_{12} + \tilde{\gamma})) - 2\tau_{ms}(\tilde{\gamma})}{\sin(2(\bar{\varphi}_{12} + \tilde{\gamma}))} \quad (23)$$

Then, if the slope of the driving force with  $\gamma_{1^m 2^m} = \tilde{\gamma}$  is greater than the minimum shear stiffness

$$-(\tilde{\sigma}_{11} - \sigma_{22}) \cos(2(\bar{\varphi}_{12} + \tilde{\gamma})) - 2\tau_{12} \sin(2(\bar{\varphi}_{12} + \tilde{\gamma})) \geq \tilde{G} \quad (24)$$

kinking may occur for the condition specified by  $\tau_{12}$ ,  $\sigma_{22}$ , and  $\bar{\varphi}_{12}$ . Otherwise kinking is not possible. Equation 24 enables a more robust and efficient numerical implementation since the kinking failure index described in the next section is only calculated when kinking is possible.

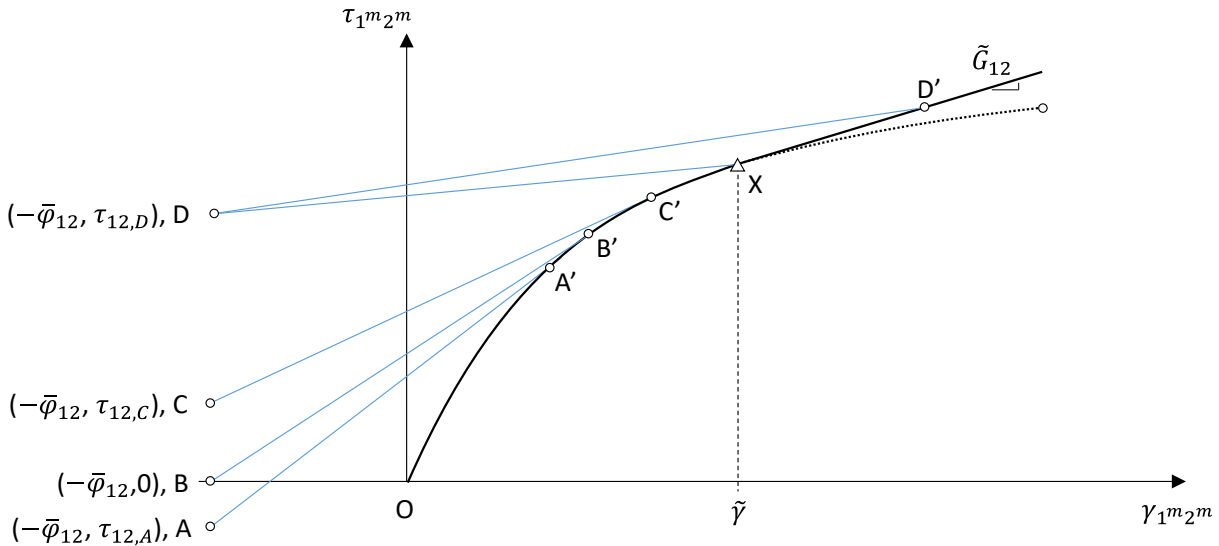


Figure 4. Considère construction considering different combinations of  $\tau_{12}$  and  $\sigma_{11}$ .

The stress state A is also included in Fig 4. to emphasize the behavior when  $\tau_{12}$  and  $\bar{\varphi}_{12}$  have opposite signs. In this case, the shear stress acts to reduce the fiber misalignment. For moderate values of  $\tau_{12}$ , the kinking stress increases above the kinking stress due to uniaxial longitudinal stress in a physically relevant way. When  $\tau_{12}$  and  $\bar{\varphi}_{12}$  have opposite signs, there is an additional condition that must be considered to ensure a physically relevant solution. When the coordinate  $(-\bar{\varphi}_{12}, \tau_{12})$  is to the left and above the shear stress-strain curve,  $\gamma_{1m_2m} > 0$ , otherwise  $\gamma_{1m_2m} < 0$ . With increasing  $\tau_{12}$  such that the fibers rotate towards a smaller misalignment, eventually the limit from Rosen [25] is reached where the kinking stress reaches a maximum of  $G_{12}$ . Of course, the fibers themselves break before reaching this theoretical limit. Therefore, a fiber rupture criterion described in section F is introduced.

### E. Kinking failure index

When kinking is possible, Eqs. (20)–(21) can be solved for  $\sigma_{11} = \sigma_{11,k}$  and  $\gamma_{1m_2m} = \gamma_k$ , thus specifying the state at kinking onset for the prescribed  $\tau_{12}$ ,  $\sigma_{22}$ , and  $\bar{\varphi}_{12}$ . From this solution, a simple failure index for kinking arises as the ratio of the current shear strain in the misaligned frame and the shear strain at kinking onset

$$FI_{\text{kink}} = \frac{\gamma_{1m_2m}}{\gamma_k} \quad (25)$$

When kinking is not possible,  $FI_{\text{kink}}$  is set to  $-1$ .

Since the kinking failure index is defined independently from matrix splitting cracks (or other relevant damage modes), the two damage modes compete until one failure index reaches a value of one, indicating damage onset. In contrast to existing approaches where the damage mode is pre-seeded early in the analysis [6] or a complicated system of equations is solved to check multiple damage modes simultaneously [7], the current approach provides a simple method to enable competition between fiber kinking and matrix splitting cracks.

### F. Fiber rupture

The fibers have an intrinsic strength beyond which they rupture [26–30], or as termed in [26–28] experience shear-driven fiber failure. Fiber rupture is considered herein as a distinct damage event that may occur before or after kinking. For the common case where kinking occurs first, after the material kinks (i.e., reaches the FKT instability) the combination of high compression and bending of the fibers leads to fiber rupture producing the characteristic kink band of misaligned broken fibers. For nearly perfectly aligned fibers or when  $\tau_{12}$  and  $\bar{\varphi}_{12}$  have opposite signs, if the loading is such that the fibers experience very high longitudinal compressive strains, eventually they rupture without kinking. These two scenarios are considered in the proposed model with a single criterion as follows.

Assume that the fiber rupture is governed by a critical compressive strain,  $\varepsilon_c$ . Strain compatibility is assumed between the ply and fibers in the longitudinal direction. A strain based criterion for fiber rupture seems appropriate since the fibers have significant nonlinearity as they approach failure [30]. The compressive strain in the fibers is assumed to be a combination of axial and bending strain. The axial strain  $\varepsilon_a$  is obtained from Eq. (7). The compressive strain due to bending  $\varepsilon_b$  can be determined from the fiber rotation  $\varphi_{1i}$ , the fiber diameter  $d_f$ , and the width of the kink band,  $w_{kb}$

$$\varepsilon_{b,i} = -\frac{d_f \tan|\varphi_{1i}/2|}{w_{kb}} \quad (26)$$

Since the model assumes that kinking occurs in either the 1-2 plane (in-plane) or the 1-3 plane (out-of-plane) [6], the bending strain is calculated separately for each case with  $i = 2, 3$ . Therefore, the fibers rupture when the following is satisfied

$$\varepsilon_a + \max(|\varepsilon_{b,2}|, |\varepsilon_{b,3}|) \geq \varepsilon_c \quad (27)$$

When Eq. (27) is satisfied, indicating that the fibers have ruptured, and  $FI_{\text{kink}} > 1$ , no further plastic strain is allowed in order to model the post-kinking response [17]. The result is that the constitutive response proceeds with the initial stiffness ( $E_{11}$ ), which results in stress transfer in the longitudinal direction to simulate the band-broadening phenomenon. When fiber rupture occurs after kinking, the contribution from bending dominates since the axial strain is small. In this case, the model predicts that fiber failure occurs when  $\varphi_{1i}$  reaches a critical value  $\varphi_{ff}$ . Therefore, either  $\varepsilon_c$  or  $\varphi_{ff}$  can be used to specify the critical condition at which the fibers break.

