Considerations for the Extension of Gas Path Analysis to Electrified Aircraft Propulsion Systems

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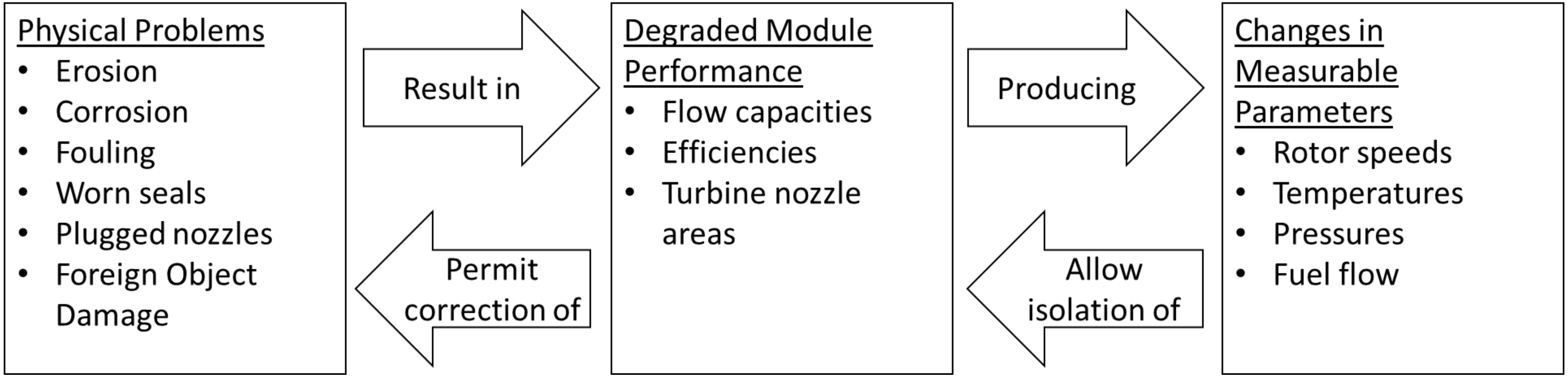
Outline

- Introduction
  - Gas Path Analysis (GPA) Process
  - Electrified Aircraft Propulsion (EAP) Systems
- Mathematical Formulation of a Performance Estimation Approach for EAP Systems and Subsystems
- Simulated Example Illustrating the Application of Estimation Approaches to an EAP Concept Aircraft
- Conclusions

The Gas Path Analysis Process

- Leverages analytical knowledge of parameter interrelationships inherent in the gas turbine cycle
- Physical problems result in degraded module performance, which results in sensed measurement changes
- Analysis of sensed measurement changes enables the isolation of degraded module performance and corrective action recommendations

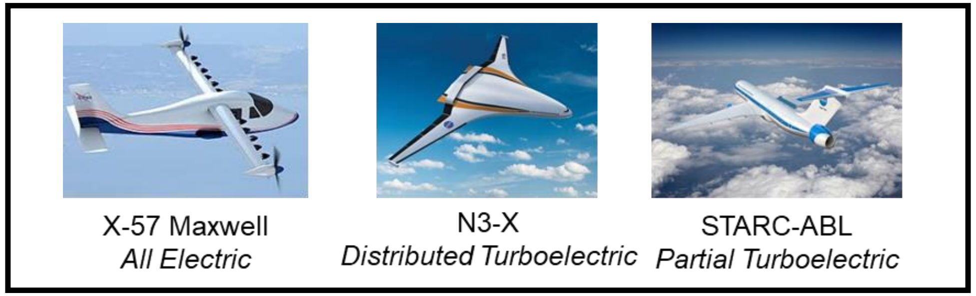


Gas Path Analysis Process

EAP System Architectures are Distributed, Interconnected Designs

- EAP relies on the generation, storage, and transmission of electrical power for aircraft propulsion
  - Enables aircraft designs that apply advanced propulsion concepts
  - Benefits include a potential reduction in emissions, fuel burn, noise, and cost

- Similarities between Gas Turbines and EAP systems
  - Designs are coupled in nature
  - Parameter interrelationships exist
  - Physical problems result in sensed measurement changes throughout the system
  - Often underdetermined (i.e., more unknowns than available sensors).



Example NASA EAP Concept Vehicles

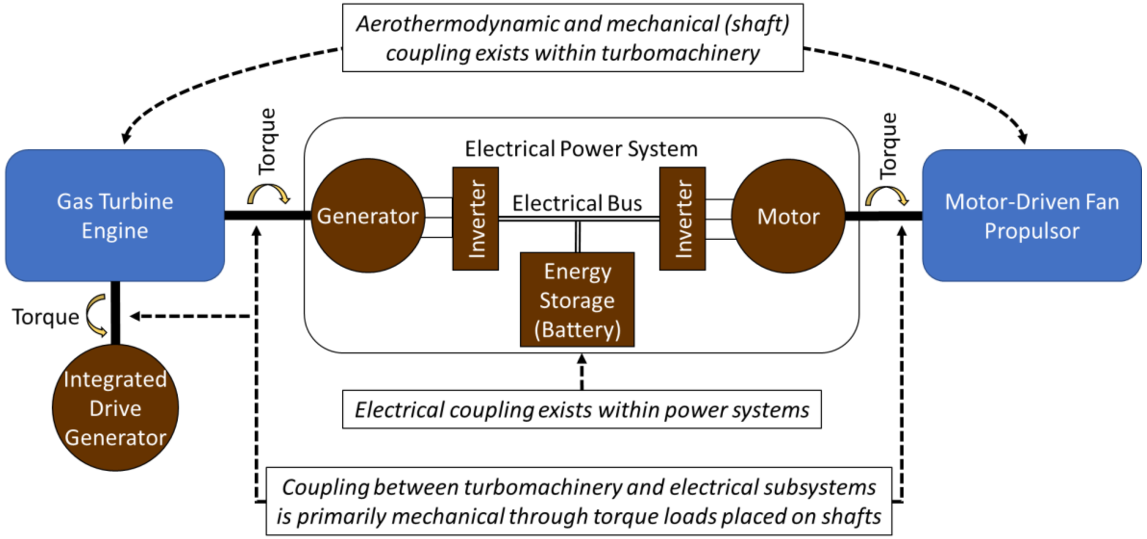


Illustration of Subsystem-to-Subsystem Coupling in a Notional EAP System

Gas Path Analysis Fundamentals

Linear steady-state measurement equation:

$$z = Hx + v$$

where:
z = sensed measurement vector deviations
x = health parameter vector deviations, representing module performance deterioration
H = Influence coefficient matrix partial derivatives relating the influence of health parameter vector changes on sensed measurements
v = zero-mean normally distributed noise with covariance *R*

