

A Low-Cost and Low-Risk Testbed for Control Design of Launch Vehicles and Landing Systems

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Abstract—This research seeks to bridge the gap between the simulation/theory and the flight-testing phases of control algorithm design for launch vehicles (LVs) and landing systems (LSs). This paper reviews contemporary testbeds for LV and LS control system design with an emphasis on reducing costs and risks. Also, a new low-cost and low-risk testbed utilizing a quadcopter, flexible inverted pendulum, and a hanging pendulum is proposed. It is shown that the planar dynamic response of this proposed testbed can match the planar dynamic response of an LV.

As mission objectives for frequent cislunar and interplanetary travel become more complex, the algorithms that control LVs and LSs need to become more advanced to ensure mission safety and success. These control systems are slow to mature because it is difficult to perform experiments in an environment relevant to the mission in a safe and cost-effective manner. A solution to this problem frequently leveraged in the aviation and satellite industries involves mimicking the dynamic response of the vehicle with a test platform that is low-cost and low-risk.

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1. INTRODUCTION

In recent years, the interest in regular cislunar and interplanetary travel has increased [1, 2]. As these missions progress, their objectives will increase in complexity. This then requires that the technology used to accomplish these objectives becomes more advanced. In particular, the Guidance, Navigation, and Control (GN&C) systems on vehicles used to deliver payloads to their destinations need to become more advanced in order to ensure mission completion and safety of the payload. The work described in this paper deals with the control systems used on launch vehicles (LVs) and landing

systems (LSs), with an emphasis on methods to efficiently develop and mature advanced control system concepts.

Motivation

A review of LV failures performed in 2002 highlights the benefits of utilizing advanced GN&C systems on LVs. The review found that 41% of all LV failures between 1990 and 2002 could have been prevented or mitigated had an advanced GN&C system been used [3]. In September 2020, an LV built by Astra Space crashed 30 seconds after liftoff due to roll oscillations caused by the guidance software forcing the launch controllers to command an engine shutdown for safety reasons [4]. If an advanced control system had been used, the damage could have potentially been mitigated while still maintaining the same safety standards.

Autonomously landing on another celestial body is challenging. Extensive research and design went into the GN&C systems used on the Apollo system, which resulted in only one failure of the primary guidance system in all of its manned missions [5, 6]. Despite this success, Apollo 11 and the five successive lunar landings were manually piloted because the astronauts felt more comfortable piloting the vehicle themselves. This can be partially attributed to limitations in the autonomous GN&C technology. Also, consider the Chandrayaan-2 mission operated by the Indian Space Research Organization (ISRO). The lander (called Vikram) that flew with the mission in late 2019 lost communication during the last 15 minutes of descent and later was found to have hard landed onto the lunar surface. One of the possible causes for the failure is a lack of robustness in the control system design [7]. This shows that as planetary landing missions become more complex, it is necessary to have an advanced control system to ensure success.

In this paper, the term “advanced control systems” refers to control systems that extend beyond the classical proportional-integral-derivative control and the use of gain and phase stability margins as a measure of robustness. Classical control methods have a lengthy history in the control design of space vehicles; control engineers typically select them for their legacy and easy quantification of robustness through stability margins. Advanced control systems for use on LVs and LSs have difficulties reaching the maturity required due to a lack in legacy, testing, and extensive theory behind frequency domain analysis. One metric that can be used to describe the maturity of a technology is known as the

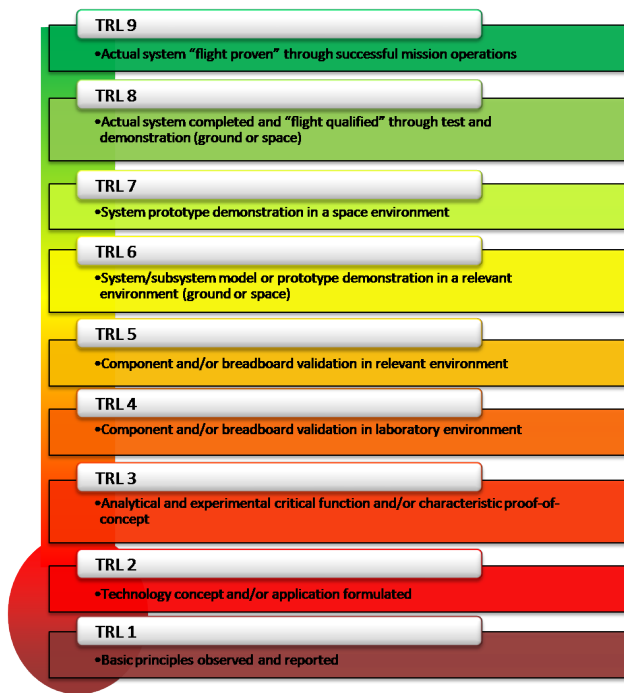


Figure 1: Definitions of the Technology Readiness Levels (TRLs). Image Credit: NASA.

Technology Readiness Levels (TRLs), as defined by the National Aeronautics and Space Administration (NASA) and seen in Figure 1. Based on this metric, LV and LS control systems often have difficulties progressing past TRLs 4-6; the so-called "technology valley of death".

Generally, full-scale flight testing is required to advance the TRL beyond 6. However, these undertakings are expensive and risky. To bridge the gap between simulation and full-scale flight testing, aviation and spacecraft industries often use low-cost and low-risk (LCR) testbeds as an intermediate step to further validate the design and mitigate potential risks.

