# *A Robust Uniform Control Approach for VTOL Aircraft*

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# *Presentation Outline*

- NASA LA-8 Introduction
- Mathematical Preliminaries
- Equilibrium Manifold Exploration
- Uniform Control Architecture
- Transition Trajectory Tracking Example









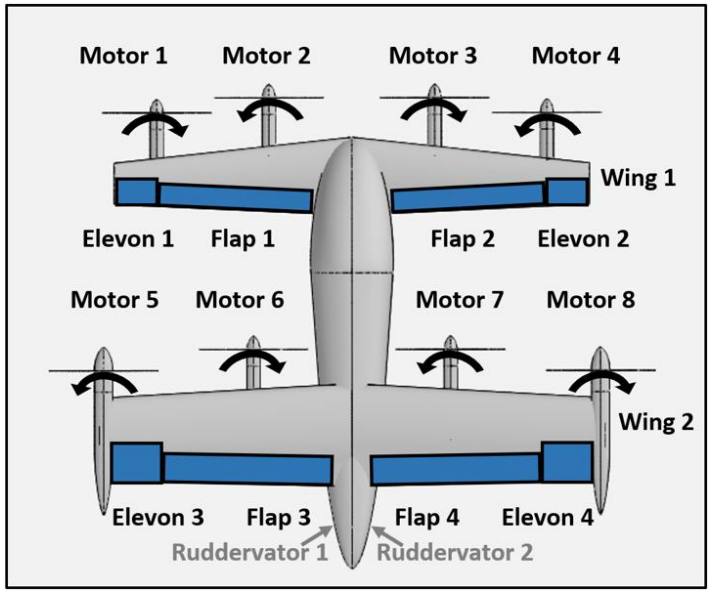


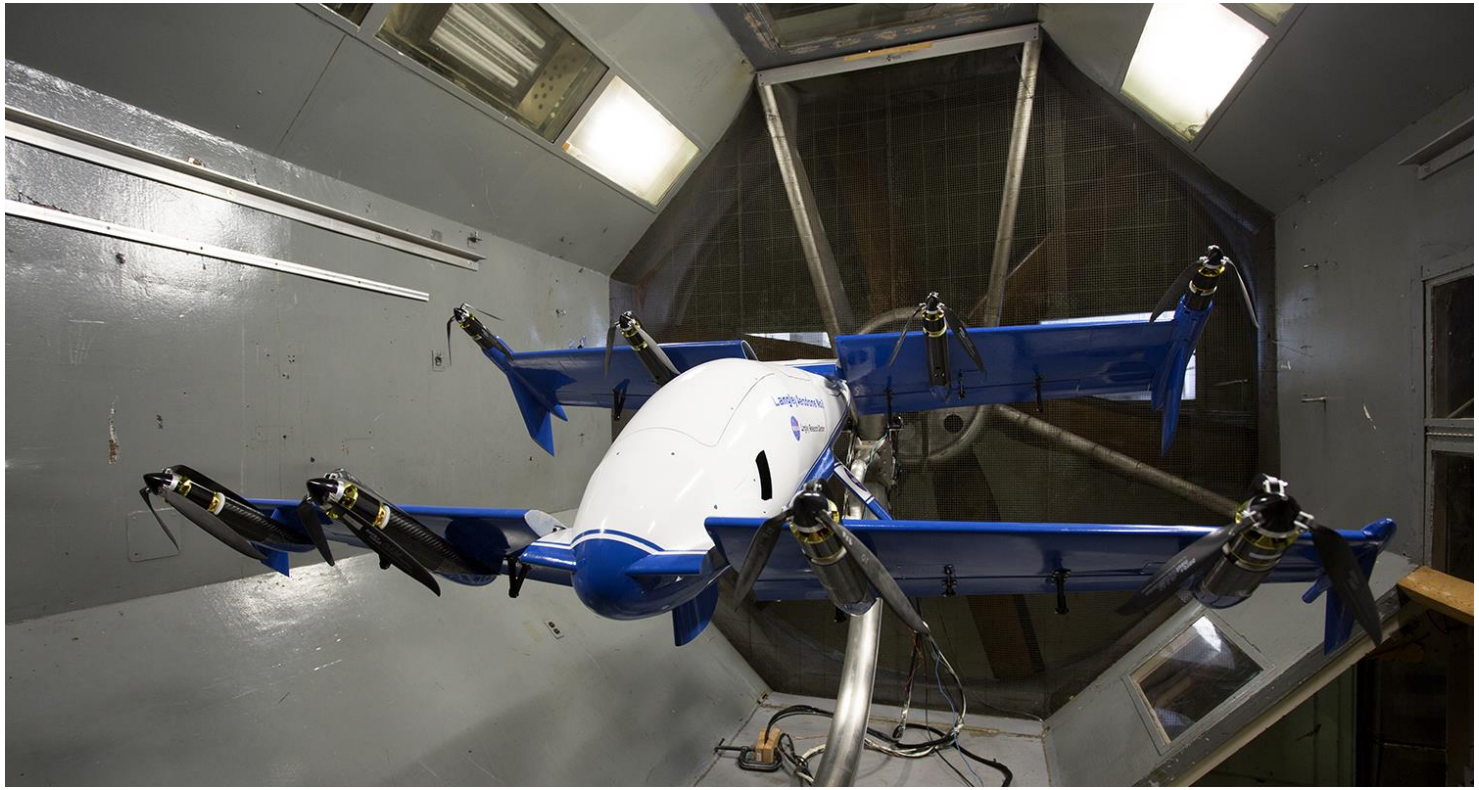


# NASA LA-8 aircraft

**Tandem Tiltwing VTOL Aircraft**

- Eight, three-bladed 16x8 propellers
- Two rotating wings
- Four elevons
- Four fowler flaps
- Two ruddervators



R. C. Busan, P. C. Murphy, D. B. Hatke, and B. M. Simmons, "Wind Tunnel Testing Techniques for a Tandem Tilt-Wing, Distributed Electric Propulsion VTOL Aircraft," in *AIAA SciTech Forum*, (VIRTUAL EVENT), American Institute of Aeronautics and Astronautics, Jan. 2021.

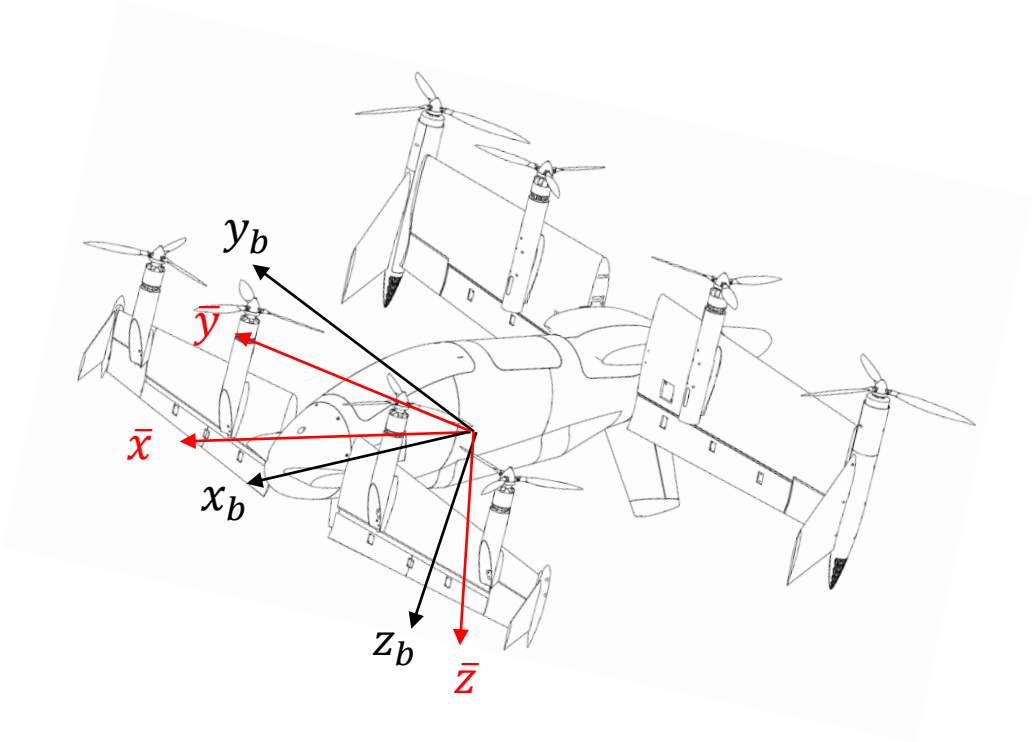
B. M. Simmons and P. C. Murphy, "Wind Tunnel-Based Aerodynamic Model Identification for a Tilt-Wing, Distributed Electric Propulsion Aircraft," in *AIAA SciTech Forum*, (VIRTUAL EVENT), American Institute of Aeronautics and Astronautics, Jan. 2021.

# Reference Frames

**Inertial North-East-Down**

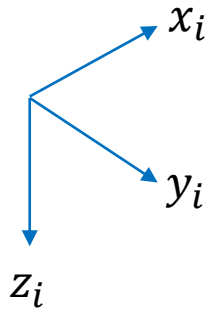
**Body Frame**
- Origin at the nominal center of mass
- X-axis out the nose
- Y-axis out the right wing
- Z-axis completes the right hand rule

**Heading Frame**
- Origin at the nominal center of mass
- X-axis points in direction of the heading on the local horizontal plane
- Z-axis points down
- Y-axis completes the right hand rule




**Body to Inertial (NED)**

$$R = \left[R_x(\phi)R_y(\theta)R_z(\psi)\right]^T$$
$$\mathrm{v}_i = R\mathrm{v}_b$$




**Body to Heading Frame**

$$\bar{R} = [R_x(\phi)R_y(\theta)]^T$$
$$\bar{\mathrm{v}} = \bar{R}\mathrm{v}_\mathrm{b}$$

**Inertial to Heading Frame**

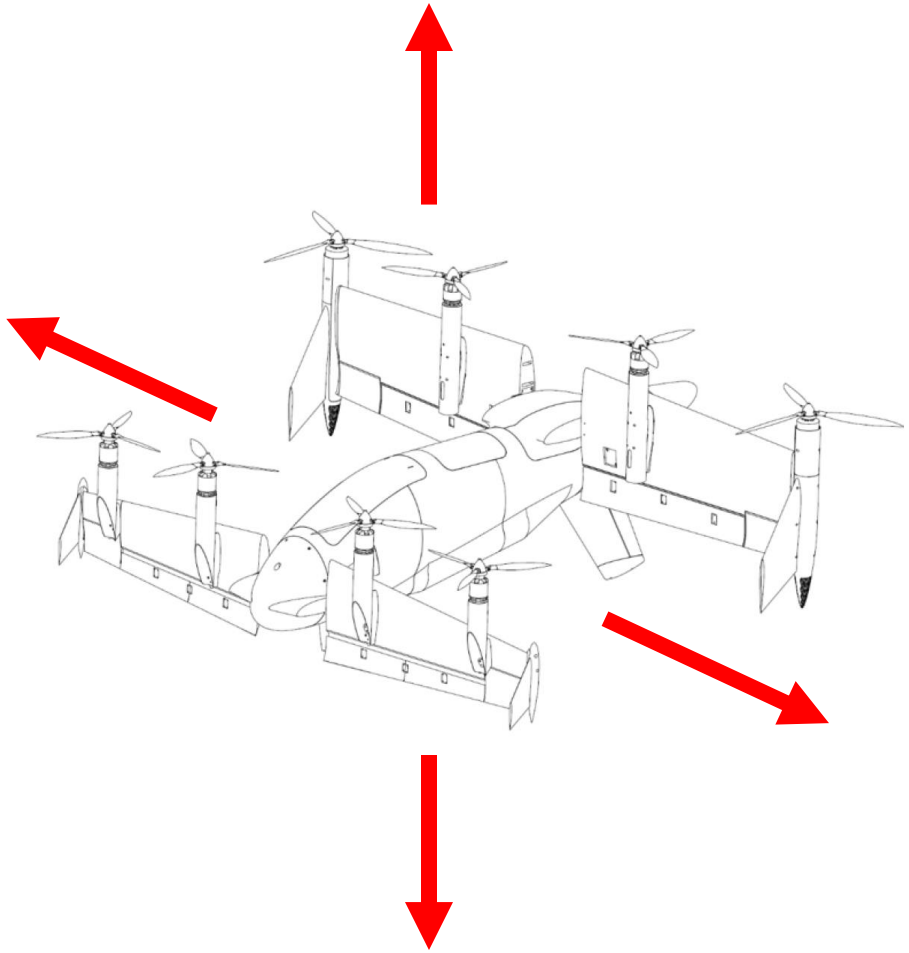
$$\bar{\mathrm{v}} = R_z(\psi)\mathrm{v}_i$$

Assuming no wind $\chi = \psi$

# Guidance Commands

## VTOL Aircraft Maneuvers

- Encompass those of rotor-craft and conventional flight
- Lateral and vertical maneuvers in hover



## Heading Frame Velocity

$$\bar{\mathrm{v}} = [\bar{u} \quad \bar{v} \quad \bar{w}]^T$$

- $\bar{u}$ - forward velocity
- $\bar{v}$ - lateral velocity
- $\bar{w}$ - vertical velocity

## Guidance Commands

$$r(t) = [\bar{\mathrm{v}}_d^T(t) \quad \dot{\psi}_d(t)]^T$$

# Heading Frame Dynamics

$$\bar{v} = [\bar{u} \quad \bar{v} \quad \bar{w}\,]^T$$

$$\omega = [p \quad q \quad r\,]^T$$

$$u = \left[\omega_p^T \quad \delta_i^T \quad \delta_e^T \quad \delta_f^T \quad \delta_r^T\right]^T$$

## Translational Dynamics

$$\dot{\bar{v}} = -\dot{\psi}\hat{e}_3 \times \bar{v} + g\hat{e}_3 + \frac{1}{m}\bar{F}(\bar{v}, \omega, \bar{R}, u)$$

## Rotational Dynamics

$$J\dot{\omega} = -\omega \times J\omega + \tau(\bar{v}, \omega, \bar{R}, u)$$

## Rotational Kinematics

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

| Control Effectors | |
|---|---|
| motor speeds | $\omega_p = [\omega_{p_1} \quad \omega_{p_2} \quad \dots \quad \omega_{p_8}]^T$ |
| wing tilt angles | $\delta_i = [\delta_{i_1} \quad \delta_{i_2}]^T$ |
| elevons | $\delta_e = [\delta_{e_1} \quad \delta_{e_2} \quad \delta_{e_3} \quad \delta_{e_4}]^T$ |
| flaps | $\delta_f = [\delta_{f_1} \quad \delta_{f_2} \quad \delta_{f_3} \quad \delta_{f_4}]^T$ |
| ruddervators | $\delta_r = [\delta_{r_1} \quad \delta_{r_2}]^T$ |

