ON THE DEVELOPMENT OF PRECONDITIONING METHODS FOR HIGH-ORDER MULTIPHYSICS PROBLEMS

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"Full-scale" aerodynamics complex

NASA Ames Research Center

- Originally founded under NACA in 1939
- Initially focused on wind tunnel research
  - National Full-scale aerodynamics complex
  - ARC unitary plan wind tunnel (supersonic flows)
  - several others

- Role expanded for spaceflight with other facilities
  - Ames Vertical Gun Range
  - Hypervelocity Free-Flight Range
  - Electric Arc Shock Tube

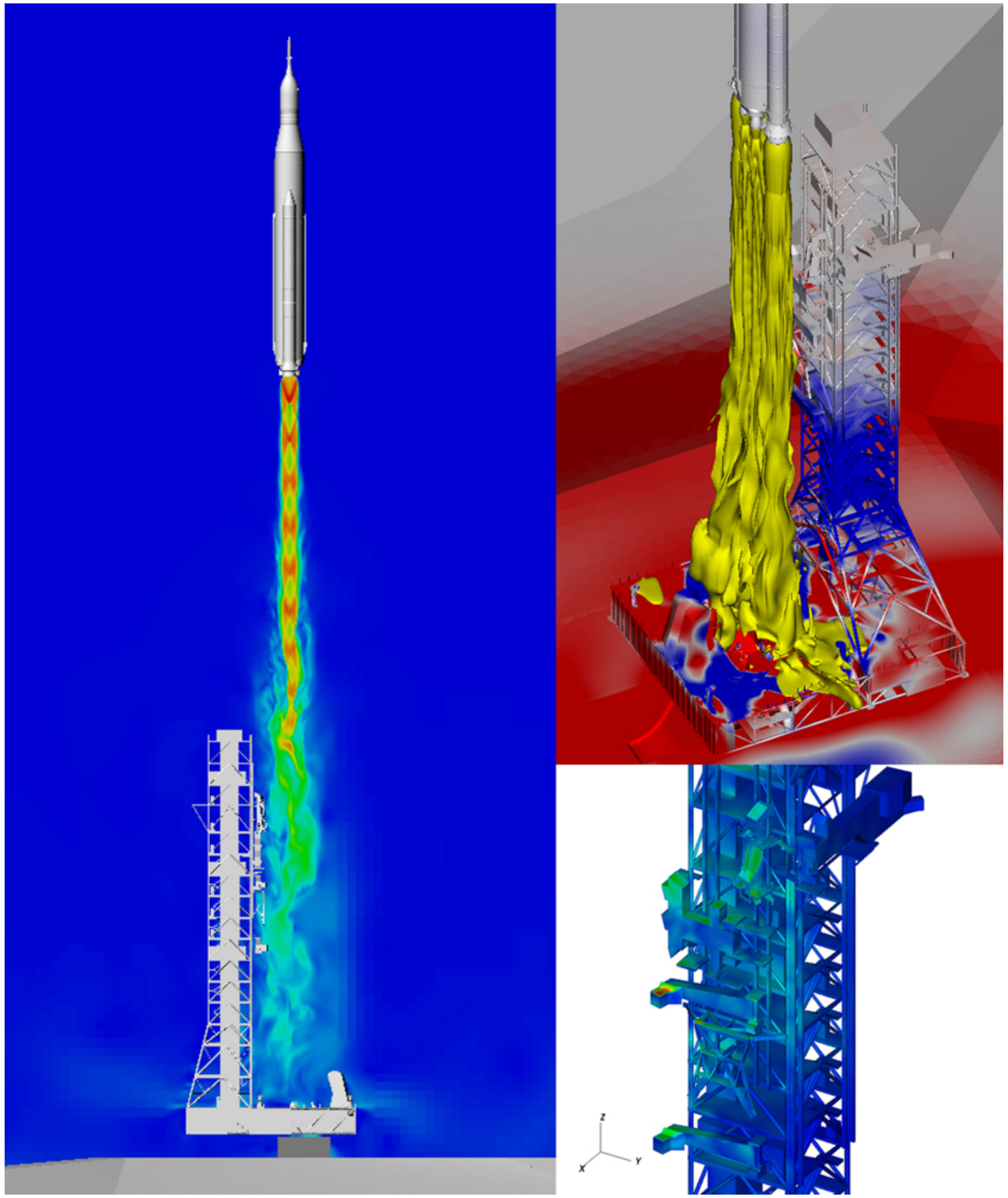
- …and supercomputing
  - NASA Advanced Supercomputing Division (NAS)