Maximum *a posteriori* (MAP) estimator:

$$\hat{x} = (P_x^{-1} + H^T R^{-1} H)^{-1} H^T R^{-1} z$$

where:
$\hat{x}$ = estimate of health parameter vector *x*
P_x = matrix containing *a priori* knowledge of expected health parameter covariance

Gas Path Analysis Fundamentals (cont.)

- Assuming normal distributions and a linear measurement process, the MAP estimator will produce the minimum theoretical sum of mean squared estimation errors (SMSEE)
- SMSEE is equal to the sum of Mean Squared Bias and Variance in $\hat{x}$:

Mean Squared Bias

$$
\begin{aligned}
\bar{\tilde{x}} &= E[\hat{x} - x] \\
&= E[\underbrace{G_x z}_{\hat{x}} - x] \\
&= E[G_x \underbrace{(Hx + v)}_{z} - x] \\
&= E[(G_x H - I)x + G_x v] \\
&= (G_x H - I)\underbrace{E[x]}_{x} + G_x \underbrace{E[v]}_{0} \\
&= (G_x H - I)x
\end{aligned}
$$

$$
\begin{aligned}
\bar{\tilde{x}}^2 &= E[\bar{\tilde{x}}^T \bar{\tilde{x}}] \\
&= E[tr\{\bar{\tilde{x}}\bar{\tilde{x}}^T\}] \\
&= E[tr\{(G_x H - I)xx^T(G_x H - I)^T\}] \\
&= tr\left\{(G_x H - I)\underbrace{E[xx^T]}_{P_x}(G_x H - I)^T\right\} \\
&= \boxed{tr\{(G_x H - I)P_x(G_x H - I)^T\}}
\end{aligned}
$$

Variance

$$
P_{\hat{x}} = E\left[\underbrace{(\hat{x} - E[\hat{x}])}_{\varepsilon}\underbrace{(\hat{x} - E[\hat{x}])^T}_{\varepsilon^T}\right]
$$

$$
\begin{aligned}
\varepsilon &= (\hat{x} - E[\hat{x}]) \\
&= G_x z - E[G_x z] \\
&= G_x(Hx + v) - E[G_x(Hx + v)] \\
&= G_x Hx + G_x v - G_x H \underbrace{E[x]}_{x} - G_x \underbrace{E[v]}_{0} \\
&= G_x v
\end{aligned}
$$

$$
\begin{aligned}
P_{\hat{x}} &= E[\varepsilon\varepsilon^T] \\
&= E[G_x v\, v^T G_x^T] \\
&= G_x E\underbrace{[vv^T]}_{R} G_x^T \\
&= G_x R G_x^T
\end{aligned}
$$

$$
\boxed{variance = tr\{G_x R G_x^T\}}
$$

$$
SMSEE(\hat{x}) = \boxed{tr\{(G_x H - I)P_x(G_x H - I)^T\}} + \boxed{tr\{G_x R G_x^T\}}
$$

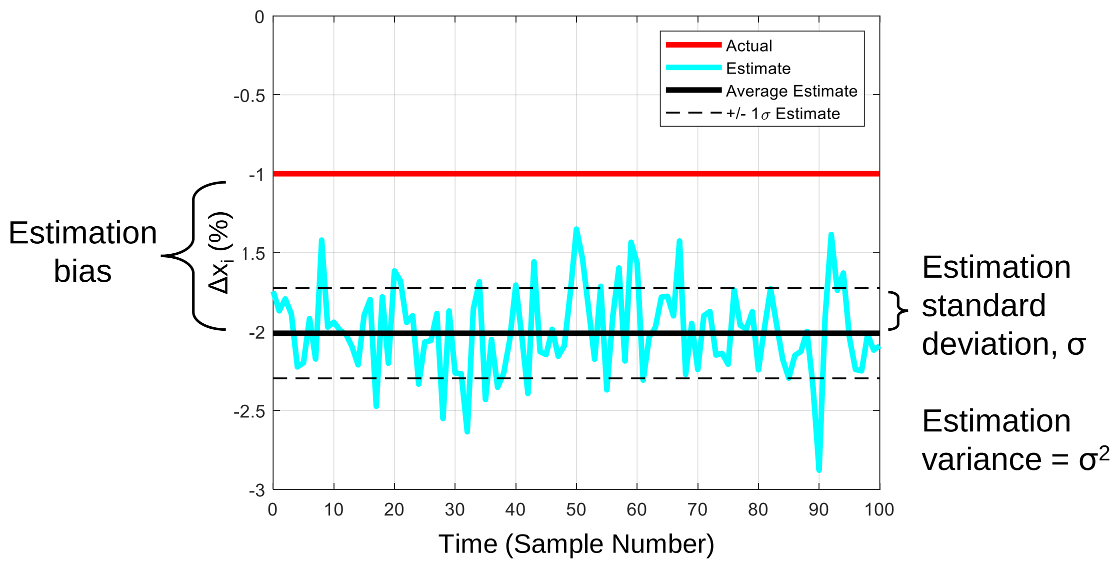


Illustration of bias and variance in the estimate of a single health parameter in a single propulsion system

