

Examination of Unified Control Approaches Incorporating Generalized Control Allocation

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Transition vehicles which combine vertical take off and landing with cruise configurations pose a unique challenge for control design and implementation. For this class of vehicle, successful control designs have historically broken the flight envelope into phases and modified the control approach for each phase. This research approaches control in a unified way across the entire envelope using a robust optimal design which provides effector weighting then implemented in a generalized Affine Generalized Inverse control allocation algorithm. System performance for a Lift plus Cruise transition vehicle is presented.

I. Introduction

The proliferation of unconventional aircraft designs emphasizes shortening the design cycle and reducing development costs, which requires simplified control approaches that are robust and require minimal modeling while still fully exploiting the unique capabilities of the unconventional designs. Transition vehicles (e.g. Vertical Take-off and Landing) are one such class of vehicle that present a substantial control design problem across a large flight regime from hover to cruise flight. Moreover, ubiquitous use of transition vehicles in the future implies a greater need for autonomous control due to an expected decrease in vehicle operator experience. Typical control approaches for transition vehicles establish several control modes based on flight conditions, with mode specific control approaches using particular effector combinations. In this work, unified control design enables a single control mode that blends optimal control with control allocation to provide control throughout the entire flight envelope without prescribed effector sets. This approach blends several design strategies into a higher-level set of control variables that can be easily integrated into a pilot-in-the-loop, a fully auto-trajectory tracking system, or some combination thereof.

This research highlights a practical approach to overcoming three major challenges to control development for transitioning VTOL aircraft. The first is the number of different control effectors available for various flight modes and how their individual effectiveness and redundancies may change over the flight envelope. The second is the changing relative degree of the control effector to the desired output in different flight regimes. For example in hover, direct control of the vertical velocity is provided by the rotor thrust, as opposed to conventional forward flight where the vertical velocity is indirectly controlled by the elevator. The third is the choice of control variables that make sense throughout the entire flight envelope and allow for an intuitive set of control commands for a pilot/operator.

One of the main features of this work is the separation of the control design from the control implementation. This is accomplished by abstracting the control design as previously discussed and coupling this with a new generalized control allocation algorithm. In this research, control allocation uses an expanded Affine Generalized Inverse (AGI) [1] algorithm which was broadened to include arbitrary positive (semi) definite effector weighting, incorporation of a preferred control, and redundant affine control effectors subject to arbitrary (e.g. one-sided) effector position and/or rate limitations. These algorithm enhancements enable the AGI algorithm to provide optimal allocation for linearized vehicle states throughout the flight envelope.

This article is organized in the following way: Section II describes preliminaries and outlines the unified control approach. Section III describes the Lift+Cruise aircraft and delineates the rigid body dynamics. Section IV details the longitudinal directional control design. Section V outlines the integration of control allocation and delineates the expansion of the Affine Generalized Inverse algorithm. The conclusions in Section VI complete this article.

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II. Control Framework

The control structure provides a unified approach to control design throughout the entire flight envelope. This approach brings together several design strategies that are described below to build a method that is applicable across all flight regimes. In this section we will provide an overview of the control methods and principals used that combine to formulate the uniform control architecture then longitudinal dynamics of the aircraft in non-turning flight will be used to demonstrate the effectiveness and practicality of the architecture.

A. Robust Servo-mechanism Linear Quadratic Regulator

The robust servo-mechanism linear quadratic (RSLQR) structure [2] builds on the well know linear quadratic optimal control theory method. When applied to the linearized dynamics of the aircraft at an equilibrium point, the linear quadratic regulator (LQR) forces the system to the origin, forming a type 0 system. When used to track a desired state of the system or reject a constant disturbance the closed loop response will have a constant steady-state offset and therefore integral error control action is needed. RSLQR makes use of the internal model principal [3], augmenting the state space representation by embedding a model of the class of signals (e.g., step, sinusoidal) to be tracked, then applying optimal control theory. The resulting control structure has two parts: a servo tracking controller for command following and state feedback for stabilization. Together they provide accurate command tracking and predictable robust performance. In practice, if a constant reference command is sufficient for obtaining the desired performance, then the system type is raised to one, providing zero steady-state error command tracking. This approach has been successfully deployed to many aerospace systems both autonomous, and pilot-in-the-loop and is quickly reviewed here for completeness.

Consider the linear time-invariant system

$$\begin{aligned}\dot{x} &= Ax + Bu, \\ y &= Cx + Du,\end{aligned}\tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the input, and $y \in \mathbb{R}^p$ is a set of outputs. We wish the output, $y(t)$, to track a constant reference signal with zero steady-state error. The error signal is defined by the difference between the reference and the output signal, $e = r - y$. To raise the system type, we add integral action to the error signal, $x_i = -\int e$, and augment the system with this new state:

$$\begin{bmatrix} \dot{x}_i \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix} \begin{bmatrix} x_i \\ x \end{bmatrix} + \begin{bmatrix} D \\ B \end{bmatrix} u + \begin{bmatrix} -I \\ 0 \end{bmatrix} r.\tag{2}$$

Taking the derivative of (2) and noting that $\dot{r} = 0$, we arrive at the servo design model

$$\dot{z} = \tilde{A}z + \tilde{B}v,\tag{3}$$

where the design model state vector $z \in \mathbb{R}^{n+p}$ is comprised of the tracking errors e and the time derivative of x , and the input $v \in \mathbb{R}^m$ is the time derivative of u such that $z = [\dot{x}_i, \dot{x}]^T$ and $v = \dot{u}$. The servo design system matrices are defined as

$$\tilde{A} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} D \\ B \end{bmatrix},\tag{4}$$

The tracking controller is obtained by applying the LQR algorithm to (3) using the quadratic cost

$$J = \int_0^\infty z^T Q z + v^T R v dt,$$

where $Q = Q^T > 0$, and $R = R^T \geq 0$, $(\tilde{A}, \tilde{B})$ is stabilizable, and $(\tilde{A}, Q^{\frac{1}{2}})$ is detectable. The optimal control is $v = -Kx$ where $K = R^{-1} \tilde{B}^T P$ and P is the unique positive definite solution to the algebraic Riccati equation

$$\tilde{A}^T P + P \tilde{A} - P \tilde{B} R^{-1} \tilde{B}^T P = 0.$$

The resulting controller drives the tracking error and the state derivatives to zero, but allows the system state to settle in at nonzero values. The design model controller $v = -Kz$ is then integrated to obtain the control input $u(t) = -K \int_0^t z(\tau) d\tau$ where the feedback gain has both integral and proportional components such that

$$u = - \begin{bmatrix} K_i & K_x \end{bmatrix} \begin{bmatrix} x_i \\ x \end{bmatrix}.\tag{5}$$

The cost functions parameters Q and R are chosen in order to elicit the desired response. A good rule of thumb is to start with weights on just the integrator states. The resultant closed loop system is shown in Figure 1.

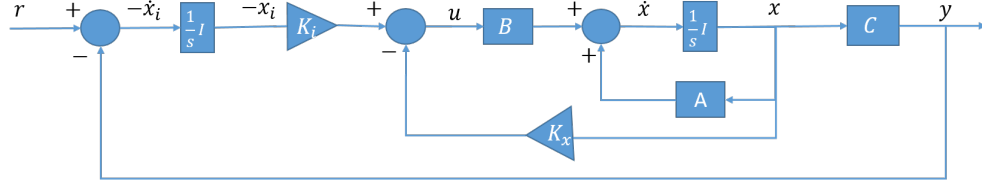


Fig. 1 Linear Quadratic Integral control model

B. Virtual Controls and Relative Degree

The relative degree of an output to an input is the number of times that output must be differentiated with respect to time such that the derivative is affected by that input. For example, for an aircraft in hover the relative degree of the vertical velocity with respect to the thrust of the rotors is one, since the thrust of the rotors directly affects the vertical acceleration. On the other hand, in cruise, the relative degree of the vertical velocity to the elevator is two. The elevator input causes a pitching moment which in turn changes the angle of attack increasing the lift production and thus the vertical acceleration.

Virtual controls are introduced to handle the changing relative degree of the aircraft in flight [4] [5] [6]. The virtual controls are equivalent to using feedback to close an inner loop with faster dynamics than a closed outer loop with slower dynamics. This time separation ensures good overall performance of the system. Here the use of virtual controls is demonstrated within the RSLQR frame work using a simplified system representative of a pitch-for-speed type dynamics.

Consider the linear system

$$\begin{aligned}\dot{x}_1 &= ax_3, \\ \dot{x}_2 &= bu, \\ \dot{x}_3 &= x_2.\end{aligned}\tag{6}$$

Expanding into state space gives

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} u.\tag{7}$$

Suppose that we would like to use the RSLQR algorithm to track the first two states such that the tracking variables are defined as

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.\tag{8}$$

Using system matrices defined in (7) and (8) and putting them into the servo design model (3), we see that the resulting system with $\tilde{A}$ and $\tilde{B}$ is not stabilizable. This is remedied by including the kinematic state x_3 as a virtual control, $v = x_3$, and defining a new state vector $\bar{x} = [x_1, x_2]^T$. The resulting design model is then

$$\begin{aligned}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}, \\ y &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.\end{aligned}\tag{9}$$

Applying the RSLQR formulation of Eq. (2) results in the feedback law

$$\begin{bmatrix} u \\ v \end{bmatrix} = -K \begin{bmatrix} x_i \\ \bar{x} \end{bmatrix}, \quad (10)$$

where the control gain matrix has the form

$$K = \begin{bmatrix} K_i^u & K_x^u \\ K_i^v & K_x^v \end{bmatrix}. \quad (11)$$

The virtual control is used as a reference for x_3 feedback, and the difference is added to the x_2 reference signal. The design relies on proper time separation of the x_1 and x_2 closed-loop dynamics to ensure good performance, which can be accomplished by weighting the cost matrices Q and R appropriately. The resulting closed loop system is depicted in figure 2, which shows the general structure of the RSLQR with virtual control feedback. The controller dynamics are defined as

$$\begin{aligned} -\dot{\hat{x}}_i &= r - K_v K_i^v x_i - [K_v K_x^v + K_v C_v + C]x, \\ u &= -K_i^u x_i - K_x^u x, \end{aligned} \quad (12)$$

where, for this particular example, $K_v = [0, 1]^T$ and $C_v = [0, 0, 1]$. For proper dimensions, zeros are appended to the right side of the K_x matrices such that $K_x^v \in \mathbb{R}^{1 \times 3}$ and $K_x^u \in \mathbb{R}^{1 \times 3}$.

