Analysis of the cost, schedule, and risk for Lynx mirror assembly production

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Abstract

This paper analyzes a recently published model for Lynx Mirror Assembly production duration, to understand the sensitivity of duration to various factors. The factors considered are finite process yield, knowledge of the individual process times and finite server reliability. In all of these cases, closed form estimates are given along with numerical examples. This initial analysis indicates that accurate and precise knowledge of the process times is fundamental to making an accurate prediction of schedule duration. Analysis of the failure of any given server is also explained and can be used as the basis of a rational sparing policy.

Key Words; Lynx X-ray Observatory, Production Duration, Schedule Risk Calculation

Introduction

A key element in the success of a large project is the ability to schedule the work needed to realize a complex system in a time and cost efficient manner. This element of success boils down to being able to accurately predict the time for completion of complex tasks and production. The Lynx study has recognized the challenge of manufacturing its x-ray mirror assembly, the Lynx Mirror Assembly (LMA) as central to mission viability. [1] As part of the CAN study an analytic model was introduced and developed, to formulate the cost, schedule and risk associated with LMA production. [2], [3], [4] This brief report builds on these results, specifically examining the sensitivity of the duration of manufacturing, D of the LMA to various factors.

The LMA is an assembly of 37,492 mirror elements. [5] Current study timelines call for this production to take 3 years. At the simplest level, this means that on average more than 34 mirror elements must be produced each day, every day, all day for the manufacturing process to complete on time. This simple analysis does take into account any variances or tolerances in the manufacturing process. Any successful manufacturing plan for Lynx must include accommodation for variance that will cause an increase in the manufacturing duration. This complete analysis is in our plan to be reported in future publications. As an initial step to that error budget for D, sensitivities must be investigated so that the budget is complete and accurate.

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In this paper we analyze the model given in [4] for D for the sensitivity of D to the various parameters of the problem. These factors include yield loss ratio, server availability and uncertainties in process times. This will help formulate the problem of schedule risk, and determine the parameters with the greatest influence, so schedule risk can be estimated on a rigorous basis. This analysis will also inform the technology development process, insofar as it will aid in the determination of requirements for the accuracy and precision requirement on process times.

Manufacturing Duration, D

The expression for the duration D, for producing Q units in a process with n step, Q>>n, as

$$D=\left(Q+F+m\sqrt{F}+(n-1)\right)t_G+S \tag{1}$$

where F is the expected number of unit that fail in process, so called yield losses, the term proportional to the square root of F is the variance in F, m is the number of standard deviation of losses mitigated, t_G is the gate or rate limiting process time, $t_G=\max\left[\frac{t_i}{w_i}\right]$, w_i is the number of servers for step i and S is the total shipping time between manufacturing steps. [4]

For the Lynx mission, Q=37,492, n=21, S is very small by s design, so (1) can be rewritten as

$$D=\left(Q+F+m\sqrt{F}\right)t_G. \tag{2}$$

Defining the yield loss, y, as F/Q, ratio of units that fail in production to the number that pass, (2) can be expressed as

$$D=\left(Q(1+y)+m\sqrt{Qy}\right)t_G. \tag{3}$$

For small y, of order of a few percent, typical of an efficient process, this definition approximates the traditional definition of production yield and simplifies the following discussion.

Sensitivity of D to Gating Time, t_G

The optimization process, adjusts the number of servers w_i, so that the effective process time, $\tau_i=\frac{t_i}{w_i}$, is largely equal across the production process, as shown in Figure 1. Figure 1 shows the effective process times for a set of w_i, that give D=3 years for 100% yield or nil losses, y=0. [4]