### G. Initial Misalignment Angle

The initial misalignment angle,  $\bar{\varphi}$ , accounts for fiber misalignments and other manufacturing anomalies that may contribute to fiber kinking initiation. The initial fiber misalignments can be implemented based on experimental

measurements (e.g., [6, 31, 32]) or synthetically fabricated based on the longitudinal compressive strength. While using measured fiber misalignments is preferred, synthetic initial fiber misalignments are used herein for simplicity. The initial fiber misalignment corresponding to the longitudinal strength is calculated by rearranging Eq. (1) as

$$\bar{\varphi}_{1i}^c = \frac{\eta_i - 1}{G_{1i}} \left( \frac{G_{1i} - X_c}{X_c \eta_i \alpha_i^{1/\eta_i}} \right)^{\frac{\eta_i}{\eta_i - 1}} \quad (28)$$

with  $i = 2, 3$ , such that  $\bar{\varphi}_{1i}^c$  is fully defined in terms of material property inputs.

A spatial perturbation in  $\bar{\varphi}$  with  $-\bar{\varphi}^c \leq \bar{\varphi} \leq \bar{\varphi}^c$  is necessary to represent the random nature of fiber alignments. It is emphasized that the initial fiber misalignment is a local feature; if the material has a uniform misalignment, global anisotropy is introduced. In this study, the two initial misalignments,  $\bar{\varphi}_{12}$  and  $\bar{\varphi}_{13}$ , are assumed to have uncorrelated random uniform distributions except where noted otherwise. The values for  $\bar{\varphi}_{12}$  and  $\bar{\varphi}_{13}$  are calculated at the start of an analysis and used as initial conditions.

### H. Failure Index for Matrix Splitting Cracks

Within the ply, fiber kinking competes with matrix cracking. For certain misalignments and stress states, the material will crack between the fibers (i.e. split) instead of kink. In many experimental observations of longitudinal compression, both kinking and splitting are noticed, e.g. [33]. Therefore, a complete model for longitudinal compression failure must account for both mechanisms and their interactions. Likewise, since  $\bar{\varphi}_{1i}$  are considered for fiber kinking, the role of  $\bar{\varphi}_{1i}$  in matrix splitting should also be considered. Following LaRC04 [23], this is accomplished by performing the matrix cracking calculations in the current fiber referenced frame.

## III. Verifications Using Single Element Models

The model as described in the previous section has been implemented in Abaqus/Explicit [34] in the CompDam VUMAT [16]. A series of single element models and analytical results are described in this section that demonstrate the model features and verify the implementation, with an emphasis on the new developments.

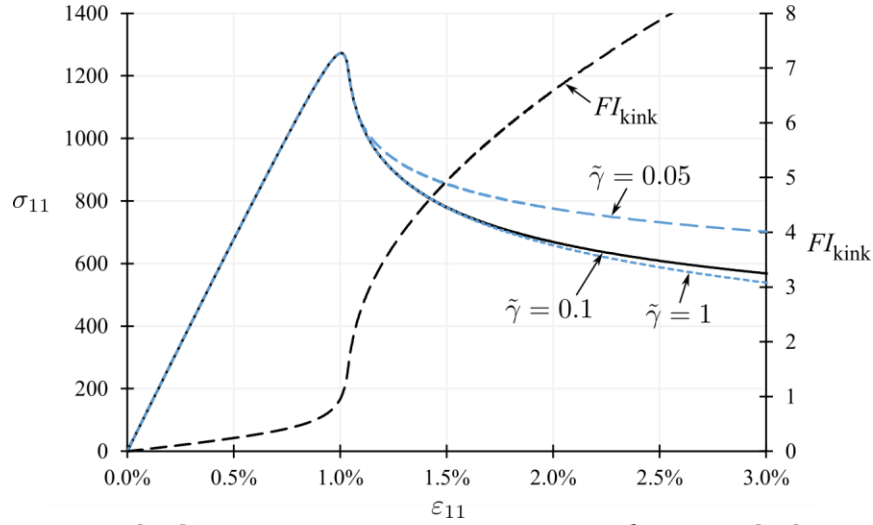
The single element models have a 3D solid continuum element with reduced integration (C3D8R) and a uniform edge length of 0.1 mm. A sigmoid shaped amplitude function (smooth-step) was used to apply the loads through a duration of 0.1 seconds. A uniform mass scaling was used with a factor of  $10^4$ . The kink band width was  $w_{kb} = 0.1$  mm [4]. The IM7/8552 material system was used with the material properties summarized in Table 1.

**Table 1. IM7/8552 material properties inputs for mesoscale model [35].**

| $E_1$<br>(MPa) | $E_2$<br>(MPa) | $G_{12}$<br>(MPa) | $\nu_{12}$ | $\nu_{23}$ | $\alpha$<br>(MPa $^{1-\eta_{PL}}$ ) | $\eta$ | $X_c$<br>(MPa) | $S_L$<br>(MPa) |
|----------------|----------------|-------------------|------------|------------|-------------------------------------|--------|----------------|----------------|
| 140653         | 8703           | 5164              | 0.32       | 0.45       | $4.06 \times 10^{-9}$               | 5.4    | 1731           | 97.6           |

### A. Longitudinal compression

The stress vs. strain response of the model is shown in Fig. 5. The solid black line shows the nominal predicted response where the characteristics of FKT theory are evident: the response is nearly linear to the peak load, after which there is a steep drop in stress, and then the curve gradually flattens out toward a residual stress level, consistent with past results [5]. The blue dashed line shows the prediction of the model with two different values of  $\tilde{\gamma}$  to demonstrate its effect on the constitutive response. The nominal stress-strain curve with  $\tilde{\gamma} = 0.1$  is nearly identical to the case where  $\tilde{\gamma} = 1$  such that the constant slope region of the shear law is never reached. However, when  $\tilde{\gamma}$  is reduced, e.g.  $\tilde{\gamma} = 0.05$ , the post-kinking response varies significantly. As  $\tilde{\gamma} \rightarrow 0$ , a linear shear response is recovered and kinking does not occur. Therefore, since the purpose of  $\tilde{\gamma}$  is to avoid numerical difficulties at large shear strains, it should be selected such that it does not significantly affect the stress-strain response. Herein,  $\tilde{\gamma} = 0.1$  is used as the nominal case. The kinking failure index for the nominal stress-strain curve is shown in Fig. 5 by the dotted line, using the right hand ordinate axis. The failure index reaches 1.0 at the peak load and continues to increase in the post-kinking regime. In this example, an initial misalignment of  $\bar{\varphi} = 2^\circ$  is used so that the snap-back instability and stress drop that occur with small initial misalignments are avoided. Therefore, a smooth response is obtained, which makes the results easier to interpret and verify. The large (arbitrary) initial misalignment produces a strength below the material strength ( $X_c$ ), highlighting the sensitivity of the model to the initial misalignment angle. When  $\bar{\varphi} = \bar{\varphi}^c = 1.2^\circ$ , the model reproduces the input strength  $X_c$ .