NASA Ames Research Center

Advanced Supercomputing Division

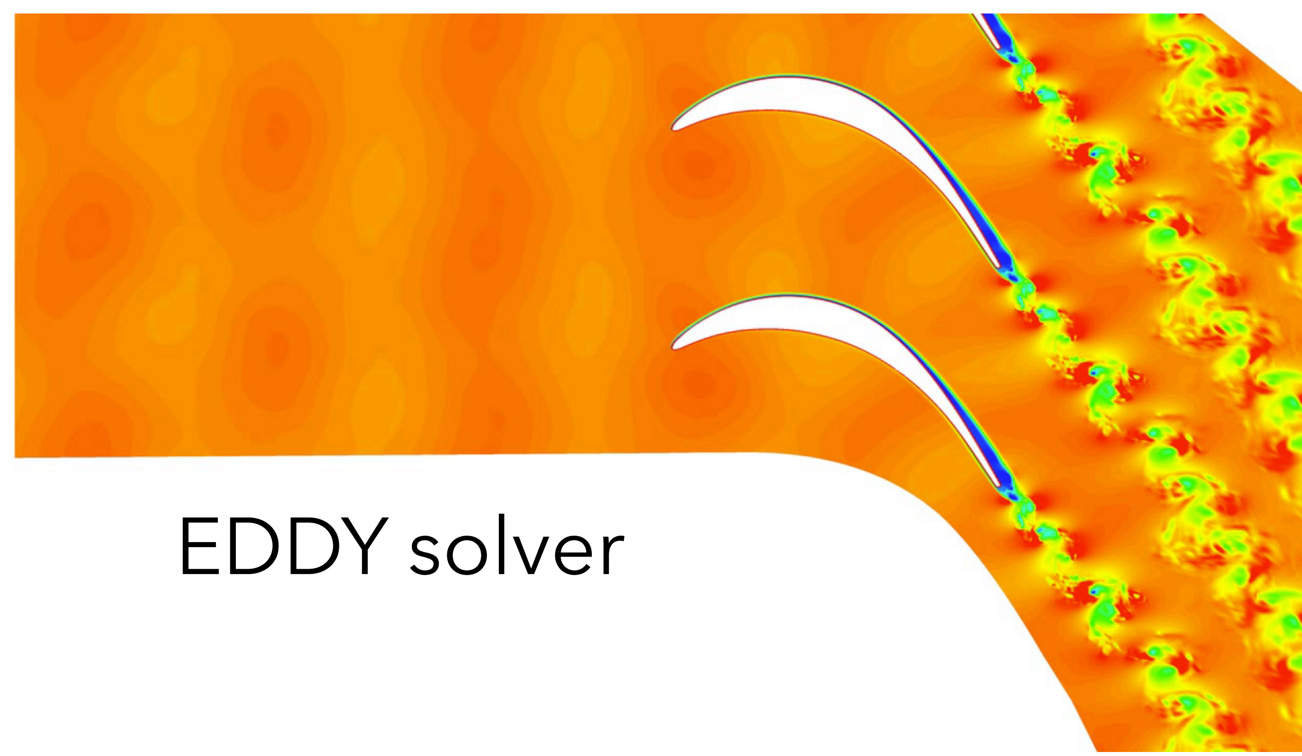
NASA Ames Research Center

Some NAS CFD solvers



LAVA solver

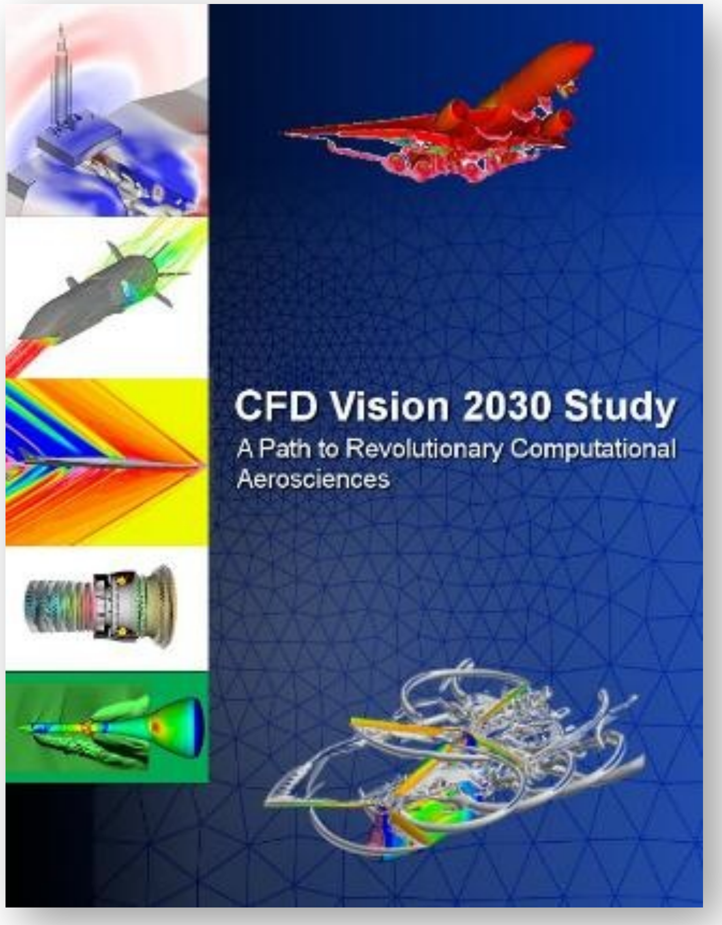
OVERFLOW solver

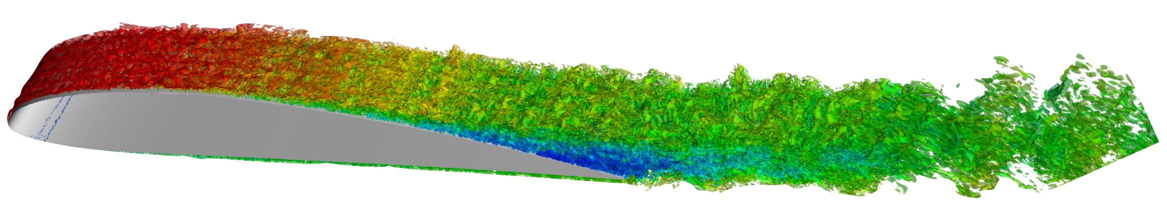


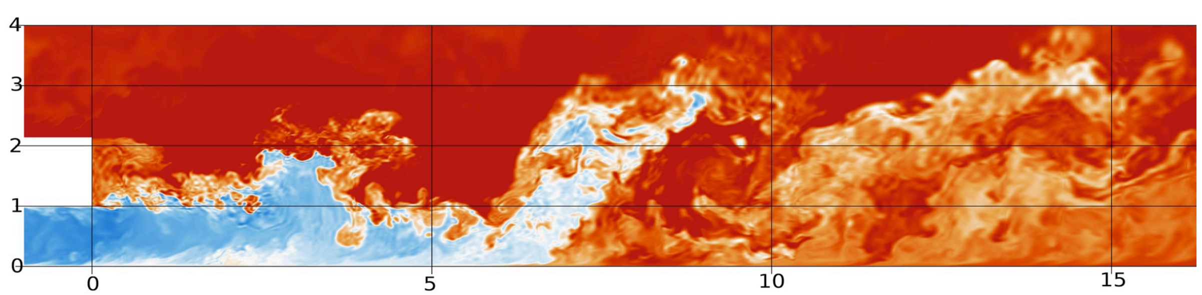
EDDY solver

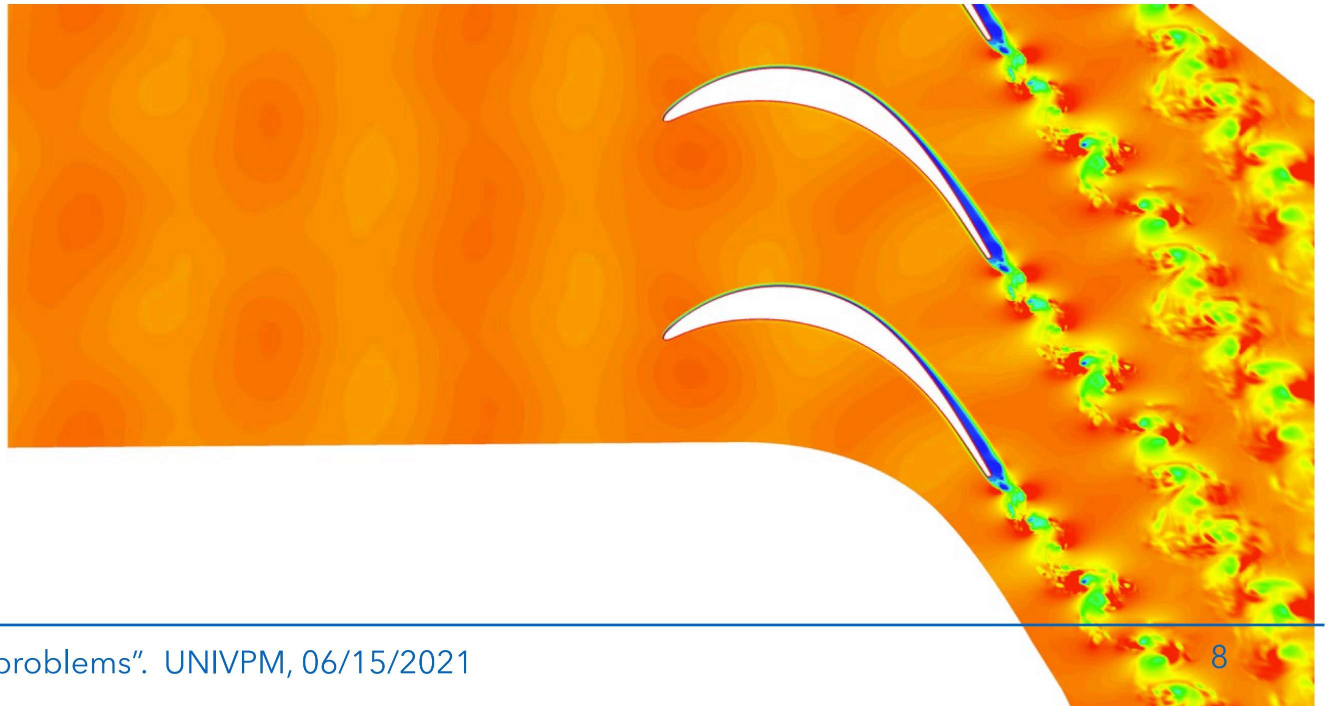
Background

- CFD vision 2030 calls for dramatic improvements in simulation capabilities
- Exascale-computing era is here (2022)
- Need to establish **new solver frameworks** to
  - Provide orders of magnitude improvements over state-of-the art
  - Leverage new super-computing architectures (GPUs)

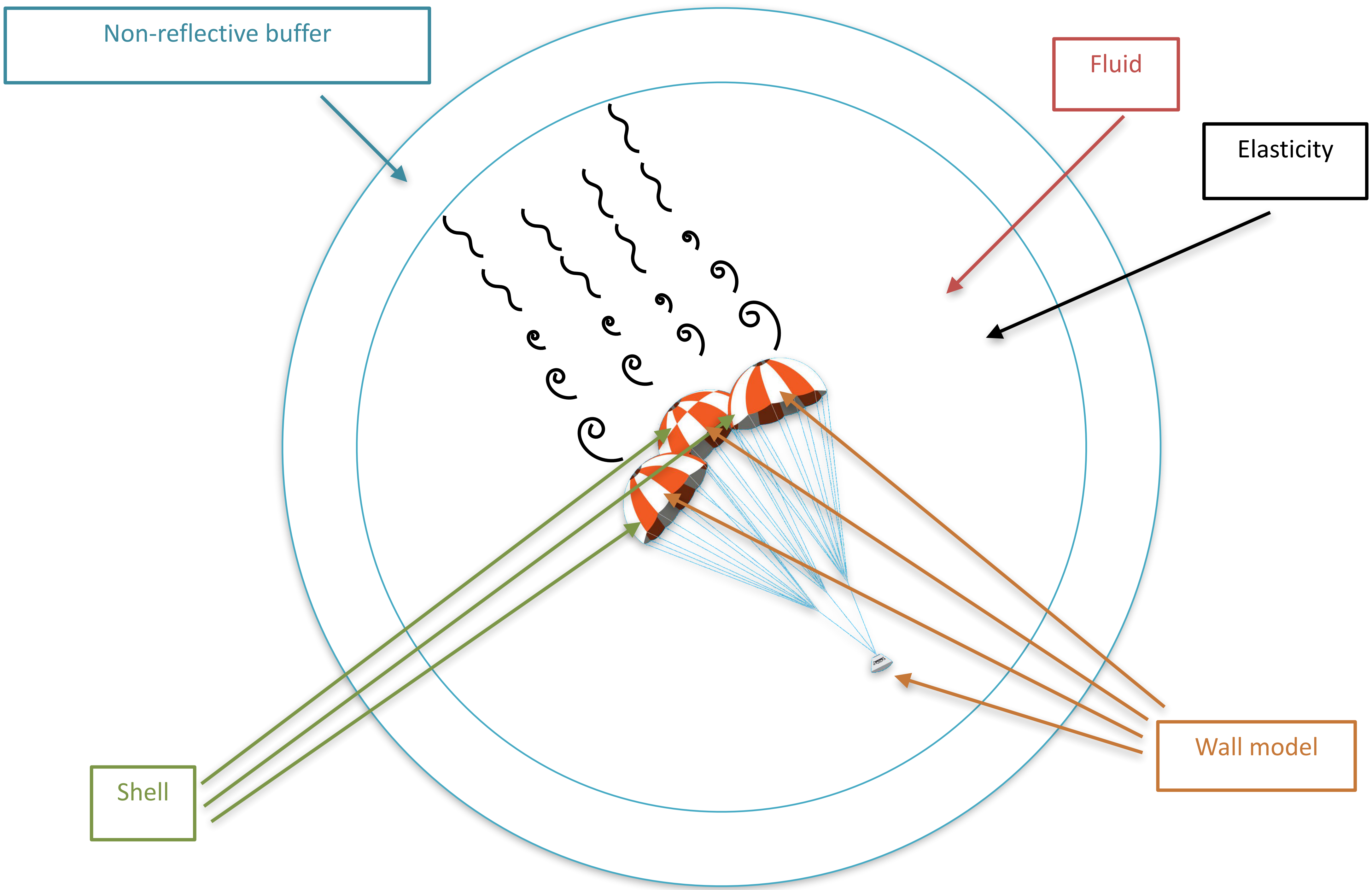








Parachute-capsule modeling



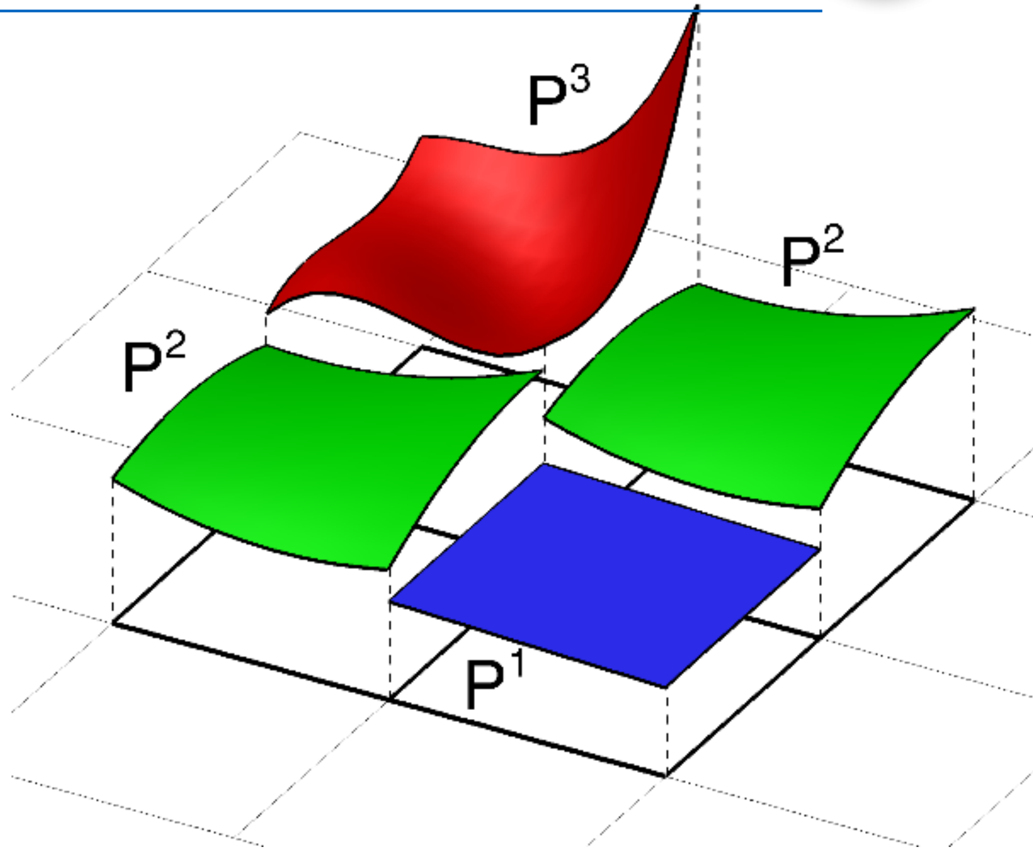
Parachute-capsule modeling

- State of the art: Partitioned Solvers
  - Solve for individual quantities, and then iterate until converged
  - Convenient approach to leverage existing solvers
  - Massive drawbacks
    - Non conservative
    - Unstable (simulation could just crash)
    - Inaccurate