# *Equilibrium Manifold Exploration*

# Trim Map for Steady Level Flight

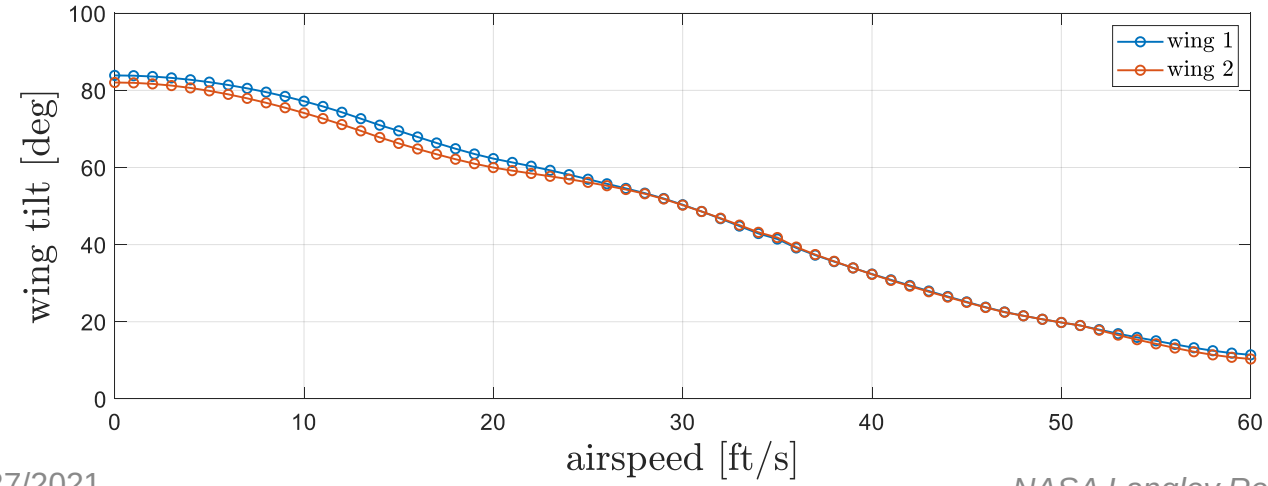
**Trimming Free variables**

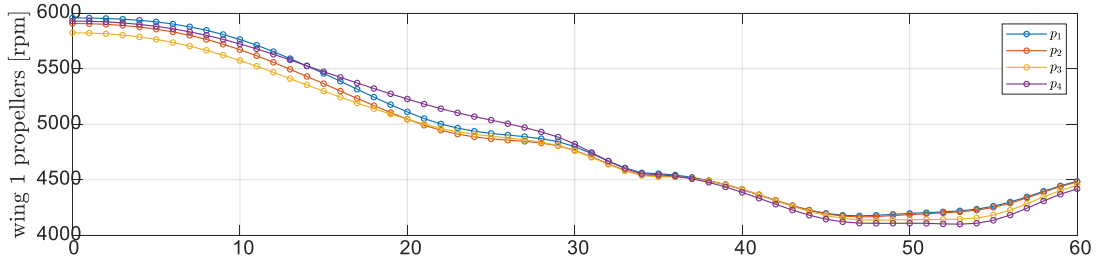
- Wing tilt $(\delta_i)$
- Propeller speed $(\omega_p)$
- Flap deflection $(\delta_f)$

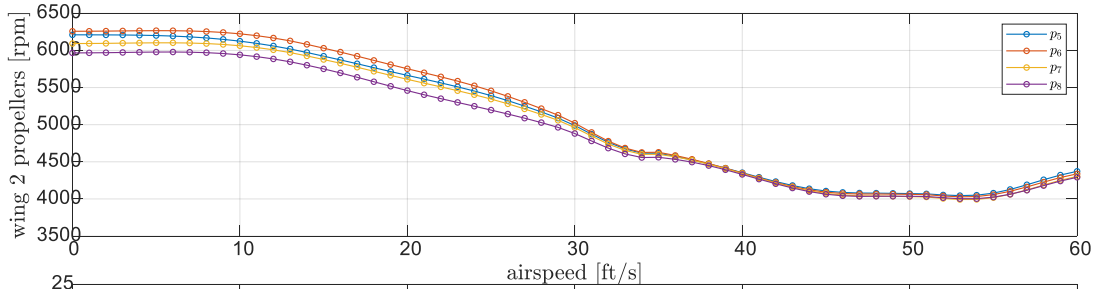
**Elevon and Ruddervators set at 0°**
**Body pitch angle held at 0°**

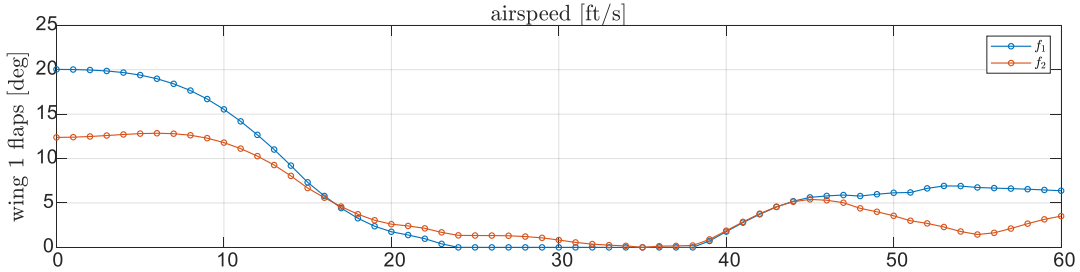
**Cost function**

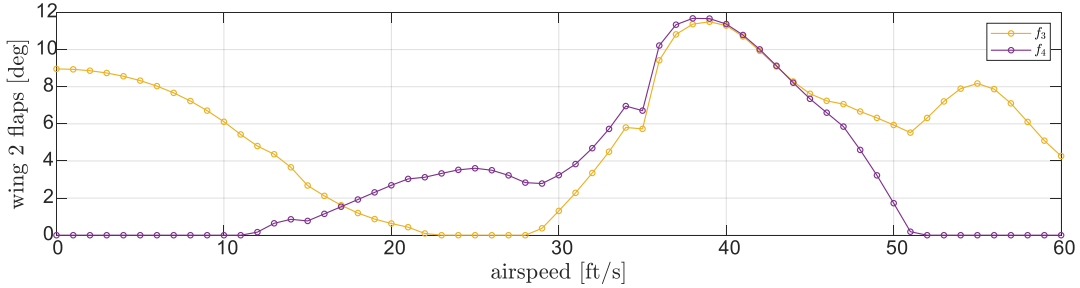
$$J(u) = w_1 \omega_p^T \omega_p + w_2 \delta_f^T \delta_f + w_3 \left(\delta_{i_2} - \delta_{i_1}\right)^2 + w_4 \left(\delta_{f_2} - \delta_{f_1}\right)^2$$
$$+ w_5 \left(\delta_{f_4} - \delta_{f_3}\right)^2 + w_6 \sum_{i=1}^{7} \left(\omega_{p_{i+1}} - \omega_{p_i}\right)^2$$



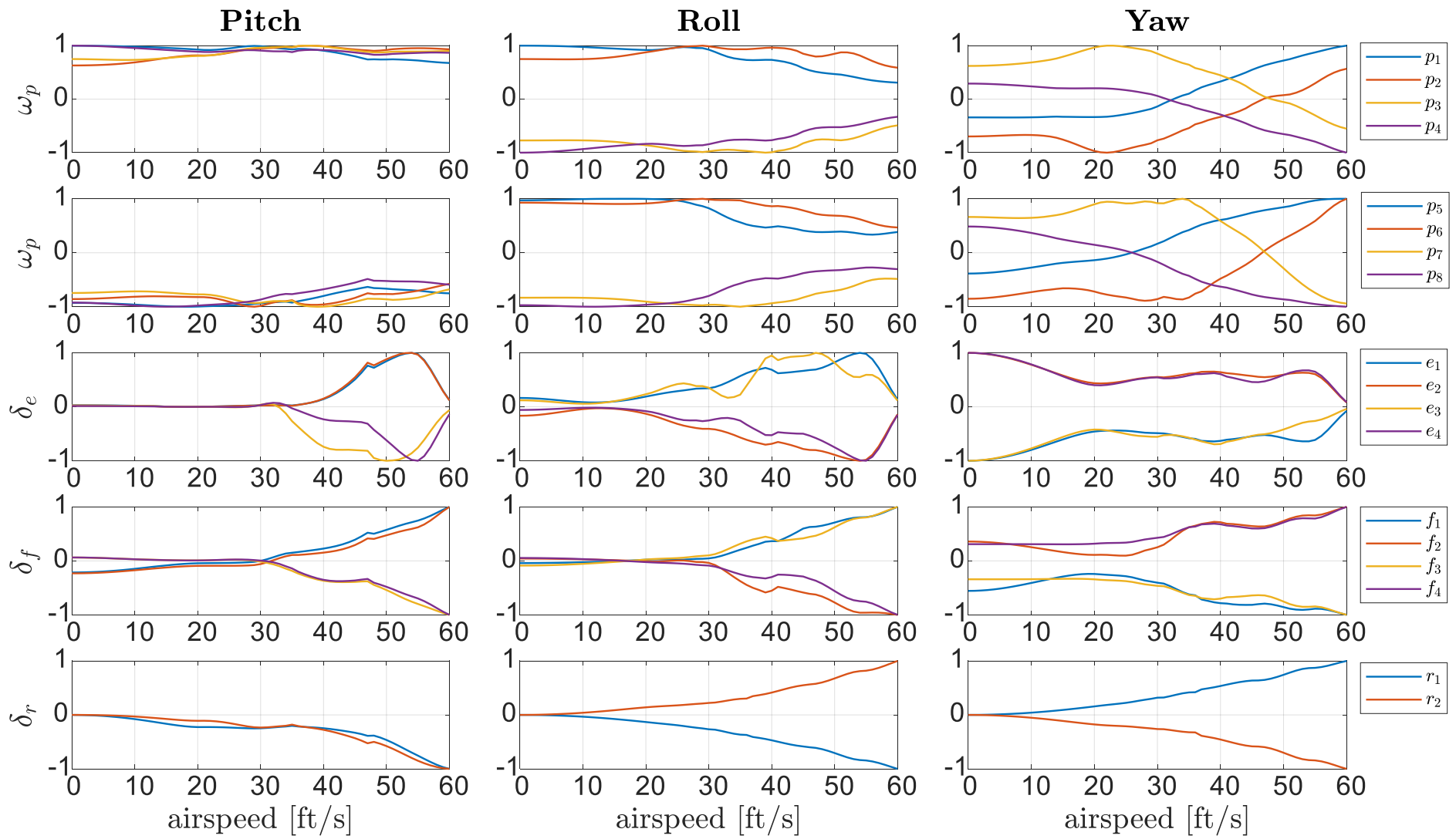



# Control Effectiveness

How does the **effectiveness** of a control actuator change?