Extension of Gas Path Analysis to EAP Systems and Their Subsystems

- The Maximum a Posteriori (MAP) estimation approach is agnostic to the number of health parameters and sensors in the system – it can be applied to complete EAP architectures if the necessary system information is available
- In some cases, it may be desirable or necessary to estimate health parameters at the subsystem level as opposed to the full-system level. In such cases, the measurement equation can be algebraically re-arranged to isolate a "subsystem of interest" from the rest of the architecture

Original Measurement Equation

$$z = Hx + v$$

→

Re-arranged Measurement Equation Isolating Subsystem of Interest

$$\underbrace{\begin{bmatrix} z_0 \\ z_\emptyset \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} H_0 & H_{0\emptyset} \\ H_{\emptyset 0} & H_\emptyset \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x_0 \\ x_\emptyset \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} v_0 \\ v_\emptyset \end{bmatrix}}_{v}$$

"<u>Subsystem of interest" parameters</u>

z_0 = sensed measurement vector
x_0 = health parameter vector
H_0 = Influence coefficient matrix
v_0 = measurement noise vector

"<u>Non-subsystems of interest" parameters</u>

$z_\emptyset$ = sensed measurement vector
$x_\emptyset$ = health parameter vector
$H_\emptyset$ = Influence coefficient matrix
$v_\emptyset$ = measurement noise vector

<u>System-level influence coefficients</u>

$H_{0\emptyset}$ = influence of non-subsystems of interest health parameters on subsystem of interest measurements
$H_{\emptyset 0}$ = influence of subsystems of interest health parameters on non-system of interest measurements

Formulation of Subsystem Health Parameter Estimators

Measurement equation

$$z_0 = H_0 x_0 + \underbrace{H_{0\emptyset} x_\emptyset + v_0}_{v_{0,\emptyset}} = H_0 x_0 + v_{0,\emptyset}$$

Influence of non-subsystem of interest health parameter variations treated as sensor measurement noise

Measurement noise covariance matrix

$$R_{0,\emptyset} = E\left[\left(v_{0,\emptyset} - \underbrace{E[v_{0,\emptyset}]}_{0}\right)\left(v_{0,\emptyset} - \underbrace{E[v_{0,\emptyset}]}_{0}\right)^T\right] = E\left[v_{0,\emptyset} v_{0,\emptyset}^T\right]$$

$$R_{0,\emptyset} = E\left[\underbrace{(H_{0\emptyset} x_\emptyset + v_0)}_{v_{0,\emptyset}} \underbrace{(H_{0\emptyset} x_\emptyset + v_0)^T}_{v_{0,\emptyset}}\right]$$

$$= E\left[H_{0\emptyset} x_\emptyset x_\emptyset^T H_{0\emptyset}^T\right] + E\left[H_{0\emptyset} x_\emptyset v_0^T\right] + E\left[v_0 x_\emptyset^T H_{0\emptyset}^T\right] + E\left[v_0 v_0^T\right]$$

$$= H_{0\emptyset} \underbrace{E\left[x_\emptyset x_\emptyset^T\right]}_{P_{x_\emptyset}} H_{0\emptyset}^T + H_{0\emptyset} \underbrace{E\left[x_\emptyset v_0^T\right]}_{0} + \underbrace{E\left[v_0 x_\emptyset^T\right]}_{0} H_{0\emptyset}^T + \underbrace{E\left[v_0 v_0^T\right]}_{R_0}$$

$$= H_{0\emptyset} P_{x_\emptyset} H_{0\emptyset}^T + R_0$$

Non-subsystem of interest health parameter covariance matrix contributes to measurement noise covariance matrix derivation

MAP estimation equation

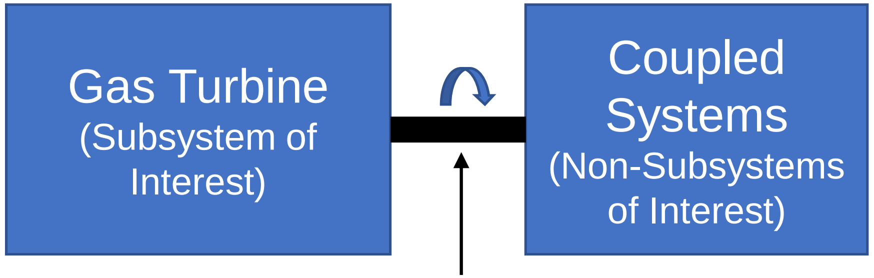
$$\hat{x}_0 = \underbrace{\left(P_{x_0}^{-1} + H_0^T R_{0,\emptyset}^{-1} H_0\right)^{-1} H_0^T R_{0,\emptyset}^{-1}}_{G_{x_0}} z_0$$

Only sensed measurements from the subsystem of interest are used to produce the estimate

Sum of mean squared estimation errors

$$SMSEE(\hat{x}_0) = tr\left\{(G_{x_0} H_0 - I) P_{x_0} (G_{x_0} H_0 - I)^T\right\} + tr\left\{G_{x_0} R_{0,\emptyset} G_{x_0}^T\right\}$$

Subsystem Estimator Adjustments Based on Available Measurements of Applied Shaft Torque



Shaft coupling with applied torque, x_Q

Note: if direct torque measurements are unavailable other system measurements such as generator or motor current may be applied as a proxy for torque.

