Simulation and Modeling of Hypersonic Turbulent Boundary Layers Subject to Adverse Pressure Gradients due to Concave Streamline Curvature

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Motivation

- Realistic flight vehicles exhibit streamline curvature that induces favorable and adverse pressure gradients.
- Mechanical nonequilibrium associated with a combination of curvature driven pressure gradients and compressibility effects poses significant challenges to turbulence modeling (Tichenor et al. 2013).
  - Adverse pressure gradients have been shown to destabilize the boundary layer and amplify turbulent fluctuations.
  - A favorable pressure gradient has been shown to reduce turbulent fluctuations and stabilize the boundary layer.
- Most turbulence models are based on compressible flow extensions to models designed for incompressible flow.
  - High-speed phenomena such as bulk dilation neglected.
  - Model assumptions not fully assessed in the hypersonic regime.
- There is a lack of detailed data to validate compressibility transformations such as the Morkovin’s scaling and strong Reynolds analogy.
Previous Work
Development of DNS database

- DNS dataset of hypersonic favorable pressure-gradient TBLs
  
  
  (Nicholson et al., AIAA Paper 2021-1672)

- Flow conditions representative of TAMU high-speed blown-down wind tunnel
  - Allow for one-to-one comparisons with wind-tunnel experiments

- Based on a high-order scheme with a large domain size ($L_x/\delta_i > 65$, $L_y/\delta_i > 7$) and sufficiently long sampling duration ($T_{fu}/\delta_i > 2.5$) to
  - Minimize any artificial effects due to inflow turbulence generation
  - Ensure the convergence of selected high-order turbulence statistics
Objectives

- Extend existing DNS database to include high speed flow under pressure gradient
  - Flow conditions representative of the APG experiment of TAMU at Mach 5 (Neel et al. 2016)

- Investigate boundary layer turbulence subject to pressure gradient ($\beta \equiv \left( \frac{d \rho_w}{d x} \frac{\delta^*_{inc}}{\tau_w} \right) \leq O(1)$)
  - Effects of pressure gradient

- Assessment of common RANS models
  - Models’ performance worsened under favorable pressure gradient
Outline

- Flow Conditions and DNS methodology
- Assessment of DNS data
- Results
  - DNS study of boundary-layer physics
  - Assessment of RANS models
- Summary and future work
Flow Conditions & Methodology

- Freestream and wall-temperature conditions match those of TAMU experiment (Tichenor et al. JFM 2013)
  - $M_\infty = 4.9$, $T_w = 317$ K, $T_w/T_r \sim 1.0$
  - Flat wall followed by curved section defined by two 8-degree arcs. (Neel et al. 2016)
  - Profiles at $\beta_{inc} = \left( \frac{dp_w}{dx} \right) \left( \frac{\delta^{*}_{inc}}{\tau_w} \right) \sim 0, 0.85, 1.05$

- DNS solves 3D compressible Navier-Stokes equations in conservative form
  - 7th-order WENO (Jiang & Shu 1996)
  - Rescaling/recycling method for inflow turbulence generation (Xu & Martin 2004)
  - “Embedded” DNS method
Boundary layer growth closely matches values reported from experiments.
Similarly good comparison is found with Clauser beta parameter.
Assessment of DNS Data
Comparison with Experiments (Location U, Xg = 0.154m)

van-Driest (VD) velocity scaling

\[ u_{VD}^+ = \int_0^\infty \bar{\rho}^{1/2} d\bar{u}^+. \]

\[ + = \text{variable in inner wall units} \]

\[ u_{VD}^+ \]

\[ z^+ \]

\[ 1 - u_{VD}^+ / \left( u_{VD}^+ \right) \]

\[ z/\delta \]

- \( u_{VD}^+ \) profile of the ZPG TBL conforms well with incompressible law of the wall and shows narrow logarithmic region with other ZPG TBLs.
- \( u_{VD}^+ \) compares well with TAMU experiments, particularly in the outer region.
- Excellent comparison with the van Driest transformed deficit velocity.
Velocity profiles compare fairly well with experimental results.

Subjecting the flow to an adverse pressure gradient results in a less full boundary layer.

For a favorable pressure gradient, boundary layer becomes fuller.

Van Driest transformation shows a good collapse of the viscous sub layer.

Similarly good collapse in the log-law layer except for strong favorable pressure gradient.
Similar with experiments there is a progressive increase in Reynolds stresses with increasing pressure gradient.

