Strategies for Uncertainty Modeling and Optimization Under Uncertainty

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GNC V&V Series

Motivation

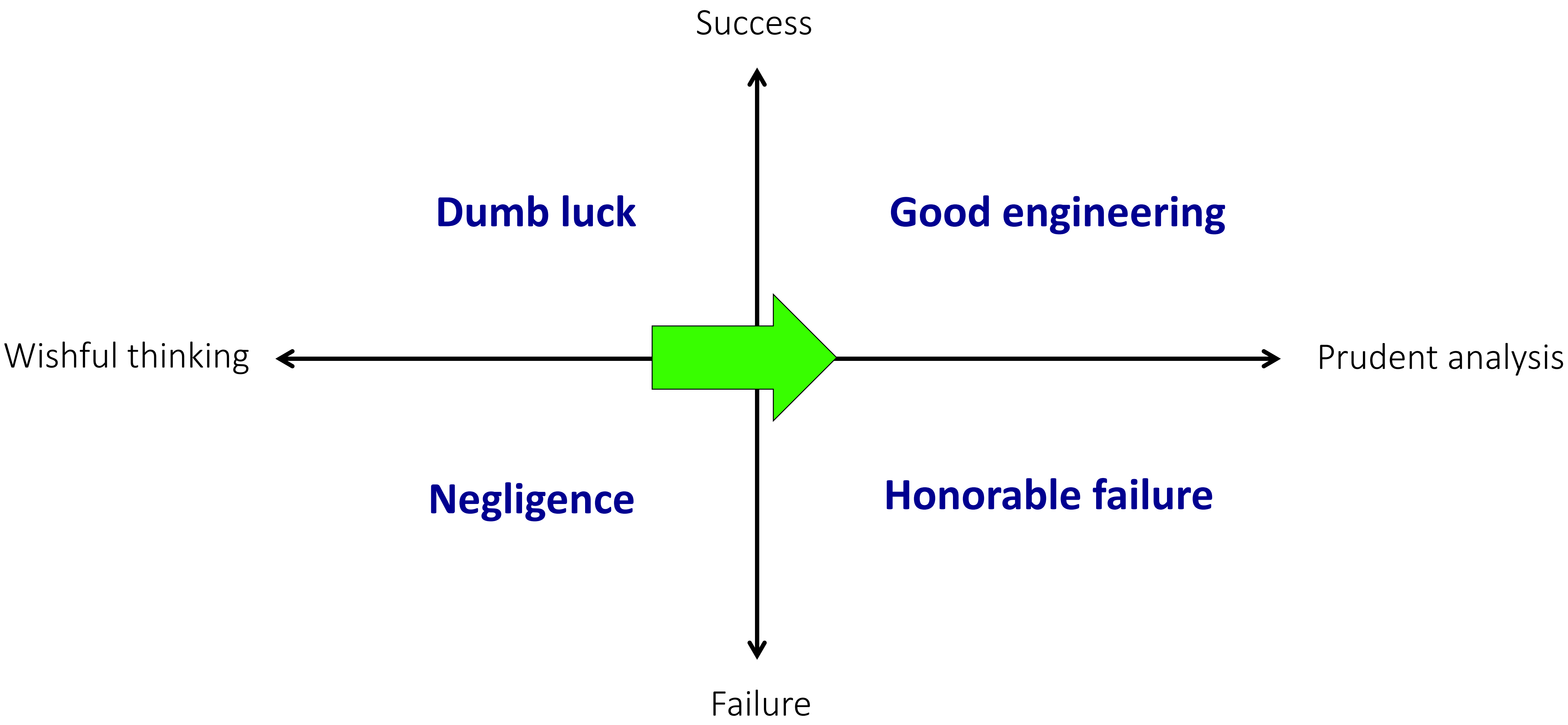
- Heavy reliance on software and simulations in engineering and science
- Third party software being continuously modified by many people
- Common practices are prone to error

Agency	Event	Cause	Cost
ESA	Loss of Ariane 5 rocket	Guidance algorithm during take-off	$1 billion
NASA & Lockheed Martin	Loss of Mars Climate orbiter	wrong units, spacecraft descended 53km more than expected	$193 million
Intel	Pentium processor gave incorrect binary floating point results when performing a division	missing entries in a look-up table	$475 million
Boeing	Two boeing 737 Max crashes	anti-stall flight control malfunction	346 dead, $18 billion

UQ, Verification & Validation

- **Goal**: to increase confidence and consistency in the analysis and design/optimization of systems subject to uncertainty and error
- This is a discipline-independent goal requiring its own framework and methods

UQ, Verification & Validation



Motivation to NASA

- NASA missions often involve the development of new vehicles and systems that must be designed to operate in harsh domains with a wide array of operating conditions
- These missions involve high-consequence and safety-critical systems for which quantitative data is either very sparse or prohibitively expensive to collect
- NASA modeling and simulation standards require estimates of uncertainty and descriptions of any processes used to obtain these estimates

Uncertainty Quantification (UQ)

- Uncertainty quantification is the science of characterizing, propagating, reducing and managing uncertainty in computational models and real-world systems
- UQ requires a model of the unknown: *uncertainty model*
- Uncertainty models constructed from data and expert opinion
- Uncertainty models are particularly inaccurate

Model Verification (V&V)

- The process of determining that a computational model accurately represents the underlying mathematical model and its solution from the perspective of the intended uses of modeling and simulation: <u>*mathematics*</u>
- Prediction depends on numerical settings and model parameters
- Formal strategies: manufactured solutions, model checking, formal methods
- Informal strategies: analysis of convergence, sensitivity analysis, code review by peers

Model Validation (V&V)

- The process of determining the degree to which a model or a simulation is an accurate representation of the real world from the perspective of the intended uses of the model or simulation: *physics*, *experiments*

UQ, V&V for GNC: Goals

- Goal: design a controller/guidance law for an uncertain plant that satisfies the system requirements robustly
- Plant model accounts for uncertainties and failures
- Requirements prescribed as a set of constraints
- Figure of merit: P[requirement violation]

UQ, V&V for GNC: Complexity

- Model complexity has a cost
  - Is the added complexity justified from the perspective of improved performance and increased risk?
- Glorify simplicity not complexity
- Evaluate novel approaches thoroughly before embracing them
  - Have a known and tested baseline solution to compare against
  - e.g., see [1] for a sobering assessment on ML and controls

[1] Plenary talk at the IFAC 2020 world congress, “Reflections on the Learning-to-Control Renaissance”, by B. Recht, https://www.youtube.com/watch?v=IEZFwh8sw8s

UQ, V&V for GNC: Framework

- V&V step 0: characterize uncertainty
  - Improper UQ has a cost: too small vs. too large (outliers)
  - Uncertainty model for robust control vs uncertainty model for UQ
- V&V step 1: use control theory to design a controller with acceptable robustness and performance characteristics
  - Identify WC combinations of uncertainties and disturbances
  - Simplifications have a cost
- V&V step 2: use best physics-based model to verify controller
  - Challenge the mathematically convenient assumptions and simplifications needed by the theory
  - Assess risk using a probabilistic uncertainty model

UQ, V&V for GNC: Framework

- Conflict of interests: the goal of the V&V 2 group is to expose the weaknesses of the design obtained by the V&V 1 group
- Such groups should work largely independently
- You can not prove your way out of V&V
- V&V outcome
  - If favorable: how do the results generalize?
  - If unfavorable: how to fix the controller?

This Talk

- Our research group has focused on the development of UQ and V&V methods for bridging the gap between theory and practice
- This principle has driven not only our own research goals but also the challenge problems we have issued
- The talk conveys key ideas through simple notional examples

Overview

- Set deformations
- Uncertainty modeling
- Optimization under uncertainty
- Challenge problems

Monte Carlo Simulations

- ✓Simple to do
- ✓Complexity is independent of the number of parameters, their distribution, and the manner by which the outputs depend on them
- × It is expensive
- × The uncertainty model is often subjective, thus, the results
- × It does not render worst-case combinations
- Use Quasi-Monte Carlo methods
- How to fix unacceptably large P[req violation]*
- What are the generalization properties of the results*?

*more on this presented later

Set Deformations

- Quantify the largest deviation in the nominal value of *x* that leads to an unacceptable output

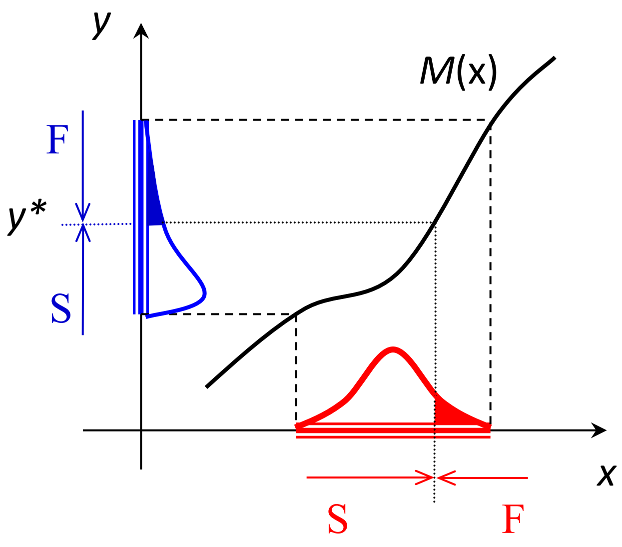
$$y = M(x)$$

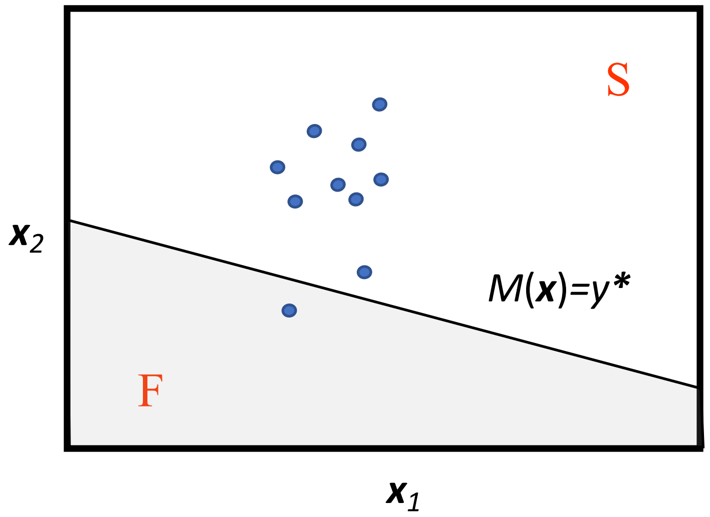
Computational model

$$g(x) = y - y^* \leq 0$$

System requirements

Set Deformations: Analysis

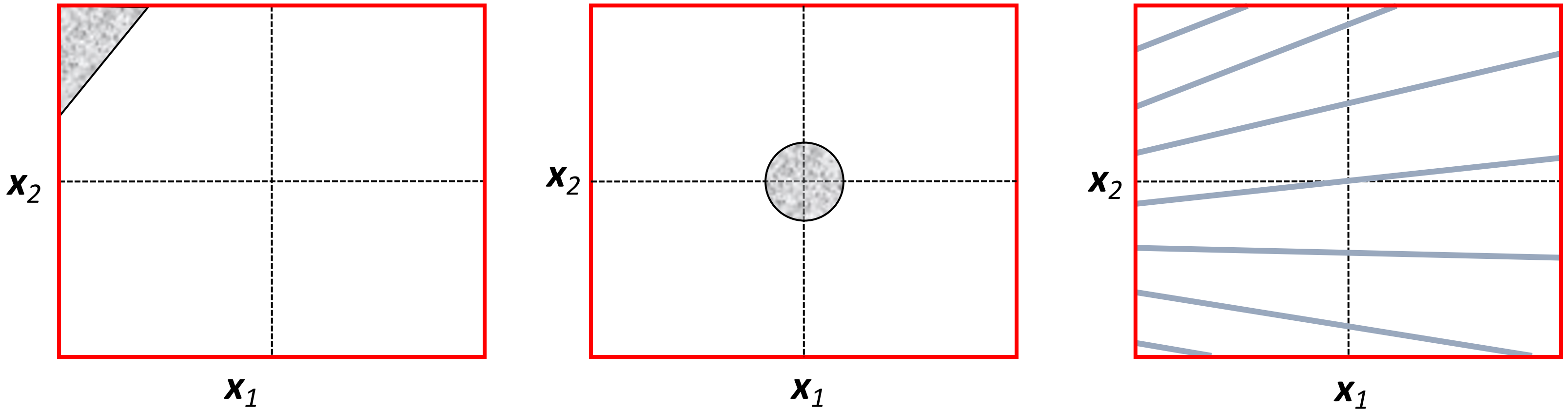




- Probability of failure by MC

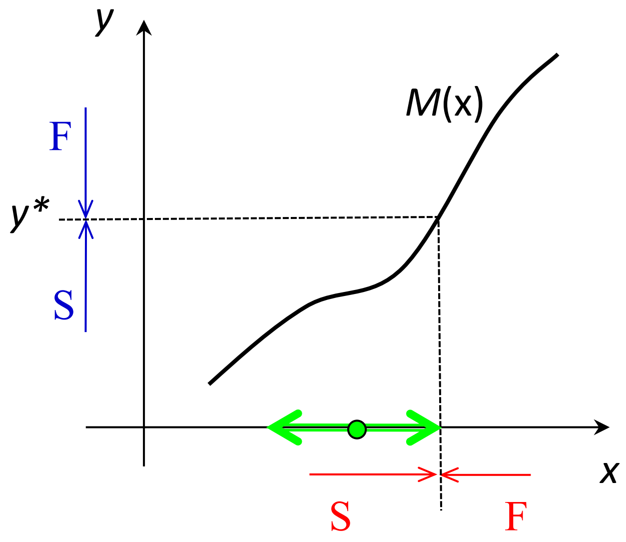
Set Deformations: Analysis

- How do we want the failure domain to look?



Set Deformations: Analysis

What is the smallest deviation from the nominal value leading to a requirement violation?



Set Deformations: Analysis

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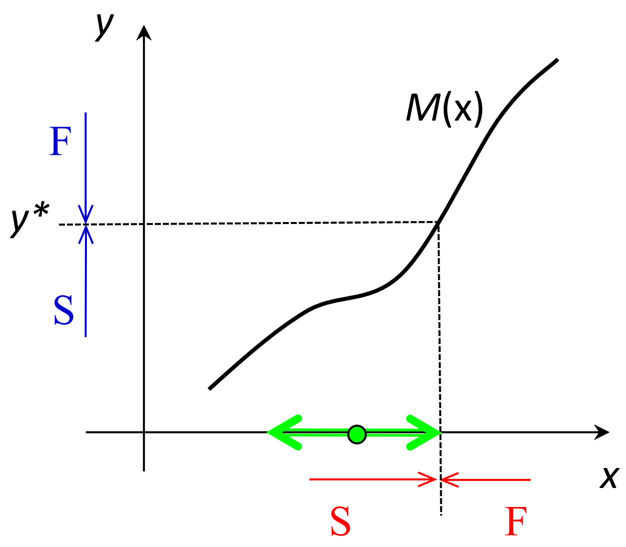
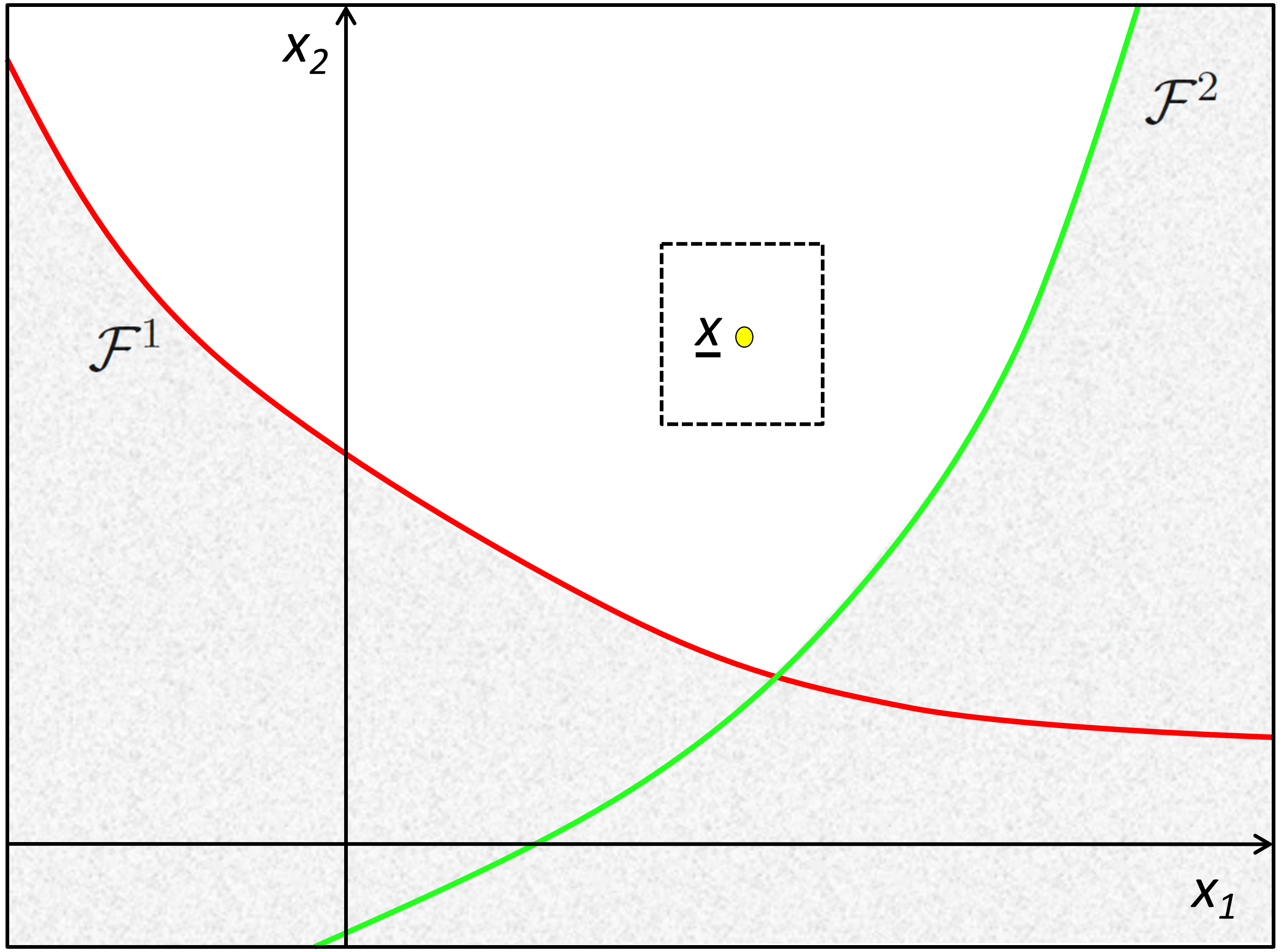
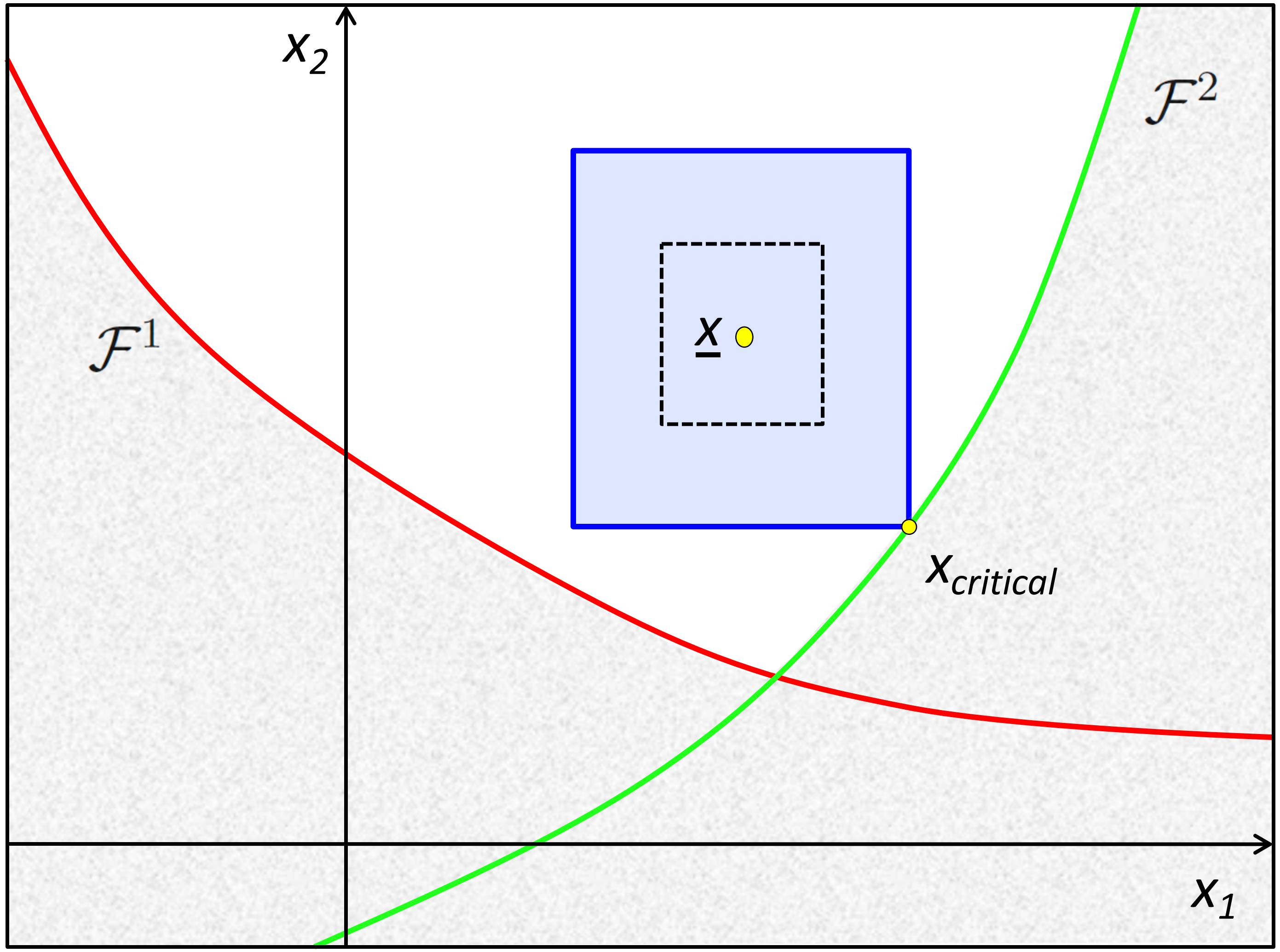


This notion of robustness is conceptually different from the probabilities of failure

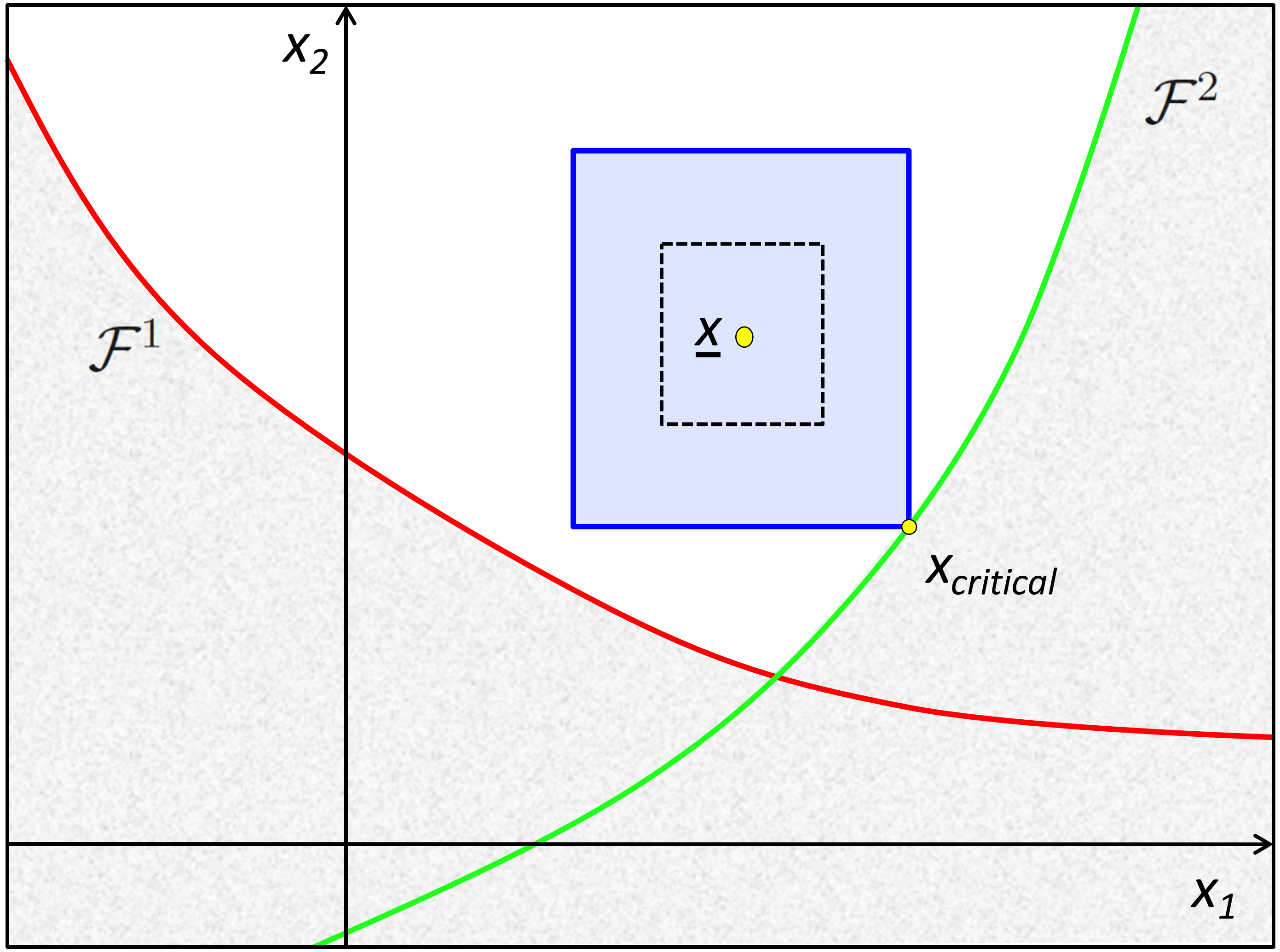
Set Deformations: Analysis



Set Deformations: Analysis



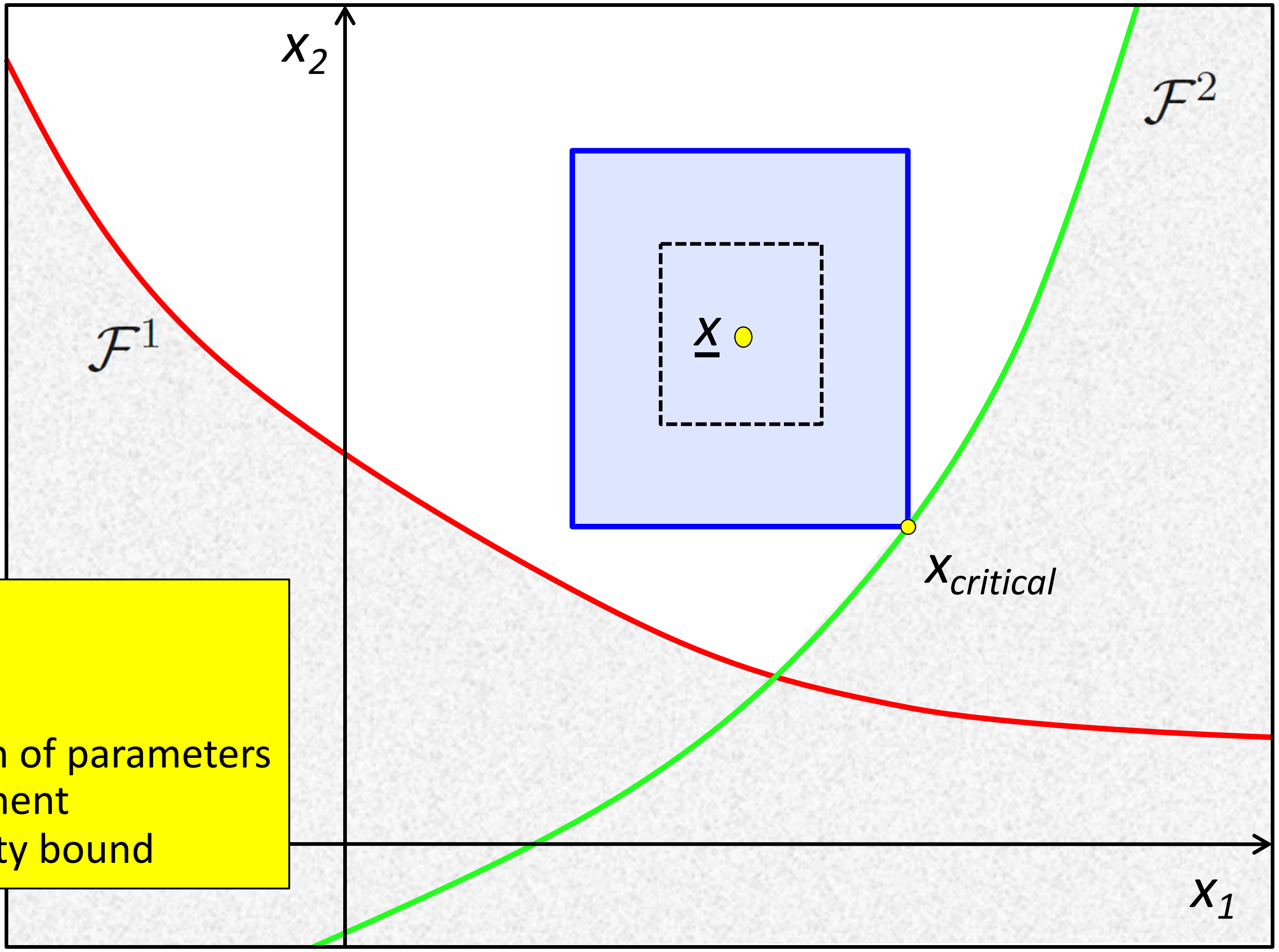
Set Deformations: Analysis



In contrast to MC, the outcome is a characterization of the uncertainty

Set Deformations: Analysis

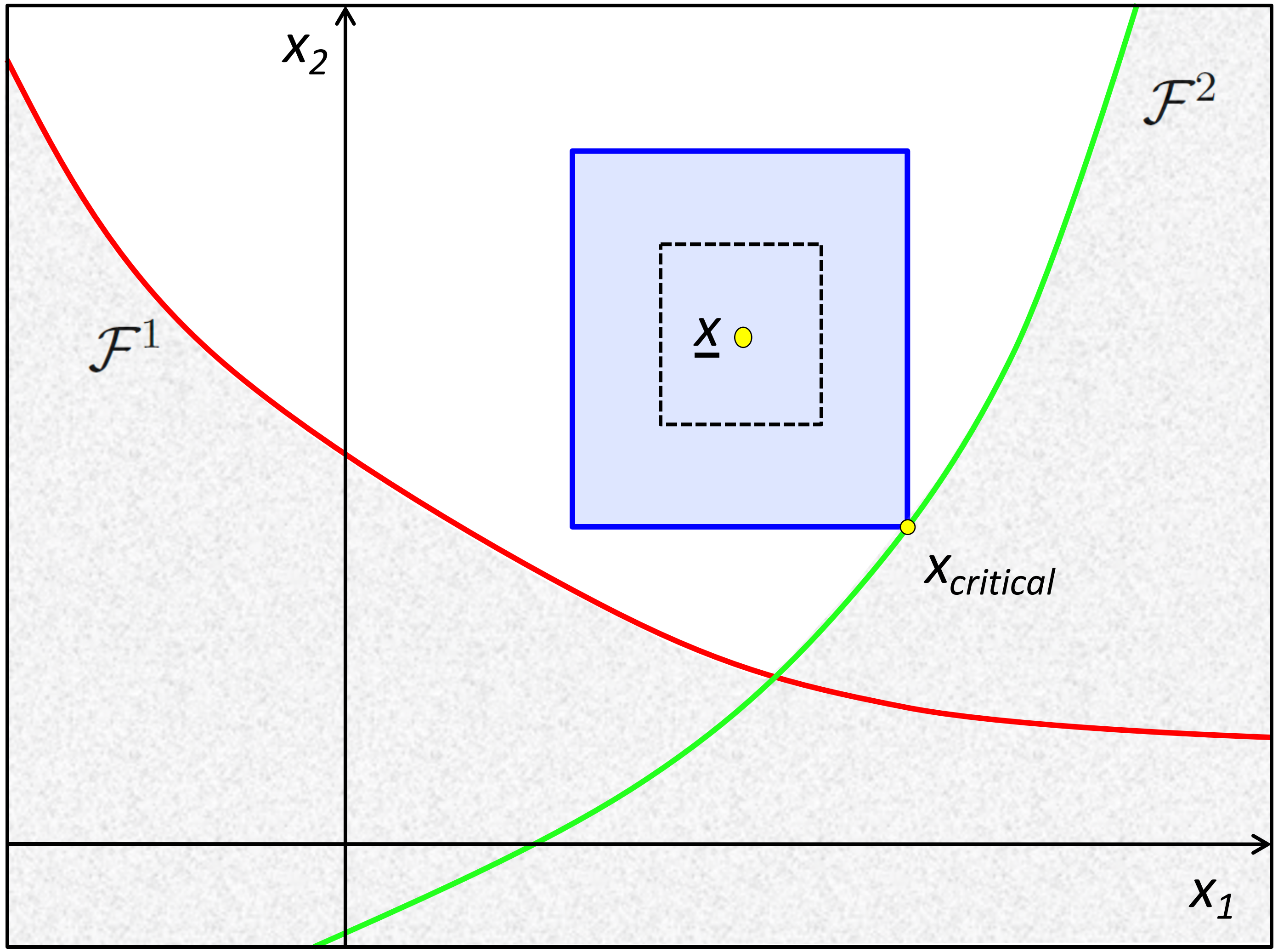
$$y = M(x)$$



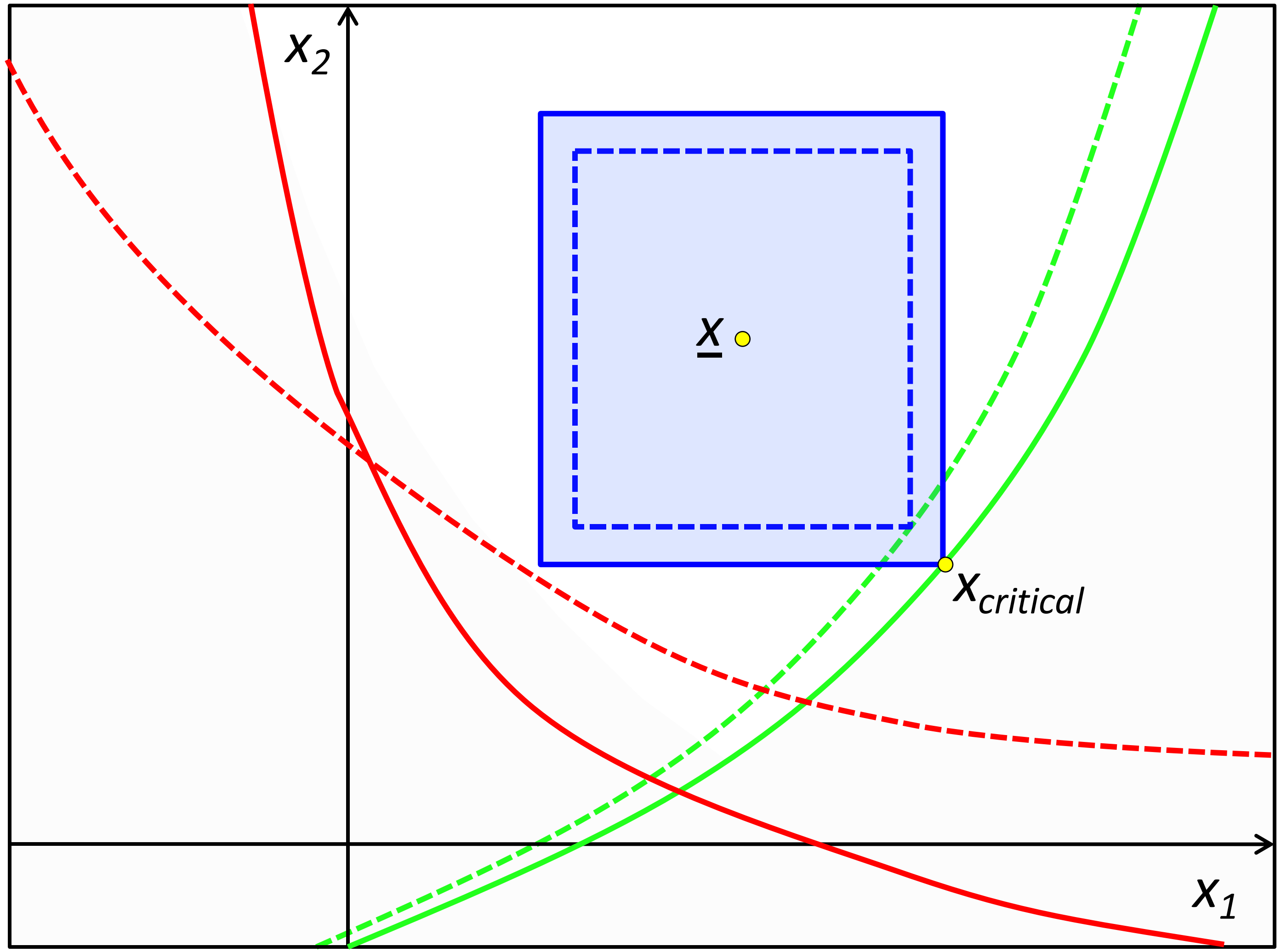
Outcomes:
- A subset of *S*
- A safety margin
- WC combination of parameters
- Critical requirement
- Cheap probability bound