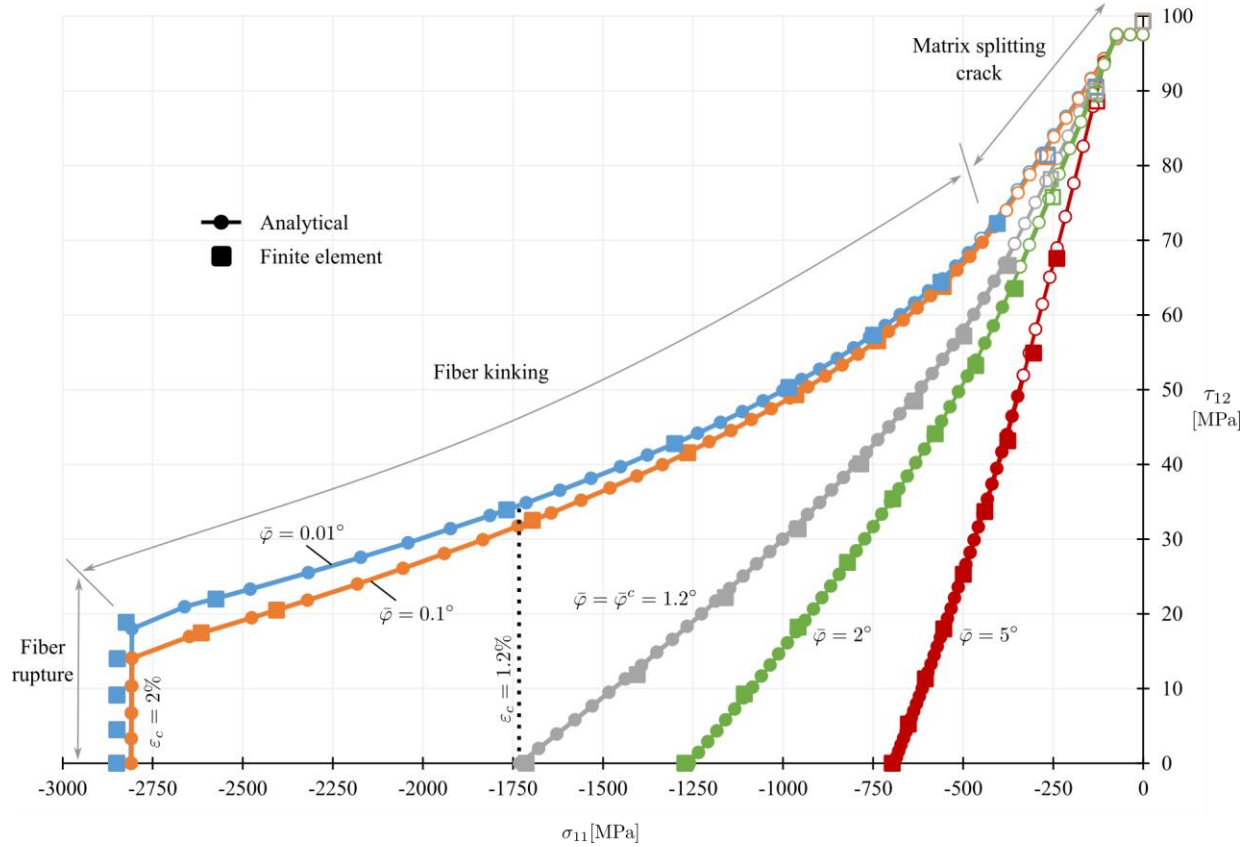


**Figure 5. Longitudinal compressive stress vs. strain response from a single element model.**

### B. Combined longitudinal compression and shear

Several failure envelopes for  $\sigma_{11}$  vs.  $\tau_{12}$  are shown in Fig. 6 to demonstrate and verify the features of the model. In the figure, the square markers are results from the single element analyses using the finite element implementation and the solid lines with circular markers are the results from the analytical solution, obtained by solving Eqs. (20)–(21) for different ratios of  $\sigma_{11}$  and  $\tau_{12}$ . The region of the curves labeled “fiber kinking” are essentially the same as obtained from the LaRC04 instability criterion. Open and closed markers are used to indicate failure predicted to occur by matrix splitting and fiber kinking, respectively. Thus, the transition between matrix splitting and fiber kinking occurs at the point along the envelopes where the symbols change style. The results are shown for several values of  $\bar{\varphi}$ . Results obtained for the nominal value  $\bar{\varphi} = \bar{\varphi}^c$ , shown in gray, confirm that the model reproduces the longitudinal strength  $X_c$  for pure longitudinal compression. Results obtained for  $\bar{\varphi} = 2^\circ$  and  $\bar{\varphi} = 5^\circ$ , shown in green and red, indicate that the failure envelope is approximately linear for larger values of  $\bar{\varphi}$ . In contrast, predictions shown in orange and blue highlight the nonlinearity of the failure envelope obtained with smaller values of  $\bar{\varphi}$ . It is also noteworthy that the failure envelope approaches a limit as  $\bar{\varphi} \rightarrow 0$ . Overall, there is good agreement between the analytical solutions and numerical solutions, which verifies the CompDam implementation. The transition between matrix splitting and fiber kinking occurs around  $\sigma_{11} = -500$  MPa. The two approaches show relatively good agreement in terms of the stress state at which the failure mode transitions. When  $\bar{\varphi} = 5^\circ$ , the finite element results indicate that the transition between fiber kinking and matrix splitting occurs at a lower longitudinal stress (higher shear stress) level than computed by the analytical solution. This difference can be seen by comparing the point where the symbols transition from solid to open. This discrepancy is thought to be due to geometric nonlinearity in the finite element result that is not included in the analytical solution.

Gutkin et al. developed a finite element model [27] and a finite fracture mechanics model [28] that describe fiber kinking under combined longitudinal compression and in-plane shear. Both models are based on micromechanical analysis of a single fiber. The models produce a similar failure envelope trending from the shear strength toward the compressive strength with the relative slope a function of the initial misalignment. Fiber rupture (shear driven fiber-failure) is accounted for in the finite element model using a stress-based CDM approach. The general shape and characteristics of the failure envelopes found by Gutkin’s micromechanical models are the same as produced by the present mesoscale model and shown in Fig. 6.



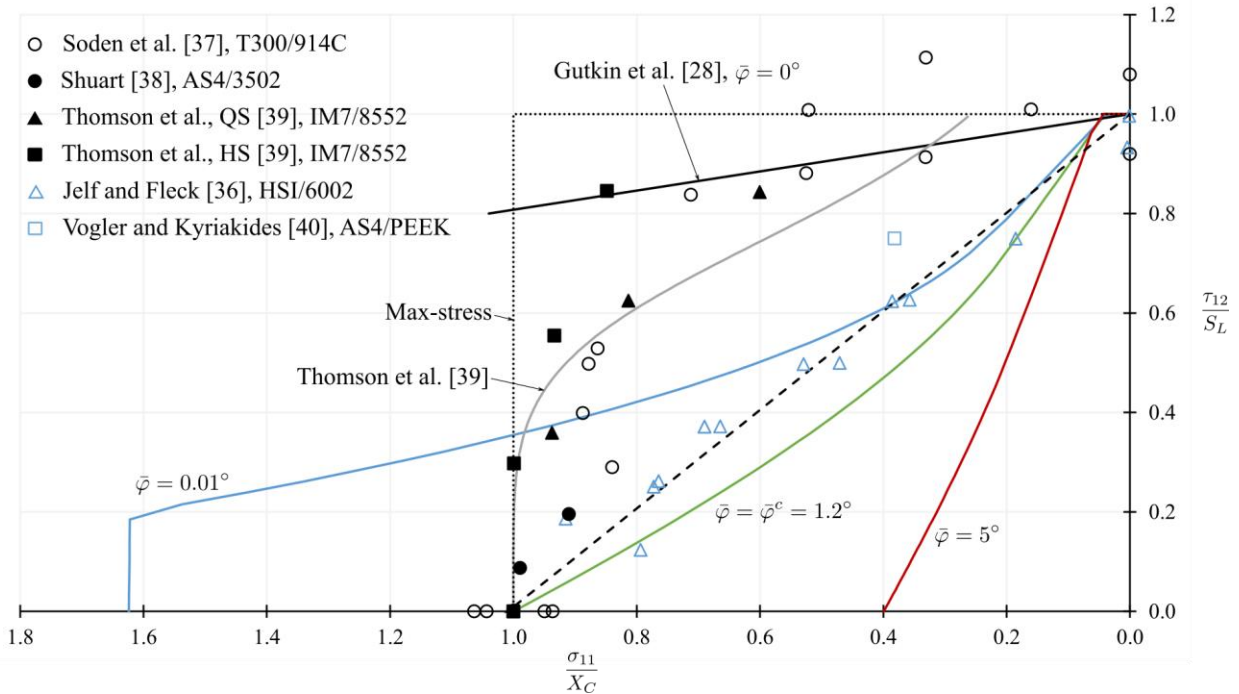
**Figure 6. Longitudinal compression vs. in-plane shear failure envelope from finite element implementation and analytical solution.**