DNS profiles for the streamwise component are close to experimental profiles except at location U in the near wall region.

DNS predicts generally higher wall-normal component of Reynolds stress.

For shear component DNS predicts higher, similar, and then lower values at locations U, D1, D2, respectively.

Variance is believed to be due to inaccuracies in the PIV measurement.
Increased pressure gradient leads to increased magnitude of turbulent fluctuations.

Unlike for previously seen ZPG profiles, Morkovin's scaling \( u^* = u_t \sqrt{\frac{\rho_w}{\rho}} \), does not provide a good collapse across different pressure gradients.

For the ZPG profile, the turbulent Prandtl number is close to unity throughout the boundary layer.

Adverse pressure gradients have little impact on \( Pr_t \) or the SRA ratio.
Assessment of several popular RANS models (Baldwin-Lomax (BL), Spalart-Allmaras (SA), and the k-ω SST) performed against the DNS.

- Both SA and k-ω SST models capture APG region well.
- k-ω SST maintains good accuracy while the SA model is not as accurate in predicting changes due to FPG.
- BL model does not preform well in the curved wall region.
- Overall, k-ω SST model provides excellent comparison.
All models underpredict the magnitude of the Reynolds normal stress regardless of pressure gradient.

For location D1, SA and BL models provide excellent predictions of Reynolds shear stress, while $k-\omega$ SST slightly underpredicts it.

At location D2 with greater pressure gradient, predictions via all models worsen significantly.
Summary

- DNS of adverse-pressure-gradient (APG) turbulent boundary layers were presented for a nominal freestream Mach number of 5
  - Flow conditions and planar curved-wall geometries matched those of the experiments at the Texas A&M University
  - DNS profiles sampled at locations with varied pressure gradient corresponding to $\beta \approx 0, 0.96, 1.22$, induced via surface curvature used during experiments

- The DNS showed decent comparisons with the measured velocity profiles and the streamwise Reynolds-stress component, but predicted higher values of wall-normal and shear components of Reynolds stresses than measurements
  - Discrepancies between the predicted and the measured wall-normal and shear components of Reynolds stress are primarily attributed to the lower than actual values inferred from typical PIV measurements of turbulent boundary layers

- Morkovin’s scaling shown to provide poor collapse amongst cases with adverse pressure gradient, while strong Reynolds analogy proves adequate with $Pr_t$ close to unity across boundary layer

- Assessment of several popular RANS models (Baldwin-Lomax, Spalart-Allmaras, Wilcox k-$\omega$, and the k-$\omega$ SST) against the DNS showed that
  - Overall, the k-$\omega$ SST model performed the best, yielding nearly DNS-like predictions for the skin friction over the curved wall.
  - Predictions of Reynolds stresses worsen with increasing pressure gradient for all models
Next Steps

- Develop DNS database of hypersonic TBLs over a 3-D ogive geometries

- Conduct one-to-one comparison of DNS against experiments at TAMU

- Characterize boundary-layer turbulence
  - Mean and turbulence statistics
  - Budgets of TKE and Reynolds-stress transport
  - Compressibility transformations

- Assess turbulence models
  - Reynolds-stress transport (RST) models
  - Algebraic energy flux model by Bowersox
Acknowledgment

- **Financial Support**
  - ONR (Award No.: N00014-20-1-2194)

- **Computational resources**
  - DoD High Performance Computing Modernization Program
  - Ohio Supercomputer Center
Backup
Existing literature on effects of streamline curvature on hypersonic boundary layer turbulence is limited

- Most studies focused on fully turbulent boundary layers at low-speed, few focus on supersonic and even fewer on hypersonic
- Early experiments limited to flow visualization and single point measurements for supersonic via hot-wire anemometry and/or laser Doppler velocimetry
- Detailed and global measurements not obtained until more recent studies based on particle image velocimetry (PIV)
  - Valuable insights into the effects of pressure gradients on behavior, distribution and scaling of mean velocity and Reynolds stresses
  - Favorable pressure gradients have a stabilizing effect, while adverse has a destabilizing effect
  - Studies have been unable to provide sufficiently well resolved measurements of both velocity and thermodynamic fields, particularly near the wall.

