PMSM Parameter Estimation using a Linear Unknown Input Interval Observer

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In this paper, the problem of permanent magnet synchronous motor (PMSM) speed and unknown load torque estimation is addressed. For this purpose, an interval unknown input observer (UIO) for linear time-invariant (LTI) systems is used. First, the PMSM model is linearized in order to make it in a suitable form for the linear interval UIO. Then, the interval UIO is applied to allow the joint estimation of the motor speed and the unknown load torque disturbance. The main advantages of this approach is that it not only allows the joint state and unknown input estimation, but also to take the different uncertainty sources into account. Indeed, taking model and measurement uncertainty into account is crucial. Assuming that the measurement noise and disturbances are bounded, lower and upper bounds are first computed for the unmeasured state (motor speed) and then for the unknown input (load torque). The proposed approach and its limitations are demonstrated with the nonlinear PMSM model derived from its equivalent electrical circuit.

I. Introduction

Emerging Urban Air Mobility (UAM) operations, are expected to revolutionize transportation in moderate to densely populated urban regions by providing an air transportation system that can safely, efficiently, and autonomously move people and cargo. These operations, in rural and urban metropolitan areas, will vary from air medical services for patients and/or medical supplies, small and large package delivery, and personal taxi services using a variety of autonomous aircraft types.

With the widespread interest stemming from the vast potential of UAM, electric propulsion powered aircraft are under investigation as their vertical takeoff and landing capabilities can enable UAM operations in small constrained spaces. Furthermore, electric propulsion systems provide the added benefits of reduced noise and pollution levels and more importantly, they provide significant mass savings with the removal/reduction in jet fuel required for typical aircraft. On the other hand, the propulsion systems of electric aircraft have significantly reduced energy in comparison to their jet fuel counterparts, thus limiting their range and endurance. As a result, it is important to accurately characterize their components to enable prediction and mitigation of hazards that could arise from their use.

NASA's Aeronautics Mission Directorate is currently funding the System-Wide Safety (SWS) project to develop and demonstrate innovative and safety-oriented solutions for emerging operations such as UAM. Through the SWS project, hazards to emerging operations are identified such as critical system failures, propulsion component degradation, or engine/power failure. For these hazards, quantifiable safety metrics are defined, models to monitor and predict their evolution are developed, and test data is generated to verify and test these models, considering the complex interplay of the different hazards that influence these safety metrics. With the goal of monitoring, assessing, predicting, and mitigating risks to safe UAM operations by first developing accurate models that represent critical UAM components, this paper investigates a technique to characterize the parameters of an electric motor with significant potential for use in UAM vehicle design, the permanent magnet synchronous motor (PMSM).

The PMSM is gaining interest in the UAM industry because it has high efficiency and high power density while remaining a relatively small motor with reduced noise levels [1],[2]. Estimating the speed of the PMSM, reduces reliance on mechanical sensors that may or may not be included in the design by the motor manufacturer. Thus, the implementation of a diagnostic technique that can enable fault detection, identification, or isolation, can be achieved

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using the estimated speed and load torque of the PMSM. In addition, these parameters will feed into the overall diagnostics of the electric power train that depends on electro-mechanical components including the motor and battery.

Although there are varying techniques of motor speed and load torque estimation, this work utilizes an observer-based approach that can handle parameter uncertainties in the motor model to improve accuracy and provide robustness. Specifically, the interval unknown input observer (UIO) technique for linear time-invariant (LTI) systems is chosen for its capability to estimate both state and unknown inputs in the presence of disturbances that cannot be measured.

This paper is organized as follows. First we give an overview of the PMSM in Section II along with its nonlinear and linearized equations. This is followed by a description of the Unknown Input Interval Observer design in Section III. Then, we describe the application of the UIO to the PMSM speed and load torque disturbance estimation in IV, after which we give a discussion and analysis of results. Finally, a conclusion and guide to future work in Section V.

II. Permanent Magnet Synchronous Motor model

In this section, the permanent magnet synchronous motor model is described in both three-phase abc frame and two-phase $d-q$ frame. Then, the linearized PMSM model used later with the interval UIO is presented.

A. PMSM motor equations

The PMSM model derived from its equivalent circuit model has been widely studied and used in the literature [3], and has been established under the following assumptions:

- Symetrical three phase-machine
- Non-salient poles (surface-mounted magnets)
- Saturation and hysteresis in the magnetic circuit, as well as eddy currents in the armature are neglected
- Induced electromagnetic force (EMF) is sinusoidal

1. Electrical equation

With these assumptions, the voltage equation of the PMSM in the abc reference frame is given by:

$$\mathbf{v}_{abc} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \mathbf{i}_{abc} + \frac{d\boldsymbol{\lambda}_{abc}}{dt} \tag{1}$$

where $\mathbf{v}_{abc}$, $\mathbf{i}_{abc}$ and λ_{abc} are respectively the voltage, current and flux linkage vectors of the three phases, and R_s is the phase resistance.

Under the previous assumptions, the flux linkage vector is expressed as

$$\lambda_{abc} = \begin{bmatrix} L_s & M & M \\ M & L_s & M \\ M & M & L_s \end{bmatrix} \mathbf{i}_{abc} + \lambda_M \begin{bmatrix} \cos(\theta_r) \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \tag{2}$$

where L_s is the phase self inductance, M is the mutual inductance, λ_M is the peak strength of the flux linkage due to the magnets, and θ_r is the electrical position of the rotor.