Set Deformations: Design

$$y = M(d, x)$$

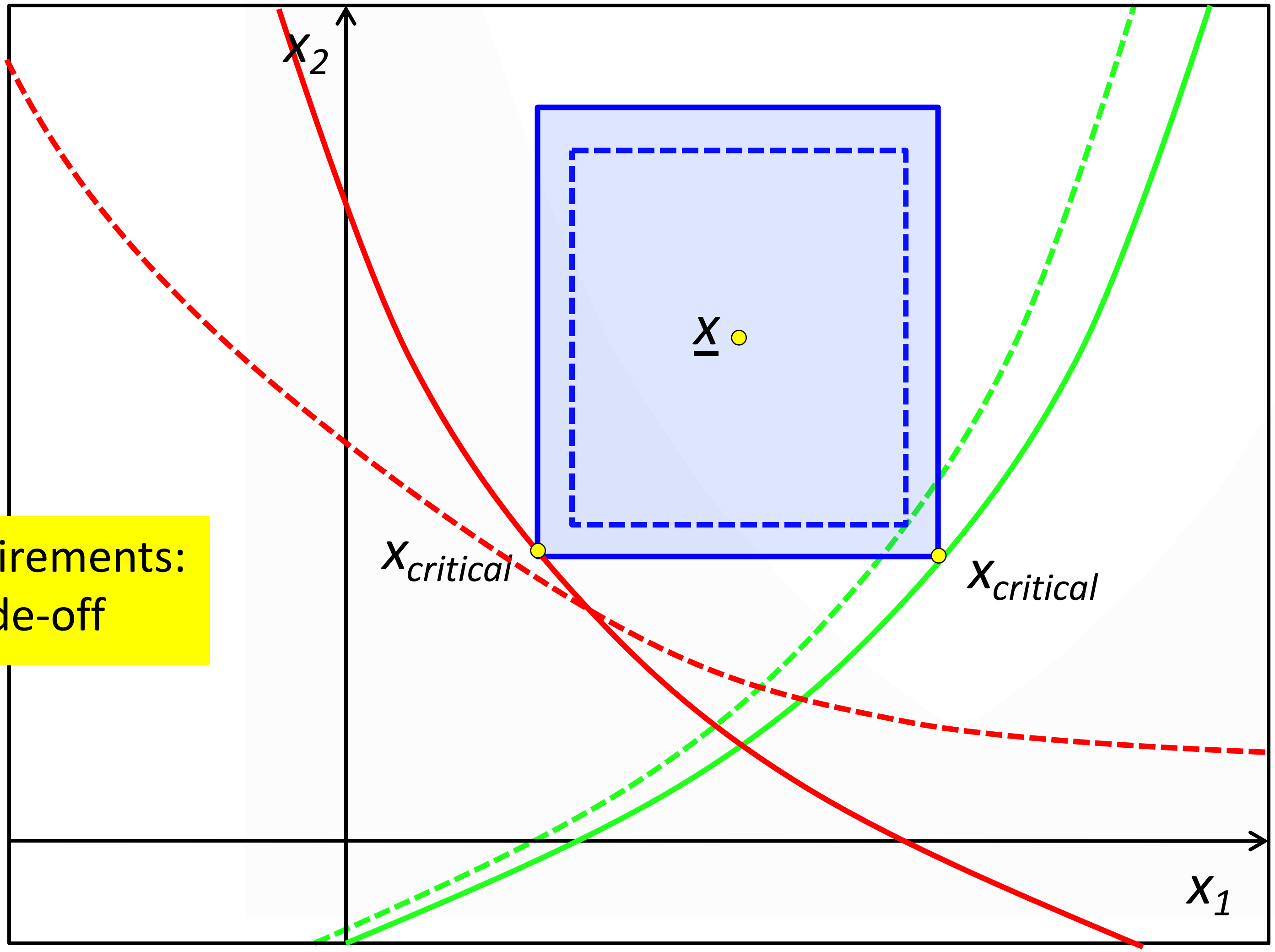


Set Deformations: Design



Set Deformations: Design

$$y = M(d, x)$$

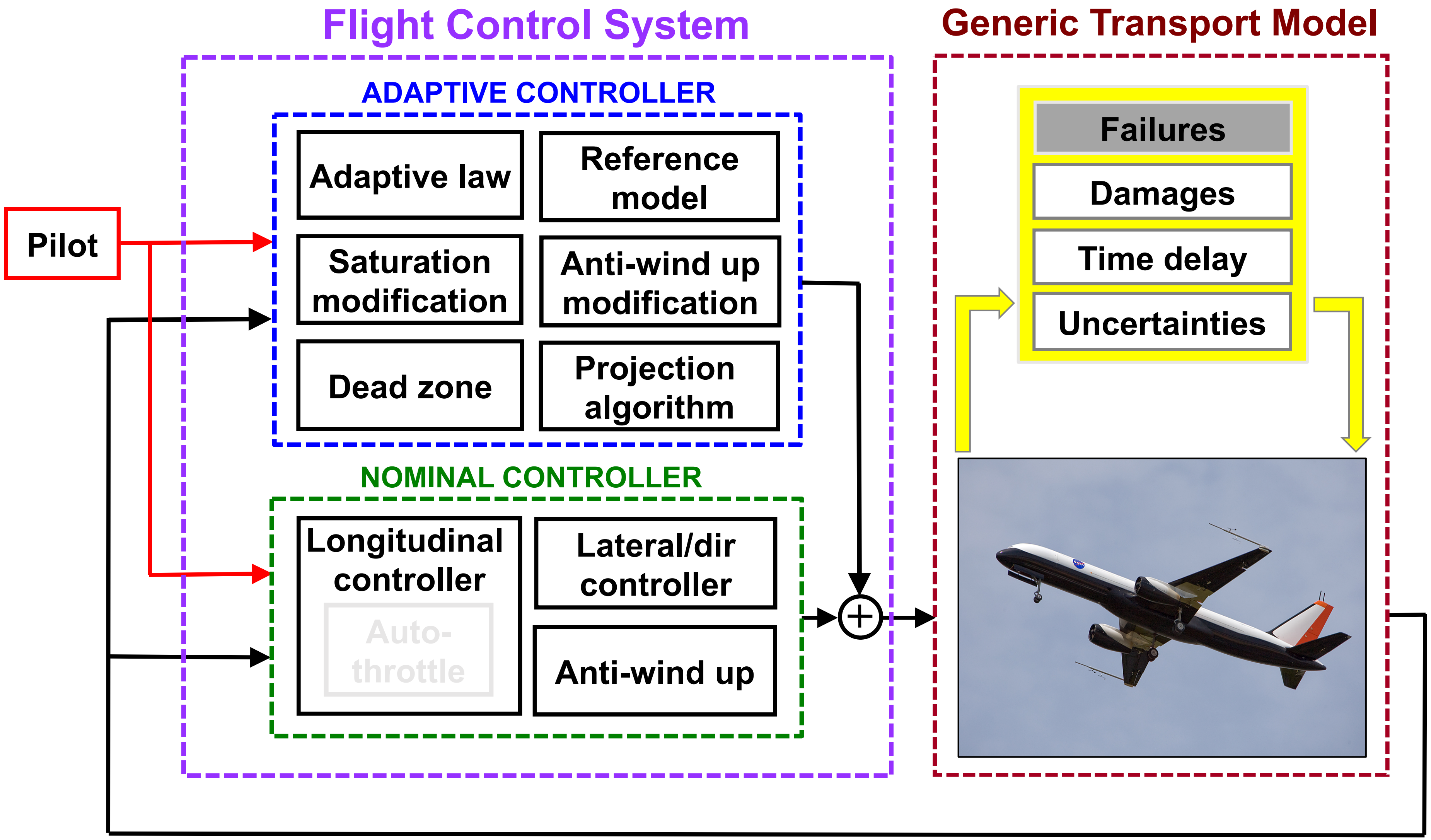


Conflictive requirements:
optimal trade-off

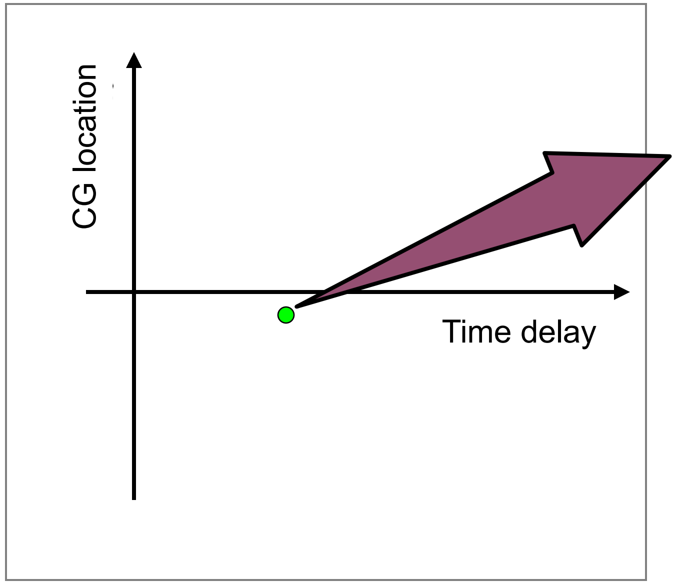
Set Deformations: GTM

- Dynamically scaled flight test article
- High-fidelity mathematical model having non-linear aerodynamics, avionics, engine and sensor dynamics, atmospheric model, telemetry effects, etc. (278 states)
- **Analysis objective**: perform a robustness analysis of a flight controller
- **Design objective**: Improve controller's robustness with respect to key uncertainties

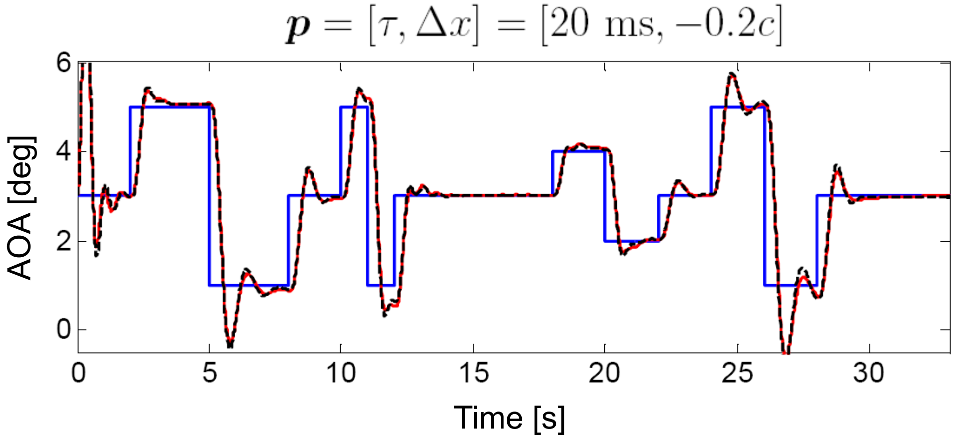
The GTM: Simulation Model



Set Deformations: GTM

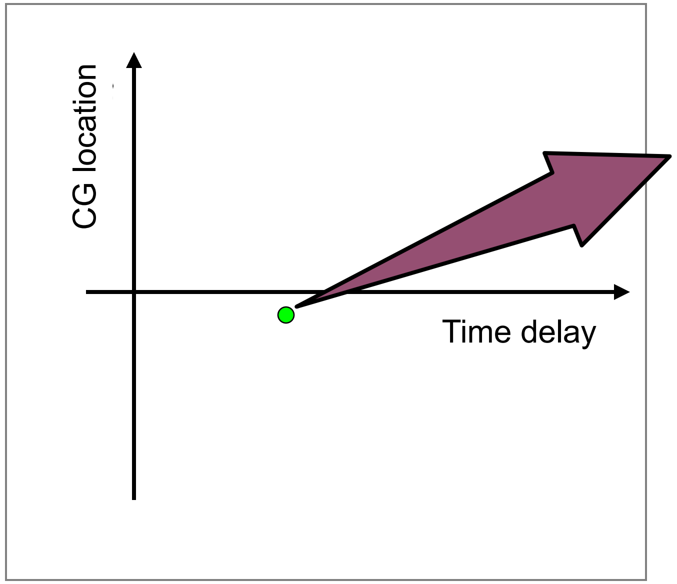


Parameter Space



Reference command, Controller 1, Controller 2

Set Deformations: GTM



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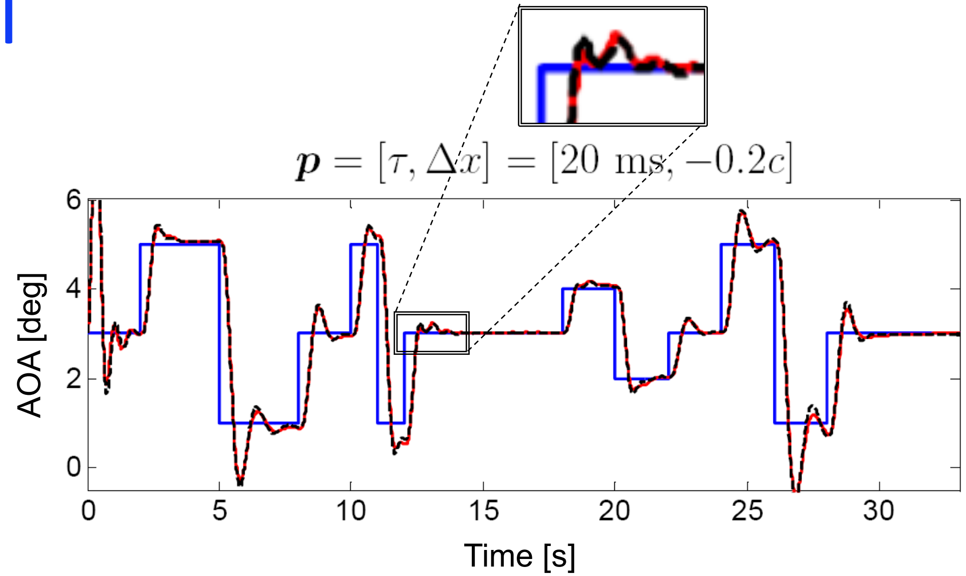

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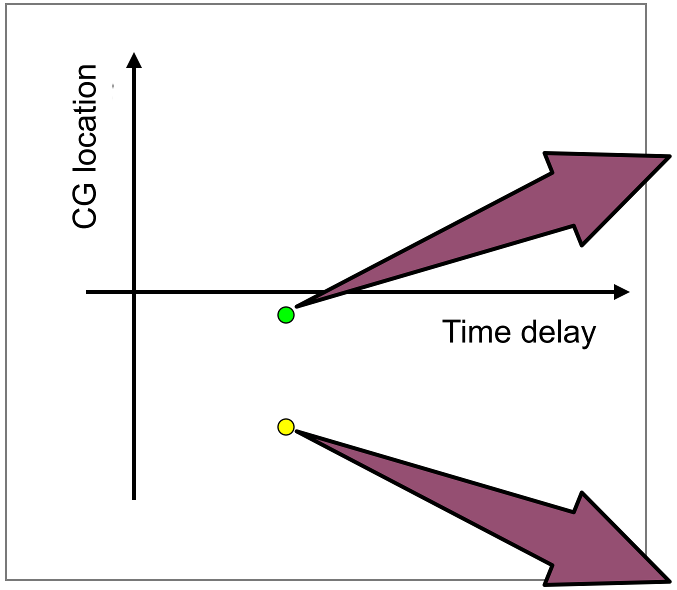

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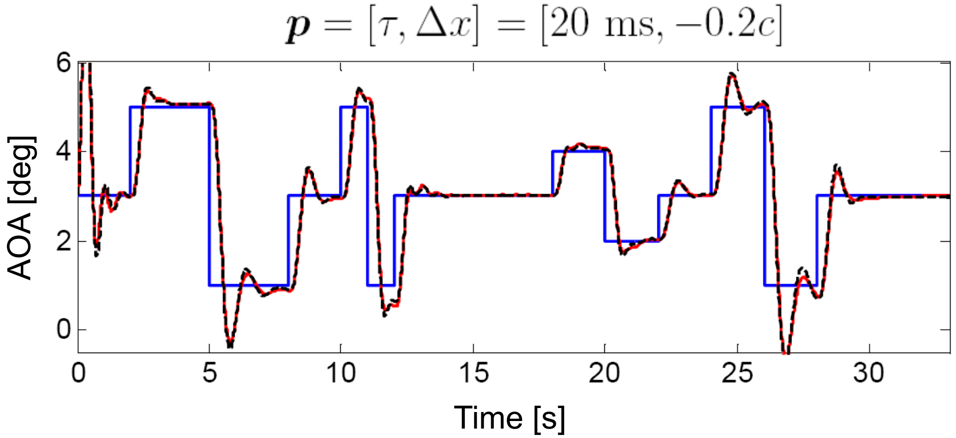
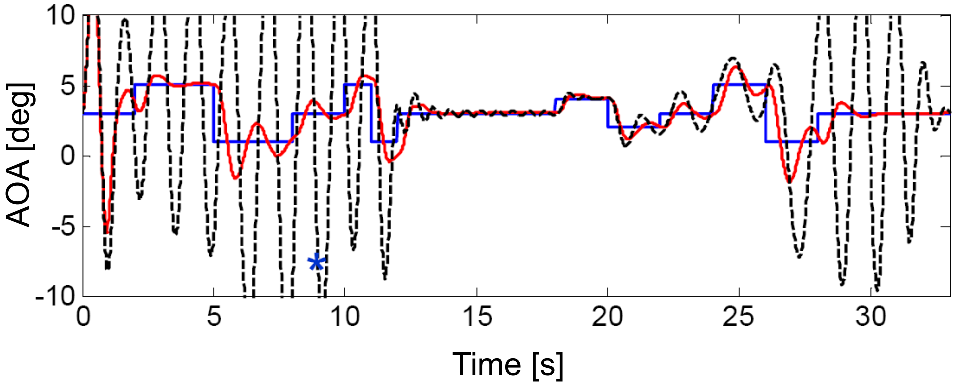
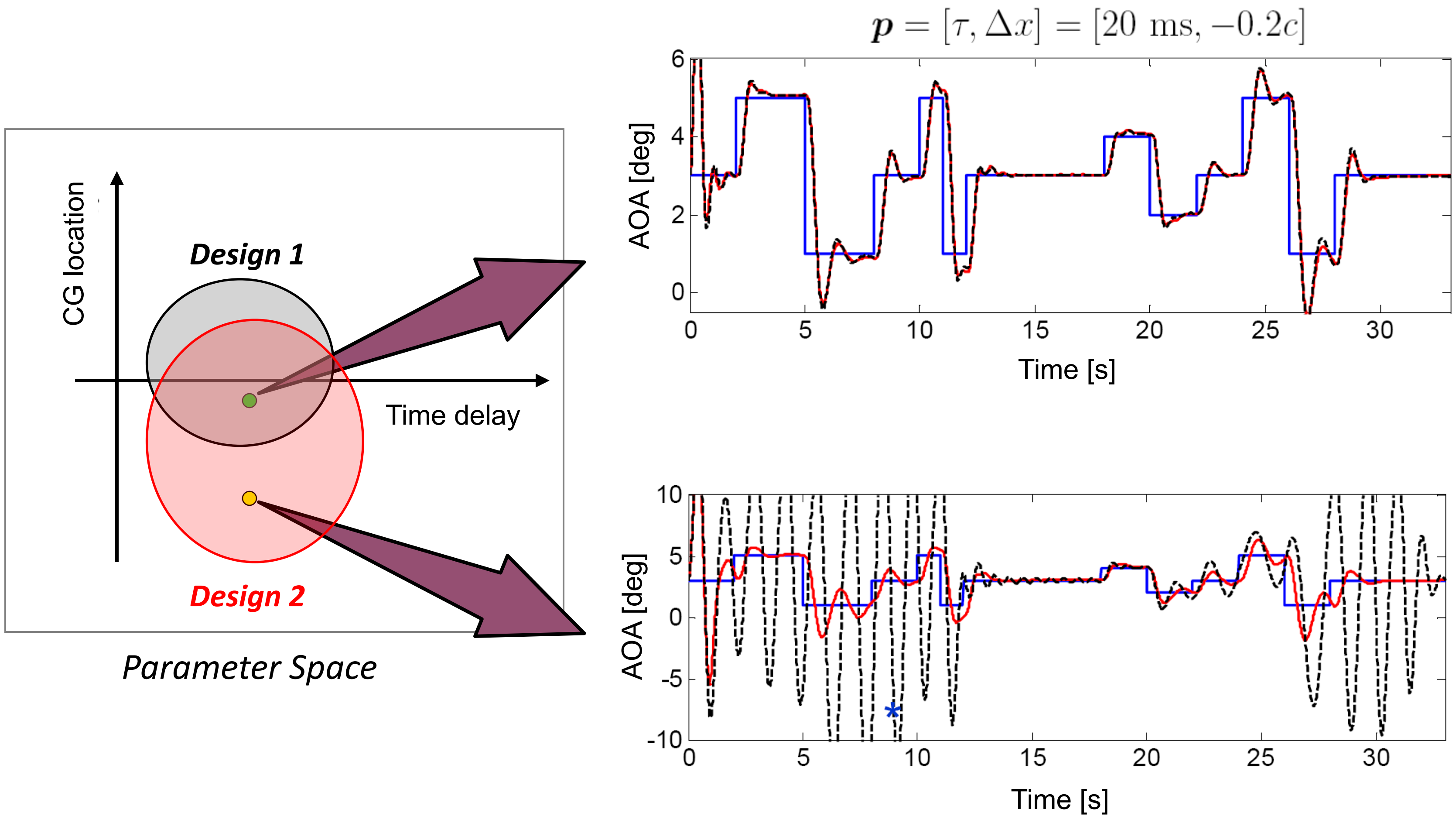




Reference command, Controller 1, Controller 2

Set Deformations: GTM

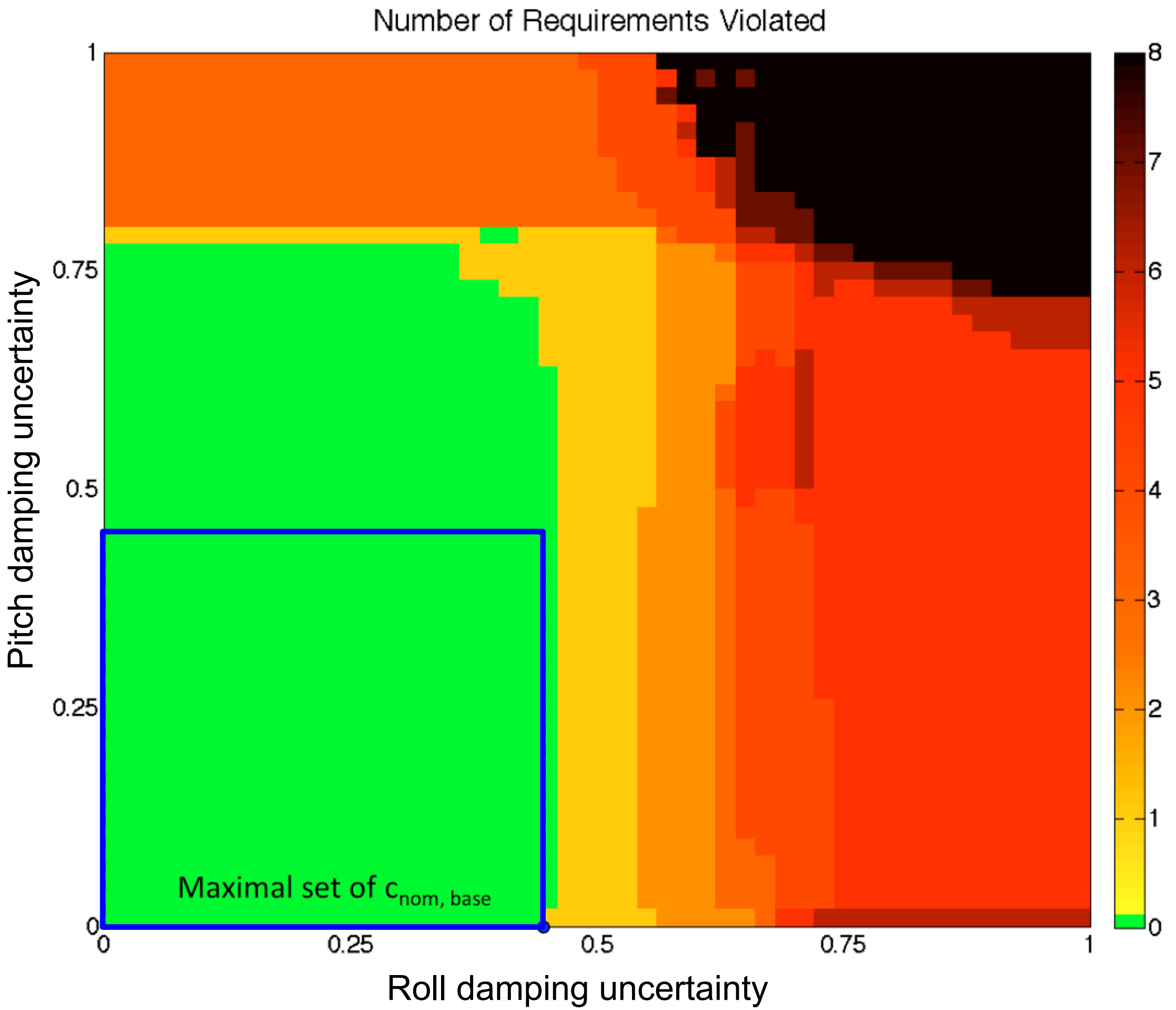


Reference command, Design 1, Design 2

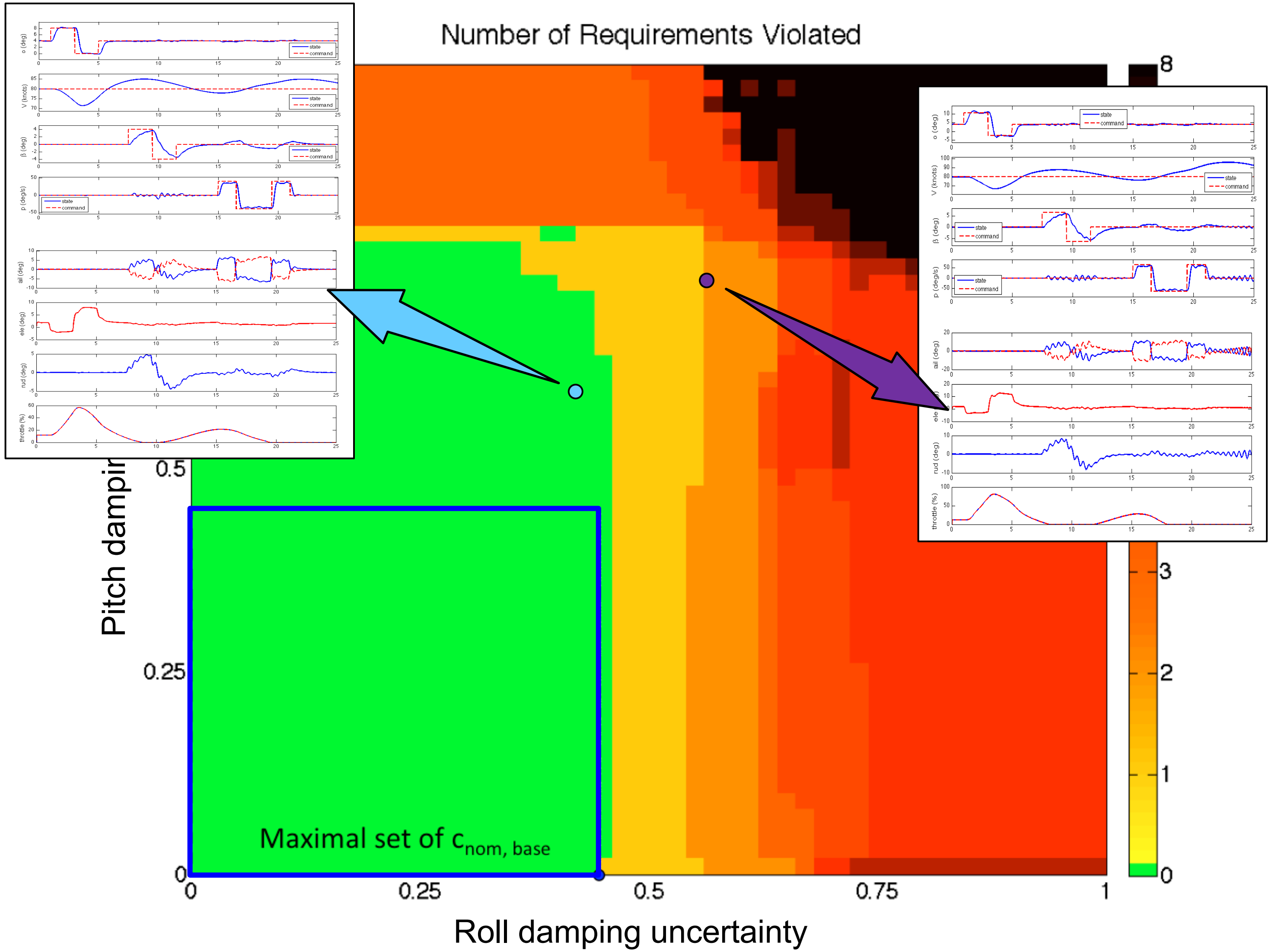

Controller 1

Controller 2

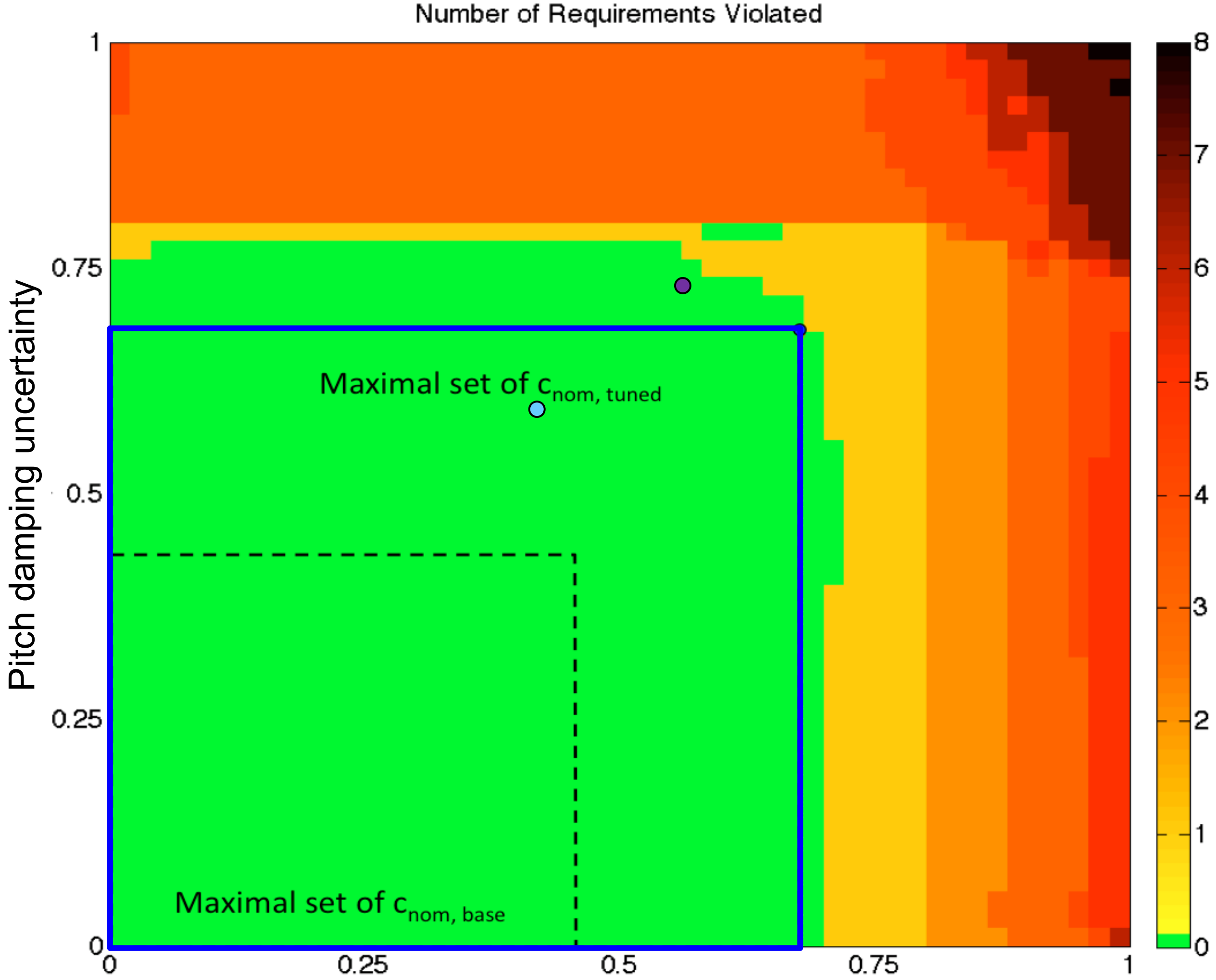
Set Deformations: GTM Analysis



Set Deformations: GTM Analysis

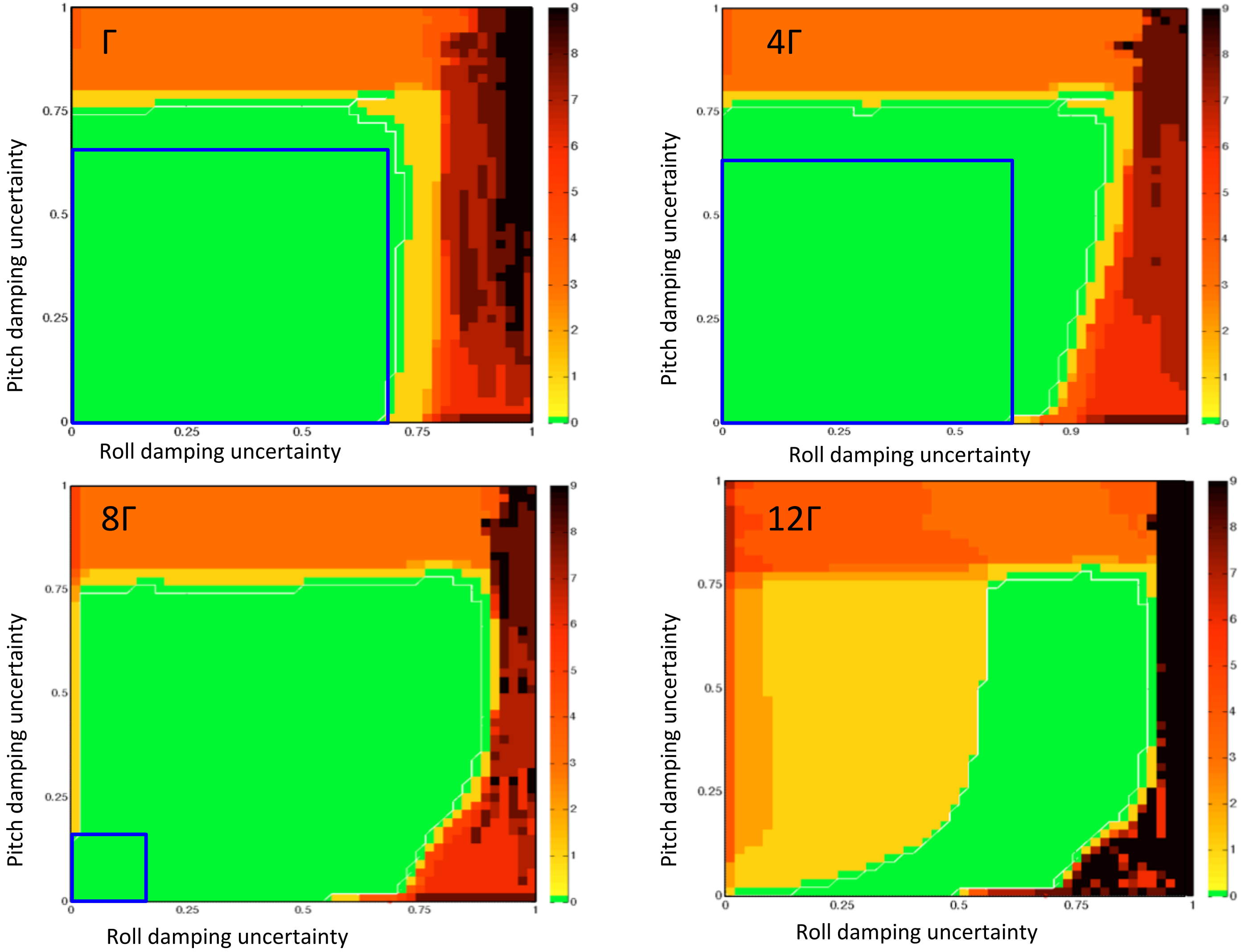


Set Deformations: GTM Design



The new design tolerates 48% more roll damping uncertainty than the baseline design