Three additional features of the failure envelopes in Fig. 6 are worth mentioning. First, the fiber kinking portion of the curves is related to the strain-hardening model used for the shear response, as was found in [28]. For typical strain-hardening shear responses, the  $\sigma_{11}$  vs.  $\tau_{12}$  failure envelopes are always the same shape, concave upward. Second, with  $\varepsilon_c = 2\%$  (from [30] for T800), failure occurs by fiber rupture for  $\bar{\varphi} \leq 0.1^\circ$  and  $\tau_{12} \leq 20$  MPa. The resulting strengths predicted due to fiber rupture appear very high. The value of  $\varepsilon_c$  shifts the vertical fiber-rupture portion of the failure envelope left (higher  $\varepsilon_c$ ) and right (lower  $\varepsilon_c$ ), similar to the behavior of the micromechanical model in [27]. For  $\varepsilon_c = 1.2\%$ , the failure envelope is truncated at  $X_c$ . It is difficult to evaluate this failure mode since fiber rupture has only been reported in notched test specimens wherein the local strains are hard to measure. Fiber rupture may only occur in certain structural configurations where kinking is delayed due to the fibers being supported or confined such that fiber rotation is resisted. Certain fibers may be sensitive to fiber rupture. Test measurements of the local longitudinal strain associated with fiber rupture could help to provide a physical basis for assessing this portion of the failure envelope. Third, when the shear stress is in the opposite direction of the misalignment ( $\tau_{12} < 0$  in Fig. 6), the model predicts the curves continue along their trajectories until intersecting either the fiber rupture curves (approximately vertical line) or the matrix splitting threshold at high shear stresses (approximately horizontal line at  $\tau_{12} = -S_L$ ). Thus, when the shear stress and misalignment angle are opposite, there is an apparent strengthening.

Numerous experiments have been reported in the literature that can be used to evaluate the failure envelopes [36–40]. The FKT failure envelopes for three values of  $\bar{\varphi}$  are repeated alongside several experimental results and other theories in Fig. 7. The axes are normalized with the longitudinal and shear strengths to show experimental results measured for different materials on the same plot. The experimental results are shown by the unconnected symbols. The test data were obtained using a variety of test configurations: 1) combined compression/torsion tests on unidirectional tubes [36, 37]; 2) compression tests on  $\pm\theta$  angle ply specimens (restricted to the ply angles that failed by fiber damage) [38]; 3) compression tests on inclined (intentionally misaligned) cross-ply specimens [39] including both quasi-static (QS) and high-strain rate (HS) tests; and 4) a combined compression/shear test on a unidirectional specimen [40]. The tests also used several different materials, as summarized in the legend in the figure. Carbon fibers

pre-impregnated with epoxy resins are used in all cases except the result from [40], wherein a thermoplastic resin was used. While each study used a different material system, it is noted that Thomson et al. [38] used the same material as used in this study, IM7/8552. Test data obtained using unidirectional and multidirectional laminate specimens are shown with open and closed symbols, respectively. While results from laminate specimens might be more difficult to reduce to apparent ply strengths (e.g., due to interacting damage modes or residual thermal stresses), no obvious difference is observed between the results from unidirectional and multidirectional specimens. Considering the experimental results plotted in Fig. 7, the data largely cluster into two groups. The results shown in blue from [36] [40] show relatively good agreement with the FKT with  $\bar{\varphi} = \bar{\varphi}^c$ . In other words, the  $\sigma_{11}$  vs.  $\tau_{12}$  interaction is approximately linear (black dashed line). The rest of the data show substantially less interaction, being closer to the max-stress (no-interaction) envelope (black dotted line) than the approximately linear interaction predicted by FKT. Classifying the results by material system, test configuration, or test setup does not explain the two groups of test results. A number of issues make interpretation of the different tests difficult and may contribute to the wide range of test data: resolving laminate failure to ply stress is nontrivial, most of the tests are unstable and rely on post-test identification of failure mechanisms, and it may be inappropriate to compare different material systems as is done here. Thus, it is unclear how to interpret the large range in the experimental data.

For the purpose of this study, it is suggested that the data from Thomson [39] are the most representative since the material system is the same as used in this study. However, the FKT model cannot match the data from [39]. Thomson recognized this discrepancy and proposed a phenomenological criterion that matches the data well, shown with the gray curve in Fig. 7 for fiber kinking. When  $\sigma_{11}$  is small, matrix splitting occurs, and a linear criterion such as in [28] with  $\bar{\varphi} = 0^\circ$  matches the test data well. Therefore, together the criteria from [39] and [28] for kinking and splitting, respectively, match the test data best. These criteria could be implemented together with a fracture based degradation approach in a CDM model. However, such an approach is partially unsatisfactory since it provides no mechanistic or kinematic understanding of fiber kinking. Despite the apparent shortcomings of the FKT model representing the compression-shear failure envelope, analyses of the OHC were conducted to evaluate its capability.



**Figure 7. Comparison of experimental and theoretical results for the longitudinal compression vs. in-plane shear failure envelope.**

#### IV. Application to Open Hole Compression (OHC)

The capability of CompDam using the FKT model described in Section II for predicting the response of standard OHC specimens with three different layups is discussed in this section. The results are compared with nearly identical models run using the conventional CDM fiber damage model described in [19]. This section describes the models and

highlights key results with emphasis placed on how the fiber kinking model differs from the traditional CDM based approach for fiber compression damage.

#### A. Test Specimens and Experimental Procedure

A series of OHC tests on three representative layups was conducted as part of the ACP [41, 19]. The layups were designated as Soft, Quasi, and Hard (Table 2), and were selected to assess model performance across a range of relevant laminate configurations that exhibit different damage behaviors. The specimens were fabricated from unidirectional IM7/8552 plies having a cured ply thickness of 0.183 mm and sized following ASTM D6484 Procedure A [42]. Loading was conducted at room temperature with a quasi-static monotonic loading at 0.6 mm/min. Digital image correlation and an extensometer were used to obtain strain and displacement data near the hole. Five specimens were tested for each layup. Three specimens were loaded until failure, and then the subsequent two specimens were loaded to 75% and 90% of the average peak load of the previous three replicates. Post-test, the interrupted test specimens were inspected via X-Ray computed tomography (CT). For complete details of the experimental procedure and test results, see [19].

**Table 2. OHC layups.**

| Designation | Stacking sequence                         |
|-------------|-------------------------------------------|
| Soft        | $[45/-45/0/45/-45/90/45/-45/45/-45]_s$    |
| Quasi       | $[45/0/-45/90/45/0/-45/90/45/0/-45/90]_s$ |
| Hard        | $[(45/-45/0_2)_3]_s$                      |

