Prandtl Propeller: Understanding, Modelling, and Evaluation

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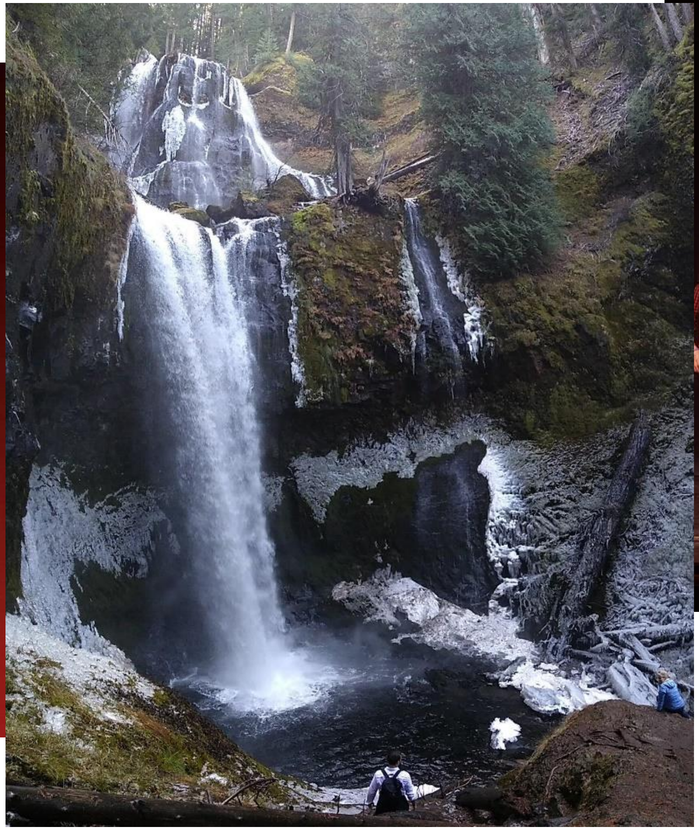
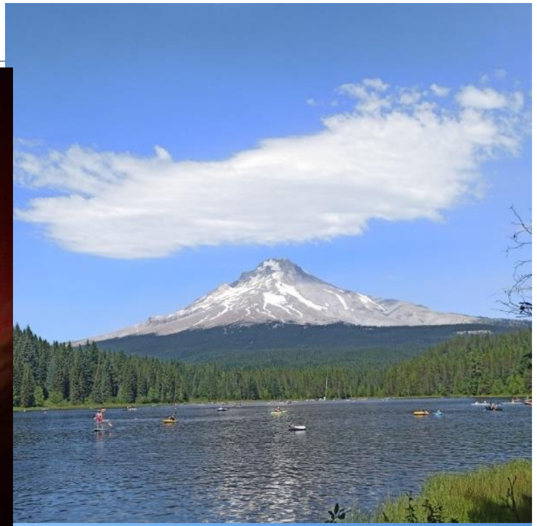
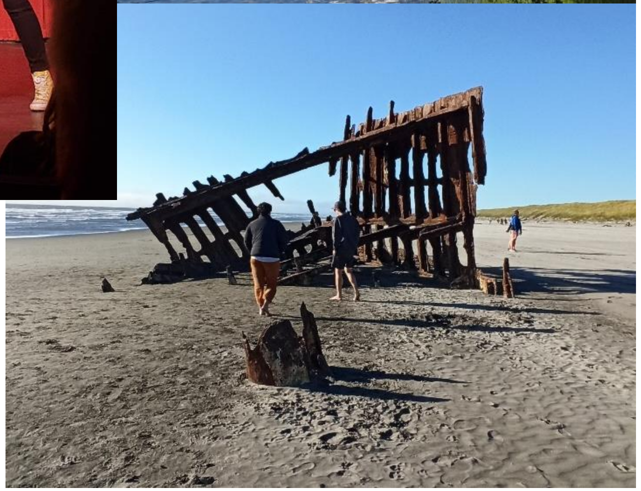






Prandtl Prop Project Overview

Apply the Prandtl Technology to the design of propellers

Goal:

Design and model propellers using the Prandtl method

Research the efficacy of using Prandtl methods on propellers

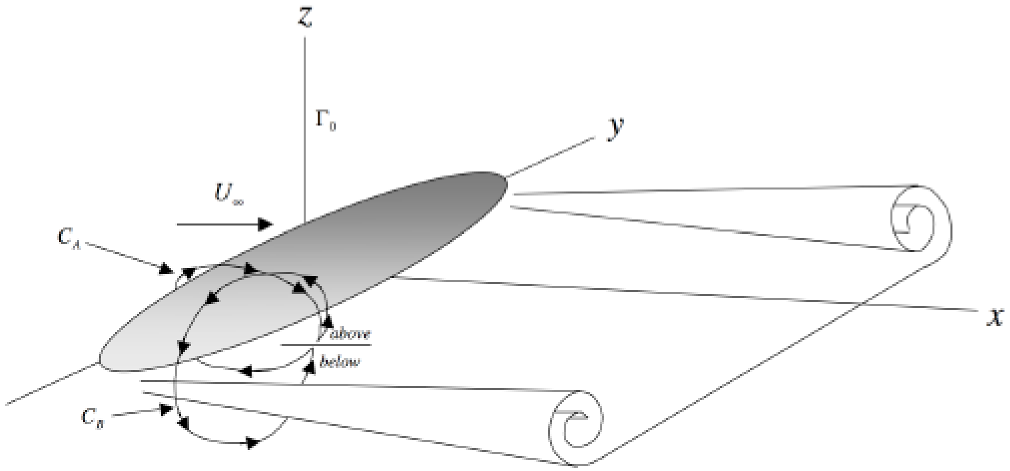
Methods:

Develop modelling software to verify the results found through previous tests

Understand the mathematics of Prandtl wing technology to develop design and optimization method for Prandtl propellers

Background on Aero Design

Methodically working through each of these and how they're designed to then understand how to design the Prandtl Prop design



Wing design –
- Elliptical Spanload(Lift Distribution)
- Elliptical Planform

Prandtl wing design –
- Bell Curve Spanload
- Pointy Wingtips Planform

Prop design –
- Elliptical Spanload
- Intense Thrust at Wingtips

Prandtl Prop Design –
- Bell Curve Spanload
- Localized Thrust in Center

Prandtl Prop Background

1933 Prandtl

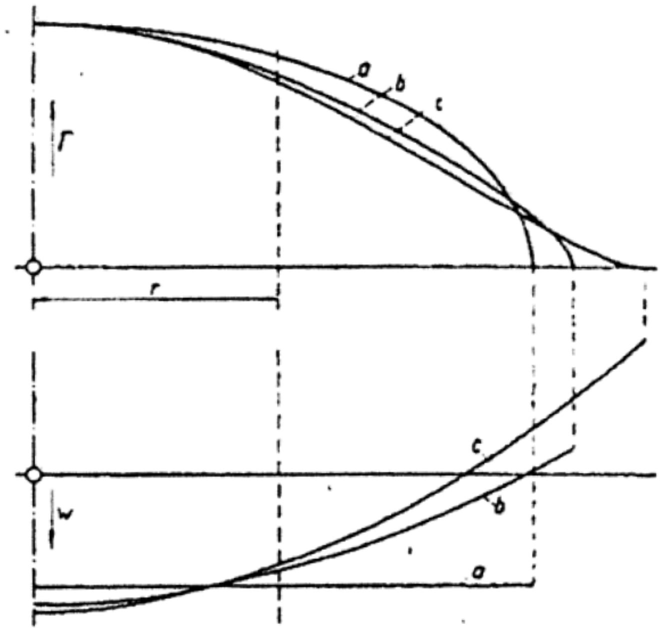
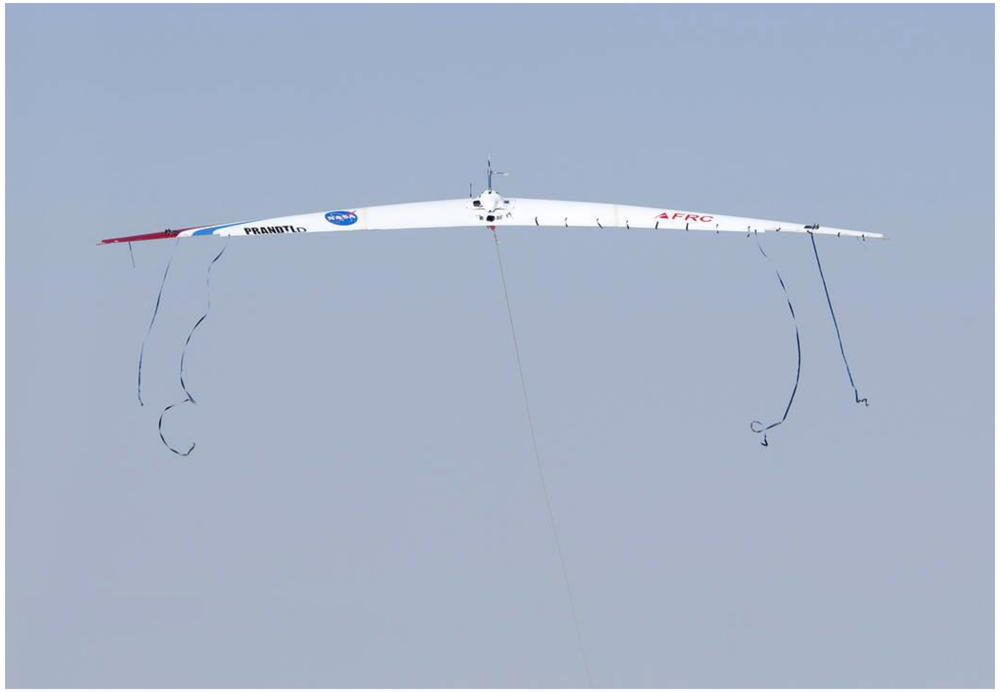