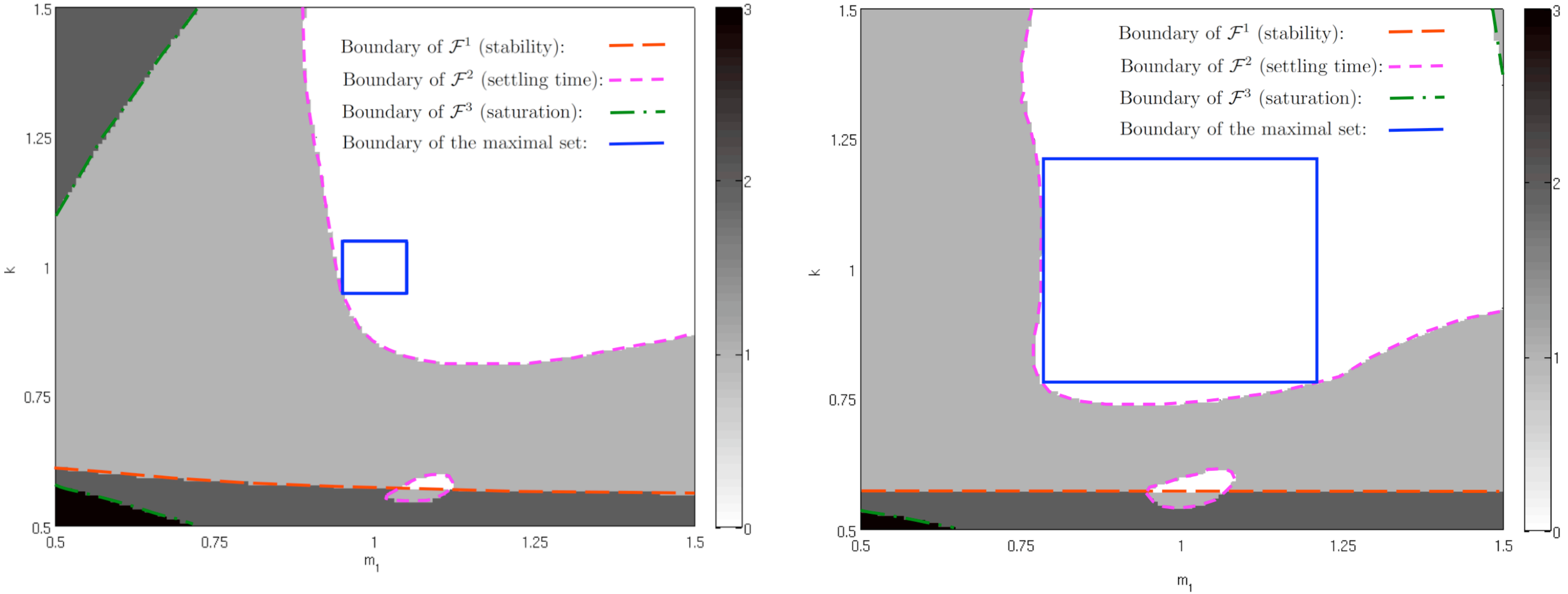
Set Deformations: PSM vs P[req-violation]



[2] Crespo et al, Design of an Adaptive Controller for a Remotely Operated Air Vehicle, 2012, AIAA JGCD

Set Deformations: Example

- See [3] for an application to a robust control challenge problem



[3] Crespo et al, A Computational Framework to Control Verification and Robustness Analysis, NASA/TP 2010-216189

[4] Crespo et al, Reliability-based analysis and design via failure domain bounding, Structural Safety, 2009

[5] Kamath, Bennani (ESA), et al, Robust safety margin assessment and constrained worst-case analysis of a launcher vehicle, IFAC Sym Robust control, 2012

Set Deformations

✓Provides WC uncertainty combination and a robustness metric
✓Uses a simpler uncertainty model
✓Sizes the uncertainty a given controller tolerates
✓Can be naturally integrated into design
× Complexity depends on the number of parameters and the manner by which the outputs depend on them
× Requires convergence to a global optimum (as WCDVV)
- What to do if parameters are dependent*?
- What to do if data is available*?

*more on this presented later

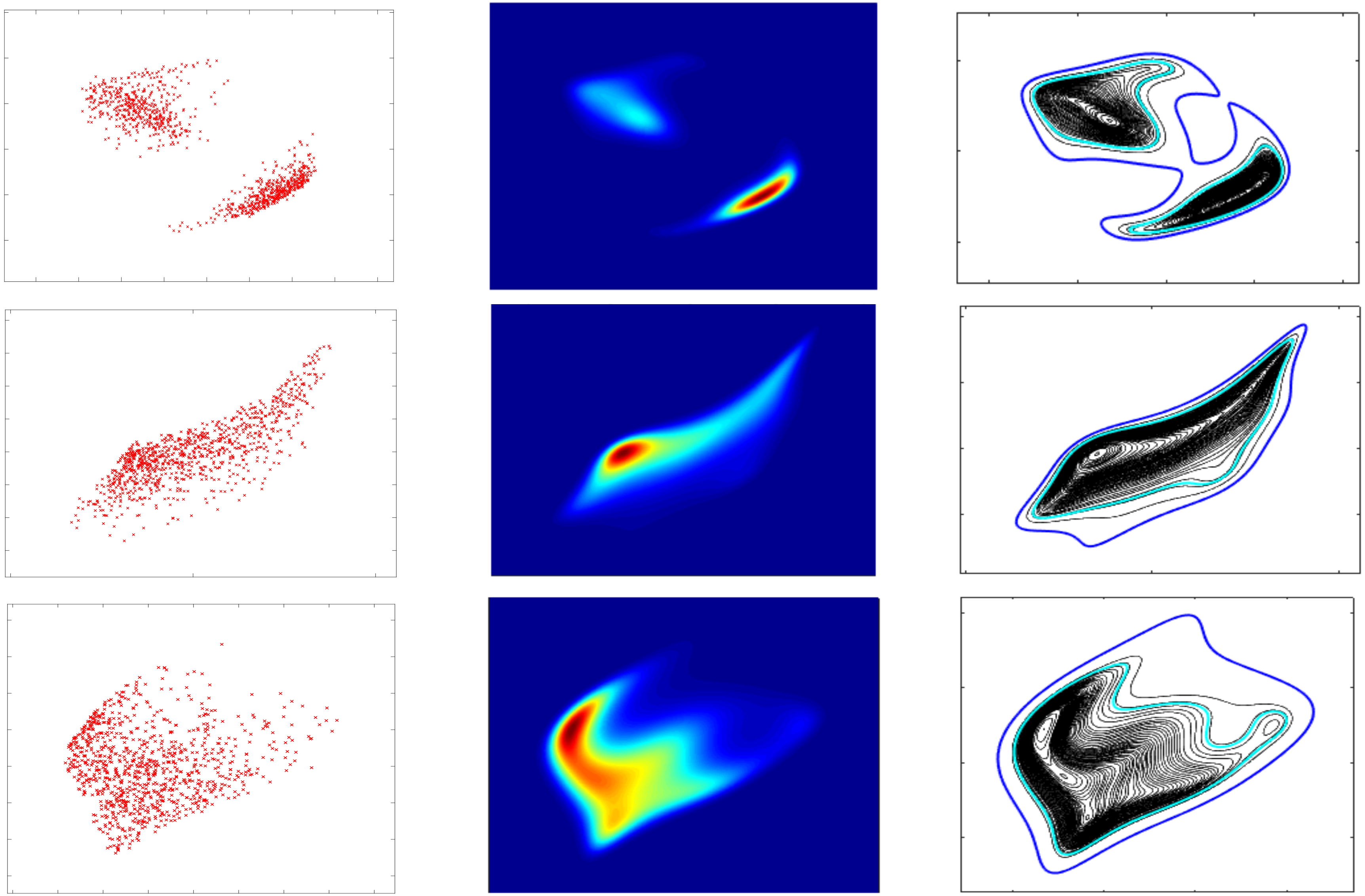
Overview

- Set deformations
- Uncertainty modeling
- Optimization under uncertainty
- Challenge problems

Uncertainty Modeling

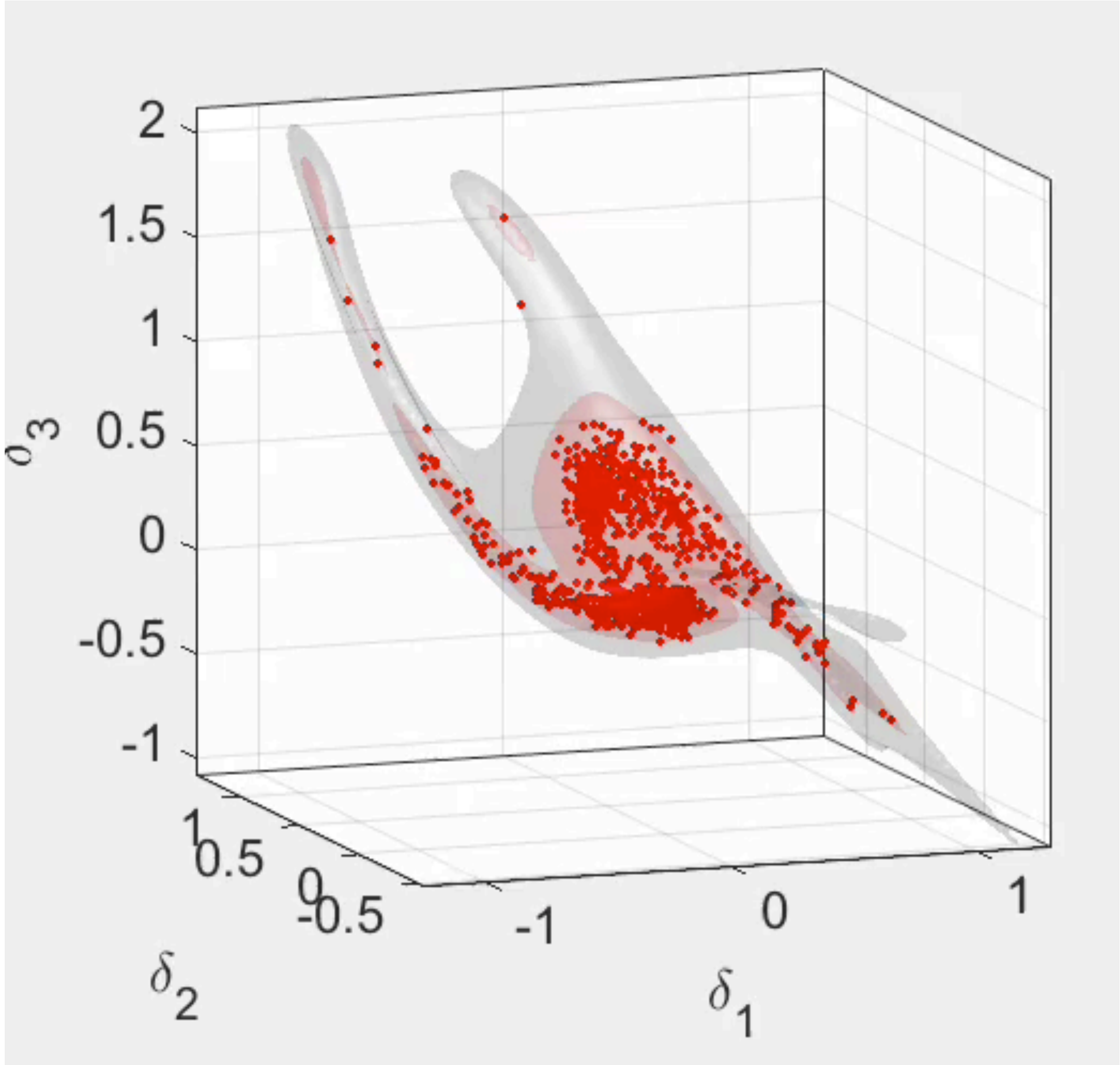
- Data is instrumental in system identification, uncertainty quantification for robust control, V&V
- Poor characterizations of the data might lead to either overly conservative designs or insufficiently robust designs
- Key question: How reliable is an uncerttainty model based on *n* observations?

Uncertainty Modeling



[6] Crespo et al, On the quantification of aleatory and epistemic uncertainty using Sliced-Normal distributions, systems and control letters, 2019

Uncertainty Modelling: Sets

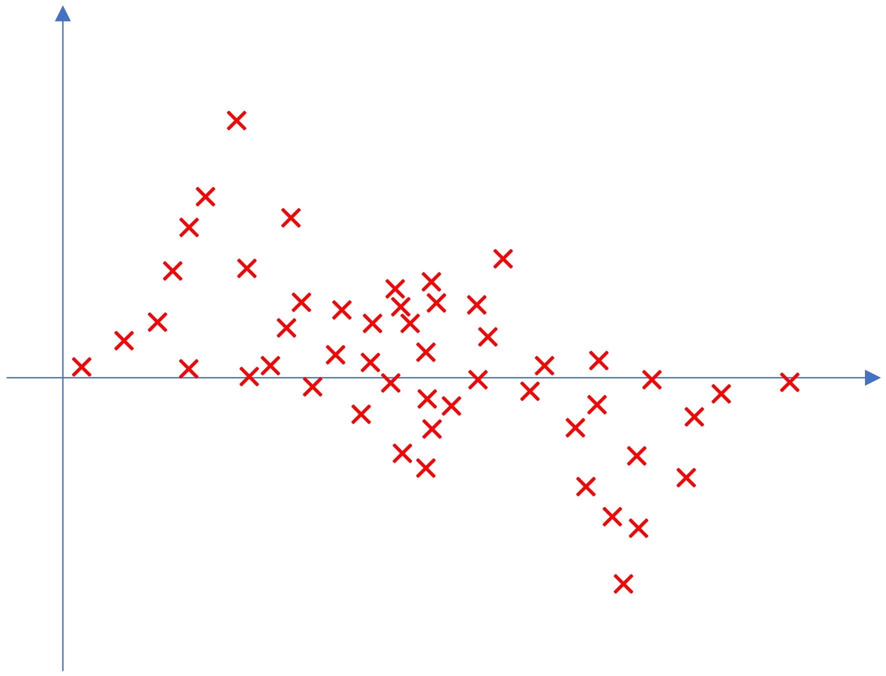


Uncertainty Modeling

Q: What can give rise to complex data-clouds?

A1: Input-dependent noise

A2: Model-form uncertainty



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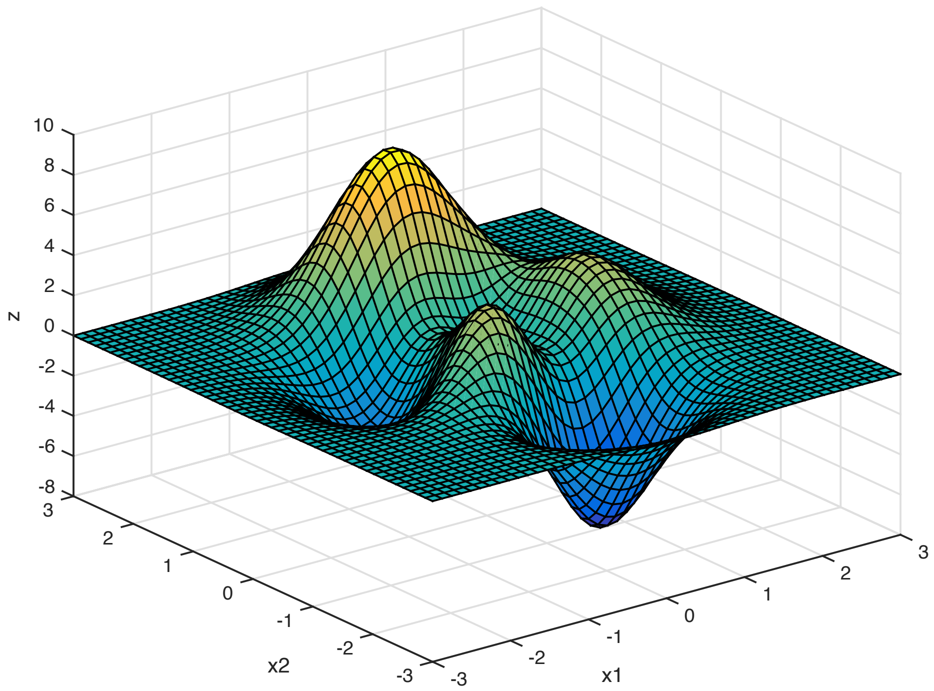

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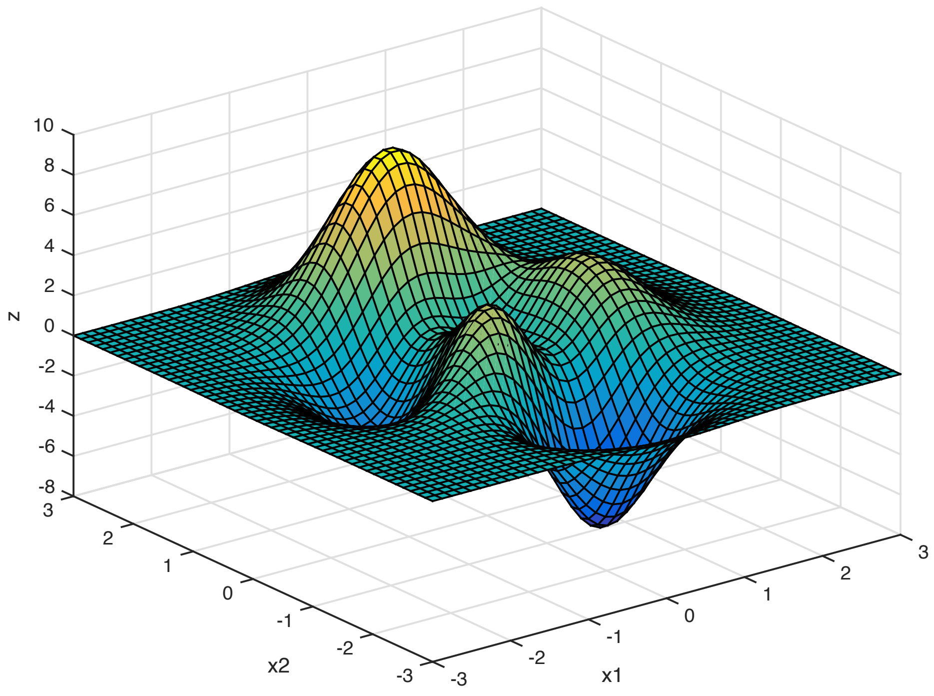

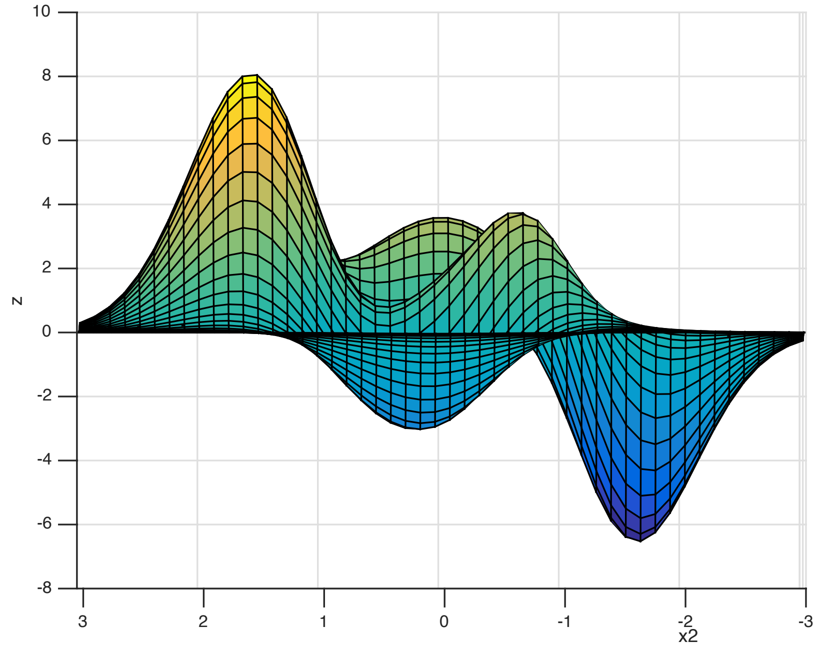

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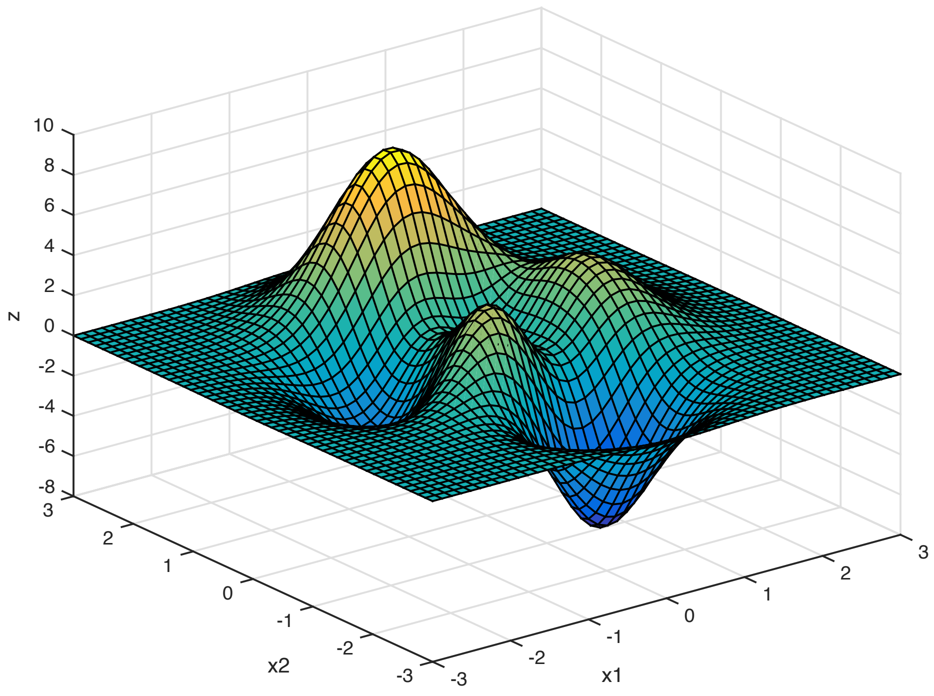
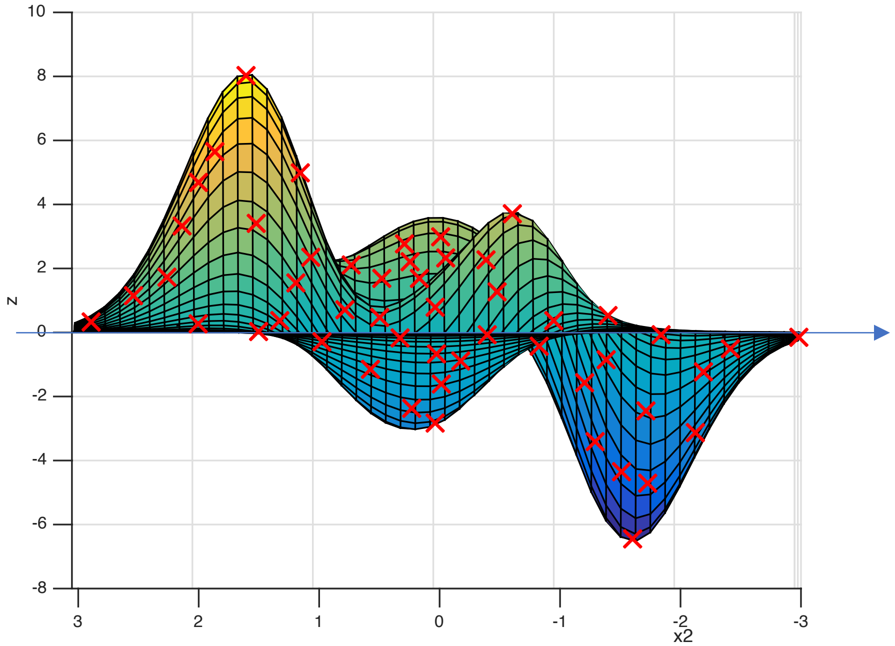
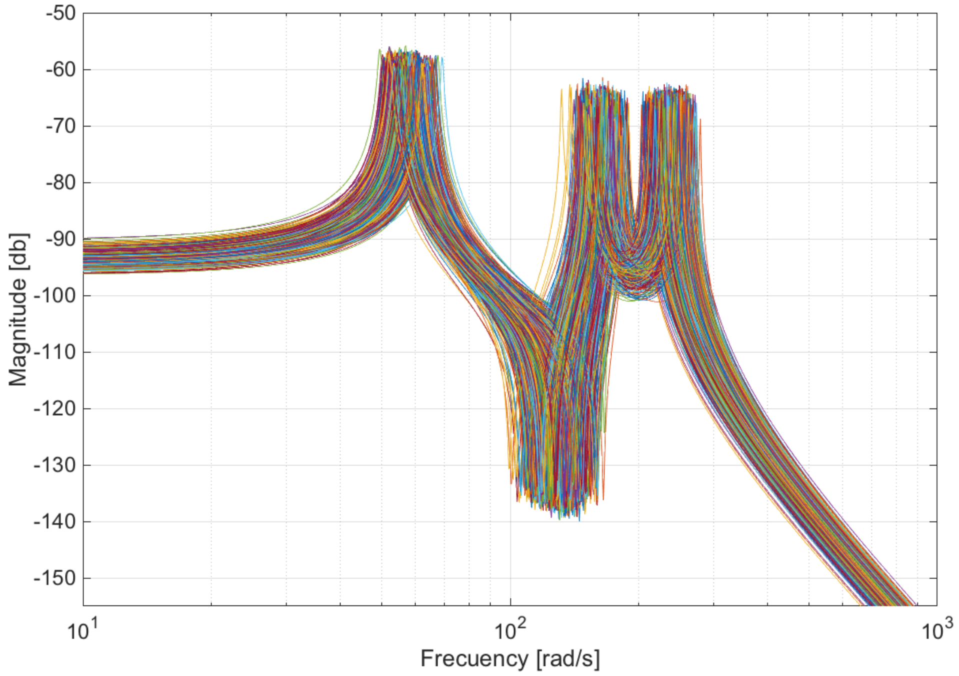




Perceived noise is not Gaussian

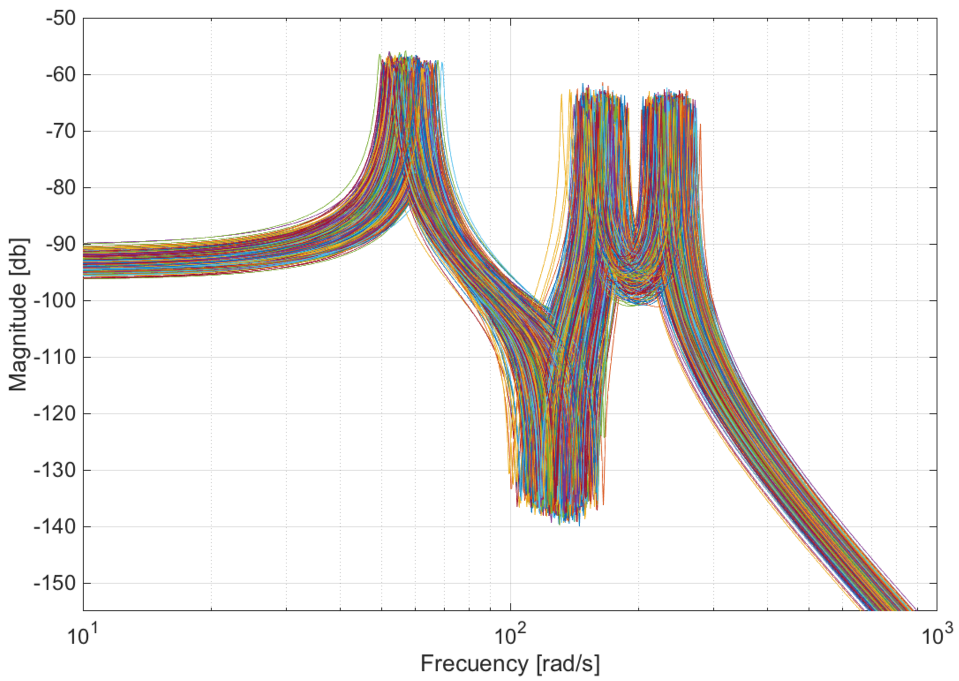
Uncertainty Modeling: Parameter Dependencies



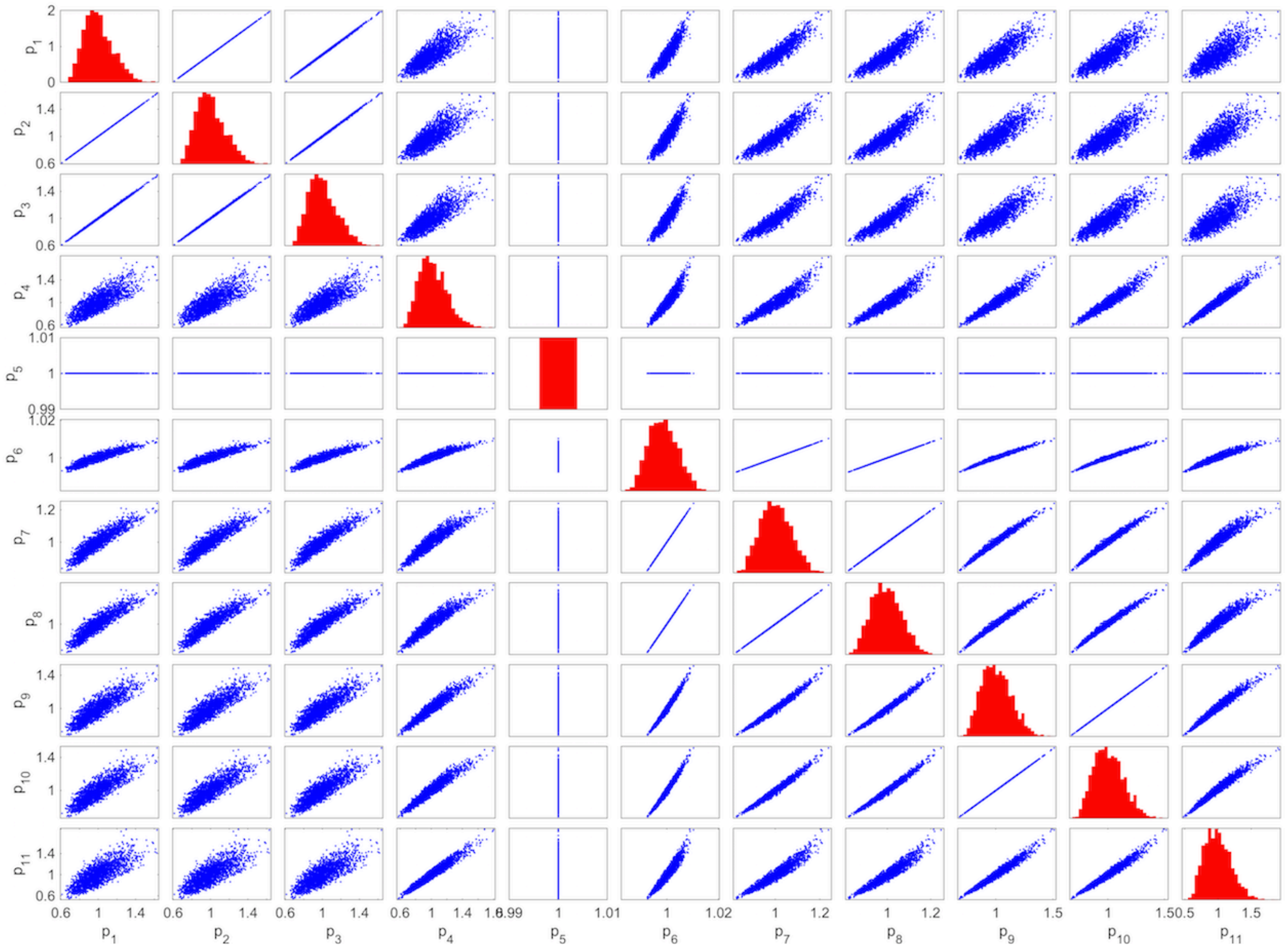
Uncertainty in masses and stiffnesses are independent

[8] Crespo et al, A Scenario Optimization Approach to System Identification with Reliability Guarantees, ACC 2019

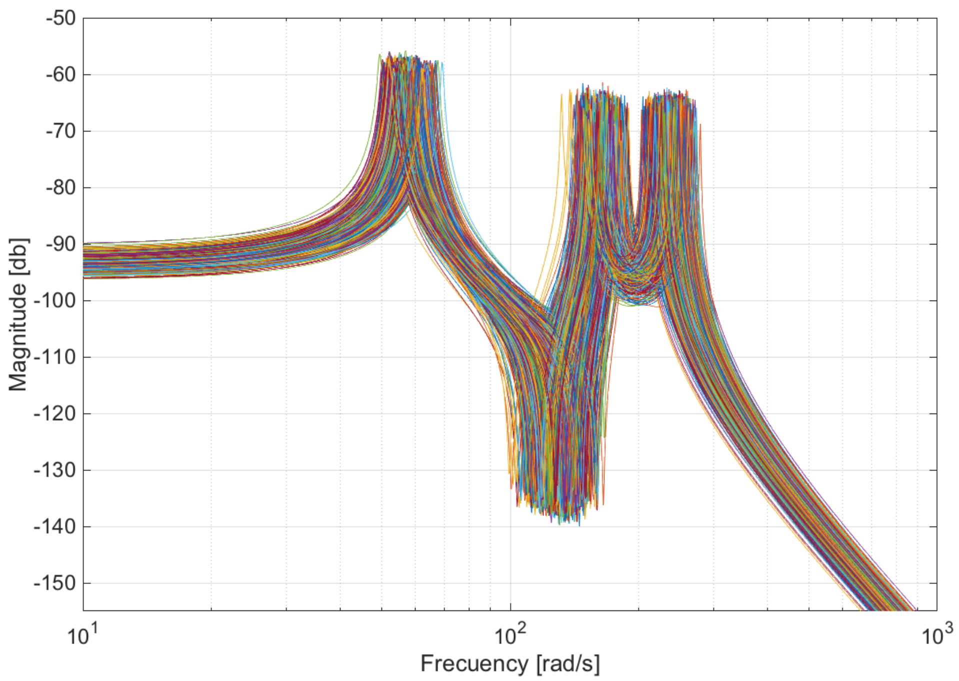
Uncertainty Modeling: Parameter Dependencies

