Estimation With Range Depended Sensor Model

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This paper focuses on the improvement of object detection accuracy taking into account the sensor's reading degradation as the relative range increases. The approach is based on the assumption that range measurement error depends on the actual range. Specifically, we model the measurement error as a proportional to the actual range term plus a zero-mean, Gaussian distributed, and uncorrelated process. The tracking problem is considered in a mixed continuous-discrete time domain, where the target dynamics is in continuous-time and the measurements are in discrete-time, which is an optimal choice in many tracking and navigation applications. We adopt a commonly used continuous time coordinate-uncoupled white-noise acceleration model for a point object to describe the target motion, and use Extended Kalman Filter (EKF) framework to estimate the target's state and the unknown proportionality coefficient.

I. Introduction

Advanced Air Nobility (AAM) is a revolutionary air transportation system for passengers and cargo in metropolitan areas, which is designed to support a variety of on-board, ground-piloted and increasingly autonomous flight operations [1]. Some future concepts for on-demand mobility include fully autonomous vehicle operation using advanced sensing, perception, and navigation algorithms capable of replacing onboard pilot's functions. These type of operations in a dense and highly dynamic aerospace require the air vehicles be equipped with reliable sense and avoid (SAA) capability to maintain safe separation from terrain and other airborne objects[2, 3].

Traditionally, radio detection and ranging (radar) sensors have been used to detect obstacles such as terrains and air vehicles. Recently, they found widespread applications in autonomous driving and driver assistive systems as they have long ranges and are robust to atmospheric attenuation and lighting conditions. Sense and avoid poses a clear challenge to autonomous operation. However, size, weight, power and cost (SWAP-C) limitations create challenges in reliably achieving desired obstacle (target) detection and estimation results for unmanned aircraft systems [4].

Radar sensors used for target tracking are conventionally modeled to provide measurements of the target's relative to the sensor range and angles, and in some cases also range rate, with uncertainties normally assumed to be zero-mean, Gaussian distributed, and uncorrelated [5]. In general, range and angle measurements may have different accuracy depending on the sensors configurations. For example, a phased array radar has more accurate range measurements than angle measurements, which can be inferred from analysis of the estimation error covariance ellipsoid that looks like a pancake normal to the range vector. In case of a continuous-wave radar, the situation is reversed and the error

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covariance ellipsoid has a cigar-shape along the range vector [6]. As it was mentioned in Ref. [1], these latter type of radars may be suitable for small aircraft systems in terrain detection and dynamic obstacle tracking for safe and reliable AAM operations. From this perspective it is worth to take a second look at radar sensor's range measurement model. As far a the target motion is concerned, a good survey of more or less standard models can be found in Ref. [9], where the control input for target maneuvers is consider as a random process belonging to three main groups of White, Markov and Semi-Markov jump processes.

This paper focuses on the improvement of object detection accuracy taking into account the sensor's reading degradation as the relative range increases. The approach is based on the assumption that range measurement error depends on the actual range. Similar idea was used in Ref. [7] to derive estimation bounds in radar target tracking assuming that the range measurement error upper bound is proportional to the actual range itself. Specifically, we model the measurement error as a proportional to the actual range term plus a zero-mean, Gaussian distributed, and uncorrelated process. The tracking problem is considered in a mixed continuous-discrete time domain, where the target dynamics is in continuous-time and the measurements are in discrete-time, which is an optimal choice in many tracking and navigation applications. We adopt a commonly used continuous time coordinate-uncoupled white-noise acceleration model for a point object to describe the target motion [8], and use Extended Kalman Filter (EKF) framework to estimate target's state and the unknown proportionality coefficient.

II. Problem Formulation

We consider a target estimation/tracking problem in continuous-discrete time domain, that is, the target dynamics in continuous-time and the measurements in discrete-time with a time step δt, which is the optimal choice in many publications for tracking and navigation application (see for example [11–13] and references therein). Let the target motion be given by the following continuous time coordinate-uncoupled white-noise acceleration model in Cartesian coordinate system $\{x,\ y,\ z\}$

$$\begin{aligned} \ddot{x}(t) &= n_x(t) \\ \ddot{y}(t) &= n_y(t) \\ \ddot{z}(t) &= n_z(t)\,, \end{aligned} \tag{1}$$

where $n_x(t),\ n_y(t),\ n_z(t)$ are zero-mean, Gaussian distributed, and uncorrelated processes with variances $\sigma_x^2,\ \sigma_y^2,\ \sigma_z^2$. The observation model is given in $3D$ spherical coordinate system $\{r,\ \theta,\ \phi\}$ as

