

Modeling the Effects of a Backward-Facing Step on Boundary-Layer Transition

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We model transition to turbulence in a two-dimensional boundary layer downstream of a backward-facing step (BFS) along a flat plate. With the goal of evaluating the available engineering models for predicting the effects of step excrescences on the transition characteristics, two separate methodologies are used to monitor the streamwise shift in the transition onset location as the step height and the flow speed are varied across the range of a previously reported experiment involving step-height-to-local-displacement-thickness ratios of $0 < h/\delta^* < 1.6$. Unlike the variable N -factor method from the previous literature, both of these methods are general in scope and do not involve any empirical correlations that are specific to step excrescences. The first of these techniques involves an N -factor method that directly accounts for the change in boundary-layer instability characteristics due to the step. Stability computations using the harmonic linearized Navier-Stokes equations (HLNSE), which fully account for the nonparallel-mean-flow effects close to the BFS, indicate that the measured transition locations at nearly all test conditions ($h/\delta^* < 1.3$) correlate well with a computed N -factor of $N_{tr} = 7.6$, demonstrating a successful stability-based transition criterion related to step excrescences. Linear stability theory, which does not account for nonparallel effects, demonstrates reasonable agreement with the HLNSE results, yielding good predictions for the overall trends, but predicts a somewhat earlier onset of transition than HLNSE. The other methodology used in this work involves transport-equation-based transition models. We first show that the Langtry-Menter $\gamma - \widetilde{Re}_{\theta t}$ transition model cannot accurately predict the location of transition onset for moderate BFS heights because it is unable to accurately account for the flow history effects. Along with the Langtry-Menter transition model, we also show the amplification factor transport model does not produce accurate transition locations for subsonic flow over steps even though it accounts for some flow history effects.

Nomenclature

h	=	step height (m)
N	=	logarithmic amplification ratio
N_{tr}	=	peak N -factor at transition onset location
p	=	pressure (Pa)
Re_∞	=	freestream unit Reynolds number (1/m)
Re_v	=	vorticity Reynolds number
Re_{θ_c}	=	critical momentum-thickness Reynolds number
Re_{δ^*}	=	Reynolds number based on δ^*
T	=	temperature (K)
(u, v, w)	=	velocity components (m/s)
(x, y, z)	=	Cartesian coordinates (m)

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x_s	=	step location (m)
y^+	=	distance from wall in viscous units
α	=	streamwise wavenumber (1/m)
β	=	spanwise wavenumber (1/m)
δ^*	=	boundary layer displacement thickness at the step location (m)
θ	=	backward-facing-step maximum slope ($^\circ$)
ρ	=	density (kg/m ³)
C_f	=	skin friction
Tu	=	freestream turbulence intensity
μ_t/μ	=	turbulent viscosity ratio
σ	=	spatial growth rate (m)
f	=	frequency (Hz)
E	=	energy norm

I. Introduction

The energy efficiency of new and existing aircraft must be significantly enhanced in order to reduce the negative impact of aviation on the environment [1]. Maximizing the laminar regions on aerodynamic surfaces by delaying the onset of transition reduces skin-friction drag, and thus, can increase the energy efficiency by reducing the fuel burn rate [2]. Laminar flow control (LFC) is one of the best ways to achieve breakthrough reductions in the aerodynamic drag. Various methods of LFC include natural laminar flow control that uses a well-crafted airfoil geometry to tailor the surface pressure gradients [3], hybrid laminar flow control (HLFC) based on either heat transfer or small holes and slots over a wing surface through which suction of the boundary layer occurs either passively or actively [2], and spanwise discrete roughness elements on the airfoil surface that reduce the growth of crossflow instabilities [4]. To successfully realize the benefit of these methods on modern aircraft, it is important to understand the effects of unavoidable surface irregularities (2D steps, surface waviness, slots in HLFC, discrete roughness elements, etc.) on the transition process. Physical mechanisms associated with these irregularities include destabilization of convective-type instability waves [5, 6], streak instabilities that form in the wakes of 3D nonuniformities [7], increased boundary-layer receptivity [8], and possibly a global instability within the separation zone [9, 10].

Many experiments on boundary-layer transition with steps have been reported in the literature [6, 11–14]. Empirical criteria based on experimental measurements are often used to predict the effects of surface roughness [15]. Recently, higher-fidelity computational approaches have been applied to roughness-induced transition. Ragab et al. [16] utilized the interactive boundary-layer equations to compute small changes in the mean flow and classical quasiparallel stability theory to obtain the amplification characteristics of TS waves. Masad and Malik [17] also used an interactive boundary-layer approach with quasiparallel stability theory to predict transition onset of subsonic flow over a hump. Perraud and Seraudie [18] found the transition location shifts progressively for backward-facing steps (BFS) and rather abruptly for forward-facing steps. Direct numerical simulation (DNS) has been applied by Edelmann and Rist [19] to model flow over forward-facing steps. Work by Ref. [20] used the parabolized stability equations (PSE) and the harmonic linearized Navier-Stokes equations (HLNSE) to examine the impact of surface indentations. DNS performed by Wise et al. [21] of boundary-layer flow over a step looked at the effects of different step heights, favorable pressure gradients, and freestream turbulence intensity levels. Most studies about roughness-induced transition have focused on TS waves, but there has also been work [22, 23] on the effects of surface excrescences on crossflow instability in three-dimensional swept-wing boundary layers.

Linear stability theory (LST), PSE, and HLNSE can account for the amplification of TS waves and crossflow vortices, but they require a detailed understanding of hydrodynamic stability analysis and transition physics. They can also lack robustness due to the assumptions made about the flow field (i.e., parallel, slowly varying, or harmonic in time/span). For the previous two reasons, and to reduce the computational cost of transition prediction, multiple Reynolds-averaged Navier-Stokes (RANS) formulations have been developed and applied to transition problems [24–29]. These models numerically solve additional transport equations and use correlations to find the onset of transition. It is important to note that RANS-based transition models do not capture the entire process of transition and oversimplify the physics. Nevertheless, these models result in a flow field that has laminar, transitional, and fully-turbulent portions. They also can produce an accurate transition location in simple flow configurations (e.g., flat plates and airfoils). There has not been a lot of work on applying RANS-based transition models to boundary-layer flows over steps.

Our recent work [30] used a combination of PSE and HLNSE to compute growth rates and N -factors of TS waves over a sharp flat plate with and without a BFS based on the experimental configuration in Ref. [6]. However, because of the increased memory and computational cost associated with HLNSE, that approach may not always be practical, especially in the context of engineering calculations. The current effort seeks to evaluate simpler methodologies to predict the effects of step excrescences on transition behavior. Specifically, we assess the accuracy of the classical quasiparallel LST in lieu of the PSE/HLNSE combination. In addition, we also examine the accuracy of RANS-based transition models, namely, the Langtry-Menter four-equation transitional shear-stress transport (SST) model [25] and Coder's amplification factor transport combined with the Spalart-Allmaras (SA) model [26, 28]. The eventual goal is to develop a reliable and cost efficient, yet robust and user-friendly approach of transition modeling. In this paper, we directly compare the measured shift in transition locations due to the presence of a BFS with the computed transition locations from both LST and HLNSE as well as those based on the Langtry-Menter transition model and Coder's version of the amplification factor transport model. We also comment on the best approach moving forward and some improvements that need to be made for the RANS-based transition models.