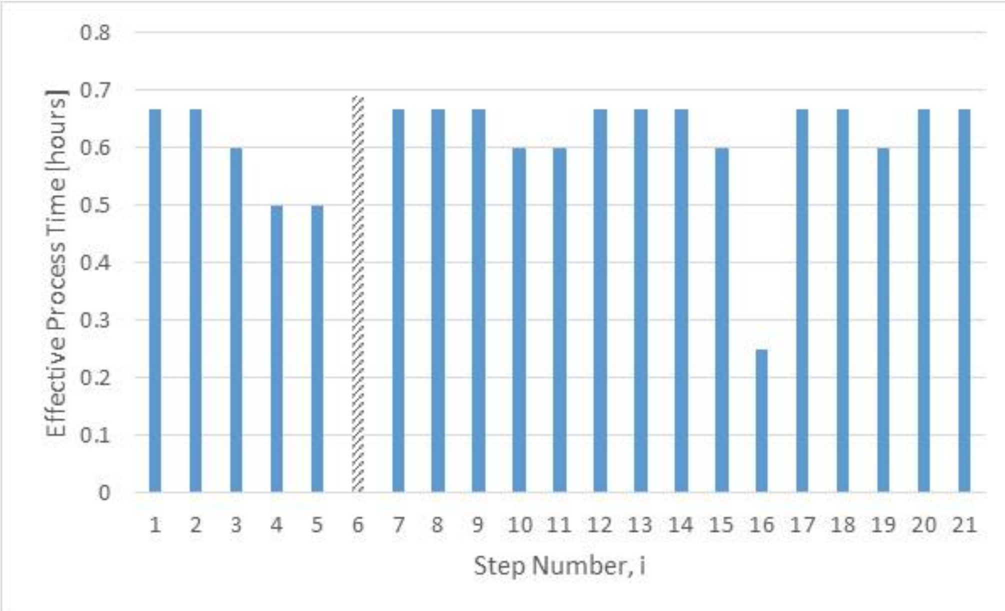

Figure 1; LMA Effective Process times for D=3 years

The schedule optimization drives the τ_i to be largely the same. In this case, t_G is step 6, the hatched bar, in Figure 1, and has the value of 0.698 hours.

There a few scenarios where the value of rate limiting or gate time, t_G, can be incorrect, such as;

1. Statistical uncertainty in the process time of the current rate limiting step
2. Statistical uncertainty in another process step such that the true rate limiting step is misidentified
3. Failure of any single server, changing its effective process time.

To understand the impact on D from scenario 1, consider the change in D from an error in t_G, δt_G which is given by

$$\delta D = \frac{\partial D}{\partial t_G}\delta t_G . \tag{4}$$

The partial derivative in (4) is

$$\frac{\partial D}{\partial t_G} = Q\left(1+y\right) + m\sqrt{Qy} . \tag{5}$$

Substitution of (5) into (4) gives

$$\delta D = \left(Q\left(1+y\right) + m\sqrt{Qy}\right)\delta t_G . \tag{6}$$

What is the impact on D from an error of 6 minutes (0.1 hours) in t_G? Assume y=0.05 and take m=3, then we find

$$\delta D = \left(37492\left(1.05\right) + 3\sqrt{37492 \cdot .05}\right)0.1 = 3949\ hours = 164\ days\,. \qquad (7)$$

The results shown by (7), show the high sensitivity of D to t_G, in rough numbers 27 days/minute! This behooves us to understand t_G as completely as possible.

Figure 1 shows Step 6, as rate limiting, with t_G=0.698 hours, but we can also be see that there are 12 other steps, that have the effective duration of 0.666 hours. An uncertainty of 2 minutes in their effective process time, would give a t_G=0.699, and impact schedule by 1.6 days! From this cursory look, we can see that understanding not only the process times, but their distributions will be necessary ingredient in rigorous design of the LMA manufacturing process.

As we have just seen, D is sensitive to t_G, exactly as intuition and nomenclature would indicate. It is equally clear that during the development and design of the LMA manufacturing process t_G should be actively managed and not just reported. This section has been directed to the statistical variation of t_G which is the maximum value of the ratio $\frac{t_i}{w_i}$. Potential variance can come from either the numerator or denominator of the ratio. One necessary element is to understand the distribution of t_i. The other factor is controlling w_i. At first look, this would seem to be trivial, w_i is of course a discrete variable since servers come in integer units. This is true only if the manufacturing schedule is continuous, so called 24/7/365. If the manufacturing time is less than full, then it is possible to run the servers in "overtime" effectively increasing w_i to manage t_G. The optimal strategy for Lynx will include the considerations discussed above for managing t_G, and therefore D, and the cost of these various strategies.