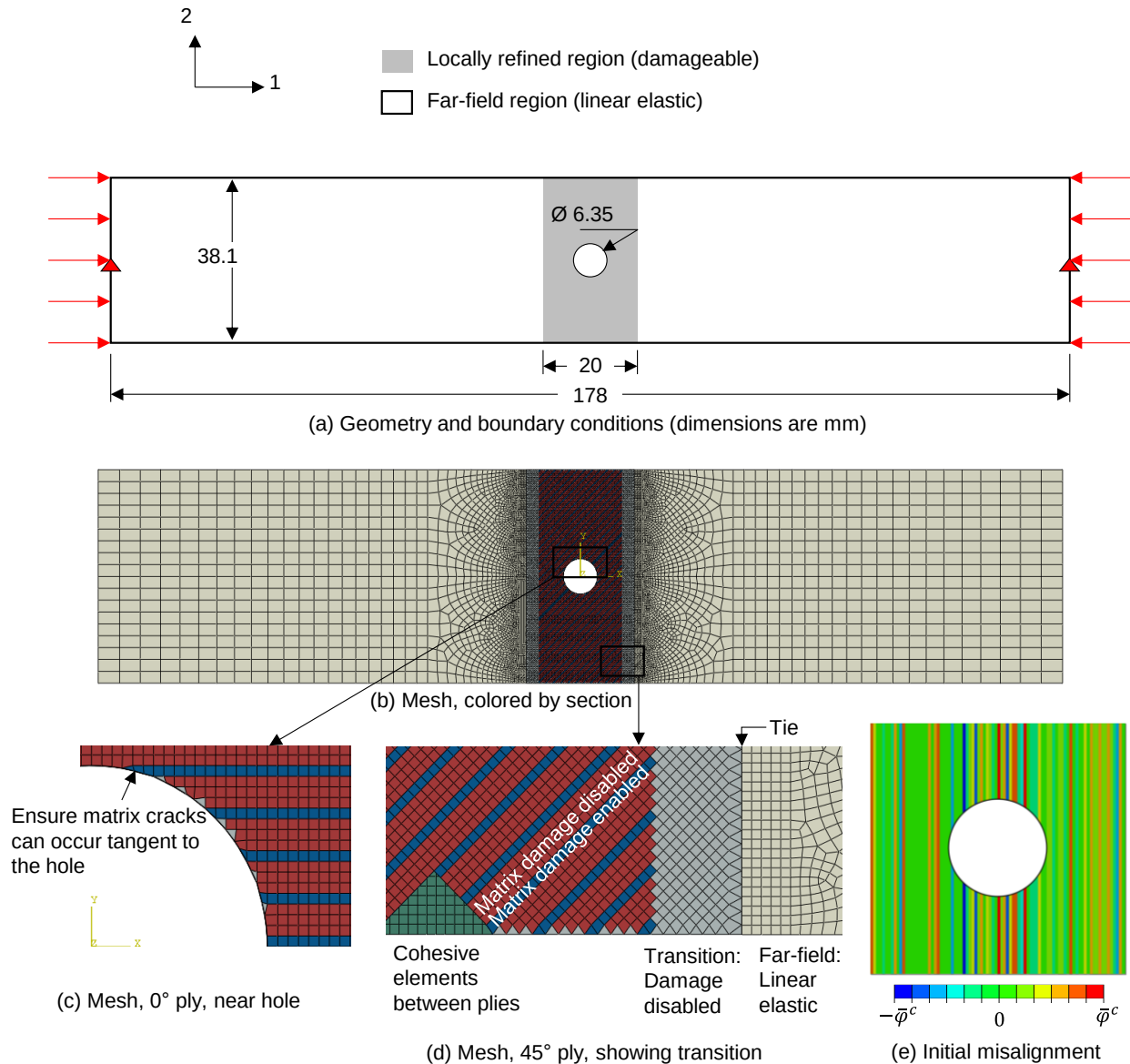
#### B. Model Description

The finite element model used for the OHC analysis is described in this section. The model geometry and boundary conditions are shown schematically in Fig. 8a. The geometry reflects the standard OHC configuration with a 6.35 mm diameter hole and 38.1 mm width, but a shorter length is modeled since it is not necessary to model the full gripping region. Uniaxial loading is applied via prescribed end shortening. Out-of-plane displacements are set to zero in the far-field to mimic the anti-buckling guides in the OHC test fixture. A local 20-mm-long gage section includes ply-by-ply discretization with the capability to damage. The remaining far-field region is modeled with one continuum shell element (SC8R) through-the-thickness and a linear elastic material behavior. The size of the locally refined region was chosen as a trade-off between computational expense and minimization of damage interaction at the local region boundary. The locally refined region is modeled following the approach in reference [2], and is shown in Fig. 8b–d. One layer of solid-continuum reduced-integration elements (C3D8R) is used for each ply, with the mesh aligned to the fiber direction. The typical in-plane element size is 0.2 mm. To facilitate the fiber aligned meshing, wedge elements are used around the perimeter of the hole and on the model boundaries. The local region is tied to the far-field region. In the local region, when two adjacent plies have the same direction, they share nodes at the ply interfaces. For dissimilar ply interfaces, the layers are connected using node-based tie constraints to a layer of zero-thickness cohesive elements to model delamination. In order to represent the discrete nature of matrix cracking, matrix damage is only allowed in every fourth row of elements [41]. A transition region is used to minimize perturbations in the stress field due to the discretization transition between the far-field and local region. The meshing and section assignments are selected such that matrix cracks can occur tangent to the hole in all plies, as show in Fig. 8c for a 0° ply.

Two variations of the model were created. The first variation uses a conventional CDM model for fiber compression damage and is referred to herein as the CDM model. The second variation of the model uses the proposed fiber kinking model, designated the FKT model. The two models are nearly identical with the main difference being the material model for fiber damage. The material properties are listed in Tables 3–6. In general, the material properties are taken from reference [35] and are the same set of properties used throughout the ACP. The material properties specific to the CDM model for fiber damage are listed in Table 5. The properties required are the compressive strength  $X_c$ , compressive fiber fracture toughness  $G_{fc}$ , and two parameters,  $m$  and  $n$ , that indicate the location of the knee in the trilinear stress-strain curve, which can be related to the R-Curve. The material properties specific to the FKT model for fiber damage are listed in Table 6, where in addition to the compressive strength, the kink band width  $w_{kb}$ , and fiber diameter  $d_f$ , are required. In the FKT model, the initial fiber misalignments ( $\bar{\varphi}$ ) are assigned with a random uniform distribution varying only in the fiber direction, as shown for one 0° ply in Fig. 8e.

The quasi-static loading was modeled in the explicit dynamic analysis by applying the prescribed displacement with a smooth step amplitude in a 0.2 second explicit dynamic step. Mass scaling was introduced with automatic mass scaling such that typical analyses used 1.5 million increments to reach peak load and the element mass in the local

region was scaled by  $\sim 10^3$ . Detailed investigation showed that this level of mass scaling was sufficient to suppress dynamic effects until after the peak load is reached. Element deletion was only used in cases where elements distorted excessively and had to be deleted for the analysis to continue. In the FKT simulations, final failure typically included several elements being deleted, resulting in an abrupt load drop.



**Figure 8. Configuration, mesh, and section assignments used in the OHC model.**

**Table 3. Elastic material properties used in all OHC analyses.**

| $E_1$<br>(MPa) | $E_2$<br>(MPa) | $E_3$<br>(MPa) | $G_{12}$<br>(MPa) | $G_{13}$<br>(MPa) | $G_{23}$<br>(MPa) | $\nu_{12}$ | $\nu_{13}$ | $\nu_{23}$ |
|----------------|----------------|----------------|-------------------|-------------------|-------------------|------------|------------|------------|
| 140653         | 8703           | 8703           | 5164              | 5164              | 3001              | 0.32       | 0.32       | 0.45       |

**Table 4. Matrix properties used in the local region: Strength, toughness, and shear-nonlinearity.**

| $Y_T$<br>(MPa) | $Y_C$<br>(MPa) | $S_L$<br>(MPa) | $G_{Ic}$<br>(kJ/m <sup>2</sup> ) | $G_{IIc}$<br>(kJ/m <sup>2</sup> ) | $\eta_{BK}$ | $\alpha$<br>(MPa <sup>1-\eta_{PL}</sup> ) | $\eta$ |
|----------------|----------------|----------------|----------------------------------|-----------------------------------|-------------|-------------------------------------------|--------|
| 80.1           | 288.2          | 97.6           | 0.24                             | 0.739                             | 2.07        | $4.06 \times 10^{-9}$                     | 5.4    |

**Table 5. Fiber properties used in the CDM model.**

| $X_c$<br>(MPa) | $G_{fc}$<br>(kJ/m <sup>2</sup> ) | $m$ | $n$ |
|----------------|----------------------------------|-----|-----|
| 1731           | 61                               | 0.5 | 0.2 |