The paper is structured in the following manner. Section II explains the numerical methodology of the linear stability analysis that includes LST, PSE, and HLNSE, along with the transport-equation-based transition models. Results from these two approaches are compared in Section III for low-speed two-dimensional flat plate and BFS configurations. We conclude the paper in Section IV by addressing the strengths and weaknesses of each transition modeling approach.

II. Numerical Methodology

We employ two different approaches to model transitional flow in subsonic flat-plate and BFS configurations. One approach uses a combination of PSE and HLNSE to compute the growth rates and N -factors of TS waves about laminar basic states [30]. This approach takes into account the nonparallel effects close to the step. We also use linear stability theory, which does not account for nonparallel effects. The application of this stability approach entails an increased computational cost and requires significant user expertise. LST also involves the inversion of very large matrices (even for 2D flow configurations). The other approach, which utilizes transport-equation-based transition models in FUN3D 13.7 and OVERFLOW 2.3d, is less computationally expensive and does not require any knowledge of hydrodynamic stability theory. In this paper, we use the 2009 version of the four-equation model by Langtry and Menter [25] with the 2003 version of Menter's SST turbulence model [31] and Coder's version [26, 28] of the amplification factor transport model with the SA turbulence model [32] to compute transitional basic states. Both of these numerical approaches will be discussed at length in the following subsections.

A. Stability Analysis

We compute the laminar basic states by solving the Navier-Stokes equations on curvilinear grids that cluster points near the bottom wall [30]. The variables (x, y, z) denote the Cartesian coordinates, and (u, v, w) are the velocity components in the streamwise, wall-normal, and spanwise directions, respectively. Note ρ is the density, p is the pressure, and T is the temperature. We assume the linear perturbations about the various basic states are harmonic in the spanwise direction and in time. These perturbation are defined as

$$\tilde{\mathbf{q}}(x, y, z, t) = \check{\mathbf{q}}(x, y) \exp [i (\beta z - \omega t)] + \text{c.c.}, \quad (1)$$

where β represents the spanwise wavenumber, ω is the angular frequency, and c.c. denotes the complex conjugate. Here, $\tilde{\mathbf{q}}(x, y, z, t) = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$ and $\check{\mathbf{q}}(x, y) = (\check{\rho}, \check{u}, \check{v}, \check{w}, \check{T})^T$. The linear disturbance amplification is computed with LST, PSE, and HLNSE [20, 33–38] over the flow configurations of interest. For LST, we assume the linear perturbations are harmonic in the streamwise direction as well. We use the PSE solution as the inflow condition for the HLNSE, which accounts for streamwise variation of the boundary-layer thickness and shape of the disturbance field. PSE solves an approximate form of the Navier-Stokes equations for a slowly-varying shape function and a complex streamwise wavenumber $\alpha(x)$ with the following relation

$$\check{\mathbf{q}}(x, y) = \hat{\mathbf{q}}(x, y) \exp \left[i \int_{x_0}^x \alpha(x') dx' \right]. \quad (2)$$

LST, PSE, and HLNSE use the same boundary conditions. They are $\tilde{u} = \tilde{v} = \tilde{w} = \tilde{T} = 0$ at the bottom wall and $\tilde{u} = \tilde{v} = \tilde{w} = \tilde{T} = \check{\rho} = 0$ at the left, right, and top edges of the computational domain.

According to the e^N method and linear stability theory, the onset of transition to turbulence occurs at the streamwise location where the logarithmic amplification ratio or N -factor of any fixed-frequency disturbance first reaches a critical value of N_{tr} . We define the N -factor as

$$N(x) = \frac{1}{2} \ln \left[\frac{E(x)}{E(x_{lb})} \right]. \quad (3)$$

Here, the subscript lb stands for lower branch, and x_{lb} is where the disturbance at any given frequency first becomes unstable. The energy norm E is given by

$$E(x) = \int_y \check{\mathbf{q}}(x, y)^H \text{diag} \left[\frac{\bar{T}(x, y)}{\gamma \bar{\rho}(x, y) M^2}, \bar{\rho}(x, y), \bar{\rho}(x, y), \bar{\rho}(x, y), \frac{\bar{\rho}(x, y)}{\gamma(\gamma - 1) \bar{T}(x, y) M^2} \right] \check{\mathbf{q}}(x, y) dy, \quad (4)$$

where the superscript H represents the Hermitian operator. In this work and in Ref. [30], we investigated the possibility of correlating the onset of transition to turbulence with the location where $N(x_{tr}) = N_{tr}$. We assume that N_{tr} in the presence of a BFS is the same as that determined for a smooth flat plate.

We discretize the Navier-Stokes equations for the laminar basic states with sixth-order finite-difference schemes in both spatial directions. This results in a nonlinear system, which we solve by using a Newton-Raphson method that converges the residual to machine zero. The LST, PSE, and HLNSE for the unsteady disturbances are discretized with second-order and sixth-order finite-differences. We solve the different systems of equations with the sparse direct solver MUMPS [39] as described in Ref. [40]. Furthermore, the discretization schemes and the computational domain sizes for the stationary Navier-Stokes, LST, PSE, and HLNSE solutions are varied in a few cases to check that the computed results do not change. For more details on the stability approach in this paper, refer to our recent express article [30].

B. Transport-Equation-Based Transition Models

The suite of codes known as FUN3D is a node-centered, unstructured-grid, upwind-biased RANS code [41]. One of the RANS-based transition models available in FUN3D 13.7 is the 2009 version of the four-equation model by Langtry and Menter [25], which is based on the 2003 version of Menter's SST model [42]. This will be referred to as LM2009 throughout the paper. It involves the addition of two transport equations to the SST $k - \omega$ formulation, which are for the intermittency and transition-onset momentum-thickness Reynolds number. The intermittency function turns on the production term of the turbulent kinetic energy downstream of the transition point in the boundary layer, while the transport equation for the transition-onset momentum-thickness Reynolds number is necessary to capture the nonlocal influence of the turbulence intensity. Rumsey and Lee-Rausch [43] demonstrated the use of LM2009 in FUN3D on a trapezoidal wing (albeit with some convergence issues).

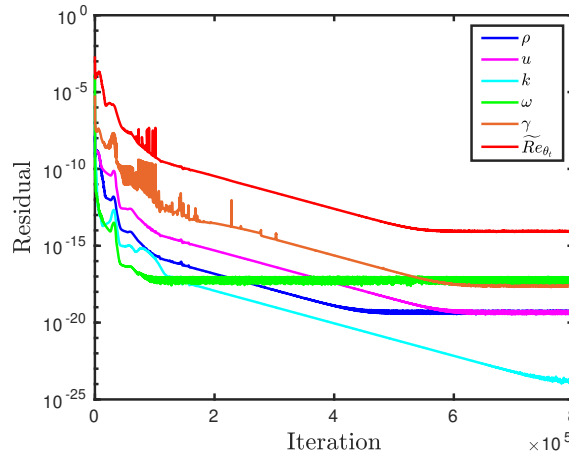


Fig. 1 Residuals of the various transport equations from a BFS simulation at $U_\infty = 34$ m/s, $Re_\infty = 2.26 \times 10^6$ m⁻¹, $\theta = 75^\circ$, and $h = 1.0$ mm with the LM2009 transition model in FUN3D.