**Effectiveness Def:**

$$eff = \frac{F_\delta}{\max(|F_\delta|)}$$

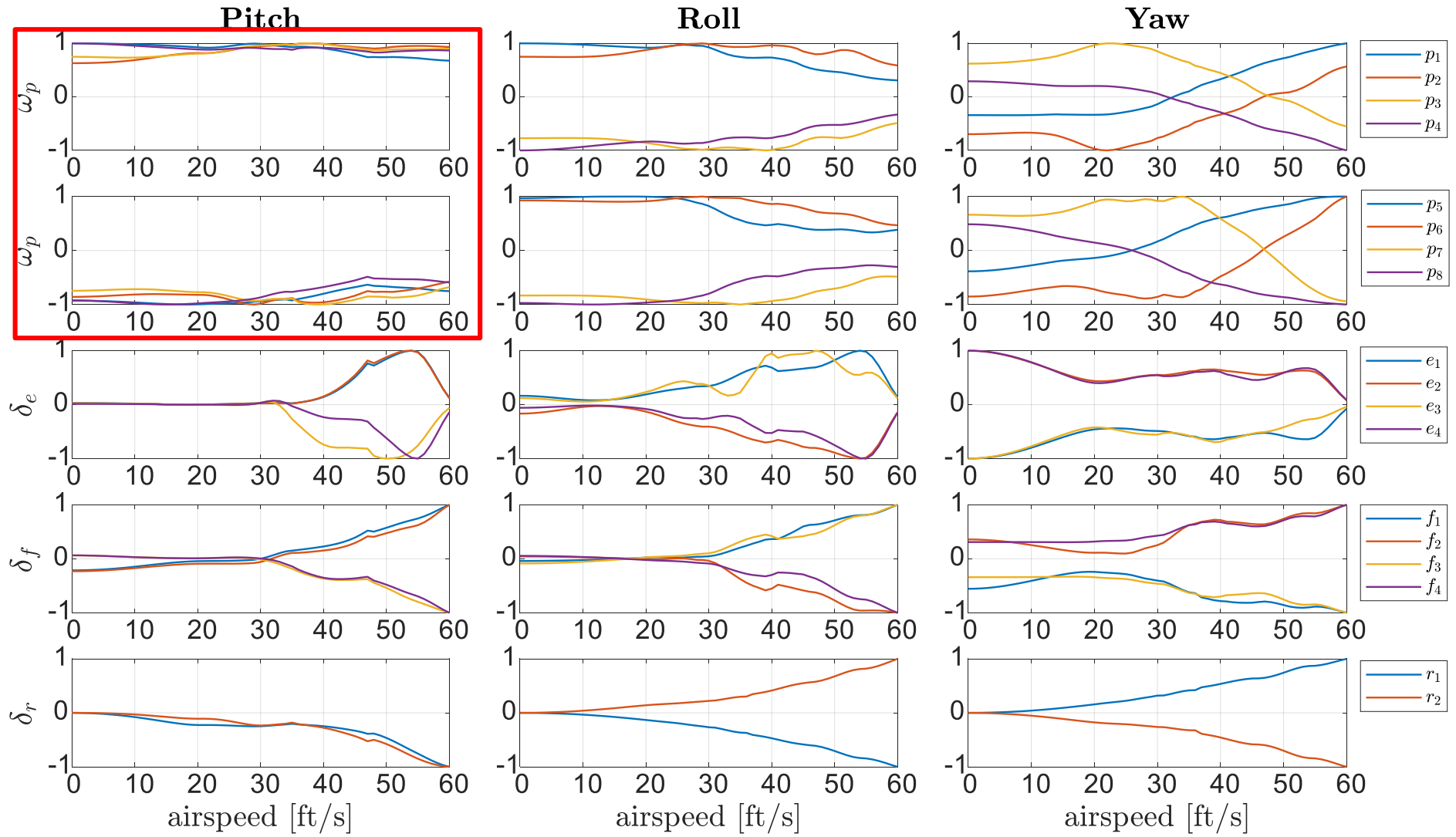


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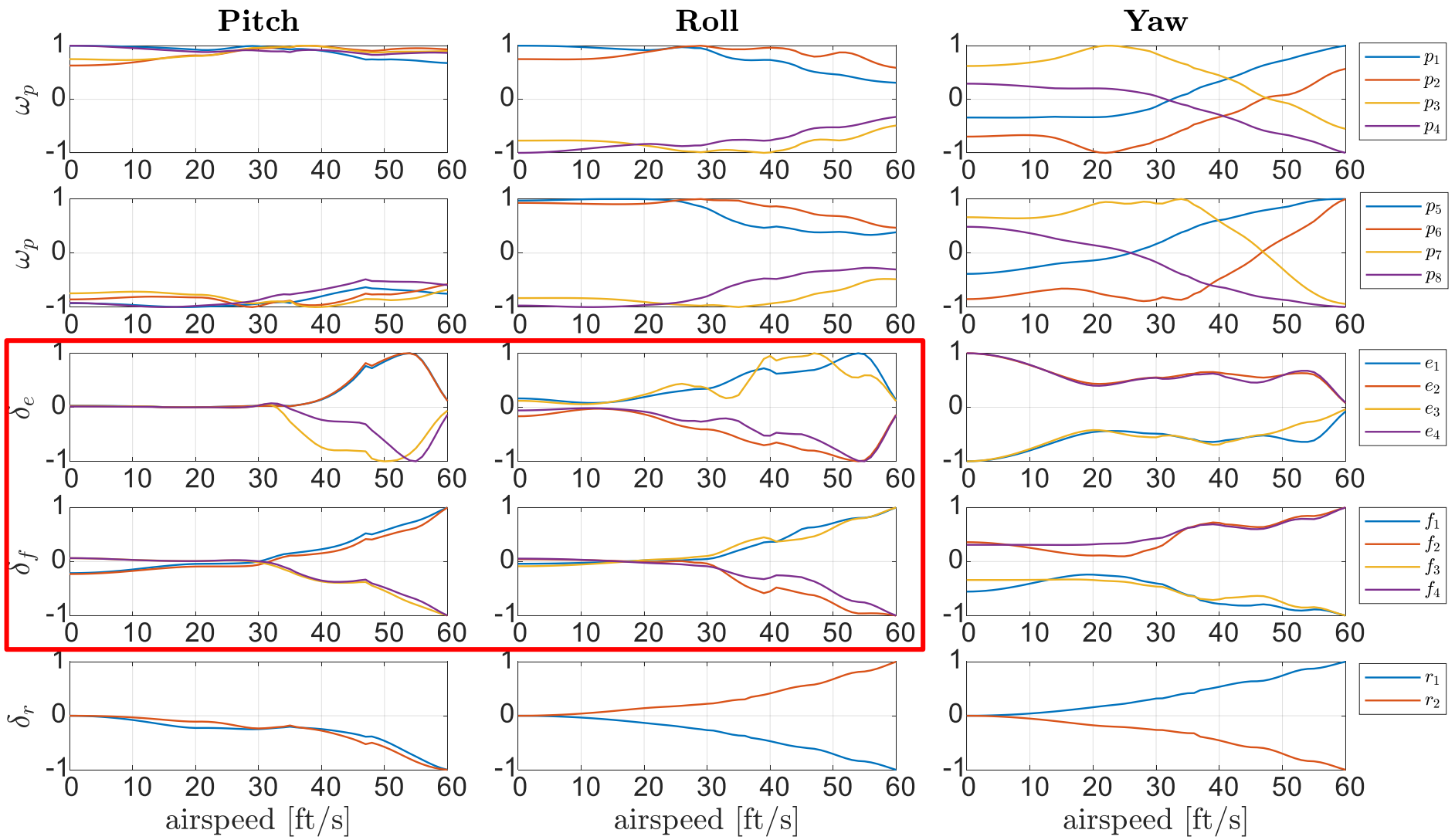


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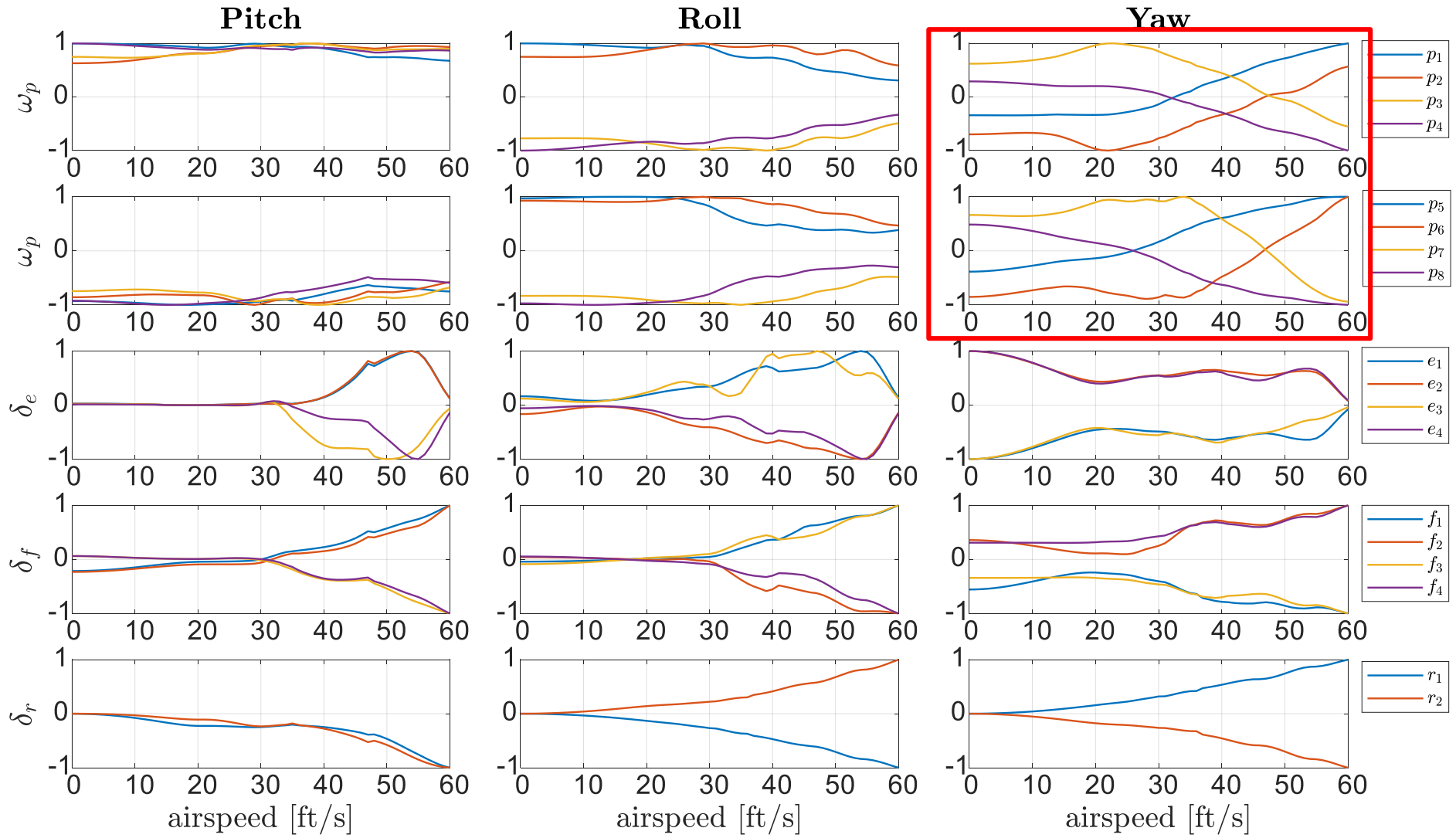


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# Control Authority

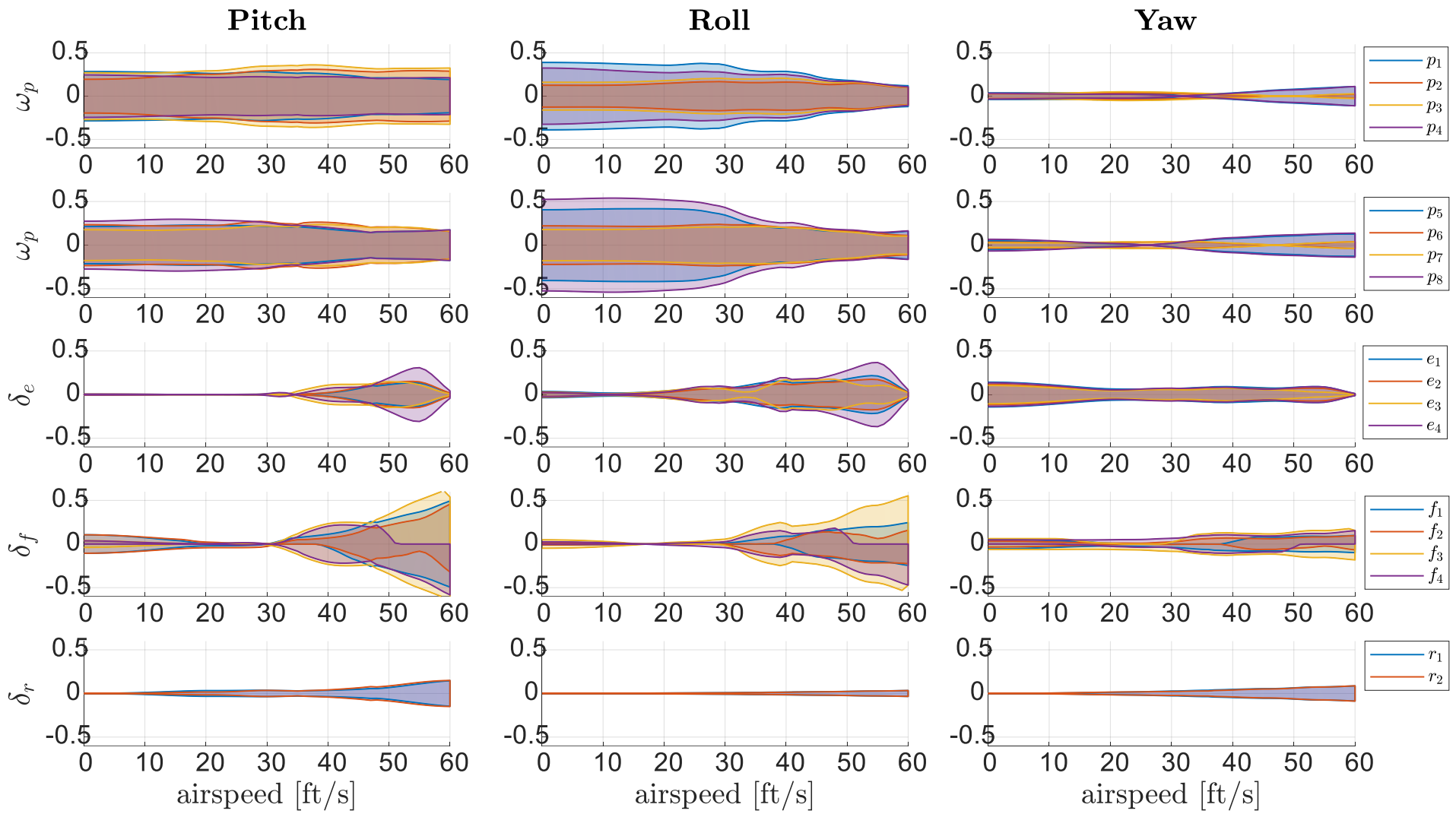
How much **control authority** does each actuator provide?

Nominal Actuator Deflections

| $\Delta\omega_p$ | $\Delta\delta_i$ | $\Delta\delta_e$ | $\Delta\delta_f$ | $\Delta\delta_r$ |
|---|---|---|---|---|
| 200 RPM | 2° | 10° | 5° | 10° |

**Control Authority Def:**

$$Auth = F_\delta \Delta\delta_{nom}$$



# Control Authority

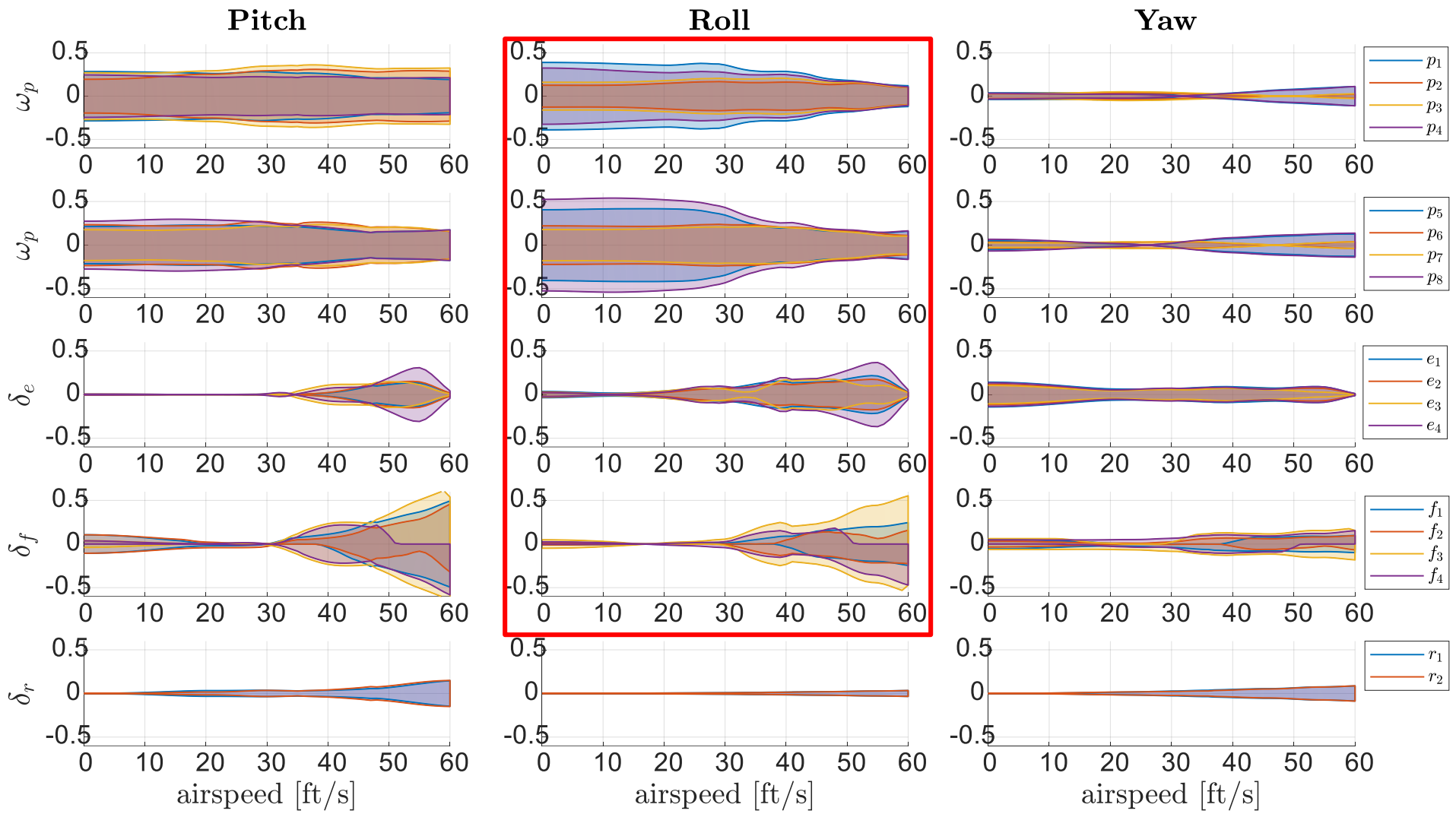
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# Control Authority