Prior Art

The aviation industry has utilized many LCR testbeds to simulate various aerodynamic effects and to design control systems for full-scale vehicles. The Airborne Subscale Transport Aircraft Research (AirSTAR) project at NASA Langley Research Center (LaRC) uses subscale aircraft to test GN&C technologies aimed at reducing loss-of-control accidents in a environment [8]. The AirSTAR infrastructure has even been used recently to test the novel control scheme of a robust linear controller with composite adaptive control augmentation [9]. The X-48B aircraft at NASA Armstrong Flight Research Center (AFRC) was an 8.5%-scale low-speed vehicle that saw success in the design and testing of a preliminary control system for the full-scale Blended Wing Body type aircraft [10]. A number of testbeds have been built for researching vehicles with high-aspect-ratio wings. This type of vehicle can experience aeroelastic flutter past a certain airspeed, which can cause devastating damage to the airframe in flight if not accounted for carefully. Examples of testbeds that study this effect include Lockheed Martin's Body Freedom Flutter vehicle [11], NASA's X-56 Multi Utility Technology Testbed (MUTT) aircraft [12], and University of Minnesota's mini MUTT [13] and mAEWing aircraft [14].

All of these vehicles feature various control techniques to suppress the aeroelastic flutter. A variety of flying unmanned aerial vehicles and autonomous cars are part of an LCR testbed at Purdue University that is used for researching the challenges faced by full-scale autonomous vehicle systems used for applications such as urban air mobility, wildfire suppression, and military missions among others [15]. NASA AFRC has been at the forefront of flight control research for decades through the use of F/A-18 jets for research purposes. Some areas of research include the effects of high angle-of-attack, autonomous formation flight, and airborne refueling; most notably for the focus of this paper, these aircraft have been used for researching a wide range of advanced control techniques [16].

The spacecraft industry has also utilized LCR testbeds when designing and validating GN&C systems. The two primary methods being: flight testing with CubeSats and ground-based facilities. By design, CubeSats are meant to be low-cost and low-risk. As such, they are used in a wide range of applications ranging from biomedical to Earth Science applications [17]. Some of the notable CubeSat missions include demonstrating GN&C functionalities for the challenging task of precise formation flying in low Earth orbit. In 2014, the CanX-4 and CanX-5 CubeSats performed a series of precise formation flying experiments after systematically eliminating large initial differences in the orbital elements [18]. Around the same time, the CubeSat Proximity Operations Demonstration mission was initiated. This mission is intended to demonstrate key rendezvous, proximity operation, and docking technologies [19, 20]. Ground-based testbeds offer an even cheaper alternative to flight tests with CubeSats. These testbeds use air bearings and are relatively accurate in the portrayal of planar dynamics in a low-gravity environment. Some early examples of these include the air spindle and triaxial air bearing testbeds built at the University of Michigan and a low-cost spacecraft simulator at the Georgia Institute of Technology in the early 2000s [21, 22]. More recently, the Spacecraft Attitude Testbed was built at San Diego State University [23]. While these testbeds are low-cost and low-risk, their usefulness beyond designing attitude control systems is limited due to their stationary nature. More advanced testbeds allow for translation and rotation; these 3 degree-of-freedom (DOF) facilities are useful for simulating close-proximity operations and docking of spacecrafts while in space [24, 25].

The way to maturing low-TRL advanced control systems for LVs and LSs is through LCR testbeds. However, there is a conspicuous dearth of LCR testbeds for LVs and LSs. This is, in part, because there are some very difficult challenges associated with replicating the conditions for LVs and LSs.

Paper Contribution

The purpose this paper is to explore the state of the art in LCR testbeds for LVs and LSs. It is also to propose a concept for such an LCR testbed and compare its dynamic similarities with an LV. Future work addressing the technical challenges associated with the design is discussed.

The remainder of the paper progresses as follows: Section 2 reviews contemporary testbeds that have been used for control design for LVs and LSs. Section 3 proposes a new LCR testbed that can be used for control design for LVs and LSs and evaluates its dynamics similitude to those systems. Section 4 provides some concluding remarks on the results of this paper and future work.

2. LITERATURE REVIEW

A review of testbeds used for the design and testing of flight controllers for LVs and LSs is presented in this section.

Launch Vehicle Testbeds

This section encompasses a review of testbeds utilized for the development of control systems for expendable LVs during ascent and reusable LVs during ascent and descent back to Earth. This section reviews the testbeds used by SpaceX, Blue Origin, the European Space Agency (ESA), and NASA.

To the best of the knowledge of the authors, SpaceX does not have any documented LCR testbeds for testing the flight controllers used for their reusable LVs during ascent or descent. The company does, however, perform tests using vehicles that are reduced cost and risk compared to the actual flight vehicle; specifically for the descent and landing of the reusable LVs. The approach involves incrementally designing and testing the vehicle in steps. This starts with a vehicle that is as low-cost and low-risk as possible and performing a basic functionality test [26]. Each subsequent step then slightly advances the system towards the full vehicle capabilities by expanding upon the test objectives. For the development of the Falcon 9 (F9) and Falcon Heavy LVs, SpaceX constructed two experimental reusable LVs named Grasshopper and the Falcon 9 Reusable Development Vehicle (F9R Dev). These test articles were intended to certify key flight hardware and GN&C algorithms [27, 28]. The eight flights of Grasshopper and five flights of the F9R Dev contributed immensely to the success of the F9 vehicle [29].

Blue Origin takes a similar incremental approach in certifying its LVs. Since its first mission in 2007, the company has stood by their belief “in incremental improvement and in keeping investments at a pace that is sustainable” [30]. Although these incremental tests are lower cost and risk compared to the full flight-ready vehicle, these testbeds still remain far too costly and risky for simply validating novel GN&C algorithms. During a flight of Blue Origin’s second test vehicle, “a flight instability drove an angle of attack that triggered the range safety system to terminate thrust on the vehicle” [31]. An advanced control system could have potentially been able to account for this anomaly and saved the mission.