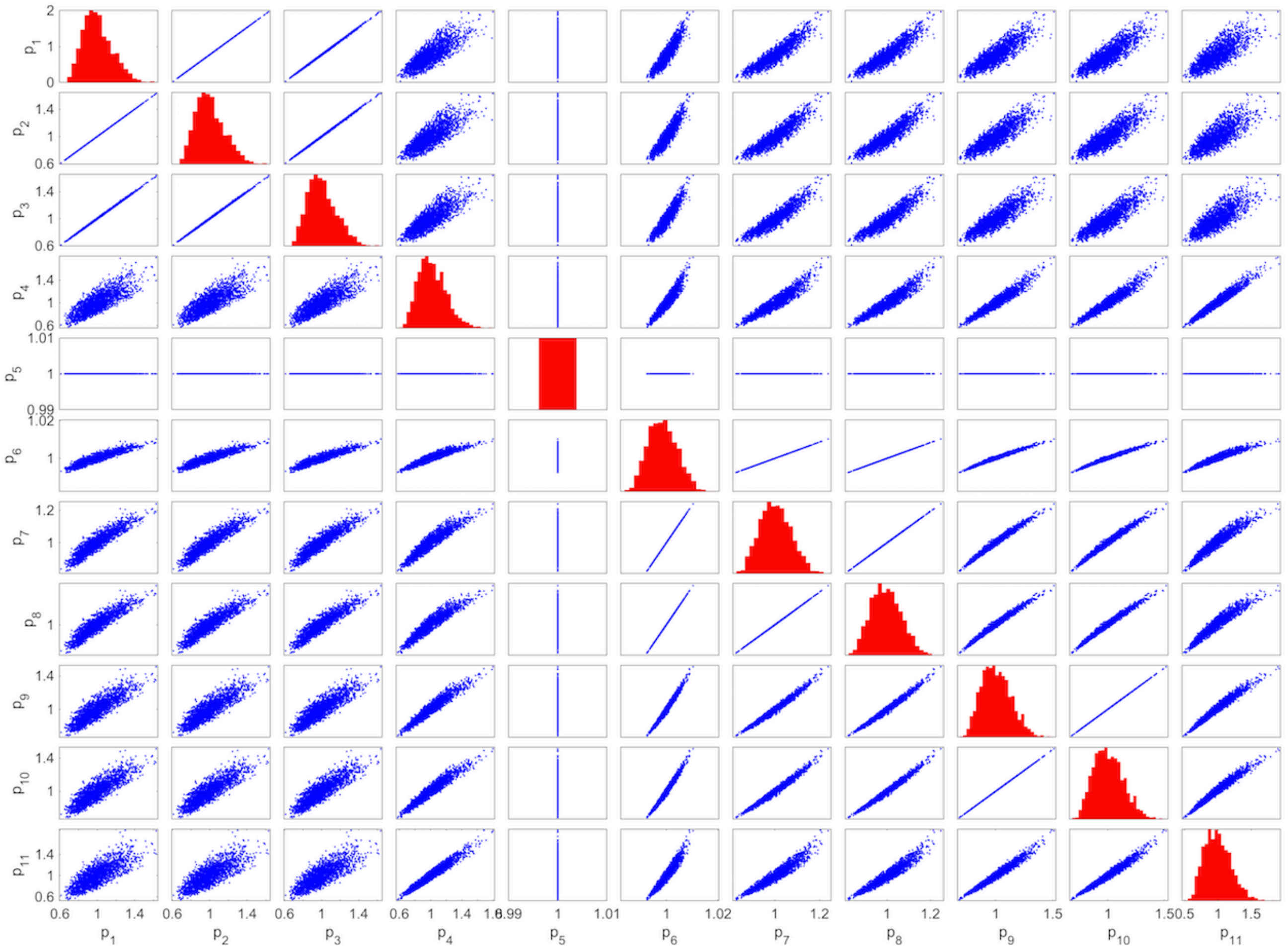


$$H(s) = \frac{p_1 s^4 + p_2 s^3 + p_3 s^2 + p_4 s + p_5}{p_6 s^5 + p_7 s^4 + p_8 s^3 + p_9 s^2 + p_{10} s + p_{11}}$$



Uncertainty Modeling: Parameter Dependencies





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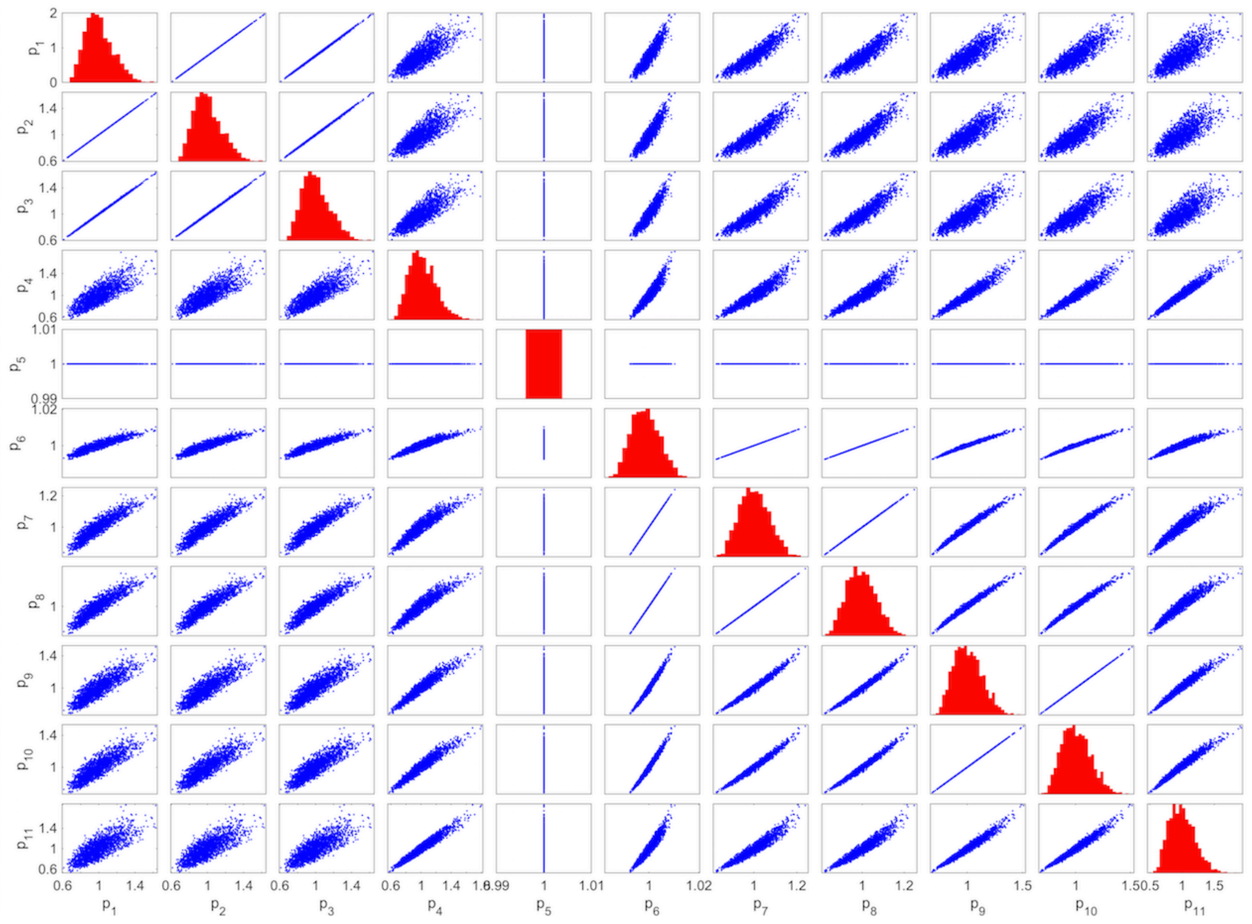
- Independent parameters in physical space become dependent in the space of the transfer function parameters
- Assuming parameter independence introduces unnecessary conservatism

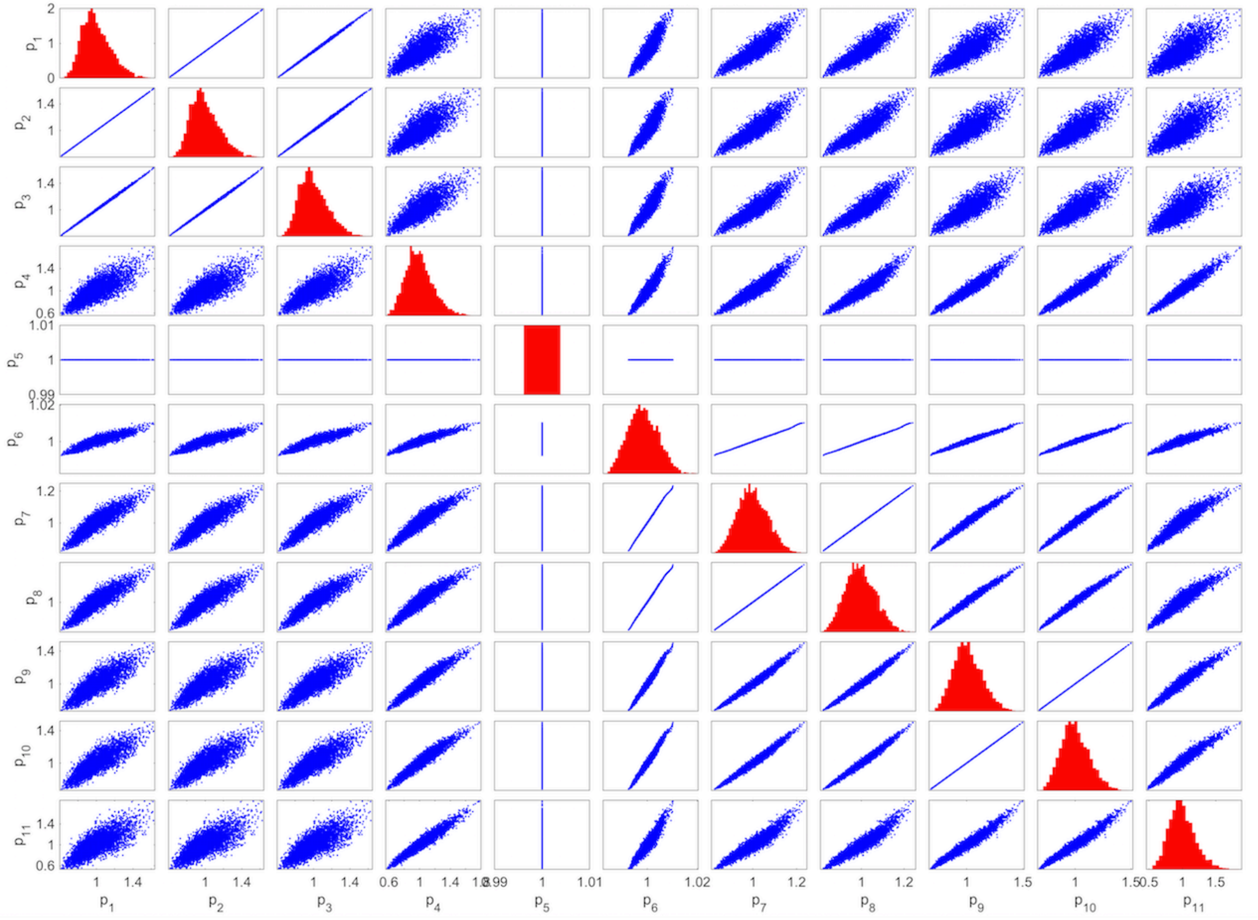
Uncertainty Modeling: Parameter Dependencies



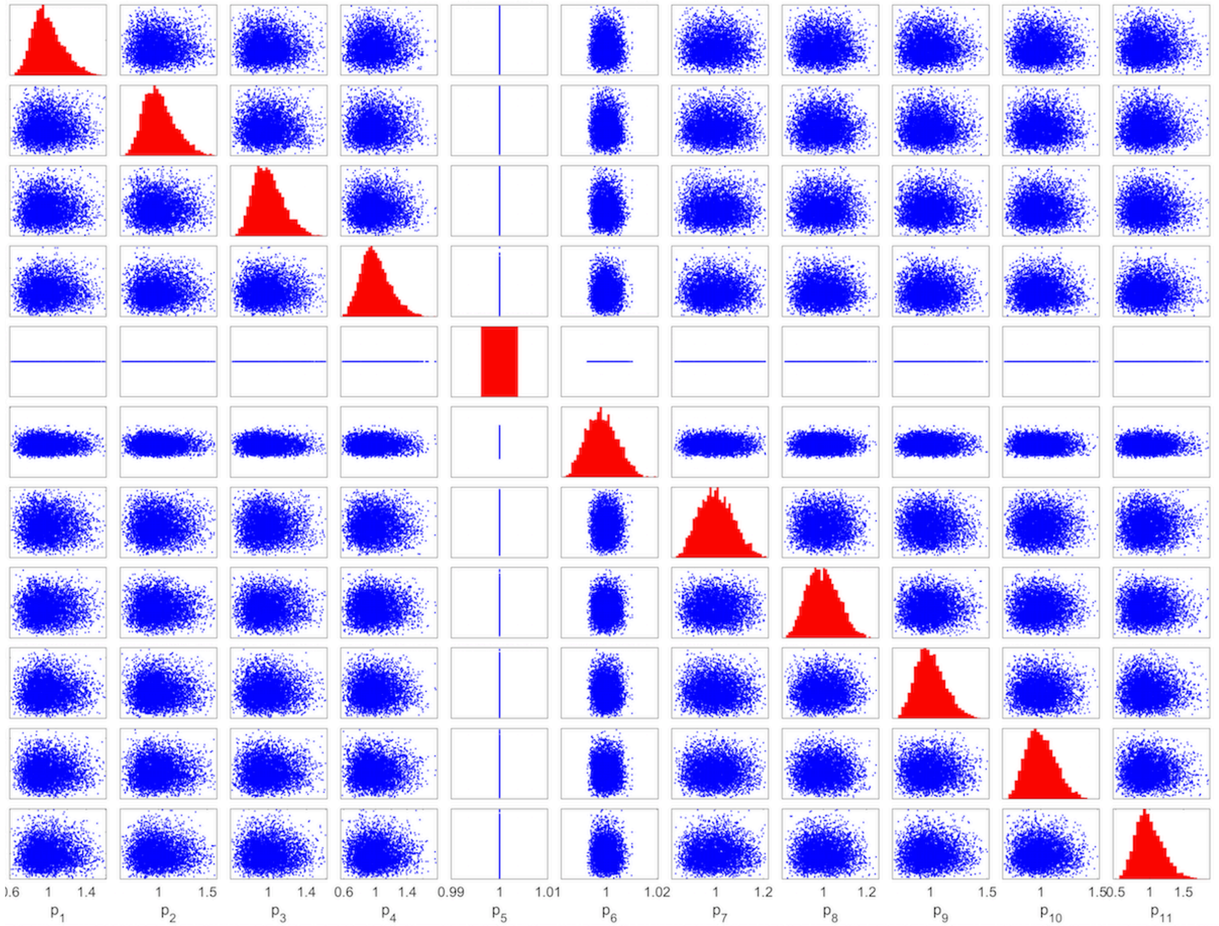
Consider two uncertainty models:

- Dependent parameters
- Independent parameters

Uncertainty Modeling: Parameter Dependencies

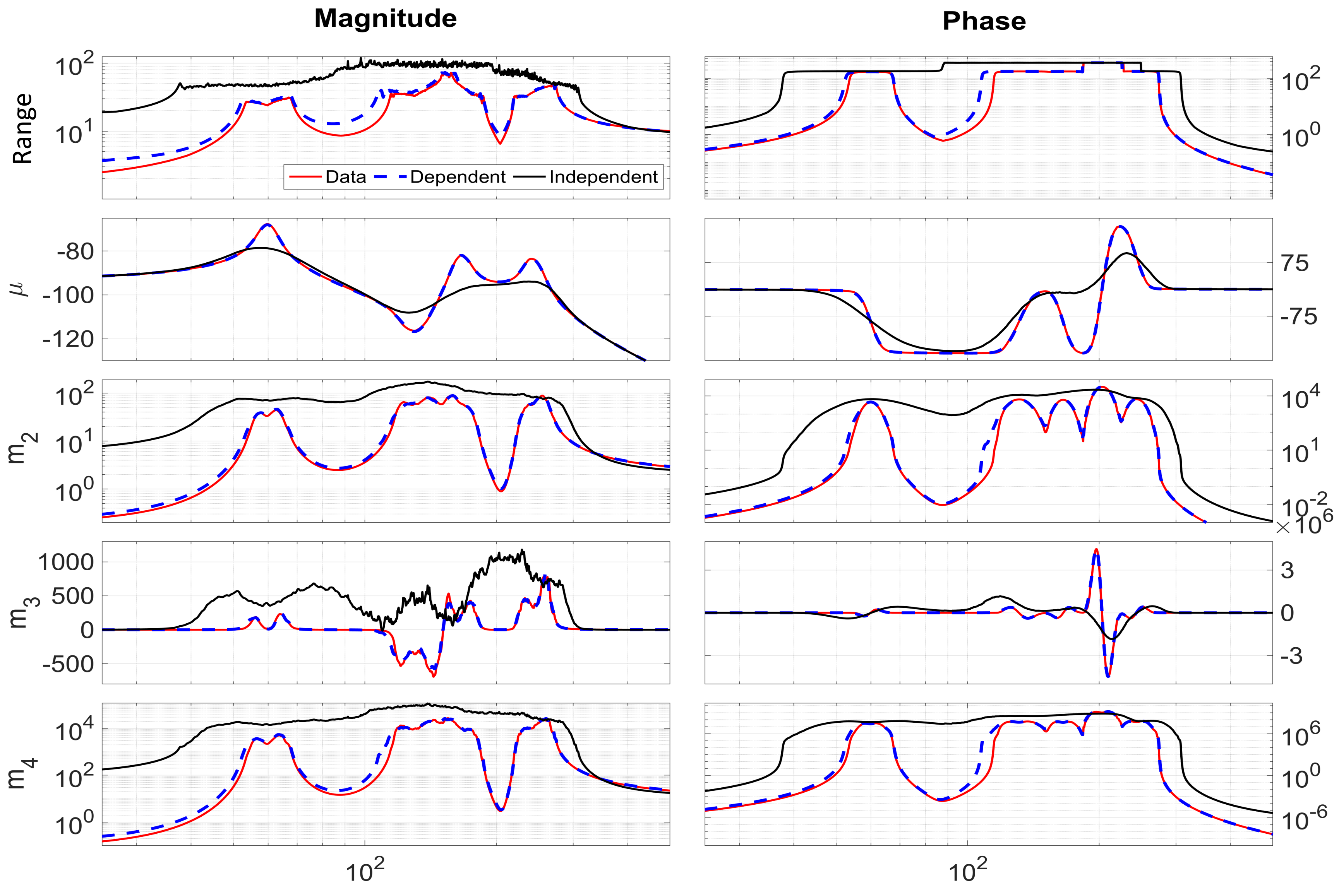


Samples of a distribution with **dependent** parameters



Samples of a distribution with **independent** parameters

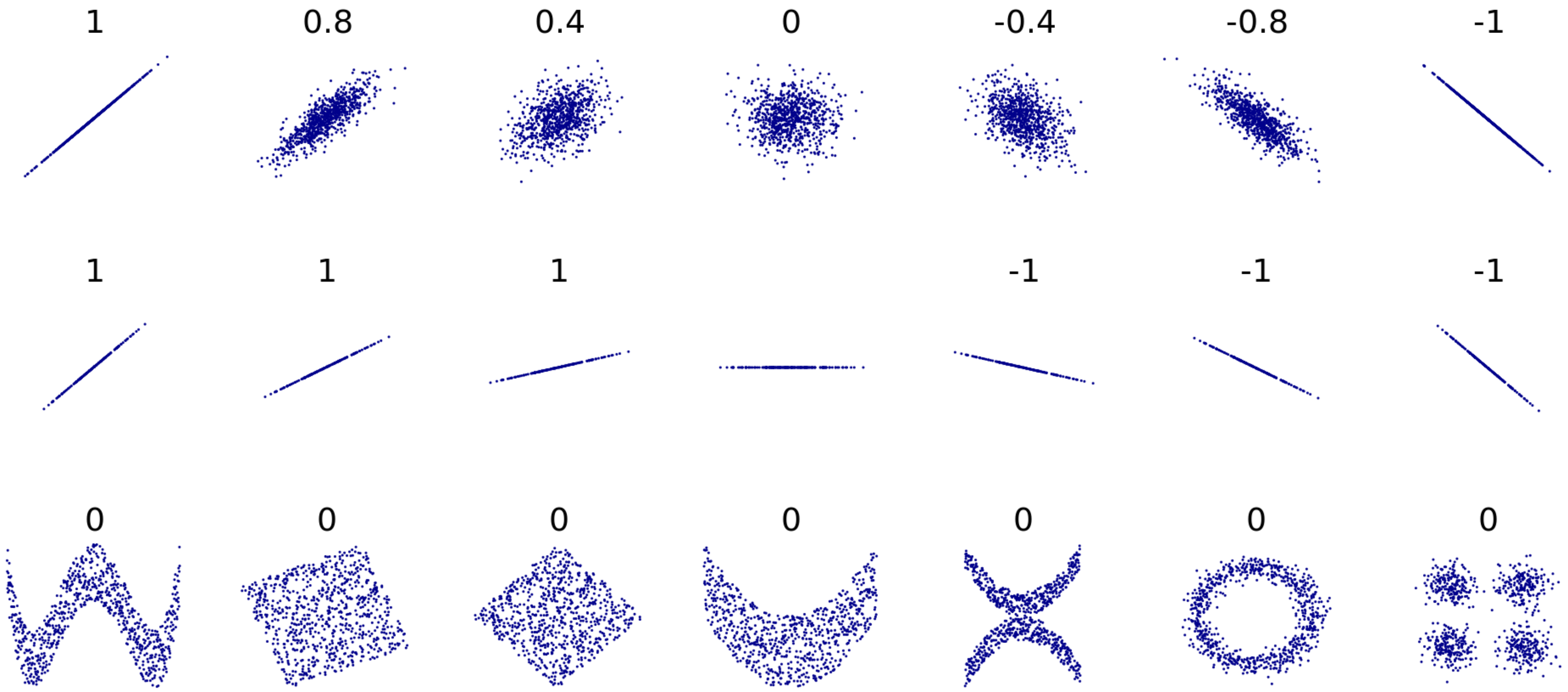
Uncertainty Modeling: Parameter Dependencies



	P[instability]
Data	0
Model with dependent parameters	0
Model with independent parameters	0.23

How to Quantify Parameter Dependencies?

- Correlation is only a measure of linear dependency/noise



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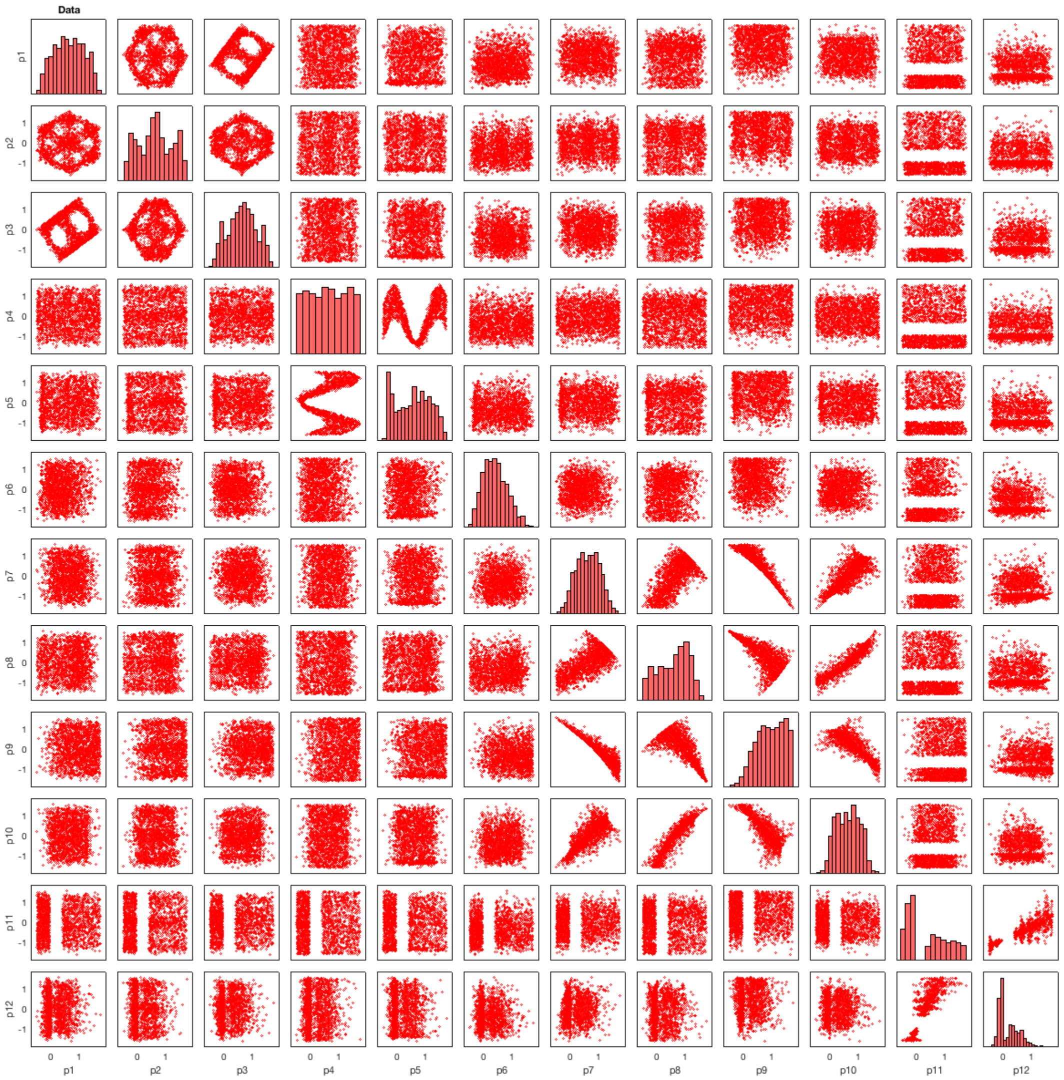
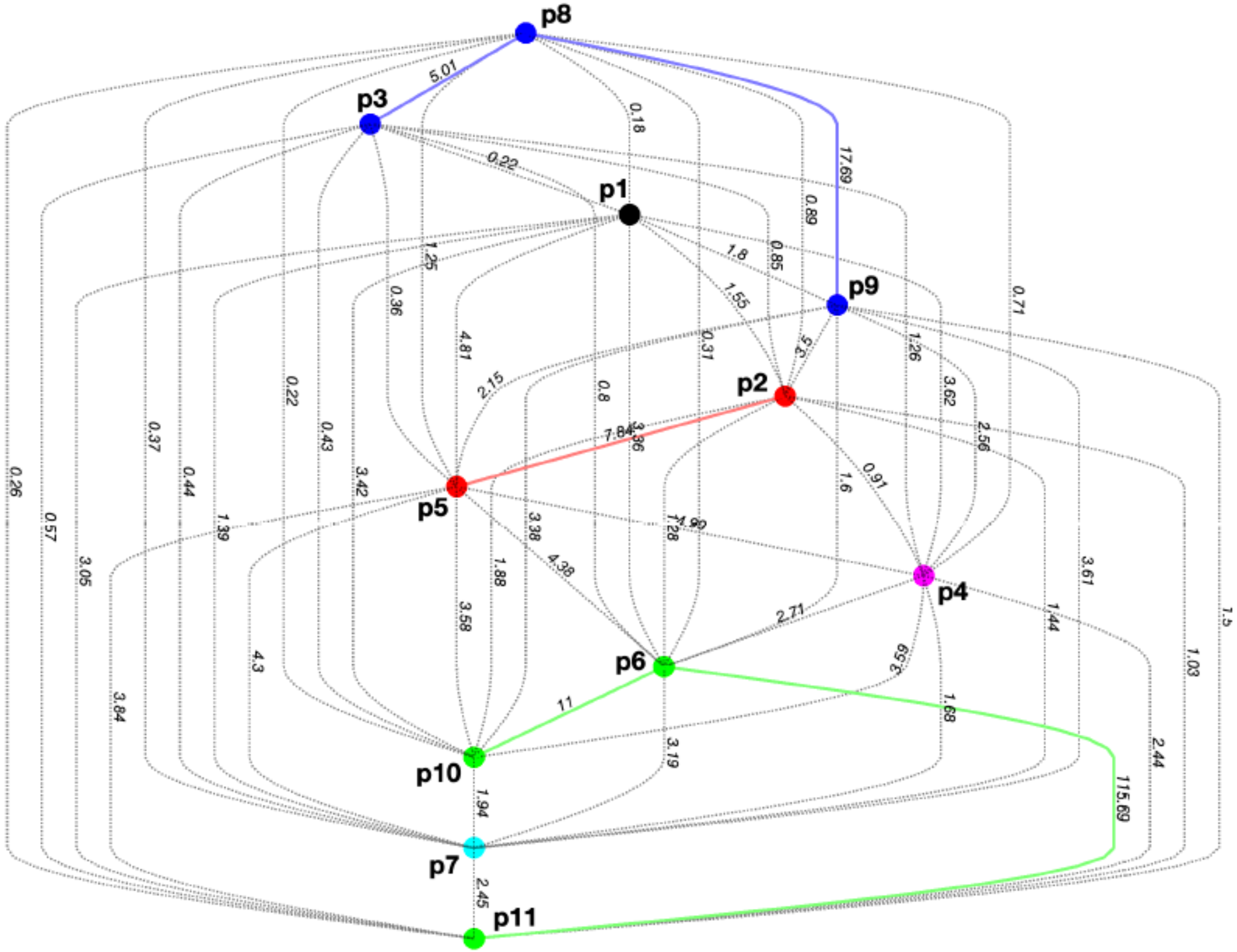


How to Quantify Parameter Dependencies?

- Use copula-based metrics for quantifying the degree of dependency among variables



[7] Crespo et al, Dimensionality Reduction of Sliced-Normal Distributions, IFAC world congress 2020

Overview

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- Challenge problems

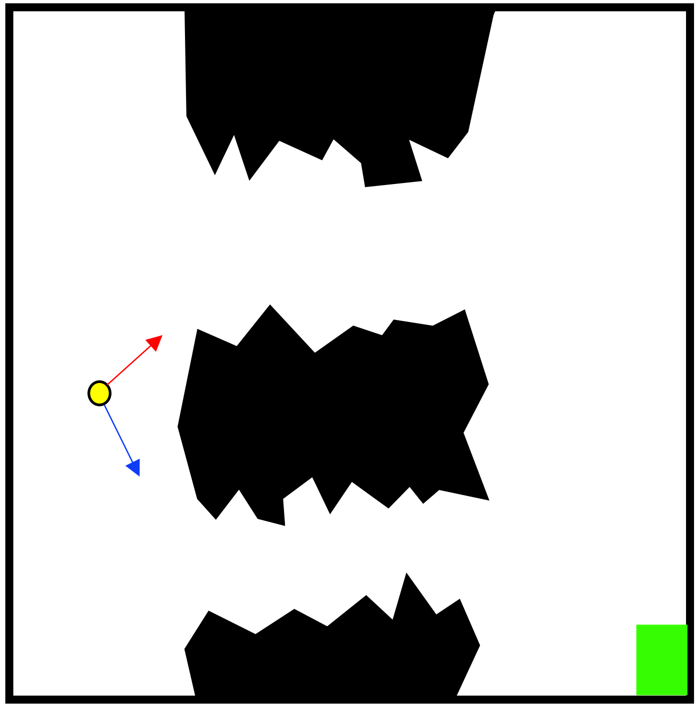
Optimization under Uncertainty

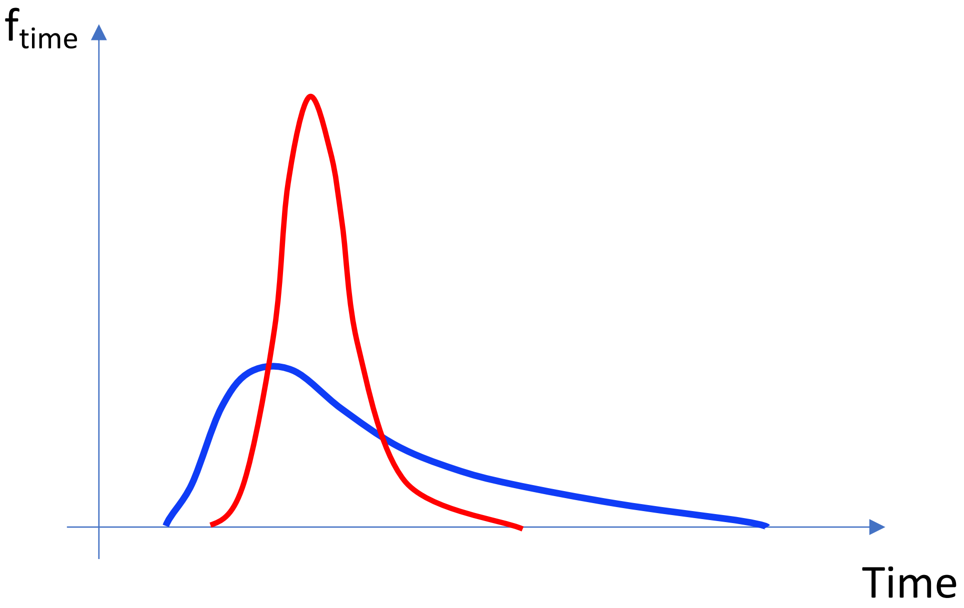
- Design processes are often performed by optimization
- Optimization is highly sensitive to what it is assumed to be known
- Plethora of applications in regression analysis, model calibration, system identification, control theory, machine learning, systems theory, autonomous systems, financial mathematics, decision making, etc.

Optimization under Uncertainty

- Ideal properties: convexity & robustness to uncertainty: it is not only about reaching to the finish line fast, it is also about getting to the right finish line
- There is a price to pay for convexification
- The dependency of the program on the decision variables and on the uncertain parameters play a key role
- Key goal in real-time optimization: satisfying safety-critical constraints during the numerical search for a robust optimum

Optimization under Uncertainty

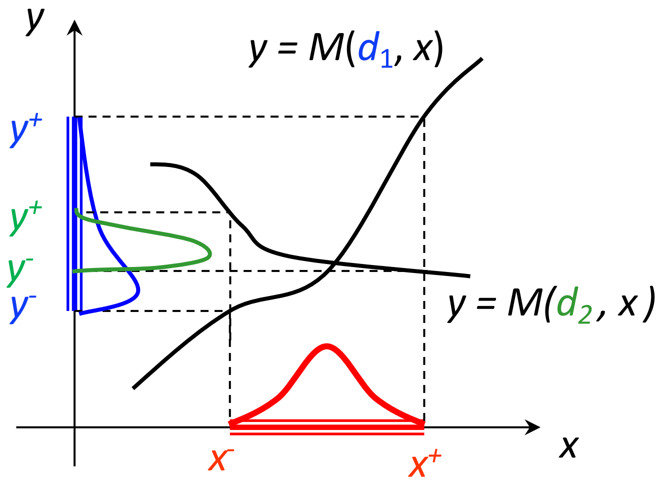




- What policy should be chosen to score in minimum time?

Optimization under Uncertainty

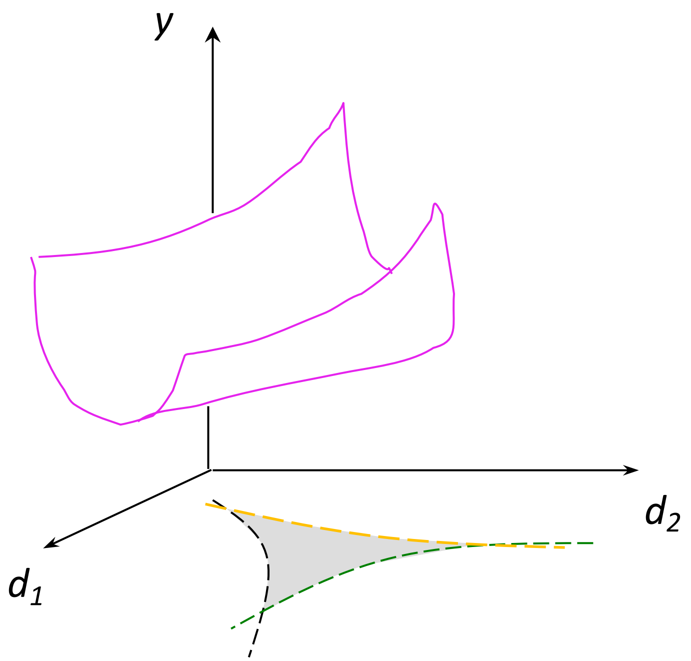
$$y = M(d, x)$$



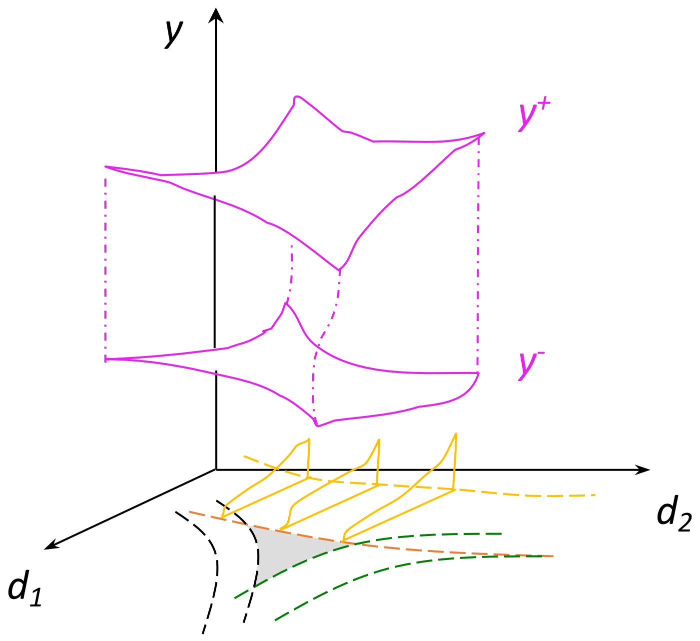
Use the decision variable d to shape f_y at will. How do we want it to look like?