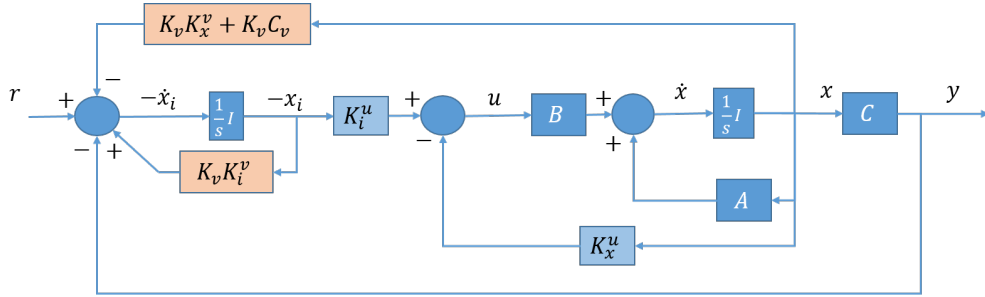


Fig. 2 RLSQR with virtual controls

C. Performance Design and Control Allocation

VTOL aircraft that transition from thrust-borne flight to the more efficient wing-borne flight must traverse a large flight envelope where the effectiveness of a control input may vary greatly and there are inevitably regions of cross-over where there are multiple effectors capable of producing similar accelerations. An example of this on a VTOL aircraft is the control effectors for the pitching moment. In hover, a pitch moment is produced by differential thrust of the rotors. As the dynamic pressure increases at the tail of the aircraft, the elevator becomes effective for producing the necessary moment, and thus some of the load may be shared between the two effectors until the aircraft has transitioned to wing-borne flight. To simplify the design process, it is helpful to break up the design into two parts: the performance and the actuation. The performance being the desired response to a command input in any part of the flight envelope, and the actuation pertaining to how the control effort is distributed among the available effectors. This may be accomplished by applying theory presented in [7], which is briefly described here for the linear quadratic case.

Consider the linear system

$$\dot{x} = Ax + B_u u, \quad (13)$$

with $A \in \mathbb{R}^{n \times n}$, $B_u \in \mathbb{R}^{k \times m}$, $x(t) \in \mathbb{R}^n$ is the state, and $u(t) \in \mathbb{R}^m$ is the control input. Assume B_u does not have full column rank, implying that it can be factorized as

$$B_u = B_\mu B,$$

where $B_\mu \in \mathbb{R}^{n \times k}$ and $B \in \mathbb{R}^{k \times m}$ both have rank k . This gives way to an alternative description

$$\begin{aligned} \dot{x} &= Ax + B_\mu \mu, \\ \mu &= Bu, \end{aligned} \quad (14)$$

where $\mu(t) \in \mathbb{R}^k$ may be interpreted as the total control effort of the effectors, most naturally thought of (in our application) as the total accelerations. Since $k < m$, B and B_u have a non-trivial null space in which u may be perturbed without producing an acceleration μ . Simply put, there are multiple ways to actuate the control inputs u that produce the same commanded total acceleration μ , and therefore there are built in redundancies in the control actuation.

When designing optimal feedback control, we explore two approaches. The first poses the optimal control policy in terms of the input u or the second poses the optimal control policy in terms of μ , and the solution is then mapped onto u by solving a quadratic optimization problem. We describe the two methods below

Method 1. Consider the linear system description (13). Determine $u(t)$ by solving

$$\min_{u(t)} \int_0^\infty [x^T Q x + u^T R_u u] dt,$$

where $Q \geq 0$, and $R_u = R_u^T > 0$. This is the standard linear quadratic regulator (LQR) problem. If the pair (A, B_u) is stabilizable and the pair $(A, Q^{\frac{1}{2}})$ is detectable, the optimal control is $u = -R_u^{-1} B_u^T P x$, where P is the unique positive definite solution to the algebraic Riccati equation

$$A^T P + P A - P B_u R_u^{-1} B_u^T P = 0.$$

Method 2. Consider the linear system description (14). Determine $\mu(t)$ by solving

$$\min_{\mu(t)} \int_0^\infty [x^T Q x + \mu^T R_\mu \mu] dt, \quad (15)$$

where $Q \geq 0$, and $R_\mu = R_\mu^T > 0$. Again solving the LQR problem with (A, B_μ) stabilizable and $(A, Q^{\frac{1}{2}})$ detectable, the solution to (15) is $\mu = -R_\mu^{-1} B_\mu^T P x$. Then $u(t)$ is determined by solving

$$\begin{aligned} \min_{u(t)} \quad & u^T W u, \\ \text{subject to} \quad & \mu = B u, \end{aligned} \quad (16)$$

where $W = W^T > 0$. The solution to this quadratic problem is the weighted generalized inverse $u = W^{-1} B^T (B W^{-1} B^T)^{-1} \mu$.

The main result of [7] states that assuming the matrices R_u and R_μ are related such that $B R_u^{-1} B^T = R_\mu^{-1}$, then u^* and μ^* are the optimal controls associated with design 1 and 2, respectively, and $\mu^* = B u^*$ and the corresponding trajectories are the same. Further if for a given R_μ and W the matrix R_u is chosen as $R_u = W + B^T [R_\mu - (B W^{-1} B^T)^{-1}] B$, the control laws for both design methods will be the same.

By splitting up the control design process, as in method two, the control engineer designs how they would like the aircraft to respond to commands before determining how that response would be physically implemented via the available effectors. Using general acceleration as inputs alleviates scaling issues that arise when using different types of effectors and helps target specific dynamics that need to be more responsive than others. Once the desired performance is achieved the second step allows control allocation solutions to be evaluated and tuned using the weighting matrix W . Because the response is the same, provided no effectors are saturated, an apples-to-apples comparison can be made on a per W basis. Going beyond the simple solution to the quadratic program (16), a more sophisticated allocation algorithm (e.g., Affine Generalized Inverse) may be employed that removes an actuator from the allocation solution once it saturates and thus retains the desired linear performance while working within the physical constraints of the aircraft

III. Vehicle Description and Rigid Body Dynamics

The Lift+Cruise aircraft, shown in figure 3, is a six passenger electric Vertical Take-off and Landing (eVTOL) reference vehicle put forth by NASA's Revolutionary Vertical Lift Technology project [8]. It is a hybrid eVTOL aircraft that operates as a fixed-wing aircraft in forward flight but has eight lifting rotors mounted on the forward wing that provide VTOL capabilities. Its most distinctive attribute, when compared to other eVTOL configurations, is the separation of the lift propulsors from the thrust propulsors such that the lifting rotors are shut off when in the forward (fixed-wing) flight. As with many proposed and under-development Advanced Air Mobility (AAM) aircraft,

the Lift+Cruise aircraft must transition from hover to forward flight and vice versa. In this phase, the lifting force being generated by the aircraft is exchanged between the rotors and the wing. In this study the rotors are assumed to be fixed pitch and speed controlled. As with many eVTOL concepts, the lifting rotors are also used as control effectors in hover, providing control moments through differential thrust of the rotor banks: forward and aft for pitch, left and right for roll, and clockwise and counterclockwise for yaw. In forward flight, deflecting surfaces are used to generate the control moments. These include flaperons on the wing and elevons and rudder on the tail. Thrust is provide by a pusher propeller that is assumed to be speed controlled.

The aerodynamic model used for this study is derived from a semi-empirical modeling tool [9] that builds the aircraft by components. This build up of the aircraft allows for a rapid creation of a dynamic model for non-conventional configurations and provides fast calculations of the six-degree-of-freedom aero-propulsive forces and moments. Based on first principles of aerodynamics and momentum theory, the modeling approach provides a competent dynamic model of a complex aircraft when access to high-fidelity wind tunnel or flight data is unavailable.



Fig. 3 Rendering of the Lift+Cruise aircraft

A. Reference Frames

We use three general reference frames in this study, the first two are the inertial NED frame, and the body frame with origin at the vehicle center of mass, x-axis pointing out the nose, y-axis pointing out the right wing, and z-axis down. The rotation from the body frame to the inertial frame is parameterized by the Euler angles, $\eta = [\phi, \theta, \psi]^T$, (roll, pitch, and yaw respectively), and the rotation matrix $R \in SO(3)$ is defined as

$$R = [R_x(\phi)R_y(\theta)R_z(\psi)]^T \quad (17)$$

where $R_x(\cdot)$, $R_y(\cdot)$, and $R_z(\cdot)$ are the intermediate single-axis rotation matrices, explicitly defined as

$$R_x(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \end{bmatrix}, \quad R_y(x) = \begin{bmatrix} \cos x & 0 & -\sin x \\ 0 & 1 & 0 \\ \sin x & 0 & \cos x \end{bmatrix}, \quad R_z(x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The third frame is the heading frame, which is defined as the inertial frame rotated about the z-axis by the heading angle ψ with the origin coincident with the vehicle center of mass. The rotation from the body frame to the heading frame is defined as the abbreviated rotation $\bar{R} = [R_x(\phi)R_y(\theta)]^T$ such that vector specified in the body frame is mapped to the heading frame through the transformation $\bar{v} = \bar{R}v_b$ where we use the subscript b to specify the body frame and the overbar to specify a vector expressed in the heading frame.

B. Control Commands

In this study we assume the aircraft is flown by an operator or is tracking a prescribed flight path. Both activities require a command set that allows the operator to intuitively fly the aircraft without the over complication of switching between flight control variables as they transition between flight modes. When considering what makes a good set

of tracking commands, we must consider how a path in space may be parameterized as a trajectory. To that effect every point in time may be described by a position, velocity, heading, and turn rate. Therefore, a set of commands that make sense are the desired velocity expressed in the heading frame and the turn rate, such that the reference vector is $r = [\bar{u}_d, \bar{v}_d, \bar{w}_d, \dot{\psi}_d]^T$. This gives an operator the ability to command the aircraft in a way that makes sense in all flight regimes.