Subsystem measurement vector, z_0, is adjusted by subtracting the influence of measured torque variations:

$$z_{0,Q} = z_0 - H_Q z_Q$$

where

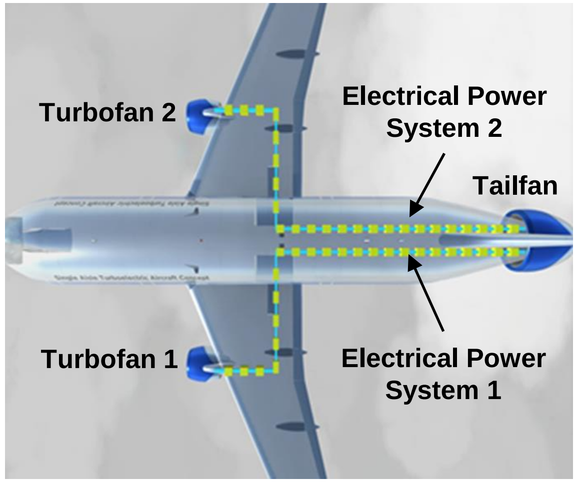
$z_{0,Q}$ = adjusted subsystem measurement vector
z_0 = original measurement vector
z_Q = measured torque variation (scalar) with covariance R_Q
H_Q = influence coefficient vector relating changes in torque to measurement changes observed in the subsystem

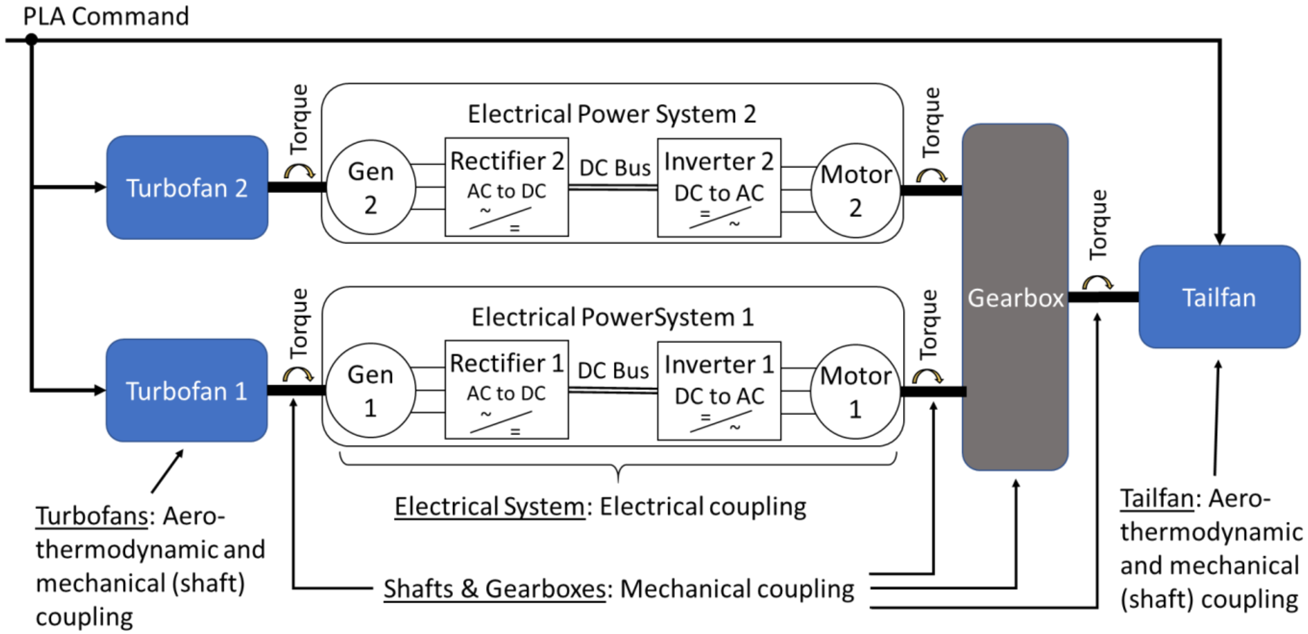
Allows construction MAP estimation equation and derivation of theoretical linear sum of squared estimation errors:

$$\hat{x}_0 = \underbrace{\left(P_{x_0}^{-1} + H_0^T R_{0,Q}^{-1} H_0\right)^{-1} H_0^T R_{0,Q}^{-1}}_{G_{x_{0,Q}}} z_{0,Q}$$

$$SMSEE(\hat{x}_0) = tr\left\{\left(G_{x_{0,Q}} H_0 - I\right) P_{x_0} \left(G_{x_{0,Q}} H_0 - I\right)^T\right\} + tr\left\{G_{x_{0,Q}} R_{0,Q} G_{x_{0,Q}}^T\right\}$$

Example Application to NASA STARC-ABL EAP Concept Aircraft





NASA Single-aisle Turboelectric AiRCraft with Aft Boundary Layer propulsor (STARC-ABL)

- Two wing-mounted geared turbofan engines produce thrust and supply mechanical offtake power to generate electricity
- Generators and rectifiers convert mechanical power into electricity
- Electricity transferred over two parallel direct current power buses
- Inverters and motors convert electricity into mechanical power supplied to tailfan propulsor
- Tailfan propulsor applies boundary layer ingestion for increased propulsive efficiency

Example Application to STARC-ABL NASA EAP Concept Aircraft (continued)

STARC-ABL Subsystem Health Parameters
(30 health parameters in entire system)

Subsystem	Health parameter		Standard deviation
Turbofans	γ_{FAN}	Fan Flow Capacity	2.5%
	η_{FAN}	Fan Efficiency	2.5%
	γ_{LPC}	LPC Flow Capacity	2.5%
	η_{LPC}	LPC Efficiency	2.5%
	γ_{HPC}	HPC Flow Capacity	2.5%
	η_{HPC}	HPC Efficiency	2.5%
	γ_{HPT}	HPT Flow Capacity	2.5%
	η_{HPT}	HPT Efficiency	2.5%
	γ_{LPT}	LPT Flow Capacity	2.5%
	η_{LPT}	LPT Efficiency	2.5%
Electrical Power Systems	η_{gen}	Generator Efficiency	0.1%
	η_{rect}	Rectifier Efficiency	0.1%
	η_{inv}	Inverter Efficiency	0.1%
	η_{mot}	Motor Efficiency	0.1%
Tailfan	γ_{FAN}	Tailfan Flow Capacity	2.5%
	η_{FAN}	Tailfan Efficiency	2.5%