**Table 6. Fiber properties used in the FKT model [4, 30].**

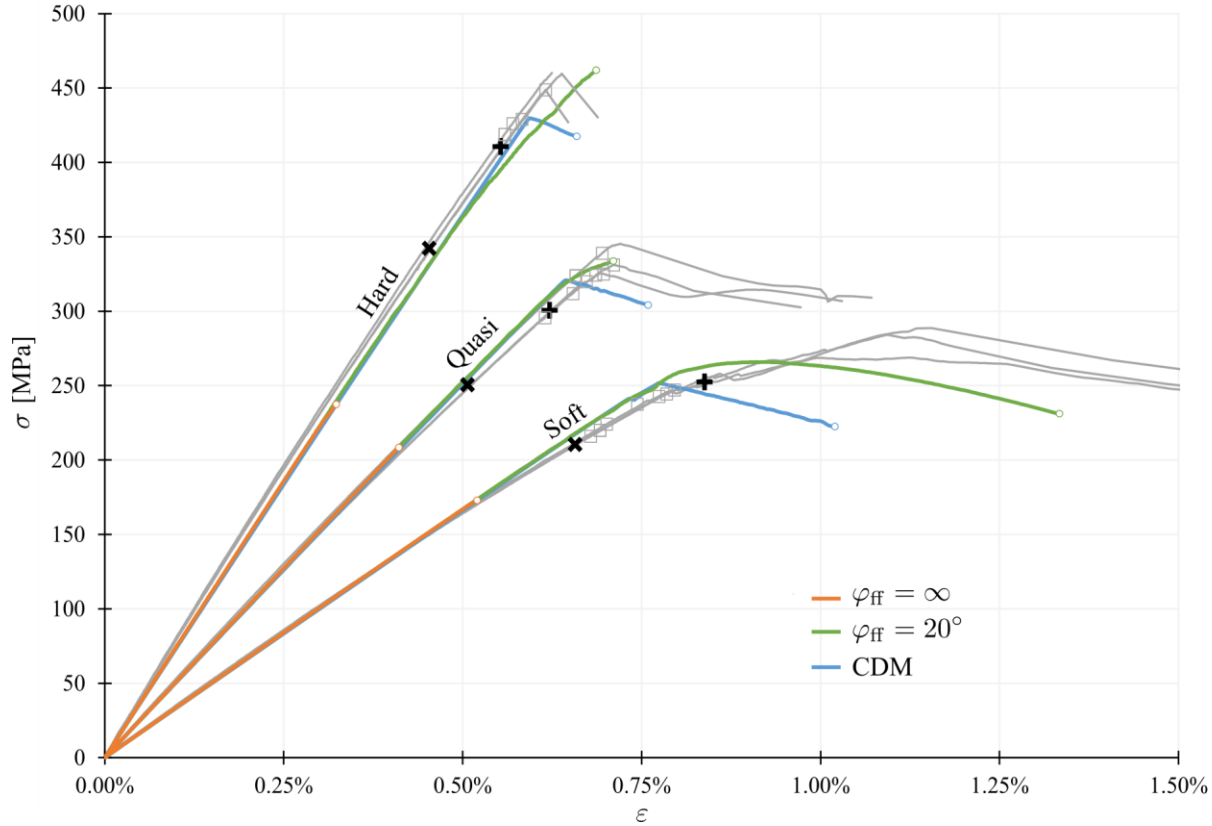
| $X_c$<br>(MPa) | $w_{kb}$<br>( $\mu$ m) | $d_f$<br>( $\mu$ m) |
|----------------|------------------------|---------------------|
| 1731           | 50                     | 5.1                 |

### C. Test and Analysis Results

The stress vs. strain curves that describe structural response of the three OHC laminates are shown in in Fig. 9 for the CDM and FKT damage models. The test data are shown with gray lines for the three replicates loaded to collapse for each layup. The gray square symbols indicate the points where evidence of fiber kinking was reported experimentally. The baseline CDM model using the trilinear stress-strain law for fiber compression is shown with the blue lines. Two variations of the FKT model are shown in orange and green with the fiber rupture feature turned off,  $\varphi_{ff} = \infty$ , and on,  $\varphi_{ff} = 20^\circ$ , respectively. In all simulations, the last point before element deletion resulting in terminate of the analysis is shown with a circular marker. After element deletion starts, several elements are deleted rapidly, and the load drops precipitously. Since this precipitous drop in load is purely numerical and not physically relevant, the curves are truncated.

As discussed in [19], good agreement is observed between the test and analysis predictions from the baseline CDM damage model in terms of stiffness and strength. The predicted strain-to-failure is in good agreement with the test results for the Hard and Quasi laminates, but is under-predicted substantially for the Soft laminate. In the Soft laminate, fiber damage occurs relatively early (starting at about 0.7% strain) and load is redistributed such that the specimen continues to carry additional load with a degraded stiffness up to the collapse at about 1.1% strain. The baseline CDM model predicts collapse occurs shortly after fiber damage becomes large, and therefore, under-predicts the strain-to-failure by about 30%.

The FKT model, without fiber rupture ( $\varphi_{ff} = \infty$ ) terminates shortly after fiber kinking onset occurs since the constitutive model leads to very large shear strains post-kinking and thus numerical instability. It is clear that the model predicts fiber kinking onset prematurely when the termination points of the  $\varphi_{ff} = \infty$  curves are compared with the experimental observations of fiber kinking onset (gray square symbols). This deficiency of the model is explored further below. By introducing the fiber rupture feature of the model, the post-kinking deformations are stabilized, allowing the analyses to proceed. A parametric study was used to calibrate  $\varphi_{ff} = 20^\circ$  ( $\approx \varepsilon_c = 1.8\%$ ), and it can be observed in Fig. 9 that the analysis predictions match the test data well for all three laminates with this  $\varphi_{ff}$ . In particular, the FKT model with  $\varphi_{ff} = 20^\circ$  reduces the error in the strain-to-failure prediction in the Soft laminate by about half compared with the baseline CDM model. The FKT model also shows a small improvement over the baseline CDM model in correlation of the strengths between test and analysis for all laminates.



**Figure 9. Nominal compressive stress vs. strain response for the three layups.**

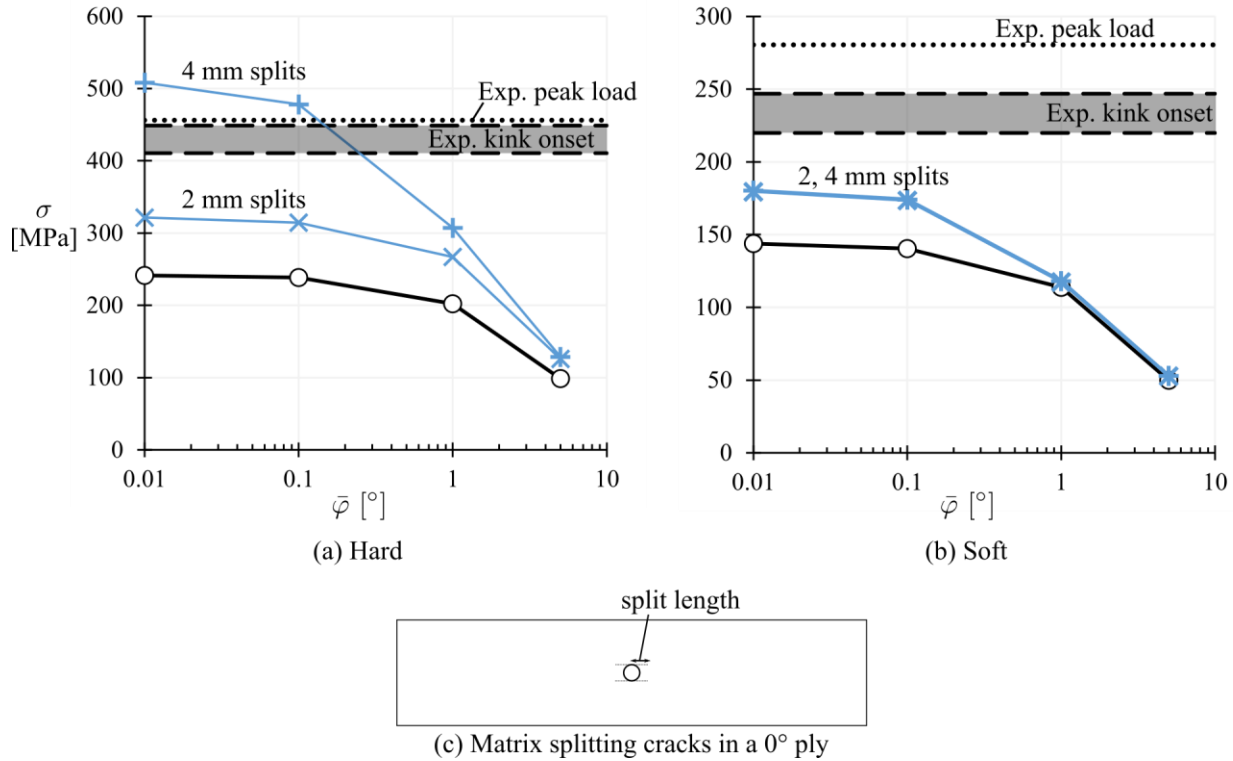
X-Ray CT scans were conducted to obtain data on the pre-peak damage morphology. The average strength,  $\sigma_{c,exp}$ , was calculated from the first three tests. The fourth and fifth test specimens were loaded to 75% and 90% of  $\sigma_{c,exp}$ , respectively, and then unloaded for subsequent inspection via X-Ray CT. For reference, the points where X-Ray CT scans were conducted are shown on the stress vs. strain data in Fig. 9 as 'x' and '+' symbols for 75% and 90%, respectively. Comparisons made between the baseline CDM model results and the X-Ray CT data in [19] show partial agreement overall. In general, the matrix cracks in the  $0^\circ$  plies were predicted accurately. Despite difficulty in establishing a value of the fiber damage variable that is expected to be associated with observable damage, the mode and size of damage predicted generally matched the experimentally observed trends.