We run the FUN3D simulations until the residual has dropped several orders of magnitude and has leveled out. An example of the residual drop is shown in Figure 1, which corresponds to a BFS simulation with $U_\infty = 34$ m/s, $Re_\infty = 2.26 \times 10^6 \text{ m}^{-1}$, and $h = 1.0$ mm. We denote k as the turbulent kinetic energy, ω as the dissipation rate, γ as the intermittency, and $\bar{Re}_\theta$ as the local transition-onset momentum-thickness Reynolds number in Figure 1(a). Notice the residual drops roughly eleven orders of magnitude to machine zero for both the density ρ and streamwise velocity u . Similar convergence is obtained for the other flat-plate and BFS simulations. Thus, we can definitively state that the LM2009 transition model in FUN3D converges well for the flat-plate and BFS configurations in this paper. The most recent FUN3D manual [44] has more information about the different numerical methods and residual calculations.

In addition to the LM2009 transition model, we also employ the amplification factor transport model due to Coder [26, 28], which uses the SA turbulence model [32]. This will be referred to as AFT2019b throughout the paper. To apply the AFT2019b transition model, we run the OVERFLOW 2.3d solver, since the amplification factor transport model is not yet available in FUN3D. OVERFLOW 2.3d is an implicit structured Navier-Stokes solver that uses overset grids [45]. Unlike the LM2009 transition model, the AFT2019b transition model takes into account history effects. It does this by solving two equations associated with the transport of intermittency and, more importantly, the amplification factor. We get similar convergence with OVERFLOW 2.3d for the transitional basic states as FUN3D 13.7. See the OVERFLOW 2.3d manual for more information [45].

III. Results

The experiment by Wang and Gaster [6] used a flat-plate model with a nominally-sharp BFS at a distance of $x_s = 0.3$ m from the plate leading edge. We define the BFS shape similar to Ref. [30] with the following equation that has a continuous slope and a compact streamwise support

$$y_{\text{wall}}(x) = \begin{cases} 0, & \text{if } x < x_s \\ -\frac{h}{2} \left(1 + \cos \left[\pi \left(\frac{x-x_s}{x_f-x_s} + 1 \right) \right] \right), & \text{if } x_s \leq x \leq x_f \\ -h, & \text{if } x > x_f \end{cases} \quad (5)$$

where x_s and $x_f \equiv x_s + h/\tan(\theta)$ are the start and end points of the step. We denote $\tan(\theta)$ as the maximum slope of the BFS and h as the height. Furthermore, we choose the flow parameters so they match those of the Wang and Gaster [6] experiment (also our previous study [30]). The freestream velocity U_∞ is varied by 2 m/s from 20 to 34 m/s. We set the freestream density and temperature to $\rho_\infty = 1.206 \text{ kg/m}^3$ and $T_\infty = 292.7$ K, respectively. The constant ratio of specific heats is denoted as γ and is set equal to 1.4. Further, the specific gas constant for air is $R = 287.15 \text{ J/K/kg}$. To compute the dynamic viscosity μ , we use Sutherland's law with the reference temperature $T_S = 110.4$ K. We define $Pr = 0.72$ as the constant Prandtl number. For this range of parameters, the unit Reynolds number varies from 1.33×10^6 to 2.26×10^6 . Refer to our previous study [30] for more information about the flow conditions.

Figure 2 shows the laminar basic state at $U_\infty = 34$ m/s, $\theta = 75^\circ$, and $h = 1.0$ mm. Our computational domain extends from $x = 0.025$ m to $x = 1.5$ m with respect to the plate leading edge for this case. A nonuniform grid is used to cluster points near the wall and step with 5001 and 351 points in the streamwise and wall-normal directions, respectively. The first grid point is placed above the wall at a distance that satisfies $y^+ = 0.1$. It was shown in Ref. [30] that the value of $\theta = 75^\circ$ yields the same results as those for a sharp BFS. Also, see Ref. [30] for a closeup view of the computational grid near the smooth BFS. We impose a compressible self-similar boundary-layer profile at the left edge of the domain. Adiabatic and no-slip boundary conditions are enforced at the bottom wall. Since the top edge of the domain is far enough from the bottom wall, we treat it as freestream. We use extrapolation at the outflow boundary. Notice that the recirculation bubble starts at the step location and extends to $(x - x_s)/\delta^* = 60$. For $U_\infty = 34$ m/s, the boundary-layer displacement thickness at the step location is $\delta^* = 6.26 \times 10^{-4}$ m. As the streamwise velocity and step height increase, the recirculation bubble grows larger. This can be seen by comparing Figure 2 in this paper to Figure 1 in our previous study [30] that shows the laminar basic state at $U_\infty = 28$ m/s, $\theta = 75^\circ$, and $h = 0.5$ mm. For $U_\infty = 28$ m/s, the boundary-layer displacement thickness at the step location is $\delta^* = 6.9 \times 10^{-4}$ m. In Figure 2, there is a favorable pressure gradient as the flow moves toward the step, which switches to an adverse pressure gradient after the step.

The computed laminar basic states are used to predict the disturbance amplification characteristics by using LST and the hybrid of PSE and HLNSE. We focus mainly on the amplification of planar Tollmien-Schlichting waves for these 2D low-speed cases. LST is applied to all of the basic states, both with and without the BFS. PSE is only applied to the flat-plate cases without the step and used as the inflow condition at $x = 0.025$ m for the HLNSE computations about the laminar basic states with a step. Figure 3 displays the spatial growth rates of TS waves from LST and HLNSE at

three representative frequencies ($f = 300, 500, \& 700$ Hz) for the BFS case with $\theta = 75^\circ$, $h = 0.5$ mm, and $U_\infty = 28$ m/s. It also displays the growth rates from PSE for the case with no step. Figure 3 shows that the LST predictions for the disturbance growth rates in the relatively unperturbed regions both upstream and downstream of the step agree well with the HLNSE results. Since there are significant nonparallel effects close to the BFS, there are observable differences between the two predictions within the intermediate region and we see larger peaks in the LST growth rates in comparison with the HLNSE growth rates, which are due to the LST being unable to model nonparallel effects. The overall trend between LST and HLNSE remains the same for all three frequencies with the TS amplification rate decreasing in front of the step. A strong destabilization occurs right behind the step, followed by a weak stabilization before the growth rates converge back to their flat-plate values.

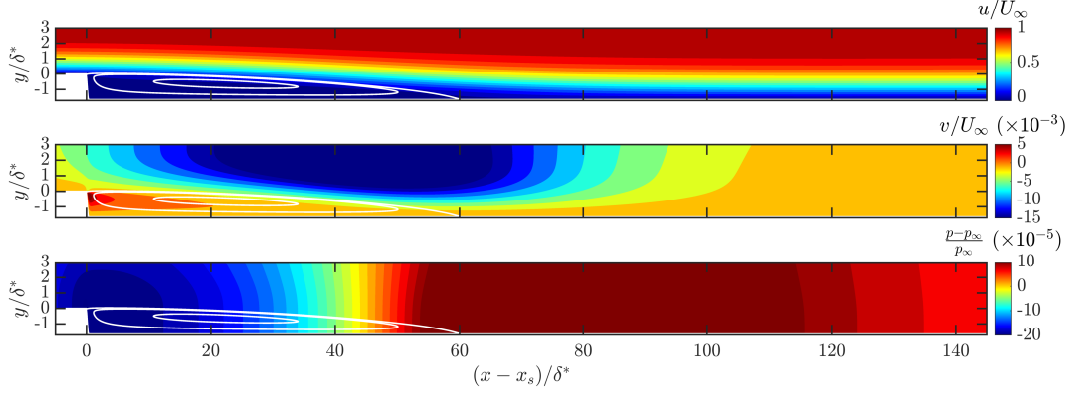


Fig. 2 Contours of nondimensional velocity and pressure of the boundary-layer flow over a BFS with $U_\infty = 34$ m/s, $\delta^* = 6.26 \times 10^{-4}$ m, $\theta = 75^\circ$, and $h = 1.0$ mm. Recirculating flow downstream of the BFS is depicted by three streamlines (white contour lines), one of which is the separation streamline.