ESA launched the Future Launchers Preparatory Programme in 2003 in order to advance launcher technologies aimed at improving performance and reliability. One area of focus of the program is the reusability of existing and new technologies including reusable rocket stages. In order to design and test the GN&C systems of the reusable boosters, the Demonstrator Technology Vehicle (DTV) seen in Figure 2 was developed and is currently undergoing qualification testing. This vehicle is an LCR platform which will allow for flight demonstrations of GN&C technologies. Recently, the DTV was flight tested in Romania in October 2020 and a larger version of the vehicle with more engines is under consideration [32]. As with the Blue Origin experimental test vehicles, the DTV is still far too risky to use strictly for validating GN&C algorithms despite the relative low cost compared to an actual flight vehicle.

NASA AFRC has been at the cutting edge of flight controls research with their use of modified F/A-18 jets for a wide range of experiments. One of these highly modified F/A-18 jets named the Full-Scale Advanced Systems Testbed (FAST) was modified in such a way to be able to research launch vehicle adaptive controllers and can be seen in Figure 3.

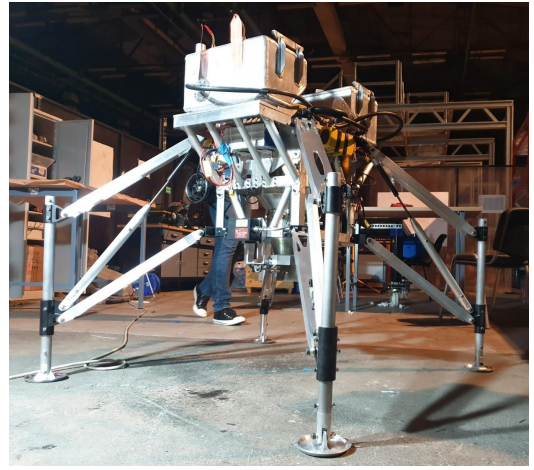


Figure 2: The Demonstrator Technology Vehicle (DTV)
Photo: ESA.



Figure 3: The F/A-18A Full-Scale Advanced Systems Testbed (FAST) aircraft No. 853 mid-flight in 2013.
Photo: NASA/Carla Thomas.

Specifically, the FAST was used to research the Adaptive Augmenting Control (AAC) algorithm for NASA’s Space Launch System (SLS). Although the FAST contains its own costs and risks, it is still far less expensive and risky than an LV. This allowed the FAST to be used to develop and test parts of the SLS GN&C system. A full breakdown on how the LV dynamics were simulated on the FAST can be found in [33]. The FAST is an aircraft, not an LV, and therefore, there are inherent limitations. In addition, an F/A-18 jet with a human pilot contains inherent costs and risks associated with each test flight that cannot be removed.

While FAST was used to validate the SLS AAC algorithm, simultaneous efforts at NASA LaRC were performed for further validation. These validation experiments were performed on LCR testbeds and are shown in Figure 4. A testbed used for LV flight control validation must characterize the fundamental dynamics of the vehicle. These dynamics include: open-loop instability caused by the aerodynamic side-force, fuel slosh dynamics, structural bending dynamics, and analogous control authority. The first of the LCR testbeds used was a 2-DOF pole-cart platform shown in Figure 4(a). This testbed involved a cart moving along a track in order to balance an inverted pendulum. The open-loop instability due to the gravity acting on the center of mass of the pole is

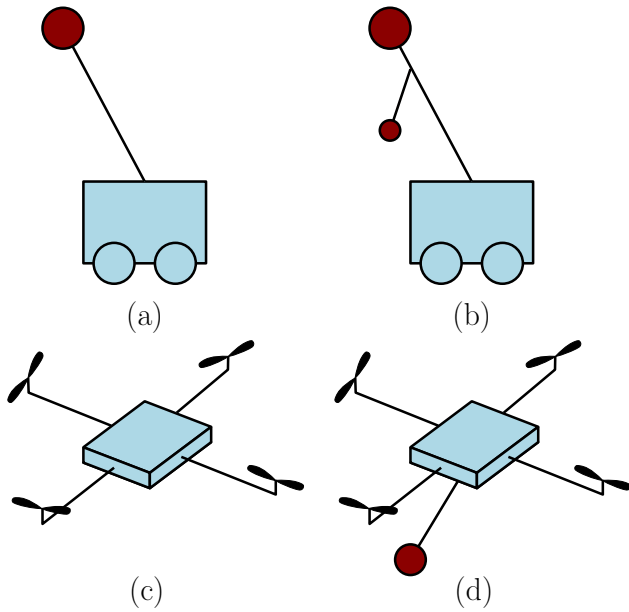


Figure 4: Schematics of the NASA LaRC LCR testbeds: (a) pole-cart, (b) pole-cart with hanging pendulum, (c) quadcopter, (d) quadcopter with hanging pendulum.

similar to the aerodynamic instability of an LV [34]. Another testbed used for the SLS AAC algorithm validation was a quadcopter shown in Figure 4(c). The rotors are analogous to gimbaled engine(s) on the aft end of an LV because they are capable of controlling thrust and producing angular acceleration. The main advantage of the quadcopter platform over the pole-cart is in its free-flying nature and the ability to replicate 6 DOF dynamics [35]. The most recent testbed that was used for these validation experiments was a quadcopter with a hanging pendulum shown in Figure 4(d). The hanging pendulum is a parallel to the fuel slosh dynamics onboard an LV [36].