Finally, after subtituting Eq. 2 into Eq. 1, the electrical equation of the PMSM in abc frame becomes

$$\mathbf{v}_{abc} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \mathbf{i}_{abc} + \begin{bmatrix} L_s & M & M \\ M & L_s & M \\ M & M & L_s \end{bmatrix} \frac{d\mathbf{i}_{abc}}{dt} - \lambda_M \omega_r \begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - \frac{2\pi}{3}) \\ \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \tag{3}$$

where ω_r is the electrical angular speed of the PMSM.

2. Mechanical equation

In order to have a complete description of the PMSM behavior which depends on the rotor position and angular speed, the mechanical equation of the PMSM derived from Newton's second law is included in the model:

$$\frac{d\omega_r}{dt} = \frac{n_p}{J}(T_e - T_l - B_f\,\omega_r) \tag{4}$$

where n_p is the number of pole pairs, J is the inertia of the rotor, B_f is the viscous friction coefficient, T_e is the electromagnetic torque, and T_l is the load torque of the motor.

The expression of the electromagnetic torque T_e is given by

$$T_e = -n_p \mathbf{i}_{abc}^{\top} \lambda_M \omega_r \begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - \frac{2\pi}{3}) \\ \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \tag{5}$$

3. PMSM equations in $d-q$ frame

The previous equations describing the PMSM behavior in abc stationary frame attached to the stator, depend on the rotor position θ_r. In order to remove this dependency and thus simplify the equations, the so-called Park transform is used to obtain a set of equations in the $d-q$ rotating frame attached to the rotor, where the voltage, current and flux vectors have two DC components instead of three AC components [4].

The Park transform to convert from the three-axis stationary frame abc to the rotating frame $d-q$ and inversely can be written as

$$\begin{aligned} \mathbf{f}_{dq} &= P_{abc\to dq}\mathbf{f}_{abc} \\ \mathbf{f}_{abc} &= P_{dq\to abc}\mathbf{f}_{dq} \end{aligned}$$

with the Park transform matrix

$$P_{abc\to dq} = \begin{bmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{bmatrix} \quad \text{and} \quad P_{dq\to abc} = \begin{bmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{bmatrix} \tag{6}$$

After applying the Park transform to the variables in abc frame, the following equations of the PMSM in $d-q$ frame are obtained:

$$v_d = R_s i_d - L_q i_q \omega_r + L_d \frac{di_d}{dt} \tag{7}$$

$$v_q = L_d \omega_r i_d + R_s i_q + L_q \frac{di_q}{dt} + \lambda_M \omega_r \tag{8}$$

$$T_e = \frac{3}{2} n_p \lambda_M i_q \tag{9}$$

$$\frac{d\omega_r}{dt} = \frac{n_p}{J}(T_e - T_l - B_f\,\omega_r) \tag{10}$$

where v_d,v_q and i_d, i_q are respectively the voltage and current components in the $d-q$ axis, L_d and L_q are respectively the direct and quadrature inductances ($L_d = L_q = L_s$ for surface-mounted PMSM [5]).

By denoting the state vector by $\mathbf{x} = [i_d \;\; i_q \;\; \omega_r]^{\top}$, the input vector by $\mathbf{u} = [v_d \;\; v_q]^{\top}$, and the unknown load torque by $d = T_L$, the state-space representation of the non-linear PMSM in $d-q$ frame is

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) + Dd \tag{11}$$

with

$$f(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} -\frac{R_s}{L_s} i_d + i_q \omega_r + \frac{1}{L_s} v_d \\ -\omega_r i_d - \frac{R_s}{L_s} i_q - \frac{\lambda_M}{L_s}\omega_r + \frac{1}{L_s} v_q \\ \frac{3n_p^2}{2J}\lambda_M i_q - \frac{B_f}{J}\omega_r \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 0 \\ 0 \\ \frac{n_p}{J} \end{bmatrix} \tag{12}$$

B. Linearization of the PMSM model

In order to apply a linear interval unknown input observer for PMSM speed and unknown load torque estimation, the Jacobian linearization of the non-linear PMSM model (11) around an operating point $\mathbf{x_0} = [i_{d_0} \;\; i_{q_0} \;\; \omega_{r_0}]$ is used:

$$\Delta\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} -\frac{R_s}{L_s} & \omega_{r_0} & i_{q_0} \\ -\omega_{r_0} & -\frac{R_s}{L_s} & -\frac{\lambda_M}{L_q} - i_{d_0} \\ 0 & \frac{3n_p^2}{2J}\lambda_M & -\frac{B_f}{J} \end{bmatrix}}_{A_J} \Delta\mathbf{x} + \underbrace{\begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \end{bmatrix}}_{B_J} \Delta\mathbf{u} - \begin{bmatrix} 0 \\ 0 \\ \frac{n_p}{J} \end{bmatrix} d \tag{13}$$

where $\Delta\mathbf{x}$ and $\Delta\mathbf{u}$ are deviation variables from the operating point, and $A_J = \frac{\partial f}{\partial \mathbf{x}}\Big|_{\substack{\mathbf{x}=\mathbf{x_0} \\ \mathbf{u}=\mathbf{u_0}}}$ and $B_J = \frac{\partial f}{\partial \mathbf{u}}\Big|_{\substack{\mathbf{x}=\mathbf{x_0} \\ \mathbf{u}=\mathbf{u_0}}}$ are the Jacobian matrices derived from $f(\mathbf{x}, \mathbf{u})$.