C. Nonlinear Dynamics

The angular kinematics of the vehicle are expressed as the time derivatives of the Euler angles

$$\dot{\eta} = S\omega, \quad (18)$$

where ω is the angular rate of the aircraft expressed in the body frame with components (p, q, r) and S is the non-orthogonal transformation from the body angular rates to the time derivatives of the Euler angles

$$S = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta \end{bmatrix}.$$

Because we seek to track heading frame velocities, it makes sense to formulate the translational dynamics in the heading frame. Given the velocity vector $\bar{v} = [\bar{u}, \bar{v}, \bar{w}]^T$ the dynamics are as follows

$$\dot{\bar{v}} = -\dot{\psi} \hat{e}_3 \times \bar{v} + g + \frac{1}{m} \bar{F}(\bar{v}, \omega, \bar{R}, u), \quad (19)$$

where $\bar{F}(\cdot)$ is the aero-propulsive forces acting on the aircraft, $g = [0, 0, a_g]^T$ is the gravitational acceleration vector, u is vector of available effectors, and $\hat{e}_3$ is the unit vector in the z direction. Expanding (19) gives

$$\begin{pmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ \dot{\bar{w}} \end{pmatrix} = \begin{pmatrix} \dot{\psi} \bar{v} + \frac{1}{m} \bar{X}(\bar{v}, \omega, \bar{R}, u) \\ -\dot{\psi} \bar{u} + \frac{1}{m} \bar{Y}(\bar{v}, \omega, \bar{R}, u) \\ a_g + \frac{1}{m} \bar{Z}(\bar{v}, \omega, \bar{R}, u) \end{pmatrix}. \quad (20)$$

The rotational dynamics are given by

$$J\dot{\omega} = -\omega \times J\omega + \tau(\bar{v}, \omega, \bar{R}, u) \quad (21)$$

where J is the inertia matrix, and τ is the aero-propulsive moments. Expanding (21) gives the following set of equations for the rotational dynamics:

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = J^{-1} \left(\begin{bmatrix} J_y r q - J_{xz} q p - J_z q r \\ J_x p^2 - J_z r^2 \\ J_x q p + J_{xz} q r - J_y p q \end{bmatrix} + \begin{bmatrix} L(\bar{v}, \omega, \bar{R}, u) \\ M(\bar{v}, \omega, \bar{R}, u) \\ N(\bar{v}, \omega, \bar{R}, u) \end{bmatrix} \right). \quad (22)$$

IV. Longitudinal Control Design

The control design relies on a set of trim points that are found at a discrete number of flight speeds, climb rates, and turn rates. At every trim point a trajectory-tracking controller is designed by linearizing the dynamics about that condition. Using a gain-scheduled approach, the control gains are then scheduled based on airspeed, climb rate, and turn rate to provide a controller with a common set of control commands over all flight regimes.

The uniform control approach described above is applied here to the longitudinal dynamics at a non-turning trim condition. Although not described in this paper, the same process can be applied to turning trim conditions and the lateral dynamics, with some considerations as to how to handle turn coordination.

For non-turning flight ($\dot{\psi} = 0$), the longitudinal dynamics simplify to the following set of equations

$$\begin{bmatrix} \dot{\bar{u}} \\ \dot{\bar{w}} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{pmatrix} \frac{1}{m} \bar{X}(x, u) \\ g + \frac{1}{m} \bar{Z}(x, u) \\ \frac{1}{J_y} M(x, u) \\ q \end{pmatrix}, \quad (23)$$

where the state vector is given by $x = [\bar{u}, \bar{w}, q, \theta]^T$, and the input u is the set of effectors consisting of the speed of the lifting rotors and pusher propeller, $\omega_r = [\omega_1, \dots, \omega_9]^T$, as well as the deflecting surfaces $\delta_s = [\delta_f, \delta_e]^T$. Given the trim condition (x_0, u_0) the Jacobian matrices are computed such that

$$A = \begin{bmatrix} \frac{1}{m}\bar{X}_{\bar{u}} & \frac{1}{m}\bar{X}_{\bar{w}} & \frac{1}{m}\bar{X}_q & \frac{1}{m}\bar{X}_\theta \\ \frac{1}{m}\bar{Z}_{\bar{u}} & \frac{1}{m}\bar{Z}_{\bar{w}} & \frac{1}{m}\bar{Z}_q & \frac{1}{m}\bar{Z}_\theta \\ \frac{1}{J_y}M_{\bar{u}} & \frac{1}{J_y}M_{\bar{w}} & \frac{1}{J_y}M_q & \frac{1}{J_y}M_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{m}\bar{X}_{\omega_r} & \frac{1}{m}\bar{X}_{\delta_s} \\ \frac{1}{m}\bar{Z}_{\omega_r} & \frac{1}{m}\bar{Z}_{\delta_s} \\ \frac{1}{J_y}M_{\omega_r} & \frac{1}{J_y}M_{\delta_s} \\ 0 & 0 \end{bmatrix}, \quad (24)$$

where the subscript denotes which variable the derivatives of the forces ($\bar{X}$, $\bar{Z}$), and moments (M) are taken with respect to.

As stated previously the horizontal and vertical velocities ($\bar{u}, \bar{w}$) are the variables to be tracked, and the pitch rate q is to be regulated, which leaves the pitch angle θ to be treated as a virtual control input

The performance design model has states $\bar{x} = [\bar{u}, \bar{w}, q]^T$, and the linear design system is

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \mu, \\ y &= \bar{x}, \end{aligned} \quad (25)$$

where $\bar{A}$ is the upper left 3×3 submatrix of A corresponding to the performance states, and μ is the general control accelerations such that $\mu = [\bar{a}_x, \bar{a}_z, \alpha_q]^T$. Applying the RSLQR structure to the system (25), the control design matrices are

$$\tilde{A} = \begin{bmatrix} 0 & I \\ 0 & \bar{A} \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ I \end{bmatrix}. \quad (26)$$

The components of the Q and R matrices are selected to obtain the desired tracking performance. The desired control accelerations are then

$$\mu = - \begin{bmatrix} K_i & K_x \end{bmatrix} \begin{bmatrix} x_i \\ \bar{x} \end{bmatrix}. \quad (27)$$

The control inputs are determined using a weighted pseudo-inverse

$$\begin{bmatrix} u \\ \theta_{des} \end{bmatrix} = W^{-1} \bar{B}^T (\bar{B}W^{-1} \bar{B}^T)^{-1} \mu, \quad (28)$$

where $W = W^T$ is a positive definite weighting matrix, and $\bar{B}$ is the input Jacobian augmented with the pitch angle derivatives

$$\bar{B} = \begin{bmatrix} \frac{1}{m}\bar{X}_{\omega_r} & \frac{1}{m}\bar{X}_{\delta_s} & \frac{1}{m}\bar{X}_\theta \\ \frac{1}{m}\bar{Z}_{\omega_r} & \frac{1}{m}\bar{Z}_{\delta_s} & \frac{1}{m}\bar{Z}_\theta \\ \frac{1}{J_y}M_{\omega_r} & \frac{1}{J_y}M_{\delta_s} & \frac{1}{J_y}M_\theta \end{bmatrix}. \quad (29)$$

The control inputs are executed while the desired pitch angle is fed back, differenced with the current pitch angle, and used to build the pitch rate reference signal.

Denoting the pseudo inverse matrix $M = W^{-1} \bar{B}^T (\bar{B}W^{-1} \bar{B}^T)^{-1}$ it can be decomposed into submatrices corresponding to the input and pitch angle

$$M = \begin{bmatrix} M_u \\ M_\theta \end{bmatrix}. \quad (30)$$

The state feedback controller is then be

$$-\dot{x}_i = -K_\theta M_\theta K_i x_i - (K_\theta M_\theta K_x + K_\theta C_\theta + C)x + Fr, \quad (31)$$

$$u = -M_u(K_i x_i + K_x x), \quad (32)$$

corresponding to the block diagram in Figure 4,

where $r(t) = [\bar{u}_d, \bar{w}_d]^T$ is the horizontal and vertical velocity commands and the F matrix maps them to the tracking variables $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T$, and C selects the tracking states from the state vector.

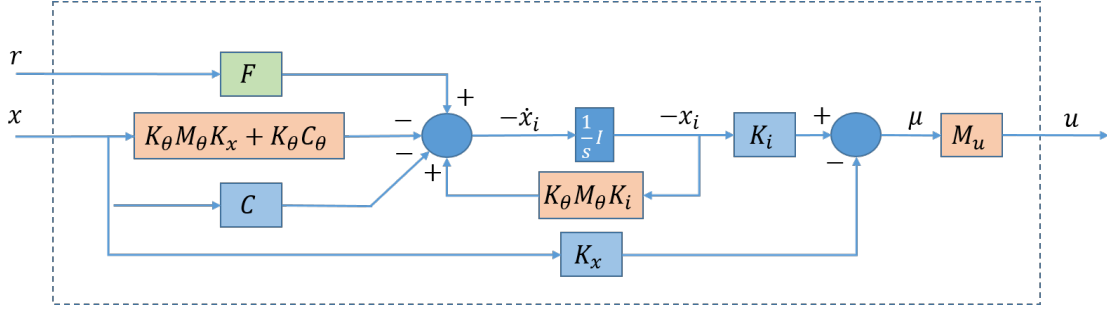
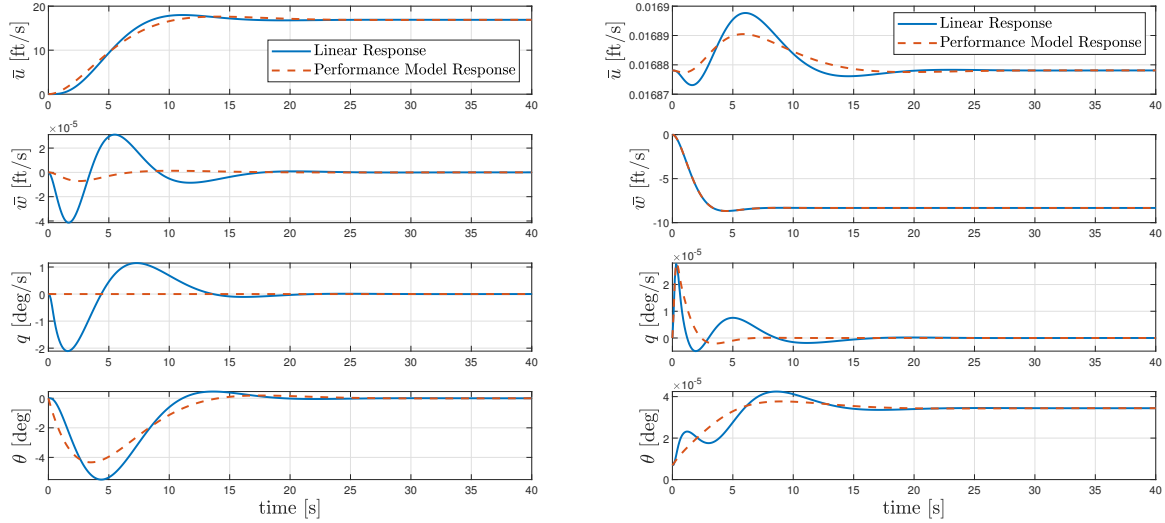