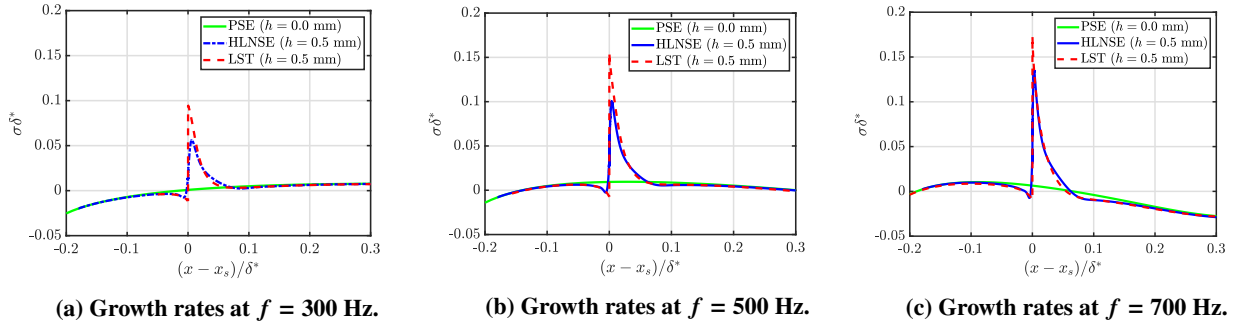
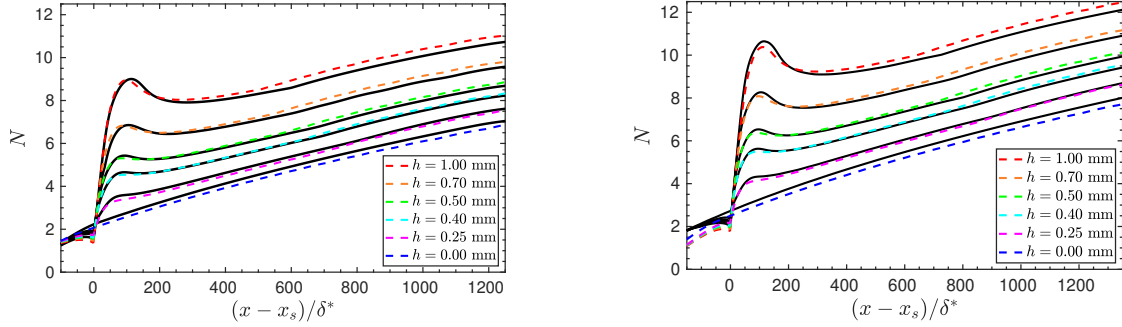


Fig. 3 Comparison of the spatial growth rates between LST and HLNSE for planar TS waves at $f = 300, 500, \& 700$ Hz for the BFS with $\theta = 75^\circ$, $\delta^* = 6.9 \times 10^{-4}$ m, $h = 0.5$ mm, and $U_\infty = 28$ m/s. The growth rates computed with PSE about a smooth flat plate with no step are shown as well.

Figure 4 depicts the N -factor envelopes at $U_\infty = 28 \& 34$ m/s for BFS heights that range from 0 to 1 mm. These envelopes are computed with LST and PSE/HLNSE for a large range of frequencies. They are defined by the local maximum at each streamwise station of the collective N -factor curves that correspond to individual frequencies. See our previous study [30] for an example of how the N -factor envelope at $U_\infty = 28$ m/s and $h = 0.5$ mm is computed with PSE and HLNSE. As seen from Figure 4, the LST and HLNSE envelopes match very well with each other in the region upstream of the step. They also remain rather close behind the BFS, and the small but notable discrepancies are due to the LST not accounting for the nonparallel effects in the vicinity of the step. Moreover, the N -factor envelopes show that the BFS-induced mean-flow deformation causes a significant increase of the N -factor across the step region as compared to the N -factor evolution over a smooth flat plate. This rise in the N -factor becomes even larger as the freestream velocity increases, which is evident in Figure 4. The results also reveal the presence of an overshoot in the N -factor envelope immediately after the BFS. This overshoot is due to the strong destabilization of the high-frequency

disturbances in the preceding region. Similar results are obtained for the N -factor envelopes at the other freestream velocities, but are not shown herein for brevity.



(a) Envelopes at $U_\infty = 28$ m/s with $\delta^* = 6.26 \times 10^{-4}$ m.

(b) Envelopes at $U_\infty = 34$ m/s with $\delta^* = 6.90 \times 10^{-4}$ m.

Fig. 4 N -factor envelopes for planar TS waves over a flat plate and BFS with different heights at $U_\infty = 28$ & 34 m/s. The dashed colored lines represent the LST results, while the solid black lines are the PSE/HLNSE results.

For a flat plate with $U_\infty = 34$ m/s, Wang and Gaster [6] measured the onset of transition at $x = 1.061$ m or $(x - x_s)/\delta^* = 1215$. This yields critical N -factor values of $N_{tr} = 7.32$ and $N_{tr} = 7.6$ for LST and PSE/HLNSE, respectively, according to Figure 4(b). The value of $N_{tr} = 7.32$ comes from the blue dashed line in Figure 4(b), and the value of $N_{tr} = 7.6$ comes from the lowest solid black line in Figure 4(b). Figure 5 shows the computed transition locations from LST and PSE/HLNSE, along with the measured transition locations from the Wang and Gaster [6] experiment, at BFS heights of 0 to 1 mm and freestream velocities of 20 to 34 m/s. We observe good agreement between the PSE/HLNSE transition locations and the experimental data, except for large step heights and freestream velocities. The error at large step heights and freestream velocities remains the same for LST. There also is a discrepancy at $h = 0.25$ mm between the computed and measured transition locations. Notice the computed and measured transition locations exactly match at $U_\infty = 34$ m/s and $h = 0$ mm because that is where the critical N -factor value is calibrated. In Figure 5, the LST transition locations are upstream of the PSE/HLNSE transition locations at moderate step heights. The reason for this upstream shift is due to the overproduction of disturbance growth rates in the vicinity of the step as seen earlier in Figure 3. LST still captures the overall trend of the experimental transition data in Figure 5. These results demonstrate that transition correlations based on linear amplification factors yield a satisfactory agreement with the measured transition data from a high-quality experiment across a broad range of the parameter space that includes significant variations in both the BFS height and the unit Reynolds number.