All three of the above testbeds successfully validated the key functionalities of the SLS AAC algorithm when applied to an open-loop unstable plant. A fourth testbed developed but not used for validation of the SLS AAC algorithm, comprised of a pole-cart platform with another pendulum hanging from the inverted pendulum shown in Figure 4(b). This was done in order to characterize fuel slosh dynamics in addition to aerodynamic instability [37].

Landing System Testbed Review

This section reviews platforms designed to test GN&C systems for vehicles with the objective of landing softly on celestial bodies. LS testbeds can be classified into three types at various levels of complexity: Pilot-in-the-Loop (PIL), Vertical Testbeds (VTBs), and low-cost-and-risk.

The class of PIL test platforms are characterized by their purpose of training a human pilot to control an LS. The two Lunar Landing Research Vehicles (LLRVs) and three Lunar Landing Training Vehicles (LLTVs) were used in the 1960s to train astronauts of the Apollo program and are currently the only testbed in the class of PIL test platforms. These vehicles were specifically designed to mimic lunar landings, therefore, they needed to simulate the lunar environment as closely as possible. This requirement led to the design of the center-mounted gimbaled engine used to compensate for the gravity

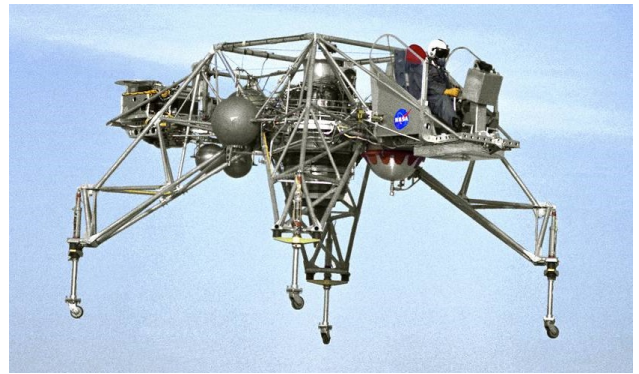


Figure 5: The Lunar Landing Research Vehicle (LLRV) #1 in flight in 1964. Photo: NASA.

difference between the Earth and the Moon. The testbed utilized hydrogen-peroxide thrusters for attitude actuation, and the pilot's cockpit was located on the vehicles as demonstrated in Figure 5 [38]. The LLRVs/LLTVs were difficult to pilot, risky, and expensive. Three of the five vehicles crashed before the end of the Apollo program. Despite this, the astronauts unanimously agreed that the training on the LLRVs/LLTVs prepared them for the actual missions [39].

Although the PIL class of testbeds may only contain the LLRVs and LLTVs currently, it will become more populated in the future to train pilots for crewed lunar landings and beyond. PIL test platforms can potentially be used to design novel LS controllers, especially for manual control considerations. However, these vehicles are inherently high-risk due to the risks associated with an onboard pilot during flight testing.

In the past decade, NASA has incorporated rocket propelled testbeds to test novel guidance and navigation systems. These testbeds are vertical take-off and landing (VTOL); henceforth, these testbeds will be referred to as vertical testbeds (VTBs).

A collaboration between Johns Hopkins Applied Physics Laboratory and NASA Marshall Space Flight Center produced a VTB named the Mighty Eagle (Figure 6) with assistance from commercial partners. The objectives of the Mighty Eagle were to test and mature the GN&C system for the final 30-50 m (98-164 ft) of the descent stage before landing with an emphasis on the optical navigation system [40]. This testbed had 20 successful flights during its 2011 and 2012 outdoor flight test campaigns in which it tested the optical navigation systems used for autonomous rendezvous and capture as well as other guidance and navigation functionalities [40].

The Morpheus VTB (Figure 7) tested guidance and navigation sensors for the Autonomous precision Landing and Hazard Avoidance Technology (ALHAT) project [41]. After Morpheus, NASA Jet Propulsion Laboratory worked with Masten Space Systems (MSS) to build the Autonomous Descent and Ascent Powered-Flight Testbed (ADAPT). The MSS Xombie vehicle was used to test terrain-relative navigation and fuel-optimal onboard planning of large divert trajectories from ADAPT [42, 43]. Subsequently, a payload called COBALT was designed to fly on the MSS Xodiac vehicle (Figure 8). Flying COBALT on Xodiac allowed



Figure 6: A close-up of the Mighty Eagle mid-flight in Huntsville, AL in 2013. Photo: NASA/MSFC/Todd Freestone.



Figure 7: NASA's Morpheus lander in Cape Canaveral, FL in 2012. Photo: NASA/Dimitri Gerondidakis.

for technologies from the ALHAT campaign and ADAPT to be blended and then tested in flight conditions unobtainable during Morpheus or ADAPT testing [44].

Recently, NASA began the Safe and Precise Landing - Integrated Capabilities Evolution (SPLICE) Project, which attempts to utilize the precision landing and hazard avoidance technology from ALHAT and COBALT on new mission concepts [45]. Two of three of the SPLICE sensors were tested on Blue Origin's New Shepard reusable LV in October 2020 [46].

All of these vertical testbeds were designed to simulate the dynamics of an LS as closely as possible. While their primary objectives were to test new sensors as well as guidance and navigation systems, these VTBs could also be used to design and test advanced control systems.