$$\begin{aligned} r_m(k) &= r(k) + f(r(k)) + n_r(k) \\ \theta_m(k) &= \theta(k) + n_\theta(k) \\ \phi_m(k) &= \phi(k) + n_\phi(k)\,, \end{aligned} \tag{2}$$

where subscript m indicates the measured value of the corresponding quantity, $f(r(k))$ represents a range dependent measurement error term, $n_r(t),\ n_\theta(k),\ n_\phi(k)$ are assumed to be zero-mean, Gaussian distributed, and uncorrelated processes with variances $\sigma_r^2,\ \sigma_\theta^2,\ \sigma_\phi^2$. Here, we assume that the error term is linear in actual range $f(r(k)) = br(k)$, where b is assumed to be unknown constant.

We use Extended Kalman Filter (EKF) framework for the target tracking. To this end, we employ the dicreteized 7th order motion model

$$\boldsymbol{s}(k) \;=\; \Phi(k-1)\boldsymbol{s}(k-1) + \Gamma(k-1)\boldsymbol{w}(k-1)\,, \tag{3}$$

where the vector $\boldsymbol{s}(k) = [x(k)\ v_x(k)\ y(k)\ v_y(k)\ z(k)\ v_z(k)\ b(k)]^\top$ represents the target state and the unknown constant b, $bfw(k) = [n_x(k)\ n_y(k)\ n_z(k)\ n_b(k)]^\top$ is the input noise vector, which also includes a zero-mean Gaussian white noise $n_b(k)$ that drives evolution of the unknown term b,

$$\Phi(k) = \begin{bmatrix} 1 & \delta t & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \Gamma(k) = \begin{bmatrix} \delta t^2/2 & 0 & 0 & 0 \\ \delta t & 0 & 0 & 0 \\ 0 & \delta t^2/2 & 0 & 0 \\ 0 & \delta t & 0 & 0 \\ 0 & 0 & \delta t^2/2 & 0 \\ 0 & 0 & \delta t & 0 \\ 0 & 0 & 0 & \delta t \end{bmatrix}$$

are the state transition and input matrices, and the linearized observation model

$$\boldsymbol{u}(k) \quad = \quad H(k)\boldsymbol{s}(k) + \boldsymbol{v}(k)\,, \tag{4}$$

where $\boldsymbol{u}(k) = [r_m(k)\ \theta_m(k)\ \phi_m(k)]^\top$ is the measurement vector, $\boldsymbol{v}(k) = [n_r(k)\ n_\theta(k)\ n_\phi(k)]^\top$ is the measurement noise vector, and

$$H(k) = \begin{bmatrix} (1+b)\cos(\phi)\cos(\theta) & 0 & (1+b)\cos(\phi)\sin(\theta) & 0 & (1+b)\sin(\phi) & 0 & r \\ -\frac{\sin(\theta)}{r\cos(\phi)} & 0 & \frac{\cos(\theta)}{r\cos(\phi)} & 0 & 0 & 0 & 0 \\ -\frac{\sin(\phi)\cos(\theta)}{r} & 0 & -\frac{\sin(\phi)\sin(\theta)}{r} & 0 & \frac{\cos(\phi)}{r} & 0 & 0 \end{bmatrix},$$

is the Jacobean. The system of equations (3) and (4) is in a standard form for the EKF application.

III. Simulation Results

We simulated the target motion $(x,\ y)x$-plane with initial speed of $400ft/sec$ in y-direction from the initial position of $(0,\ 0)ft$ with $\sigma_x = \sigma_y = 0.001$. The measurement noise was set to $\sigma_r = 300,\ \sigma_\theta = 0.01$ and the unknown constant $b = 0.01$. Fig 1 shows EKF estimation results in $(x,\ y)x$-plane. It can be observed that a satisfactory estimation was achieved. Fig. 2 displays the estimate of unknown constant b.