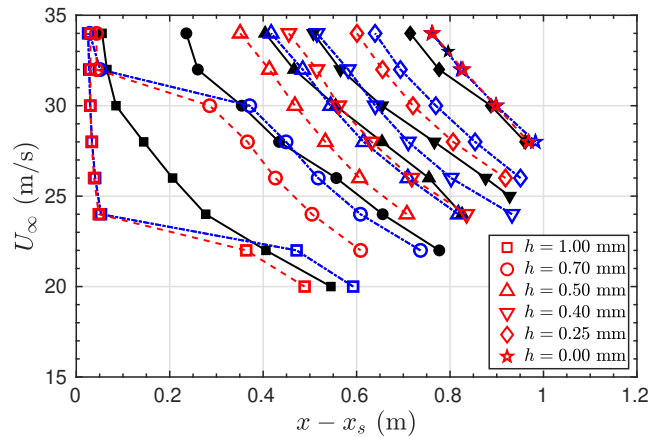


Fig. 5 Comparison of predicted transition locations based on LST using $N_{tr} = 7.32$ (red open symbols/dashed lines) and PSE/HLNSE using $N_{tr} = 7.6$ (blue open symbols/dashed-dotted lines) with measurements [6] (black closed symbols/solid lines) for $h = 0$ to 1 mm and $U_\infty = 20$ to 34 m/s.

Having established that both LST and HLNSE can successfully model the trends in transition of a two-dimensional boundary layer over a moderate height backward-facing step at different flow conditions, we now turn our attention to RANS-based transition models. We start with the Langtry-Menter four-equation transitional SST model [25] that requires the specification of a turbulent viscosity ratio and a freestream turbulence intensity denoted by μ_t/μ and Tu , respectively. Figure 6 displays the different combinations of μ_t/μ and Tu that yield the same transition location as the flat-plate case at $U_\infty = 34$ m/s from the Wang and Gaster [6] experiment. Here, μ_t/μ ranges from 1 to 100, and Tu ranges from 0.33% to 1.05%. We choose the combination of $\mu_t/\mu = 5$ and $Tu = 0.37\%$ for the purposes of this paper. Other combinations of μ_t/μ and Tu give similar results in terms of the skin friction and transition locations for the LM2009 model, but these results are not shown for brevity.

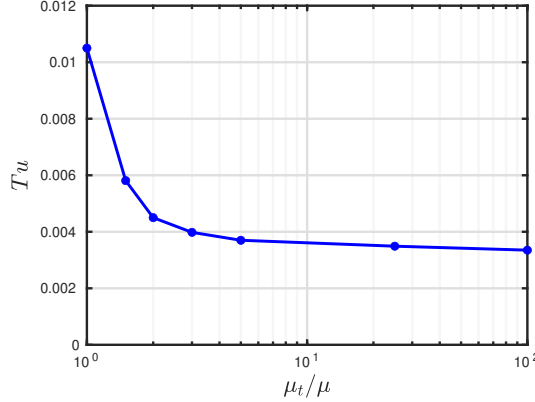


Fig. 6 Combinations of turbulent viscosity ratios and freestream turbulent intensities that result in the same transition location as the Wang and Gaster [6] experiment, corresponding to a flat plate at $U_\infty = 34$ m/s.

Figure 7 shows the LM2009 basic states corresponding to $U_\infty = 34$ m/s, $\theta = 75^\circ$, and BFS heights of 0, 0.5, and 1 mm. A sharp BFS with $\theta = 90^\circ$ yields similar results for the transport-equation-based transition models. The left edge of the computational domain at $x = -0.15$ m is a subsonic inflow, and the right edge at $x = 2$ m is a back-pressure outflow. We enforce adiabatic and no-slip boundary conditions along the bottom wall. There is a small part of the bottom boundary before the plate that is set to wall-normal symmetry. We employ a freestream boundary condition at the top edge of the domain at $y = 0.1$ m. The plate with a BFS starts at $x = 0$ m. We use a grid with 4000 points in the streamwise direction and 400 points in the wall-normal direction. The first grid point is placed above the flat plate at a distance that satisfies $y^+ = 0.05$, and the wall-normal grid spacing increases gradually from there to the freestream. We also use grid stretching in the streamwise direction to resolve where the boundary layer forms at $x = 0$ m and close to the step at $x = 0.3$ m. For the flat-plate simulation in Figure 7(a) at $U_\infty = 34$ m/s, we see boundary-layer transition occurs at roughly $(x - x_s)/\delta^* = 1300$, which is similar to experiments and stability analysis. In Figure 7(b), notice boundary-layer transition occurs slightly downstream of $(x - x_s)/\delta^* = 1300$ for a BFS height of 0.5 mm. This location of transition onset is far downstream of the experimental transition location from the Wang and Gaster [6] experiment and stability analysis at the same flow conditions (refer to Figure 5). For the case with a BFS height of 1 mm in Figure 7(b), transition occurs shortly after the step. To gain a better understanding of how the transition locations change for the RANS-based transition models, we compute the LM2009 basic states for the entire parameter space in Figure 5.

We display the skin-friction distributions for grid resolutions of 500×50 (tiny), 1000×100 (coarse), 2000×200 (medium), and 4000×400 (fine) that correspond to flat plate and BFS simulations at $U_\infty = 34$ m/s, $\mu_t/\mu = 5$, and $Tu = 0.37\%$ in Figure 8. The corresponding y^+ values are 1.0, 0.5, 0.1, and 0.05, respectively. The location of transition onset in the flat-plate simulation is converged with respect to the grid in Figure 8(a). Similarly, we see in Figure 8(b) the BFS simulation with $h = 0.5$ mm is close to grid converged on the finest two grids. The skin-friction curves from the LM2009-based boundary-layer flows over a BFS at $U_\infty = 28$ m/s and $U_\infty = 34$ m/s are shown in Figure 9 for the fine grids. According to Figure 9(a), transition is predicted to occur at roughly the same location for every BFS height. Indeed, contrary to the trend in the measured transition location as a function of the BFS height, when h increases from 0.25 to 1 mm, the predicted onset of transition shifts downstream, albeit in relatively small increments. This behavior is very different from the LST, PSE, and HLNSE transition results. It is also a departure from the Wang and Gaster [6] experiment. For $U_\infty = 28$ m/s, the LM2009 prediction for the transition location over a smooth plate agrees with

the computational and experimental results in Figure 5; however, it is unable to predict the transition locations in the presence of a BFS. We display the skin-friction curves from the LM2009 solutions of boundary-layer flow over a BFS at $U_\infty = 34$ m/s in Figure 9(b). We see in Figure 9(b) boundary-layer transition occurs just downstream of the step for $h = 1$ mm with the LM2009 model. This agrees with the computational and experimental results in Figure 5. All the other BFS heights result in behavior that is similar to that in Figure 9(a). Thus, we may conclude from Figures 7 and 9 that the LM2009 predictions for the transition onset locations for moderate BFS heights are not accurate.