STARC-ABL Subsystem Sensor Measurements
(27 sensors with torque measurements included, 24 without)

Subsystem	Sensed parameter		Noise Standard deviation
Turbofans	N_3	Core speed	0.0477%
	P_{13}	Bypass duct total pressure	0.1671%
	P_{25}	LPC exit total pressure	0.1205%
	T_{25}	LPC exit total temperature	0.0306%
	P_{s3}	HPC exit static pressure	0.0620%
	T_3	HPC exit total temperature	0.0292%
	T_{48}	Exhaust gas temperature	0.0271%
	W_f	Fuel flow	0.4687%
Electrical Power Systems	Q_{turb}	Turbofan low pressure shaft torque	0.0984%
	I_{gen}	Generator current	0.0145%
	I_{bus}	Bus current	0.0147%
	I_{mot}	Motor current	0.0149%
Tailfan	Q_{tail}	Tailfan shaft torque	0.0486%
	P_5	Exit total pressure	0.1732%
	T_5	Exit total temperature	0.0430%

Example Application to STARC-ABL NASA EAP Concept Aircraft (continued)

STARC-ABL Measurement Equation

$$z = Hx + v$$

- Sensed measurements, ***z***
- Health parameters, ***x***
- Sensor noise, ***v***

STARC-ABL Influence Coefficient Matrix, *H* (27 x 30)

		Health Parameters																													
		Turbofan 1										Turbofan 2										Power System 1				Power System 2				Tailfan	
		γ_{FAN}	η_{FAN}	γ_{LPC}	η_{LPC}	γ_{HPC}	η_{HPC}	γ_{HPT}	η_{HPT}	γ_{LPT}	η_{LPT}	γ_{FAN}	η_{FAN}	γ_{LPC}	η_{LPC}	γ_{HPC}	η_{HPC}	γ_{HPT}	η_{HPT}	γ_{LPT}	η_{LPT}	η_{gen}	η_{rec}	η_{inv}	η_{mot}	η_{gen}	η_{rec}	η_{inv}	η_{mot}	γ_{FAN}	η_{FAN}
Sensors: Turbofan 1	N_3	0.313	-0.282	-0.274	-0.142	-0.399	0.491	-0.297	0.671	0.591	-0.472											-0.156	-0.156	-0.157	-0.157					0.260	-0.159
Turbofan 1	P_{13}	0.311	-0.010	-0.016	0.000	-0.001	-0.002	0.001	-0.003	-0.003	0.002											0.000	0.000	0.000	0.000					-0.001	0.001
Turbofan 1	P_{25}	-0.106	0.364	1.401	-0.149	-0.501	-0.924	0.355	-1.358	-1.153	0.792											0.205	0.205	0.206	0.206					-0.342	0.209
Turbofan 1	T_{25}	0.000	0.008	0.233	-0.310	-0.096	-0.173	0.068	-0.237	-0.207	0.150	[0]										0.040	0.040	0.040	0.040	[0]				-0.066	0.040
Turbofan 1	P_{s3}	0.568	-0.267	0.465	-0.098	-0.029	0.039	-1.190	0.159	0.274	-0.618											-0.184	-0.184	-0.185	-0.185					0.306	-0.188
Turbofan 1	T_3	0.325	-0.293	-0.212	-0.268	-0.146	-0.143	-0.473	0.541	0.500	-0.477											-0.149	-0.149	-0.149	-0.149					0.247	-0.151
Turbofan 1	T_{48}	0.534	-0.508	-0.598	-0.232	-0.125	-0.225	0.088	-0.311	0.034	-0.928											-0.281	-0.281	-0.282	-0.282					0.467	-0.287
Turbofan 1	W_f	1.100	-0.786	-0.247	-0.232	-0.101	-0.181	0.066	-0.242	0.266	-1.612											-0.483	-0.483	-0.483	-0.483					0.800	-0.492
Turbofan 2	N_3											0.313	-0.282	-0.274	-0.142	-0.399	0.491	-0.297	0.671	0.591	-0.472					-0.156	-0.156	-0.157	-0.157	0.260	-0.159
Turbofan 2	P_{13}											0.311	-0.010	-0.016	0.000	-0.001	-0.002	0.001	-0.003	-0.003	0.002					0.000	0.000	0.000	0.000	-0.001	0.001
Turbofan 2	P_{25}											-0.106	0.364	1.401	-0.149	-0.501	-0.924	0.355	-1.358	-1.153	0.792					0.205	0.205	0.206	0.206	-0.342	0.209
Turbofan 2	T_{25}	[0]										0.000	0.008	0.233	-0.310	-0.096	-0.173	0.068	-0.237	-0.207	0.150	[0]				0.040	0.040	0.040	0.040	-0.066	0.040
Turbofan 2	P_{s3}											0.568	-0.267	0.465	-0.098	-0.029	0.039	-1.190	0.159	0.274	-0.618					-0.184	-0.184	-0.185	-0.185	0.306	-0.188
Turbofan 2	T_3											0.325	-0.293	-0.212	-0.268	-0.146	-0.143	-0.473	0.541	0.500	-0.477					-0.149	-0.149	-0.149	-0.149	0.247	-0.151
Turbofan 2	T_{48}											0.534	-0.508	-0.598	-0.232	-0.125	-0.225	0.088	-0.311	0.034	-0.928					-0.281	-0.281	-0.282	-0.282	0.467	-0.287
Turbofan 2	W_f											1.100	-0.786	-0.247	-0.232	-0.101	-0.181	0.066	-0.242	0.266	-1.612					-0.483	-0.483	-0.483	-0.483	0.800	-0.492
Power System 1	Q_{turb}																					-1.001	-1.001	-1.003	-1.003					1.661	-1.020
Power System 1	I_{gen}	[0]										[0]										0.000	-1.001	-1.003	-1.003	[0]				1.661	-1.020
Power System 1	I_{bus}																					0.000	0.000	-1.003	-1.003					1.661	-1.020
Power System 1	I_{mot}																					0.000	0.000	0.000	-1.001					1.657	-1.018
Power System 2	Q_{turb}																									-1.001	-1.001	-1.003	-1.003	1.661	-1.020
Power System 2	I_{gen}	[0]										[0]										[0]				0.000	-1.001	-1.003	-1.003	1.661	-1.020
Power System 2	I_{bus}																									0.000	0.000	-1.003	-1.003	1.661	-1.020
Power System 2	I_{mot}																									0.000	0.000	0.000	-1.001	1.657	-1.018
Tailfan	Q_{tail}																													1.657	-1.018
Tailfan	P_5	[0]										[0]										[0]				[0]				0.291	-0.008
Tailfan	T_5																													0.082	-0.061