Improvements in the test vs. analysis correlation in the structural response obtained with the FKT model are not accompanied by improvements in terms of damage states. Since the FKT model predicts kinking onset at a relatively low load, the subsequent damage develops differently than in the baseline CDM model and in the test. In general, the FKT model predicts more damage over a more widespread area than is observed, with large fiber kinking areas at peak load for all laminates. Based on these observations, it is concluded that more accurate prediction of the fiber kinking onset load is required.

A series of analyses were run to help understand why the FKT model under-predicts the fiber kinking onset load, and to evaluate potential model enhancements. In these analyses, the model was simplified by initializing  $\bar{\varphi}$  to the same value for all elements. The Hard and Soft laminates were analyzed with different values of  $\bar{\varphi}$  to determine the sensitivity of the kinking onset load to  $\bar{\varphi}$ . The results are shown with the black curves in Fig. 10. As the value of  $\bar{\varphi}$  is reduced toward zero, the fiber kinking onset load converges to values well below the experimentally observed range for both layups. Therefore, it is clear that the present FKT model under-predicts the kinking onset load for all values of  $\bar{\varphi}$ , including zero. The relative insensitivity to  $\bar{\varphi}$  can be explained by the fact that the stress state where fiber kinking is predicted to initiate includes large shear stress ( $>30$  MPa) in addition the axial compression stress. Referring to Fig. 6, it is clear that there is relatively little difference in the kinking onset stress ( $\sigma_{11}$ ) levels when  $\tau_{12} > 30$  MPa when  $\bar{\varphi} \ll 1^\circ$ .

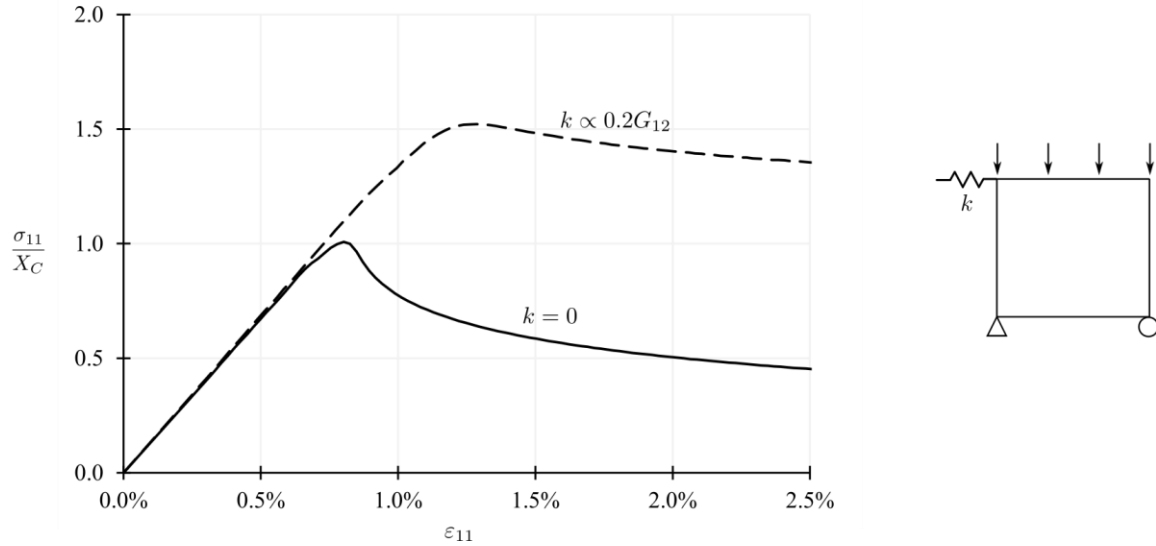
One mechanism that could increase the fiber kinking onset load is matrix splitting cracks. Splitting cracks in the  $0^\circ$  plies blunt the axial stress concentration and result in a more uniform stress state ahead of the hole, which may

delay fiber kinking onset. The present models predict splitting cracks, but they are very short at kinking onset,  $< 1$  mm. To evaluate the sensitivity of the model predictions to longer splitting cracks, 2 mm and 4 mm long splits were artificially pre-seeded in the  $0^\circ$  plies at four locations around the hole. The fiber kinking onset results with the artificial splits are shown with the blue lines in Fig. 10. The results for the Hard laminate in Fig. 10a show that relatively long splits and small  $\bar{\varphi}$  can agree with the test observations. For the 4 mm long splits, fiber kinking onset agrees with the test observations when  $\bar{\varphi} \approx 0.2^\circ$ . For the Soft laminate in Fig. 10b, the splits alone are insufficient to explain the difference between the model results and the test observations. In both cases, under-prediction of splitting crack lengths may contribute to the test/analysis discrepancy, but it does not appear to fully explain the differences. X-Ray CT scans before experimentally observed fiber kinking onset (at the 75% load level) show split lengths of about 1 mm. The observed lengths may be underestimated since the splitting cracks are sliding (Mode II) cracks that are hard to see and the scans were conducted in the unloaded state.



**Figure 10. Nominal stress level when fiber kinking onset occurs vs. the initial misalignment angle,  $\bar{\varphi}$ .**

Another mechanism that has been reported to stabilize fibers and delay fiber kinking is referred to as the “strain gradient effect” [43] or, more broadly, the “structure effect” [44–46]. Wisnom conducted tests with combined bending and compression and observed an increase in the compressive failure strain with increasing strain gradient [43]. He hypothesized that, in the presence of a strain gradient, neighboring fibers experiencing less strain are less likely to microbuckle and therefore provide some support to highly strained fibers, thus stabilizing the fibers locally and delaying fiber kinking. Others have reported that when the  $0^\circ$  fibers experience confinement or constraints (e.g., from neighboring plies or bolt clamp-up), fiber kinking is delayed and the compressive strength increases [44–46]. In a simplistic way, it is shown that the present FKT model can capture these effects. The single element model used in Section III is augmented with a spring in the transverse direction providing resistance to rotation that occurs as longitudinal compression is applied. The baseline stress vs. strain result is shown as the solid line in Fig. 11 (spring stiffness is zero). When the spring stiffness is equivalent to 20% of the shear modulus times the element length, the stress vs. strain curve is the dashed line, predicting a 50% increase in the kinking strength. Thus, the model predicts a relatively small amount of support from neighboring material can delay fiber kinking. Nonetheless, detailed verification and validation of this effect is still needed.