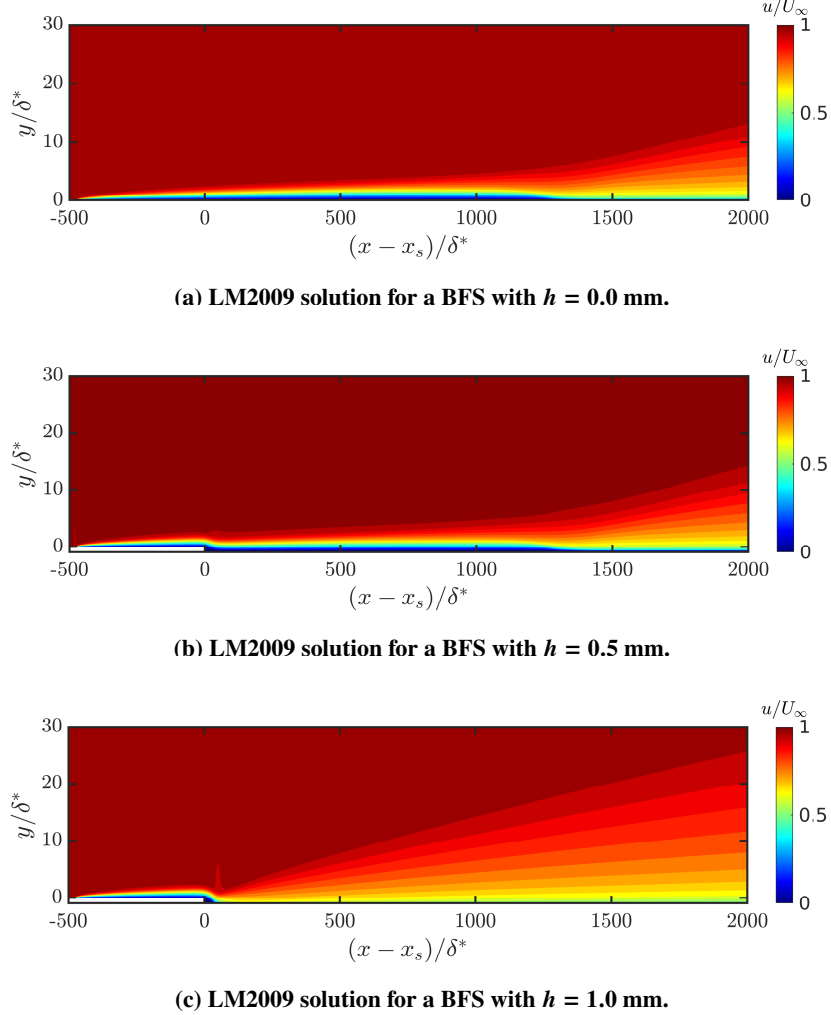


Fig. 7 Contours of nondimensional streamwise velocity of boundary-layer flow over a BFS computed with the LM2009 transition model at the following conditions: $U_\infty = 34$ m/s, $\delta^* = 6.26 \times 10^{-4}$ m, $\theta = 75^\circ$, $\mu_t/\mu = 5$, and $Tu = 0.37\%$. The step heights are 0, 0.5, and 1 mm.

Figure 10 depicts the maximum vorticity Reynolds number divided by the critical momentum-thickness Reynolds number at each streamwise station for step heights varying from 0 to 1 mm at $\mu_t/\mu = 5$, $Tu = 0.37\%$, and $U_\infty = 34$ m/s. Transition onset is controlled by the ratio Re_v/Re_{θ_c} in the LM2009 model [25]. We see in Figure 10 that Re_v/Re_{θ_c} is small for BFS heights of 0, 0.25, 0.5, and 0.7 mm before $(x - x_s)/\delta^* = 1300$. However, Re_v/Re_{θ_c} is very large for a BFS height of 1 mm shortly after the step location. Also, Re_v/Re_{θ_c} is smaller for BFS heights of 0.25, 0.5, and 0.7 mm than it is for a smooth flat plate before transition occurs near $(x - x_s)/\delta^* = 1300$. In reality, for moderate step heights transition onset moves gradually upstream as a result of the cumulative effect of the locally-perturbed boundary-layer profiles on the integrated amplification ratio. Transition still occurs substantially downstream of the step, but the TS wave remembers the upstream history associated with increased amplification near the step. Since the LM2009

model compares the surrogate momentum thickness (based on vorticity Reynolds number) with the critical momentum thickness to account for transition to turbulence, we know it is unable to account for the history effect of the step. We can see in Figure 10 the boundary-layer flow returns to its unperturbed state downstream of the step for moderate heights. Therefore, the LM2009 model will not respond to the BFS until the perturbation becomes large enough so that the submodel for the separation-induced transition gets activated and predicts transition in the step vicinity.

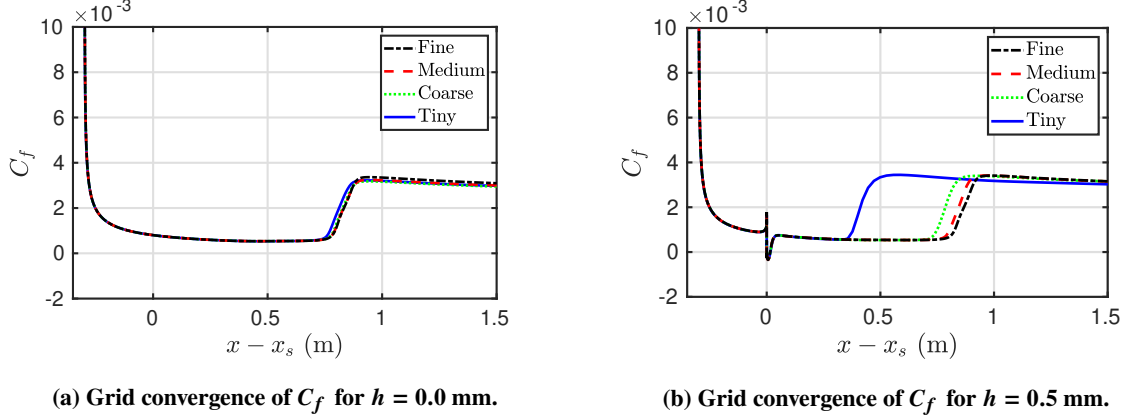


Fig. 8 Convergence of the skin friction with respect to the computational grid for boundary-layer flow over a flat plate and BFS with the LM2009 transition model at $\theta = 75^\circ$, $\mu_t/\mu = 5$, and $Tu = 0.37\%$.

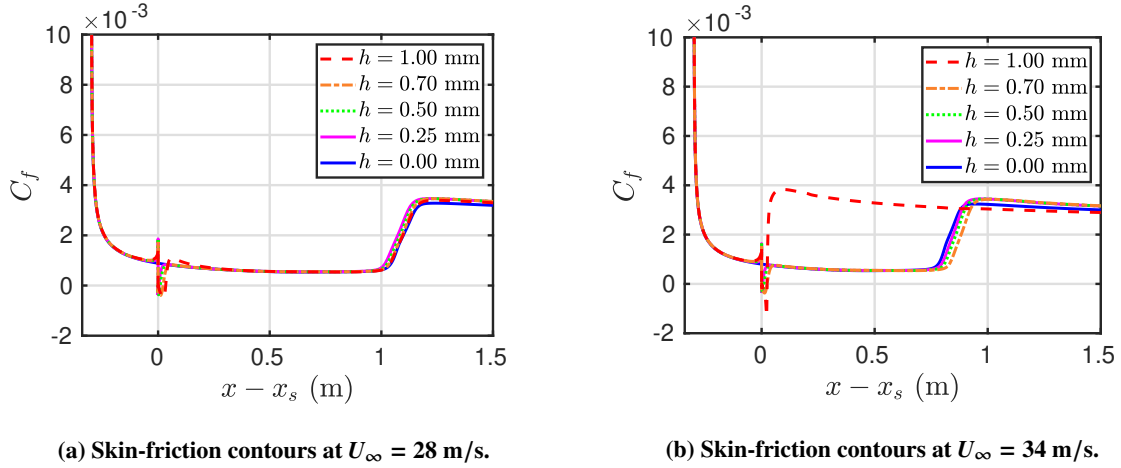


Fig. 9 Skin friction along the bottom wall for boundary-layer flow over a BFS computed with the LM2009 transition model at $\theta = 75^\circ$, $\mu_t/\mu = 5$, and $Tu = 0.37\%$.