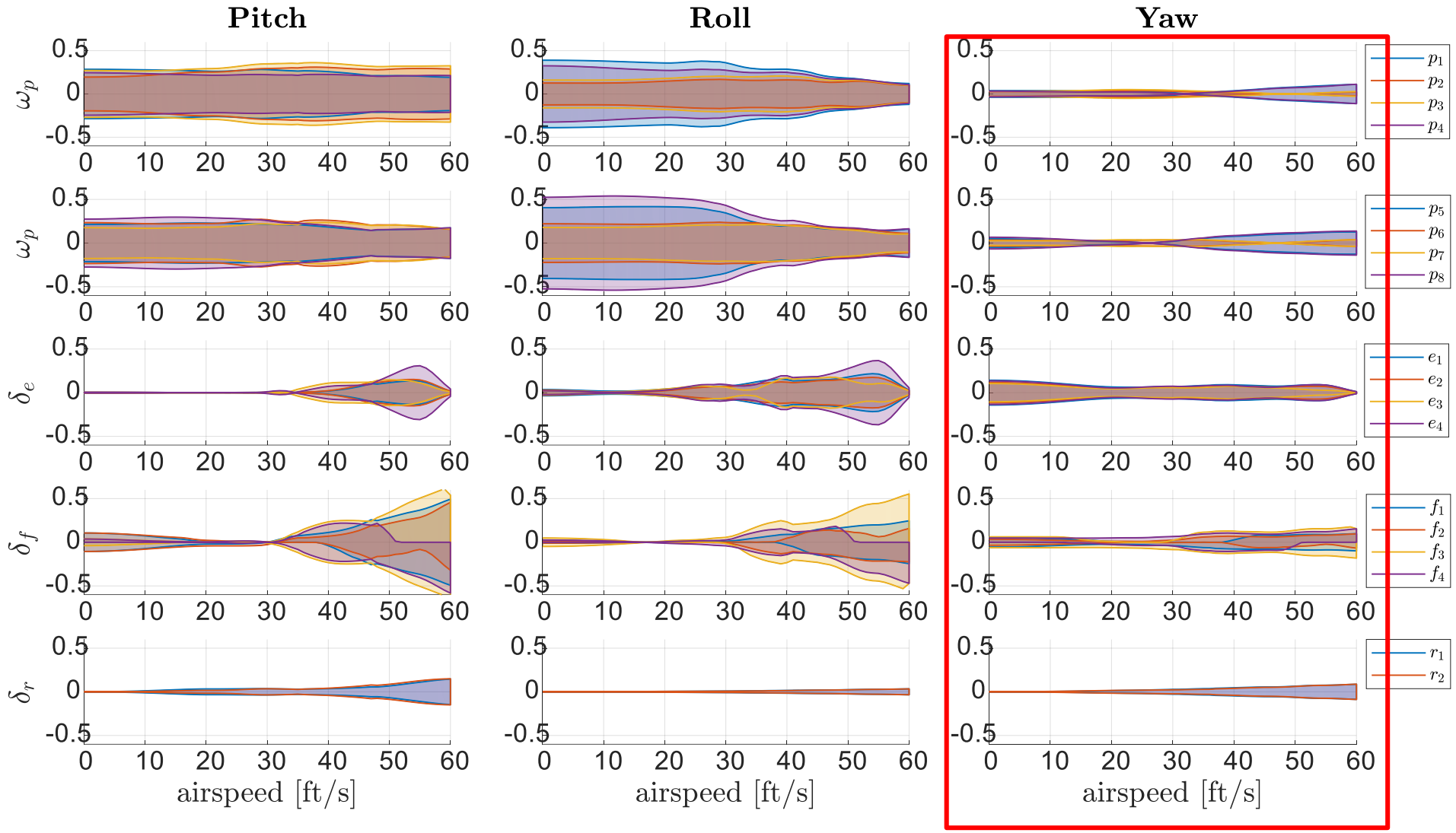
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**Control Authority Def:**

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# *Uniform Control Architecture*

# Optimal Control and Control Allocation

Given the linear system

$$\dot{x} = Ax + B_u u,$$
$$x \in R^n, u \in R^m$$

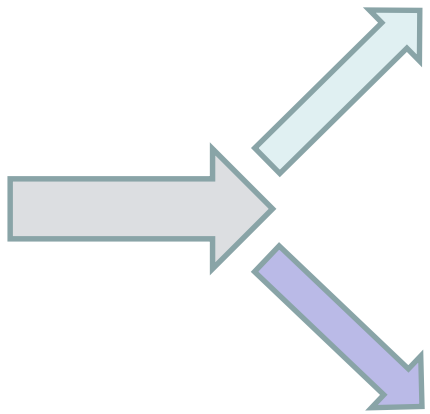
Determine $u(t)$ by solving

$$\min_{u(\cdot)} \int_0^\infty [x^T Q x + u^T R_u u] dt$$

Where $Q \geq 0$ and $R_u = R_u^T > 0$

$$\text{rank } B_u = k < m$$

- Factorize: $B_u = B_\mu B$
- $B_\mu \in R^{n \times k}$, $B \in R^{k \times m}$ both have rank $k$



Alternative Description

$$\dot{x} = Ax + B_\mu \mu$$
$$\mu = Bu$$

**Performance:**
Determine $\mu(t)$ by solving

$$\min_{v(\cdot)} \int_0^\infty \left[x^T Q x + \mu^T R_\mu \mu\right] dt$$

Where $Q \geq 0$ and $R_\mu = R_\mu^T > 0$

**Actuation:**
Then determine $u(t)$ by solving

$$\min_u \quad u^T W u$$
$$\text{subject to} \quad Bu = \mu$$

Where $W = W^T > 0$

# Feedback Performance Design

Robust Servomechanism LQR
- Embeds a model of the class of signals to be tracked to the state space representation

Applying optimal control provides:
- Accurate tracking
- Predictable and robust performance

Use of General Control Inputs
- Removes ambiguity due to relative effectiveness of propulsive and aerodynamic actuators

**Linear System with General Control Inputs**

$$\dot{x} = Ax + B_\mu \mu$$
$$y = Cx$$

**Robust Servo Mechanism Control (State Feedback)**

$$\underbrace{\begin{bmatrix}\dot{x}_i \\ \dot{x}\end{bmatrix}}_{\dot{\tilde{x}}} = \underbrace{\begin{bmatrix}0 & I \\ 0 & A\end{bmatrix}}_{\tilde{A}} \begin{bmatrix}x_i \\ x\end{bmatrix} + \underbrace{\begin{bmatrix}0 \\ B\end{bmatrix}}_{\tilde{B}} \mu$$

**LQR Control Design**

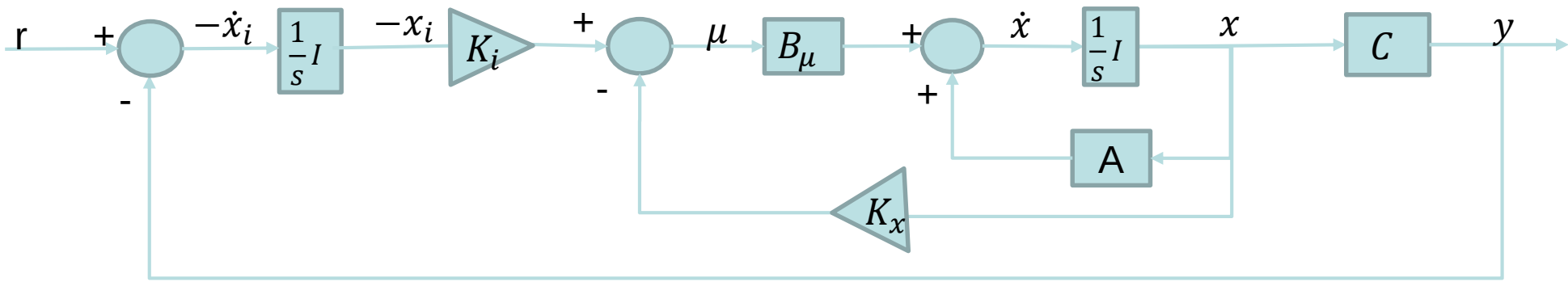
$$J = \int_0^\infty \tilde{x}^T Q \tilde{x} + \mu^T R \mu \, dt$$

For $Q = Q^T > 0$ and $R = R^T \geq 0$

**Optimal General Control**

$$\mu = -K\tilde{x} = -[K_i \quad K_x] \begin{bmatrix}x_i \\ x\end{bmatrix}$$

where $K = R^{-1}\tilde{B}^T P$ and $P$ is the solution to the algebraic Riccati equation



# Control Allocation

- Uses a weighted pseudoinverse
- Use virtual controls, $\phi$ and $\theta$, to handle relative control degree greater than zero
- Easily incorporate more sophisticated control allocation algorithms

**Actuation:**
Then determine $u(t)$ by solving

$$\min_{u(t)} \quad u^T W u$$
$$\text{subject to} \quad Bu = \mu$$

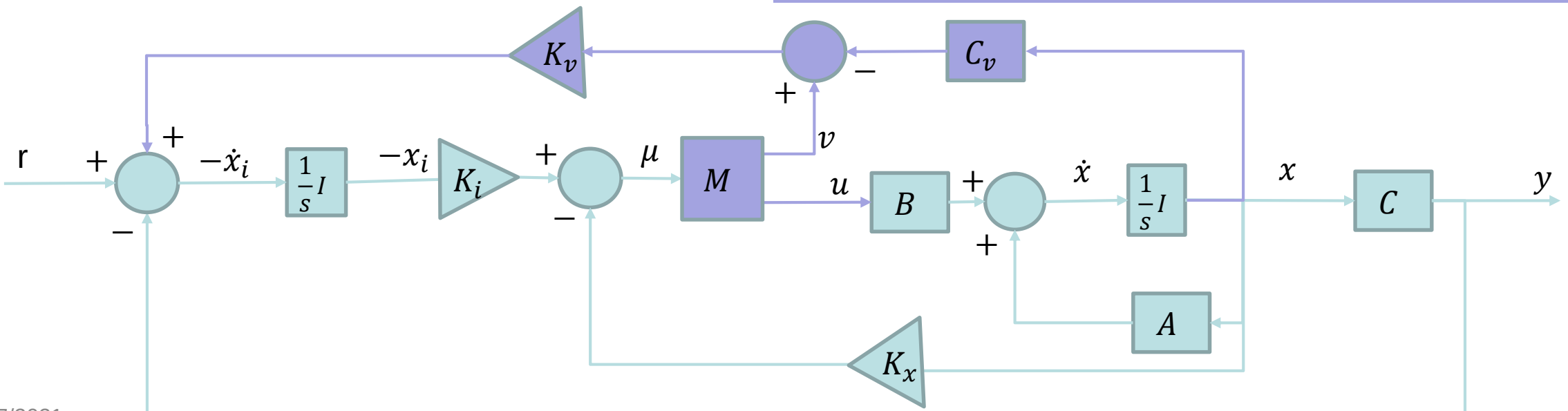
Where $W = W^T > 0$

**Actuator Input**

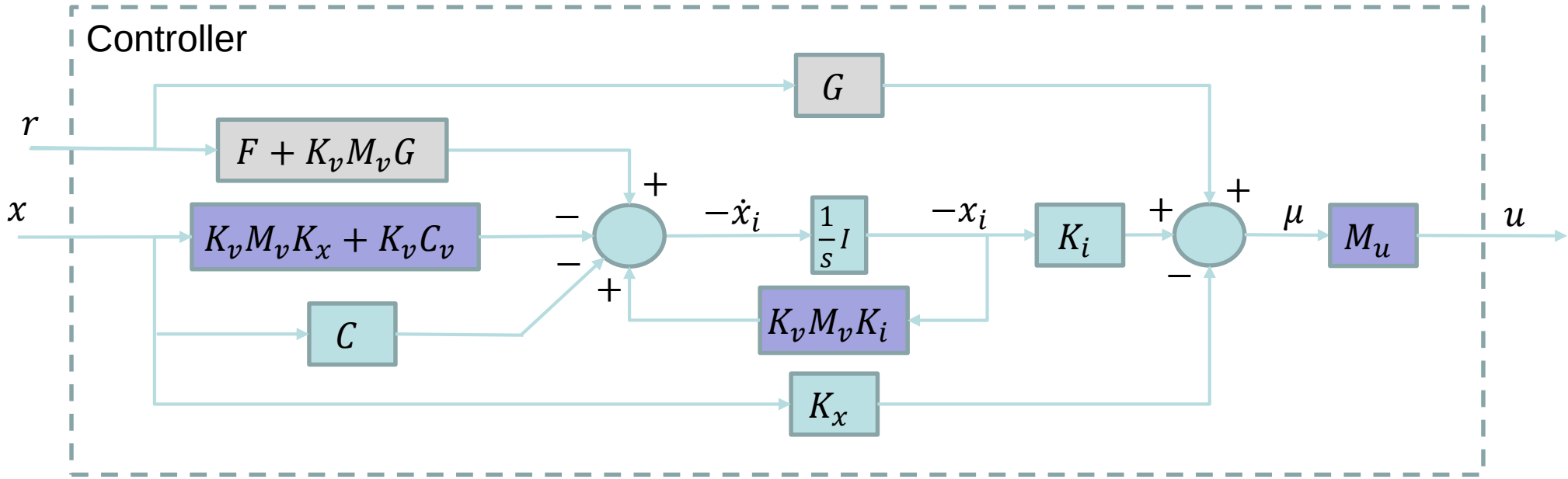
$$u = \underbrace{W^{-1}B^T(BW^{-1}B^T)^{-1}}_{M_u}\mu$$