The obtained linearized model (13) is in a suitable form for state and unknown input bounds estimation with the linear interval unknown input observer presented in the next section. In this case, $\mathbf{x}$ is the state vector and d is the unknown input associated to the load torque disturbance.

III. Unknown input interval observer design

A. Problem statement and Methodology

Consider the following LTI discrete-time system:

$$\begin{cases} x(k+1) & = Ax(k) + Bu(k) + Dd(k) + \omega(k) \\ y(k) & = Cx(k) + \delta(k) \end{cases} \tag{14}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are respectively the state, the input and the measurement vectors; $d \in \mathbb{R}^q$ is the unknown input vector which does not affect the outputs. A, B, C and D are contant matrices of appropriate dimensions. Finally, $\omega \in \mathbb{R}^n$ and $\delta \in \mathbb{R}^p$ are the state and measurement noises which are assumed to be bounded with a priori known bounds $|\omega| \leq \overline{\omega}$ and $|\delta| \leq \overline{\delta}$ where $\overline{\omega} \in \mathbb{R}^n$ and $\overline{\delta} \in \mathbb{R}^p$ are constant component-wise positive vectors and $|\cdot|$ is the component-wise absolute value for vectors. Moreover, it is assumed that $n \geq q$ and $p \geq q$.

The methodology proposed to jointly estimate the state and the unknown inputs of a LTI discrete-time system is splitted in two steps. First, two bounds $\underline{x}_k, \overline{x}_k \in \mathbb{R}^n$ for the state are estimated. Then, a technique to build the lower and upper bounds $\underline{d}_k, \overline{d}_k \in \mathbb{R}^q$ for the unknown input is described.

The complete design procedure of the interval UIO is described in [6], and was successfully applied to the prognosis of a suspension system degradation in [7].

The unknown input interval observer designed here is based on the UIO proposed in [8], whose methodology is extended to systems with bounded disturbances and noise. Following a change of coordinates, the state is divided into two subsytems, one affected by the unknown input and the second one is unknown input-free. This allows the design of an interval observer in the new coordinate basis to estimate the upper and lower bounds of the state. Then, by returning into the initial coordinates, the upper and lower bounds of the unknown input can be computed. First of all, the following assumption is required.

Assumption 1 *C is a full row rank matrix and D is a full column rank matrix.*

Under Assumption 1, there exists an orthogonal matrix $H \in \mathbb{R}^{n\times n}$ and matrices $R_0 \in \mathbb{R}^{q\times q}$ and $K \in \mathbb{R}^{q\times q}$ such that:

$$D = H \begin{bmatrix} R_0 \\ 0 \end{bmatrix} K^T \tag{15}$$

This leads to the transformation of system (14) into an equivalent one:

$$\begin{cases} z(k+1) & = \tilde{A}z(k) + \tilde{B}u(k) + \begin{bmatrix} R_0 \\ 0 \end{bmatrix} \tilde{d}(k) + \tilde{\omega}(k) \\ y(k) & = \tilde{C}z(k) + \delta(k) \end{cases} \tag{16}$$

where:

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}, \quad \tilde{A} = H^T A H = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}$$
$$\tilde{B} = H^T B = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}, \quad \tilde{C} = CH = \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \end{bmatrix}$$
$$z(k) = H^T x(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}, \quad \tilde{d}(k) = K^T d(k)$$
$$\tilde{\omega}(k) = H^T \omega = \begin{bmatrix} \tilde{\omega}_1(k) \\ \tilde{\omega}_2(k) \end{bmatrix}$$

H^T is supposed to be bounded, therefore $|\tilde{\omega}| \leq \overline{\tilde{\omega}}$ where $\overline{\tilde{\omega}}$ is a constant positive vector. The system (16) is decomposed into an unknown input depending subsystem and an unknown input-free subsytem described by:

$$\begin{cases} z_1(k+1) & = \tilde{A}_{11} z_1(k) + \tilde{A}_{12} z_2(k) + \tilde{B}_1 u(k) + R_0 \tilde{d}(k) \\ & +\tilde{\omega}_1(k) \\ z_2(k+1) & = \tilde{A}_{21} z_1(k) + \tilde{A}_{22} z_2(k) + \tilde{B}_2 u(k) + \tilde{\omega}_2(k) \\ y(k) & = \tilde{C}_1 z_1(k) + \tilde{C}_2 z_2(k) + \delta(k) \end{cases} \tag{17}$$

where $\tilde{C}_1 \in \mathbb{R}^{p\times q}$ and $\tilde{C}_2 \in \mathbb{R}^{p\times(n-q)}$.