Two LCR testbeds specifically designed and built for the control design of LSs are the Extraterrestrial Surface Landing

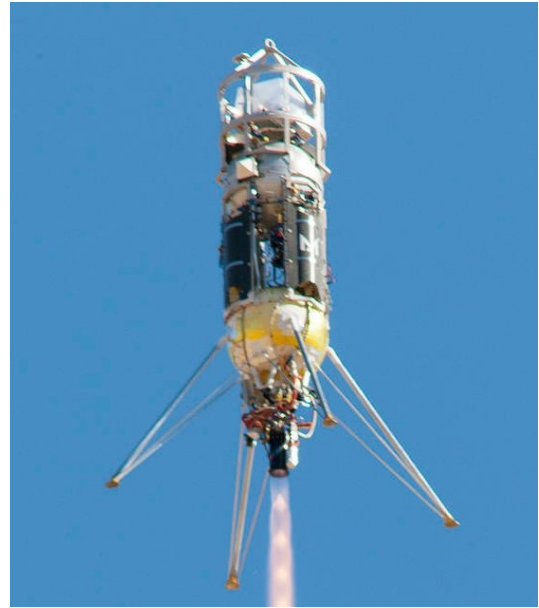


Figure 8: Masten Space Systems Xodiac launching out of Mojave Air and Space Port in 2016. Photo: NASA/Ken Ulbrich.

Research Vehicle (ESLRV) and the Environment for Autonomous GN&C Landing Experiments (EAGLE) testbeds.

The ESLRV was designed in 2010 to replicate the benefits of the LLTV while eliminating certain risks involved during LLTV flight tests. The ESLRV is a scaled down and less expensive version of the LLTV and could be piloted remotely. Like the LLTV, it utilizes a vertically-mounted, gimbaled jet engine to simulate lunar gravity. It uses four propellers mounted like a quadcopter for attitude control instead of using hydrogen peroxide thrusters [39]. Other improvements made to the system came from using contemporary technology. A full design and development breakdown of the ESLRV can be found in [47]. The major drawback is that the jet engine has inherent costs and risks.

The EAGLE was designed for control system design for LSs as well as to mature other GN&C technologies. This testbed utilizes a thrust-vector-controlled jet engine with cold gas thrusters that makes this system comparable to an LV or LS. This testbed also contains liquid fuel that would capture the effect of the slosh dynamics onboard an LV or LS. A full design and development breakdown of the EAGLE can be found in [48]. Similar to the ESLRV, the major drawback to this design is the risk and cost associated with the jet engine.

Summary

Simulation and theory alone can bring the TRL of an advanced control algorithm for an LV or LS to a 4 or 5. To advance the TRL to 6 or beyond, experimental flight testing is required. This, however, comes with its own risks and often the cost is upwards of millions of dollars. As illustrated in Figure 9, there is currently a lack of LCR testbeds for maturing mid-TRL flight control algorithms. In this research, we seek a high-fidelity LCR testbed to bridge this gap between simulation and flight testing. This work builds off the work described in [34–36]. The proposed testbed is described in Section 3.

TRL	1	2	3	4	5	6	7	8	9
	Software Environments			- LaRC testbeds - EAGLE - ESLRV			- ESA DTV - FAST - LLTVs/LLRVs - VTBs - Mighty Eagle - Morpheus - MSS vehicles		
				- SpaceX Test Vehicles - Blue Origin Test Vehicles					

Figure 9: Testbeds categorized by the TRL ranges each testbed can be used for.

3. PROPOSED TESTBED

A brief description of the proposed LCR testbed for LV and LS control system design is described in this section. Additionally, the dynamics of an LV and the proposed testbed are described.

Setup

The intent of the proposed testbed is not necessarily to have an exact one-to-one representation of the dynamics of a particular LV or LS configuration, but rather to be able to capture the fundamental dynamics of these systems such that it can be used to improve the TRL of novel control algorithms.

For an LV, the testbed must have the capability to characterize the following fundamental dynamics: aerodynamic instability, structural bending (flexible) dynamics, and fuel slosh [49–51]. LVs are typically aerodynamically unstable due to the center of pressure being ahead of the center of mass as shown in Figure 10. Also, the vehicle itself is not a perfect rigid body thus susceptible to control-structure interactions. The first bending mode of a typical LV is illustrated in Figure 10. Extensive literature exists on mechanical models for fuel slosh and their impact on flight control design [52–54]. The described fundamental dynamics of an LV share similarities with the dynamics of an LS, which are described in [38, 55–57].

A schematic of the proposed testbed is shown in Figure 11. The testbed consists of a quadrotor helicopter (quadcopter) with a flexible inverted pendulum pinned to the top of the vehicle and a hanging pendulum pinned to the bottom. Vehicle aerodynamic instability can be simulated through the inherently unstable dynamics of the inverted pendulum. The flexible dynamic effects are mimicked via the flexibility of the inverted pendulum. Furthermore, the degree of aerodynamic instability and natural frequencies of the bending modes can be adjusted by varying the length of the inverted pendulum. The control effectors of the LV or LS are represented by the quadcopter rotors; the quadcopter allocates speed commands to each rotor for attitude and throttle control similar to a thrust vector control system. Lastly, the fuel-slosh dynamics are mimicked by the hanging pendulum. Recognizable limitations of the testbed include: not being able to simulate a vehicle configuration with multiple propellant tanks, slosh damping via baffles, localized paneling bending, tail-wags-dog, and dog-wags-tail effects [58].

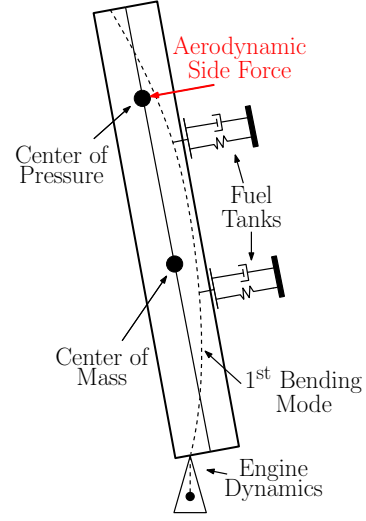


Figure 10: Model of an LV during flight.