Fig. 4 Longitudinal control block diagram

A. Control Performance

The response of the closed-loop system is evaluated against that of the performance design model Eq. (25) with the feedback law Eq. (27) in two flight conditions, forward flight and hover. In each flight condition, the response to two step commands are generated, the first is a 16.9 ft/s (≈ 10 knot) increase in $\bar{u}$, and the second is a -8.33 ft/s (≈ 500 ft/min climb) step in $\bar{w}$. In hover, the plots in figure 5a show the aircraft must pitch the nose down to produce the horizontal acceleration. This necessary use of the pitch angle results in a slight deviation of the linear response from the desired performance model response. The response to the vertical velocity step in figure 5b shows that because the aircraft has direct control of the vertical velocity with the lifting rotors, the full linear response is almost identical to that of the performance model.



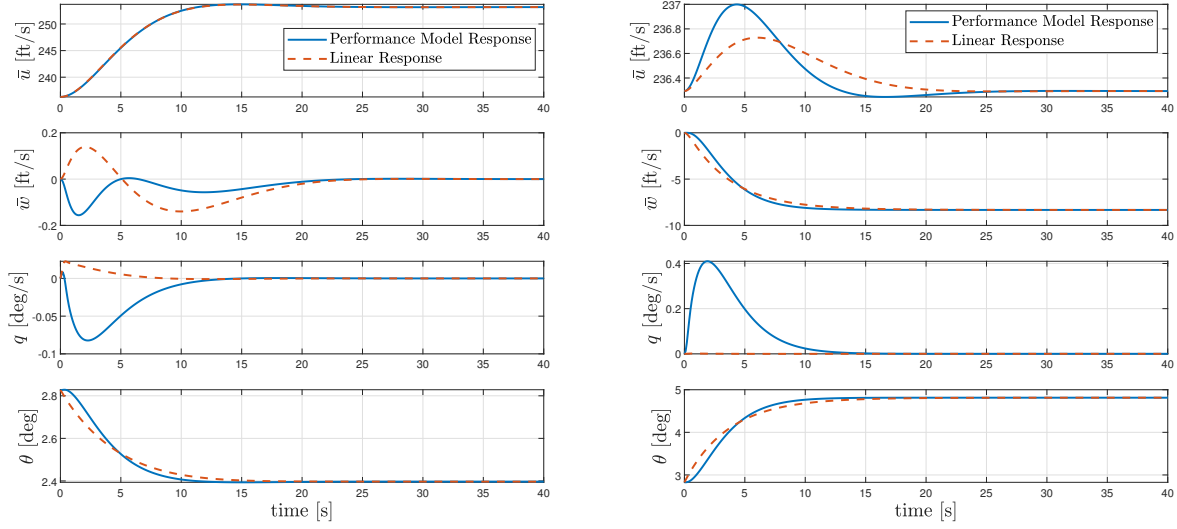
(a) Response to 16.9 ft/s step command in forward velocity

(b) Response to -8.3 ft/s step command in vertical velocity

Fig. 5 Comparison of the full linear closed loop system response to that of the closed loop performance model in hover

In forward flight, the response in figure 6a shows that the two models produce almost identical results in the horizontal velocity state $\bar{u}$ with only small differences in the other three states. This is attributed to the fact that the aircraft has almost direct control of the horizontal velocity through the use of the pusher propeller. Figure 6b shows some larger differences in the two responses since the aircraft must pitch the nose up to increase the angle of attack in order to produce the necessary lift for the increase in vertical velocity. Despite the extra dynamics the response still matches well with the desired performance model.

To demonstrate how the control architecture handles different flight regimes with different dynamics and available control effectors the same maneuver is performed at three different flight speeds that represent hover, mid-transition, and

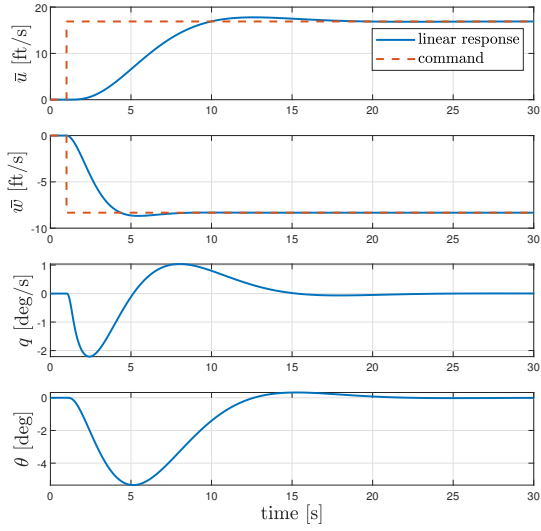


(a) Response to 16.9 ft/s step command in forward velocity (b) Response to -8.3 ft/s step command in vertical velocity

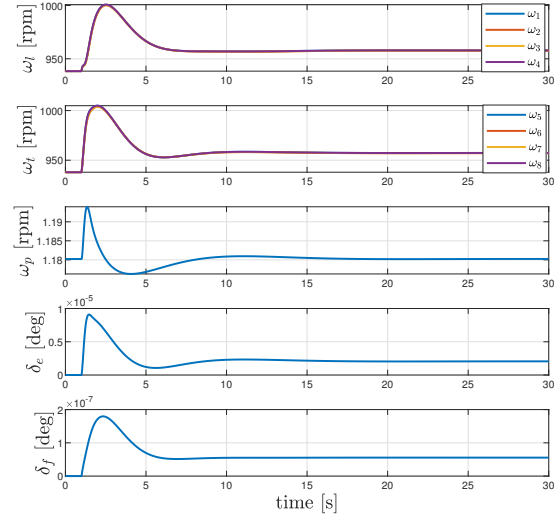
Fig. 6 Comparison of the full linear closed loop system response to that of the closed loop performance model in forward flight

forward flight. The linear responses are plotted in Figures 7, 8, and 9, where the commanded velocity is shown in the red dashed line. In all three flight conditions, the response of the vertical and horizontal velocity is relatively similar while the manner in which that response is affected is markedly different in each regime. In hover, control actuation is almost exclusively limited to the lifting rotors, which provide lift and the control moment to affect the pitch angle change. In mid-transition the control actuation is split between all effectors. The rotors and pusher propeller provide the majority of the control accelerations but are augmented by the control surfaces, which require large deflections due to the relatively low dynamic pressure at the 60-knot flight condition. Also of note is that less pitch angle is required here than in other flight conditions since both the pusher propeller and lifting rotors can provide direct control accelerations in the horizontal and vertical direction. The response in forward flight is characteristic of a conventional aircraft. The lifting force is generated by increasing the angle of attack while the pusher propeller provides the necessary thrust to increase the forward velocity.

The results demonstrate how the control architecture unifies the control design approach in all flight regimes allowing a single structure to be used without necessitating any switching between modes or command variables. In addition, the selected commands simplify the pilot interaction and provide an automated means for flight envelope protection (for θ and ϕ) through control allocation.