Legend: Turbofan Influence Coefficients; Power System Influence Coefficients; Tailfan Influence Coefficients; Inter-Subsystem Influence Coefficients

Summary of Estimator Designs Evaluated

This study applied and evaluated both **system level** and **partitioned (subsystem)** health parameter estimator designs:

- **System level estimator designs** (both with and without torque measurements) enable estimation of all 30 STARC-ABL health parameters with a single estimator
- **Partitioned (subsystem) estimator designs** (both with and without torque measurements) estimate health parameters within a specific subsystem

Summary of Estimator Designs

	Estimator	System level -or- Partitioned?	Sub-system	# of sensors	*R* size	*H* size
System level estimator without torque measurements	1	System level	N/A	24	24×24	24×30
System level estimator with torque measurements	2	System level	N/A	27	27×27	27×30
Partitioned (subsystem) estimators without torque measurements	1a	Partitioned	Turbofan	8	8×8	8×10
	1b	Partitioned	Power system	3	3×3	3×4
	1c	Partitioned	Tailfan	2	2×2	2×2
Partitioned (subsystem) estimators with torque measurements	2a	Partitioned	Turbofan	9	8×8	8×10
	2b	Partitioned	Power system	5	4×4	4×4
	2c	Partitioned	Tailfan	3	3×3	3×2

Estimation Accuracy (Theoretical Prediction vs. Monte Carlo Simulation Results)

- Predicted theoretical accuracy obtained analytically and validated through a Monte Carlo simulation study

Mean Squared Estimation Errors

		Without torque measurements				With torque measurements			
		System-Level Estimator #1		Subsystem Estimators #1a, #1b, and #1c		System-Level Estimator #2		Subsystem Estimators #2a, #2b, and #2c	
		Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo
Turbofan	γ_{FAN}	0.2797	0.2804	0.2798	0.2805	0.2797	0.2804	0.2797	0.2804
	η_{FAN}	4.2644	4.2720	4.3266	4.3341	4.2644	4.2719	4.2644	4.2720
	γ_{LPC}	0.2566	0.2563	0.2566	0.2564	0.2566	0.2563	0.2566	0.2563
	η_{LPC}	0.1964	0.1969	0.2001	0.2006	0.1964	0.1969	0.1964	0.1969
	γ_{HPC}	0.1248	0.1248	0.1467	0.1466	0.1248	0.1248	0.1248	0.1248
	η_{HPC}	0.0119	0.0119	0.0121	0.0121	0.0119	0.0119	0.0119	0.0119
	γ_{HPT}	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711
	η_{HPT}	1.3022	1.3044	1.3691	1.3709	1.3022	1.3043	1.3022	1.3044
	γ_{LPT}	2.8149	2.8176	2.9518	2.9540	2.8149	2.8175	2.8149	2.8176
	η_{LPT}	0.8717	0.8737	1.4804	1.4815	0.8714	0.8734	0.8717	0.8737
	SMSEE	**10.29**	**10.31**	**11.19**	**11.21**	**10.29**	**10.31**	**10.29**	**10.31**
Power System	η_{gen}	0.0100	0.0100	0.0100	0.0100	0.0050	0.0050	0.0050	0.0050
	η_{rec}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{inv}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{motor}	0.0051	0.0051	0.0100	0.0100	0.0018	0.0018	0.0020	0.0020
	SMSEE	**0.016**	**0.016**	**0.021**	**0.021**	**0.008**	**0.008**	**0.008**	**0.008**
Tailfan	γ_{FAN}	0.3141	0.3148	0.3402	0.3410	0.3140	0.3147	0.3140	0.3147
	η_{FAN}	0.8344	0.8361	1.0213	1.0233	0.8331	0.8348	0.8334	0.8351
	SMSEE	**1.15**	**1.15**	**1.36**	**1.36**	**1.15**	**1.15**	**1.15**	**1.15**
Total SMSEE		**21.77**	**21.79**	**23.79**	**23.82**	**21.75**	**21.77**	**21.75**	**21.78**

Estimation Accuracy (Theoretical Prediction vs. Monte Carlo Simulation Results)

- Predicted theoretical accuracy obtained analytically and validated through a Monte Carlo simulation study
- Results
  - Theoretical and Monte Carlo simulation accuracy exhibit good agreement