Optimization under Uncertainty

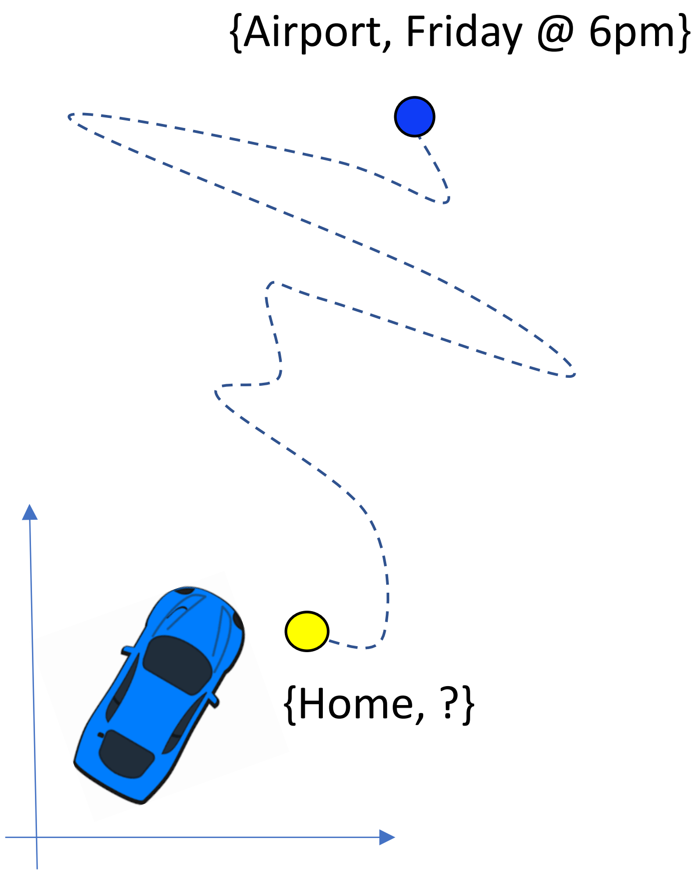
Deterministic Setting



Probabilistic Setting



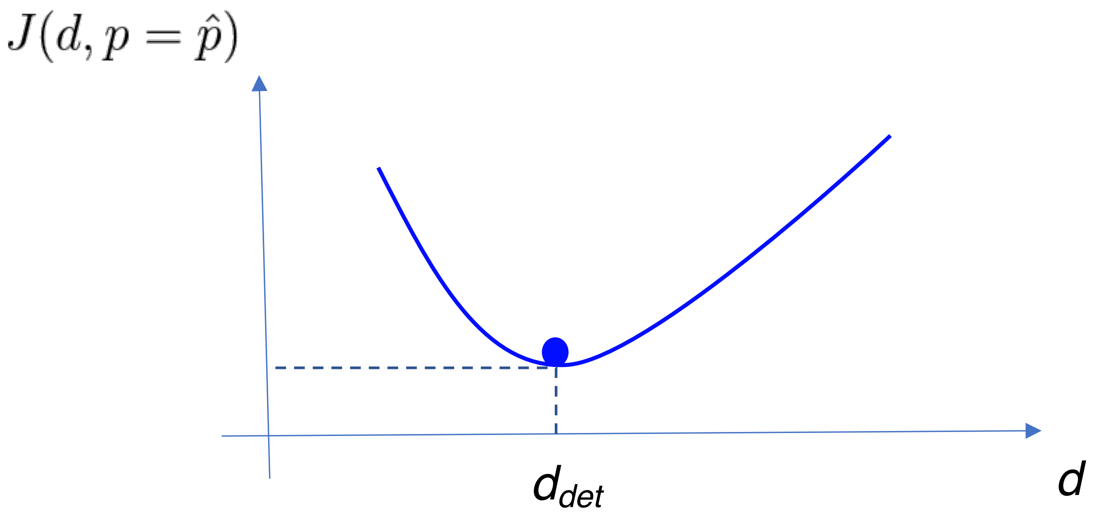
Motivational Example

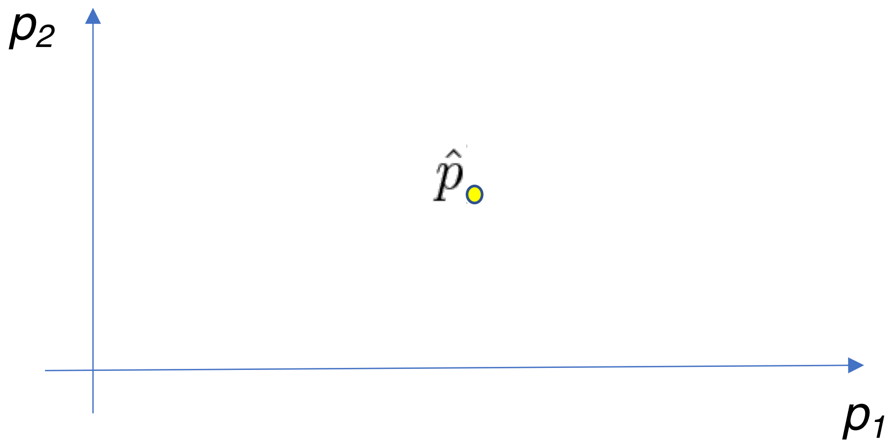


- Cost J: Time waiting in the airport
- Design variable d: time leaving home
- Uncertainty p: {traffic, road/lane closures}

Optimization under Uncertainty: $\min_d J(d, p)$

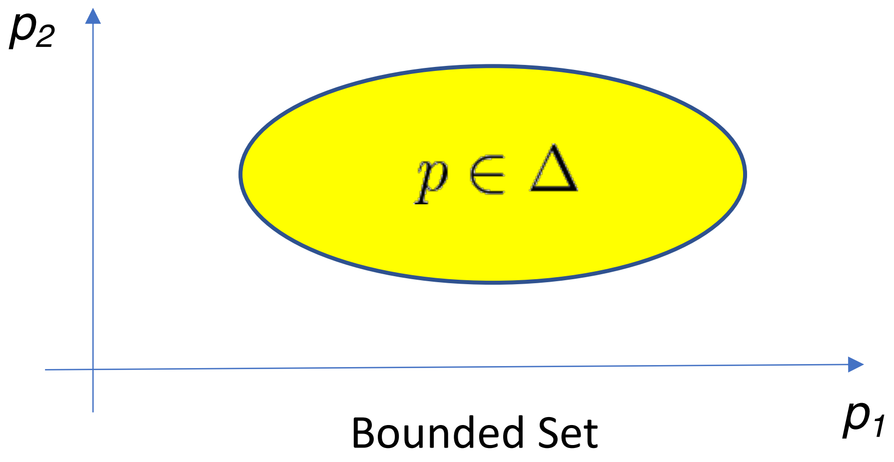
- The value of *p* is known





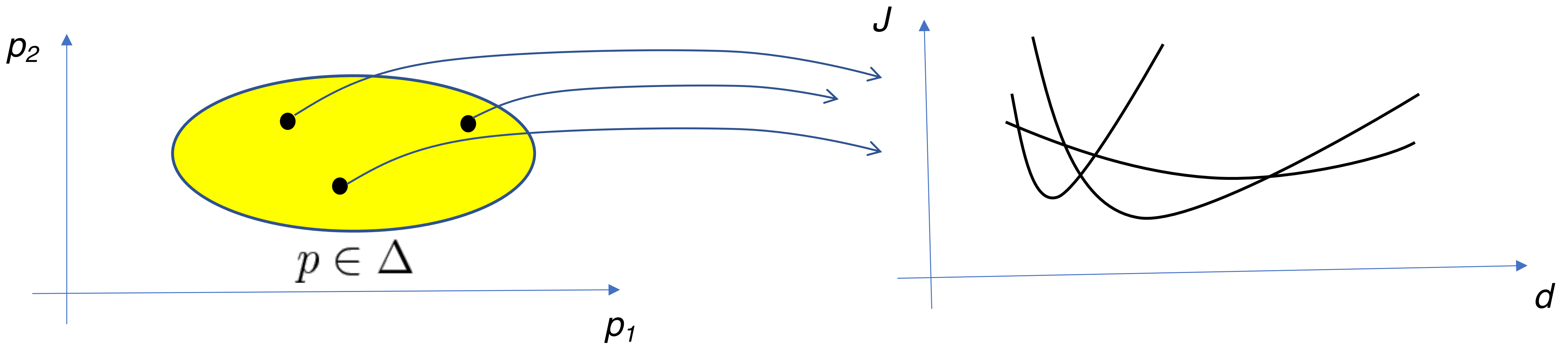
Optimization under Uncertainty: $\min_{d} J(d, p)$

- The value of p is known
- The value of p is unknown



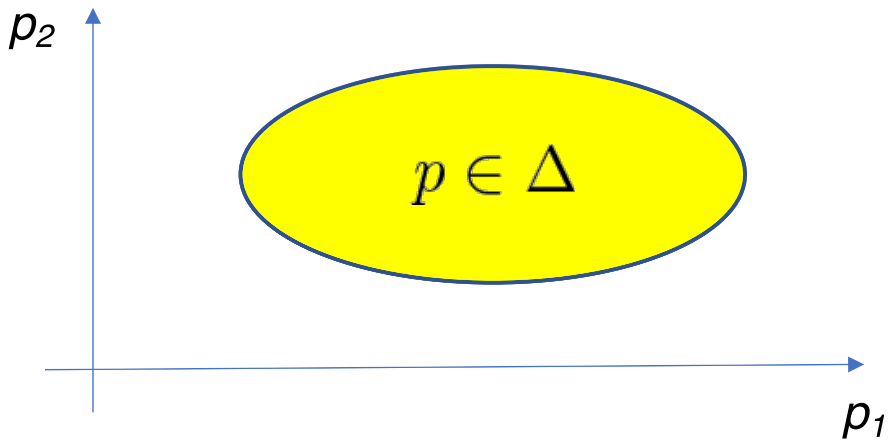
Optimization under Uncertainty: $\min_d J(d, p)$

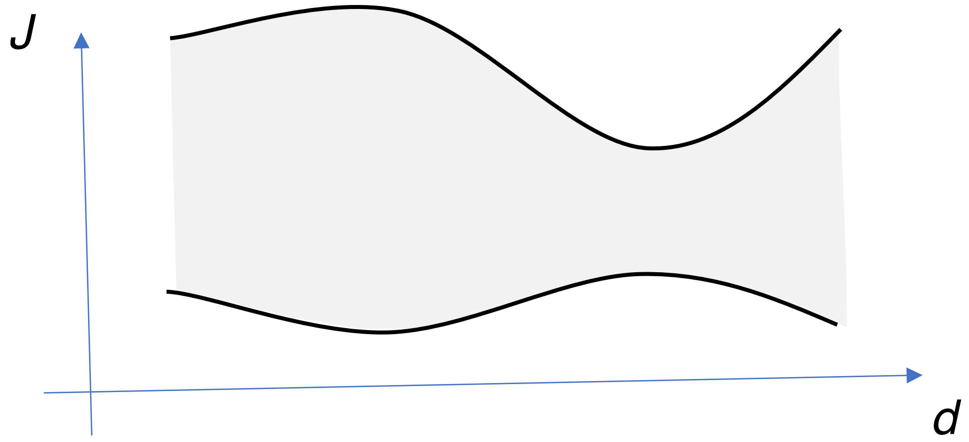
- The value of p is known
- The value of p is unknown



Optimization under Uncertainty: $\min_d J(d,p)$

- The value of p is known
- The value of p is unknown

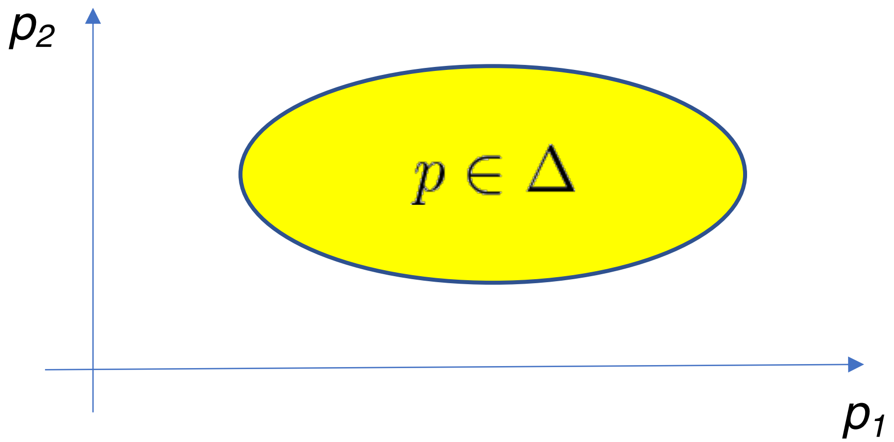


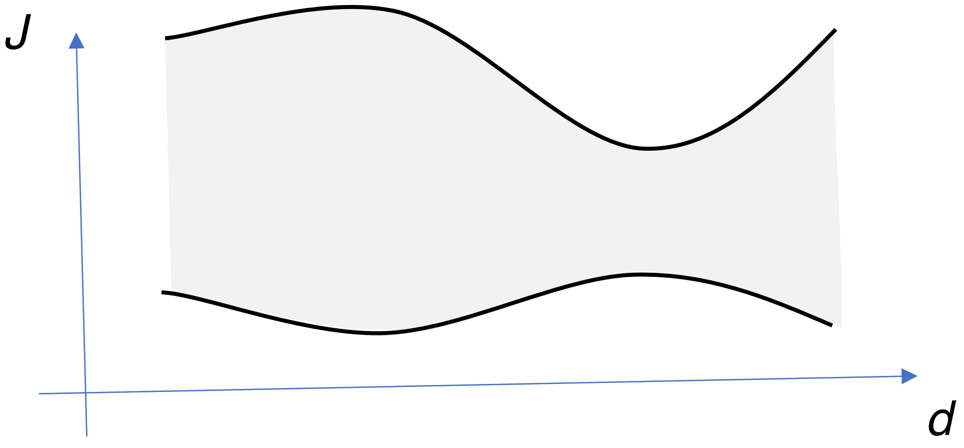


Interval-valued function

Optimization under Uncertainty: $\min_d J(d, p)$

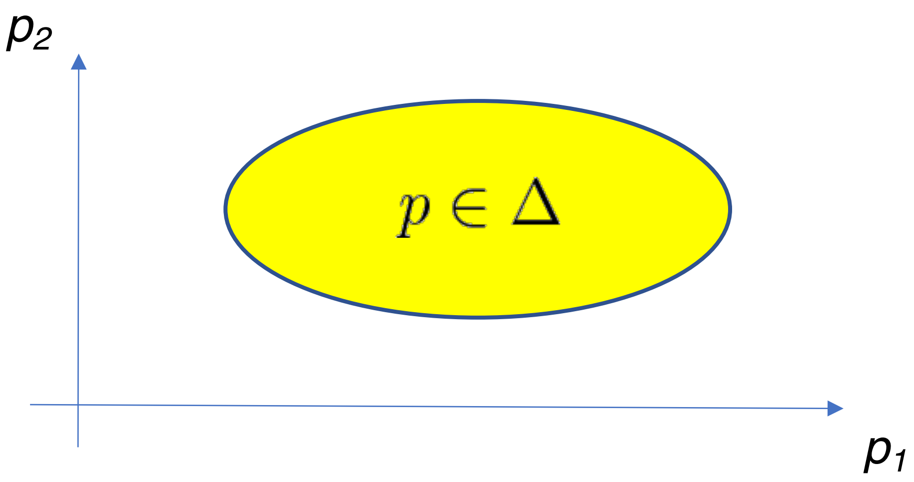
- The value of *p* is known
- The value of *p* is unknown: worst-case, average, and chance-constrained

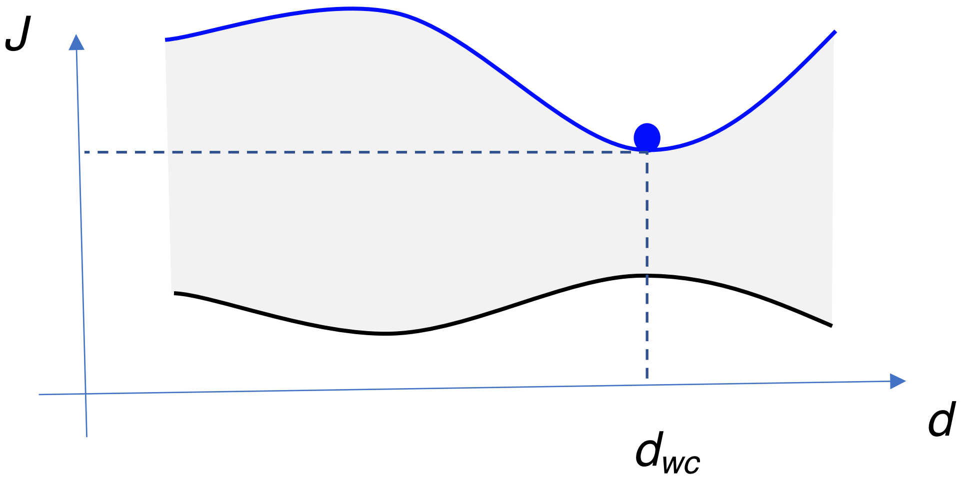




Interval-valued function

Worst-Case Optimization

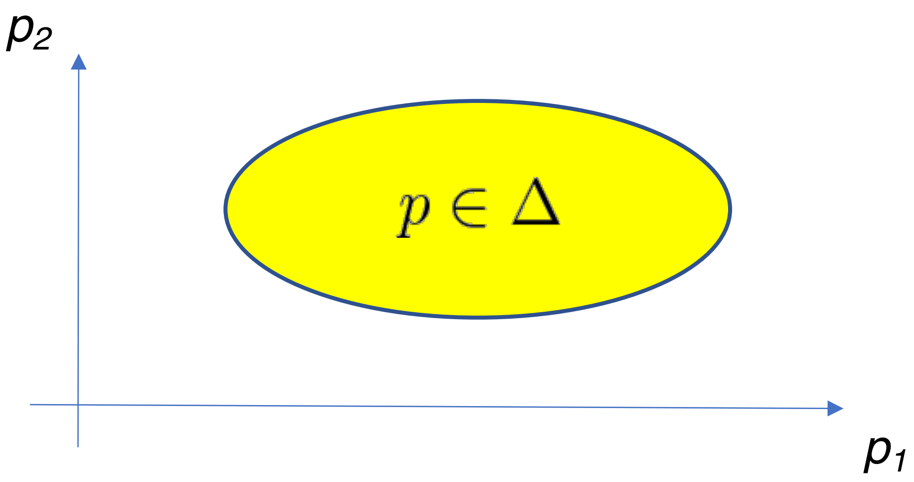


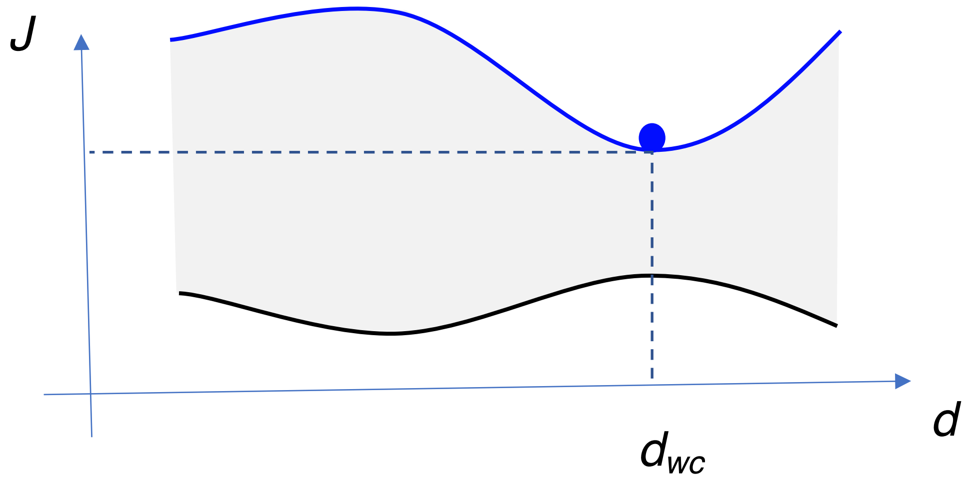


- Formulation

$$\min_{d} \left[\max_{p \in \Delta} J(d, p) \right]$$

Worst-Case Optimization



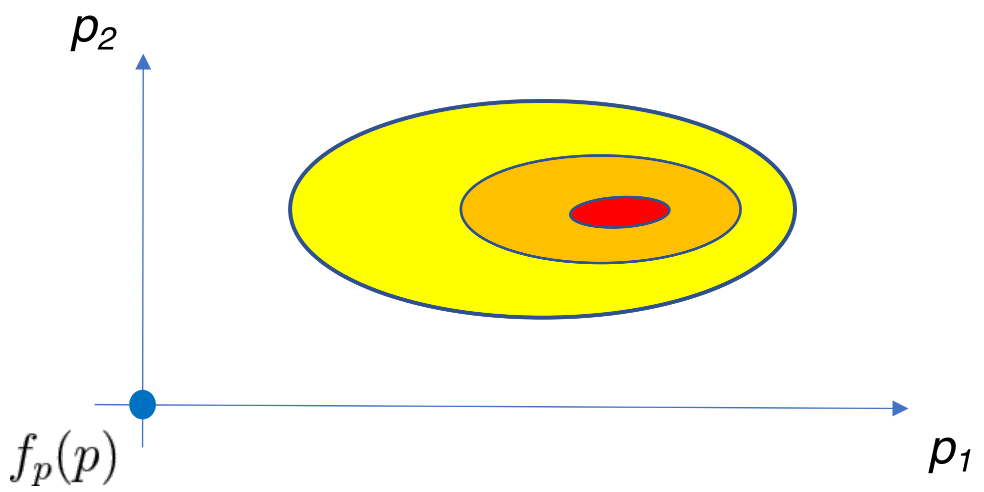


- Formulation

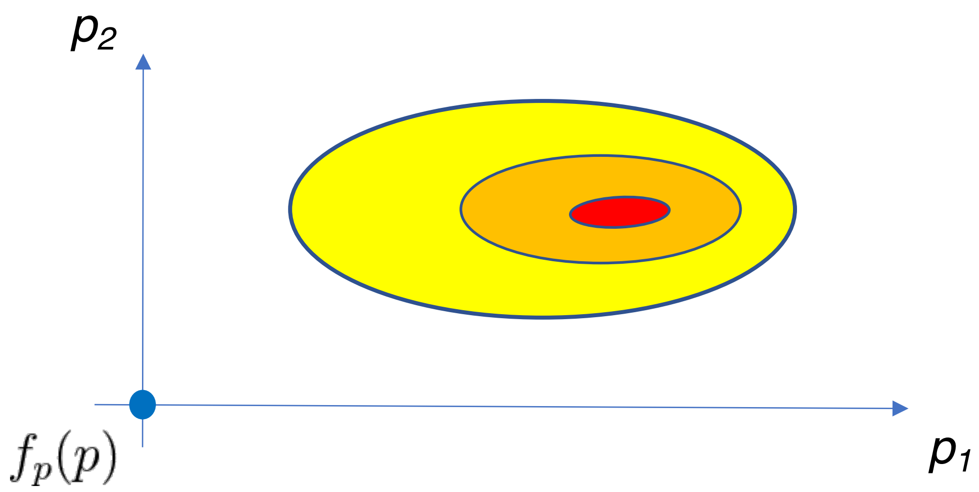
$$\min_{d} \left[\max_{p \in \Delta} J(d, p) \right]$$

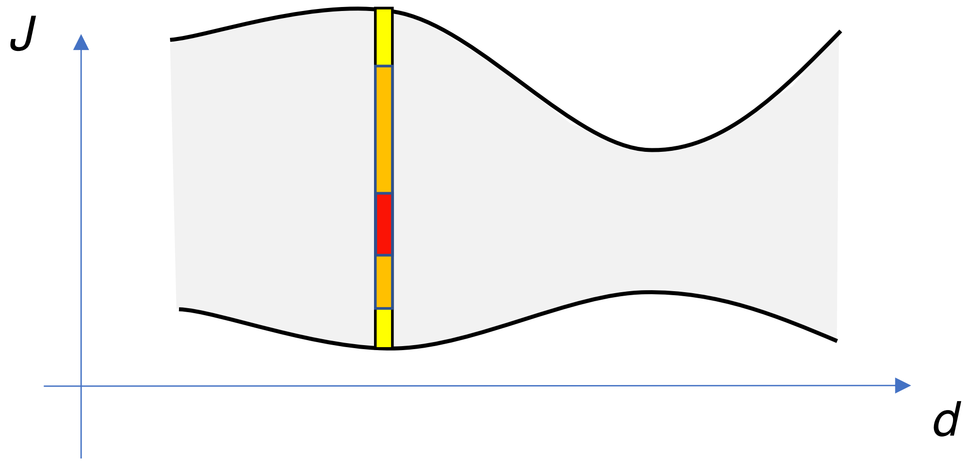
- Performance guarantees, conservative policy

Probabilistic Uncertainty



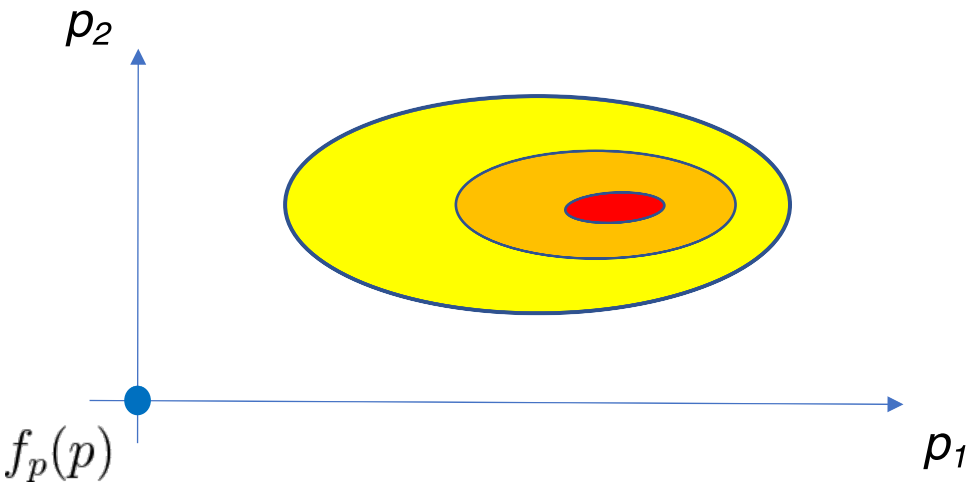
Probabilistic Uncertainty

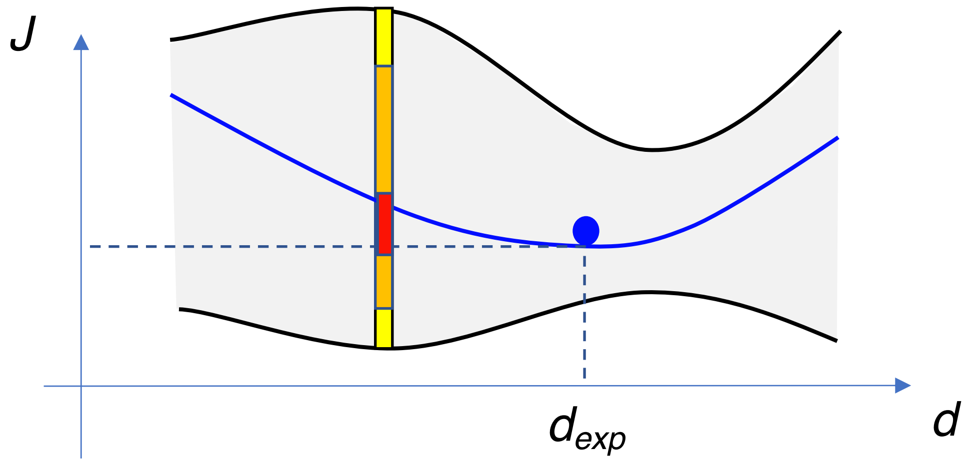




Random Process

Optimal Expected Cost

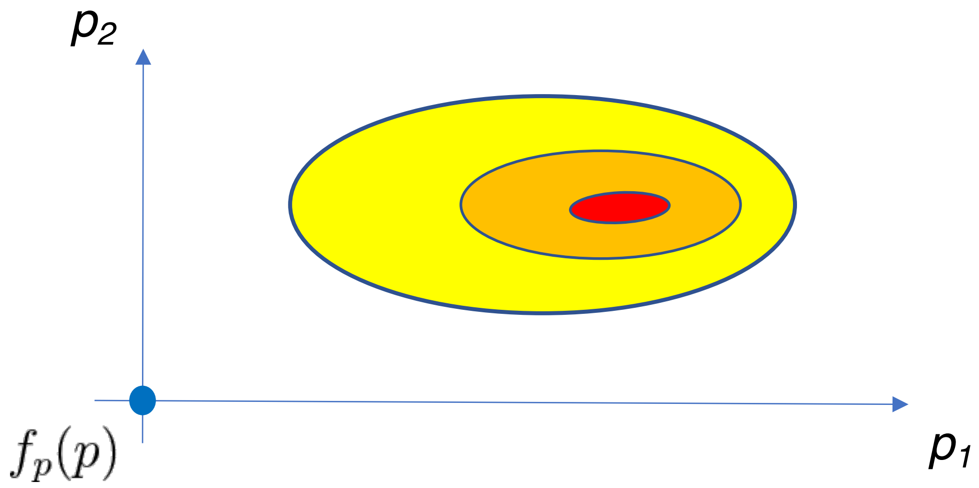


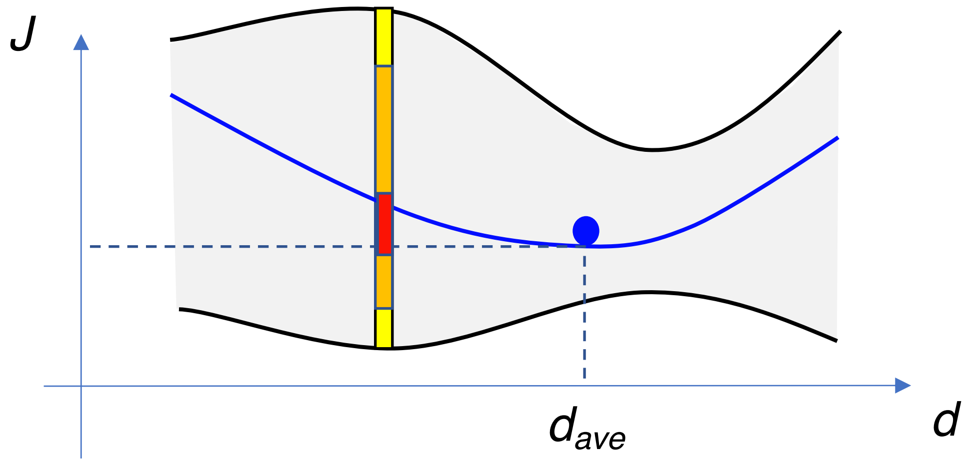


- Formulation

$$\min_d \mathbb{E}\left[J(p,d)\right]$$

Optimal Expected Cost



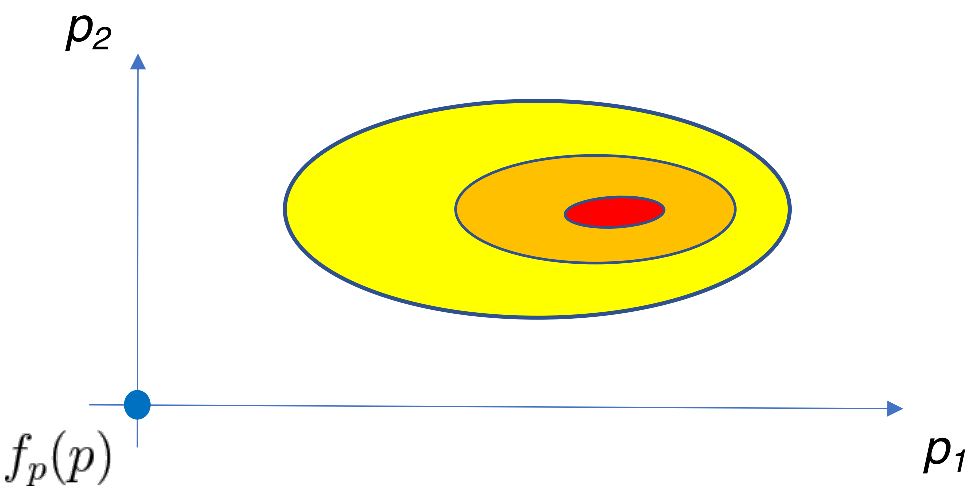


- Formulation

$$\min_{d} \mathbb{E}\left[J(p, d)\right]$$

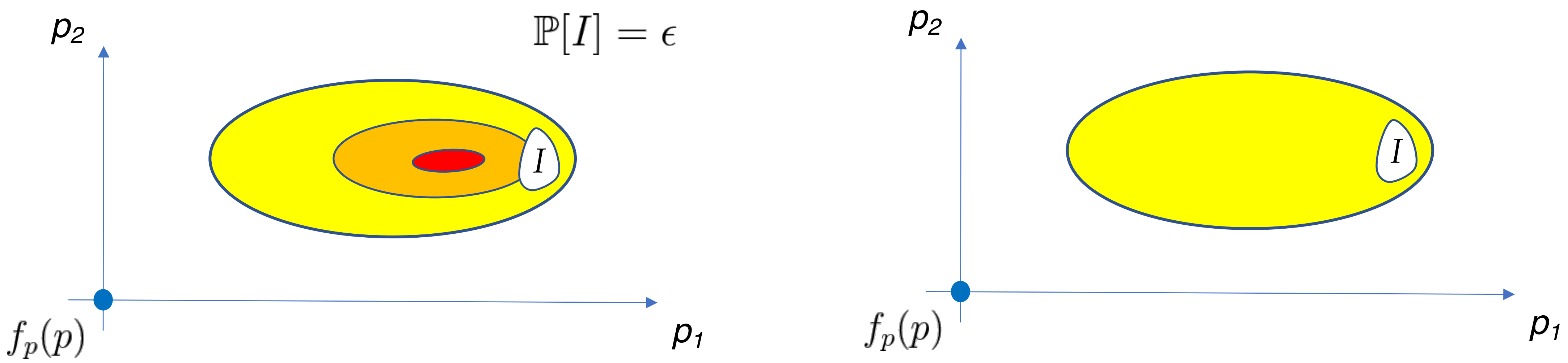
- Note $\mathbb{E}[J(p, d)] \neq J(\mathbb{E}[p], d)$ in general

Chance-Constrained Optimization



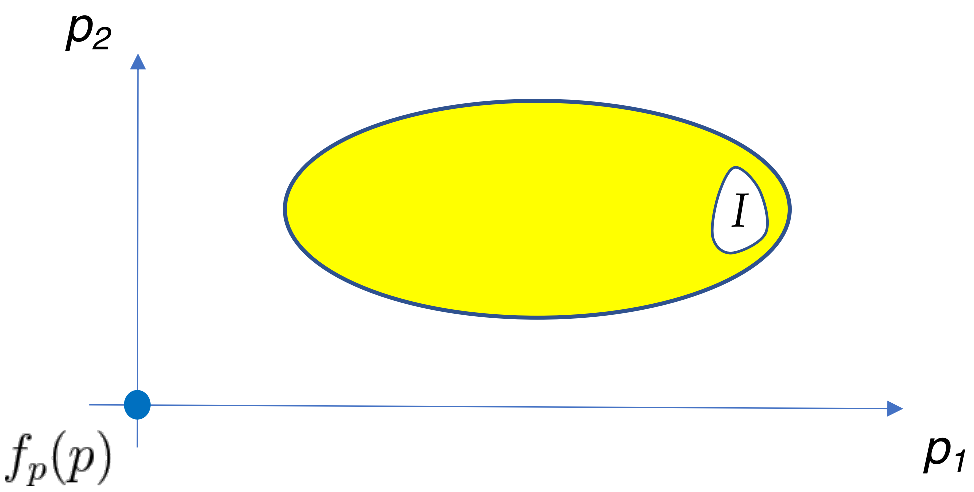
- Goal: to trade-off robustness for performance
- We need to accept some level of risk: science of gambling
- How can we do this?