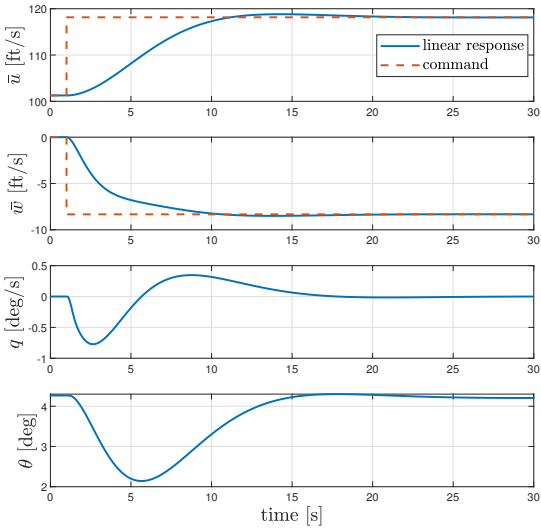


(a) States

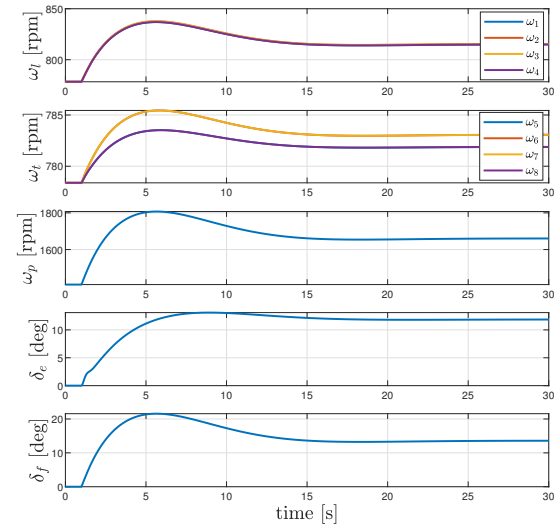


(b) Inputs

Fig. 7 Hover maneuver, 0 kts

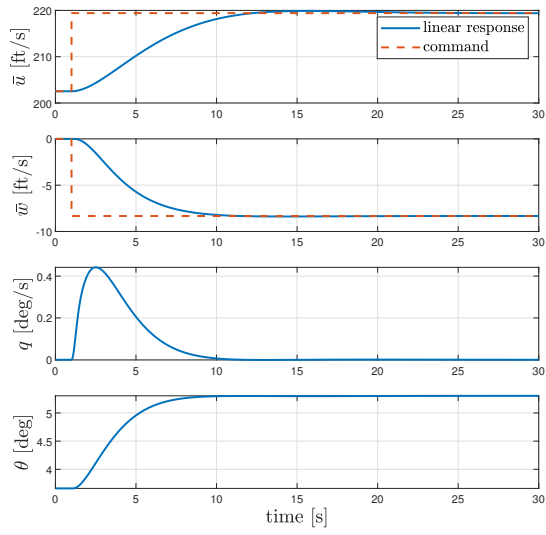


(a) States

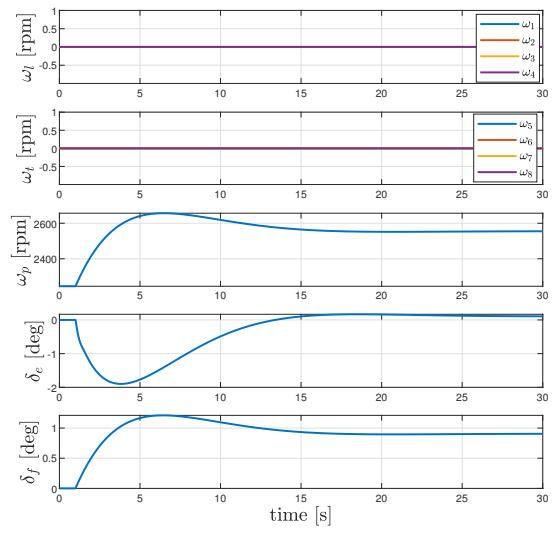


(b) Inputs

Fig. 8 Mid transition maneuver, 60 kts



(a) States



(b) Inputs

Fig. 9 Forward flight maneuver, 120 kts

V. Generalized Control Allocation

A. Allocation Implementation and Theory

The robust control delineated in section II, designed the longitudinal control channel. The lateral control channel (while not detailed in this research) was also designed separately. The output of these controllers are three desired moments (accelerations) that are used as inputs to a control allocator. Three desired accelerations ($\vec{m}_{lon}$ and $\vec{m}_{lat}$) for each channel are listed in Table 1. The control effectors, augmented by a pseudo control, for each channel are shown in Table 2.

Table 1 Desired Control Accelerations by Control Channel

Control Variable	Longitudinal	Lateral
Control Frame Velocities	x, z	y
Body Rates	q	p, r

Table 2 Control Effectors and Pseudo Control by Control Channel

Effectors	Longitudinal	Lateral
Lifting Rotors (8)	Yes [†]	Yes [†]
Pusher Propeller (1)	Yes [†]	No
Differential Ailerons (1)	No	Yes
Symmetric Ailerons (1)	Yes	No
Symmetric Elevators (1)	Yes	No
Rudder (1)	No	Yes
Pseudo Control	Lateral	Longitudinal
Theta	Yes	No
Phi	No	Yes

[†] Effectors not used in all phases of flight (hover, transition and cruise)

In this paper, two control allocation algorithms are used. The first baseline algorithm uses a single weighted generalized inverse for each allocation channel (one generalized inverse for longitudinal and a separate one for lateral). This algorithm was used to design the controllers (gain scheduling, trim conditions etc.) The second algorithm is the Affine Generalized Inverse (AGI) iterative algorithm first described in [1]. The AGI algorithm used in this research has been improved to incorporate the most general linear case (affine control effectors, preferred control, and dual-sided effector position/rate limits).

For the baseline allocation algorithm, the previously discussed linear interpolation is performed at each simulation time step to determine the required linear quantities. These quantities necessary for control allocation include control trim state $\vec{v}_0$, longitudinal and lateral control effectiveness matrices ($B_{lon} \in \mathbb{R}^{3 \times 12}$, $B_{lat} \in \mathbb{R}^{3 \times 11}$), and the corresponding control effector weighting matrices ($W_{lon} \in \mathbb{R}^{12 \times 12}$, $W_{lat} \in \mathbb{R}^{11 \times 11}$). The generalized allocation per channel for the baseline allocation is:

$$P_{lon} = W_{lon}^{-1} B_{lon}^T \left(B_{lon} W_{lon}^{-1} B_{lon}^T \right)^{-1} \in \mathbb{R}^{12 \times 3}, \quad (33)$$

$$P_{lat} = W_{lat}^{-1} B_{lat}^T \left(B_{lat} W_{lat}^{-1} B_{lat}^T \right)^{-1} \in \mathbb{R}^{11 \times 3}. \quad (34)$$

The generalized inverses of Eqs. (33 and 34), enable direct computation of the change in controls based on the desired

moment change:

$$\Delta \vec{u}_{lon} = P_{lon} \Delta \vec{m}_{lon}, \quad (35)$$

$$\Delta \vec{u}_{lat} = P_{lat} \Delta \vec{m}_{lat}. \quad (36)$$

These delta control quantities are added (addition for like effectors) to generate a total delta control which is in turn added to the trim control to yield the control allocation as follows:

$$\Delta \vec{u}_{tot} = \Delta \vec{u}_{lon} + \Delta \vec{u}_{lat}, \quad (37)$$

$$\vec{u} = \vec{u}_0 + \Delta \vec{u}_{tot}. \quad (38)$$

Effector position limits and rate limits are then enforced on the control allocation $\vec{u}$. The drawbacks of using a single generalized inverse are well known. In particular, a single generalized inverse only yields allocations within effector position limits over a strict subset of the attainable moment set [10], [11]. Moreover, in this application since longitudinal and lateral allocations occur separately and both channels have some effectors in common, effector position saturation has a more unpredictable impact on the total allocation obtained by their addition. As the baseline method doesn't incorporate effector position and rate limits during allocation, any effector saturation that occurs after allocation changes both the magnitude and direction of the generated moment. These resulting moment impacts can be quite dramatic, even resulting in system instability.

The preferred allocation algorithm used in this research is the Affine Generalized Inverse. The research in this paper has broadened the scope of the original algorithm introduced in [1] and the use of the null space of the control effectiveness matrix B to yield minimal weighted allocations with the use of effector unsaturation [12], [13]. Specifically, in this research, the AGI algorithm incorporates real-world affine effectors $\vec{m} = \vec{m}_0 + B\vec{u}$ [14], preferred control $\vec{v}_0$ and arbitrary effector position ($\vec{u}_{upr}$ and $\vec{u}_{lwr}$) and rate ($\vec{r}_{upr}$ and $\vec{r}_{lwr}$) limitations. The arbitrary limits can be two-sided (e.g., $\vec{u}_{upr} > 0 > \vec{u}_{lwr}$) or one-sided (e.g., $\vec{u}_{upr} \geq \vec{u}_{lwr} \geq 0$) determined by whether or not the limits contain the origin [15]. The appendix of this paper contains two theorems that support the ability to broaden the original scope of the AGI algorithm. Theorem 1 generalizes the optimal allocation problem to incorporate affine moment generation and preferred control for the unconstrained case, in agreement with [15]. Theorem 2 extends the previous theorem to the constrained (saturated effector) case.

The allocation problem for the AGI algorithm in this research is defined as:

$$\vec{u}_{opt} = \underset{\vec{u}}{\operatorname{argmin}} (\vec{u} - \vec{v}_0)^T W (\vec{u} - \vec{v}_0) \text{ such that} \quad (39)$$

$$\vec{m} = \vec{m}_0 + B\vec{u} \text{ subject to} \quad (40)$$

$$\vec{u}_{upr} \geq \vec{u} \geq \vec{u}_{lwr} \text{ (elementwise effector position limits),} \quad (41)$$

$$\vec{r}_{upr} \geq \frac{d}{dt} \vec{u} \geq \vec{r}_{lwr} \text{ (elementwise effector rate limits).} \quad (42)$$

The broadening of the AGI algorithm in this research is beneficial in multiple ways over the original algorithm. For instance, the incorporation of the affine effectors provides a substantial improvement in linear modeling as the intercept term ($\vec{m}_0$) allows for more accurate local derivative of change in moment per change in effector [14]. Additionally, the incorporation of the moment intercept term $\vec{m}_0$ together with the preferred control $\vec{v}_0$ enables a change of variable (which is explained subsequently) that facilitates handling one-sided position limits by converting them to two-sided limits [15]. The ability to handle one-sided position limits is important for two reasons. First this broadens the applicability of the method to a larger class of real-world systems (e.g., thrusters whose lower limit is zero). Also, the effector rate limits are implemented as modified effector position limits at each simulation time step, which commonly results in one-sided effector position limits. In the sequel, the superscript i is used to denote the i^{th} simulation time step value whereas the argument j refers to the j^{th} component of a vector quantity. For example, $\vec{u}^i(j)$ is the j^{th} component of the vector $\vec{u}$ at the i^{th} simulation time step. Using this notation then given the simulation time step Δt , the enforcement of both effector position and rate limits at the current simulation step i is performed using modified position limits as:

$$\vec{u}_{upr}^i(j) = \min\{\vec{u}_{upr}(j), \vec{u}_{opt}^{i-1}(j) + \vec{r}_{upr}(j)\Delta t\} \quad \forall j, \quad (43)$$

$$\vec{u}_{lwr}^i(j) = \max\{\vec{u}_{lwr}(j), \vec{u}_{opt}^{i-1}(j) + \vec{r}_{lwr}(j)\Delta t\} \quad \forall j. \quad (44)$$

As discussed, the preferred control not only enables one-sided position limits, it also makes physical sense and has utility in many situations. For instance, horizontal stabilizers often have a non-zero preferred position which is readily accommodated with $\vec{v}_0$.

Finally, the generalization of the AGI algorithm enables it to be used as a delta or absolute allocation algorithm. Both formulations use a linear approximation around a trim condition $\vec{u}_0$, which yields a control effectiveness matrix B , and a trim moment $\vec{m}_0 \neq \vec{0}$. The effector trim positions are required to generate equal and opposite moments to the non-aerodynamic generated moments (e.g. gravitational, inertial) to balance the equations of motion for trimmed flight. The delta allocation formulation is often used in allocation literature [16], [17].