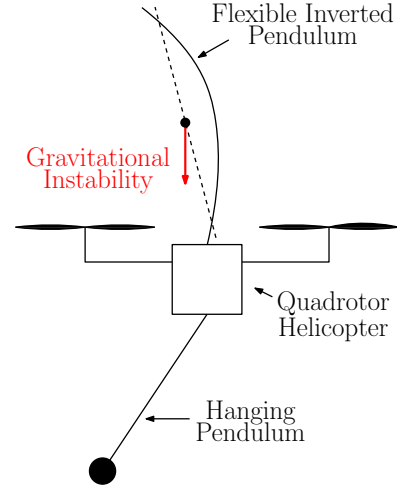


Figure 11: Model of the proposed testbed.

Modeling

Equations of motions for an LV and the proposed quadcopter testbed were developed to further understand the dynamic similarities between the two systems. This preliminary analysis is limited to planar dynamics.

The Lagrangian approach used in [49–51] is followed, which results in the linearized LV equations of motion shown in (1), (2), and (3):

$$I\ddot{\theta} + \sum_{j=1}^n m_{sj} l_{sj} \ddot{\delta}_{sj} = \frac{\partial C_N}{\partial \alpha} \bar{q} S l_a \theta - R \sum_{i=1}^k \phi_{pi} \eta_i - R l_p \left(\beta + \sum_{i=1}^k \psi_{pi} \eta_i \right) - \bar{g} \sum_{j=1}^n m_{sj} \delta_{sj}, \quad (1)$$

$$\mu_i (\ddot{\eta}_i + 2\zeta_{Bi} \omega_{Bi} \dot{\eta}_i + \omega_{Bi}^2 \eta_i) = R \phi_{pi} \left(\beta + \sum_{i=1}^k \psi_{pi} \eta_i \right) - \sum_{j=1}^n m_{sj} \phi_{sij} \ddot{\delta}_{sj}, \quad (2)$$

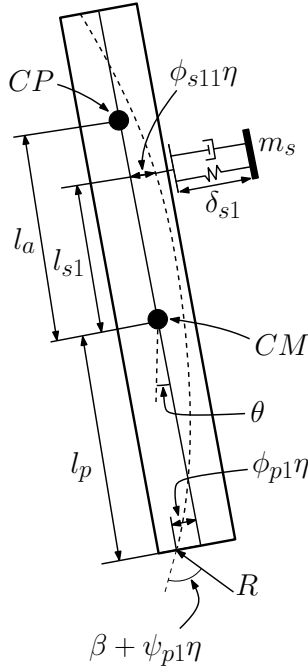


Figure 12: Schematic of LV with parameters.

$$\ddot{\delta}_{sj} + 2\zeta_{sj}\omega_{sj}\dot{\delta}_{sj} + \omega_{sj}^2\delta_{sj} = -l_{sj}\ddot{\theta} + \bar{g}\theta - \sum_{i=1}^k \phi_{sij}\ddot{\eta}_i, \quad (3)$$

where the generalized coordinates are composed of the body angular perturbation in the trajectory frame θ , the time-dependent flexible coordinate of the i^{th} eigenmode η_i , and the slosh mass perturbation of the j^{th} fuel tank δ_{sj} . The input to the system is the engine gimbal angle β . Translational and engine dynamics are not included. The remaining vehicle parameters are defined in Table 1 located at the end of the appendix and shown in Figure 12.

Similarly, the Lagrangian approach is used to derive the equations of motion of the quadcopter testbed shown in (4), (5), (6), and (7):

$$(I_q + m_b l_{pb}^2 + m_s l_{ps}^2) \ddot{\gamma} - m_b l_{pb} \sum_{i=1}^k (\phi_{bi} - \phi_{pbi}) \ddot{\eta}_i + m_b l_{pb} l_b \ddot{\varepsilon} + m_s l_{ps} l_s \ddot{\alpha} = (F_1 - F_2) l_d, \quad (4)$$

$$m_b l_b l_{pb} \ddot{\gamma} + m_b l_b^2 \ddot{\varepsilon} - m_b l_b \sum_{i=1}^k (\phi_{bi} - \phi_{pbi}) \ddot{\eta}_i = m_b g \sum_{i=1}^k \phi_{pbi} \eta_i + m_b l_b g \varepsilon, \quad (5)$$

$$\mu_i (\ddot{\eta}_i + 2\zeta_{Pi}\omega_{Pi}\dot{\eta}_i + \omega_{Pi}^2\eta_i) = m_b g \phi_{pbi} \varepsilon + m_b l_b (\phi_{bi} - \phi_{pbi}) \ddot{\varepsilon} + m_b l_{pb} \ddot{\gamma} (\phi_{bi} + \phi_{pbi}), \quad (6)$$

$$m_s l_s^2 \ddot{\alpha} = -m_s l_s g \alpha - m_s l_{ps} l_s \ddot{\gamma}, \quad (7)$$

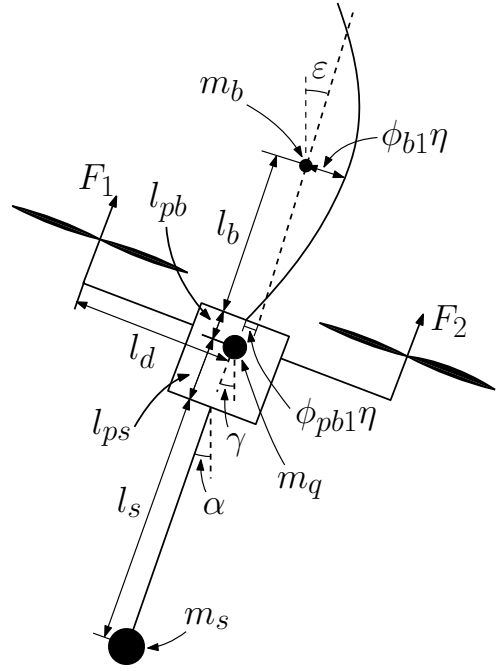


Figure 13: Schematic of testbed with parameters.

where the states of the quadcopter are: angular perturbation γ , the inverted pendulum angular perturbation ε , the time-dependent flexible coordinate of the i^{th} eigenmode η_i , and the hanging pendulum angular perturbation α . The inputs to the system are the rotor thrusts F_1 and F_2 . Once again, the translational and rotor dynamics are not included. The remaining vehicle parameters are defined in Table 2 located at the end of the appendix and shown in Figure 13. Additional details on the derivation of the equations of motion are provided in the Appendix.