Mean Squared Estimation Errors

		Without torque measurements				With torque measurements			
		System-Level Estimator #1		Subsystem Estimators #1a, #1b, and #1c		System-Level Estimator #2		Subsystem Estimators #2a, #2b, and #2c	
		Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo
Turbofan	γ_{FAN}	0.2797	0.2804	0.2798	0.2805	0.2797	0.2804	0.2797	0.2804
	η_{FAN}	4.2644	4.2720	4.3266	4.3341	4.2644	4.2719	4.2644	4.2720
	γ_{LPC}	0.2566	0.2563	0.2566	0.2564	0.2566	0.2563	0.2566	0.2563
	η_{LPC}	0.1964	0.1969	0.2001	0.2006	0.1964	0.1969	0.1964	0.1969
	γ_{HPC}	0.1248	0.1248	0.1467	0.1466	0.1248	0.1248	0.1248	0.1248
	η_{HPC}	0.0119	0.0119	0.0121	0.0121	0.0119	0.0119	0.0119	0.0119
	γ_{HPT}	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711
	η_{HPT}	1.3022	1.3044	1.3691	1.3709	1.3022	1.3043	1.3022	1.3044
	γ_{LPT}	2.8149	2.8176	2.9518	2.9540	2.8149	2.8175	2.8149	2.8176
	η_{LPT}	0.8717	0.8737	1.4804	1.4815	0.8714	0.8734	0.8717	0.8737
	SMSEE	**10.29**	**10.31**	**11.19**	**11.21**	**10.29**	**10.31**	**10.29**	**10.31**
Power System	η_{gen}	0.0100	0.0100	0.0100	0.0100	0.0050	0.0050	0.0050	0.0050
	η_{rec}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{inv}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{motor}	0.0051	0.0051	0.0100	0.0100	0.0018	0.0018	0.0020	0.0020
	SMSEE	**0.016**	**0.016**	**0.021**	**0.021**	**0.008**	**0.008**	**0.008**	**0.008**
Tailfan	γ_{FAN}	0.3141	0.3148	0.3402	0.3410	0.3140	0.3147	0.3140	0.3147
	η_{FAN}	0.8344	0.8361	1.0213	1.0233	0.8331	0.8348	0.8334	0.8351
	SMSEE	**1.15**	**1.15**	**1.36**	**1.36**	**1.15**	**1.15**	**1.15**	**1.15**
Total SMSEE		**21.77**	**21.79**	**23.79**	**23.82**	**21.75**	**21.77**	**21.75**	**21.78**

Estimation Accuracy (Theoretical Prediction vs. Monte Carlo Simulation Results)

- Predicted theoretical accuracy obtained analytically and validated through a Monte Carlo simulation study

- Results
  - Theoretical and Monte Carlo simulation accuracy exhibit good agreement
  - *Without torque measurements*: System-level estimation approaches found to yield improved accuracy (lower error) compared to subsystem estimators

Mean Squared Estimation Errors

		Without torque measurements				With torque measurements			
		System-Level Estimator #1		Subsystem Estimators #1a, #1b, and #1c		System-Level Estimator #2		Subsystem Estimators #2a, #2b, and #2c	
		Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo
Turbofan	γ_{FAN}	0.2797	0.2804	0.2798	0.2805	0.2797	0.2804	0.2797	0.2804
	η_{FAN}	4.2644	4.2720	4.3266	4.3341	4.2644	4.2719	4.2644	4.2720
	γ_{LPC}	0.2566	0.2563	0.2566	0.2564	0.2566	0.2563	0.2566	0.2563
	η_{LPC}	0.1964	0.1969	0.2001	0.2006	0.1964	0.1969	0.1964	0.1969
	γ_{HPC}	0.1248	0.1248	0.1467	0.1466	0.1248	0.1248	0.1248	0.1248
	η_{HPC}	0.0119	0.0119	0.0121	0.0121	0.0119	0.0119	0.0119	0.0119
	γ_{HPT}	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711
	η_{HPT}	1.3022	1.3044	1.3691	1.3709	1.3022	1.3043	1.3022	1.3044
	γ_{LPT}	2.8149	2.8176	2.9518	2.9540	2.8149	2.8175	2.8149	2.8176
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Power System	η_{gen}	0.0100	0.0100	0.0100	0.0100	0.0050	0.0050	0.0050	0.0050
	η_{rec}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{inv}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{motor}	0.0051	0.0051	0.0100	0.0100	0.0018	0.0018	0.0020	0.0020
	SMSEE	**0.016**	**0.016**	**0.021**	**0.021**	**0.008**	**0.008**	**0.008**	**0.008**
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Estimation Accuracy (Theoretical Prediction vs. Monte Carlo Simulation Results)

- Predicted theoretical accuracy obtained analytically and validated through a Monte Carlo simulation study

- Results
  - Theoretical and Monte Carlo simulation accuracy exhibit good agreement
  - *Without torque measurements*: System-level estimation approaches found to yield improved accuracy (lower error) compared to subsystem estimators
  - *With torque measurement*: System-level and subsystem estimators provide similar accuracy