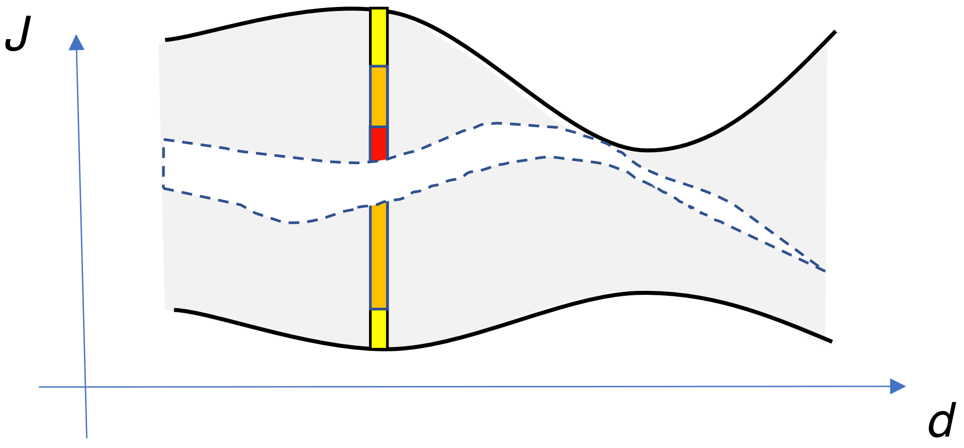
Chance-Constrained Optimization



- Idea: find the WC solution after ignoring a low-probability region
- Option 1:
  - Make a “hole” in $\boldsymbol{\Delta}$ of known probability
  - All the *p*’s within the hole will be ignored
  - Search for the optimal *d*

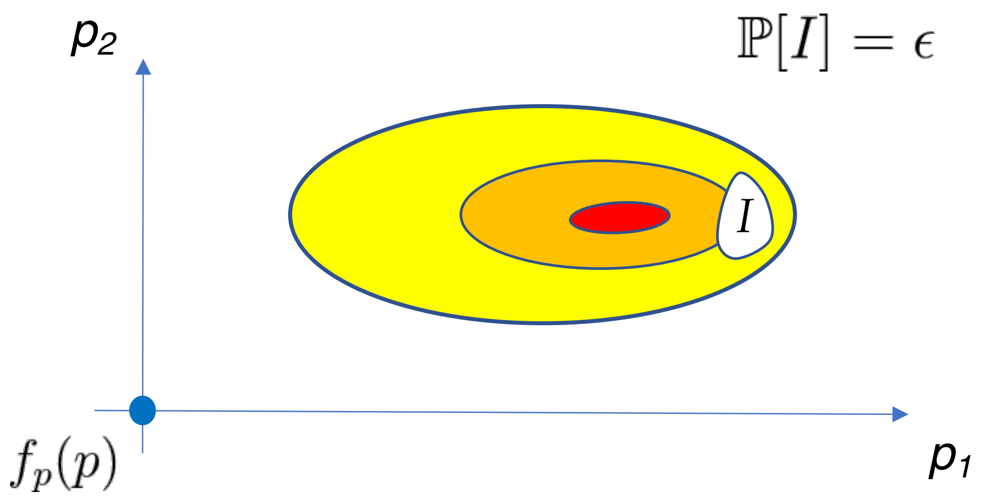
Chance-Constrained Optimization





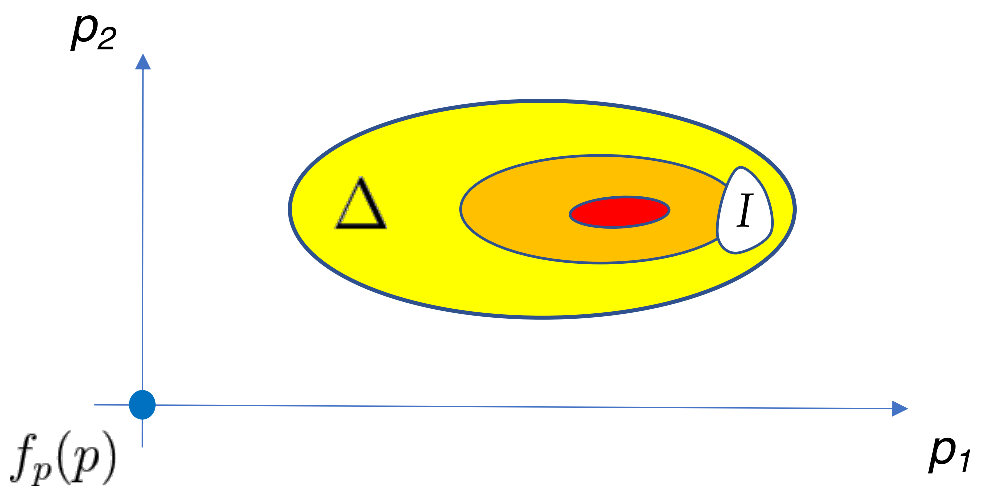
- The worst-performing cost depends on d
- The choice of I might make the optimal design subpar
- This is just another worst-case problem!!

Chance-Constrained Optimization



- Option 2:
  - Make a “hole” in **Δ** of known probability
  - All the *p*’s within the hole will be ignored
  - *d* as well as the “best” hole are searched for

Chance-Constrained Optimization

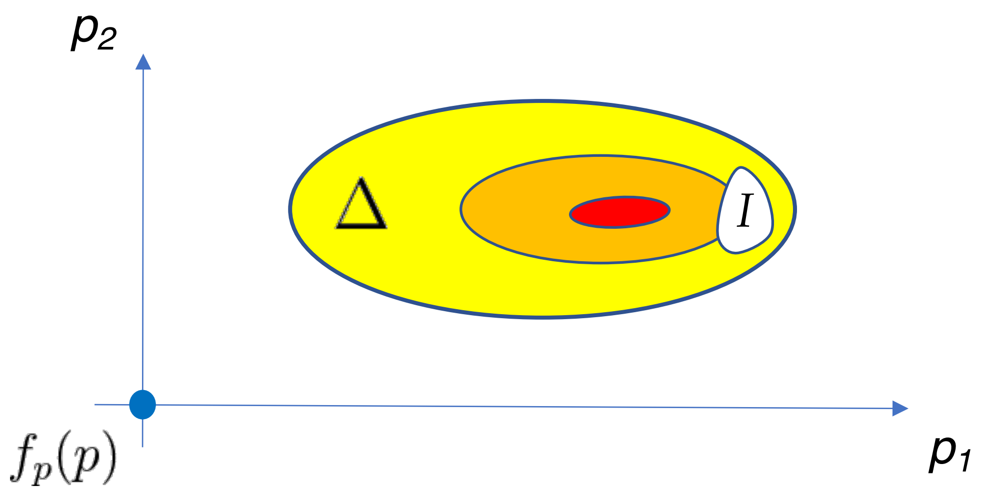


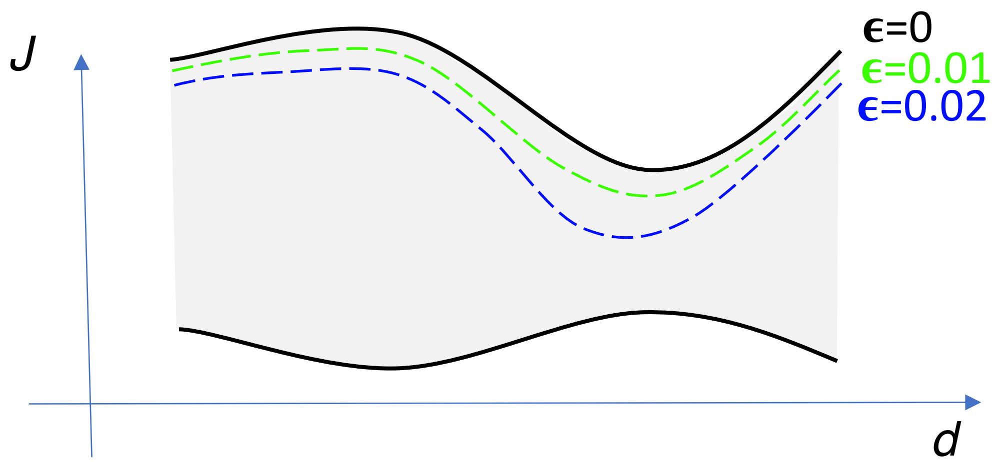
- Formulation

Risk

$$\min_{I,\,d}\left[\max_{p\in\Delta-I} J(d,p) : \mathbb{P}[I]=\epsilon\right]$$

Chance-Constrained Optimization



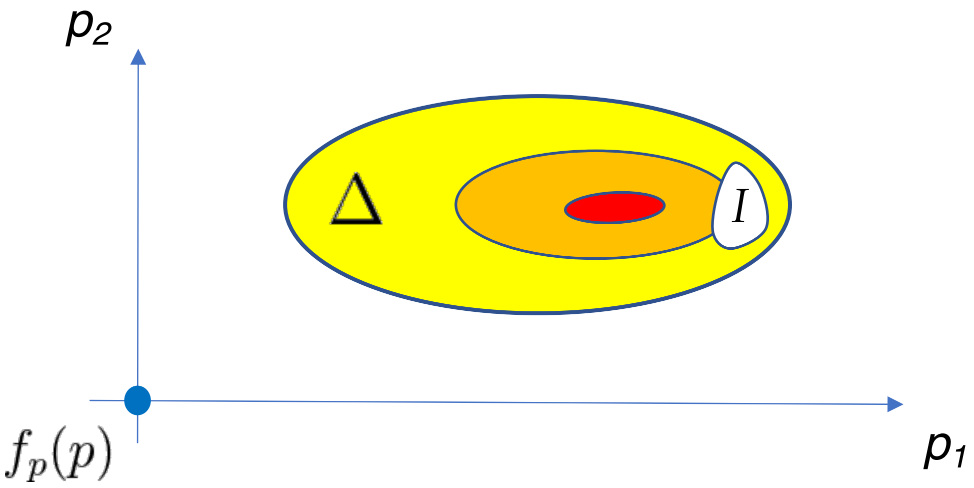


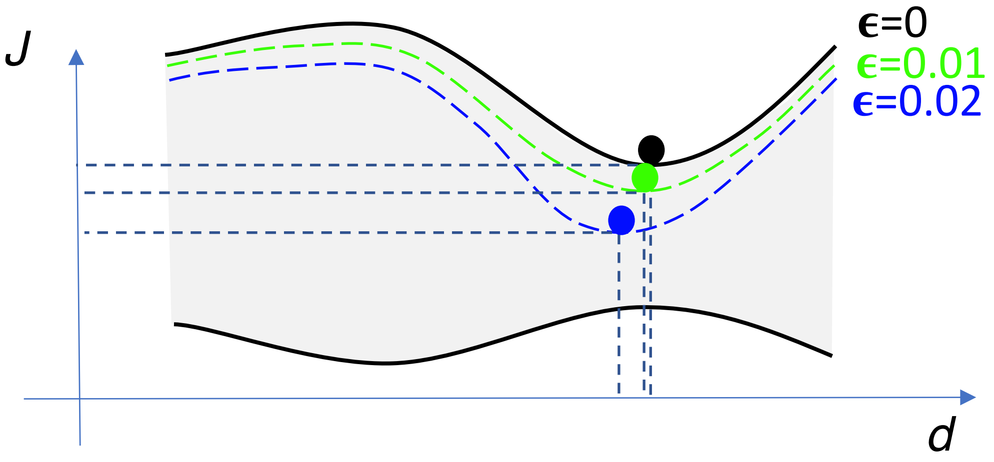
- Formulation

Risk

$$\min_{I,\, d} \left[\max_{p \in \Delta - I} J(d, p) : \mathbb{P}[I] = \epsilon \right]$$

Chance-Constrained Optimization



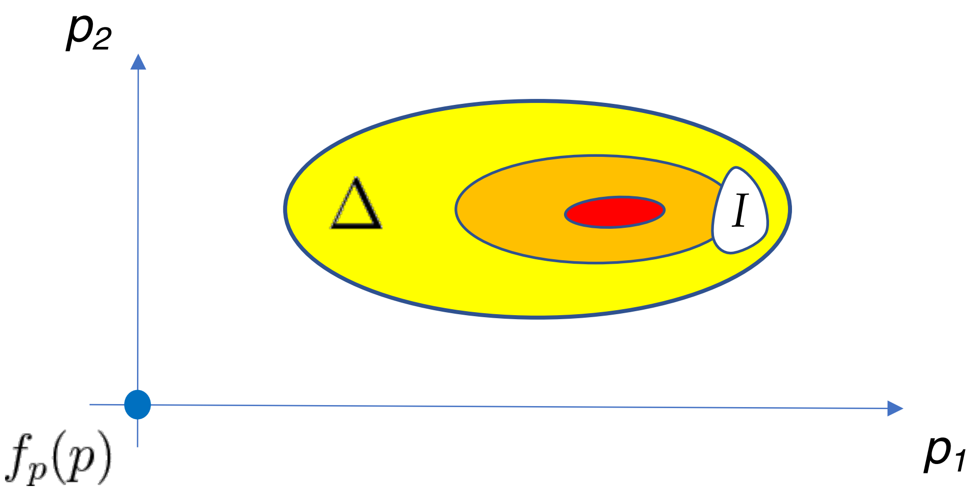


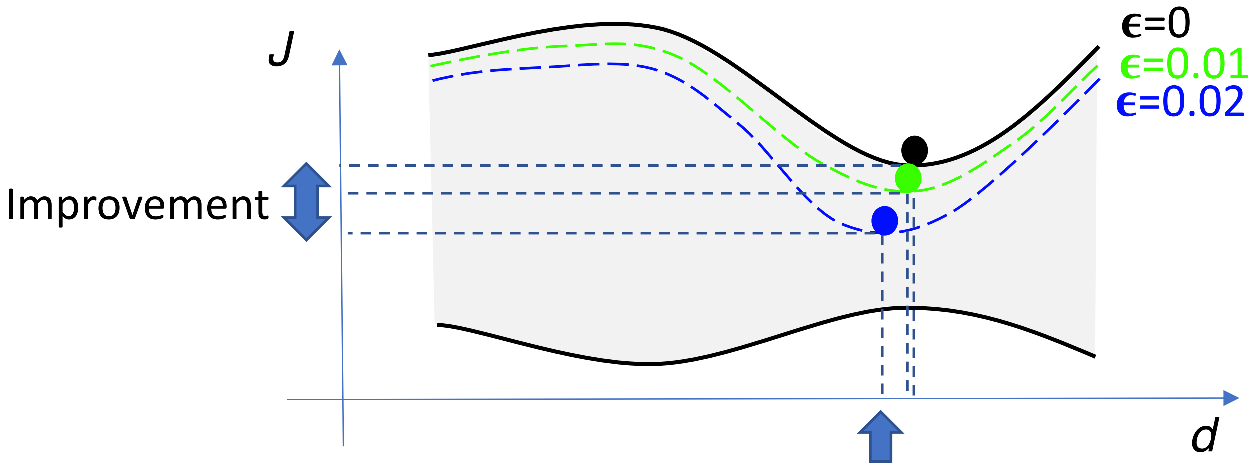
- Formulation

Risk

$$\min_{I,\,d}\left[\max_{p\in\Delta-I} J(d,p) : \mathbb{P}[I]=\epsilon\right]$$

Chance-Constrained Optimization





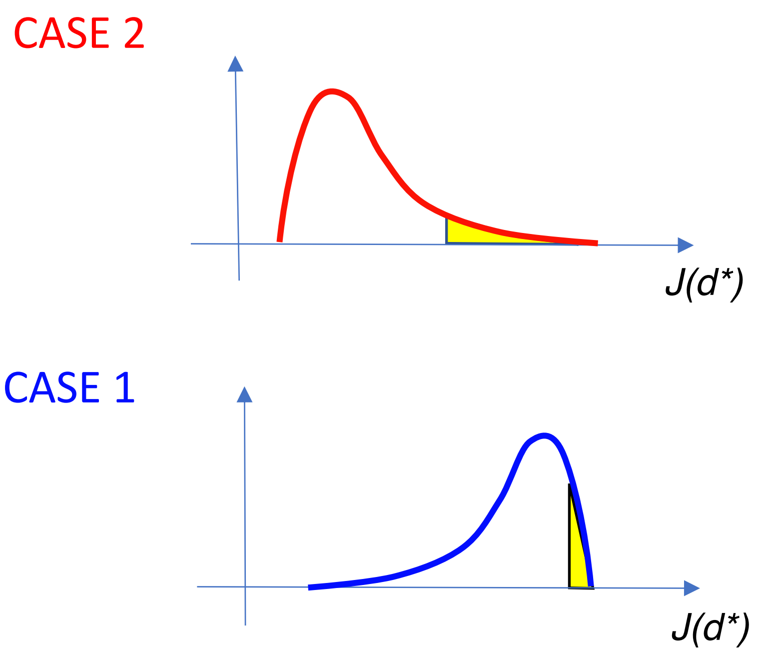
- Formulation

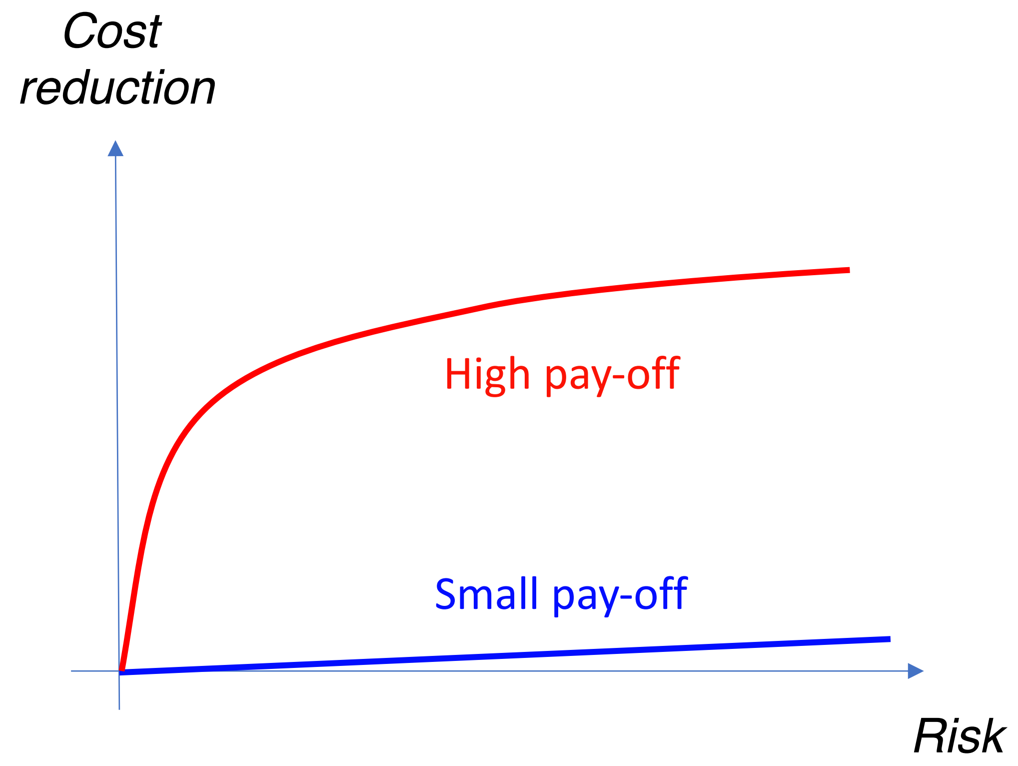
Risk

$$\min_{I,\, d} \left[\max_{p \in \Delta - I} J(d,p) : \mathbb{P}[I] = \epsilon \right]$$

Chance-Constrained Optimization

- When does Chance-Constrained Optimization (CCO) pay off?

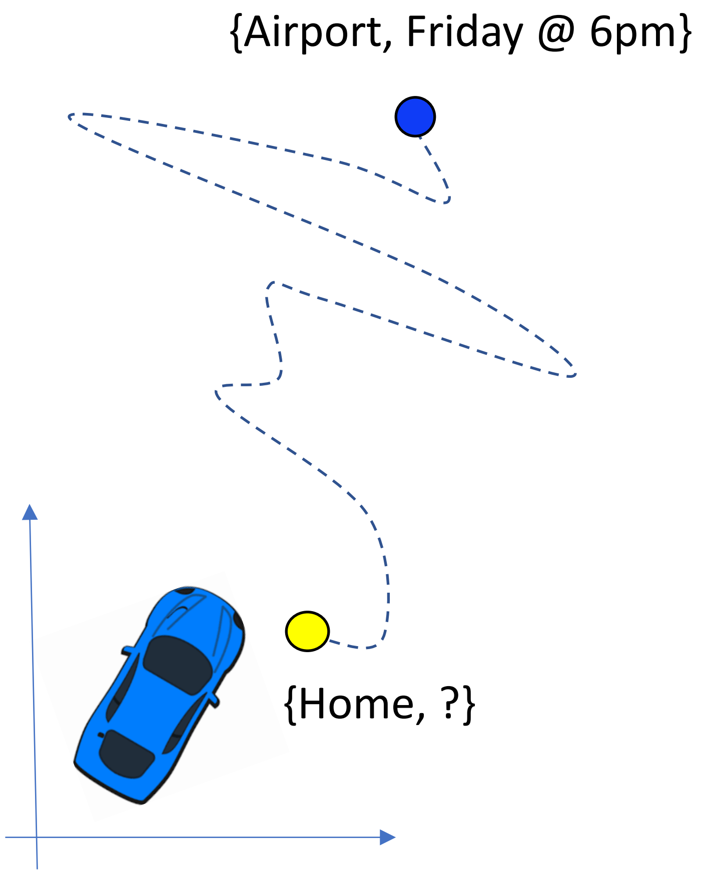




Optimization Under Uncertainty

- The computational complexity of WC is often exponential in the number of inputs and outputs whereas that of CCO is polynomial

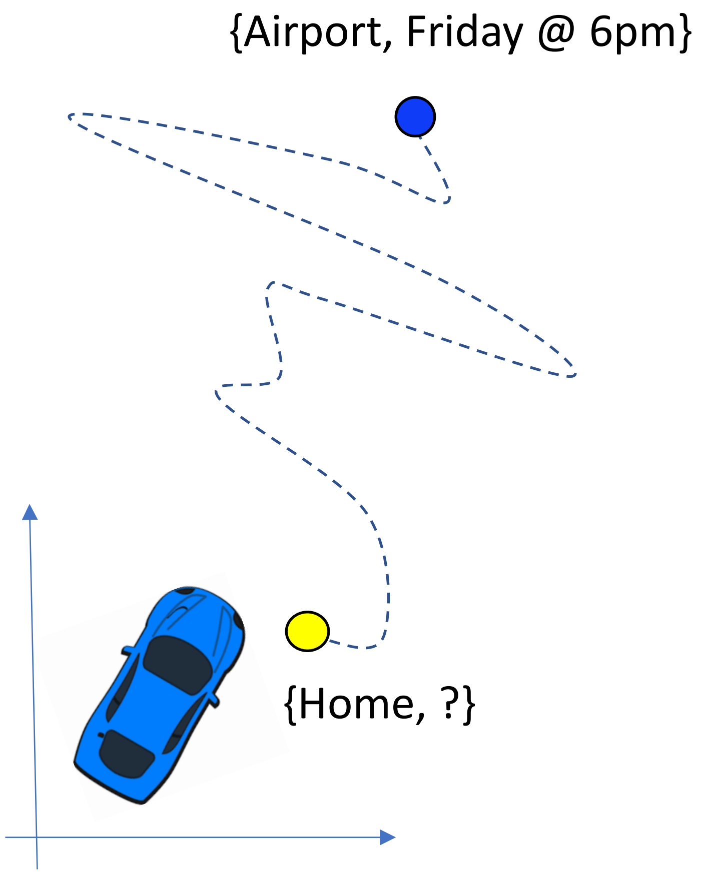
Example

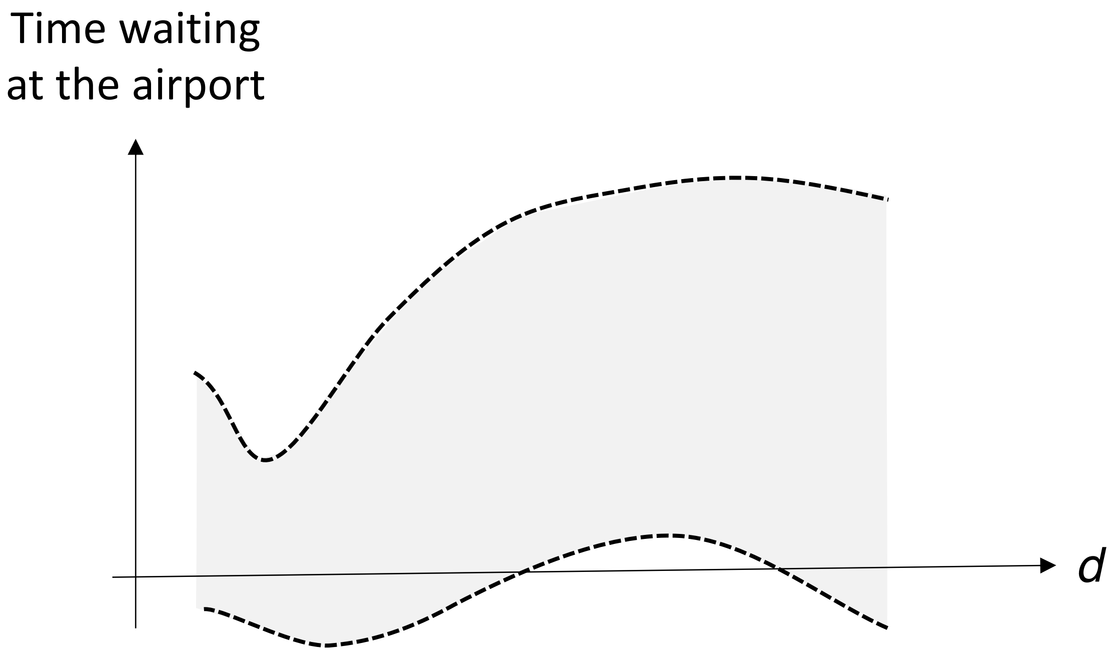


$$\min_{d}\{J(d,p) : \mathbb{P}[J(d,p) < 0] \leq \epsilon\}$$

- Cost *J*: Time waiting in the airport
- Design variable *d*: Time leaving home
- Uncertainty *p*: {traffic, road/lane closures}
- Ill-posed problem as stated

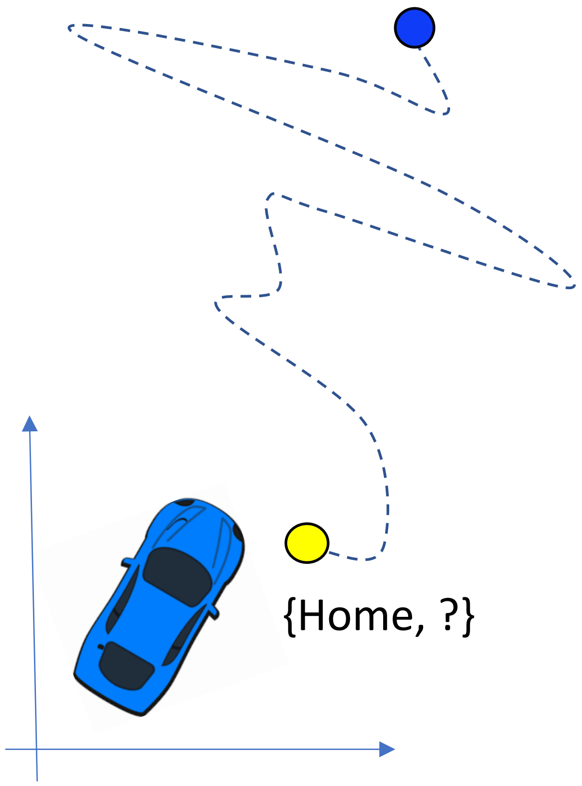
Example: WC

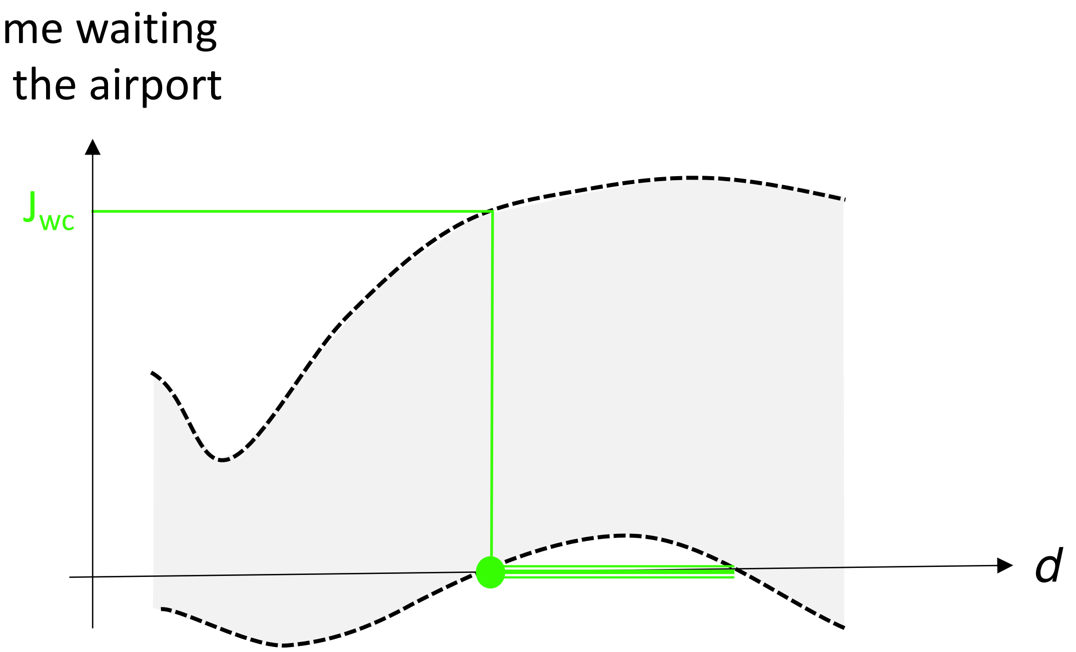




- What is the optimal worst case?
- Infinitely many options to not miss the flight

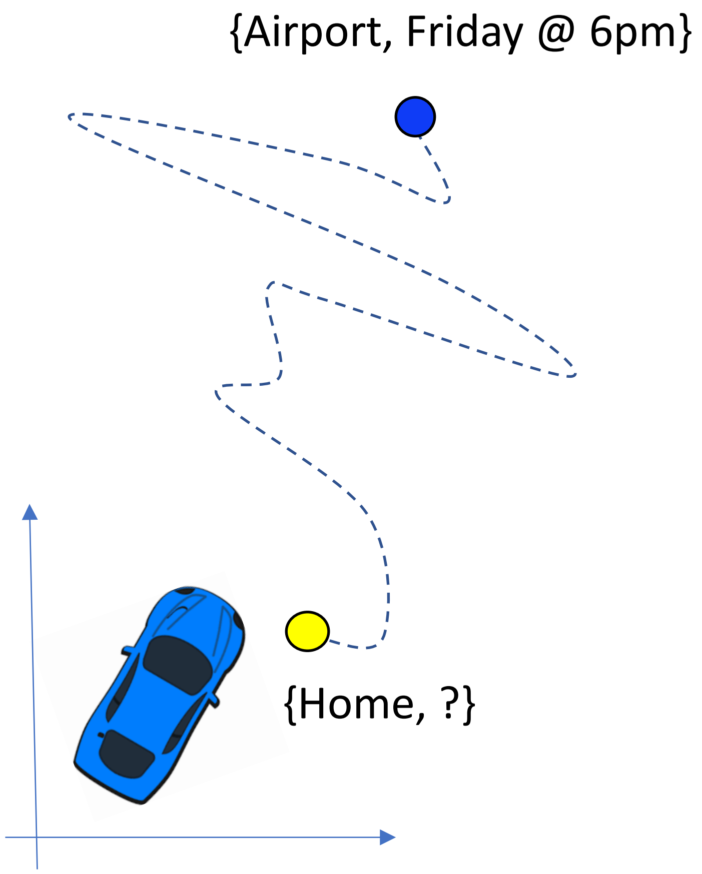
Example: WC

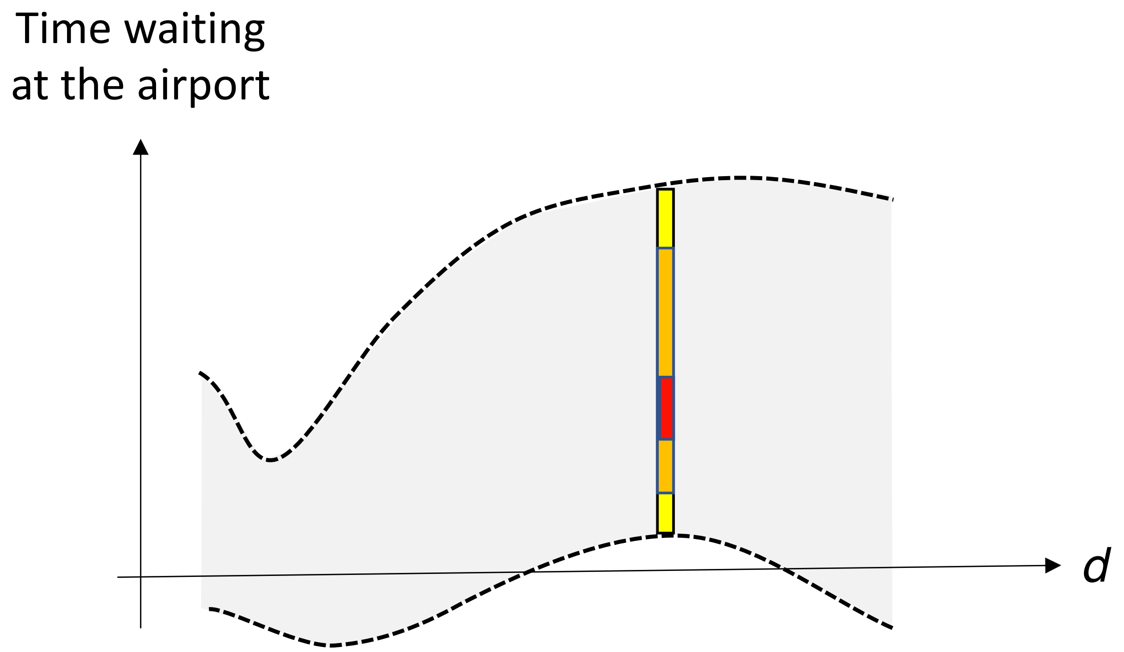




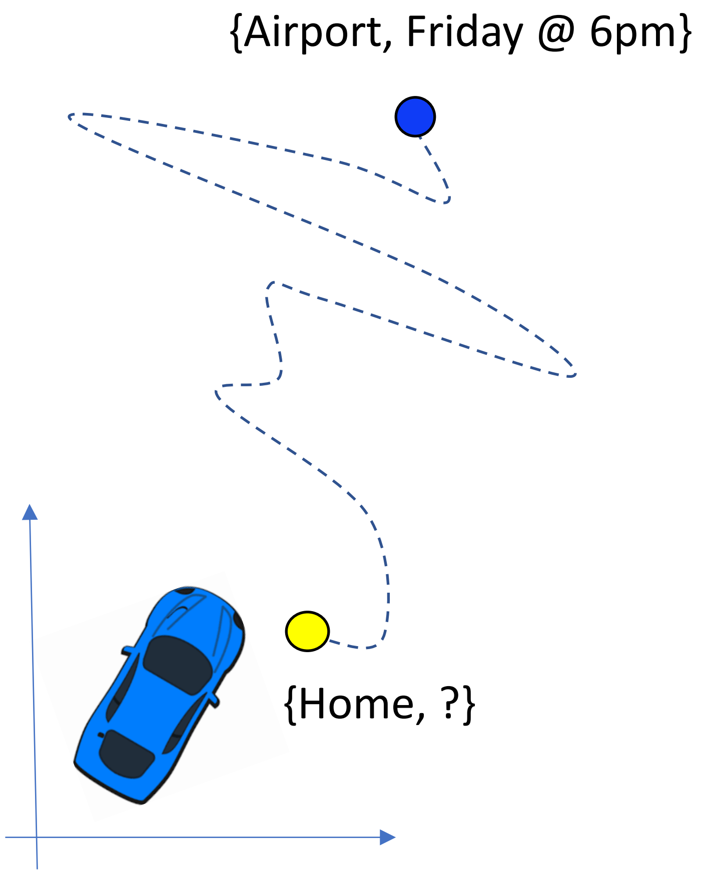
You will make the flight but might have to wait a lot

