

Adjoint-Based Minimization of X-59 Sonic Boom Noise Via Control Surfaces

David L. Rodriguez¹

Science & Technology Corp., Moffett Field, CA 94035

Michael J. Aftosmis² and Marian Nemec³

NASA Ames Research Center, Moffett Field, CA 94035

Wade M. Spurlock⁴

Science & Technology Corp., Moffett Field, CA 94035

A multidisciplinary design optimization methodology to directly minimize the ground-level noise generated by a supersonic aircraft's sonic boom is presented. A gradient-based optimizer is coupled with a Cartesian Euler flow solver and an atmospheric propagation tool to forge a unique design capability. Adjoint formulations for both the flow solver and propagation tool are also coupled to provide sensitivities in an efficient manner. The design method is demonstrated on the X-59 Low-Boom Flight Demonstrator by optimizing control surface deflections to improve ground-level noise while maintaining trimmed cruise flight. Two noise minimization examples and a noise maximization are presented. All optimized designs result in measurable improvement in the ground-level noise objective. Extensive surveys of the design space confirm that the optimization method is effective in finding a local optimum. Moreover, repeated application of the method with varying initial design points also demonstrates the robustness of the method.

I. Nomenclature

$ASEL$	=	A-weighted Sound Exposure Levels
C_L	=	aircraft lift coefficient
C_M	=	aircraft pitching moment coefficient
h	=	cruise altitude
J_{ns}	=	near-field pressure signature functional
l_{ns}	=	length of near-field pressure signature
L	=	aircraft lift force
Q	=	Cart3D flow solution
p	=	local pressure
p_∞	=	freestream pressure
p_{gs}	=	ground-level pressure signature
p_{ns}	=	near-field pressure signature
α	=	angle of attack
δ_a	=	aileron deflection

¹ Senior Research Scientist, Computational Aerosciences Branch; david.l.rodriguez@nasa.gov, Senior Member AIAA.

² Aerospace Engineer, Computational Aerosciences Branch; michael.aftosmis@nasa.gov, Associate Fellow AIAA.

³ Aerospace Engineer, Computational Aerosciences Branch; marian.nemec@nasa.gov, Senior Member AIAA.

⁴ Research Scientist, Computational Aerosciences Branch; wade.m.spurlock@nasa.gov, Senior Member AIAA.

- δ_f = flap deflection
- δ_s = stabilator deflection
- δ_t = T-tail deflection

II. Introduction

Over the past decade, increased commercial and government interest in overland supersonic flight has spawned tremendous progress in accurate sonic boom analysis. Effective methods have been developed for near-field sampling with CFD (Computational Fluid Dynamics) tools ranging from Euler to Reynolds-averaged Navier-Stokes solvers using unstructured and structured meshes [1–6]. In particular, output-based mesh adaptation, which exploits adjoint flow formulations, has proven to be an enabling technology for sufficiently accurate near-field pressure signature predictions [7, 8]. Concurrently, atmospheric propagation tools have matured to the point where ground noise can be predicted within a decibel, even for low-boom aircraft designs [9–13]. When combined, these tools provide a powerful approach (depicted in Fig. 1) for the complete ground-level noise analysis of low-boom aircraft. The CFD solver is used to compute a highly accurate near-field pressure signature that is far enough from the aircraft to minimize three-dimensional, aircraft geometry effects. This pressure signature is then processed by an atmospheric propagation tool to compute the ground-level pressure signature. Furthermore, the ground signal is then appropriately weighted and used to compute ground-level noise metrics.

While high-fidelity sonic boom analysis has matured significantly, its applicability to design optimization has mostly been relegated to an inverse design process where the near-field pressure signature is first proposed as a goal [14]. A CFD-based shape optimization method is then applied to an aircraft with the hope of actually producing this target pressure signature. While the signature is chosen so that it results in low ground noise, it can almost never be realized by an actual complex aircraft design. Also, an optimal target pressure signature is rarely apparent. Inverse design methods really only work well when the optimal target is known ahead of time. Nevertheless, this technique has been shown to be effective in reducing ground-level noise of low-boom aircraft designs [14–18]. More recent work has begun to address direct minimization of ground-level noise metrics through aircraft shape optimization [19]. The work presented here expands upon those efforts through the design optimization of highly-complex aircraft configurations followed with detailed validation of the optimum designs.

