Supporting Material for:
Direct Multipoint Observations Capturing the Reformation of a Supercritical Fast Magnetosonic Shock

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Calculating the Local Bow Shock Orientation:
Local bow shock normal direction from the Fairfield [1971] model:

$$\mathbf{n}_{bs} = [0.974, 0.190, 0.127] \text{ in GSE}$$

Upstream magnetic field (average from MMS-1):

$$\mathbf{B} = [2.00, 2.37, 0.35] \text{ nT in GSE}$$

Angle between bow shock normal and upstream B-field:

$$\theta_{Bn} = 38.5 \text{ degrees}$$

So, MMS were in the quasi-parallel foreshock, consistent with plasma and field observations and the presence of the foreshock transient structure.

Calculating the Foreshock Transient's Shock Normal Direction and Orientation:
Using coplanarity with B and V from Schwartz [1998]:

$$\mathbf{n} = (\Delta\mathbf{B} \times \Delta\mathbf{V} \times \Delta\mathbf{B})/|(\Delta\mathbf{B} \times \Delta\mathbf{V} \times \Delta\mathbf{B})|$$

where $\Delta\mathbf{X} = \mathbf{X}_{downstream} - \mathbf{X}_{upstream}$, $\mathbf{X} = \mathbf{B}$ or $\mathbf{V}$

Coplanarity, B and V:	Downstream Times: 04:UT+	Upstream Times: : 04:UT+	n-GSE
MMS-1	39:24.0 - 39:26.0	39:30.0 - 39:32.0	[0.389, -0.315, -0.866]
MMS-2	39:25.0 - 39:27.0	39:31.0 - 39:33.0	[0.590, -0.551, -0.620]
MMS-3	39:20.5 - 39:23.0	39:27.0 - 39:29.0	[0.670, -0.338, -0.661]
MMS-4	39:22.5 - 39:25.3	39:28.0 - 39:30.0	[0.512, -0.313, -0.800]

Average **n** from all four s/c ± 1 standard deviation on the mean:

$$\mathbf{n}_{sh} = [0.540, -0.379, -0.737] \pm [0.104, 0.010, 0.100] \text{ in GSE}$$

Magnetic field upstream of the transient's shock:

$$\mathbf{B} = [1.94, 1.16, 0.30] \text{ nT in GSE}$$

Angle between transient shock normal and upstream B-field:

$$\theta_{Bn} = 80.3 \text{ degrees}$$

So, the transient shock was in a quasi-perpendicular orientation

Calculating the Foreshock Transient's Shock Speed:

Spacecraft:	Shock Ramp t	Location in GSE [km]	Velocity [km/s]
MMS-1	04:39:26.320 UT	[92772.145, 32264.144, 16086.952]	[-1.715, 0.155, -0.562]
MMS-2	04:39:27.310 UT	[92579.885, 32282.023, 16023.796]	[-1.719, 0.154, -0.563]
MMS-3	04:39:23.600 UT	[93264.432, 32218.352, 16248.785]	[-1.704, 0.159, -0.560]
MMS-4	04:39:25.550 UT	[92915.644, 32250.873, 16134.104]	[-1.712, 0.156, -0.561]

Shock speed: $V_{sh} * \Delta t = \mathbf{\Delta X} \cdot \mathbf{n}_{sh}$

Spacecraft Pairs:	$|\Delta\mathbf{X}|$ [km]	Δt [s]	Shock Speed [km/s]

MMS-3 to -4	368.6	1.94	-59.9
MMS-3 to -1	520.2	2.72	-60.5
MMS-3 to -2	723.4	3.70	-61.7
MMS-4 to -1	151.6	0.78	-61.7
MMS-4 to -2	354.8	1.79	-63.6
MMS-1 to -2	203.2	0.99	-65.1

Average shock speed along shock normal ± 1 standard deviation:

$$V_{sh} = -62.1 \pm 1.9 \text{ km/s}$$

Note, the shock is apparently accelerating along the shock normal direction at an average rate of:

$$a = 2.93 \text{ km/s}^2$$

Shock velocity in spacecraft frame (GSE):

$$\mathbf{V_{sh}|sc} = [-33.5, 23.5, 45.7] \pm [2.1, -1.5, -2.9] \text{ km/s}$$

Shock velocity in solar wind frame (GSE):

$$\mathbf{V_{sh}|sw} = [207.5, -1.1, -20.5] \pm [2.1, -1.5, -2.9] \text{ km/s}$$

Mach Numbers in background solar wind:

$$M_{Alfvén} = 9.9, M_{fast} = 4.2$$

Converting MMS Time Series to Distance Along Shock Normal Vector:

$\mathbf{V_{MMS}|sh} = \mathbf{V_{MMS}} - \mathbf{V_{sh}}$: MMS velocity in shock reference frame

$v_{MMS}|sh = \mathbf{V_{MMS}|sh} \cdot \mathbf{n_{sh}}$: MMS apparent speed along shock normal

$t_3(0)$: time at which MMS-3 observed the original shock ramp (see "Shock Ramp t" in table above)

$$\Delta t_i = t_i(t) - t_i(0), i = \{1, 2, 3, 4\}$$

$$\Delta S_i = v_{MMS}|sh * \Delta t_i$$

For each spacecraft, $i = \{1, 2, 3, 4\}$, ΔS_i is then the distance along the shock normal direction from the original shock ramp location; thus, S = 0 is where each MMS spacecraft first observed the original shock ramp.

Note that for the results shown in Figure 2 of the paper, S were calculated using $t_i(0)$ for each spacecraft. See Figure SM1 below for example of the spatial series plotted vs. S where all four spacecraft are referenced to the location (S=0) of the original shock ramp when/where it was first observed by MMS-3 at $t_3(0)$. That conversion showcases the expansion speed of the foreshock transient but does not align common features between all four spacecraft.

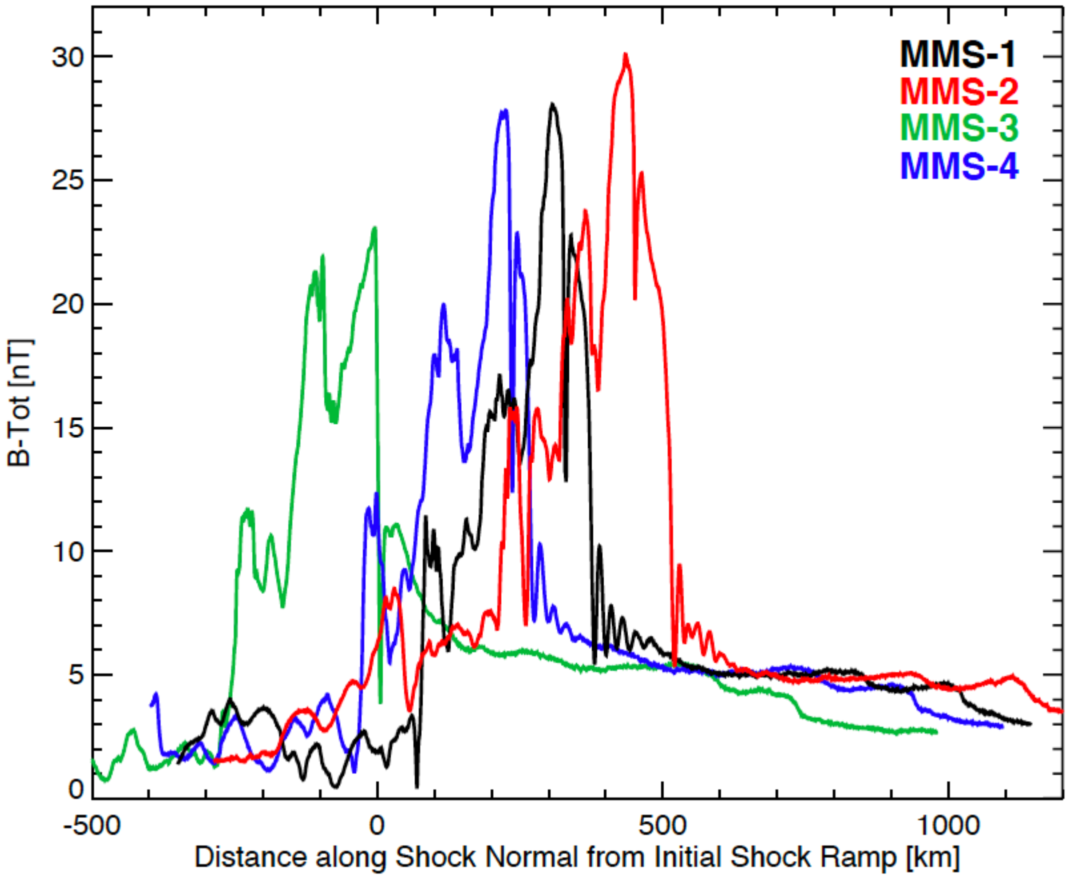

Figure SM1: |**B**| time series from each MMS spacecraft converted to distance along shock normal direction using $t_3(0)$ for all four spacecraft, i.e., instead of $t_i(0)$. This conversion captures the expansion of the foreshock transient but does not align common features along this version of S.