- Our approach: fully-coupled monolithic solver
  - All physics converge at once
  - Solves the drawbacks of partitioned solvers
  - Much harder to implement
  - Talk addresses fluid and structural solver
    - Work done within the single-physics mode
    - Coupling the monolithic system under development

High-order methods

- High-order obtained through increasing the degree of piece-wise polynomial functions defined in each element of the mesh



- Discrete variational formulation reads:

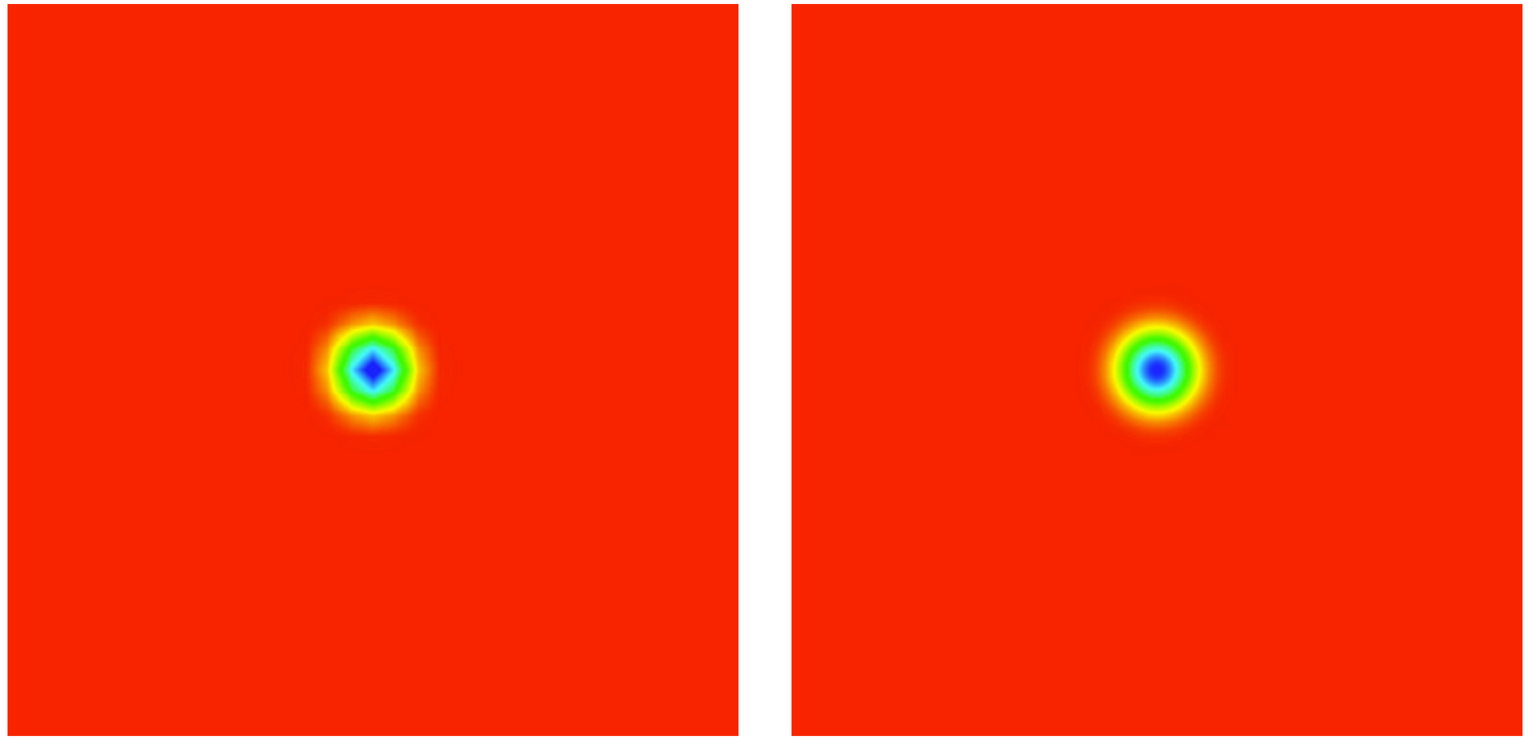
$$\sum_{K \in \mathcal{T}_h} \int_K \phi_h \frac{\partial \widehat{\mathbf{w}}_h}{\partial t} d\mathbf{x} - \sum_{K \in \mathcal{T}_h} \int_K \nabla_h \phi_h \cdot \mathbf{F}(\mathbf{w}_h, \nabla \mathbf{w}_h + \mathbf{r}([\![\mathbf{w}_h]\!]))d\mathbf{x}$$

$$+ \sum_{F \in \mathcal{F}_h} \int_F [\![\phi_h]\!] \widehat{\mathbf{F}} \left(\mathbf{w}_h^{\pm}, (\nabla \mathbf{w}_h + \eta_F \mathbf{r}_F([\![\mathbf{w}_h]\!]))^{\pm}\right) d\sigma = \sum_{K \in \mathcal{T}_h} \int_K \phi_h \mathbf{s}(\mathbf{w}_h) d\mathbf{x}$$

- Solution is obtained by finding the root of a nonlinear function accounting for multi-physics: $\mathbf{R}(\mathbf{U}) = 0$

High-order methods

low-order (standard)



high-order

- **High-order**: higher resolution capability per given problem size
  - potentially more efficient by orders of magnitudes

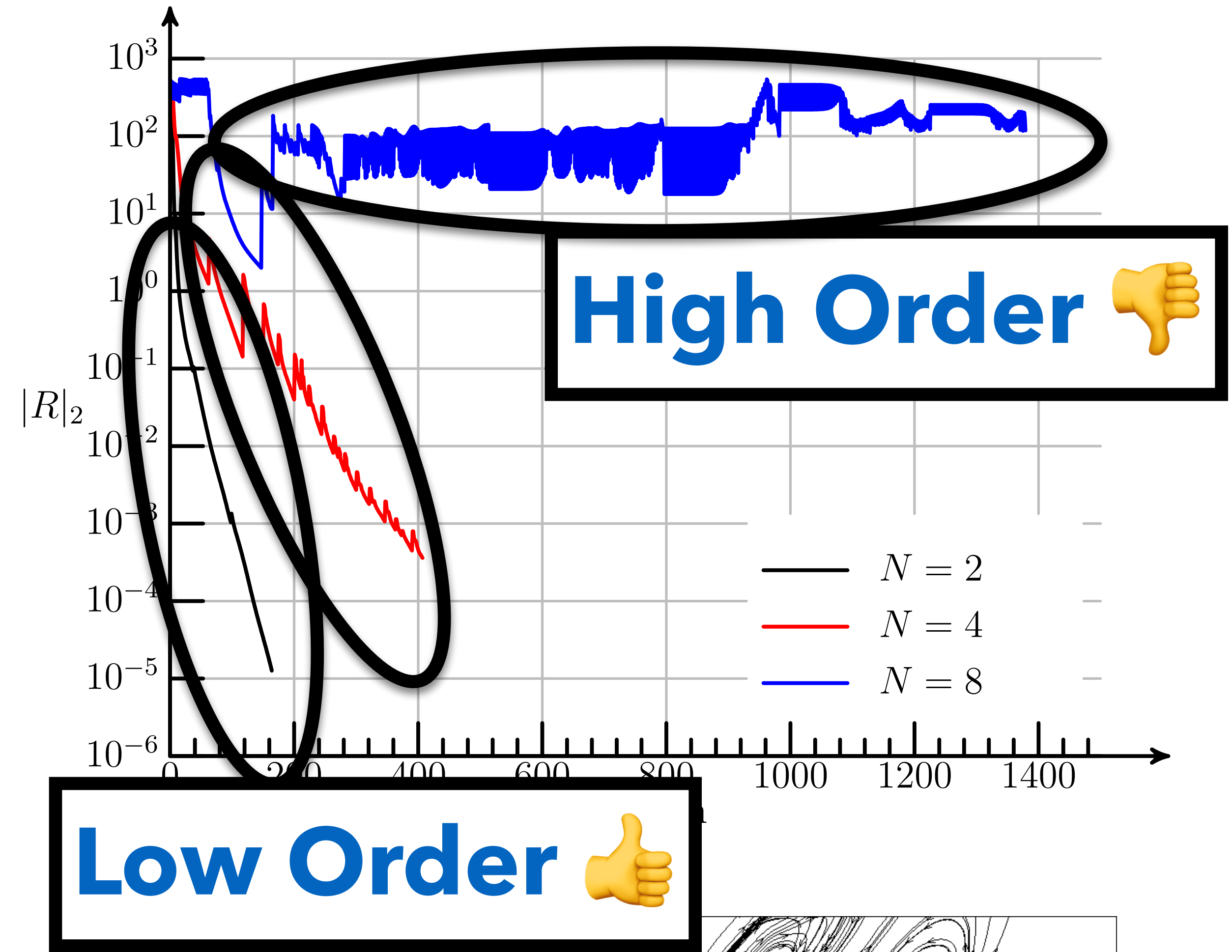
- Computational costs of standard algorithms is prohibitive:

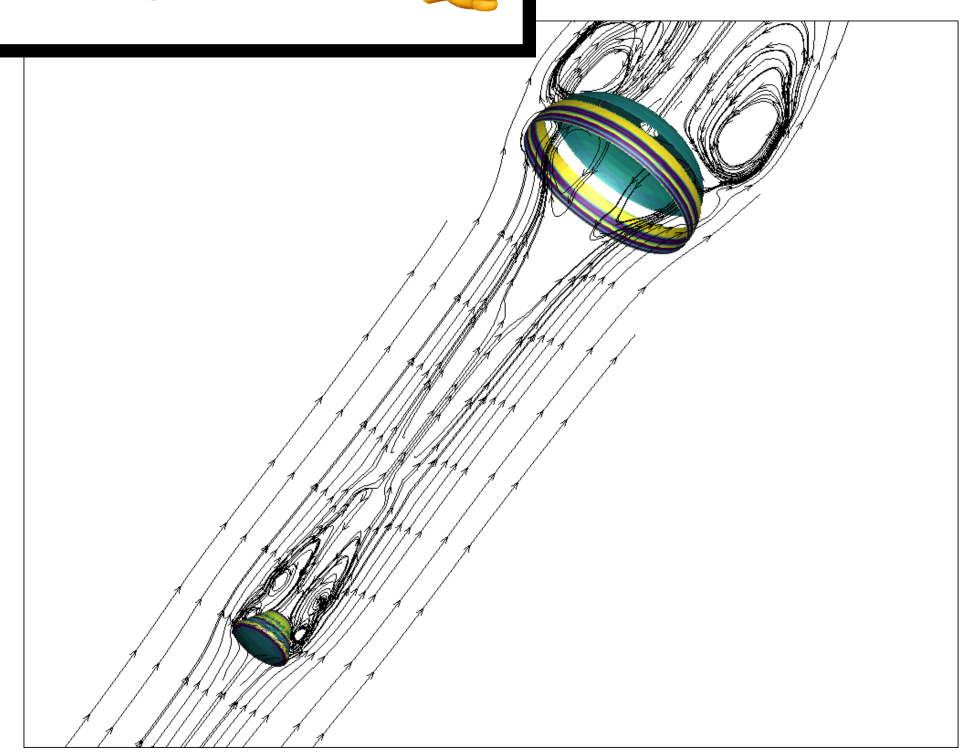
$$\boxed{\textbf{CPU time} = O(\text{order})^{12}}$$

- Need to develop faster algorithms tuned for high-order
  - my research topic since my PhD at UNIVPM

High order methods

- Finding a solution means finding a minimum
- Orion capsule simulation: low order is ok, but high order?
- The key to finding a solution is a good preconditioner: a **hint** to the algorithm that helps to find the minimum
- Problem dependent: a good preconditioner is expensive, while a cheap preconditioner is ineffective!