$\tilde{C}_1$ is supposed to be a full column rank matrix [9] and can be decomposed as:

$$\tilde{C}_1 = H_1 \begin{bmatrix} R_1 \\ 0 \end{bmatrix} K_1^T \tag{18}$$

with $H_1 = \begin{bmatrix} H_{011} & H_{012} \end{bmatrix}$ ($H_{011} \in \mathbb{R}^{p\times q}$ and $H_{012} \in \mathbb{R}^{p\times(p-q)}$) and $\tilde{y}(k) = H_1^T y(k)$; the measurements equation can be decomposed as

$$\begin{cases} \tilde{y}_1(k) & = R_1 K_1^T z_1(k) + H_{011}^T \tilde{C}_2 z_2(k) + H_{011}^T \delta(k) \\ \tilde{y}_2(k) & = H_{012}^T \tilde{C}_2 z_2(k) + H_{012}^T \delta(k) = C_2 z_2(k) + H_{012}^T \delta(k) \end{cases} \tag{19}$$

As $\tilde{y}_1(k) = G_s^T \tilde{y}(k)$ with $G_s^T = \begin{bmatrix} I_q & O_{q\times(p-q)} \end{bmatrix}$, the expression of z_1 is extracted from (19):

$$z_1(k) = E(y(k) - \tilde{C}_2 z_2(k) - \delta(k)) \tag{20}$$

with $E = K_1 R_1^{-1} G_s^T H_1^T$.

By replacing this expression of $z_1(k)$ in the second equation of (17) we obtain:

$$z_2(k+1) = \tilde{A}_{21} E[y(k) - \tilde{C}_2 z_2(k) - \delta(k)] + \tilde{A}_{22} z_2(k) + \tilde{B}_2 u(k) + \tilde{\omega}_2(k) \tag{21}$$

Finally we have the following dynamical system:

$$\begin{cases} z_2(k+1) & = A_2 z_2(k) + B_2 u(k) + D_2 y(k) - D_2 \delta(k) + \tilde{\omega}_2(k) \\ \tilde{y}_2(k) & = C_2 z_{2(k)} + H_{012}^T \delta(k) \end{cases} \tag{22}$$

where $A_2 = \tilde{A}_{22} - \tilde{A}_{21} E \tilde{C}_2$, $B_2 = \tilde{B}_2$, $C_2 = H_{012}^T \tilde{C}_2$ and $D_2 = \tilde{A}_{21} E$.

In order to be able to design an interval observer for the discrete-time system (22), the following assumption which is standard in the field of observer design is required [9].

Assumption 2 *The pair (A_2, C_2) is detectable.*

Based on Assumption 2, the following lemma allows the transformation of system (22) into a suitable form for interval observer design [10].

Lemma 1 *There exists a gain $L \in \mathbb{R}^{(n-q)\times(p-q)}$ and a transformation matrix P of appropriate dimensions such that $(A_2 - LC_2)$ is Schur stable and $R = P(A_2 - LC_2)P^{-1}$ is nonnegative.*

Such a transformation always exists, and in the case where the eigenvalues of $(A_2 - LC_2)$ are real, R can be chosen as diagonal or as Jordan form of $A_2 - LC_2$ [10]. After the change of coordinates $r_2 = Pz_2$, the system (22) is described in the new coordinates by:

$$\begin{cases} r_2(k+1) & = Rr_2(k) + PB_2u(k) + My(k) - M\delta(k) + P\tilde{\omega}_2(k) \\ \tilde{y}_2(k) & = C_2P^{-1}r_{2(k)} + H_{012}^T\delta(k) \end{cases} \tag{23}$$

where $M = P(D_2 + LH_{012}^T)$.

B. Design of the interval observer

In this section, an interval observer is designed for state estimation and then for unknown input estimation.

1. State estimation

The state estimation is first performed in the coordinates r_2. In the sequel, we define $\overline{\Delta}^T = \begin{bmatrix} \overline{\delta} & -\overline{\delta} \end{bmatrix}$, $\underline{\Delta}^T = \begin{bmatrix} -\overline{\delta} & \overline{\delta} \end{bmatrix}$ and $\overline{\Omega}^T = \begin{bmatrix} \overline{\omega} & -\overline{\omega} \end{bmatrix}$, $\underline{\Omega}^T = \begin{bmatrix} -\overline{\omega} & \overline{\omega} \end{bmatrix}$.

The following theorem allows to carry out an interval state estimation in the coordinates r_2.

Theorem 1 *Assume that $\underline{r}_2(0) \leq r_2(0) \leq \overline{r}_2(0)$. Then, for all $k \in \mathbb{Z}_+$ the estimates $\underline{r}_2(k)$ and $\overline{r}_2(k)$ given by*

$$\begin{cases} \overline{r}_2(k+1) & = R\overline{r}_2(k) + PB_2u(k) + My(k) + (-M)^*\overline{\Delta} + P^*\overline{\tilde{\Omega}}_2 \\ \underline{r}_2(k+1) & = R\underline{r}_2(k) + PB_2u(k) + My(k) + (-M)^*\underline{\Delta} + P^*\underline{\tilde{\Omega}}_2 \end{cases} \tag{24}$$

are bounded and verify

$$\underline{r}_2(k) \leq r_2(k) \leq \overline{r}_2(k) \tag{25}$$

In addition, if the gain L is chosen such that $(A_2 - LC_2)$ is Schur stable, then $\overline{r}_2$ and $\underline{r}_2$ are bounded.

Furthermore, since $r_2 = Pz_2$, the bounds of $z_2(k)$ are given by the following corollary.

