

Article

Introduction to the Physics of Ionic Conduction in Narrow Biological and Artificial Channels

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“There is plenty of room at the bottom”

Richard Feynman

Understanding, predicting and optimising the ionic selective transport properties of nanopores remains a critical challenge, both to nanotechnology and to biophysics. The last few decades have witnessed substantial progress in the analysis of such transport based on the use of a variety of experimental, numerical, and theoretical methods. Indeed, it would require several books to do full justice to the current state of the art in the field.

In some cases, the crystal structures, e.g. those of potassium, sodium, and calcium voltage-gated channels, have been discovered. This has provided invaluable insight but has also thrown into sharp relief the structure-function problem: how to predict the conduction/selectivity properties of a known structure; or conversely how to design a structure with required properties. A reliable solution to the problem offers to open new horizons in terms of pharmaceutical applications and in improved fabrication of solid-state nanopores for the sensing of molecules, desalination, DNA sequencing and the other developments that together are marking a new era in nanotechnology.

Novel numerical methods and computer hardware nowadays enable microsecond-long simulations of systems with hundreds thousands of atoms and the exploration of polarizable and quantum mechanical force fields. They provide unprecedented capabilities for reaching an understanding of experimental data and for the development of novel devices and techniques. Theoretical advances not only underlie many developments in molecular dynamics, including enhanced sampling and advanced force fields, but are also opening up new research frontiers and shedding fresh light on a number of longstanding problems such as binding probabilities, knock-on mechanisms of conduction, gating, electric double-layers, and local dielectric permittivity, just to mention a few.

It is now appreciated that selective conduction in biological ion channels has a great deal in common with that in artificial nanopores. In each case, there are intriguing analogies with the physics of quantum dots leading to the development of the theory of ionic Coulomb blockade. We dedicate this issue to the memory of our colleague Dr Igor Kh. Kaufman who developed an elegant theory of ionic Coulomb blockade in biological ion channels and suggested a simple classification of voltage-gated channels based on the charge of the selectivity filter.

At the same time it is known that specific features of ionic conduction – such as dehydration, ion-specific binding affinities, protonation, the multicomponent and competitive nature of ion

34 dynamics, the complex and adaptive structure of the ionic pathway, long range interaction, local
35 variation of the effective dielectric constant, highly correlated motion of more than one ion within a
36 narrow channel, electric double layers, and water layering at the channel entrances – add many layers
37 of complexity to the fundamental physics analogies.

38 This Special Issue brings together original high-quality papers on ionic permeation through
39 narrow water-filled channels, both biological and artificial, from some of the best researchers in the
40 field. It includes papers on the statistical physics of the process, on molecular dynamics and Brownian
41 dynamics simulations, and on relevant experiments. Although any selection of papers can only be
42 a narrow slice of the field, our aim is to emphasize the complexity and mutual interdependence of
43 recent multifaceted progress in understanding the physics of ion channels and nanopores. The time is
44 ripe for bringing together these complementary approaches, and we anticipate that they will facilitate
45 major breakthroughs, enabling the design of nanopores to meet particular technological requirements
46 as well as improvements in drug design and perhaps in personalised medicine.

47 Importantly, the Poisson-Nernst-Planck (PNP) and kinetic models remain among the principle
48 tools for predicting current through nanopores both in biology and nanotechnology . An example
49 of the classical application of the PNP model to the analysis of reversal potentials and zero-current
50 fluxes, in a system with a fixed profile of permanent charges and two mobile ion species, is provided
51 by the paper by Mofidi et al [1]. Rigorous analytic and numerical results establish the dependence of
52 the electric and chemical potential profiles on voltage and permanent charge.

53 At the same time it is well known that classical Poisson-Boltzmann (PB) and PNP theories do
54 not take account of short-range ion-ion, ion-wall, or ion-water interactions in ion channels. Efforts
55 to alleviate this deficiency of the continuous models have a long history . This stream of research is
56 represented by the interesting paper of J.-L. Liu and R.S. Eisenberg [2] featuring the development of a
57 molecular mean-field theory – a fourth-order Poisson–Nernst–Planck–Bikerman theory for modeling
58 ionic and water flows in biological ion channels. The theory treats ions and water molecules, in
59 channels of any volume or shape, with interstitial voids, polarization of water, and ion-ion and
60 ion-water correlations. It can be applied to electrolyte solutions in the nanopores of batteries and fuel
61 cells.