Unlike the LM2009 transition model, Coder's version [26, 28] of the amplification factor transport-equation-based transition model might be expected to provide a better account of the BFS-induced boundary-layer distortion on the amplification factors. We use $\mu_t/\mu = 0.001$ and $Tu = 0.17\%$ to match the flat-plate transition location from experiments and stability analysis at $U_\infty = 34$ m/s. The value of $Tu = 0.17\%$ is slightly greater than $Tu = 0.1257\%$ obtained from substituting $N_{lr} = 7.6$ into $N_{lr} = -8.43 - 2.4\ln(Tu)$ from Mack [46]. Figure 11 shows the AFT2019b solutions corresponding to $U_\infty = 34$ m/s with BFS heights of 0 and 0.5 mm. The left boundary is a subsonic inflow, and the right boundary at $x = 2$ m is a back-pressure outflow. We enforce adiabatic and no-slip boundary conditions along the bottom wall similar to the LM2009 model. A small component of the bottom edge of the computational domain in front of the plate ($x = -0.15$ to 0 m) is set to wall-normal symmetry. The top edge of the computational domain at $y = 0.1$ m is a freestream boundary condition. A grid with 4000 points in the streamwise direction and 400 points in the wall-normal direction is used to resolve the domain, similar to the grid employed for the LM2009 model. We see in Figure 11(a) that boundary-layer transition occurs at roughly $(x - x_s)/\delta^* = 1300$, which is similar to experiments, stability analysis, and

the LM2009 model. For the case with a BFS height of 0.5 mm in Figure 11(b), transition occurs slightly downstream of $(x - x_s)/\delta^* = 1300$ similar to the LM2009 transition model. This location of transition onset does not agree with the Wang and Gaster [6] experiment and stability analysis at the same flow conditions (refer to Figure 5). To further investigate the performance of the AFT2019b model, we also examined additional computations

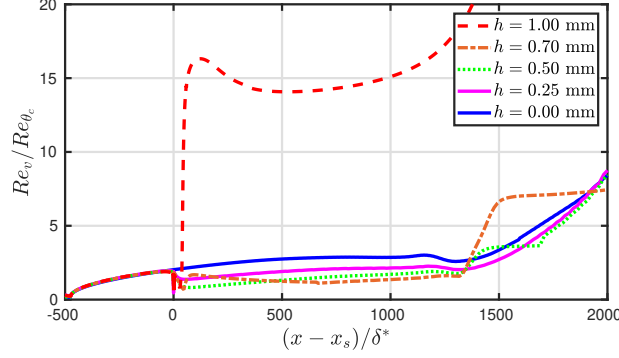
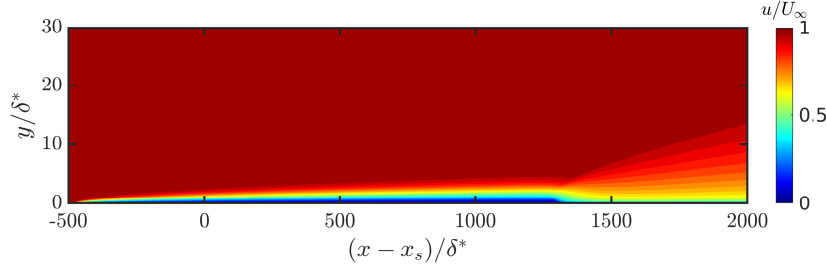
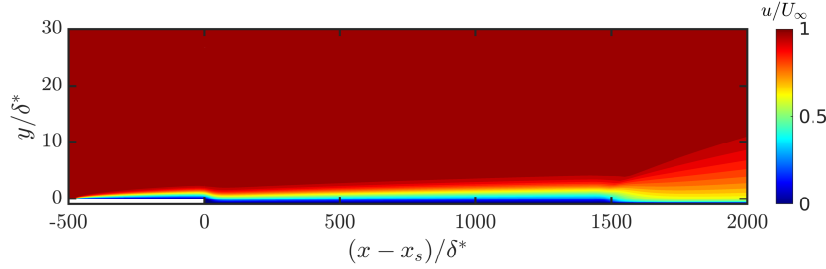


Fig. 10 Distribution of the maximum vorticity Reynolds number divided by the critical momentum thickness Reynolds number for boundary-layer flow over a BFS computed with the LM2009 transition model at $U_\infty = 34$ m/s, $\delta^* = 6.26 \times 10^{-4}$ m, $\theta = 75^\circ$, $\mu_t/\mu = 5$, and $Tu = 0.37\%$.



(a) AFT2019b solution for a BFS with $h = 0.0$ mm.



(b) AFT2019b solution for a BFS with $h = 0.5$ mm.

Fig. 11 Contours of nondimensional streamwise velocity of boundary-layer flow over a BFS computed with the AFT2019b transition model at the following conditions: $U_\infty = 34$ m/s, $\delta^* = 6.26 \times 10^{-4}$ m, $\mu_t/\mu = 0.001$, and $Tu = 0.17\%$. The step heights are 0 and 0.5 mm.

The skin-friction distributions for grid resolutions of 500×50 (tiny), 1000×100 (coarse), 2000×200 (medium), and 4000×400 (fine) that correspond to flat plate and BFS simulations at $U_\infty = 34$ m/s, $\mu_t/\mu = 0.001$, and $Tu = 0.17\%$ computed with the AFT2019b transition model are shown in Figure 12. We see the location of transition onset in the flat-plate simulation is grid converged on all but the tiny grid. Also, the BFS simulation with $h = 0.5$ mm is very close to grid converged. Figure 13 displays the skin-friction curves from the AFT2019b-based boundary-layer flows over a BFS at $U_\infty = 28$ m/s and $U_\infty = 34$ m/s for the fine grids. Notice in Figure 13 transition occurs at roughly the same

location for every BFS height from 0.25 to 1 mm, similar to results in Figure 9(a). This does not agree with the Wang and Gaster [6] experiment or the results from stability analysis. The flat-plate transition locations for $U_\infty = 28$ m/s and $U_\infty = 34$ in Figure 13 do agree with the other methods. In Figure 13, the location of transition onset for a BFS height of 1 mm does start to move upstream compared to the other step heights, but not by a significant margin. We may conclude from Figures 11 and 13 that the AFT2019b predictions of the transition onset locations for subsonic boundary-layer flow over backward-facing steps are not accurate.