Corollary 1 *Under the conditions of Theorem 1, we have $\underline{z}_2(k) \leq z_2(k) \leq \overline{z}_2(k)$ with*

$$\begin{cases} \overline{z}_2(k) = (P^{-1})^+\overline{r}_2(k) + (P^{-1})^-\underline{r}_2(k) \\ \underline{z}_2(k) = (P^{-1})^+\underline{r}_2(k) + (P^{-1})^-\overline{r}_2(k) \end{cases} \tag{26}$$

The last step consists in computing the bounds for the whole state in the original coordinates $\underline{x}^T = \begin{bmatrix} \underline{x}_1 & \underline{x}_2 \end{bmatrix}$ and $\overline{x}^T = \begin{bmatrix} \overline{x}_1 & \overline{x}_2 \end{bmatrix}$. Based on Theorem 1 and Corollary 1, the following theorem ensures the interval estimation of the state x.

Theorem 2 *Assume that the conditions of Theorem 1 are satisfied and $\underline{x}(0) \leq x(0) \leq \overline{x}(0)$. Then, for all $k \in \mathbb{Z}_+$ the estimates $\underline{x}(k)$ and $\overline{x}(k)$ given by*

$$\begin{cases} \overline{x}_1(k) & = H_{11}Ey + (H_{12})^*\overline{Z}_2(k) + (-E_1)^*\overline{Z}_2(k) + (-H_{11}E)^*\overline{\Delta} \\ \underline{x}_1(k) & = H_{11}Ey + (H_{12})^*\underline{Z}_2(k) + (-E_1)^*\underline{Z}_2(k) + (-H_{11}E)^*\underline{\Delta} \\ \overline{x}_2(k) & = H_{21}Ey + (H_{22})^*\overline{Z}_2(k) + (-E_2)^*\overline{Z}_2(k) + (-H_{21}E)^*\overline{\Delta} \\ \underline{x}_2(k) & = H_{21}Ey + (H_{22})^*\underline{Z}_2(k) + (-E_2)^*\underline{Z}_2(k) + (-H_{21}E)^*\underline{\Delta} \end{cases} \tag{27}$$

are bounded and verify

$$\underline{x}(k) \leq x(k) \leq \overline{x}(k) \tag{28}$$

with $E_1 = H_{11}E\tilde{C}_2$ and $E_2 = H_{21}E\tilde{C}_2$.

2. Unknown input estimation

In this subsection, the upper and lower bounds of the unknown input d will be estimated. The expression of d is expressed from the first equation of (17):

$$d(k) = KR_0^{-1}[z_1(k+1) - \tilde{A}_{11}z_1(k) - \tilde{A}_{12}z_2(k) - \tilde{B}_1u(k) - \tilde{\omega}_1(k)] \quad (29)$$

By replacing z_1 with its expression in (20), equation (29) becomes:

$$d(k) = KR_0^{-1}[Ey(k+1) - E\tilde{C}_2z_2(k+1) - E\delta(k+1) - \tilde{A}_{11}(Ey(k) - E\tilde{C}_2z_2(k) - E\delta(k)) - \tilde{A}_{12}z_2(k) - \tilde{B}_1u(k) - \tilde{\omega}_1(k)] \quad (30)$$

The following theorem ensures the interval estimation of the unknown input d.

Theorem 3 *Assume that the conditions of Theorem 1 are satisfied. Then, for all $k \in \mathbb{Z}_+$ the estimates $\underline{d}(k)$ and $\overline{d}(k)$ given by*

$$\begin{cases} \overline{d}(k) &= Qy(k+1) - Q\tilde{A}_{11}Ey(k) - Q\tilde{B}_1u(k) + G_1^*\overline{Z}_2(k+1) + G_2^*\overline{Z}_2(k) + G_3^*\overline{\Delta} + G_4^*\overline{\Delta} + G_5^*\overline{\tilde{\Omega}}_1 \\ \underline{d}(k) &= QEy(k+1) - Q\tilde{A}_{11}Ey(k) - Q\tilde{B}_1u(k) + G_1^*\underline{Z}_2(k+1) + G_2^*\underline{Z}_2(k) + G_3^*\underline{\Delta} + G_4^*\underline{\Delta} + G_5^*\underline{\tilde{\Omega}}_1 \end{cases} \quad (31)$$

are bounded and verify

$$\underline{d}(k) \leq d(k) \leq \overline{d}(k) \quad (32)$$

With $Q = KR_0^{-1}$, $G_1 = -QE\tilde{C}_2$, $G_2 = Q(\tilde{A}_{11}E\tilde{C}_2 - \tilde{A}_{12})$, $G_3 = -QE$, $G_4 = Q\tilde{A}_{11}E$ and $G_5 = -Q$.

IV. Application of the interval UIO for PMSM speed and unknown load torque disturbance estimation

In this section, the interval UIO presented in Section III.A is used to estimate the interval that contains the uncertainty on the PMSM speed and load torque. For this purpose, the linearized PMSM model 13 presented in Section II is considered, and only the current measurements are assumed to be available. Moreover, modeling and measurement uncertainties are taken into account in the form of additive process and measurement noises that are respectively denoted as ω and δ. Finally, the discrete-time form of the continuous-time model is used for the interval UIO implementation.

A. Simulation parameters

The parameters of the considered PMSM were taken from [11] and are listed in Table 1.