**Virtual Control**

$$\bar{B} = [B \mid A_v], \qquad A_v \subset A$$
$$\begin{bmatrix} u \\ v \end{bmatrix} = M\mu = \begin{bmatrix} M_u \\ M_v \end{bmatrix}\mu$$



# General VTOL Control Structure – State Feedback




- Provide a uniform approach to control design throughout the entire flight envelope
- Make use of well established robust control design techniques
- Handle redundant actuators in an intuitive way
- Applicable to all most VTOL configurations

# Longitudinal Dynamics and Control Design

**Steady Level Flight** ($\bar{v} = p = r = 0$)

$$\begin{bmatrix} \dot{\bar{u}} \\ \dot{\bar{w}} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{pmatrix} \dot{\psi}\bar{v} + \frac{1}{m}\bar{X}(x,u) \\ g + \frac{1}{m}\bar{Z}(x,u) \\ \frac{1}{J_y}M(x,u) \\ q \end{pmatrix}, \qquad x = \begin{bmatrix} \bar{u} \\ \bar{w} \\ q \\ \theta \end{bmatrix}, \qquad u = \begin{bmatrix} \omega_p \\ \delta_e \\ \delta_f \\ \delta_r \end{bmatrix}$$

**Control Design States**

$$\xi = [\bar{u} \quad \bar{w} \quad q]^T$$

**General Control *Inputs***

$$\mu = [\bar{a}_x \quad \bar{a}_z \quad \alpha_q]^T$$

**Physical Control Inputs**

$$u = \left[\omega_p^T \quad \delta_e^T \quad \delta_f^T \quad \delta_r^T\right]^T$$

**Virtual Control Inputs**

$$v = \theta$$

**Reference Inputs**

$$r = [\bar{u}_{des} \quad \bar{w}_{des}]$$

## RSLQR Control Parameters

$$A = \begin{bmatrix} \frac{1}{m}\bar{X}_{\bar{u}} & \frac{1}{m}\bar{X}_{\bar{w}} & \frac{1}{m}\bar{X}_q \\ \frac{1}{m}\bar{Z}_{\bar{u}} & \frac{1}{m}\bar{Z}_{\bar{w}} & \frac{1}{m}\bar{Z}_q \\ \frac{1}{J_y}M_{\bar{u}} & \frac{1}{J_y}M_{\bar{w}} & \frac{1}{J_y}M_q \end{bmatrix}$$

$$Q = diag([q_{\bar{u}} \quad q_{\bar{w}} \quad q_q \quad 0 \quad 0 \quad 0])$$

$$R = diag([r_{\bar{a}_x} \quad r_{\bar{a}_z} \quad r_{\alpha_q}])$$

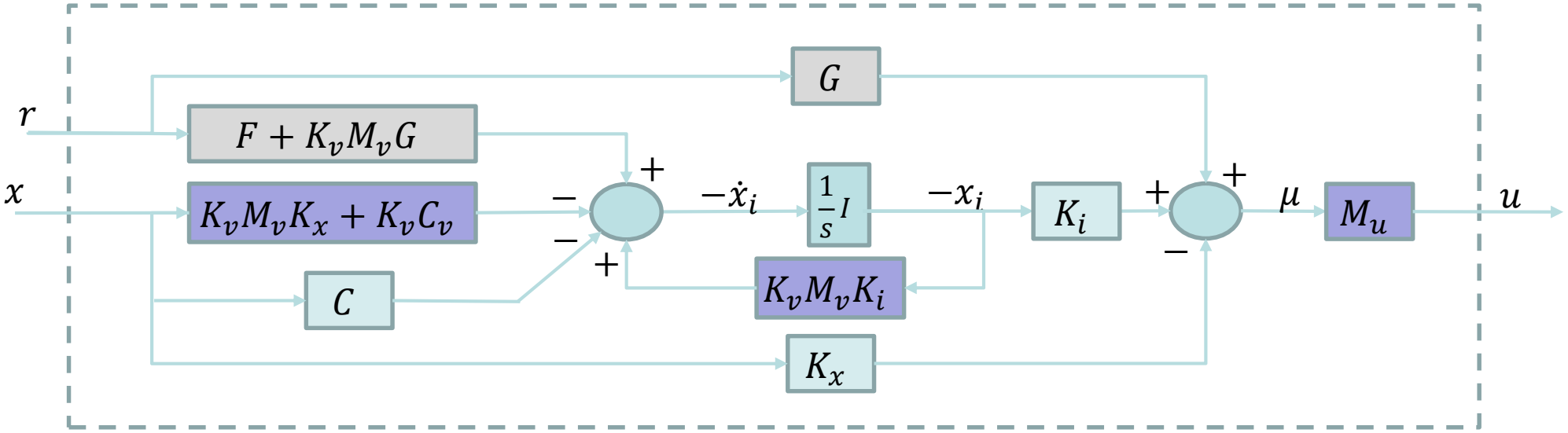
## Control Allocation Parameters

$$\bar{B} = \left[\begin{array}{ccc|c} \frac{1}{m}\bar{X}_{\omega_r} & \frac{1}{m}\bar{X}_{\omega_p} & \frac{1}{m}\bar{X}_{\delta_e} & \frac{1}{m}\bar{X}_\theta \\ \frac{1}{m}\bar{Z}_{\omega_r} & \frac{1}{m}\bar{Z}_{\omega_p} & \frac{1}{m}\bar{Z}_{\delta_e} & \frac{1}{m}\bar{Z}_\theta \\ \frac{1}{J_y}M_{\omega_r} & \frac{1}{J_y}M_{\omega_p} & \frac{1}{J_{yy}}M_{\delta_e} & \frac{1}{m}M_\theta \end{array}\right]$$

$$W = diag([w_{\omega_p} \quad w_{\delta_e} \quad w_{\delta_f} \quad w_{\delta_r} \mid w_\theta])$$

$$M = W^{-1}\bar{B}^T(\bar{B}W^{-1}\bar{B}^T)^{-1}$$

# Longitudinal Dynamics and Control Design




RSLQR

$$x = [\bar{u} \quad \bar{w} \quad q \quad \theta]^T$$

$$x_i = \left[\int e_{\bar{u}} \quad \int e_{\bar{w}} \quad \int e_q\right]^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Control Allocation

$$u = \left[\omega_p^T \quad \delta_e^T \quad \delta_f^T \quad \delta_r^T\right]^T$$

$$v = \theta$$

$$M = \begin{bmatrix} M_u \\ M_v \end{bmatrix}$$

$$C_v = [0 \quad 0 \quad 0 \quad 1]$$

$$K_v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Reference

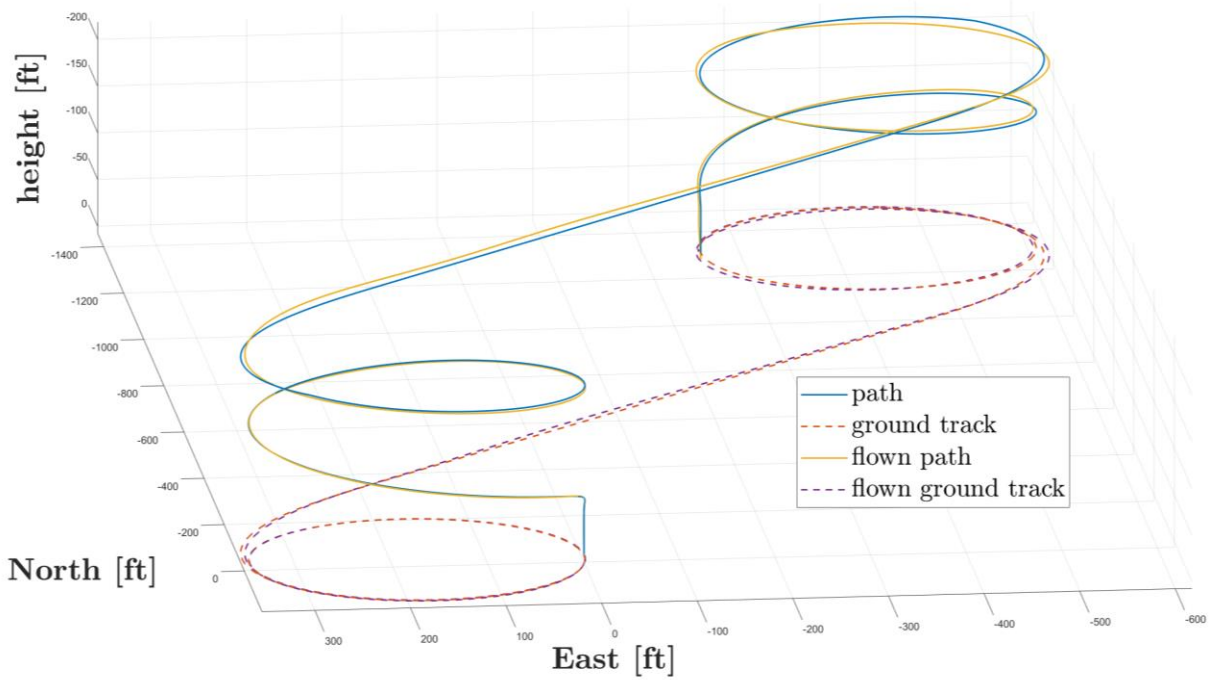
$$r = [\bar{u}_{des} \quad \bar{w}_{des}]^T$$

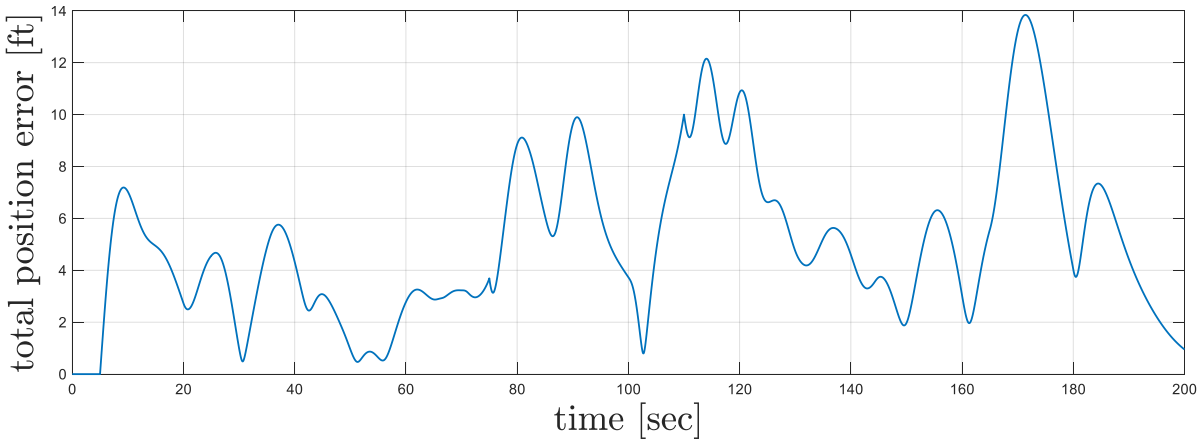
$$F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

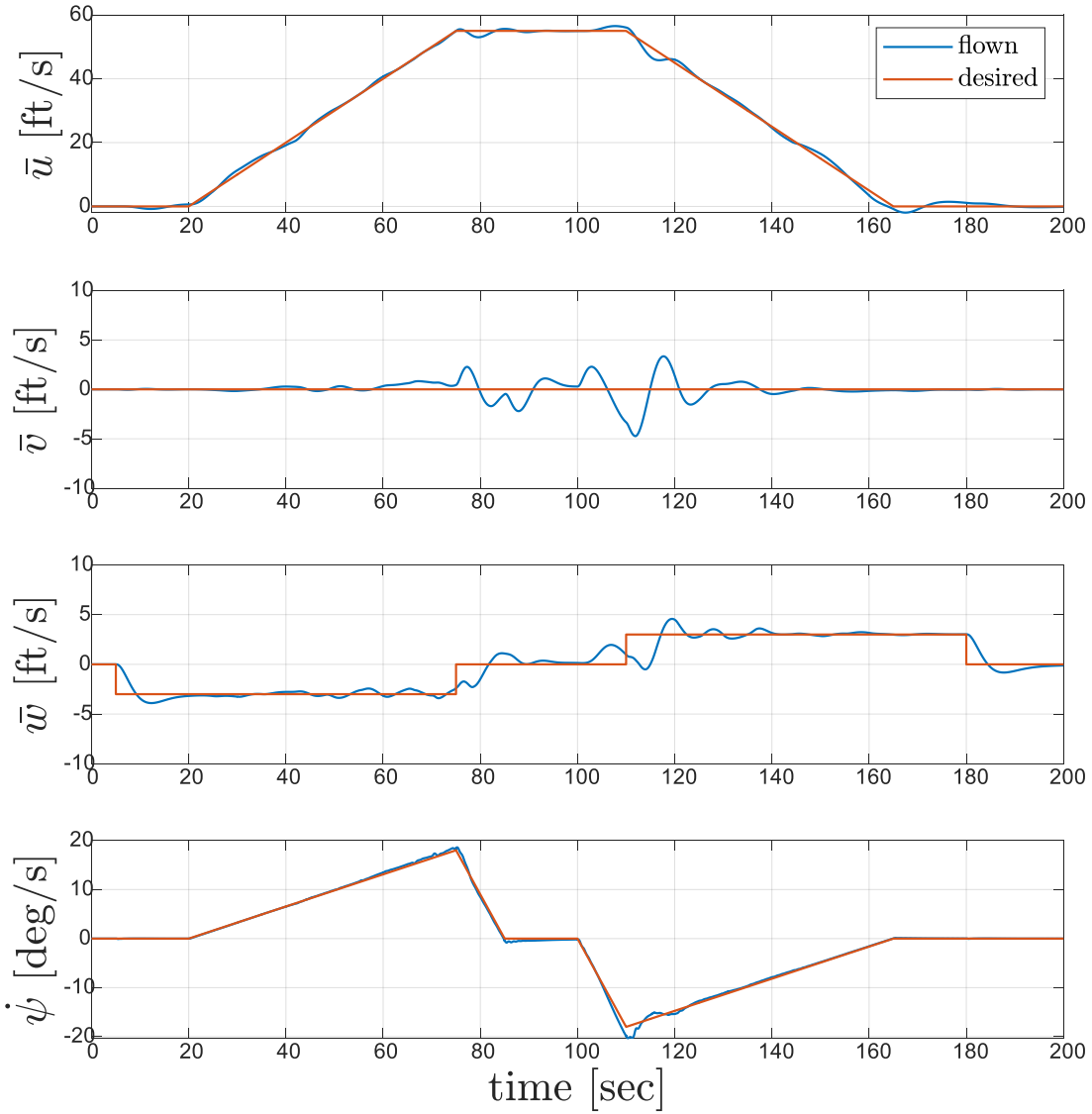
$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# *Transition Trajectory Tracking*