Solution process

- Multiple time steps to advance the numerical solution in time
- Minimization of the nonlinear residual function $\mathbf{R}(\mathbf{U}) = 0$ through the Newton algorithm at each time step:

(1) **do** until converged:

(2) $(\partial\mathbf{R}/\partial\mathbf{U})\delta\mathbf{U} = -\mathbf{R}(\mathbf{U}_n)$

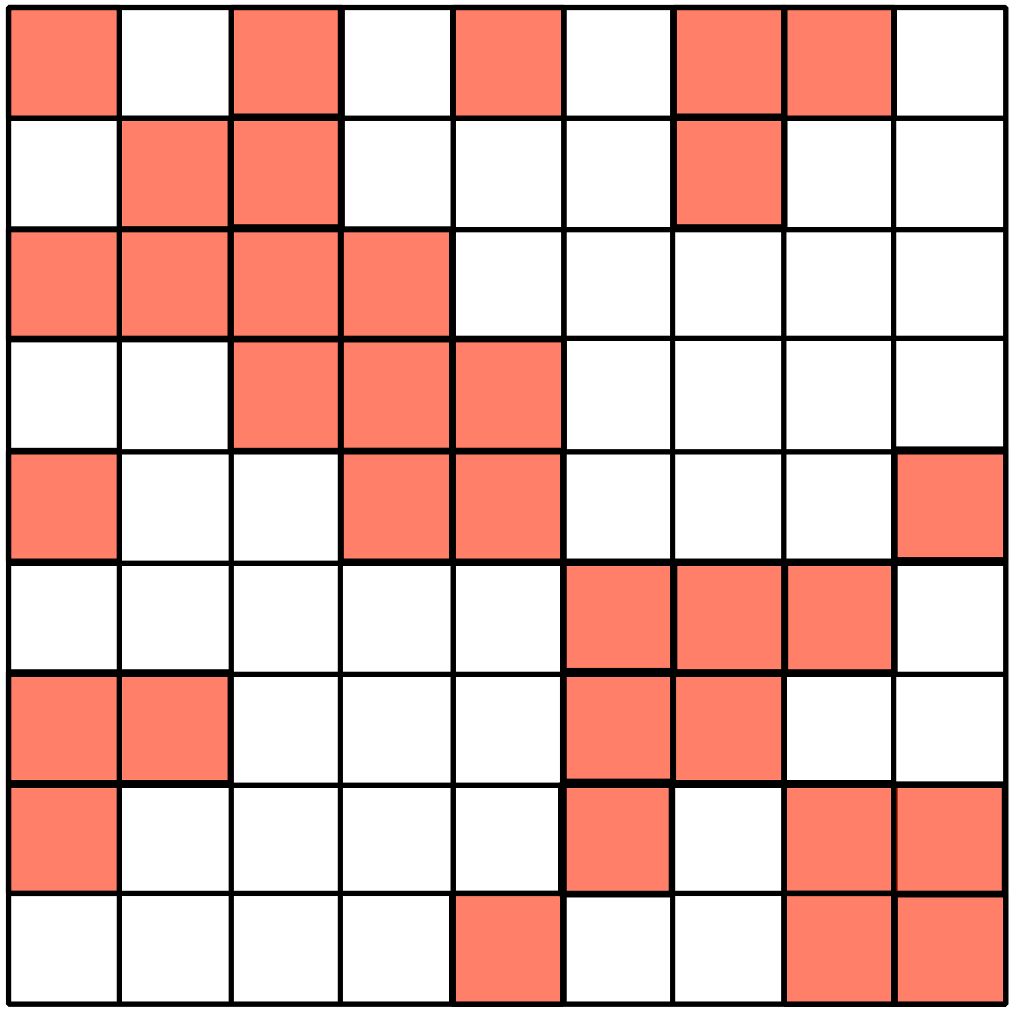
(3) **do** until tolerance reached:

(4) Solve linearized problem $\mathbf{A}\mathbf{x} = \mathbf{b}$, with with (flexible) GMRES

(5) Apply preconditioner $\mathbf{M} \approx \mathbf{A}^{-1}$

- Focus of current work is the application of the preconditioner $\mathbf{M}$ in step 5 for solving linear system
- Strong preconditioning provides good convergence rates at a potentially high cost – **tradeoff is problem-dependent**

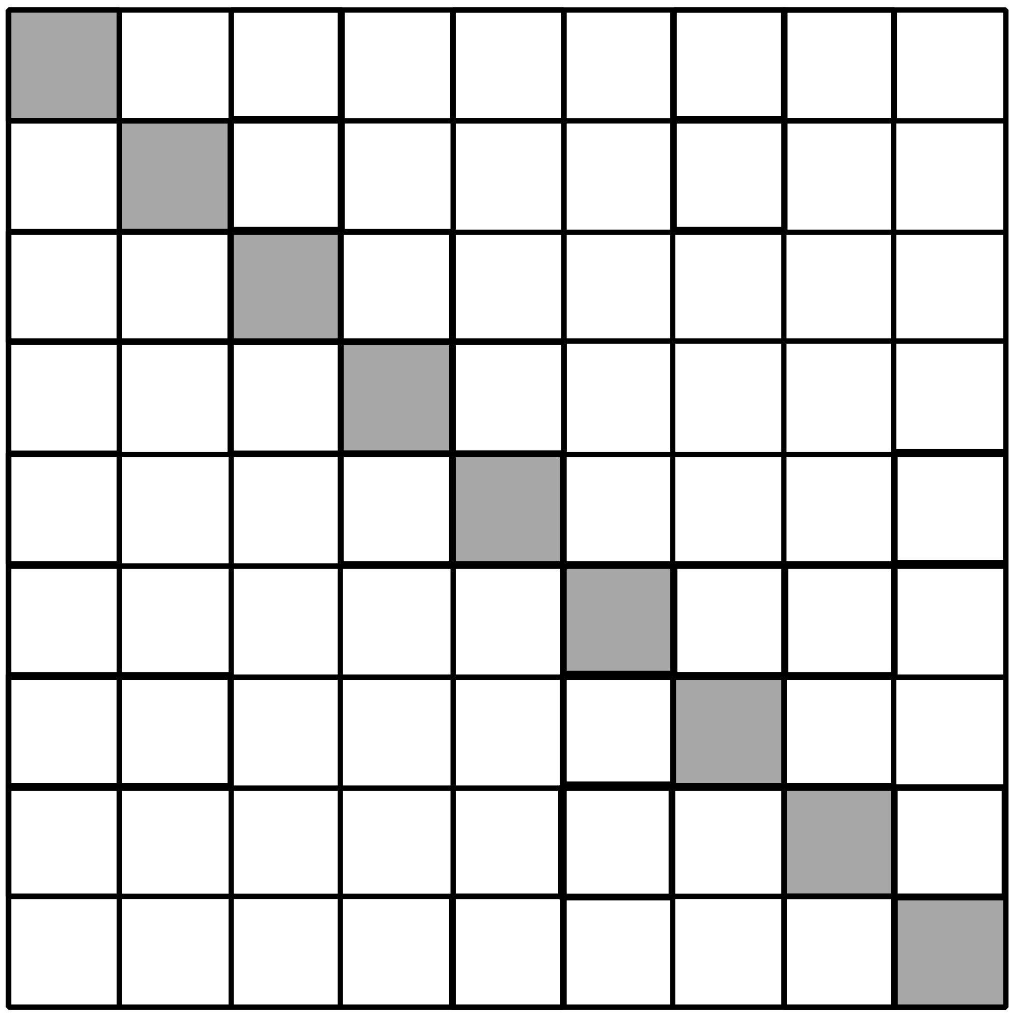
Tensor-product preconditioning (fluid solver)



Diosady & Murman, AIAA 2013-2870

Diosady & Murman, JCP 2017-330, 296-318

Tensor-product preconditioning (fluid solver)



- Key idea 1: neglect off-diagonal blocks for preconditioning

Diosady & Murman, AIAA 2013-2870

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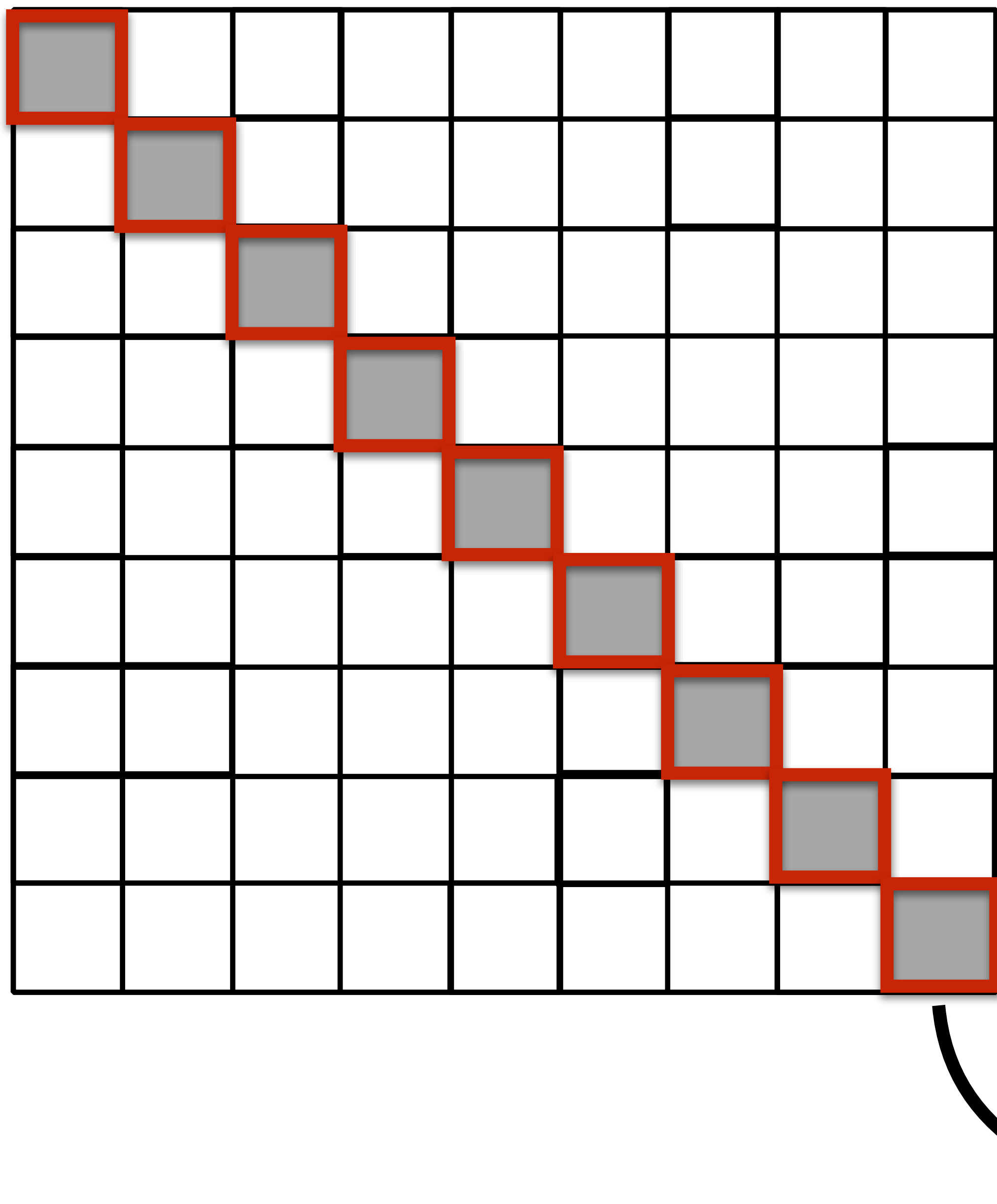
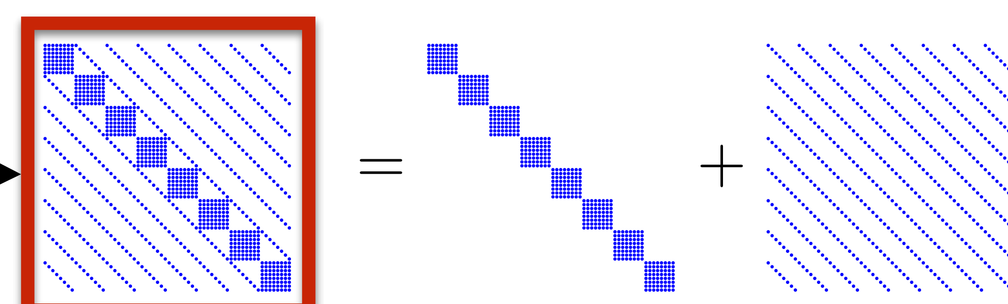
Tensor-product preconditioning (fluid solver)