Analysis

Following the derivation of the linear equations of motion for the two systems, the poles of the open-loop plants are compared in an effort to identify similarities in the dynamic responses. Figure 14 provides an illustration of the fundamental dynamics of a typical LV and the quadcopter testbed. Only the first bending modes and a single fuel tank are considered for this analysis. Table 1 and Table 2 (both located at the end of the appendix) contain the values of the LV and testbed parameters respectively. The testbed has two extra poles both located at $s = 0$, representing the neutral stability of the quadcopter attitude dynamics. The poles located at $s = \pm 0.62$ correspond to the aerodynamic instability of the LV and the gravitational instability of the inverted pendulum. The poles at $s = -0.06 \pm 12.5j$ correspond to the flexible dynamics of the LV and the inverted pendulum. The poles located at $s \approx \pm 3.1j$ correspond to the dynamics of the fuel slosh and hanging pendulum. Note in Figure 14 some unrealistic parameters are chosen for the testbed to exactly match the dynamics of the LV. In practice, the choice of these parameters will be largely limited by operational constraints.

4. CONCLUSIONS & FUTURE WORK

This paper surveyed the literature for LCR testbeds used to mature flight controllers for LVs and LSs. Despite the

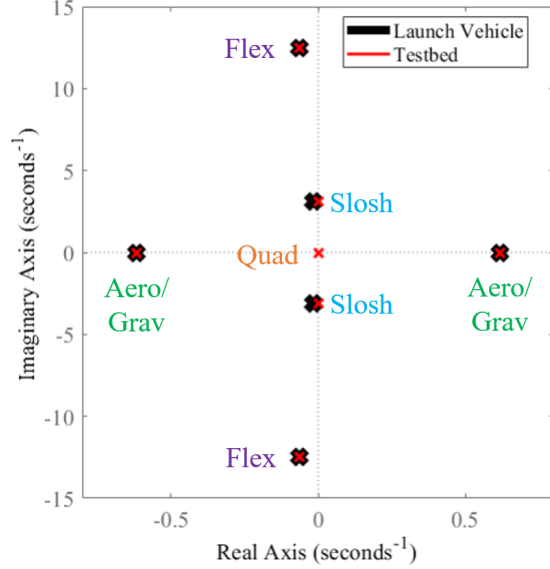


Figure 14: Pole map comparing the pole locations of the LV and quadcopter testbed models.

numerous potential benefits, it appears that there is a lack of such testbeds applicable to TRLs 4-6 across academia, government, and industry research laboratories. A high-fidelity LCR testbed was proposed for maturing the LV and LS flight controllers through TRLs 4-6. The proposed testbed consists of a quadcopter, flexible inverted pendulum, and hanging pendulum. The system accurately characterizes the fundamental dynamics of an LV and LS configuration. This would allow hardware-in-the-loop testing of novel LV/LS GN&C systems on a dynamically similar platform for a fraction of the cost.

Future work includes a comparison between the three-dimensional nonlinear dynamics of an LV, an LS, and the proposed testbed, as well as construction of the testbed.

APPENDIX

This appendix contains the derivation of the equations of motion of the testbed found in (4), (5), (6), and (7). This derivation does not include the effects of the translational motion of the quadcopter. The inertial frame $\mathcal{F}_a$ used in this analysis is a North-East-Down frame where all motion occurs in the North-Down frame. The kinetic energy of each of the components relative to fixed point w with all derivatives taken with respect to inertial frame $\mathcal{F}_a$ are

$$T_{Qw/a} = \frac{1}{2} I_q \dot{\gamma}^2, \quad (8)$$

$$T_{Bw/a} = \frac{1}{2} m_p (l_{pb}^2 \dot{\gamma}^2 + \left(\sum_{i=1}^k (\phi_{bi} - \phi_{pbi}) \dot{\eta}_i - l_p \dot{\varepsilon} \right)^2 - 2l_{pb} \dot{\gamma} \left(\sum_{i=1}^k (\phi_{bi} - \phi_{pbi}) \dot{\eta}_i - l_p \dot{\varepsilon} \right) \cos(\gamma - \varepsilon)), \quad (9)$$

$$T_{Sw/a} = \frac{1}{2} m_p (l_{ps}^2 \dot{\gamma}^2 + 2l_{ps} l_s \gamma \alpha \cos(\gamma - \alpha) + l_s^2 \dot{\alpha}^2), \quad (10)$$

where $T_{Qw/a}$ is the kinetic energy of the quadcopter, $T_{Bw/a}$ is the kinetic energy of the inverted pendulum, and $T_{Sw/a}$ is the kinetic energy of the hanging pendulum. Definitions of the remaining parameters are found in Table 2 located at the end of the appendix.