The model and measurement noises are assumed to be uniformly distributed and bounded with $\overline{\omega} = 10^{-2}\begin{bmatrix}1 & 1 & 1\end{bmatrix}^\top$ and $\overline{\delta} = 10^{-2}\begin{bmatrix}1 & 1 & 1\end{bmatrix}^\top$. Finally, the bounds of the initial state vector are chosen as $\underline{x_0} = 10^{-2}\begin{bmatrix}1 & 1 & 1\end{bmatrix}^\top$ and $\overline{x_0} = 10^{-2}\begin{bmatrix}1 & 1 & 1\end{bmatrix}^\top$.

The motor speed and unknown load torque bounds were computed under different scenarios to see the effects on the estimations:

1) Step load torque from 0 to 2 Nm is applied at 0.25 sec.
2) Initial constant load torque of 2 Nm, then sinusoidal disturbance is applied at 0.25 sec.
3) Ramp speed variation

Table 1 PMSM parameters

Parameter	Symbol	Value
Number of pole pairs	n_p	1
Stator resistance	R_s	1.4 Ω
Stator inductance	L_s	0.73 mH
Stator mutual inductance	M_s	73 μH
Permanent magnet flux	λ_M	0.1546 Wb
Rotor inertia	J	$60 \cdot 10^{-6}$ kgm^2

B. Results

In this section, the interval estimations of the motor speed and unknown load torque under the three different scenarios are depicted. Matlab was used to run the simulations.

The results obtained for the first scenario are shown in Fig. 1 for the load torque and in Fig. 2 for the speed. The upper and lower bounds of the load torque are well estimated, they include the reference unknown load torque despite the step perturbation applied at 0.25 sec. As for the speed interval estimation (Fig. 2), even if the estimated bounds are very close to the reference speed, they do not always include it. However, it is shown in Fig. 3 that when the load torque perturbation is applied, the interval UIO is still able to estimate the speed perturbation.