Foreshock Transient Shock Orientation Sketches:

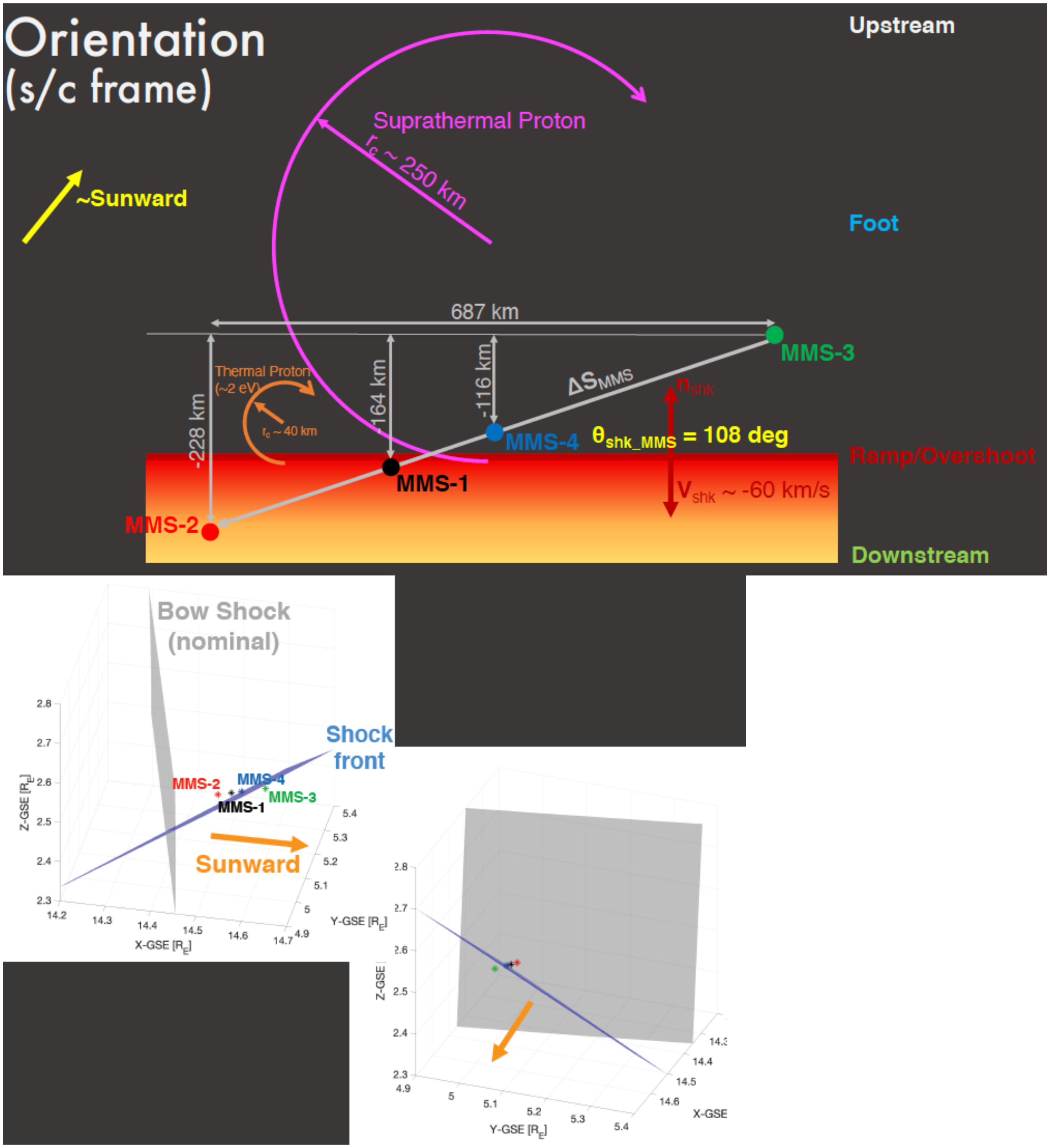

Figure SM2: Sketches of the orientation and relative size scales of the foreshock transient shock and MMS spacecraft and the bow shock.