- Key idea 1: neglect off-diagonal blocks for preconditioning
- Key idea 2: approximate block-diagonal with a tensor-product
- Orders of magnitude faster than standard factorizations
- Ineffective for diffusion-dominated regions

$$\left(\frac{\partial \mathbf{R}}{\partial \mathbf{U}}\right)_{\kappa} = (\mathbf{D}_x \otimes \mathbf{I}) + (\mathbf{I} \otimes \mathbf{D}_y)$$



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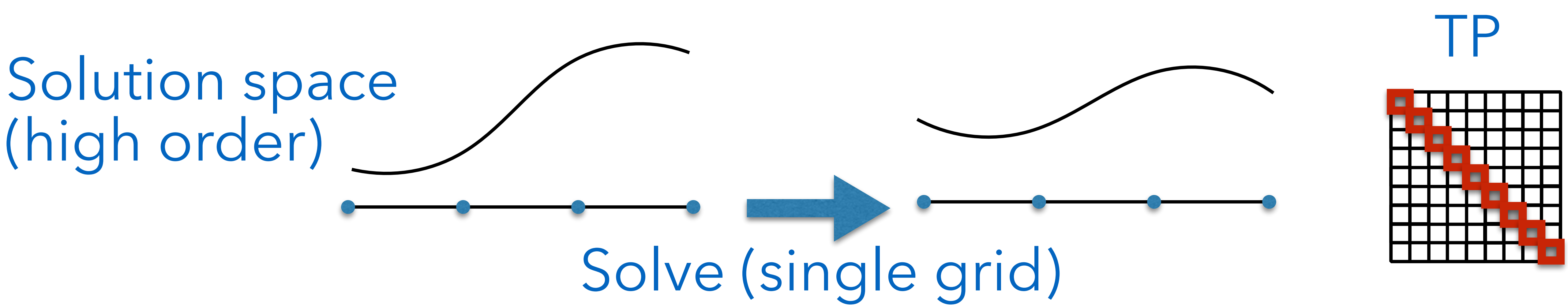
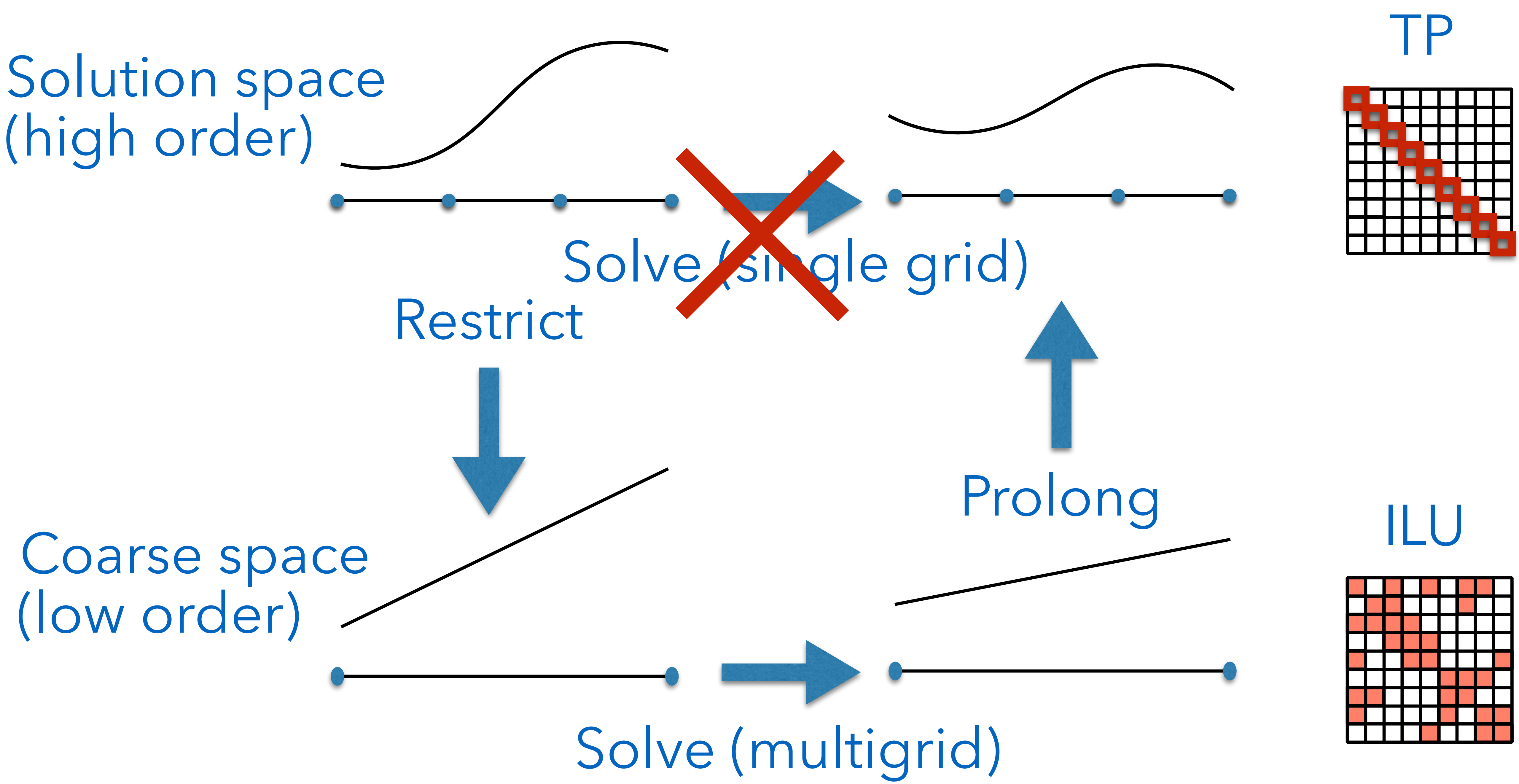
Spectral multigrid method (fluid solver)