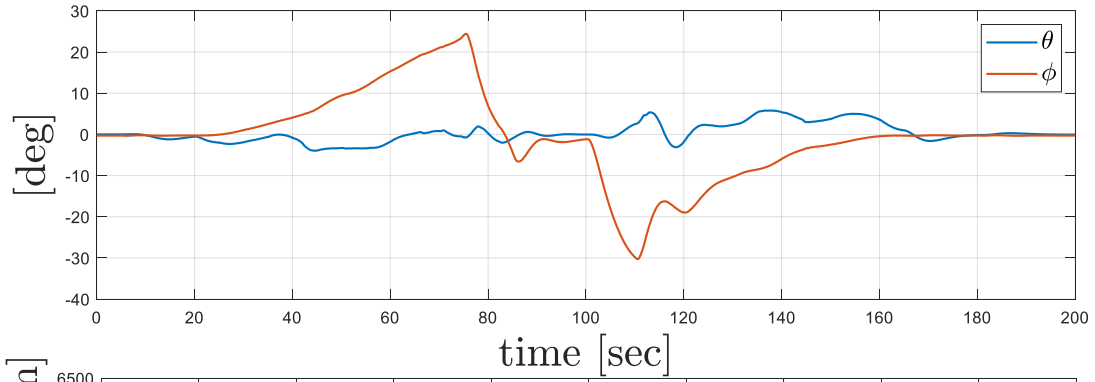
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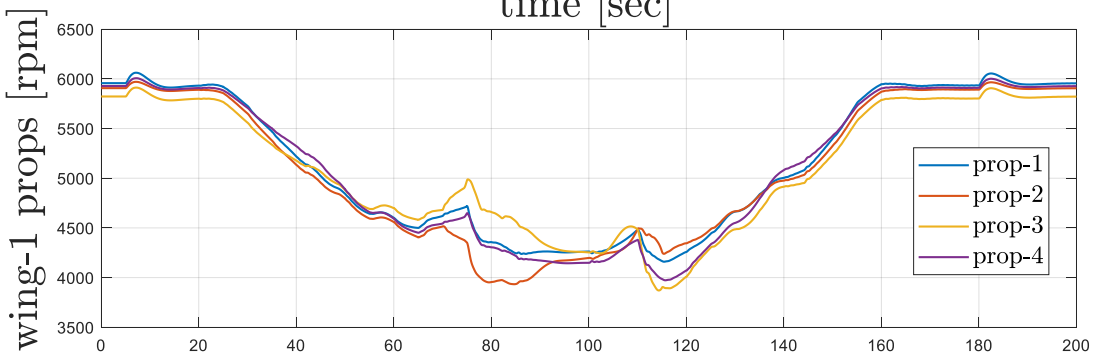