For the delta formulation, the allocator requires no knowledge of the real world linearized trim moment $\vec{m}_0$ nor of the effector trim positions $\vec{u}_0$. Therefore, we specify $\vec{m}_0 = \vec{0}$ and $\vec{v}_0 = \vec{0}$ and for a desired moment $\vec{m}$ (referred to as the delta moment $\Delta\vec{m}$) we obtain the delta allocation $\Delta\vec{u}$ from Eqs. (39,40) and the total control, respectively, as:

$$\Delta\vec{u}_{opt} = \underset{\Delta\vec{u}}{\operatorname{argmin}} \Delta\vec{u}^T W \Delta\vec{u} \text{ such that } B\Delta\vec{u} = \Delta\vec{m}, \quad (45)$$

$$\vec{u}_{total} = \vec{u}_0 + \Delta\vec{u}_{opt} \text{ (total allocation)}. \quad (46)$$

Note, that while the delta allocator had no knowledge of either the effector trim vector $\vec{u}_0$, nor the trim moment $\vec{m}_0$, the total allocation shifts the allocation delta solution to achieve them.

Conversely for the absolute allocation formulation, the goal of allocation is to yield the desired total allocation directly from the allocation algorithm. Therefore, in Eqs. (39,40) we specify the moment intercept term as $\vec{m}_0 = -B\vec{u}_0$ and the absolute moment desired as $\vec{m} = \Delta\vec{m} - \vec{m}_0$. Next, the preferred control $\vec{v}_0$ is set identically as the effector trim vector $\vec{u}_0$, and we obtain the total control directly as:

$$\vec{u}_{total} = \vec{u}_{opt} = \underset{\vec{u}}{\operatorname{argmin}} (\vec{u} - \vec{u}_0)^T W (\vec{u} - \vec{u}_0) \text{ such that } B\vec{u} = (\Delta\vec{m} + B\vec{u}_0) \quad (47)$$

For this research, the delta formulation is used. However, the absolute formulation is valid and may be better suited for some situations. For example, for the case of steady-state trim of a flight vehicle (e.g. steady level flight), then while keeping the trimmed intercept moment ($\vec{m}_0$) constant, we can retrim to a new preferred trim effector allocation simply by changing to the new preferred control $\vec{v}_0$ with the control regulator engaged. Additionally, if we seek the smallest magnitude control (irrespective of initial trim effector positions), then the absolute allocation method might be better suited.

The process to convert one-sided effector position limits to two-sided limits is facilitated with a change of variable which requires determining a control vector offset. While many choices of change of variable would suffice, in this research it is computed as follows:

$$\vec{o}(j) = \begin{cases} -(\vec{u}_{upr}(j) - \vec{u}_{lwr}(j)) / 2, & \vec{u}_{upr}(j) \geq 0 \text{ and } \vec{u}_{lwr}(j) \geq 0 \\ (\vec{u}_{upr}(j) - \vec{u}_{lwr}(j)) / 2, & \vec{u}_{upr}(j) < 0 \text{ and } \vec{u}_{lwr}(j) < 0 \\ 0 & \text{otherwise} \end{cases} \quad (48)$$

The offset vector $\vec{o}$ is then used to compute the change of variable as follows:

$$\tilde{\vec{u}}_0 = \vec{u}_0 + \vec{o} \quad (49)$$

$$\tilde{\vec{m}}_0 = \vec{m}_0 + B\vec{o} \quad (50)$$

$$\tilde{\vec{u}}_{upr} = \vec{u}_{upr} + \vec{o} \quad (51)$$

$$\tilde{\vec{u}}_{lwr} = \vec{u}_{lwr} + \vec{o} \quad (52)$$

The allocation problem of Eqs. (39-42) is then solved using the tilde quantities of Eqs. (49-52). Assuming that the problem is feasible (specifically that the moment is obtainable), then the solution $\tilde{\vec{u}}_{opt}$ obtained using Eqs. (39-42) must be shifted back such that $\vec{u}_{opt} = \tilde{\vec{u}}_{opt} - \vec{o}$. The resulting solution is optimal and within the original one-sided limits.

An issue can occur with the above change of variable when the desired moment $\vec{m} - \vec{m}_0$ is not obtainable (i.e., the problem is not feasible). The baseline AGI algorithm [1] is known to preserve moment direction with an optimal solution within effector two-sided limits that provides the desired moment (if feasible) or the maximal moment that is feasible along the desired moment direction. Therefore, the change of variable ensures that the solution $\vec{u}_{opt}$ is

contained within the two-sided limits ($\tilde{u}_{upr} \geq \tilde{u}_{opt} \geq \tilde{u}_{lwr}$ element wise). However, to preserve the unshifted moment direction $\tilde{m} - \tilde{m}_0$ in Eq. (50) when shifted back, the $\tilde{u}_{opt}$ solution requires that:

$$\tilde{u}_{opt} = \tilde{u}_{opt} - \frac{\|B\tilde{u}_{opt}\|_2}{\|\tilde{m} - \tilde{m}_0\|_2} \tilde{o}. \quad (53)$$

The effect of Eq. (53) is that for non-feasible desired moments, the offset vector $\tilde{o}$ is scaled by a factor less than one. In this case, there is no guarantee that $\tilde{u}_{upr} \geq (\tilde{u}_{opt} - \|B\tilde{u}_{opt}\|_2 \tilde{o} / \|\tilde{m} - \tilde{m}_0\|_2) \geq \tilde{u}_{lwr}$. This undesired consequence can readily be addressed if the controller is limited to only ask for feasible moments. Moreover, in practice the issue is often ameliorated for the AGI delta formulation as the total allocation is achieved by adding back the trim controls (i.e., $\tilde{u}_{tot} = \tilde{u}_{opt} + \tilde{v}_0$) which may restore the total allocation back within the absolute effector position limits.

B. Allocation Performance

In this section, we display some basic Lift+Cruise aircraft performance for the longitudinal and lateral controllers using the baseline and AGI allocators. The allocators both used the delta allocation method (delta not total moment) around a nominal trim condition. This trim condition is obtained at each time step by linearly interpolating between a series of gain scheduled lookup tables, and the interpolation provides the allocators the necessary trim effector positions $\tilde{v}_0$, the linearized control effectiveness matrices (B_{lat} and B_{lon}), and the designed effector weighting matrices (W_{lat} and W_{lon}). As denoted previously, the baseline allocator yields allocation solutions for each controller separately and subsequently sums the allocations and then enforces effector position/rate limits. Conversely, the AGI allocator, combines the desired moments of both controllers and allocates simultaneously. Moreover, the AGI algorithm enforces effector position/rate limits as part of the allocation process.

As the allocators are using the same gain scheduled matrices, for simulations that do not result in any effector saturating, the baseline and AGI algorithms yield identical allocations. However, in Figures 10 and 11, a scenario is presented that results in elevator saturation. In this scenario, the aircraft starts trimmed at a forward velocity of 60 ft/sec and undergoes a step change in forward velocity of -8 ft/sec and vertical velocity of 10 ft/sec (descent). As Figure 11b shows, the elevator saturates shortly after the step inputs occur (≈ 6.5 seconds). Prior to this elevator saturation, the baseline and AGI allocations are identical, but diverge from each other afterwards. The principal effect of saturation is seen in Figure 10a which shows a substantial delay in the baseline allocator reaching the desired descent rate of 10 ft/sec. In Figure 11b, the bottom right plot shows the magnitude of the control requested ($\|\Delta\tilde{m}\|_2$) by each allocator at each time step. This plot shows that the baseline allocator has to request substantially more control than the AGI allocator, even to achieve its reduced performance.

A main goal of this research is to establish a simulation architecture that can be readily applied to a wide variety of aerospace vehicles that will facilitate autonomous decision research [18]. We next present an example of the impact that a simple change in weighting matrix could make on vehicle performance. In this example, we demonstrate how a change in the weighting matrix (to the identity matrix) using the same controller gains, trim condition and control effectiveness matrices can substantially improve performance. As shown in Figure 12a, we start from the same forward velocity of 60 ft/sec, but introduce an additional side velocity (4 ft/sec) and increase the descent velocity to 15 ft/sec. The elevator again saturates, but the introduction of the side velocity step results in an instability in the baseline controller. Conversely, the AGI allocator readily achieves the requested inputs in a stable manner. While this research documents the control and allocation applicable to any vehicle within the simulation architecture, the autonomous decision processes applicable to this control framework, alluded to by the previous example, are subjects of future papers.

VI. Conclusions

The principal focus of this research was to establish a unified control architecture necessary to support the revolutionary aeronautics transformation incorporating unconventional transition aircraft. The foundational elements required of this architecture include robustness, control uniformity across all phases of flight, simplicity to implement given modeling of sufficient fidelity, and the potential to fully utilize the capabilities of these novel aircraft with a variety of control effectors. This research documents the successful achievement of these elements with the establishment of a simulation framework supporting research of transition vehicles within the NASA Intelligent Contingency Management (ICM) project. This simplified control approach across all phases of flight has been successfully applied to efficient universal trajectory language (EUTL) trajectory tracking [19] and successfully applied to other transition vehicles (e.g., LA-8). While not documented in this research, this architecture is already being used in support of a host of ICM's

autonomy goals [18], including collision avoidance, dynamic flight envelope assessment, and trajectory re-planning. With a solid simulation control architecture in place, we expect further research to focus on higher-level decision processes which make use of the flexibility this architecture provides.