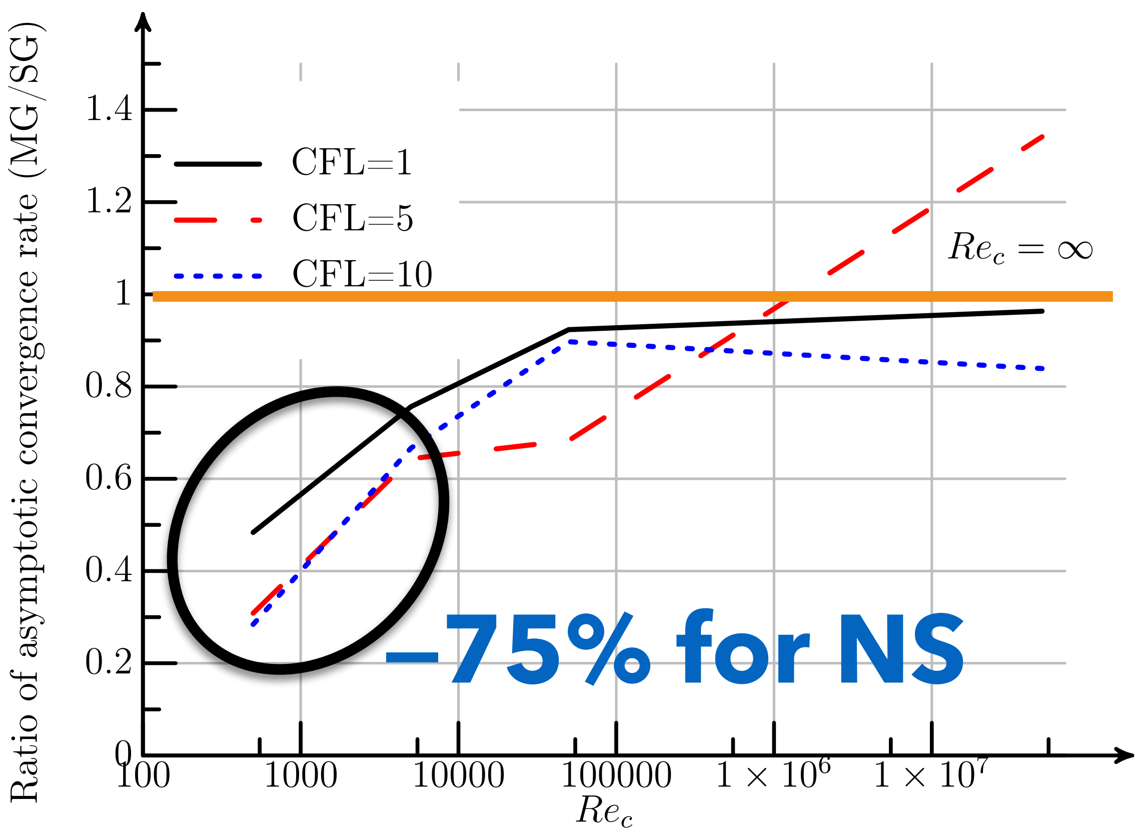
Franciolini & Murman, AIAA 2020-1314

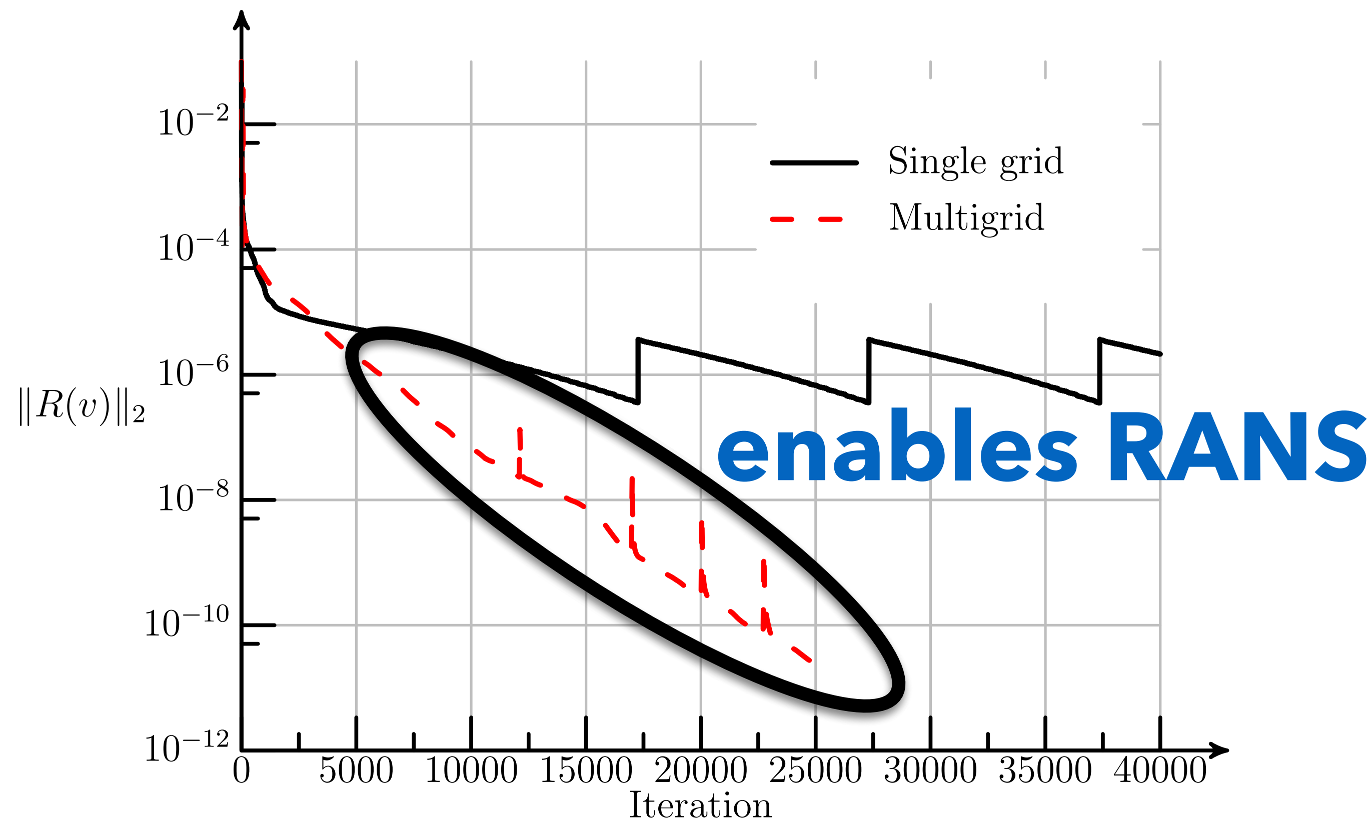
Spectral multigrid method (fluid solver)



Franciolini & Murman, AIAA 2020-1314

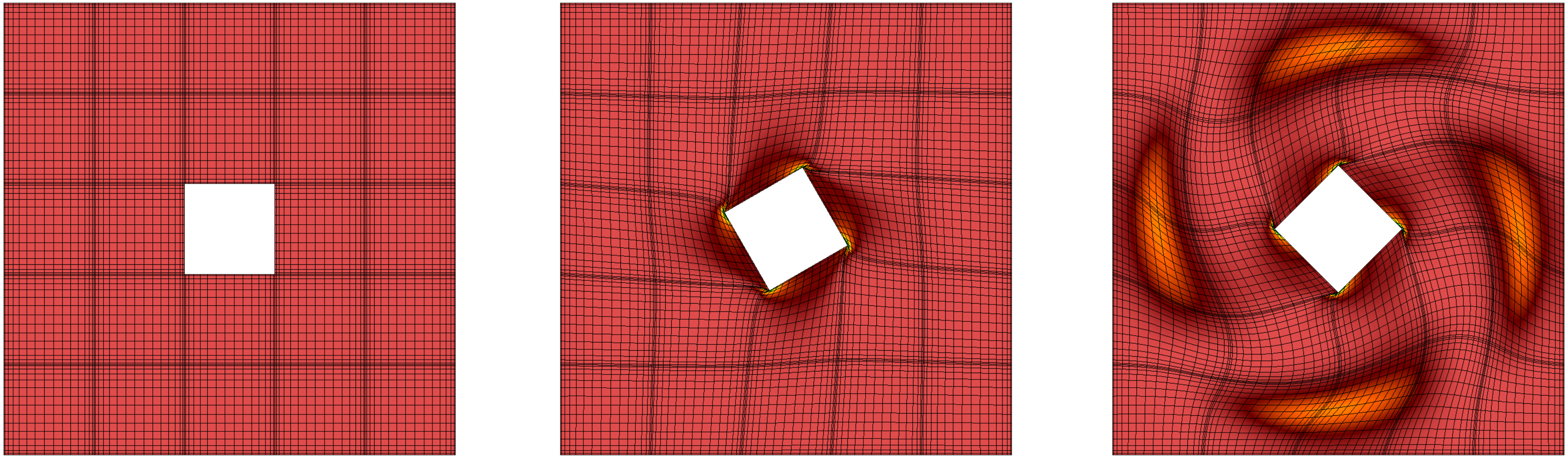
Multigrid performance (fluid solver)





- Plot of performance metric ratio over Reynolds number for range of CFL
- Improvement up to 4x for diffusion dominated regimes
- Performance decrease by increasing Re
- Enabled the solution of different physical models
  - RANS/Transition
  - Chemistry/Hypersonics

Mesh deformation (structural solver)



$\theta = 0^\circ$ $\theta = 30^\circ$ $\theta = 45^\circ$

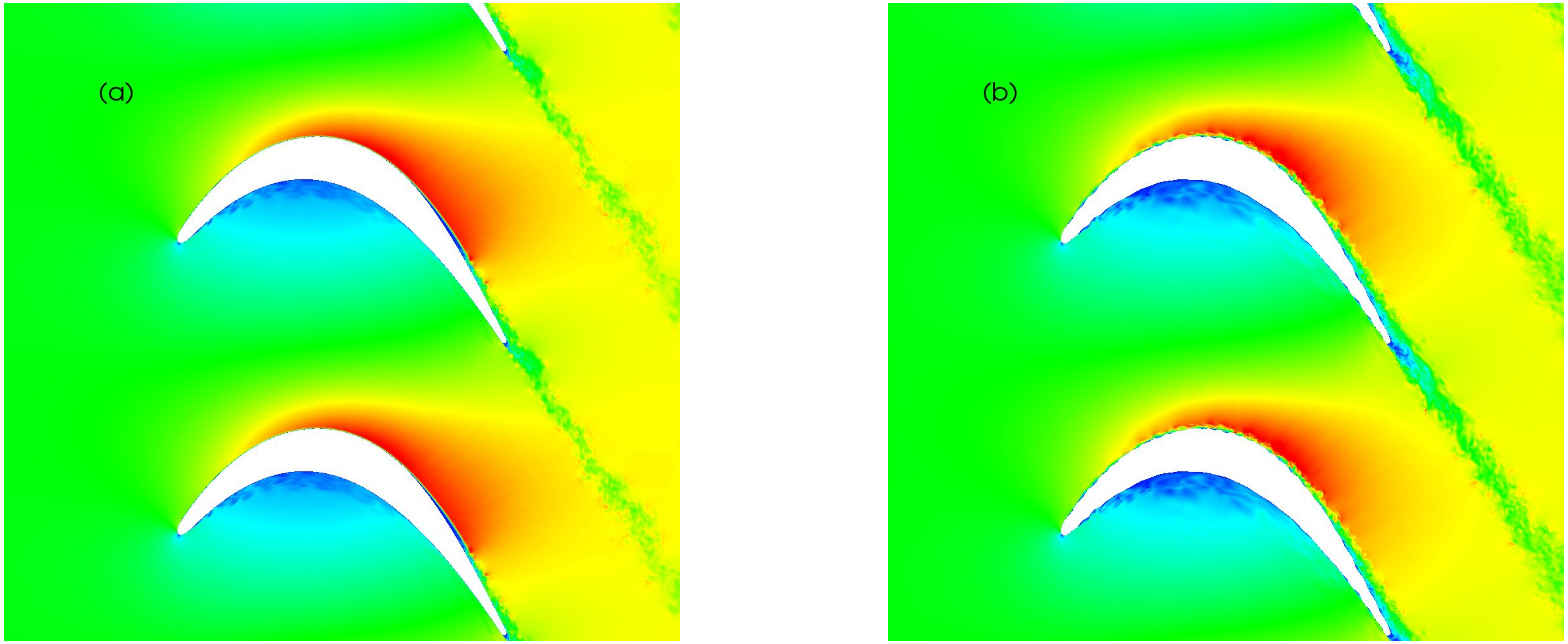
- Applied movement on fluid walls
  - Need to maintain a valid fluid mesh
  - Propagate motion throughout the domain by solving elasticity equations:

$$\sigma_{ij} = 2\mu\epsilon_{ij} - \lambda\epsilon_{kk} = 0 \qquad \mu = \frac{E}{2(1+\nu)}$$

$$\epsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad \lambda = \mu\frac{2\nu}{1-2\nu}$$