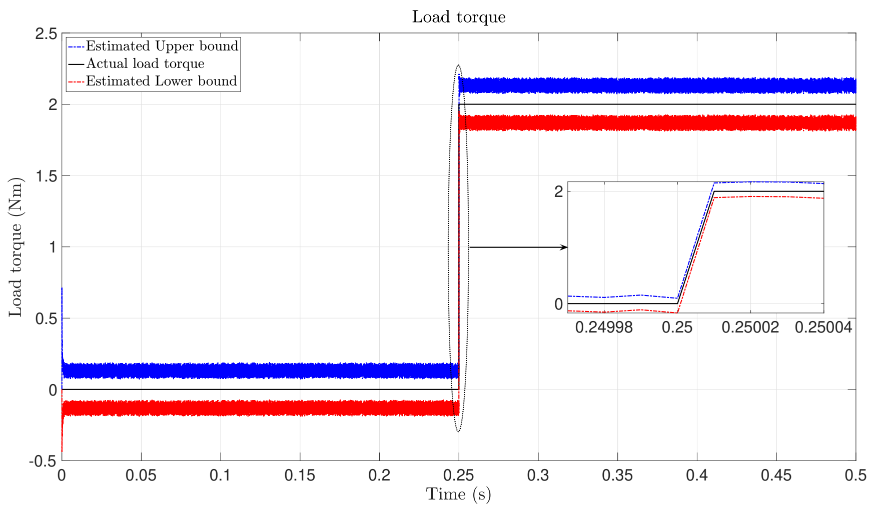

Fig. 1 Step load torque

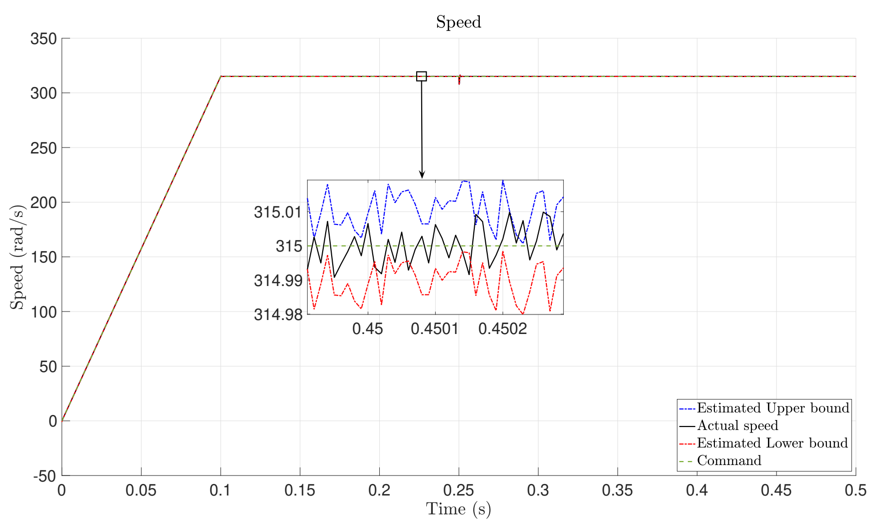

Fig. 2 Speed when load torque is a step

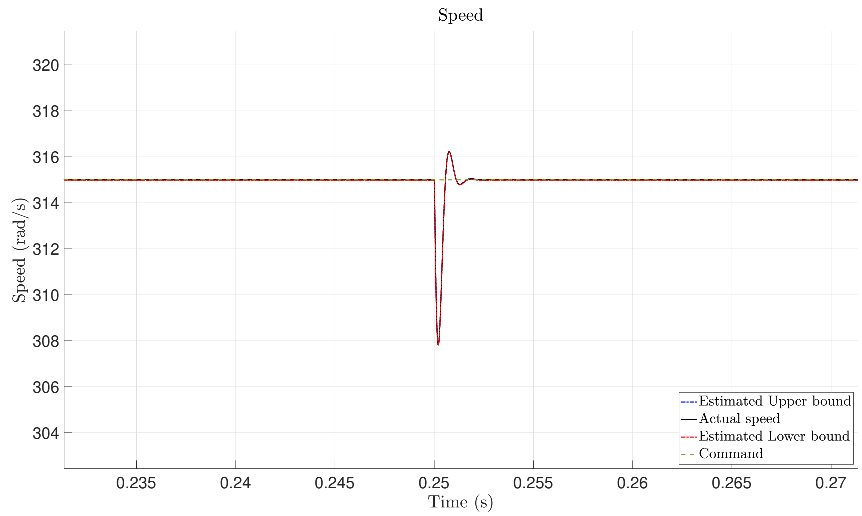

Fig. 3 Close up the speed when load torque is a step

Then, the simulation results for scenario 2 where the load torque is sinusoidal are depicted in Fig. 4 and Fig. 5. The interval UIO managed to reconstruct the constant then sinusoidal shape of the unknown load torque.