Impact of Server Failure on D

The value of t_G also has a discrete element to it. Namely the impact of server failure, namely, for some i,

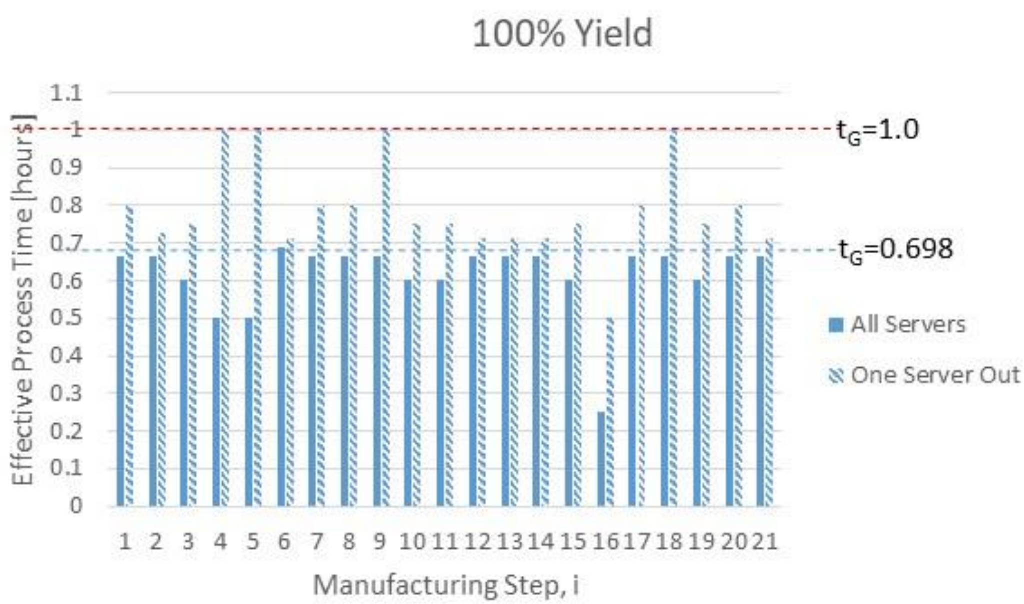

Figure 2: Effective Process Times for 100% yield: All servers functioning (solid bars) and one server failed (hatched bars)

a single server fails, leaving wi-1 operational servers for the ith step. This changes the effective process time discontinuously from $\frac{t_i}{w_i}$ to $\frac{t_i}{w_i - 1}$. Figure 2 shows the effective process time for all of the process steps, y=0, in a group bar graph. The left or solid bar, is the effective process time when all of the servers are functioning. Step 6 is the rate limiting of gating process, with an effective process time of 0.698 hours. The right or hatched bars, are the effective process time if there are w_i-1 servers for each i, one server failed. This means that if one server becomes inoperative at step i, the hatched bar, indicates its effective process time. For the Lynx process all the hatched bars are higher than the solid bars for each step i, i≠16. Moreover, for each of the hatched bases, save step 16, the hatched bar is above than the dashed line indicating the t_G for all servers operating. This means that a failure of any one server, other than that the one for Step 16, the gate time gets longer and D increases.

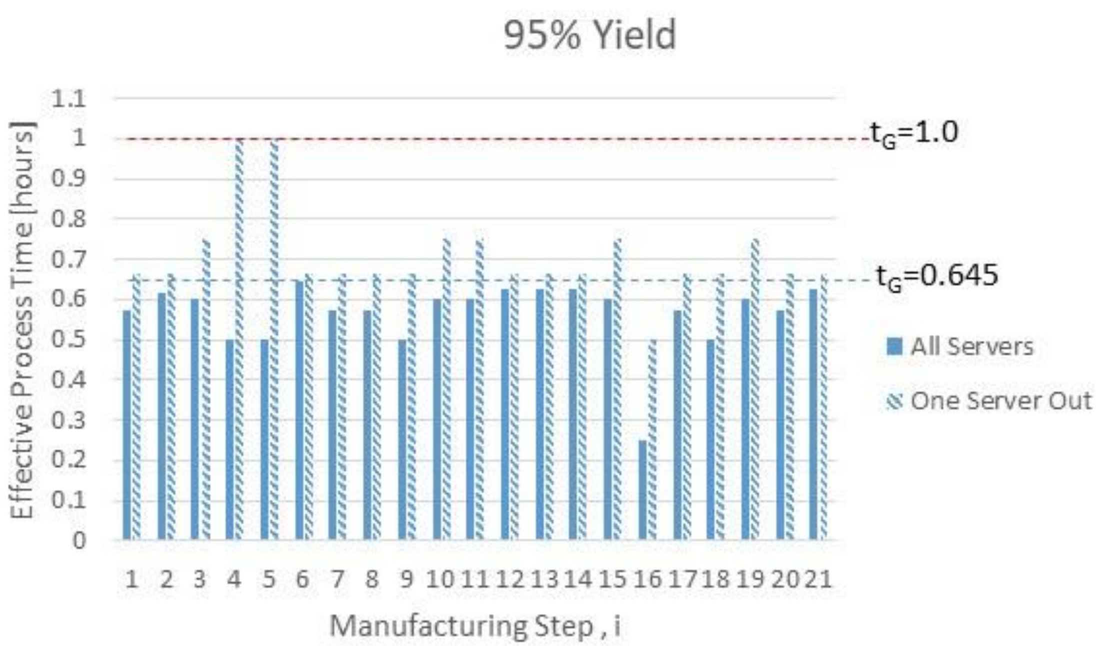

Figure 3: Effective Process Times for 95% average manufacturing yield: All servers functioning (solid bars) and one server failed (hatched bars)