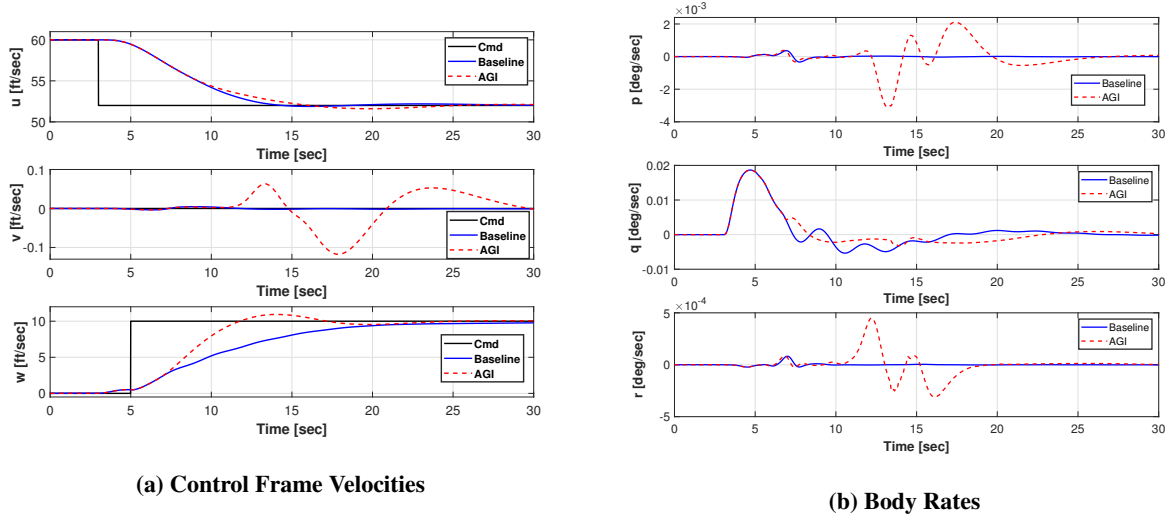


Fig. 10 Command Step Response Comparison between Baseline and AGI (Identical Weighting)

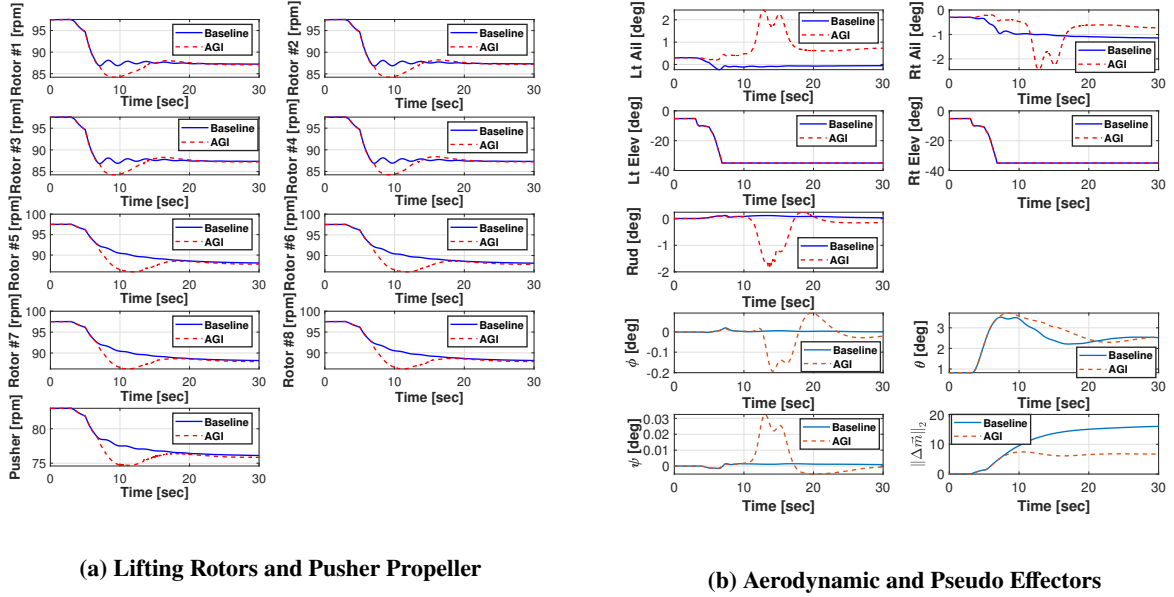
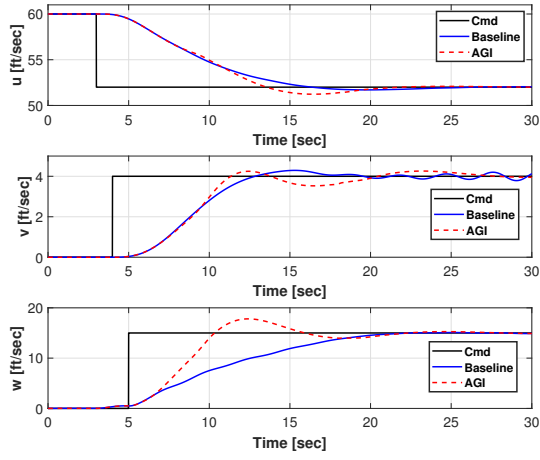
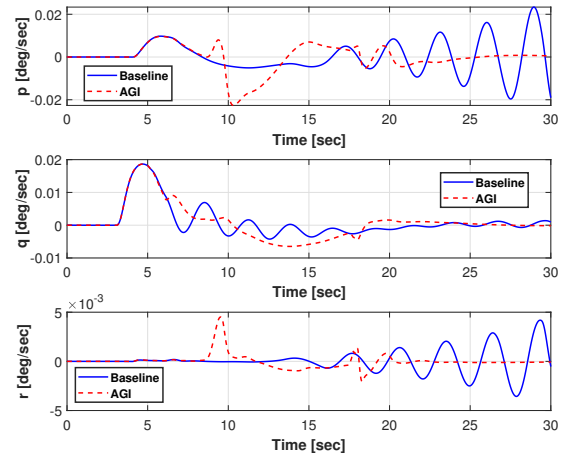


Fig. 11 Effector Step Response Comparison between Baseline and AGI (Identical Weighting)

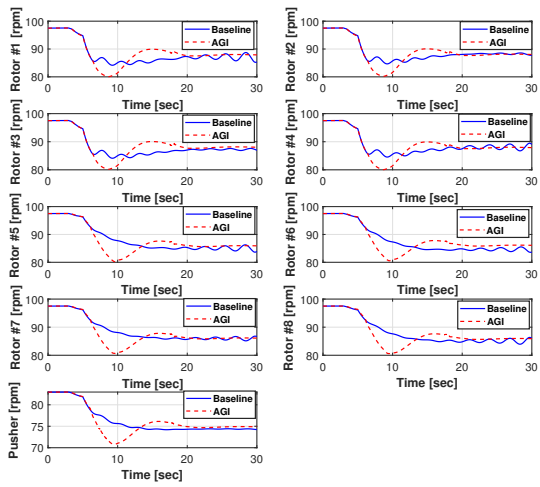


(a) Control Frame Velocities

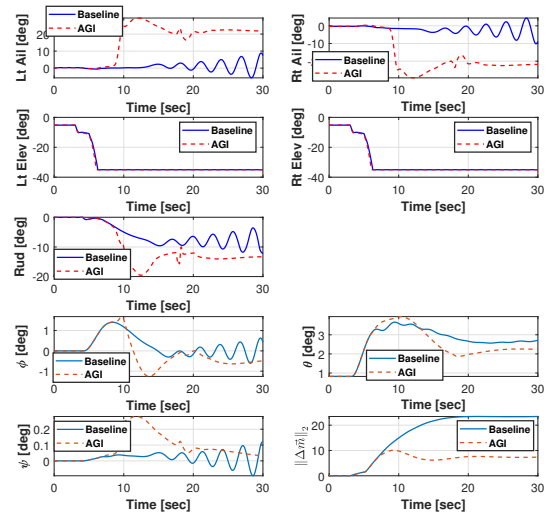


(b) Body Rates

Fig. 12 Command Step Response Comparison between Baseline and AGI (Modified Weighting)



(a) Lifting Rotors and Pusher Propeller



(b) Aerodynamic and Pseudo Effectors

Fig. 13 Effector Step Response Comparison between Baseline and AGI (Modified Weighting)

Appendix

Theorem 1. *The unconstrained weighted control allocation problem with affine control effectiveness (e.g. $\vec{m} = \vec{m}_0 + B\vec{u}$), preferred control $\vec{u}_0$, and positive(semi) definite weighting matrix $W > 0$ ($W \geq 0$) seeks the optimal allocation $\vec{u}^*$:*

$$\vec{u}^* = \underset{\vec{u}}{\operatorname{argmin}} \|W (\vec{u} - \vec{u}_0)\|_2 \text{ such that } \vec{m} = \vec{m}_0 + B\vec{u} \quad (54)$$

The solution to Eq.(54) is:

$$\vec{u}^* = \left(I - W^{-1} \left(BW^{-1} \right)^{\ddagger} B \right) \vec{u}_0 + W^{-1} \left(BW^{-1} \right)^{\ddagger} (\vec{m} - \vec{m}_0), \text{ where} \quad (55)$$

$$\left(BW^{-1} \right)^{\ddagger} = \left(BW^{-1} \right)^T \left(BW^{-1} \left(BW^{-1} \right)^T \right)^{-1}. \quad (56)$$

Proof. We define the cost function with Lagrange multiplier $\vec{\mu}$ as:

$$J(\vec{u}, \vec{\mu}) = \frac{1}{2} (\vec{u} - \vec{u}_0)^T W^T W (\vec{u} - \vec{u}_0) + \vec{\mu}^T (B\vec{u} + \vec{m}_0 - \vec{m}). \quad (57)$$

Now expanding Eq.(57) and solving for the first-order necessary conditions gives:

$$J(\vec{u}, \vec{\mu}) = \frac{1}{2} \vec{u}^T W^T W \vec{u} - \frac{1}{2} \vec{u}^T W^T W \vec{u}_0 - \frac{1}{2} \vec{u}_0^T W^T W \vec{u} + \frac{1}{2} \vec{u}_0^T W^T W \vec{u}_0 + \vec{\mu}^T (B\vec{u} + \vec{m}_0 - \vec{m}), \quad (58)$$

$$\frac{\partial J}{\partial \vec{u}} = \vec{u}^T W^T W - \vec{u}_0^T W^T W + \vec{\mu}^T B = 0, \quad (59)$$

$$\frac{\partial J}{\partial \vec{\mu}} = (B\vec{u} + \vec{m}_0 - \vec{m})^T = 0. \quad (60)$$

Right multiplying Eq.(59) by $W^{-1} (W^{-1})^T B^T$ and rearranging yields:

$$\vec{\mu}^T = (\vec{u}_0 - \vec{u})^T B^T \left(BW^{-1} \left(BW^{-1} \right)^T \right)^{-1}. \quad (61)$$

Rearranging Eq.(60) we obtain $\vec{u}^T B^T = (\vec{m} - \vec{m}_0)^T$ and substituting into Eq.(61) yields:

$$\vec{\mu}^T = \vec{u}_0^T B^T \left(BW^{-1} \left(BW^{-1} \right)^T \right)^{-1} - (\vec{m} - \vec{m}_0)^T \left(BW^{-1} \left(BW^{-1} \right)^T \right)^{-1}. \quad (62)$$