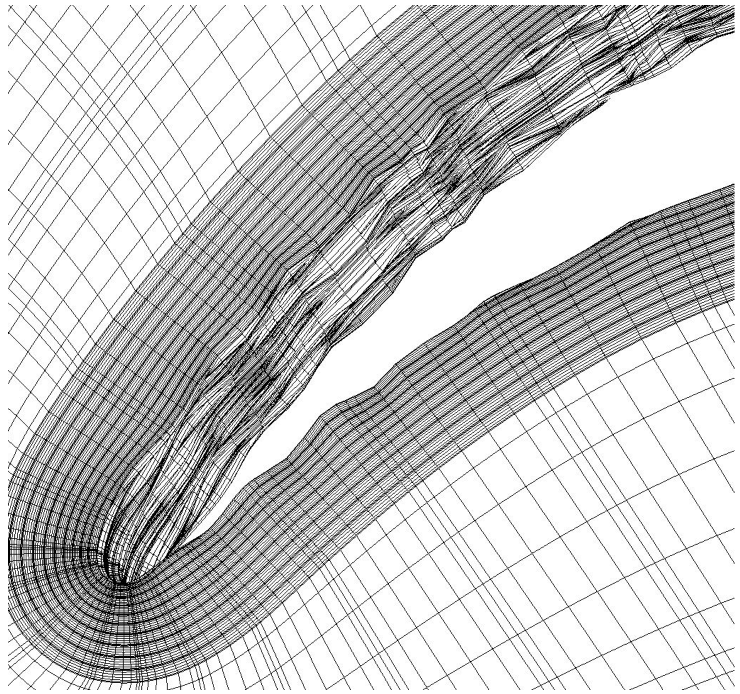
Diosady & Murman, AIAA 2018-0919

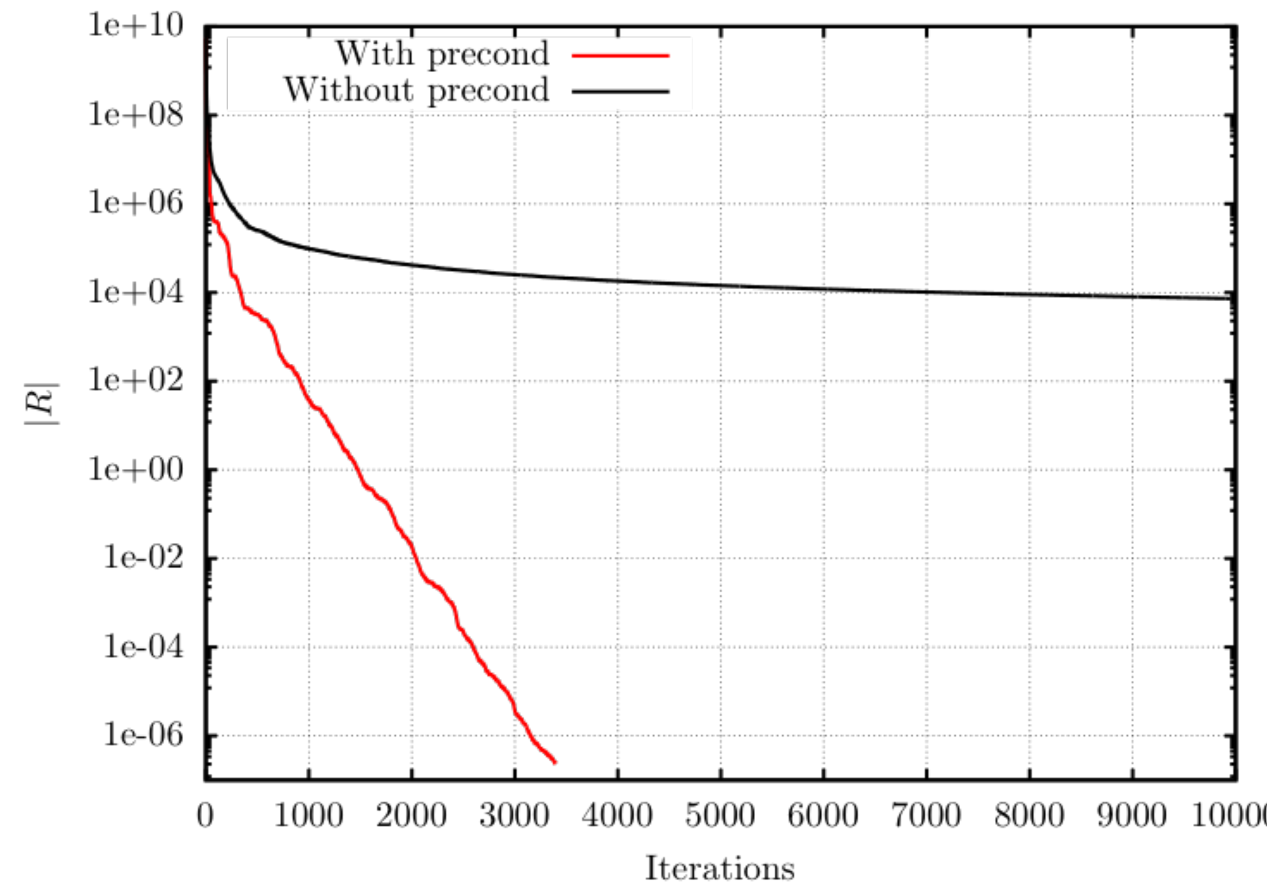
Mesh deformation (structural solver)



- Presence of modeled surface roughness results in different transition behavior and size of separation bubble
- Better agreement with experiments
  - Garai et al. (2018 IGTI)
- Obtaining deformed mesh is **very expensive**

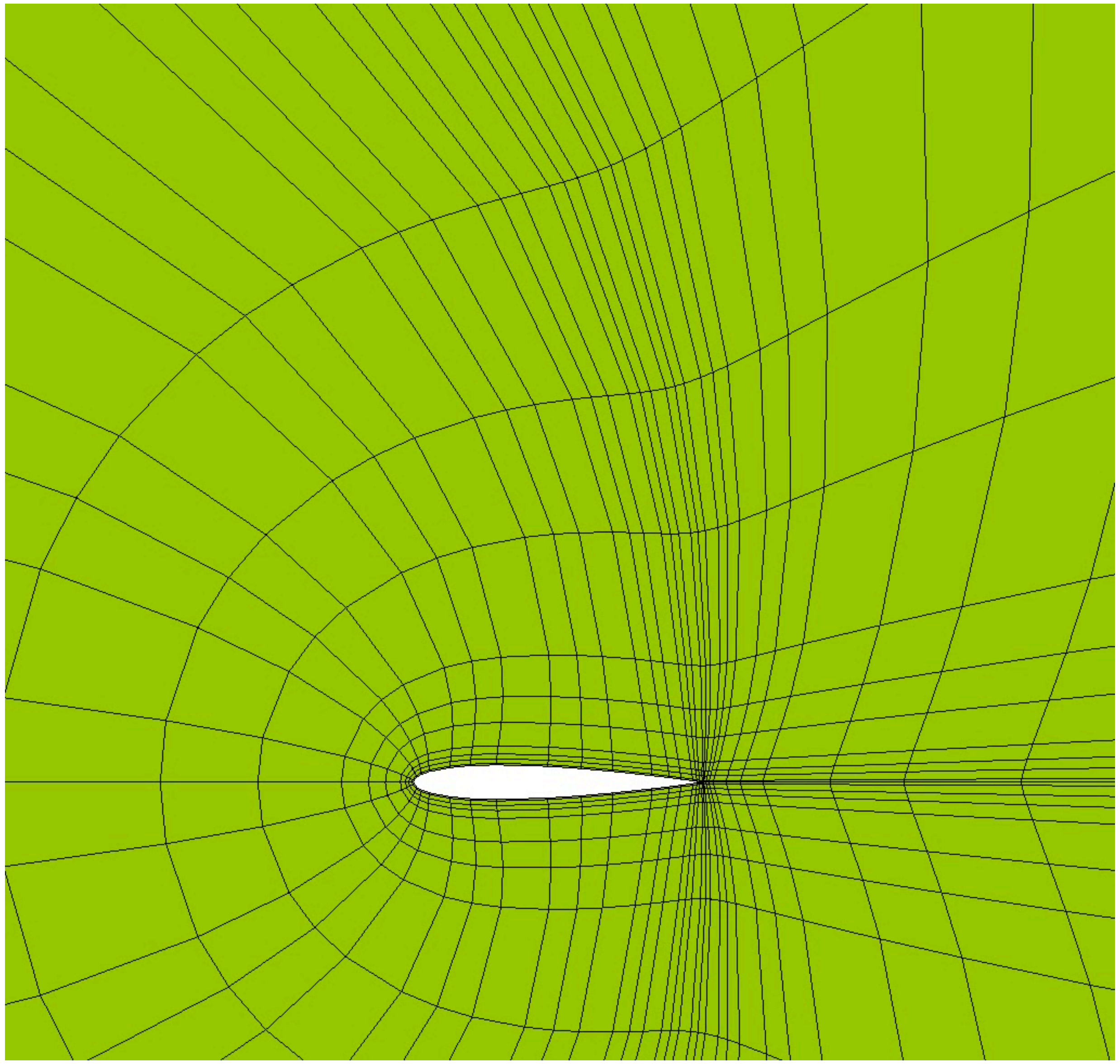
Mesh deformation (structural solver)





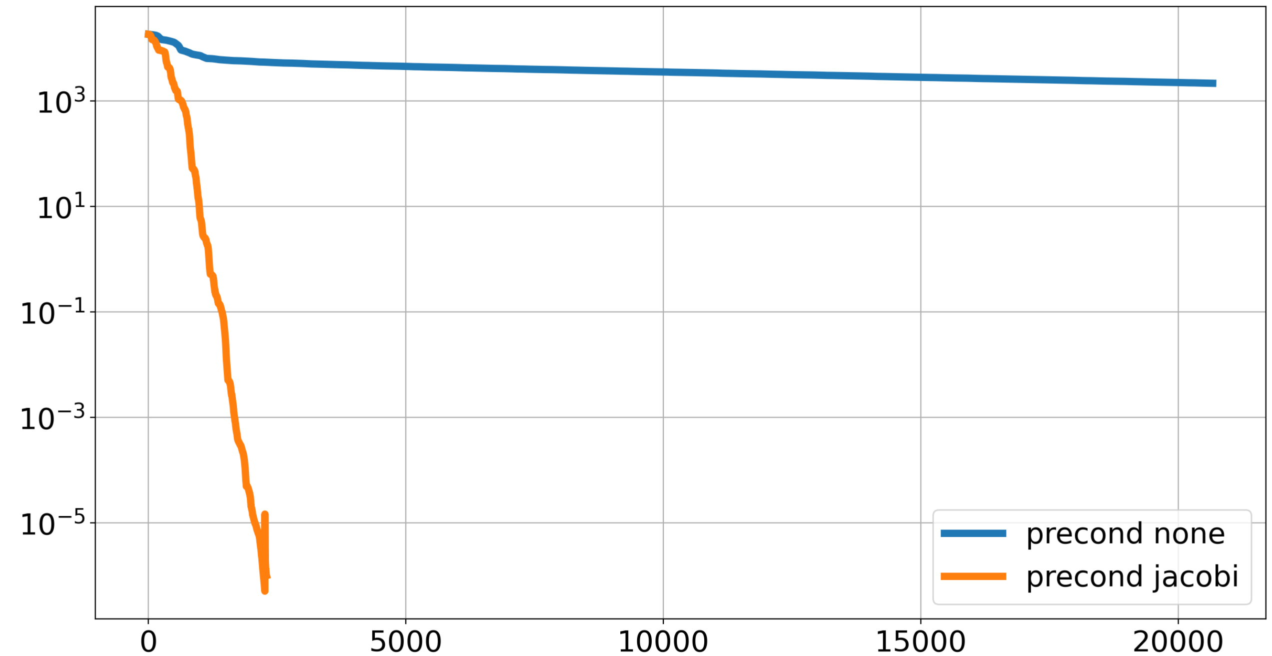
- 8th-order accurate deformation applied to mesh nodal points
- Up to **200X** improvements w.r.t. unpreconditioned case
- Parallel implementation shows linear scaling with number of cores
  - 8th order, 14 Million Dof converges in seconds