The potential energy of each of the components relative to fixed point w is

$$U_{Qw} = 0, \quad (11)$$

$$U_{Bw} = m_b g \left(l_{pb} \cos \gamma - \sum_{i=1}^k \phi_{pbi} \eta_i \sin \varepsilon + l_b \cos \varepsilon \right) + \frac{1}{2} \omega_{Bi}^2 \eta_i^2, \quad (12)$$

$$U_{Sw} = -m_s g (l_{ps} \cos \gamma + l_s \cos \alpha), \quad (13)$$

where U_{Qw} is the potential energy of the quadcopter, U_{Bw} is the potential energy of the inverted pendulum, and U_{Sw} is the potential energy of the hanging pendulum. Definitions of the remaining parameters are found in Table 2 located at the end of the appendix.

The kinetic and potential energies are used to form the Lagrangian

$$L = T_{Qw/a} + T_{Bw/a} + T_{Sw/a} - U_{Qw} - U_{Bw} - U_{Sw}. \quad (14)$$

The generalized forces and moments acting on the system, $\mathbf{f}_x$, are determined using the method of virtual work, where the virtual work is found as

$$\delta W = \mathbf{f}_x^T \delta \mathbf{q} = \begin{bmatrix} (F_1 - F_2) l_d \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \begin{bmatrix} \delta \gamma \\ \delta \varepsilon \\ \delta \alpha \\ \delta \eta_1 \\ \vdots \\ \delta \eta_k \end{bmatrix}, \quad (15)$$

where F_1 and F_2 are the thrust forces generated by the quadcopter's propellers, and l_d is the distance between the quadcopter center of mass and each propeller.

The equations of motion are then derived using Lagrange's equation, given by

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{f}_x, \quad (16)$$

where

$$\mathbf{q} = \begin{bmatrix} \gamma \\ \varepsilon \\ \alpha \\ \eta_1 \\ \vdots \\ \eta_k \end{bmatrix}. \quad (17)$$

Simplifying assumptions and linearizations are applied to the resulting nonlinear equations of motion, including small angle approximations, the inclusion of a small amount of internal damping in the flexible inverted pendulum, and that the center of mass of the testbed is at the same location as the center of mass of the quadcopter, which yields the equations of motion in (4), (5), (6), and (7).

Table 1: Launch Vehicle Parameters

Parameter	Definition	Value
θ	Body angular perturbation in the trajectory frame	– rad
η_i	State variable of the i^{th} eigenmode	–
δ_{sj}	Slosh mass perturbation of the j^{th} fuel tank	– m
β	Effective rigid-body gimbal angle	– rad
I	Vehicle total moment of inertia about the center of mass	$1 \times 10^8 \text{ kg}\cdot\text{m}^2$
l_g	Length from gimbal point to vehicle center of mass	75 m
m_{s1}	Mass of fuel in the single fuel tank	3000 kg
l_{s1}	Length from vehicle center of mass to the slosh mass of the tank	10 m
$\frac{\partial C_N}{\partial \alpha}$	Linearized aerodynamic normal force coefficient	8 N
$\bar{q}$	Dynamic pressure	1000 m^{-2}
S	Span reference area	100 m^2
l_a	Aerodynamic moment arm	50 m
R	Gimbaled thrust force	$1 \times 10^6 \text{ N}$
$\bar{g}$	Acceleration along the longitudinal axis of the vehicle body	64 m/s^2
ϕ_{p1}	Mode shape of the 1^{st} mode at the gimbal point	$-5 \times 10^{-4} \text{ m}$
ψ_{p1}	Mode slope of the 1^{st} mode at the gimbal point	1×10^{-4}
ϕ_{s11}	Mode shape of the 1^{st} mode at the single slosh mass location	$1 \times 10^{-4} \text{ m}$
μ_1	Generalized mass of the 1^{st} elastic eigenmode	1 kg
ζ_{B1}	Damping ratio of 1^{st} bending mode	5×10^{-3}
ω_{B1}	Natural frequency of 1^{st} bending mode	12.5 radians
ζ_{s1}	Damping ratio of the slosh mode	5×10^{-3}
ω_{s1}	Natural frequency of the slosh mode	0.9π radians

Table 2: Testbed Parameters

Parameter	Definition	Value
γ	Quadcopter angular perturbation	– rad
ε	Inverted pendulum angular perturbation	– rad
η_i	State variable of the i^{th} eigenmode	–
α	Hanging pendulum angular perturbation	– rad
I_q	Quadcopter moment of inertia about the center of mass	$0.0181 \text{ kg}\cdot\text{m}^2$
m_b	Mass of inverted pendulum	0.01 kg
l_b	Length from inverted pendulum center of mass to inverted pendulum pin joint	35.115 m
l_{pb}	Length from inverted pendulum pin joint to quadcopter center of mass	0.05 m
m_s	Mass of hanging pendulum	0.11 kg
l_s	Length from hanging pendulum pin joint to hanging pendulum center of mass	2 m
l_{ps}	Length from quadcopter center of mass to hanging pendulum pin joint	0.1 m
F_1	Thrust force of rotor 1	– N
F_2	Thrust force of rotor 2	– N
l_d	Moment arm from quadcopter center of mass to rotors	0.1651 m
g	Gravitational constant	9.81 m/s^2
ϕ_{b1}	Mode shape of the 1^{st} mode at the inverted pendulum center of mass	$1 \times 10^{-4} \text{ m}$
ϕ_{pb1}	Mode shape of the 1^{st} mode at the inverted pendulum pin	$-5 \times 10^{-4} \text{ m}$
μ_1	Generalized mass of the 1^{st} elastic eigenmode	1 kg
ζ_{P1}	Damping ratio of 1^{st} bending mode	5×10^{-3}
ω_{P1}	Natural frequency of 1^{st} bending mode	12.5 radians

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