62 Modelling ionic currents with reduced models is extensively analysed by Boda et al [3]. They
63 show that channels are especially amenable to reduced modelling because their functions and
64 the relationships between input parameters (e.g. applied voltage, bath concentrations) and output
65 parameters (e.g. current, rectification, selectivity) are well-defined, allowing one to focus on the physics
66 of input-output relationships rather than on the atomic-scale physics inside the pore. Based on decades
67 of research, the authors propose four rules of thumb for constructing good reduced models of ion
68 channels and nanopores, focusing on the physics of input-output relationships rather than on atomic
69 structure. The proposed rules relate to the importance (1) of the axial concentration profiles, (2) of
70 the pore charges, (3) of choosing the right explicit degrees of freedom, and (4) of creating the proper
71 response functions. Examples demonstrating the application of these rules are provided. Further
72 improvements in predicting the capabilities of reduced models can be achieved by incorporating into
73 the solution of the PNP model the potential of the mean force obtained from MD simulations. The
74 performance of two such methods is examined by A. Pohorille and M. A. Wilson [4] using stochastic
75 simulations. These methods require neither knowledge of the diffusivity nor simulations at multiple
76 voltages, which greatly reduces the computational effort needed to probe the electrophysiology of ion
77 channels. They can be used to determine the free energy profiles from either forward or backward
78 one-sided properties of ions in the channel, such as ion fluxes, density profiles, committor probabilities,
79 or from their two-sided combination. By generating large sets of stochastic trajectories, which are
80 individually designed to mimic the molecular dynamics crossing statistics of models for channels of
81 trichotoxin, p7 from hepatitis C and a bacterial homolog of pentameric ligand-gated ion channel, GLIC,
82 the authors found that the free energy profiles obtained from the generated trajectories corresponding
83 to molecular dynamics (MD) simulations.

84 There follow two papers reporting MD simulations of ion currents in biological and artificial
85 channels. First, S.M. Cosseddu et al [5] present an extended MD-based analysis of ion motion within
86 the KcsA channel. They reveal complicated patterns of potassium currents that are governed by
87 the structural variability of the selectivity filter. They show that ion motion involves the complex
88 dynamics of a strongly-correlated network of residues and water molecules. Intriguing features of
89 self-organisation and readjustment of the network are analysed statistically and discussed in detail.

90 Secondly, we note that ionic transport in nano- to subnano-scale pores is highly dependent on
91 translocation barriers and potential wells. These features in the free-energy landscape are primarily
92 the result of ion dehydration and electrostatic interactions. For pores in atomically thin membranes,
93 the ionic dynamics both inside and outside the geometrical volume of the pore can be critical in
94 determining its transport properties. S. Sahu and M. Zwolak [6] examine regimes of transport that are
95 highly sensitive to pore size due to the interplay of dehydration and interaction with pore charge, where
96 picometer changes in the size, e.g. due to a minute strain, can lead to a large change in conductance.

97 We have already remarked upon the crucial importance of water, the electric double-layer,
98 water-layering, polarisation, and the resultant changes of local dielectric permittivity at the entrances
99 of nanopores. Another approach to this problem is illustrated in the paper by T.-L. Horng [7]. Starting
100 from the classical Helmholtz free energy functional for an electrolyte, including the solvation energies
101 for anions and cations, the author follows the Bikerman modification by adding an entropy term to the
102 functional, and he then extends the Bikerman approach by introducing ion-size-specific corrections to
103 the theory.

104 The approach based on density functional theory (DFT), which works well near charged walls
105 and in bulk electrolytes, can be extended to the analysis of the orientational ordering of water dipoles
106 in membrane nanotubes. M. Drab et al [8] analyse water ordering in nanotubes by minimizing the
107 corresponding Helmholtz free energy functional, also including the orientational entropy contribution
108 of water dipoles, and deriving the modified Langevin Poisson-Boltzmann (MLPB) model of the
109 electric double-layer. The MLPB equation is solved in cylindrical coordinates to determine the spatial
110 dependences of the electric potential, relative permittivity and average orientations of water dipoles
111 within charged tubes of different radii. Results show that, for tubes of a large radii, the macroscopic
112 (net) volume charge density of cations and counterions is zero on the geometrical axis. This is attributed
113 to effective electrolyte charge screening in the vicinity of the charged inner surface of the tube. For
114 tubes of small radii, the screening region extends into the whole inner space of the tube, leading to
115 non-zero net volume charge density and non-zero orientational ordering of water dipoles near the axis.

116 The DFT results mentioned above are examples of statistical physics yielding insight into the
117 function of ion channels and nanopores. This theme is continued and extended, first by Gibby et
118 al [9] who apply their recent derivation of an effective grand canonical ensemble and linear response
119 theory of ion channels to analyse the conduction of the bacterial NaChBac selectivity filter. The authors
120 compare their theory to experimental current-voltage and current-concentration dependences for
121 a single channel and for a whole cell. They find that the statistical theory in the linear response
122 regime predicts correctly many important properties of the NaChBac filter including the concentration
123 dependence of the reversal potential and the current-voltage relations. They also show that the
124 theoretical results are consistent with MD simulations of the filter population at each binding site.