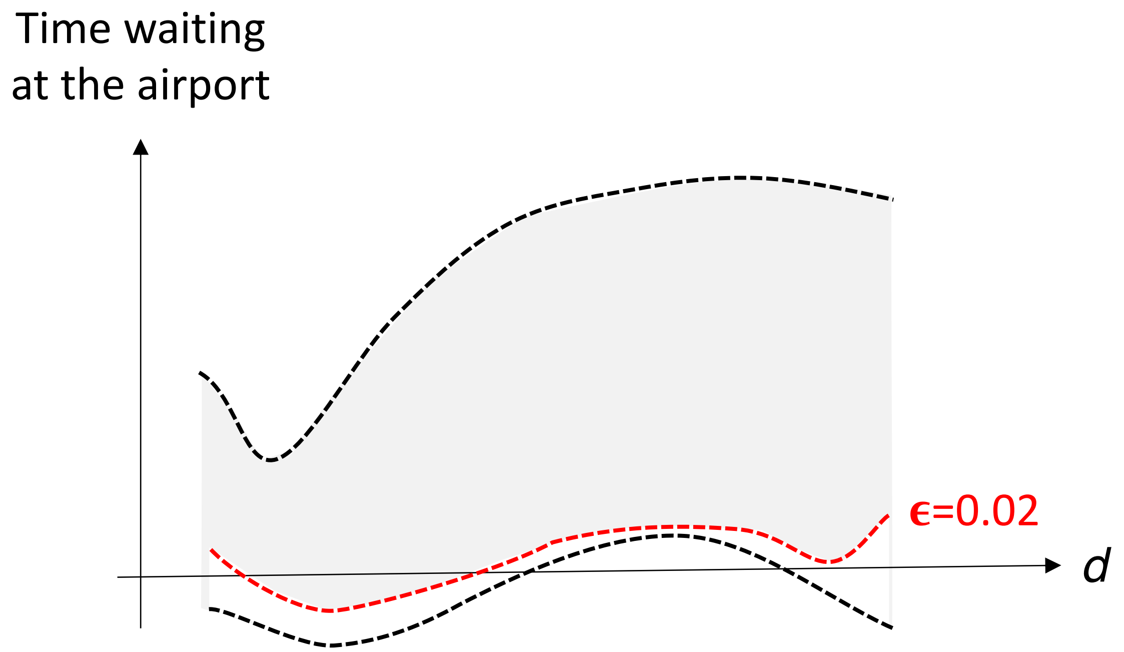
Example: WC vs. CCO





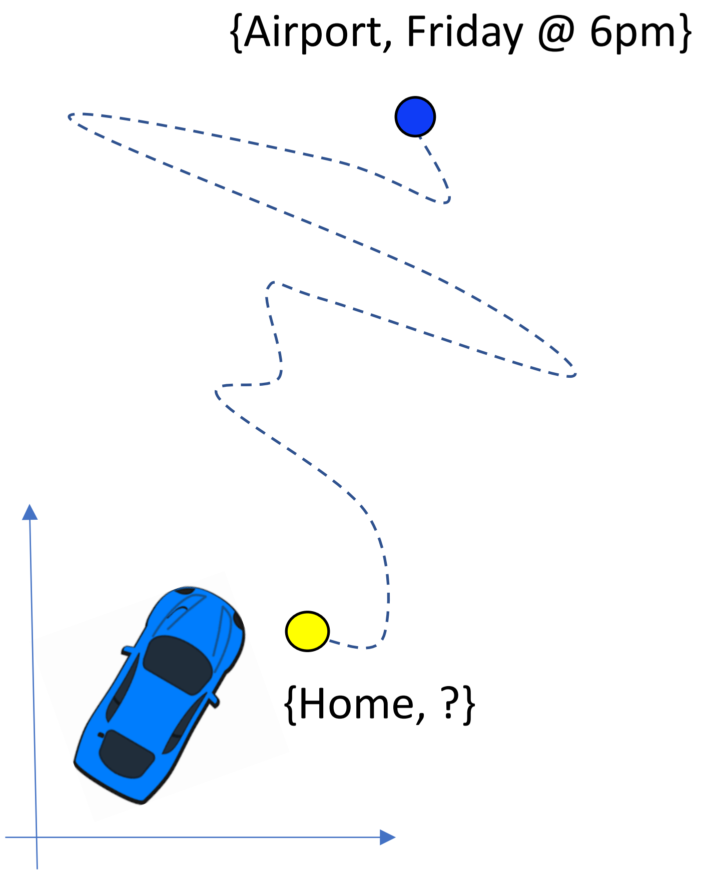
Example: WC vs. CCO

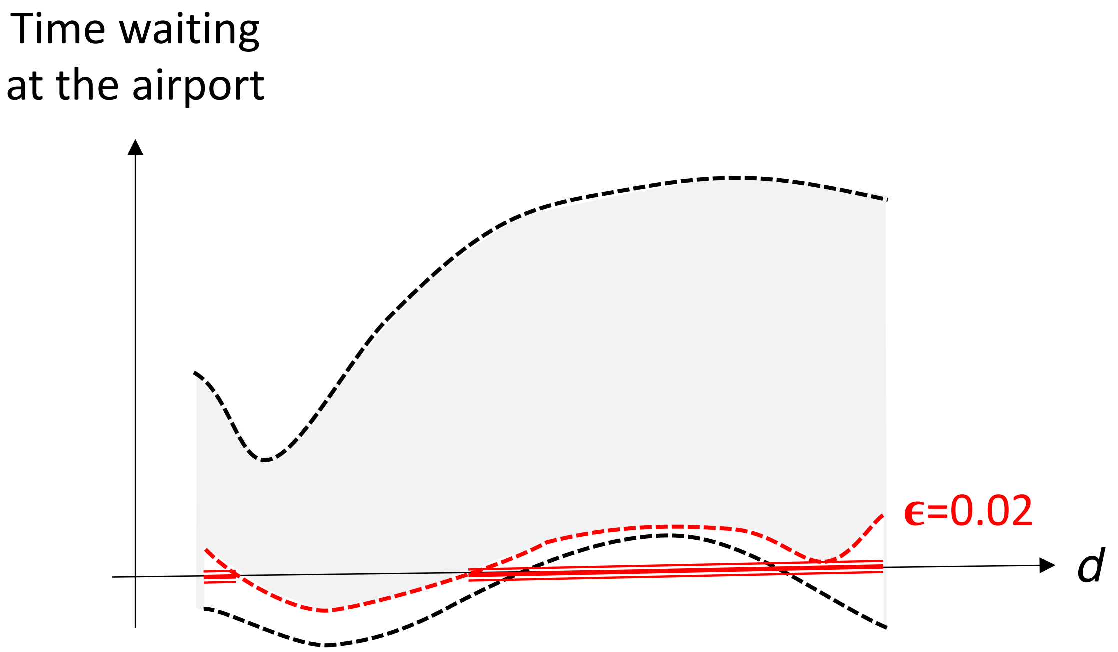




Which designs are feasible?

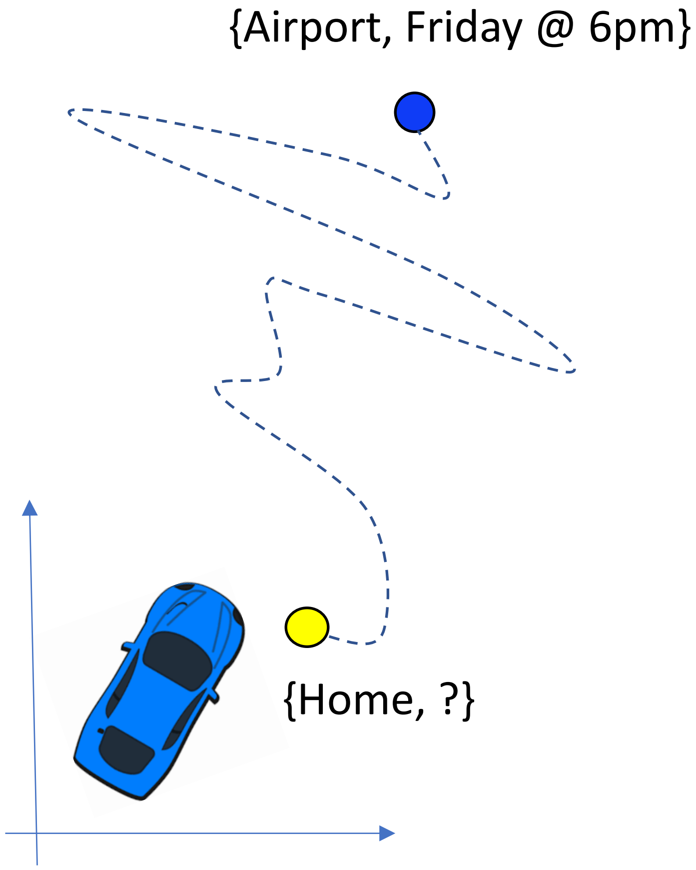
Example: WC vs. CCO

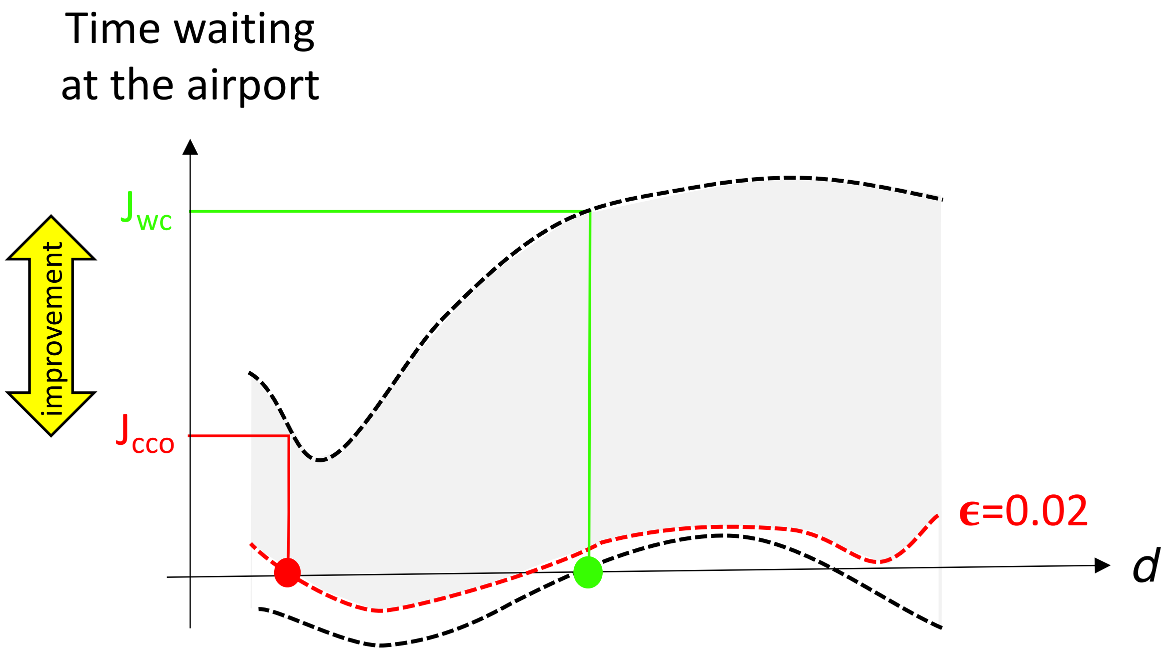




Which designs are feasible?

Example: WC vs. CCO

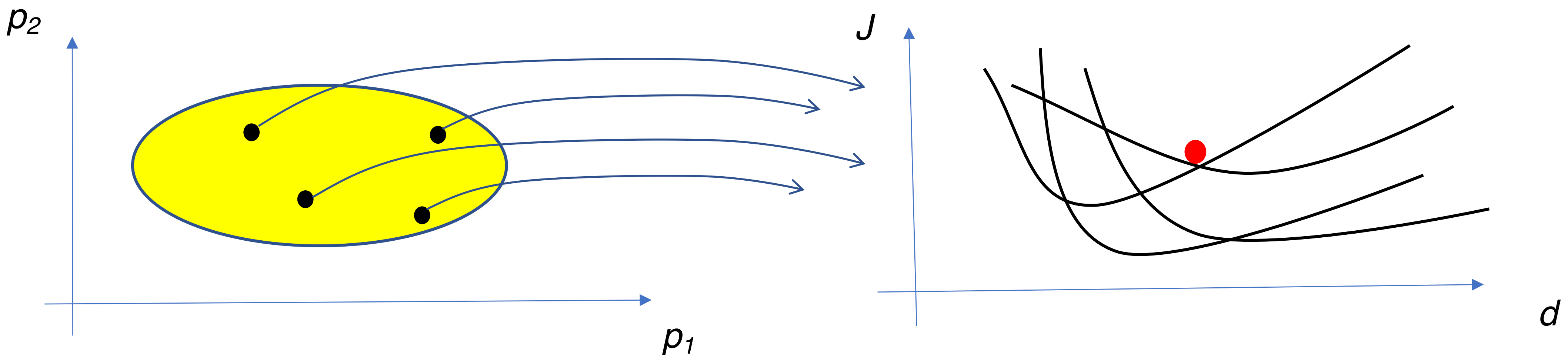




You might miss the flight with 0.02 probability
but will wait considerably less

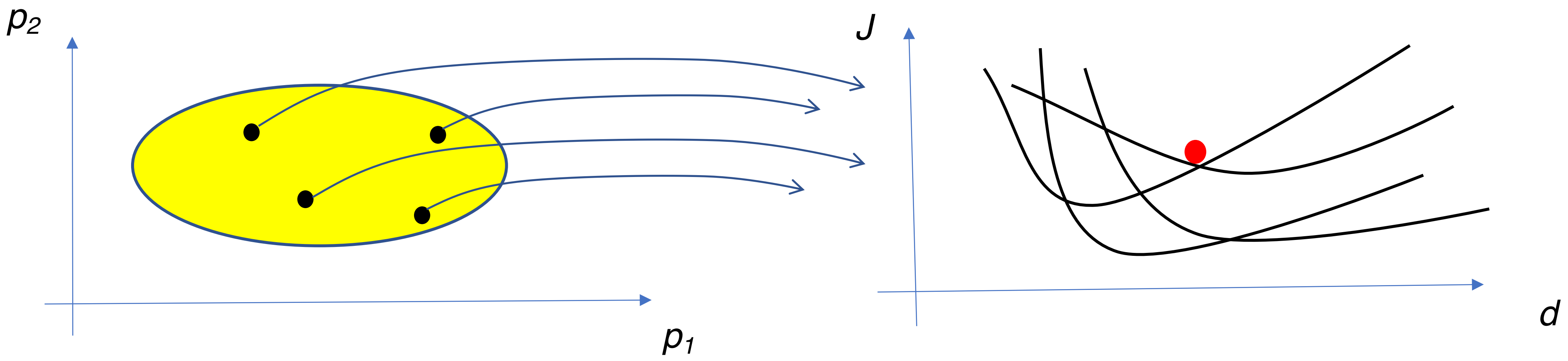
Solving WC and CCO by Scenario Optimization

Scenario Optimization: Worst-Case



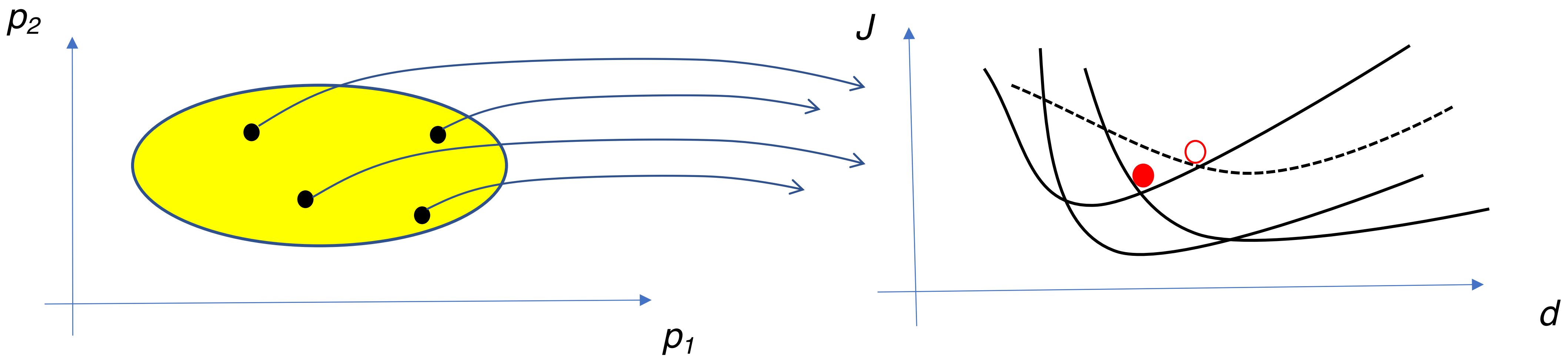
- Solve the problem for *n* scenarios

Scenario Optimization: Chance Constrained



- Solve the problem for *n* scenarios
- Loop: Identify a worst-case scenario, remove it, and solve again. Stop when $k=floor(n\epsilon)$ scenarios are removed

Scenario Optimization: Chance Constrained



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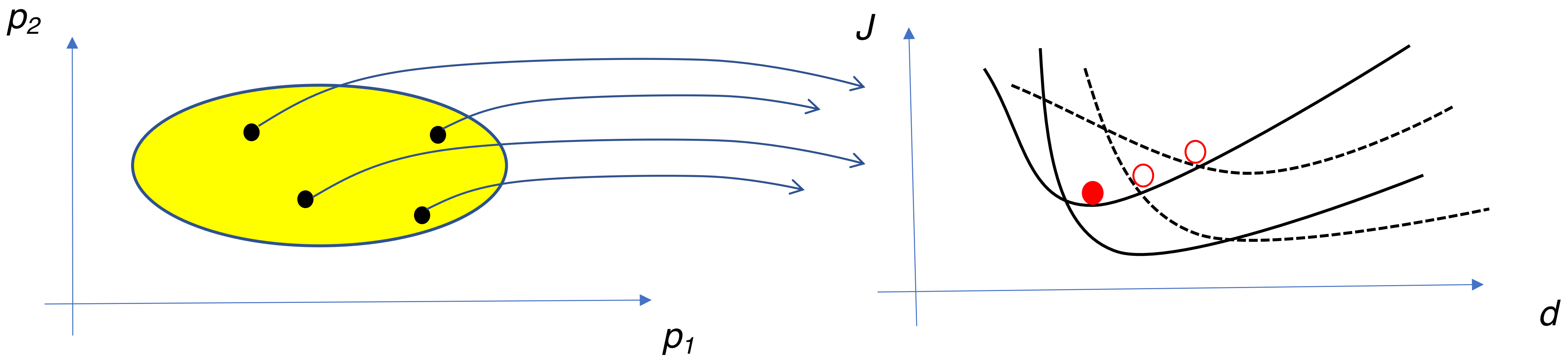
Scenario Optimization: Chance Constrained



- Solve the problem for *n* scenarios
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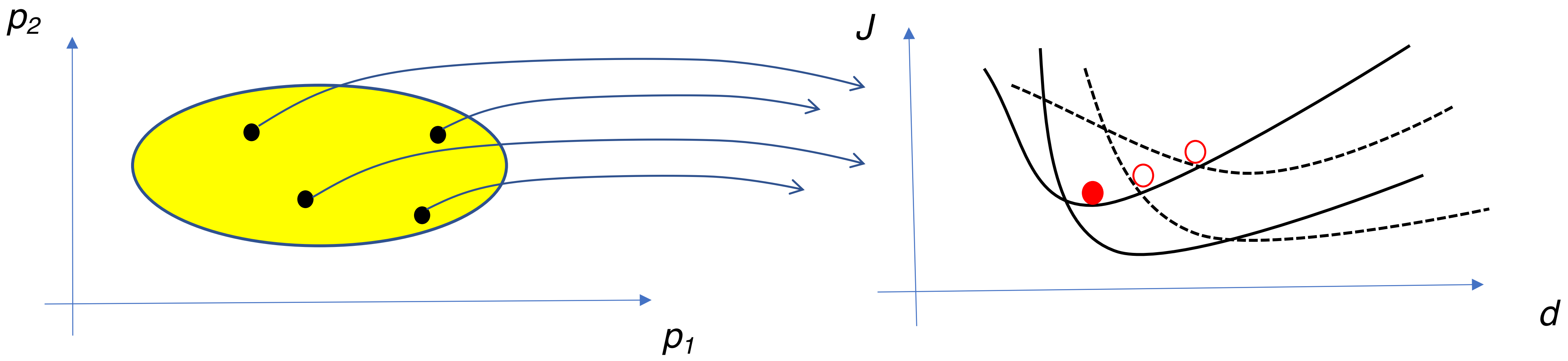
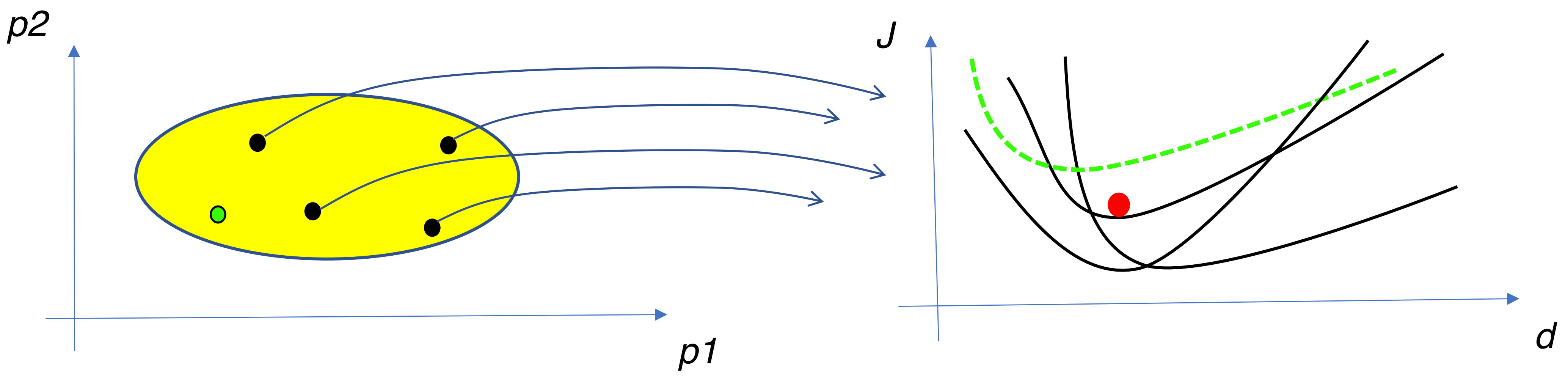
Scenario Optimization: Chance Constrained



- Arbitrary dependency of the cost/constraints on *p*

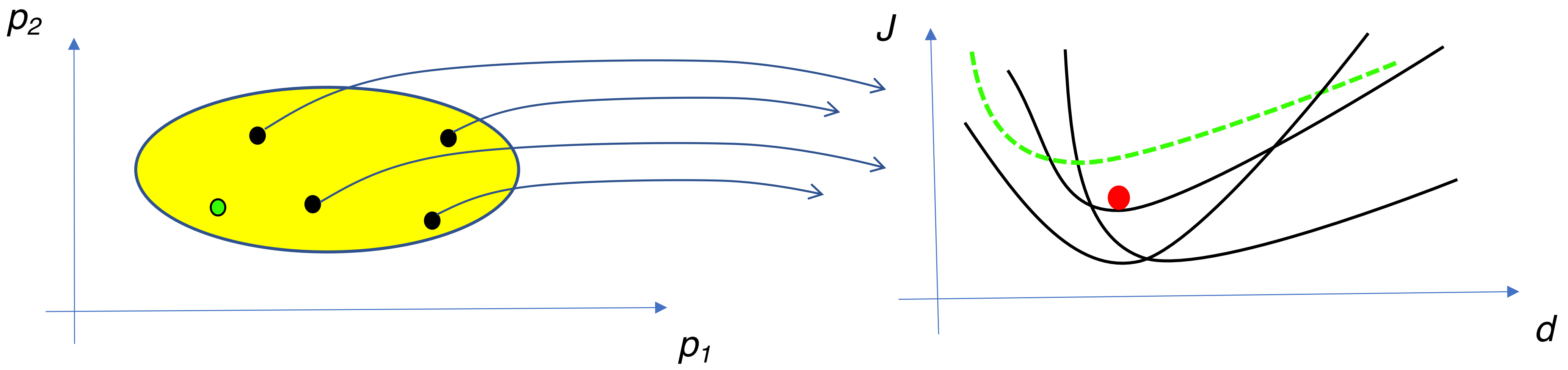
How reliable is the resulting data-based optimum
(i.e. how do the results generalize to unseen data)?

Scenario Optimization: Chance Constrained



Q: What is the probability of a future scenario increasing the optimum cost?

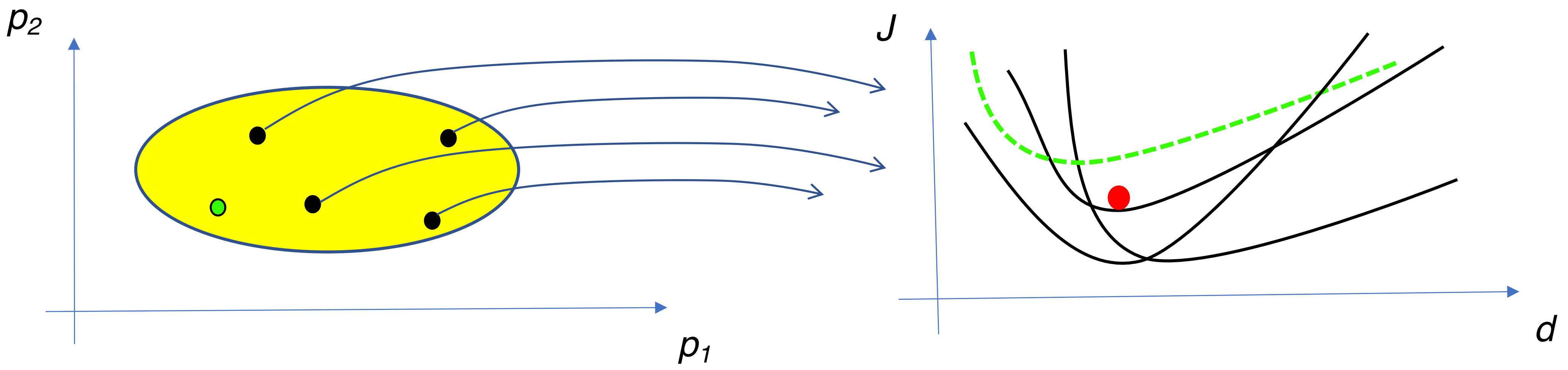
Scenario Optimization: Chance Constrained



Q: What is the probability of a future scenario increasing the optimum cost?

A1: Start by modeling the distribution of the data….

Scenario Optimization: Chance Constrained



Q: What is the probability of a future scenario increasing the optimum cost?

A1: Start by modeling the distribution of the data…

A2: Use **Scenario Theory** and avoid modeling the distribution

Scenario Theory

- Scenario theory provides a tight bound on the probability that an unseen datum will violate the requirements imposed upon a data-based optimum. When the optimization program is convex:

$P_{violation}$ < Bound (amount of data, models' complexity, number of outliers, confidence)

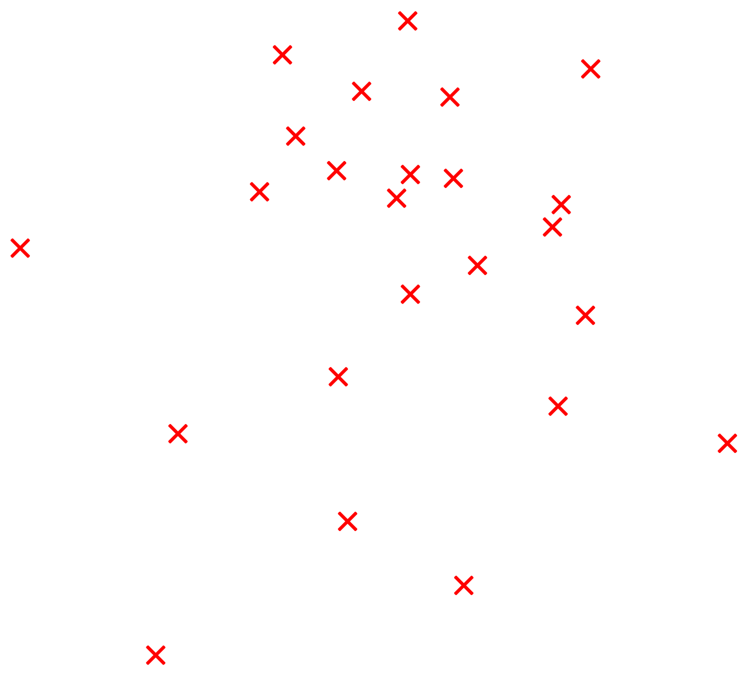
Scenario Theory

- Scenario theory provides a tight bound on the probability that an unseen datum will violate the requirements imposed upon a data-based optimum. When the optimization program is convex:

$P_{violation}$ < Bound (amount of data, models' complexity, number of outliers, confidence)

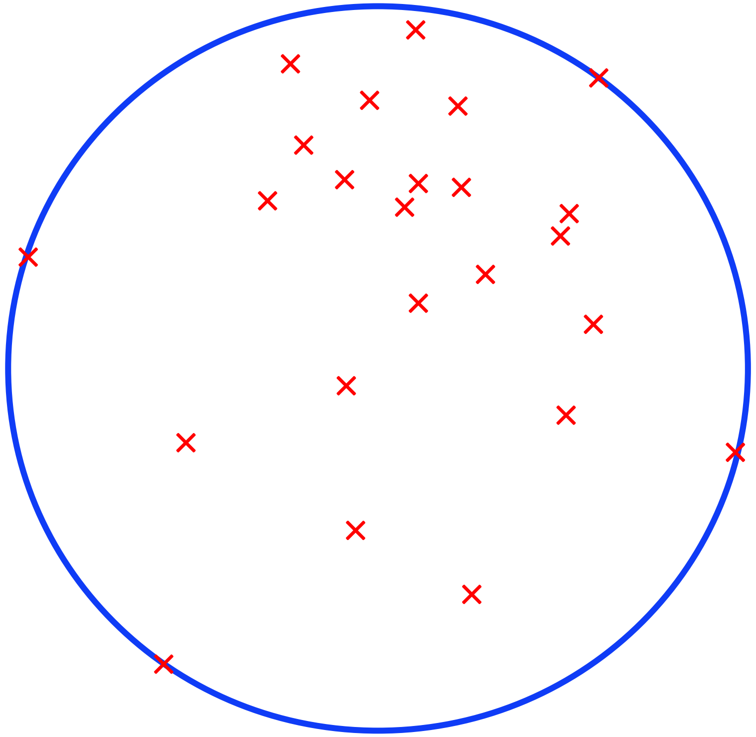
- Features
  - Bound is non-asymptotic and distribution-free
  - There is no need to model the uncertainty!!
  - Arbitrary dependency of the CCO on the uncertainty & design variable
  - Convexity in the design variables helps
  - No need to make the problem analytically tractable

Example 1: Compute Data-enclosing Set



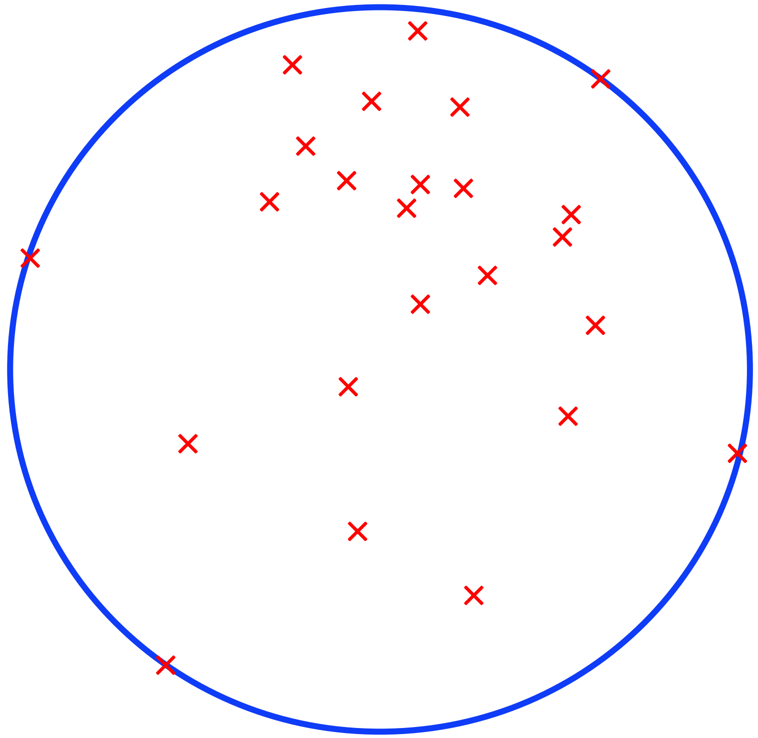
Compute the data-enclosing circle of minimum volume?

Example 1: Compute Data-enclosing Set



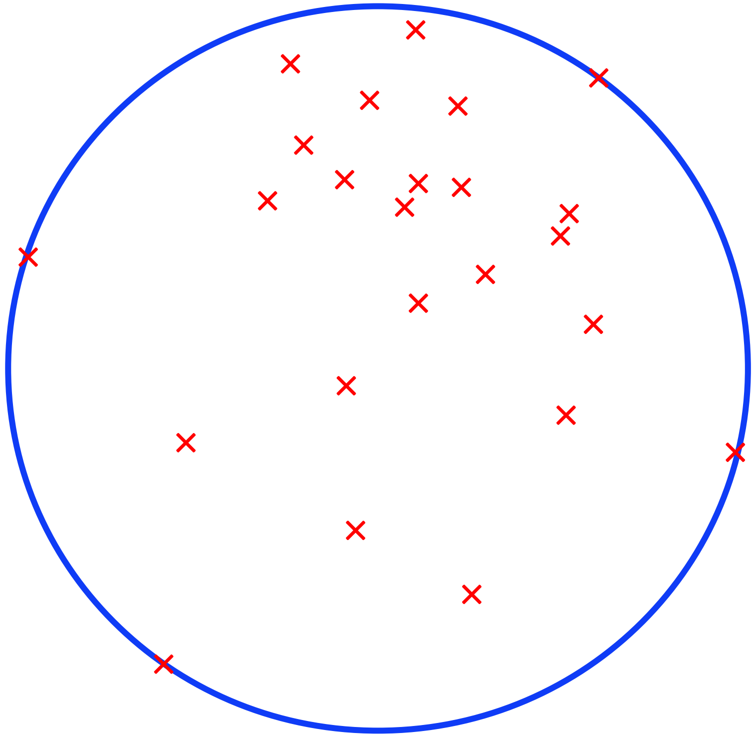
Compute the data-enclosing circle of minimum volume?

Example 1: Compute Data-enclosing Set



What is the probability of a
new datum falling outside the circle?