Figure 1. Progression of the Circulation and the Downwash Velocity. A. Elliptical Dist., C. Best Distribution, B. Intermediate Case.

- Prior to this 1933 paper, the proposed best lift distribution was the elliptical spanload
- Prandtl proposes that a bell curve lift distribution is more efficient
- Extend windspan and minimize the structure (weight)

Prandtl Prop Background

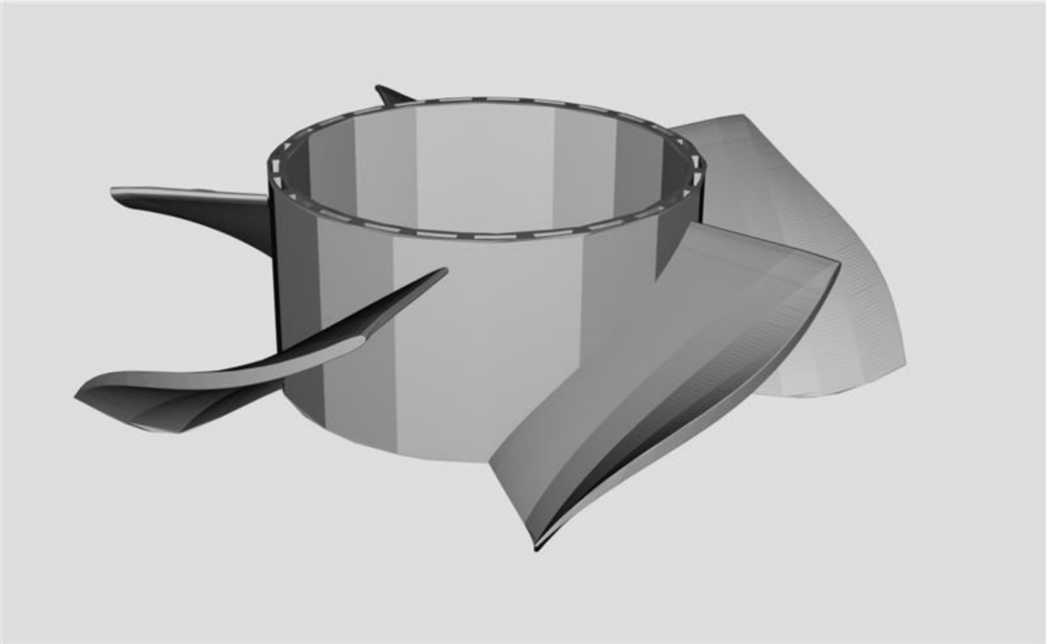
PRANDTL-D



- How does Prandtl Wing apply to props?
- 2015-16 AFRC Flight Tests
- Proof of concept of Prandtl's Paper
- Wingtip Vortices move inboard (Weaker force = more efficient)