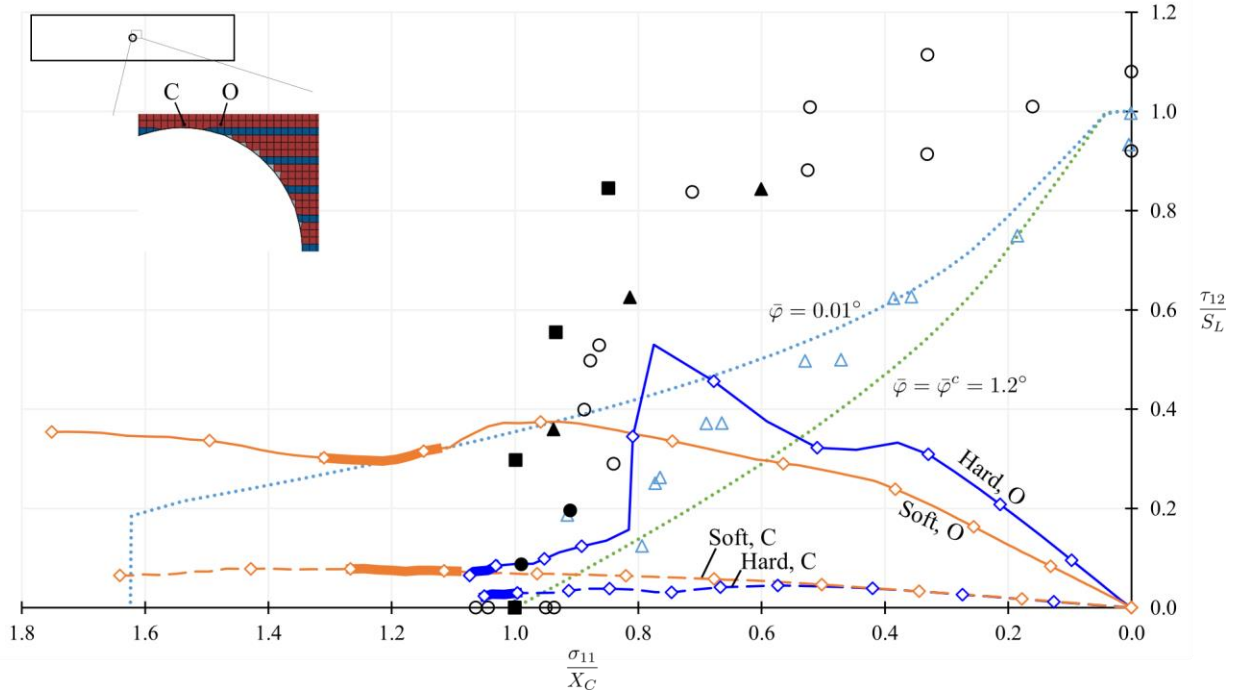


**Figure 11. Longitudinal compressive stress vs. strain response from a single element model showing the effect of a spring resisting fiber rotation.**

Perhaps the most likely explanation for the difference in fiber kinking onset in the test and analysis is related to the inability of the FKT to match the experimental data in the  $\sigma_{11}$  vs.  $\tau_{12}$  failure envelope (Fig. 7). To investigate this possibility, selected results from OHC analyses are shown on the  $\sigma_{11}$  vs.  $\tau_{12}$  failure envelope in Fig. 12. The test data shown in Fig. 7 from the literature are repeated in Fig. 12 with the same unconnected symbols. The theoretical FKT envelopes are shown for  $\bar{\varphi} = 0.01^\circ$  and  $\bar{\varphi} = \bar{\varphi}^c = 1.2^\circ$  with the dotted lines. The Soft and Hard laminate models were modified such that fiber kinking is prevented in order to extract the stress history in selected elements up to the average experimentally measured peak load. The matrix damage and delamination were not modified and thus free to develop naturally. Since the matrix cracking damage was similar to that predicted in the CDM models and thus in relatively good agreement with experimental observations, it is assumed that stress redistributions due to matrix cracking are represented accurately. Stresses were extracted from two locations: offset, ‘O’, where the FKT models predict kinking starts and center, ‘C’, where the CDM models predict fiber damage initiates, as shown in the schematic in the upper left of Fig. 12. The integration point stress histories are plotted from the ‘O’ and ‘C’ elements as the solid and dashed lines, respectively. The Hard and Soft laminate results are shown as blue and orange, respectively. Diamond shaped markers are shown on the finite element model stress histories at 10% increments in applied load. Since the local stresses evolve nonlinearly through the loading history, the approximate far-field load at any point along the curve can be inferred by these diamond markers. For example, for the Hard laminate, offset shear stress decreases precipitously at a load level between 50% and 60% of the experimental peak load (due to growing matrix splitting cracks). Finally, in the range of loads where experimental observations of fiber kinking were recorded, the stress-histories are shown with a thick line.

By comparing the finite element results with the theoretical FKT failure envelopes and the experimental data from the literature, several observations can be made. First, these results explain the predicted fiber kinking locations. The FKT models predict fiber kinking starts at location ‘O’ since the stress at this location reaches the FKT failure envelope at a lower applied load than the stress extracted at location ‘C’. Second, looking at the stress history results for the ‘O’ location, the stress has already exceeded the failure envelope for  $\bar{\varphi} = 0.01^\circ$  before kinking is observed experimentally (thick lines). Therefore, if it is assumed that the stress fields in the model are accurate, it appears that the fiber kinking observed in the test is less sensitive to shear stress than FKT predicts. The observation of relative insensitivity of fiber kinking to shear stresses is consistent with the experimental data in the literature (e.g., Thomson et al. [39]). A physical explanation for the apparent insensitivity of fiber kinking to shear stresses is needed. Third, the longitudinal stresses extracted from the model at the load levels when fiber kinking was observed experimentally are always greater than  $X_C$  and, in the case of the Soft laminate, significantly greater than  $X_C$ . This can be explained by the structural configuration. In both laminates, the notch strain gradient and neighboring plies provide a confining effect that resists fiber rotation and therefore delays microbuckling. The effect is more pronounced in the Soft laminate compared with the Hard laminate because the Soft laminate has a 1-ply-thick  $0^\circ$  ply whereas the Hard laminate has a 4-ply-thick  $0^\circ$  ply-block. The supporting effect of neighboring plies is more significant for thin  $0^\circ$  plies [46]. Therefore, it is expected

that the  $0^\circ$  plies in the Soft laminate would have a higher apparent strength. Considering the three observations made in this paragraph, modifications to the current FKT damage model that can accurately account for kinking onset under combined stress states, and the apparent strengthening effects of confining conditions are needed to improve the model capabilities relative to OHC.



**Figure 12. Stress histories from OHC simulations shown on the longitudinal compression vs. in-plane shear failure envelope.**

## V. Summary

The CompDam fiber kinking model has been augmented with additional developments to facilitate analysis of coupons that fail under compressive loads, such as the open hole compression specimen. A procedure was introduced for the robust calculation of a fiber kinking failure criterion to provide a numeric indication of the onset of fiber kinking. Using the fiber kinking failure criterion, competition between fiber kinking and matrix cracking is allowed at each integration point until the failure criteria is reached for one of the two mechanisms. A second enhancement is a strain-based fiber rupture criterion that can be triggered before or after kinking. When the fiber rupture criterion is satisfied, the shear response is modified.

The fiber kinking criterion is examined in the context of combined longitudinal compression and in-plane shear loading. The finite element implementation was verified through comparison with the analytical solution and found to be in excellent agreement with different combinations of model parameters. By comparing the failure envelopes with test data from the literature, it is apparent that the kinking model can only match a subset of the test data. The test data suggests less sensitivity to in-plane shear than the fiber kinking theory predicts. The strain-based fiber rupture criterion suggests that fiber rupture does not explain the experimentally observed insensitivity to shear stress.

The model was applied to predict the behavior of open hole compression (OHC) specimens. Mesoscale models were constructed of three OHC laminates and analyzed using a conventional trilinear constitutive law and the proposed fiber kinking model. Both models were able to predict the overall structural response of the three laminates. However, neither model was able to predict the point of fiber kinking onset accurately. A detailed investigation into the results from the fiber kinking model was conducted. It was found that two mechanisms play important roles in the discrepancy between experimentally observed and predicted fiber kinking onset load. First, the stress histories of the elements that kink show the kinking failure criterion is satisfied under combined compression and shear, a condition for which it is apparently overly conservative. Although it is unclear at this stage why fiber kinking is less sensitive to shear stress than the theory predicts, it is clear that an improved mechanistic understanding of fiber kinking under combined compression and shear stress will benefit predictions of fiber kinking in OHC specimens. Second, the structural effects of the laminate stacking sequence, ply thickness, strain gradients, and confinement all contribute to fiber kinking. The

stress histories from the OHC show that for accurate prediction of fiber kinking onset, the relatively thin 0° plies in the Soft laminate must sustain significantly higher stress than the thick 4-ply-block in the Hard laminate. The model is shown to predict some of these structural effects since it is sensitive to the freedom of the neighboring material to allow fiber rotation. However, further assessment and developments are needed.

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