To consider the effects of finite yield, the average production yield loss of 5%, y=0.05, is chosen. Figure 3 shows the effective process times the case of all servers functioning, again the left or blue bars. Step 6 is again the gate, but the t_G is now 0.645 hours, reflecting a larger w_i to make up for the fact that 5% more items must be made in the same time. As can be seen from Figure 3, like the case of perfect yield, if any server other than Step 16 fails, the revised gate time, $t_G^{[1]}$, the superscript indicating the failure of 1 server is larger than t_G, meaning an increase in D.

To estimate the increase in production time D, δD due to a server outage, we determine the time to complete Q units with Z units being completed with one failed server, and subtracting the value of D when there are no server failures

$$\delta D = \left\{\left(Q+F-Z\right)t_G + Zt_G^{[1]}\right\} - \left\{\left(Q+F\right)t_G\right\}. \tag{8}$$

The left most term in (8) in the curly braces is the time to complete Q+F-Z elements with the original gate time, and Z elements under the longer gate time, $t_G^{[1]}$, from this is subtracted the baseline time to complete Q+F units. Equation (8) becomes

$$\delta D = Z\left(t_G^{[1]} - t_G\right). \tag{9}$$

In order to estimate Z, the number of units produced while the failed server is not available, we must calculate the time it takes to return it to service. This will be sum of the mean time to repair (MTTR) and the mean logistic delay time (MLDT). MTTR is time to do the work on the server, once the necessary part and expertise is on hand, MLDT is the time spent waiting for either parts or trained personnel to arrive. So,

$$Zt_G^{[1]} = MTTR + MLDT\ . \tag{10}$$

Substitution of (10) into (9) gives

$$\delta D = \frac{t_G^{[1]} - t_G}{t_G^{[1]}}\left(MTTR + MLDT\right). \tag{11}$$

Which simplifies to

$$\delta D = \left(1 - \frac{t_G}{t_G^{[1]}}\right)\left(MTTR + MLDT\right). \tag{12}$$

Figure 4, shows the evaluation of the factor $1-\frac{t_G}{t_G^{[1]}}$ in (12), for a failed server for each step i except for Step 16, which even in the case of a failure of a server does not become the gating step. Viewed in this way, Figure 4 shows δD in units of MRRT+MLDT. Imagine it took one month to replace either server 4 or 5, in round units δD would be approximately 11 days.

There are several means of managing the impact from a failed server. The most obvious is on site spares and service, reducing MLDT and if the spare is integrated into the production line, a "hot spare", MTTR can be reduced as well. The number of servers could be increased over the minimum needed to complete the LMA in D, in other words, planned early delivery. Alternately, extra servers could be purchased for process steps that have smaller numbers of wi, so the failure of a single unit causes a greater change in $\frac{t_i}{w_i}$. The specific plan for mitigating the impact of a failed server, is entwined with cost, enabling Lynx engineering and management a quantitative management tool to optimize cost, schedule and risk.