Prandtl Prop Background

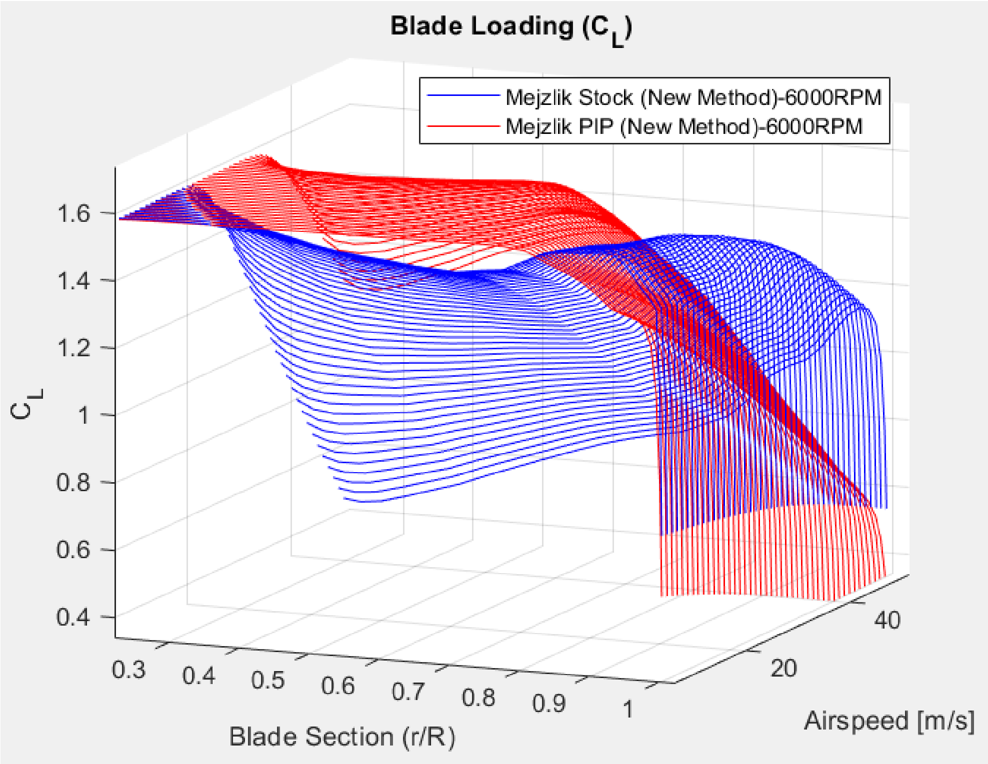
International Space Station Fan



- ISS noise issue because lots of computer fans
- Twist the blade to create the bell curve distribution
- Project transferred to Johnson Space Center after initial test

Prandtl Prop Background

Prandtl Modelling and Design Software



- Pivot to modelling and analysis instead of testing
- Matlab interfaced with XROTOR
- Comparing traditional propellers to Prandtl Inspired Props (PIPs)
- Indicated same thrust at lower RPM (More Quiet)

My Contributions

Matlab:

- Expanded on previous Matlab work
- Decoupled from XROTOR and XFOIL
  - Wanted to Control Optimization
- User Guide

```
%%  Running XROTOR/XFOIL
%%  Plotting XROTOR/XFOIL Results
%%  Make Propeller(s)
%%  Propeller Comparisons (Non-XROTOR)
%%  Supplemetary Scripts
```

XROTOR/XFOIL Running Inside Matlab

Order of Operations for the Code

This is a supplementary document to the document Notes on Code by Chapter and Sections. This is meant to be a comprehensive User Manual of sorts that should explain the order of operations for the things you want to do in the code.

Notes on Code by Chapter and Sections

Notes on overall code: This code is living and ever changing, as is this document. As the one working with it, you know the code better than anyone else. This document is to better assist those using this code in navigating, using, and editing the code. There're essentially three portions of this code: the XROTOR section, the graphing section, and the Experimental "Let's try to do what XROTOR does ourselves" section. The majority of the sections are graphing the information that XROTOR gave us.