Mean Squared Estimation Errors

		Without torque measurements				With torque measurements			
		System-Level Estimator #1		Subsystem Estimators #1a, #1b, and #1c		System-Level Estimator #2		Subsystem Estimators #2a, #2b, and #2c	
		Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo	Theo-retical	Monte Carlo
Turbofan	γ_{FAN}	0.2797	0.2804	0.2798	0.2805	0.2797	0.2804	0.2797	0.2804
	η_{FAN}	4.2644	4.2720	4.3266	4.3341	4.2644	4.2719	4.2644	4.2720
	γ_{LPC}	0.2566	0.2563	0.2566	0.2564	0.2566	0.2563	0.2566	0.2563
	η_{LPC}	0.1964	0.1969	0.2001	0.2006	0.1964	0.1969	0.1964	0.1969
	γ_{HPC}	0.1248	0.1248	0.1467	0.1466	0.1248	0.1248	0.1248	0.1248
	η_{HPC}	0.0119	0.0119	0.0121	0.0121	0.0119	0.0119	0.0119	0.0119
	γ_{HPT}	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711	0.1711
	η_{HPT}	1.3022	1.3044	1.3691	1.3709	1.3022	1.3043	1.3022	1.3044
	γ_{LPT}	2.8149	2.8176	2.9518	2.9540	2.8149	2.8175	2.8149	2.8176
	η_{LPT}	0.8717	0.8737	1.4804	1.4815	0.8714	0.8734	0.8717	0.8737
	SMSEE	**10.29**	**10.31**	**11.19**	**11.21**	**10.29**	**10.31**	**10.29**	**10.31**
Power System	η_{gen}	0.0100	0.0100	0.0100	0.0100	0.0050	0.0050	0.0050	0.0050
	η_{rec}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{inv}	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
	η_{motor}	0.0051	0.0051	0.0100	0.0100	0.0018	0.0018	0.0020	0.0020
	SMSEE	**0.016**	**0.016**	**0.021**	**0.021**	**0.008**	**0.008**	**0.008**	**0.008**
Tailfan	γ_{FAN}	0.3141	0.3148	0.3402	0.3410	0.3140	0.3147	0.3140	0.3147
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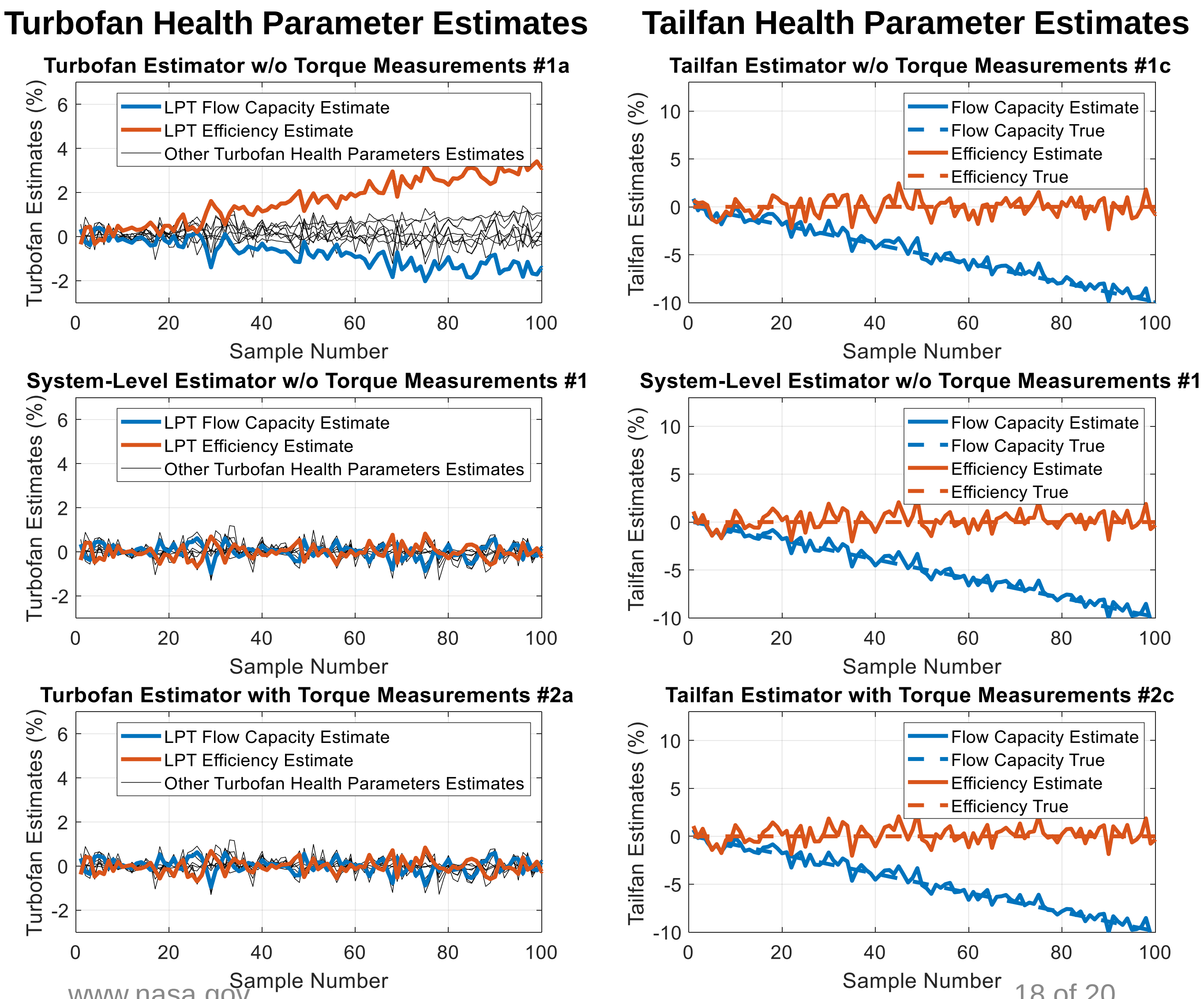
Total SMSEE		**21.77**	**21.79**	**23.79**	**23.82**	**21.75**	**21.77**	**21.75**	**21.78**

Visual Illustration of STARC-ABL Estimation Results (Tailfan Flow Capacity Degradation Case)

- A linear decrease in Tailfan flow capacity degradation is simulated while all other health parameters remain at zero (affects the amount of torque extracted from turbofans)
- Comparison of estimator designs:
  - Subsystem estimator without torque measurements (#1a) produces biased turbofan estimates
  - System-level estimator without torque measurements (#1) and subsystem estimator with torque measurements (#2a) produce more accurate turbofan health parameters estimation results
  - All estimators (#1, #1c, #2c) provide comparable Tailfan health parameter estimation accuracy



Conclusions

- Demonstrated application of a conventional gas path analysis performance estimation approach to electrified aircraft propulsion systems
  - Mathematical formulation of performance estimation equations presented
  - Theoretical estimation accuracy derived and validated through a linear Monte Carlo simulation study

- A partitioned approach towards the estimation problem setup can be applied enabling health parameter estimates at the subsystem level
  - Knowledge of torque applied to mechanical shafts that couple subsystems is key

- Presented approach and equations are generic and applicable to any electrified aircraft propulsion architecture of a distributed and interconnected nature

Acknowledgments

- This work was conducted under the NASA Advanced Air Vehicles Program, Advanced Air Transport Technology Project