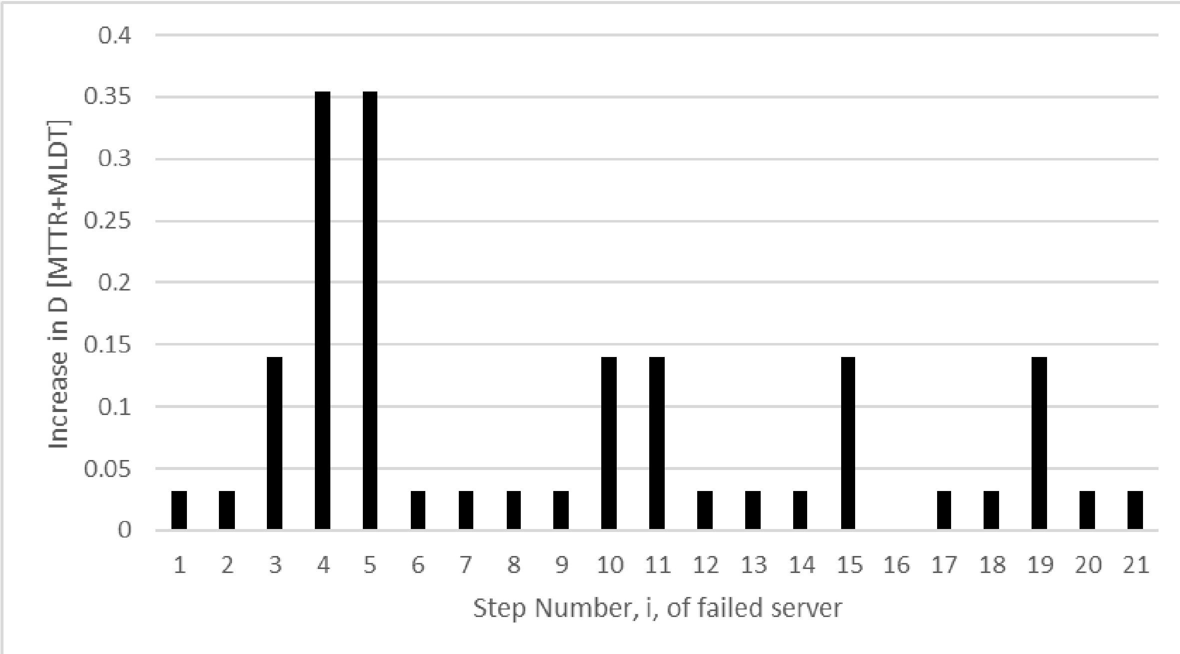

Figure 4: Impact of a Failed Server

In the cases of the very small impacts, such as Steps 1 and 2 (and many more) the net impact is 1.2 days.

Sensitivity of D to average yield loss, y

To calculate δD for a change (error) in y, y>0, begin with

$$\delta D = \frac{\partial D}{\partial y}\delta y\,. \tag{13}$$

The partial derivative in (13) is

$$\frac{\partial D}{\partial y} = t_G\left(Q - \frac{m}{2}\sqrt{\frac{Q}{y}}\right). \tag{14}$$

For Lynx, (14) can be approximated

$$\frac{\partial D}{\partial y} = t_G\left(Q - \frac{m}{2}\sqrt{\frac{Q}{y}}\right) \approx t_G Q \tag{15}$$

since $Q \gg \frac{m}{2}\sqrt{\frac{Q}{y}}$ as $37,492 \gg 3\sqrt{\frac{37,492}{0.01}}$, namely $37,492 \gg 2,704$.

Substitution of (15) into (13) gives

$$\delta D = t_G Q \delta y\,. \tag{16}$$

From (16) we can estimate the schedule impact of a 1%, 0.01, error in y,

$$\delta D = 0.698 \cdot 37492 \cdot 0.01\,hours = 261.7\,hours = 10.9\,days\,. \tag{17}$$

This schedule delay (or advancement) is interpreted at the time to make Q δy more (or less) mirror elements. Since there will be a programmatic penalty for being late, the upper bound on y should be used and that way any non-zero error in y is negative, resulting in an early finish to schedule.

The yield loss ratio, y, is largely a statistical property of the manufacturing process. Its precise finite value, which should be small, can be accommodated into the design of LMA manufacture through F and its variance through (3). As part of the development the values for the y_i and their distribution should be known and the variance accommodated through a rigorous and bounding estimate of F. During manufacture, monitoring the y_i will be key check on the health and status of LMA production.

Summary and a Look Ahead

This very short paper has presented an initial analysis of the existing model of LMA production time, for schedule risk to uncertainty in process times, yield and finite server availability. This analysis clearly shows the high degree of sensitivity of the predicted value of D, the manufacturing duration to process times.

The results presented in this paper clearly inform the Lynx project and teams developing the optics that the validity and credibility of any manufacturing schedule for Lynx must be based on very sound statistics. Knowing this crucial fact will allow for the necessary data to refine the t_i and understand their distributions to be collected and analyzed during process development. This information will be used to further refine the selection of the number of servers needed to complete the LMA in time D, with the proper balance of risk, cost and schedule.

Our results motivate us to further explore the analytic model for further insights. Understanding the fundamental causes of variance in D will allow the design of the Lynx manufacturing process to complete its task, including variance in D within the allotted programmatic schedule.

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