The experimental section starts at the "Make Propeller(s)" chapter and below. This is where you get to play around and experiment and really mess around with the design of a propeller. The comparison chapter is exactly what it sounds like, remember to compare props that are either all without induced velocities or all are with induced velocities.

Another point of note is that the section *Propeller Performance (Non-XROTOR)* is the attempt to do what XROTOR does for us but without XROTOR. The appeal hear is that we then know exactly what's going on, what the math is behind the conclusions, what assumptions are being made.

NOTE ON ALL GRAPHS: The graphs are color-coded in a way that relates similar data. For example, props at the same RPM will be the same color.

My Contributions

Mathematics:

- Worked on verifying the mathematics behind Prandtl bell curve
- Closed gaps between Prandtl wing and Prandtl propeller understanding

$$\int_0^{\frac{b}{2}} M dx = \rho \upsilon \int_0^{\frac{b}{2}} \frac{x^2}{2} \Gamma dx \,,$$

$$w = \frac{\Gamma_0}{2b}\left(1 + \frac{\mu}{2} - 3\mu\xi^2\right)$$

$$\Gamma = \Gamma_0\left(1 - \mu\xi^2\right)\sqrt{1-\xi^2}$$

$$L = \frac{\pi}{4}\rho b v \Gamma_0\left(1 - \frac{\mu}{4}\right)$$

$$\int_0^{b/2} M dx = xM\Big|_0^{b/2} - \int_0^{b/2} x \frac{dM}{dx} dx \,, \quad \frac{dM}{dx} = -\rho v \int_x^{b/2} \Gamma dx'$$

$$M(x) = \rho v \int_x^{b/2} \Gamma (x' - x) dx'$$

$$\int_0^{b/2} M(x) dx = \int_0^{b/2} \underbrace{\rho v \int_x^{b/2} \Gamma (x' - x) dx'}_{u} \underbrace{dx}_{dv}$$

$$\Rightarrow \int_0^{b/2} M(x) dx = uv - \int_0^{b/2} v du$$

$$= x(M(x))\Big|_0^{b/2} - \int_0^{b/2} -x\rho v \int_x^{b/2} \Gamma dx' dx$$

$$= 0 + \rho v \int_0^{b/2} x \int_x^{b/2} \Gamma dx' dx$$

since $uv = 0$ at $\frac{b}{2}$ and x

$$= \rho v \int_0^{b/2} \underbrace{x}_{dv} \underbrace{\left[\int_x^{b/2} \Gamma dx'\right]}_{u} dx$$

$$= \frac{x^2}{2} \int_x^{b/2} \Gamma dx' \Big|_0^{b/2} - \int_0^{b/2} \frac{x^2}{2}(-\Gamma) dx$$

$$= 0 - - \int_0^{b/2} \frac{x^2}{2} \Gamma dx$$

$$\int_0^{b/2} M(x) = \int_0^{b/2} \frac{x^2}{2} \Gamma dx \quad \checkmark$$

$$u = \rho v \int_x^{b/2} \Gamma x' - \Gamma x \, dx'$$

$\frac{du}{dx}$ = Partial derivative $\frac{d}{dx}$

$$= \frac{d}{dx} \rho v \int_x^{b/2} \Gamma x' - \Gamma x dx'$$

$$= \rho v \left[\int_x^{b/2} \underbrace{\frac{d}{dx}\Gamma x'}_{\text{const.}} - \frac{d}{dx}\Gamma x dx'\right]$$

$$= \rho v \int_x^{b/2} 0 - \Gamma dx'$$

$$\frac{du}{dx} = -\rho v \int_x^{b/2} \Gamma dx' \quad \checkmark$$

$$dv = dx$$

$$v = x$$

$$v = \frac{x^2}{2} \quad dv = x$$

$$u = \int_x^{b/2} \Gamma dx' = \frac{\Gamma b}{2} - \Gamma x$$

$$\frac{du}{dx} = 0 - \Gamma = -\Gamma$$

Results

- Began new Matlab code that helps in designing, modelling, and comparing Prandtl Props to traditional propellers