125 Secondly, analysis of quantum mechanical effects in ion channels is another important direction,
126 supported by the extended capabilities of modern quantum mechanics/molecular mechanics
127 simulations. In this respect, interesting perspectives are opened by mapping the statistical mechanics
128 of ion channels onto an effective quantum mechanics. Such investigations are reviewed by T. Gulden
129 and A. Kamenev [10], who study the dynamics and thermodynamics of ion channels, considered as
130 effective 1D Coulomb systems whose statistical mechanics is dominated by entropic effects that may
131 be taken accurately into account by mapping onto an effective quantum mechanics. The corresponding
132 semiclassical calculations for non-Hermitian Hamiltonians are conducted by applying tools from

133 algebraic topology. The relationship of the solutions to the thermodynamics and correlation functions
134 of multivalent solutions within long water-filled channels is discussed.

135 The actual properties of real nanopores are, of course, discovered by experiment, which has been
136 leading the research in this area especially since the discovery of the structure of the KcsA channel . In
137 our Special Issue, experimental insight is provided by three leading researchers in the field.

138 O. Fedorenko et al [11] discuss the properties of voltage-gated sodium channels (Navs).
139 These channels play fundamental roles in eukaryotes but lack structural resolution, which renders
140 understanding their structure-function relationships a challenging problem. Bacterial Navs,
141 representing simplified homologues of their eukaryotic counterparts, have enabled both structural
142 resolution and electrophysiological characterization. However, their homotetrameric structure leads to
143 an EEEE locus in the SF that is at odds with the DEKA locus of eukaryotic Navs. Indeed, prokaryotic
144 Navs have long been considered more similar to eukaryotic calcium channels (Cavs) than to Navs,
145 leading to the formulation of the “EEEE paradox”. This was arguably solved by Kaufman et al by the
146 realisation that there is a critical D residue close to the EEEE ring of eukaryotic Cavs generating an
147 effective EEEED locus of charge $-5e$. Fedorenko et al. present a follow-up of a previous study , aimed
148 at mimicking the SF of eukaryotic Navs by engineering radial asymmetry into the SFs of bacterial
149 channels. This goal was pursued with two approaches: co-expression of different monomers of the
150 NaChBac bacterial channel in mammalian cells, to induce the random assembly of heterotetramers;
151 and the concatenation of four bacterial monomers to form a concatemer that can be targeted by
152 mutagenesis on specific strands of the SF, thereby introducing asymmetry. Patch-clamp measurements
153 and MD simulations showed that an additional gating charge in the SF leads to a significant increase
154 of Na^+ and a modest increase in Ca^{2+} conductance in the NavMs concatemer in agreement with
155 the behaviour of the population of random heterotetramers with the highest proportion of channels
156 with charge $-5e$. This study confirms that, although the charge at the SF is important, it is not the
157 only factor affecting conduction and selectivity. It also offers new tools extending the use of bacterial
158 channels as models of eukaryotic ones.

159 The work by A. Chernev et al [12] reviews the most promising approaches to the fabrication of
160 artificial nanofluidic devices capable of reproducing properties of single ion channels. It is shown
161 that modern technologies have great potential in allow one to test various theoretical models of ion
162 channels. The review aims to highlight ionic Coulomb blockade — the phenomenon which (see above)
163 can often be key player in ion channel selectivity. The authors discusses the most critical obstacles
164 associated with these studies and suggest possible solutions to further advance the field.

165 The rapid interdisciplinary advances in nanotechnology can be characterised as the beginning of
166 a new industrial revolution, where novel devices and materials are fabricated and controlled on the
167 atomic level. Ion- and water-selective nanopores represent an important frontier in these advances.

168 The selected papers in this Special Issue provide both a snapshot of the present and strong
169 indications of how the subject is likely to evolve over the coming years. We may, for example, anticipate:
170 major developments in the theory at a fundamental level, based on statistical mechanics and quantum
171 mechanics; substantial improvements in “intermediate level”s theories like PNP, modified CKE and
172 DFT which promise quantitative predictions of the properties of real channels; together with much
173 faster and more capacious MD modelling of larger ensembles of atoms on longer timescales, more
174 accurate due to use of polarizable force field and QM/MM, encompassing gating and permeation
175 events at a statistically useful level. This progress is expected to lead to the first-principles design
176 and fabrication of structures optimised for many important applications including ion pumps, energy
177 harvesting, and field effect ionic transistors as well as those mentioned above at the beginning. Many
178 of these will require theory and experiment on small scales where disciplinary distinctions have mostly
179 faded away, but where physics predominates.

180 An additional impulse propelling these developments forwards is expected due to the fusion of
181 physics-based approaches with artificial intelligence. The latter has already been proven to be a very
182 useful for accelerated learning of the force fields in MD, as well as for reconstruction of the potentials

183 of the mean force and neural-network-based discovery of partial differential equations. Remarkably, it
184 also underlies a recent breakthrough in the solution of the protein-folding problem.

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