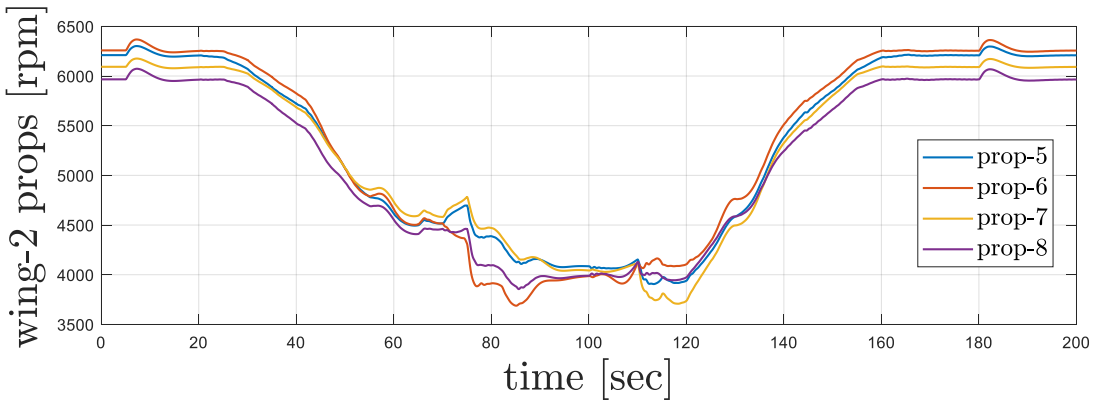



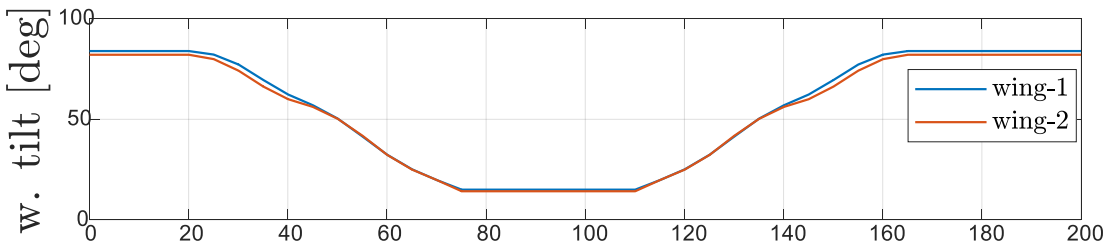



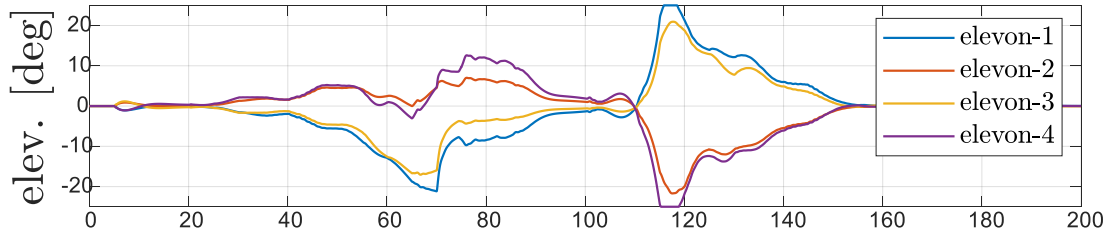
# Transition Trajectory Tracking

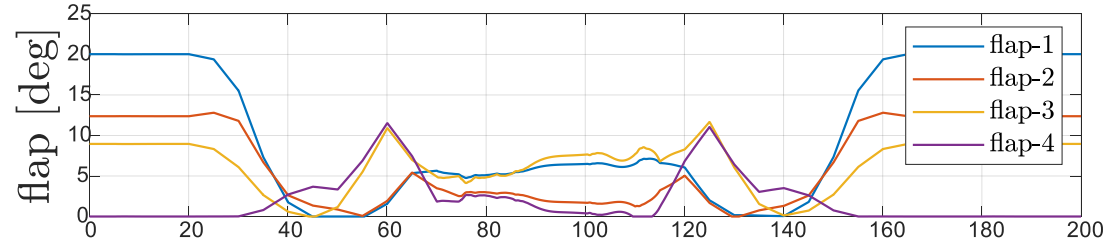


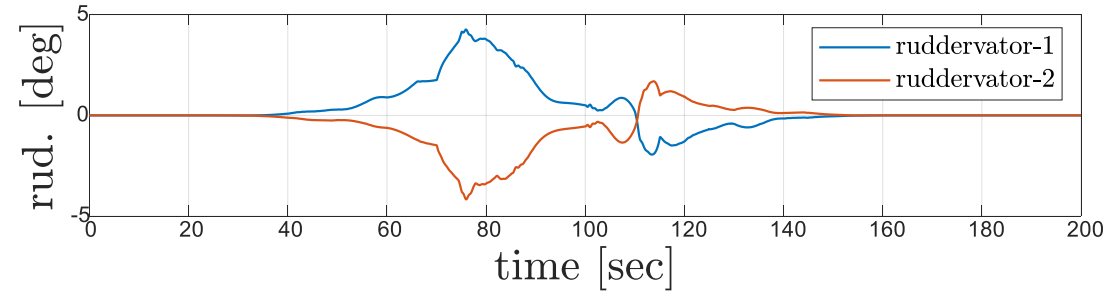



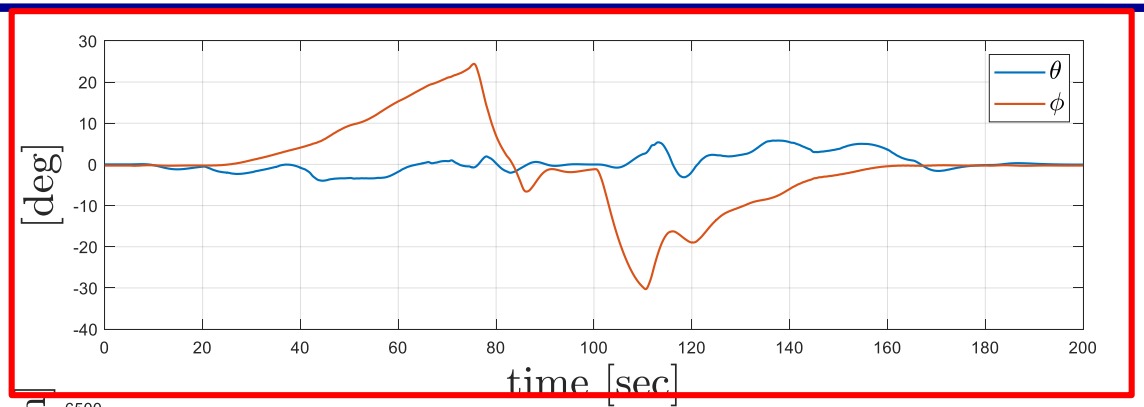



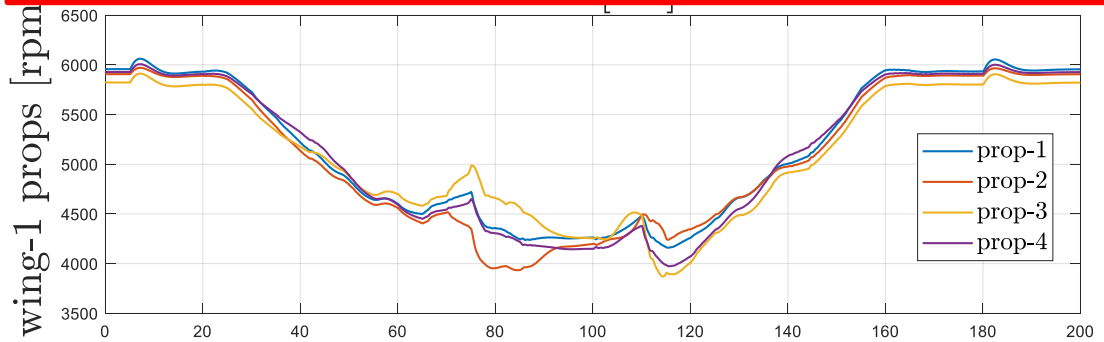



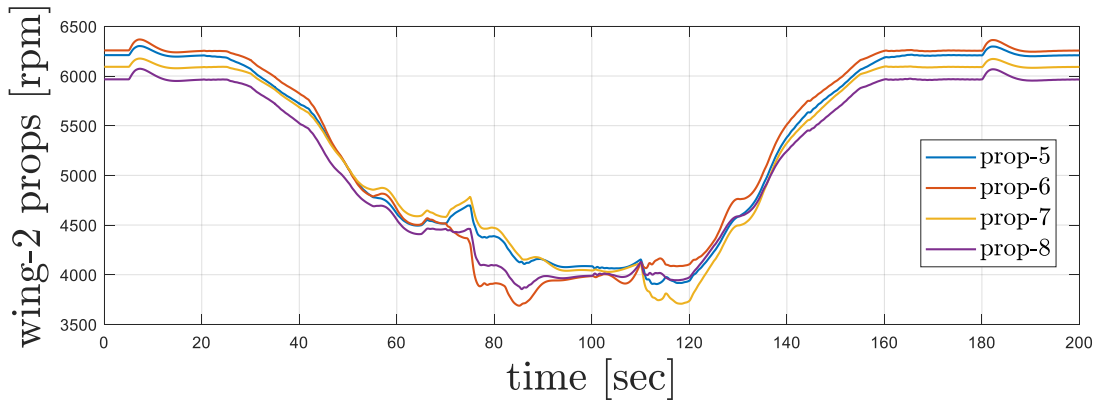



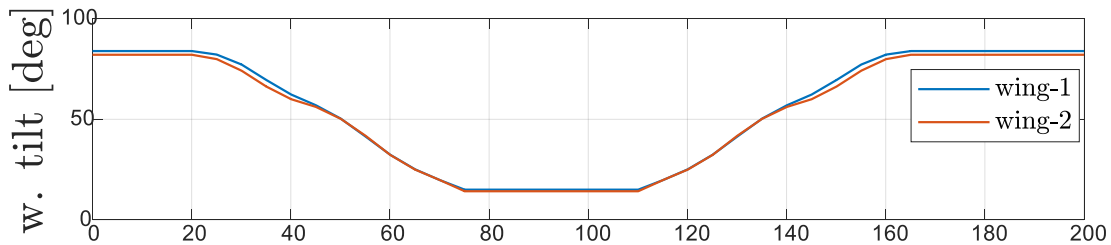



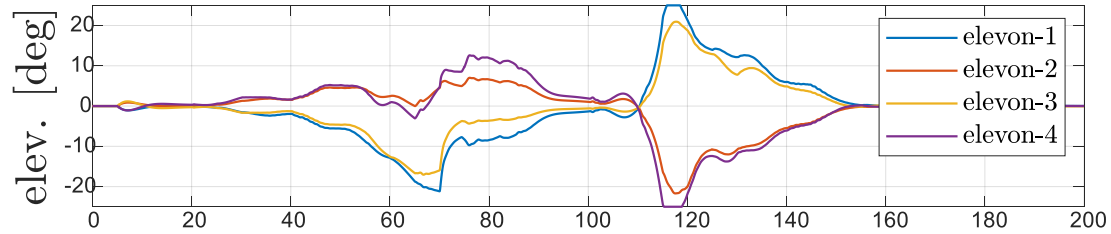
# Transition Trajectory Tracking

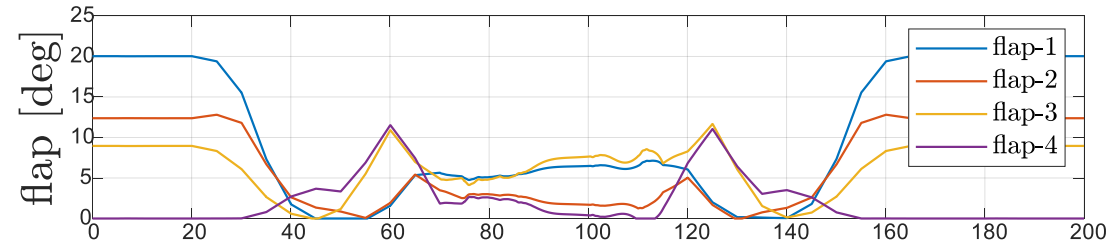


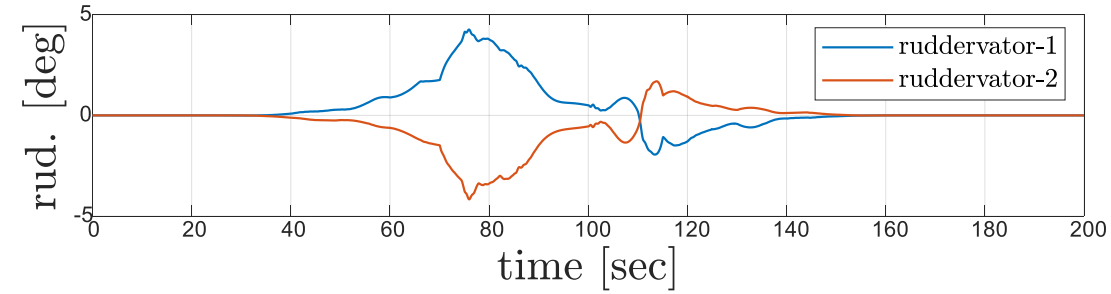



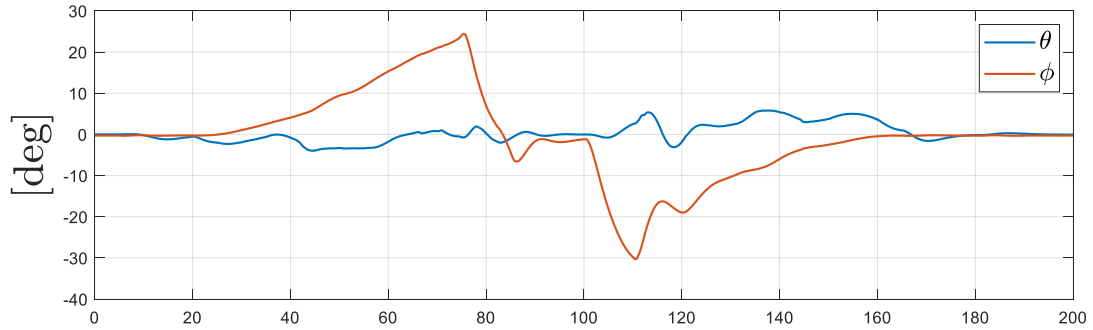



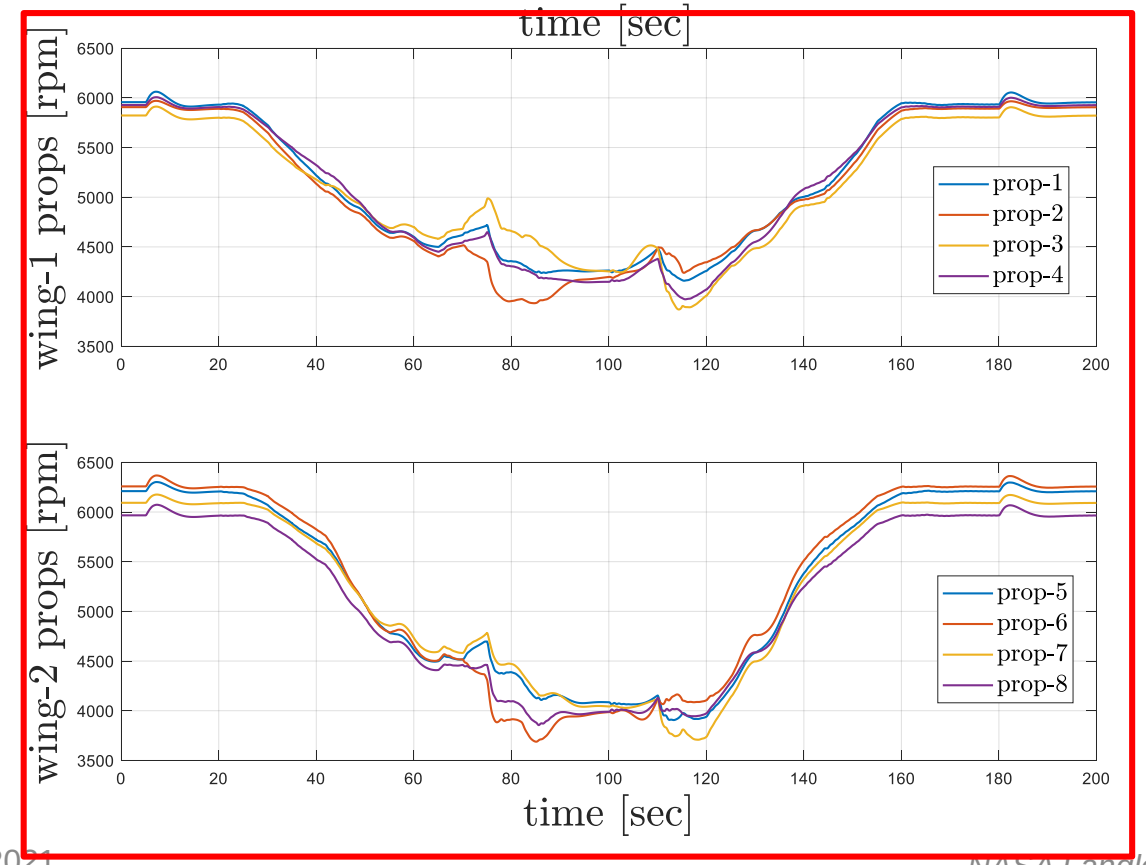



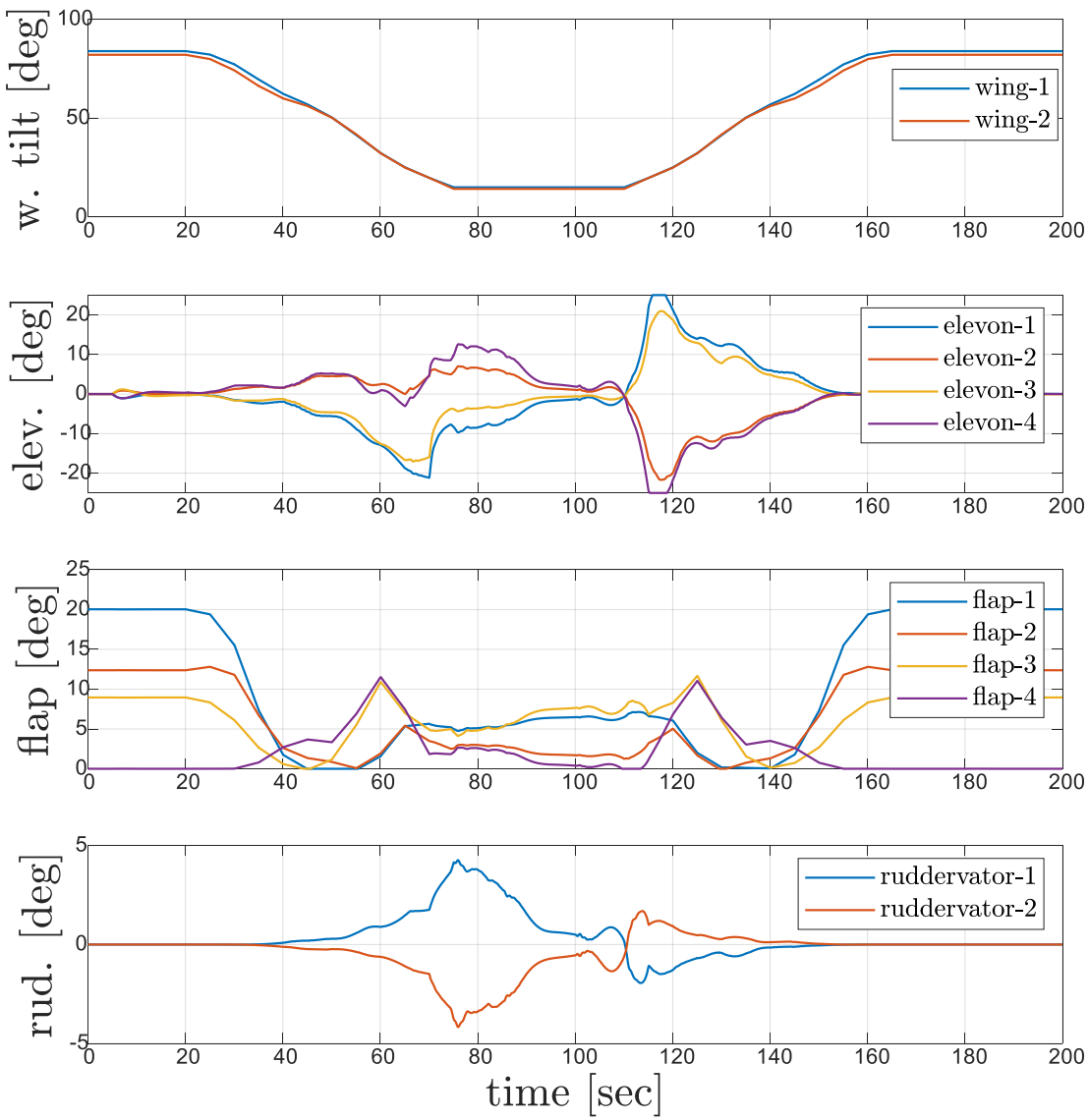



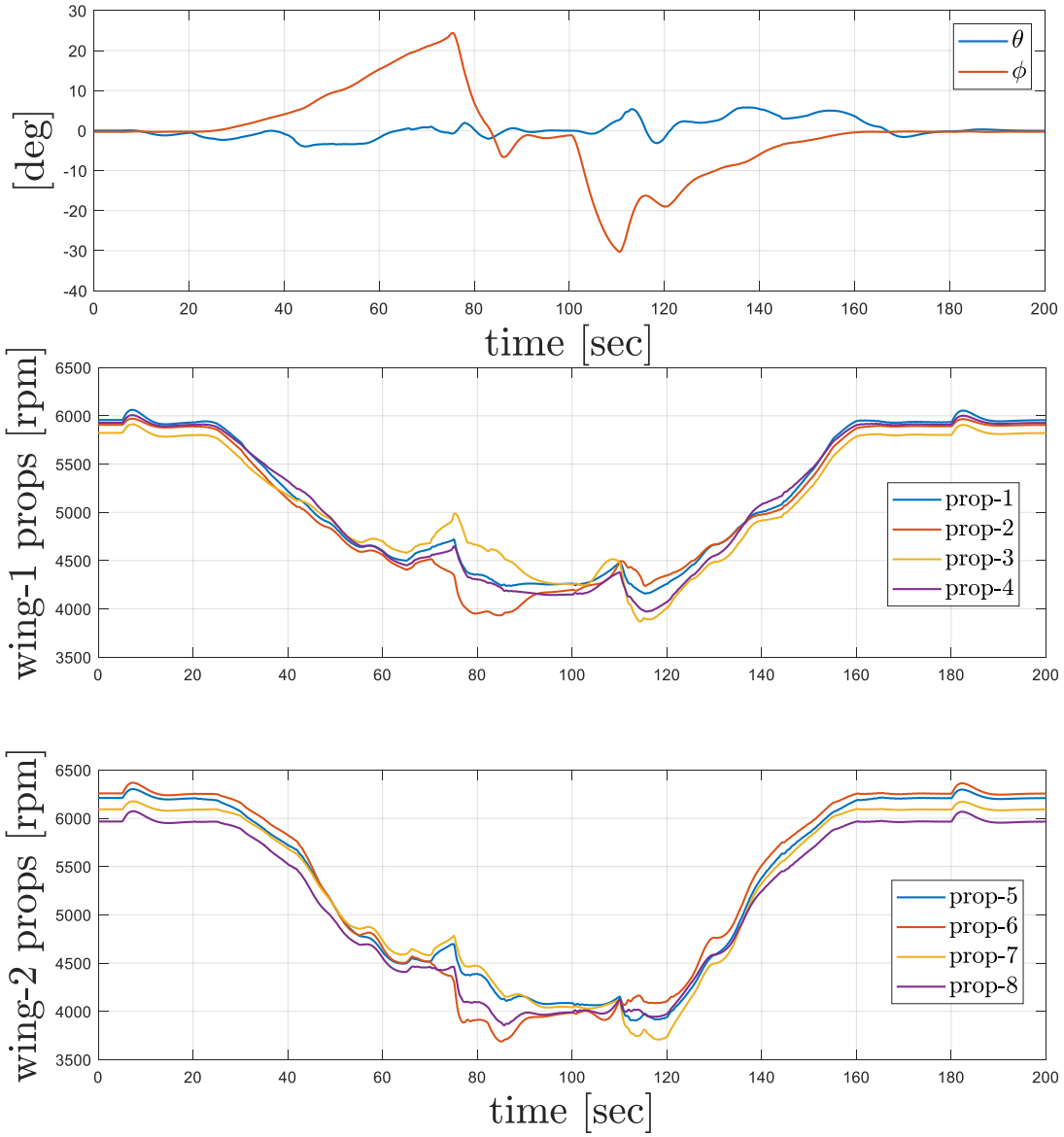
# Transition Trajectory Tracking

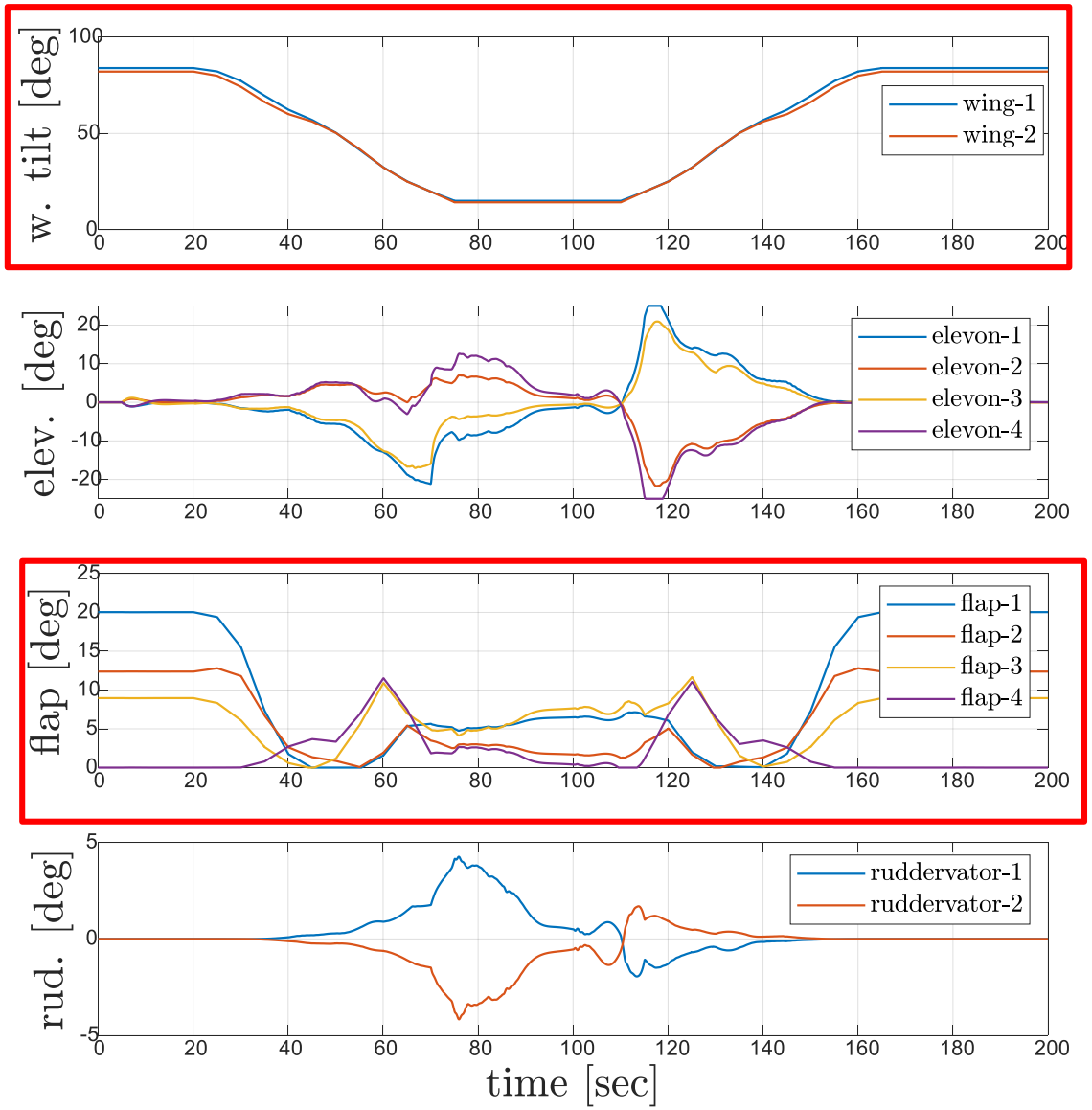


# Transition Trajectory Tracking



# Conclusions

- Robust Uniform Control architecture that simplifies the control design process for complex transitioning VTOL aircraft
- Formulation of the dynamics in the heading frame allowed for a uniform set of reference commands in all flight regimes
- Redundancy of control was handled by splitting control design into two processes, feedback performance, and control allocation
- Applicable to all UAM configurations, and demonstrated on a tandem tiltwing VTOL aircraft






# Questions

# Lateral Dynamics and Control Design

**Steady Level Flight** ($\bar{w} = q = 0$)

$$\begin{bmatrix} \dot{\bar{v}} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{pmatrix} -\dot{\psi}\bar{u} + \frac{1}{m}\bar{Y}(x,u) \\ \frac{1}{J_{xz}^2 - J_x J_z}\left(J_{xz}N(x,u) - J_z L(x,u)\right) \\ \frac{-1}{J_{xz}^2 - J_x J_z}\left(J_x N(x,u) - J_{xz} L(x,u)\right) \\ p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \end{pmatrix}, \quad x = \begin{bmatrix} \bar{v} \\ p \\ r \\ \phi \end{bmatrix}, \quad u = \begin{bmatrix} \omega_p \\ \delta_e \\ \delta_f \\ \delta_r \end{bmatrix}$$

**Control Design States**

$$\xi = [\bar{v} \quad p \quad r]^T$$

**General Control Inputs**

$$\mu = [\bar{a}_y \quad \alpha_p \quad \alpha_r]^T$$

**Physical Control Inputs**

$$u = \left[\omega_p^T \quad \delta_e^T \quad \delta_f^T \quad \delta_r^T\right]^T$$

**Virtual Control Inputs**

$$v = \phi$$

**Reference Inputs**

$$r = [\bar{v}_{des} \quad \dot{\psi}_{des}]$$