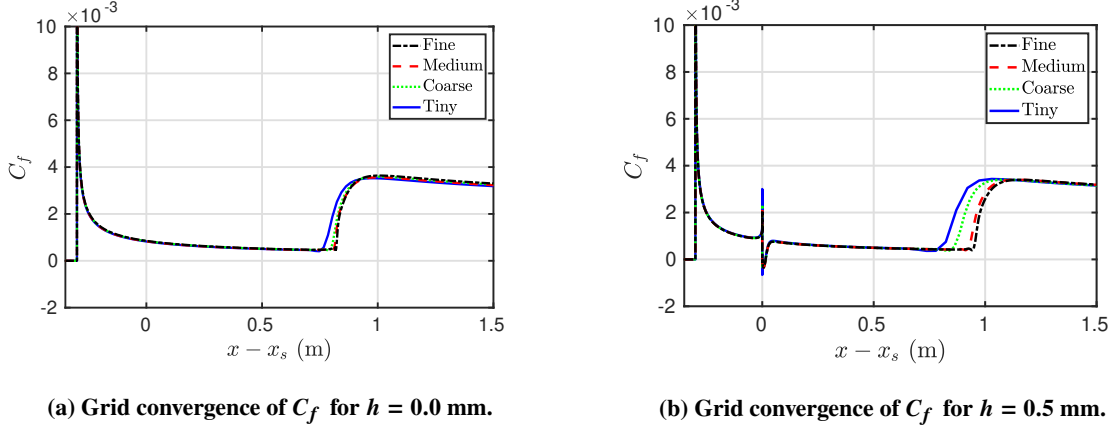


Fig. 12 Convergence of the skin friction with respect to the computational grid for boundary-layer flow over a flat plate and BFS with the AFT2019b transition model at $\theta = 75^\circ$, $\mu_t/\mu = 0.001$, and $Tu = 0.17\%$.

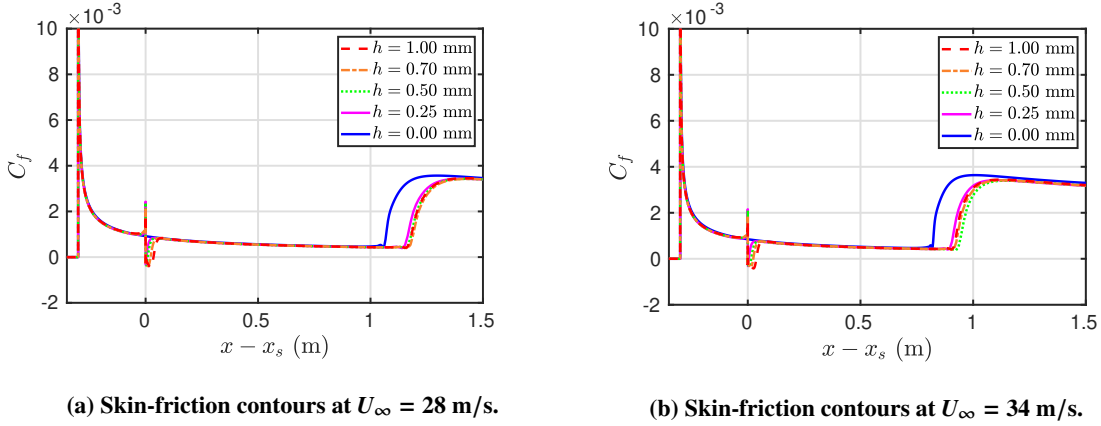


Fig. 13 Skin friction along the bottom wall for boundary-layer flow over a BFS computed with the AFT2019b transition model at $\theta = 75^\circ$, $\mu_t/\mu = 0.001$, and $Tu = 0.17\%$.

Figure 14 depicts the maximum amplification factor at each streamwise station for boundary-layer flow over a BFS computed with the AFT2019b transition model at $U_\infty = 34$ m/s. Notice there is a very localized peak in $\tilde{N}$ close to the step, but it quickly diminishes. This localized peak gets larger with increasing BFS height, but even at $h = 1$ mm it still is not enough to cause transition. One might expect the AFT2019b transition model to account for the effect of favorable and adverse pressure gradients near the backward-facing step on the N -factor envelope, but Figure 14 indicates that the model predicts a net decrease in the envelope across the step region, and thus, fails to capture the upstream shift in transition location as the BFS height increases.

IV. Conclusion

The objective of this paper was to assess the accuracy of different engineering methods in predicting the effects of surface excrescences on transition in 2D boundary layers. The methodologies of interest include both linear stability

correlations and phenomenological models that augment the RANS framework with transition models based on auxiliary transport equations. In contrast to the variable N -factor criteria based on empirical correlations that are specific to step excrescences, both of these methodologies provide a direct account of changes in the laminar basic state as a result of the step excrescences and how those changes may influence the transition process. In assessing the linear stability correlations, we used both LST and HLNSE to predict the amplification of instability waves in the boundary-layer flow perturbed by a backward-facing step. The computed N -factor curves were used to predict the streamwise shift in the transition location across the various flow speeds and BFS heights considered during the high-quality wind-tunnel experiment performed by Wang and Gaster [6]. The range of flow parameters includes freestream velocities from 20 to 34 m/s and BFS heights between 0 and 1 mm. By virtue of its accurate modeling of the disturbance evolution across the nonparallel mean flow in the vicinity of the step, the HLNSE agreed the best with experiments, but it had discrepancies at large step heights and higher freestream velocities [30]. The agreement between LST predictions and the experimental measurements was not as good as that with the HLNSE, mainly because it does not account for those nonparallel effects. However, LST did capture the overall trends from the experiments quite well. It also had a similarly large error as HLNSE in predicting the onset of transition at large step heights and freestream velocities.

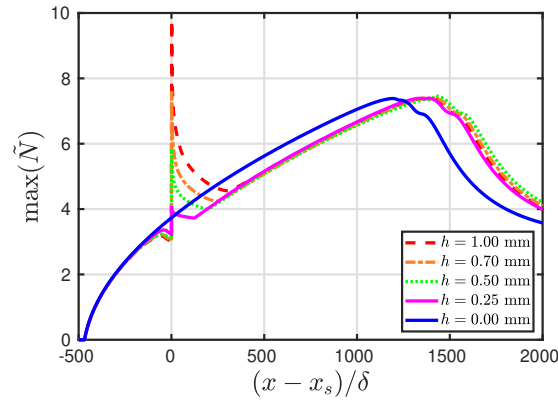


Fig. 14 Distribution of the maximum amplification factor at each streamwise station for boundary-layer flow over a BFS computed with the AFT2019b transition model at $U_\infty = 34$ m/s, $\delta^* = 6.26 \times 10^{-4}$ m, $\theta = 75^\circ$, $\mu_t/\mu = 0.001$, and $Tu = 0.17\%$.

Along with LST and HLNSE, we computed transition locations by using transport-equation-based models. These include the Langtry-Menter four-equation transitional SST model and Coder's amplification factor transport model combined with the SA turbulence model. The local-correlation-based transition model in LM2009 is unable to account for the history effect related to the increased disturbance amplification near the BFS because it is based on a comparison of the surrogate momentum thickness with the critical momentum thickness to determine if the flow is laminar or turbulent. For moderate step heights, transition occurs far downstream of the BFS, where the boundary-layer flow has returned to its unperturbed state. The local correlation underlying the LM2009 model does not retain the memory of the step-induced perturbation, and hence, is unable to predict the transition accurately at these conditions. Coder's amplification factor transport model seeks to account for the flow history effects by solving a transport equation for the amplification factor, but it is still not able to predict the measured shift in the transition location as a function of the BFS height. Further details on why these particular transport-equation-based models do not predict the correct transition locations without modifications (according to experiments and stability analysis) for boundary-layer flows over surface excrescences will be included in a forthcoming paper.

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