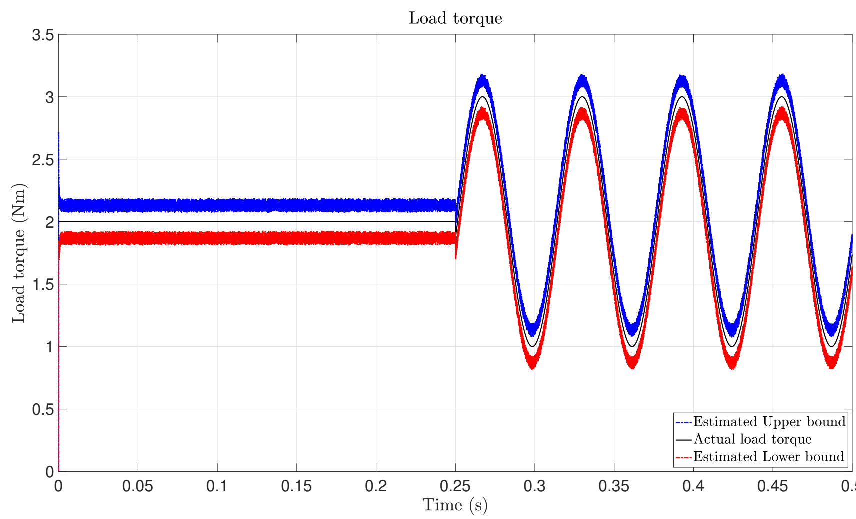

Fig. 4 Sinusoidal load torque

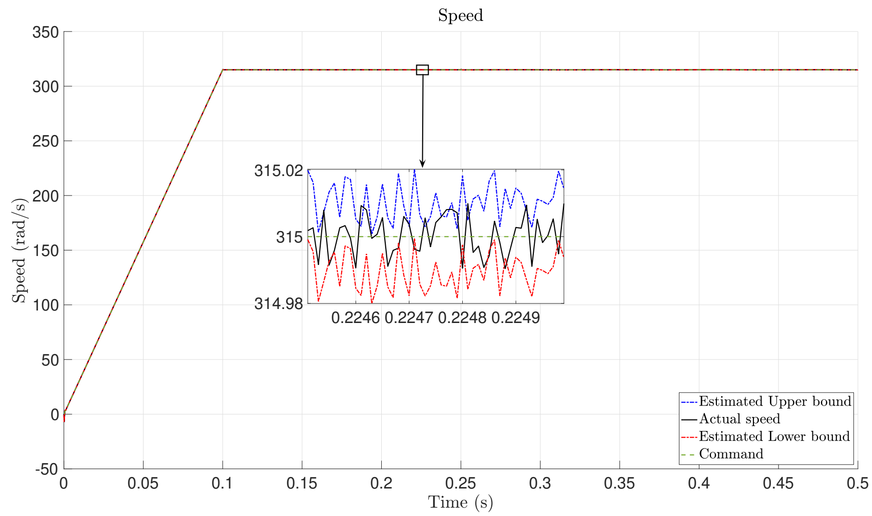

Fig. 5 Speed when load torque is sinusoidal

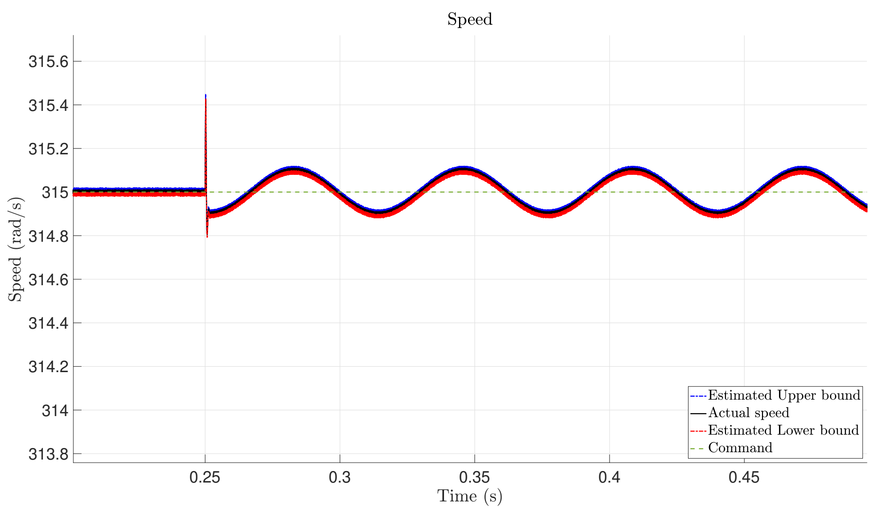

Fig. 6 Close up of the speed when load torque is sinusoidal

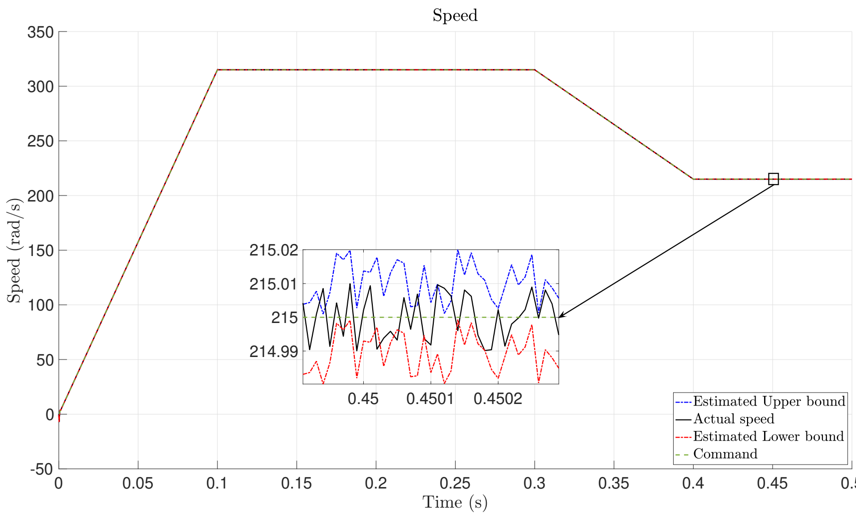

Fig. 7 Variable speed

C. Discussion

V. Conclusion

Acknowledgments

This work was supported by the System-Wide Safety (SWS) project under the Airspace Operations and Safety Program within the NASA Aeronautics Research Mission Directorate (ARMD).

References

[1] Colucci, B. F., "Turning Volts to VTOL," 2016, pp. 30–33.

[2] Henke, M., Narjes, G., Hoffmann, J. M., Wohlers, C., Urbanek, S., Heister, C., Steinbrink, J., Canders, W.-R., and Ponick, B., "Challenges and Opportunities of Very Light High-Performance Electric Drives for Aviation," 2018.

[3] Bowen, C., Jihua, Z., and Zhang, R., "Modeling and simulation of permanent magnet synchronous motor drives," *ICEMS'2001. Proceedings of the Fifth International Conference on Electrical Machines and Systems (IEEE Cat. No. 01EX501)*, Vol. 2, IEEE, 2001, pp. 905–908.

[4] Park, R. H., "Two-reaction theory of synchronous machines generalized method of analysis-part I," *Transactions of the American Institute of Electrical Engineers*, Vol. 48, No. 3, 1929, pp. 716–727.

[5] Vyncke, T., Boel, R., and Melkebeek, J., "A comparison of stator flux linkage estimators for a direct torque controlled PMSM drive," *35th Annual Conference of IEEE Industrial Electronics*, 2009, pp. 971–978.

[6] Robinson, E. I., Marzat, J., and Raïssi, T., "Interval observer design for unknown input estimation of linear time-invariant discrete-time systems," *IFAC-PapersOnLine*, Vol. 50, No. 1, 2017, pp. 4021–4026.

[7] Robinson, E., Marzat, J., and Raïssi, T., "Prognosis of uncertain linear time-invariant discrete systems using unknown input interval observer," *International Journal of Control*, 2019, pp. 1–17.

[8] Maquin, D., Gaddouna, B., and Ragot, J., "Estimation of unknown inputs in linear systems," *Proceedings of American Control Conference, Baltimore, MD*, Vol. 1, 1994, pp. 1195–1197.

[9] Hou, M. and Muller, P., "Design of observers for linear systems with unknown inputs," *IEEE Transactions on Automatic Control*, Vol. 37, No. 6, 1992, pp. 871–875.

[10] Efimov, D., Perruquetti, W., Raïssi, T., and Zolghadri, A., “On interval observer design for time-invariant discrete-time systems,” *European Control Conference (ECC), Zurich, Swiss*, 2013, pp. 2651–2656.

[11] Otava, L. and Buchta, L., “Implementation and verification of the PMSM stator interturn short fault detection algorithm,” *19th European Conference on Power Electronics and Applications (EPE’17 ECCE Europe)*, IEEE, 2017, pp. 1–10.