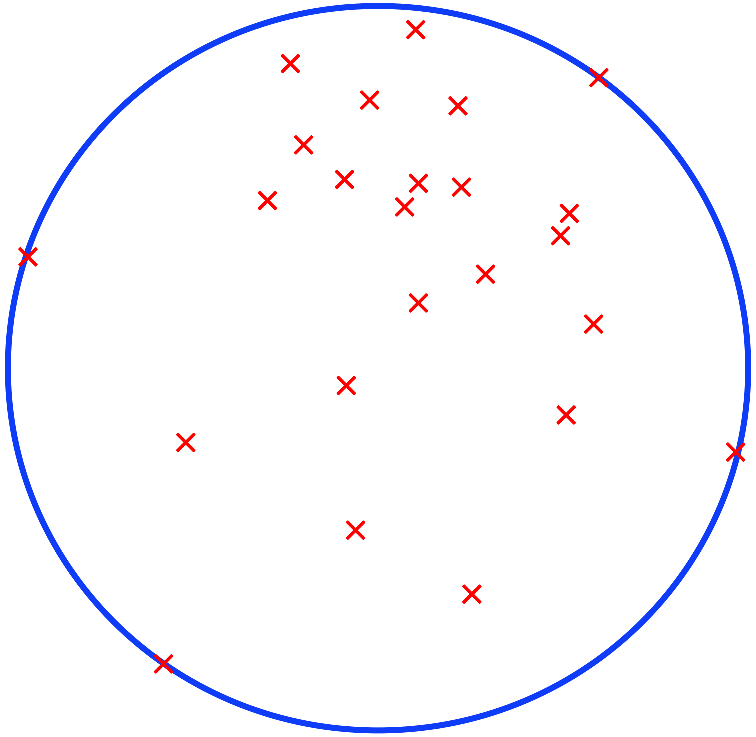
Example 1: Compute Data-enclosing Set



Bound only requires evaluating an algebraic expression

n=24, n_d=3, k=0, β=1e-3: $P[x \notin C] \leq 0.38$

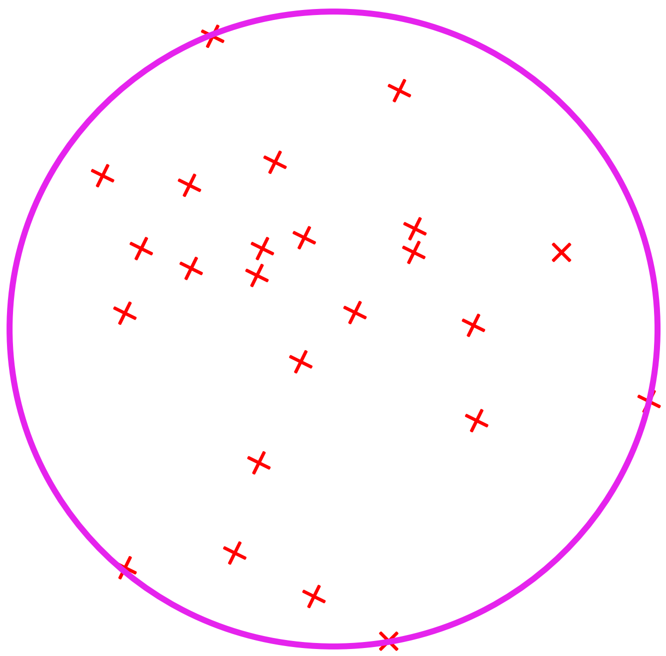
Example 1: Compute Data-enclosing Set



No need to model the distribution of the data!!

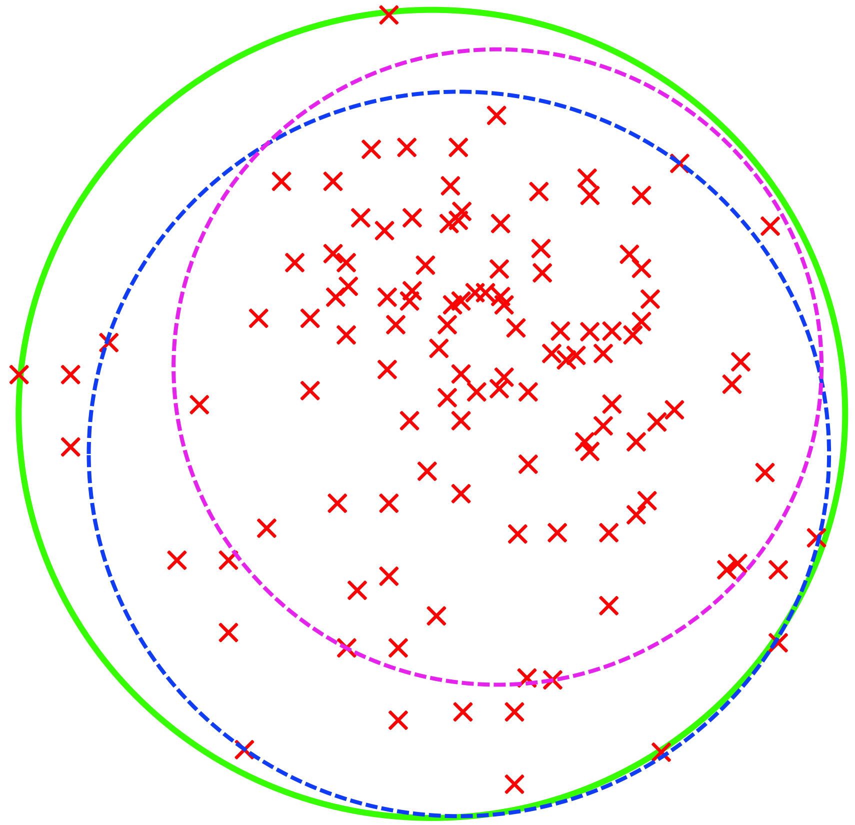
n=24, n_d=3, k=0, β=1e-3: $P[x \notin C] \leq 0.38$

Example 1: Compute Data-enclosing Set



n=24, n_d=3, k=0, β=1e-3: $P[x \notin C] \leq 0.38$

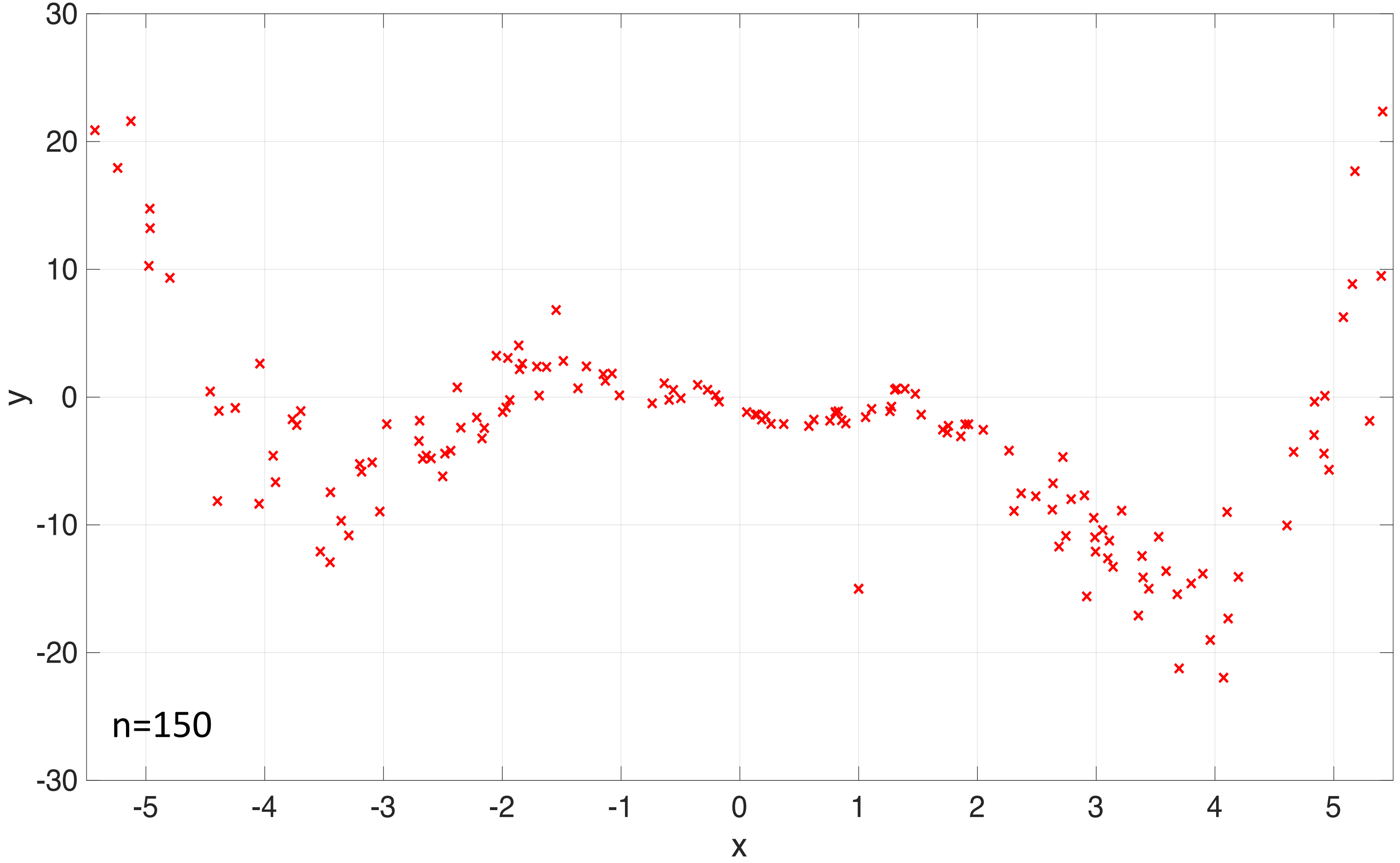
Example 1: Compute Data-enclosing Set



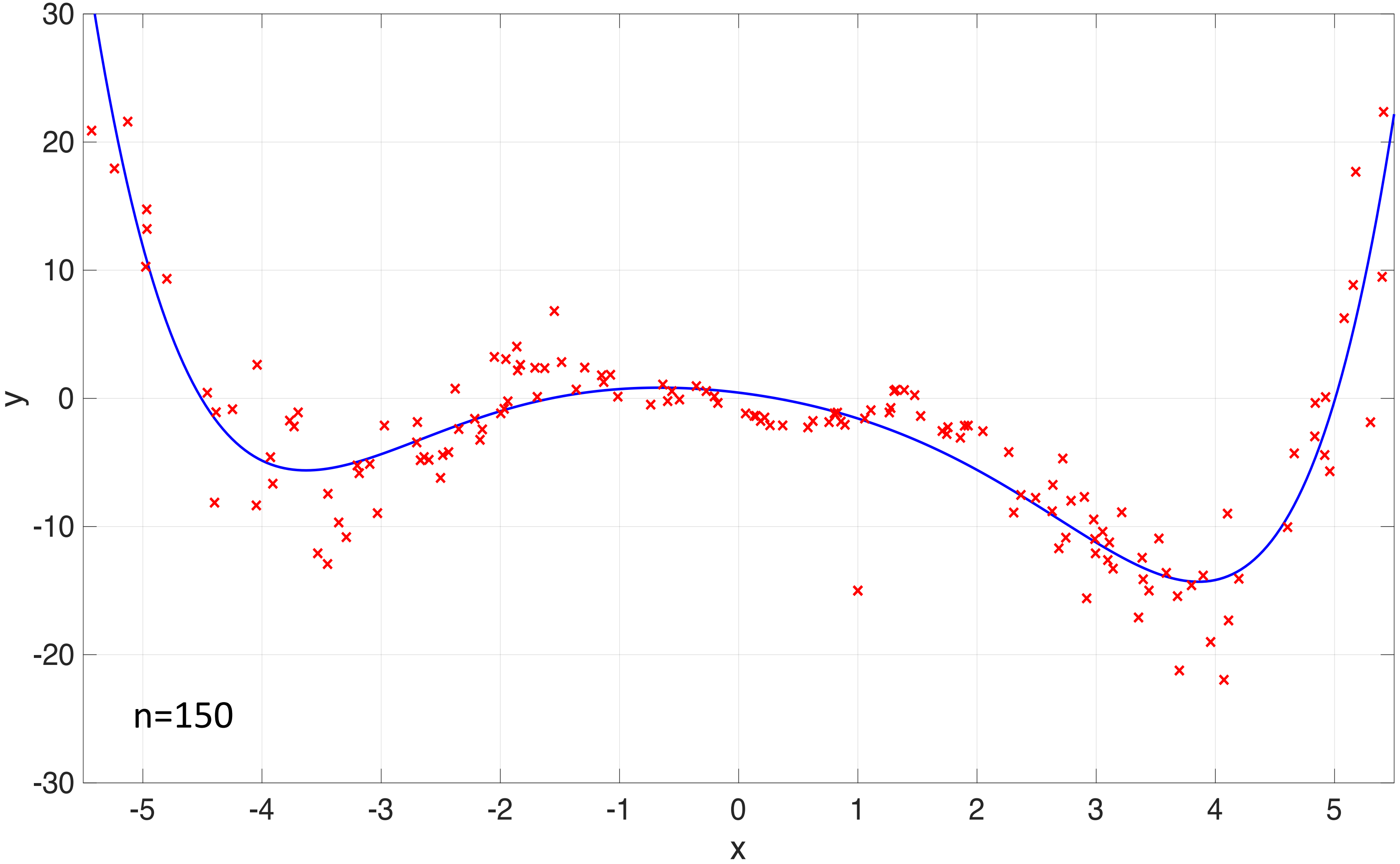
- Bound is tight
- Tighter bound can be computed by making additional calculations

n=120, n_d=3, k=0, β=1e-3: $P[x \notin C] \leq 0.129$

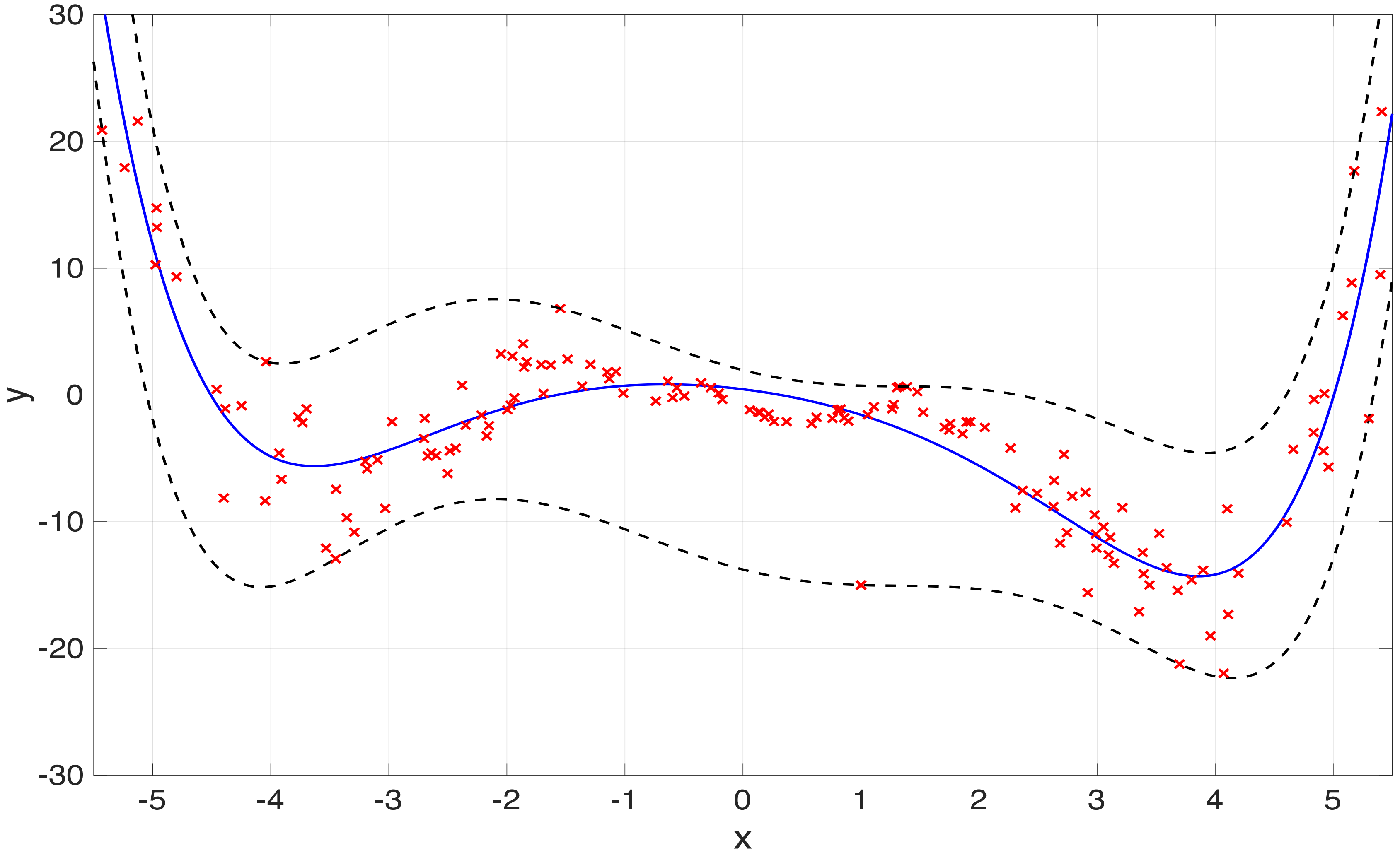
Example 2: Interval Predictor Models



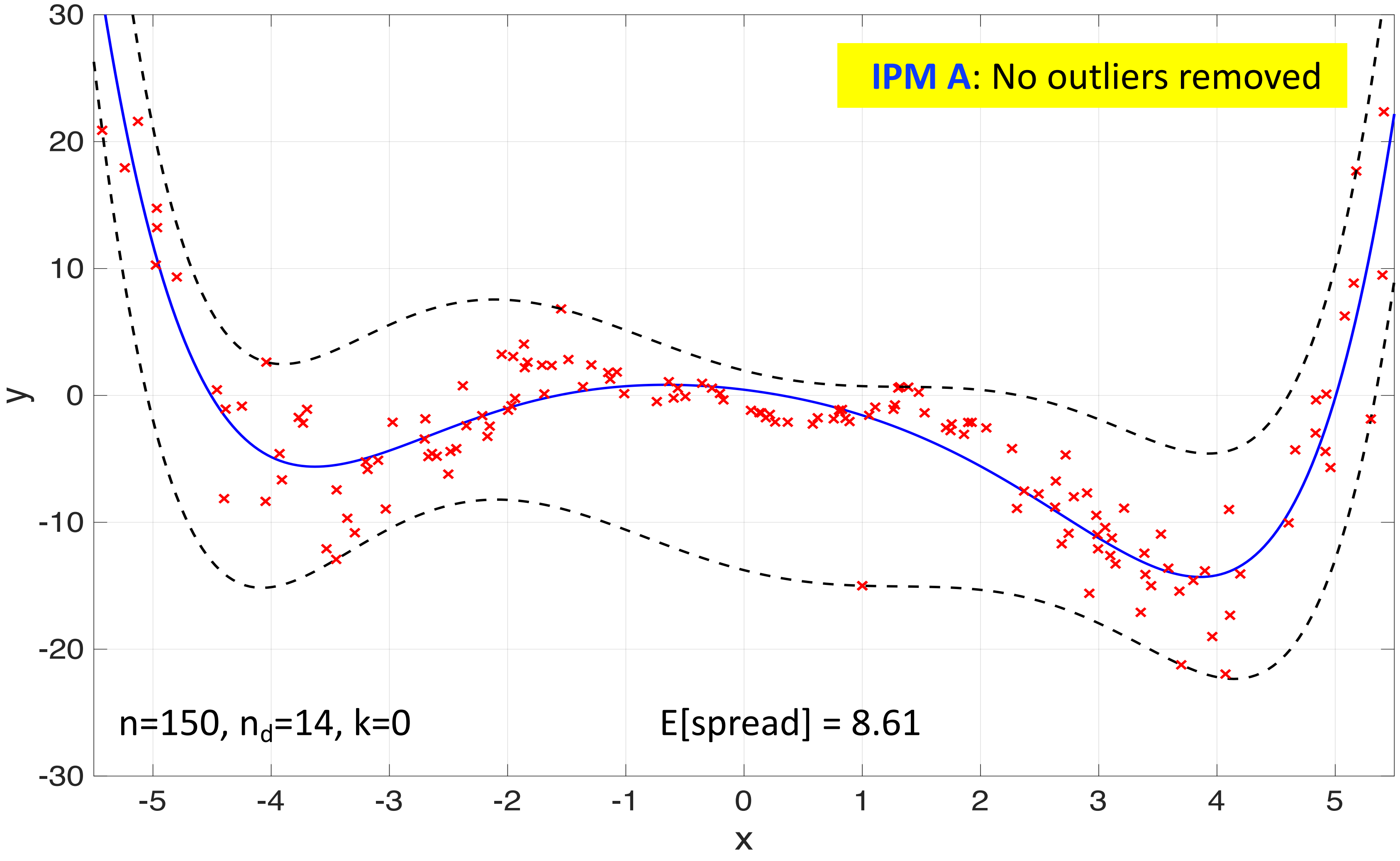
Example 2: Interval Predictor Models



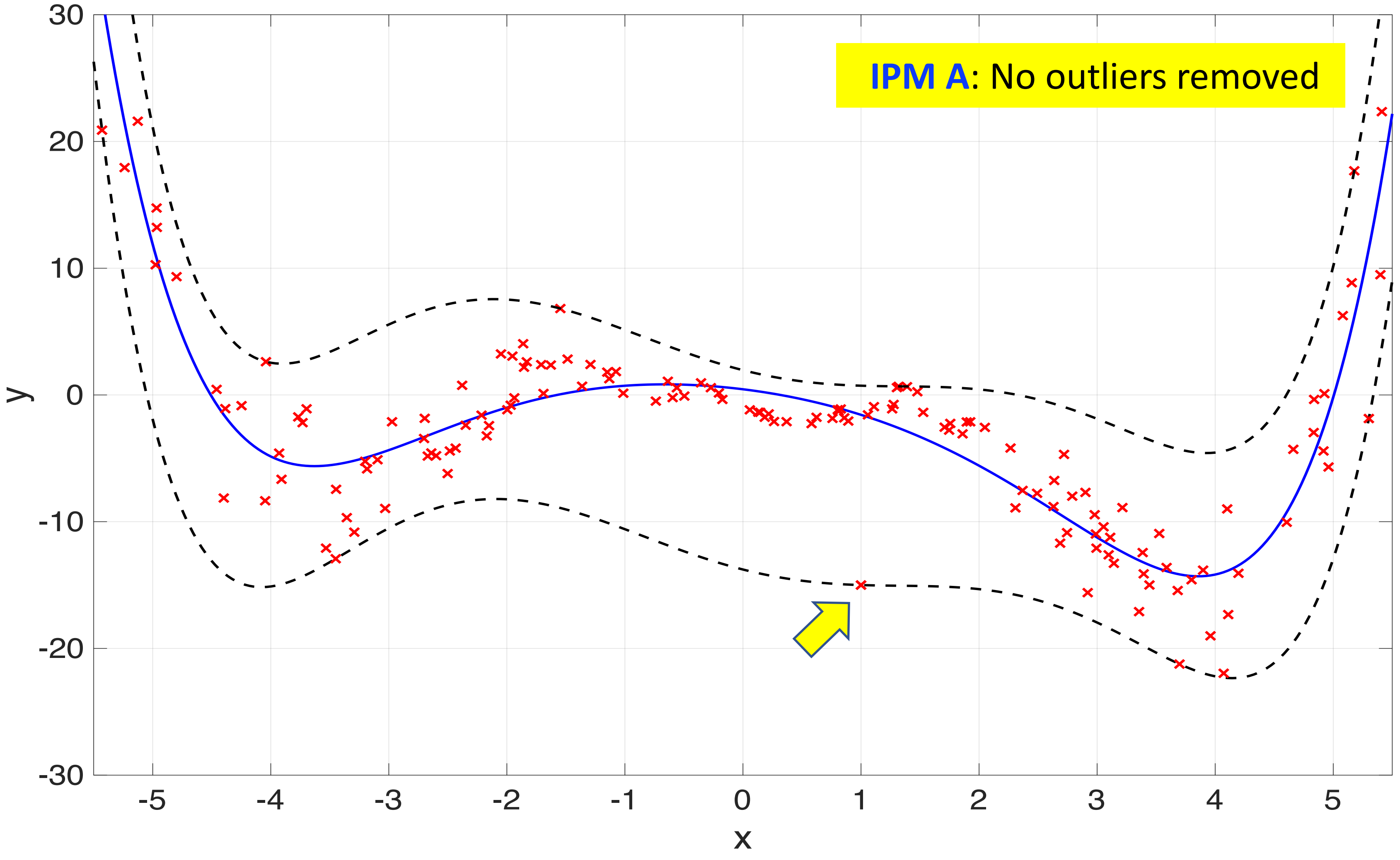
Example 2: Interval Predictor Models



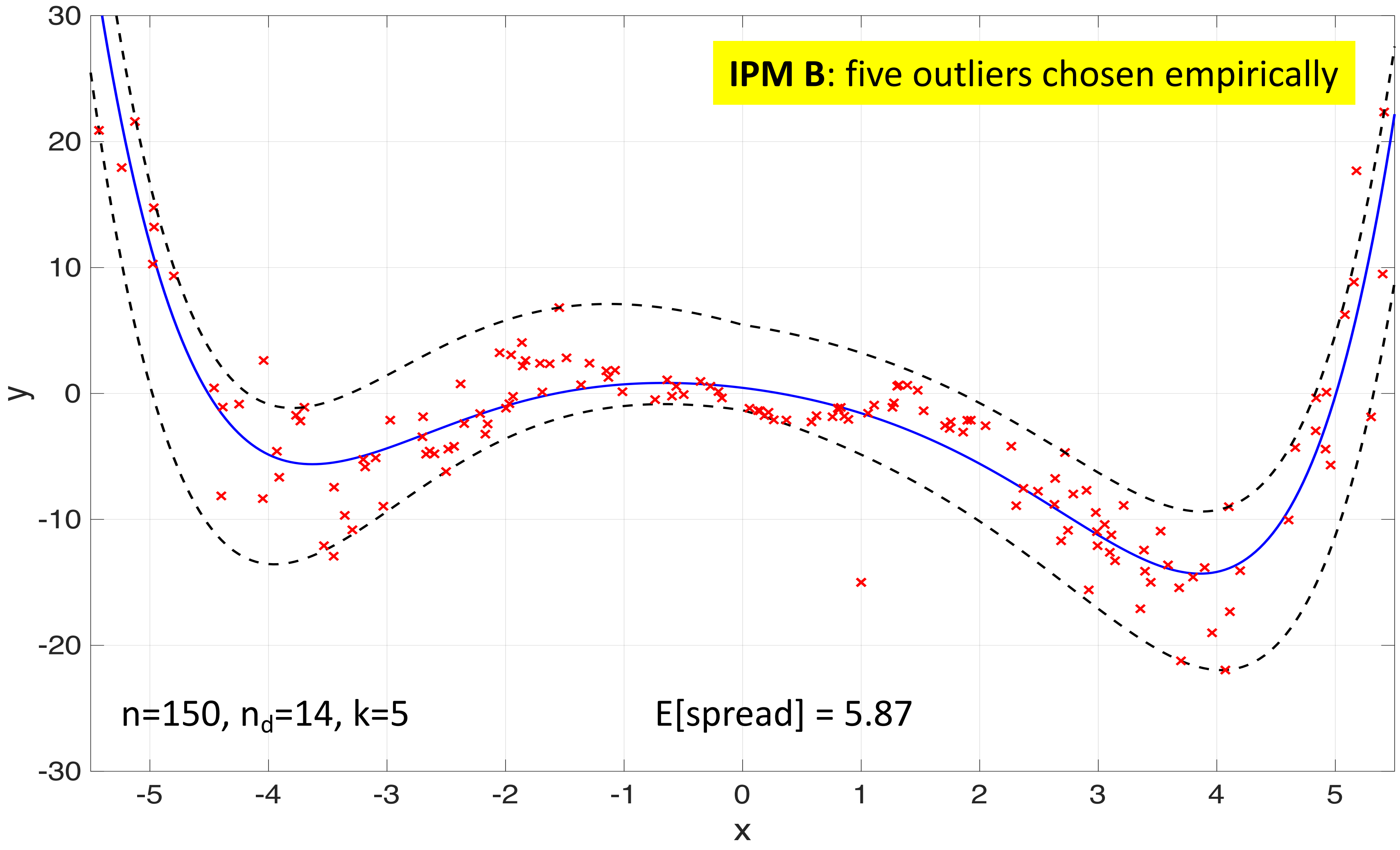
Example 2: Interval Predictor Models



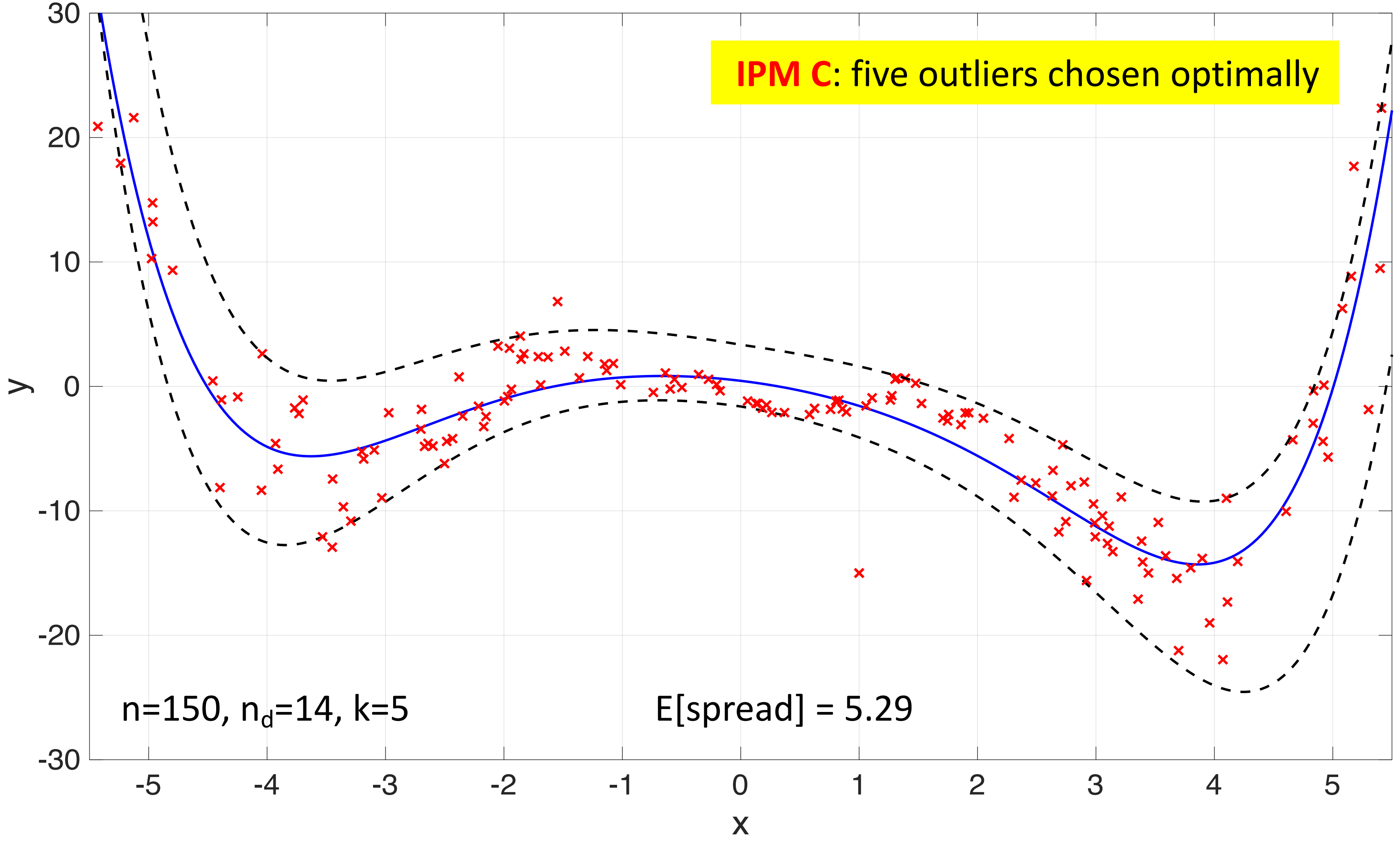
Example 2: Interval Predictor Models



Example 2: Interval Predictor Models



Example 2: Interval Predictor Models



Example 2: Interval Predictor Models Reliability

	PERFORMANCE	RISK (β=0.99)
IPM A (n=150, d=14, k=0)	8.61	ϵ<0.155
IPM B (n=150, d=14, k=5)	5.87	ϵ<0.285
IPM C (n=150, d=14, k=5)	5.29	ϵ<0.285

- A performance improvement of 32% increased the risk 0.13

- Bound is validated by using more data (we might not have it in practice)
- Dividing the data set into a training set & a validation set is not needed

[10] Crespo et al, Interval Predictor Models With a Linear Parameter Dependency, Journal of V&V and UQ, 2016

Overview

- Set deformations
- Uncertainty modeling
- Optimization under uncertainty
- Challenge problems

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- Set deformations
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Uncertainty Classification:

- Epistemic: caused by ignorance, reducible
- Aleatory: caused by randomness, irreducible

The NASA Langley UQ Challenge

Dynamic Systems & Control Branch

UQ Challenge: High-level Objectives

- To determine limitations and ranges of applicability of existing UQ and V&V methods
- To develop new methods suitable for engineering applications having a direct interest to NASA

[11] Special edition, AIAA Journal of Aerospace Information Systems 2015, Vol 12

[12] Special edition, Mechanical Systems and Signal Processing, 2021 (call for responses out)

UQ Challenge: Technical Objectives

- Perform UQ in the presence of epistemic and aleatory uncertainties given data drawn from both a subsystem and the integrated system
- Account for the impact of having limited data
- Refine uncertainty models given observations
- Rank the uncertain parameters according to the global sensitivity of key figures of merit and determine which uncertainty models to refine
- Perform optimization under uncertainty

	Organization	Title	Authors
1	Sandia National Labs	UQ methods for model calibration, validation and risk analysis	C. Safta, H. Najm, B. Debusschere, K. Sargsyan, K. Chowdhary, M. Eldred, L. Swiler
2	Los Alamos National Lab	Robust design applied to the NASA Langley UQ challenge	Kendra Van Buren, Francois Hemez
3	Ecole Centrale Paris/Supelec	Uncertainty and sensitivity analysis of the mathematical model of a…	Nicola Pedroni, Enrico Zio
4	Swiss Federal Institute of Technology	The Bayesian multilevel framework for the NASA multidisciplinary…	Joseph Nagel
5	Stinger and Ghaffarian Technologies	Subjective approach to UQ: solution to the NASA UQ challenge	Shankar Sankaraman
6	Institute for Risk and Uncertainty, U. Liverpool	An integrated and efficient numerical framework for UQ…	Edoardo Patelli, Mateo Broggi, Marco de Angelis
7	University of Florida	Prioritized information based UQ: the NASA UQ challenge…	A. Chaudhuri, G. Waycaster, N. Price, T. Matsumara, C. Park, R. Haftka
8	Vanderbilt University	Bayesian method framework for multidisciplinary UQ and optimization	Chen Liang, Snakaran Mahadevan
9	University of Southern California & Sandia National Labs	A probabilistic approach to the NASA Langley multidisciplinary UQ…	R. Ghanem, H. Meidani, E. Kalligiannaki, C. Thimmisetty, V. Keshavarzzadeh, I. Yadegaran, et. al
10	General Electric Global Research	A hybrid Bayesian solution to the NASA Langley multidisciplinary UQ	Ankur Srivastava, Arun Subramaniyan
11	Southwest Research Institute	A Bayesian probabilistic treatment of multiple uncertainty types	John McFarland, Barron Bichon, David Riha

UQ Challenge: Lessons Learned

- UQ Method verification: perform convergence studies
- Global sensitivities for different metrics are different
- Refine epistemic uncertainties in series: do full loop
- Beware of the cascading effect: results depend on the methods, numerical setup and decisions made upstream: calibration strategy, sensitivities, chosen parameters, and propagation method, etc.
- The error resulting from using a surrogate model should be quantified
- A considerable spread in the results observed
- Key UQ needs are not fully addressed by existing methods
- Further R&D needed

Conclusions 1/2

- Underestimation of the uncertainty might lead to the wrong decision
- Overestimation of the uncertainty might prevent making a decision
- State of knowledge might lead to wide predicted ranges
  - The UQ analyst does not need to apologize for such an outcome!!
- Decision maker should accept an inconclusive state
- Means to narrow the predicted uncertainty ranges
  - Refine epistemic uncertainty: resources required
  - Use optimization under uncertainty
- Good engineering must have precedence over the urgency of making a decision

Conclusions 2/2

- UQ analyst vs. decision maker
  - UQ analyst: ask for resources, know what to ask for
  - Decision maker: challenge the uncertainty model, provide resources
- UQ provides a systematic approach for uncertainty management, i.e. characterization, propagation, decomposition and design
- UQ methods are discipline independent
- UQ methods may pose stringent demands on conventional analysis and design practices
- UQ methods provide unbiased means to evaluate and compare solutions in terms of their robustness, risk and performance; regardless of the way they were derived

Strategies for Uncertainty Modeling and Optimization Under Uncertainty

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NASA Langley Research Center

GNC V&V Series