Mesh deformation (structural solver)



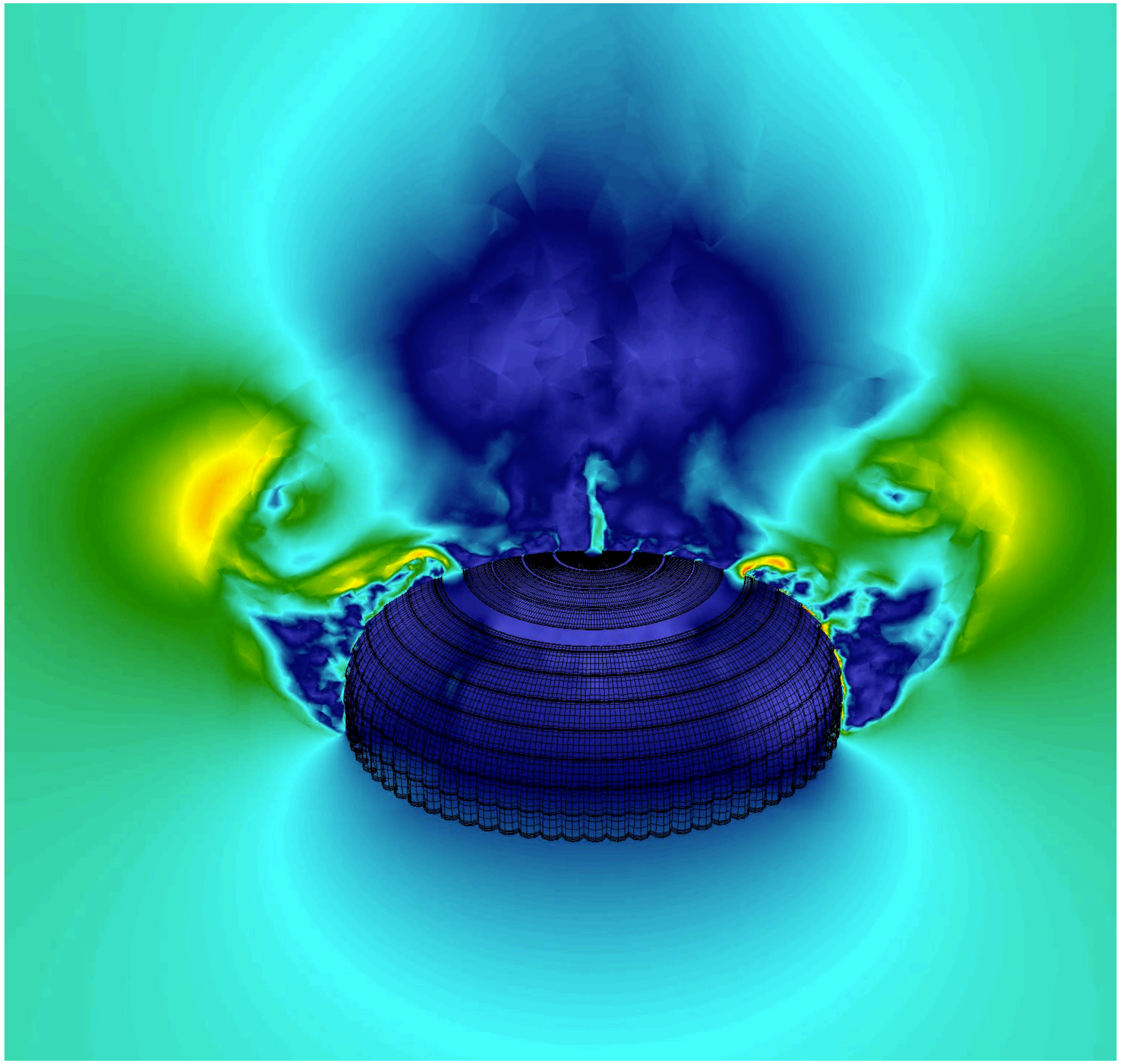
- Extension to time-dependent problems
  - Heaving/pitching airfoil test case
  - Analytical movement prescribed to the airfoil to deform the mesh

Membrane solver



- 2D linear shell case:
  - two pressure pulses moving on a 2D fabric
  - 16th order accurate deformation applied to mesh nodal points
  - preconditioning enables to run the simulation within seconds

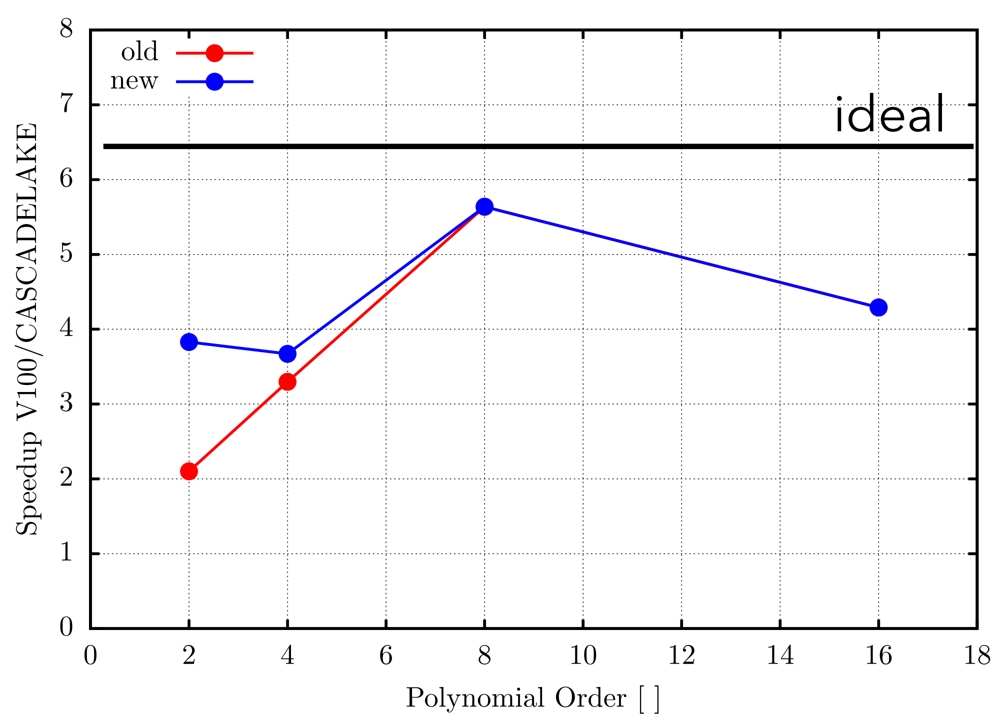
Fluid solver in free flight mode

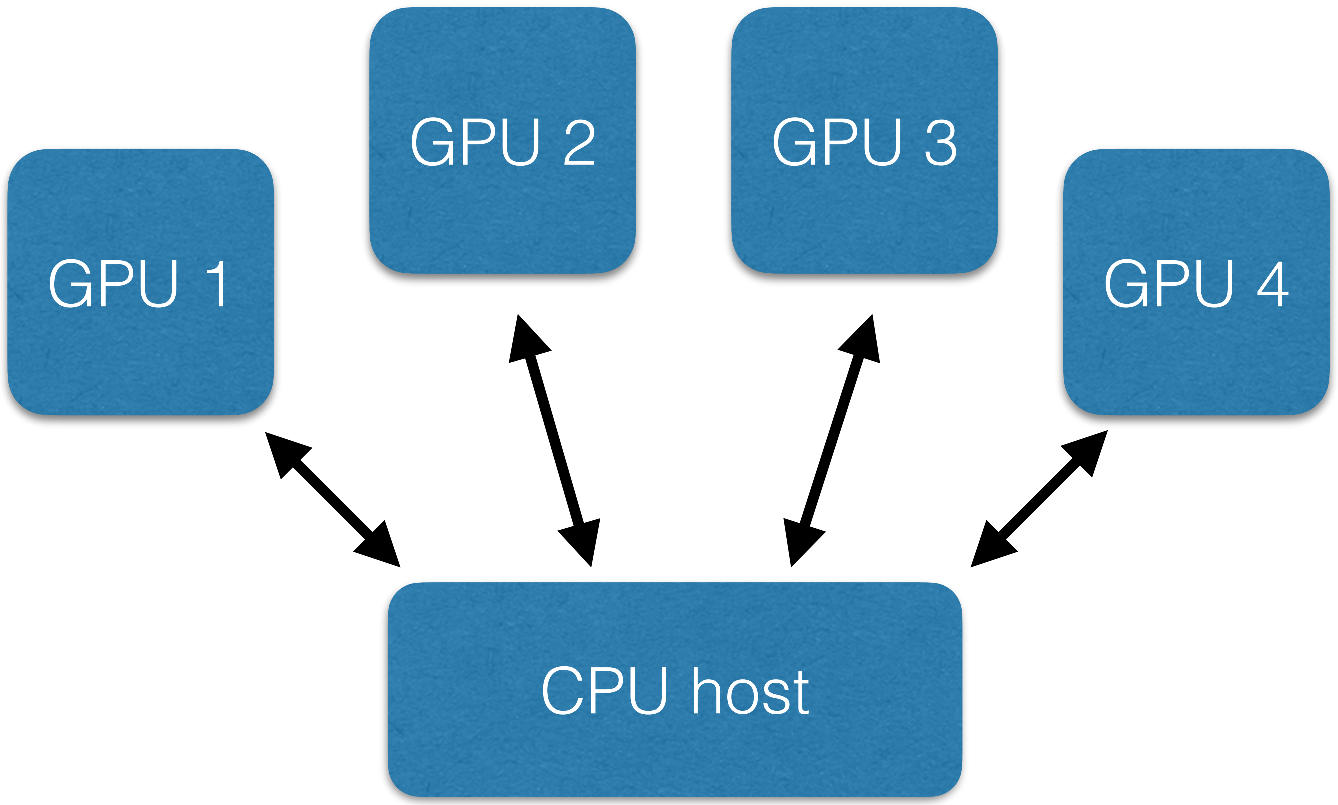


- Fluid + free flight (6 Dofs) solver demo

GPU Acceleration

- Next-generation computing platoforms are heterogeneous
  - several GPUs attached to CPU host
- Traditional simulation algorithms require complete rewrite
  - not clear how to benefit from increased compute capacity
- Preliminary studies show optimal performance
  - Same holds true for Performance/Watt metric





Summary

- Developed a suite of preconditioning strategies for different physics/discretizations
  - Required to enable FSI simulations
- Improved solver robustness using multigrid preconditioning with block factorization of coarse space Jacobian matrices
  - cost reduction for ill-conditioned cases (diffusion)
  - enabling technology for solving stiff source terms (RANS, hypersonics)
- Implementation of preconditioning operator for structural solver
  - enabling use of mesh deformation as high order grid generation tool
  - enabling use of a linear shell structural solver for parachute fabric
- Efforts to port multi-physics applications on GPUs
  - Theoretical performance gains possible with heavy code-rewrite

Acknowledgements

- NASA STMD/ESM
- NASA NAS
- Dr. Scott Murman and the eddy group