DNS of supersonic and hypersonic flow

- Can provide detailed global measurements of global fields and distributions of turbulent fluctuations
- Enable systematic investigations into
  - Turbulence scaling laws
  - Evaluations of exact equations of turbulent kinetic energy and Reynolds-stress transport
- Help facilitate development of and improvements to turbulence modeling.
Previous Work
Development of DNS database

- DNS datasets of supersonic and hypersonic zero-pressure-gradient TBLs (Zhang et al., AIAAJ, 2018) and (Huang et al., AIAA Paper 2020-0571)

- Spatially developing, zero-pressure gradient, flat plate

- Covers a wide range of freestream Mach numbers ($M_\infty = 2.5 - 14$) and wall-to-recovery temperature ratios ($T_w/T_r = 0.18-1.0$)

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_\infty$</th>
<th>$U_\infty$ (m/s)</th>
<th>$\rho_\infty$ (kg/m$^3$)</th>
<th>$T_\infty$ (K)</th>
<th>$T_w$ (K)</th>
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- Friction Reynolds number ($Re_τ$) recently increased to include moderately high Reynolds number (up to 1138)

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</table>
Previous Work
Development of DNS database

- **DNS datasets of supersonic and hypersonic zero-pressure-gradient TBLs** (Zhang et al., AIAAJ, 2018, Huang et al., AIAA Paper 2020-0571)

  - Flow conditions representative of several hypersonic wind tunnels
    - Purdue Quiet Tunnel at Mach 6, Sandia Hypersonic Wind Tunnel at Mach 8, CUBRC at Mach 11, and AEDC Tunnel 9 at Mach 14.
    - Allow for one-to-one comparisons with wind-tunnel experiments

  - Based on a high-order scheme with a large domain size \( \frac{L_x}{\delta_i} > 50, \frac{L_y}{\delta_i} > 8 \) and sufficiently long sampling duration \( \frac{T_{fu}}{\delta_i} > 5 \) to
    - Minimize any artificial effects due to inflow turbulence generation
    - Ensure the convergence of selected high-order turbulence statistics

DNS of ZPG TBL over a highly-cooled flat plate

*CUBRC Run 7*

Huang et al., AIAA Paper 2020-0571

Gnoffo, JSR 2013
DNS Methodology

- DNS solves 3D compressible Navier-Stokes equations in conservative form
  - 7th order WENO (Jiang & Shu 1996)
  - rescaling/recycling method for inflow turbulence generation (Xu & Martin 2004)
  - carried out in 3 stages involving overlapping streamwise domains
    - Inflow plane is greater than $80\delta_i$ from the first sample to allow DNS to achieve fully developed equilibrium state of a ZPG TBL
    - sufficiently long sampling duration ensures the convergence of selected high-order turbulence statistics

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<th>$x_{beg}$-$x_{end}$, m</th>
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<th>$\Delta z^+$</th>
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<th>$L_y/\delta_i$</th>
<th>$L_z/\delta_i$</th>
<th>$N_x \times N_y \times N_z$</th>
<th>$T_f u_\tau/\delta$</th>
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<td>6.9</td>
<td>5.2</td>
<td>0.56-8.8</td>
<td>69.3</td>
<td>7.0</td>
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<td>4500 $\times$ 600 $\times$ 550</td>
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<td>7.0</td>
<td>50.0</td>
<td>6500 $\times$ 600 $\times$ 550</td>
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DNS Results
Boundary Layer Parameters

- Effect of pressure gradient is investigated by comparing boundary-layer profiles at three downstream locations ($x_g = 0.154 \text{ m}, 0.292 \text{ m}, \text{ and } 0.308 \text{ m})
  - Three profiles referred to as U, D1, D2 respectively
  - Clauser equilibrium pressure gradient parameter $\beta \sim 0$ to 1.22

<table>
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<tr>
<th>Dataset</th>
<th>$x_{\text{beg}}$-$x_{\text{end}}$, m</th>
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<th>$\Delta y^+$</th>
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<td>-</td>
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<td>1.05</td>
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Numerical Methodology

- Direct numerical simulation (DNS): three-dimensional compressible Navier-Stokes in Cartesian coordination
  - inviscid flux: 7th-order weighted essentially non-oscillatory scheme (WENO, Jiang & Shu 1996)
  - viscous flux: 4th-order central difference scheme
  - time advancement: 3rd-order low-storage Runge-Kutta scheme (Williamson 1980)
  - spanwise: periodic boundary condition
  - outflow: nonreflecting characteristic boundary condition (Thompson 1987)
  - wall: no-slip isothermal wall boundary, $T_w=317$ K
  - inflow: rescaling/recycling inflow turbulence generation method (Xu & Martin 2004)
References