## RSLQR Controller Parameters

$$A = \begin{bmatrix} \frac{1}{m}\bar{Y}_{\bar{v}} & \frac{1}{m}\bar{Y}_p & \frac{1}{m}\bar{Y}_r \\ \frac{1}{\bar{J}}\left(J_{xz}N_{\bar{v}} - J_z L_{\bar{v}}\right) & \frac{1}{\bar{J}}\left(J_{xz}N_p - J_z L_p\right) & \frac{1}{\bar{J}}\left(J_{xz}N_r - J_z L_r\right) \\ \frac{-1}{\bar{J}}\left(J_x N_{\bar{v}} - J_{xz} L_{\bar{v}}\right) & \frac{-1}{\bar{J}}\left(J_x N_p - J_{xz} L_p\right) & \frac{-1}{\bar{J}}\left(J_x N_r - J_{xz} L_r\right) \end{bmatrix}$$

$$Q = diag([q_{\bar{v}} \quad q_p \quad q_r \quad 0 \quad 0 \quad 0])$$

$$R = diag([r_{\bar{a}_y} \quad r_{\alpha_p} \quad r_{\alpha_r}])$$

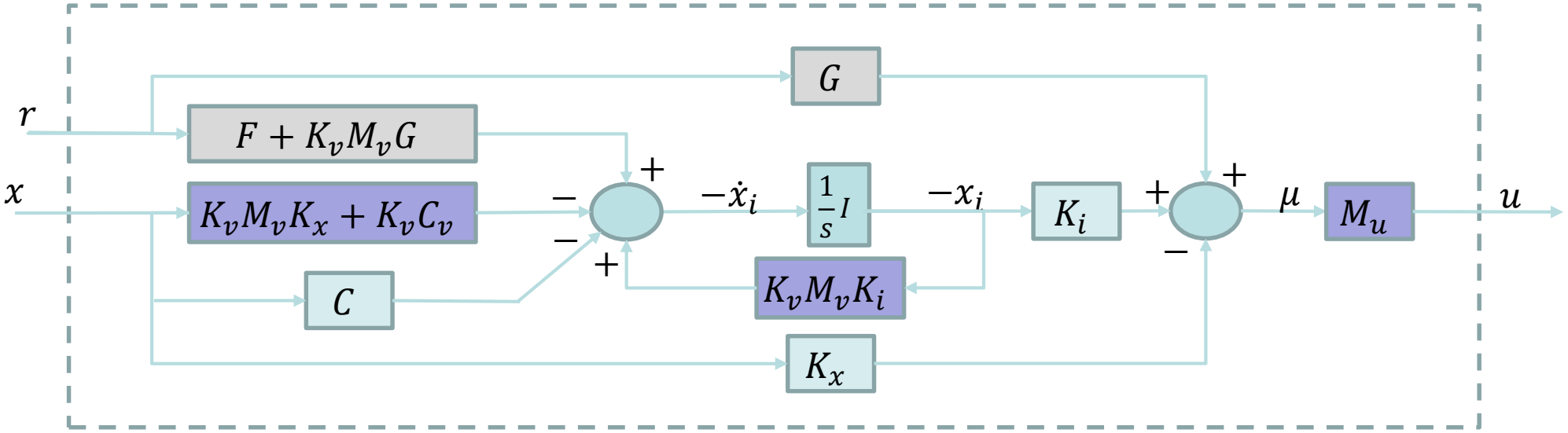
## Control Allocation Parameters

$$\bar{B} = \left[\begin{array}{ccc|c} \frac{1}{m}\bar{Y}_{\boldsymbol{\omega}_r} & \frac{1}{m}\bar{Y}_{\delta_a} & \frac{1}{m}\bar{Y}_{\delta_r} & \frac{1}{m}\bar{Y}_{\phi} \\ \frac{1}{\bar{J}}\left(J_{xz}N_{\boldsymbol{\omega}_r} - J_z L_{\boldsymbol{\omega}_r}\right) & \frac{1}{\bar{J}}\left(J_{xz}N_{\delta_a} - J_z L_{\delta_a}\right) & \frac{1}{\bar{J}}\left(J_{xz}N_{\delta_r} - J_z L_{\delta_r}\right) & \frac{1}{\bar{J}}\left(J_{xz}N_{\phi} - J_z L_{\phi}\right) \\ \frac{-1}{\bar{J}}\left(J_x N_{\boldsymbol{\omega}_r} - J_{xz} L_{\boldsymbol{\omega}_r}\right) & \frac{-1}{\bar{J}}\left(J_x N_{\delta_a} - J_{xz} L_{\delta_a}\right) & \frac{-1}{\bar{J}}\left(J_x N_{\delta_r} - J_{xz} L_{\delta_r}\right) & \frac{-1}{\bar{J}}\left(J_x N_{\phi} - J_{xz} L_{\phi}\right) \end{array}\right]$$

$$W = diag([w_{\omega_p} \quad w_{\delta_e} \quad w_{\delta_f} \quad w_{\delta_r} \quad w_{\phi}])$$

$$M = W^{-1}\bar{B}^T(\bar{B}W^{-1}\bar{B}^T)^{-1}$$

# Lateral Dynamics and Control Design




RSLQR

$$x = [\bar{v} \quad p \quad r \quad \phi]^T$$

$$x_i = [\int e_{\bar{v}} \quad \int e_p \quad \int e_r]^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Control Allocation

$$u = [\omega_p^T \quad \delta_e^T \quad \delta_f^T \quad \delta_r^T]^T$$

$$v = \phi$$

$$M = \begin{bmatrix} M_u \\ M_v \end{bmatrix}$$

$$C_v = [0 \quad 0 \quad 0 \quad 1]$$

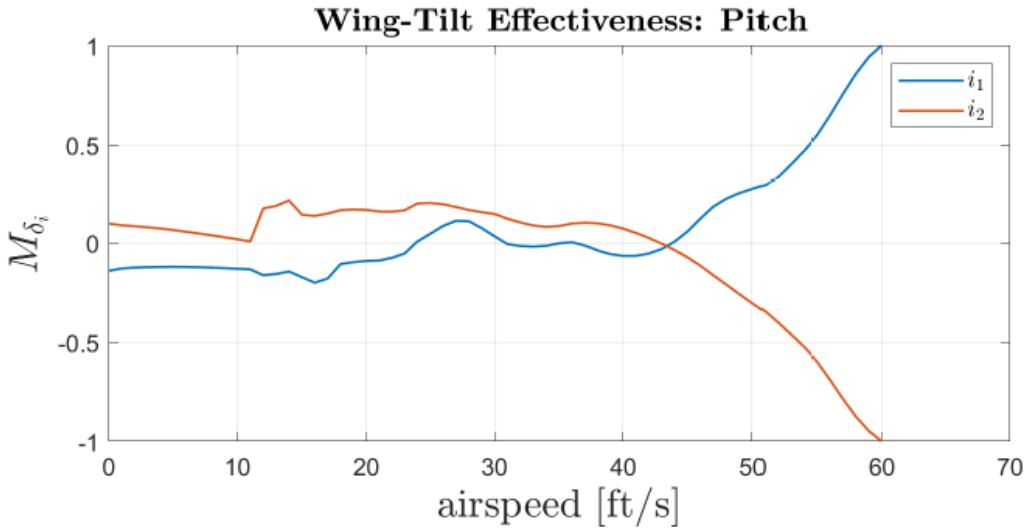
$$K_v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

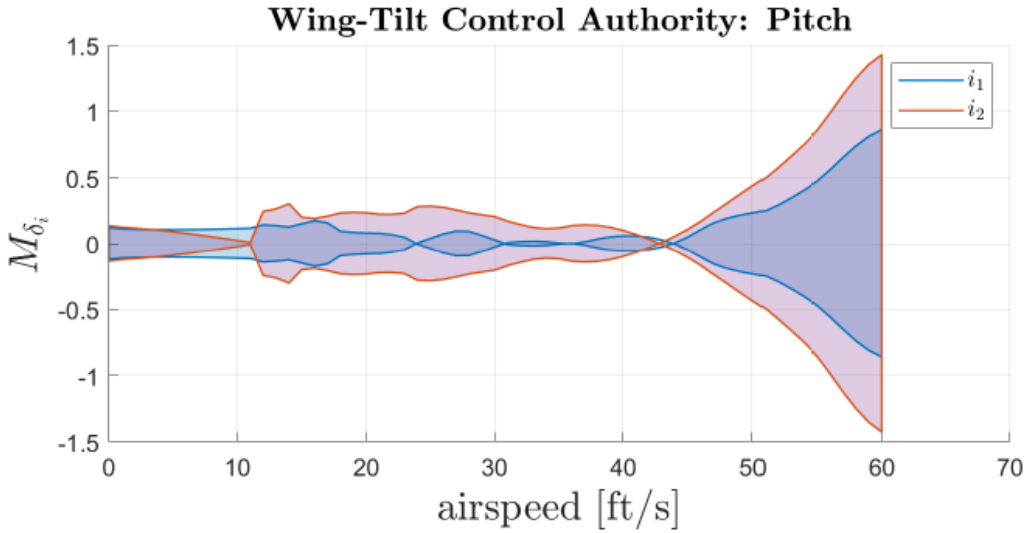
Reference

$$r = [\bar{v}_{des} \quad \dot{\psi}_{des}]^T$$

$$F = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 \\ 0 & \bar{u}_0 \\ 0 & 0 \end{bmatrix}$$


Wing-Tilt Effectiveness: Pitch
$M_{\delta_i}$
airspeed [ft/s]
$i_1$
$i_2$


Wing-Tilt Control Authority: Pitch
$M_{\delta_i}$
airspeed [ft/s]
$i_1$
$i_2$