Now substituting Eq.(62) back into Eq.(59) with yields:

$$\vec{u}^T W^T W - \vec{u}_0^T W^T W + \vec{u}_0^T B^T \left(BW^{-1} \left(BW^{-1} \right)^T \right)^{-1} B - (\vec{m} - \vec{m}_0)^T \left(BW^{-1} \left(BW^{-1} \right)^T \right)^{-1} B = 0. \quad (63)$$

Right multiplying Eq.(63) by $W^{-1} (W^{-1})^T$, substituting Eq.(56), and rearranging yields Eq.(54) as claimed. $\square$

The above proof agrees with [15]. The applicability of Theorem 1 is severely limited in real-world applications. Specifically, it precludes saturating one or more effectors, which precludes controls on the boundary of the allowable control set (i.e., $\vec{u} \notin \delta(\Omega)$). This results in the use of the unconstrained formulation applicable over a strict subset of the Attainable Moment Set (Φ). Next, we derive the optimal allocation but for the case of saturated effectors and therefore find an allocation with applicability over the entire AMS. For a given system the set S is defined as the indices of the system effectors (i.e., $S = \{1, 2, \dots, m\}$). Then we define the effector sets S_1 and S_2 to denote the unsaturated and saturated effector sets respectively, such that the union $S_1 \cup S_2 = S$ and intersection $S_1 \cap S_2 = \emptyset$. In the sequel, we will use the subscripts 1 and 2 to denote, for vector quantities, the components associated with the unsaturated and saturated effectors, respectively. For example, $\vec{u}_1$ is the unsaturated components of $\vec{u}(i)$ such that $i \in S_1$. Additionally, we similarly decompose $B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$ (using the unsaturated and saturated effector columns of B), the generalized

inverse $P = \begin{bmatrix} P_1^T & P_2^T \end{bmatrix}^T$ (for the unsaturated and saturated rows of P), and the weighting matrix $W = \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix}$.

Theorem 2. The constrained weighted control allocation problem with affine control effectiveness (e.g. $\vec{m} = \vec{m}_0 + B\vec{u}$), preferred control $\vec{v}$, and positive(semi) definite weighting matrix $W > 0$ ($W \geq 0$) seeks the optimal allocation $\vec{u}^*$:

$$\vec{u}^* = \underset{\vec{u}}{\operatorname{argmin}} \|W (\vec{u} - \vec{v})\|_2 \text{ such that } \vec{m} = \vec{m}_0 + B\vec{u} \text{ and } \vec{u}_2 = \vec{s}_2, \quad (64)$$

where $\vec{s}_2$ is the vector of position limits for the saturated effectors of S_2 . The solution to Eq.(64) is $\vec{u}^* = \begin{bmatrix} \vec{u}_1^T & \vec{u}_2^T \end{bmatrix}^T$:

$$\vec{u}_1^* = (I - P_1 B_1) \left(\vec{v}_1 + \left(W_1^{-1} \right)^T W_2 (\vec{v}_2 - \vec{s}_2) \right) - P_1 B_2 \vec{s}_2 + P_1 (\vec{m} - \vec{m}_0) \quad (65)$$

$$\vec{u}_2^* = \vec{s}_2 \quad (66)$$

Proof. Using the Lagrange multipliers $\vec{\mu}$ and $\vec{\lambda}$, we define the cost function:

$$J(\vec{u}, \vec{\mu}, \vec{\lambda}) = \frac{1}{2} (\vec{u} - \vec{v})^T W (\vec{u} - \vec{v}) + \vec{\mu}^T (B\vec{u} + \vec{m}_0 - \vec{m}) + \vec{\lambda}^T (\vec{u}_2 - \vec{s}_2) \text{ which expands to} \quad (67)$$

$$\begin{aligned} J(\vec{u}_1, \vec{u}_2, \vec{\mu}, \vec{\lambda}) &= \frac{1}{2} \begin{bmatrix} \vec{u}_1^T & \vec{u}_2^T \end{bmatrix} \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \begin{bmatrix} \vec{u}_1 \\ \vec{u}_2 \end{bmatrix} - \begin{bmatrix} \vec{v}_1^T & \vec{v}_2^T \end{bmatrix} \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \begin{bmatrix} \vec{u}_1 \\ \vec{u}_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \vec{v}_1^T & \vec{v}_2^T \end{bmatrix} \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \end{bmatrix} \\ &+ \vec{\mu}^T \left(\begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} \vec{u}_1 \\ \vec{u}_2 \end{bmatrix} + \vec{m}_0 - \vec{m} \right) + \vec{\lambda}^T (\vec{u}_2 - \vec{s}_2) \end{aligned} \quad (68)$$

After expansion, the first-order necessary conditions are:

$$\frac{\partial J}{\partial \vec{u}_1} = \vec{u}_1^T W_1 + \vec{u}_2^T W_2^T - \left(\vec{v}_1^T W_1 + \vec{v}_2^T W_2^T \right) + \vec{\mu}^T B_1 = 0, \quad (69)$$

$$\frac{\partial J}{\partial \vec{u}_2} = \vec{u}_1^T W_2 + \vec{u}_2^T W_3 - \left(\vec{v}_1^T W_2 + \vec{v}_2^T W_3 \right) + \vec{\mu}^T B_2 + \vec{\lambda}^T = 0, \quad (70)$$

$$\frac{\partial J}{\partial \vec{\mu}} = (B_1 \vec{u}_1 + B_2 \vec{u}_2 + \vec{m}_0 - \vec{m})^T = 0, \quad (71)$$

$$\frac{\partial J}{\partial \vec{\lambda}} = (\vec{u}_2 - \vec{s}_2)^T. \quad (72)$$

Equation (72) directly yields:

$$\vec{u}_2 = \vec{s}_2. \quad (73)$$

Rearranging Eq.(71) and substituting $\vec{u}_2 = \vec{s}_2$ yields:

$$\vec{u}_1^T B_1^T = \vec{m}^T - \vec{m}_0^T - \vec{s}_2^T B_2^T. \quad (74)$$

Now substituting $\vec{u}_2 = \vec{s}_2$ into Eq.(69) and right multiplying by W_1^{-1} yields:

$$\vec{u}_1^T + \vec{s}_2^T W_2^T W_1^{-1} - \vec{v}_1^T - \vec{v}_2^T W_2^T W_1^{-1} - \mu^T B_1 W_1^{-1} = 0, \quad (75)$$

which after transposing and rearranging yields

$$\vec{u}_1 = - \left(W_1^{-1} \right)^T W_2 S_2 + \vec{v}_1 + \left(W_1^{-1} \right)^T W_2 \vec{v}_2 - \left(W_1^{-1} \right)^T B_1^T \mu. \quad (76)$$

Now right multiplying Eq. (75) by B_1^T and $(B_1 W_1^{-1} B_1^T)$ and rearranging yields:

$$\begin{aligned} \vec{\mu}^T &= -\vec{u}_1^T B_1^T \left(B_1 W_1^{-1} B_1^T \right)^{-1} - \vec{s}_2^T W_2^T W_1^{-1} B_1^T \left(B_1 W_1^{-1} B_1^T \right)^{-1} + \vec{v}_1^T B_1^T \left(B_1 W_1^{-1} B_1^T \right)^{-1} \\ &+ \vec{v}_2^T W_2^T W_1^{-1} B_1^T \left(B_1 W_1^{-1} B_1^T \right)^{-1}. \end{aligned} \quad (77)$$

Now substituting $\vec{u}_1^T B_1^T$ from Eq. (74) into Eq. (77) yields:

$$\begin{aligned} \vec{\mu}^T = & \left(\vec{s}_2^T B_2^T + \vec{m}_0^T - \vec{m}^T \right) \left(B_1 W_1^{-1} B_1^T \right)^{-1} - \vec{s}_2^T W_2^T W_1^{-1} B_1^T \left(B_1 W_1^{-1} B_1^T \right)^{-1} + \vec{v}_1^T B_1^T \left(B_1 W_1^{-1} B_1^T \right)^{-1} \\ & + \vec{v}_2^T W_2^T W_1^{-1} B_1^T \left(B_1 W_1^{-1} B_1^T \right). \end{aligned} \quad (78)$$

Defining $B^\ddagger = (B_1 W_1^{-1} B_1^T)^{-1}$ and substituting into Eq.(78) and transposing yields:

$$\vec{\mu} = B^\ddagger (B_2 \vec{s}_2 + \vec{m}_0 - \vec{m}) - B^\ddagger B_1 \left(W_1^{-1} \right)^T W_2 \vec{s}_2 + B^\ddagger B_1 \vec{v}_1 + B^\ddagger B_1 \left(W_1^{-1} \right)^T W_2 \vec{v}_2. \quad (79)$$

Next substituting Eq.(79) into Eq. (76) yields:

$$\begin{aligned} \vec{u}_1 = & \vec{v}_1 + \left(W_1^{-1} \right)^T W_2 \vec{v}_2 - \left(W_1^{-1} \right)^T W_2 \vec{s}_2 - \left(W_1^{-1} \right)^T B_1^T B^\ddagger (B_2 \vec{s}_2 + \vec{m}_0 - \vec{m}) + \left(W_1^{-1} \right)^T B_1^T B^\ddagger B_1 \left(W_1^{-1} \right)^T W_2 \vec{s}_2 \\ & - \left(W_1^{-1} \right)^T B_1^T B^\ddagger B_1 \vec{v}_1 - \left(W_1^{-1} \right)^T B_1^T B^\ddagger B_1 \left(W_1^{-1} \right)^T W_2 \vec{v}_2. \end{aligned} \quad (80)$$

Defining $P_1 = (B_1 W_1^{-1})^T (B_1 W_1^{-1} B_1^T)^{-1} = (B_1 W_1^{-1})^T B^\ddagger$ then using Eq. (80) and Eq. (73) we obtain the desired solution:

$$\vec{u}_1^* = (I - P_1 B_1) \left(\vec{v}_1 + \left(W_1^{-1} \right)^T W_2 (\vec{v}_2 - \vec{s}_2) \right) - P_1 B_2 \vec{s}_2 + P_1 (\vec{m} - \vec{m}_0), \quad (81)$$

$$\vec{u}_2^* = \vec{s}_2. \quad (82)$$

□

Acknowledgements

This work was supported with funding from NASA Transformational Tools and Technologies.

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