- Write-up of foundational concepts and mathematics that are behind Prandtl Tech

Previously on Prandtl Prop Project…

```
%% PROPELLER DESIGN AND ANALYSIS
%% ------ Design ------
%% Matt Method (no Induced velocities) [Used mainly for testin
%% Matt Method (Induced Velocities) [Used for actually designi
%% Larrabee Method (Has induced velocities) [Used for actually

%% ------ Analysis ------
%% Matt Method (no Induced velocities) [Used for testing this
%% Matt Method (Induced Velocites) [Used for actually analyzin
%% Larrabee  Method (Has Induced Velocities) [Used for actuall
%% ------ Graphing ------
%% Matt Method
%%      ----Propeller Performance (XROTOR)
%%      ----Section Values (XROTOR) (3D-Plots)
%%      ----Section Values (XROTOR) (2D-Plots) (Might Need Som
%%      ----Section Values 2 (XROTOR) (2D-Plots)
%%      ----Cy and Cx Prediction (XROTOR)
%%      ----Cl/Cd Over Blade (XROTOR)
%%      ----Propeller Airflow (XROTOR) (MIGHT NEED SOME TWEEKS
%%      ----Airfoil Performance - Reynolds Number (XFOIL)
%%      ----Airfoil Performance - Mach Number (XFOIL)
%% Not sure what to do here%%...%%
%% Larrabee Method
```

Lessons Learned

- **Keep your math resources nearby**
- **Record Record Record**
- **Bring up ideas in meetings**
- **Big brain work is hard work**

Integration by Parts

Partial Derivatives

Pen & Paper = Best Friend

Also Word

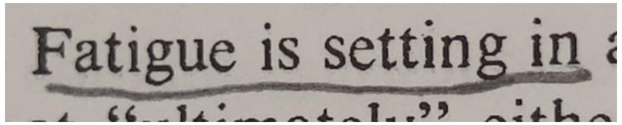
You are a part of the team

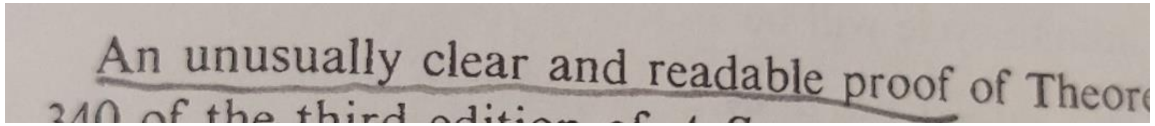
You are doing hard work, Be nice to yourself!

Future Work

Personal:

- Another NASA Internship
- Finish my Mathematics Degree
- Peace Corps
- Mutual Aid Work
- Write a Math Reference Book


Fatigue is setting in


An unusually clear and readable proof of Theor

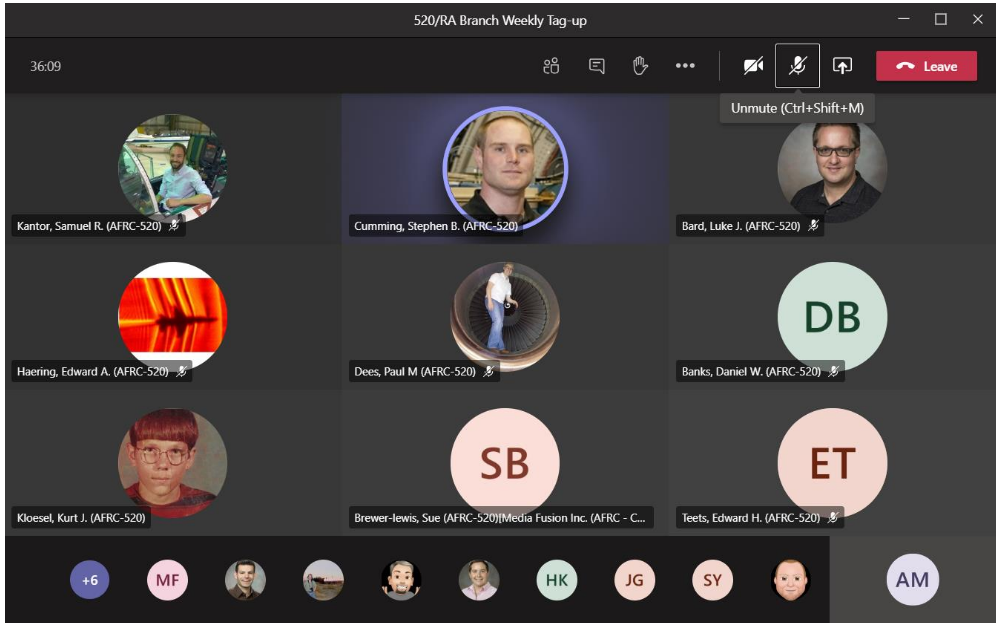
Project:

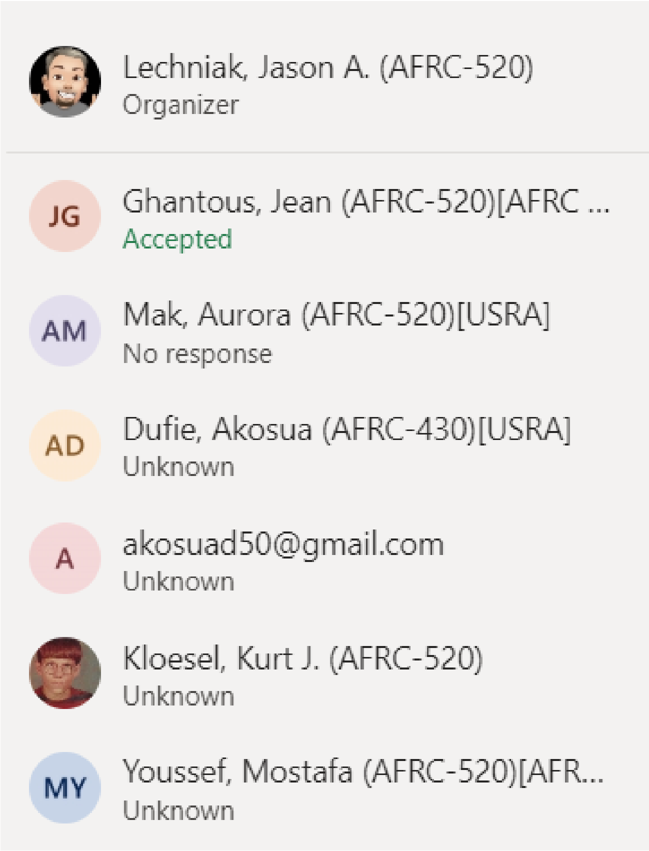
- Continue developing the design and analysis algorithm
- Continue creating modelling software

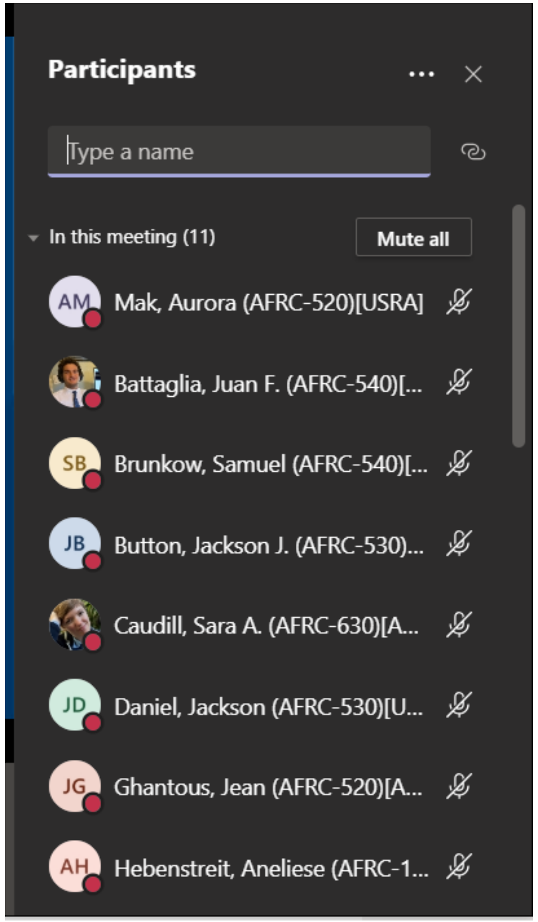
Just Keep Swimming...

Internship Experience

So many memories, so many friendships







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Questions?

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