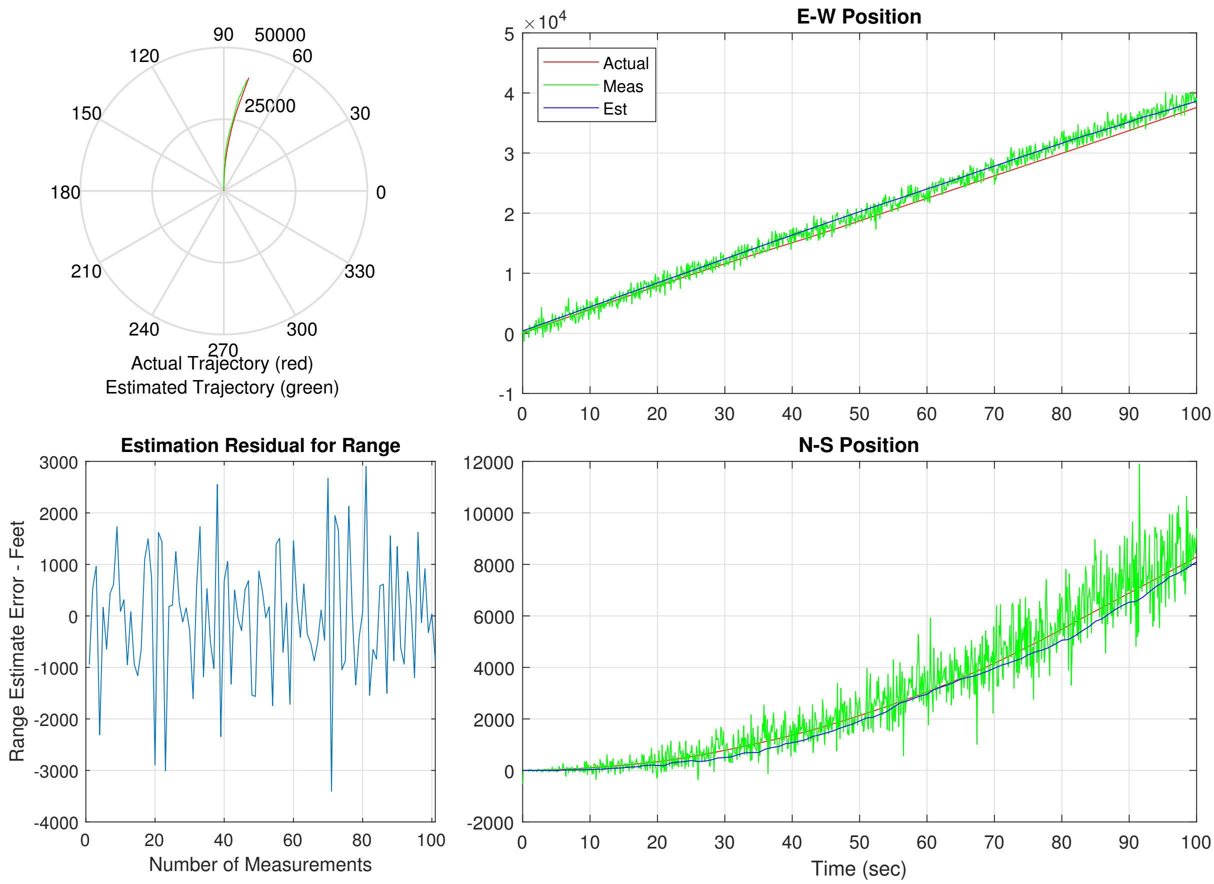

Fig. 1 EKF estimation results for the position tracking

The full paper will include comprehensive and comparative analysis of target tracking with range dependent sensor model for several estimation/filtering frameworks.

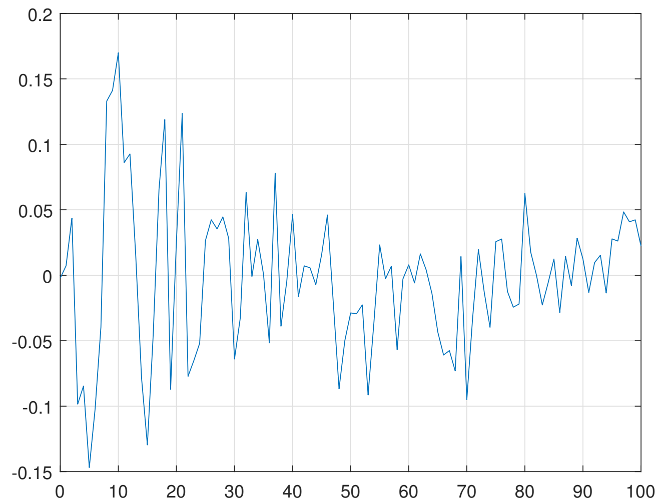

Fig. 2 EKF estimation results for the unknown constant

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