At the publication time of this paper, the X-59 Low-Boom Flight Demonstrator (LBFD) [20] has been essentially fully designed. While shape optimization techniques cannot productively be applied to the X-59 outer mold line, avenues to lessen sonic boom noise remain. The X-59 has four sets of control surfaces and at cruise is over-controlled, meaning that there are multiple combinations of surface deflections that still allow the aircraft to maintain trimmed, level, unaccelerated flight. The control surfaces also have an appreciable effect on the sonic boom produced by the aircraft. This presents an opportunity to partially manage the ground-level noise of the aircraft by modifying control surface deflections and flight conditions. Therefore, to demonstrate its capability, the design optimization process discussed above is applied to this real-world problem.

III. Methodology

To directly minimize the ground-level noise produced by the X-59 aircraft flying in supersonic cruise, a multidisciplinary design optimization method was built using already existing and well-validated tools. **Cart3D** [21] is an aerodynamics simulation package that solves the Euler equations on an adaptive Cartesian mesh. The user provides a watertight surface triangulation, and the solver generates a Cartesian mesh automatically using the cut-cell approach [22]. The package also includes an adjoint solver [23] which allows for both output-driven mesh adaptation [24] and sensitivity calculation [25]. **Cart3D** has proven to be excellent at computing accurate near-field pressure signatures which can be used to compute the sonic boom of a vehicle [26, 27]. In fact, the near-field pressure signature error is usually used to drive the mesh adaptation to guarantee accuracy. A second tool,

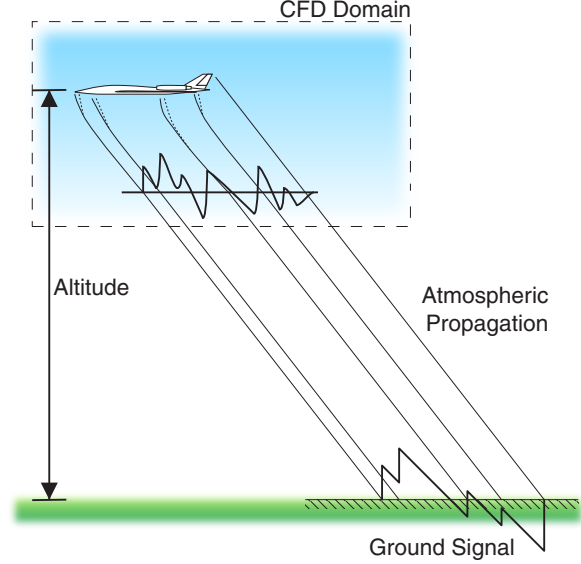


Fig. 1. Sonic boom prediction process.

trix, is provided in the **Cart3D** package and allows the user to rotate geometry components about arbitrary axes. In this work, **trix** is used to deflect the control surfaces of the triangulated aircraft geometry.

The third tool used to compute ground-level noise is **sBOOM** [10]. The solver uses an augmented Burgers equation to take a near-field pressure signature at altitude and propagate it through the atmosphere to the ground. The solver also incorporates an adjoint formulation that can compute the sensitivity of the propagated ground signature to the near-field pressures [28]. This capability is critical for gradient-based optimization. **sBOOM** also computes ground-level noise metrics from the ground signature, including *ASEL* [29, 30] and *PLdB* [31]. Metrics like this transform the waveform of the ground signature into quantifiable measures of noise experienced by an observer on the ground. The adjoint formulation of **sBOOM** also incorporates the noise metric computation, thus providing the user with sensitivities of noise to the provided pressure signature. Note that sensitivities for *PLdB* are not computed by **sBOOM** because that metric is not easily differentiable.

The final tool used in the design method is **SNOPT** [32], which uses a sequential-quadratic-programming (SQP) algorithm to conduct gradient-based numerical optimization. **SNOPT** works with both linear and nonlinear objective and constraint functions. The adjoint solvers provided by both **Cart3D** and **sBOOM** are exploited to evaluate the gradients of any computed functions with respect to all design variables. For the direct minimization of noise from a sonic boom, the objective function would naturally be the ground-level noise metric. Ground-level noise gradients with respect to control surface deflections can be computed using the chain rule. For example, the gradient of *ASEL* with respect to flap deflection can be computed using Eq. (1):

$$\frac{d(ASEL)}{d\delta_f} = \frac{\partial(ASEL)}{\partial p_{gs}} \frac{\partial p_{gs}}{\partial p_{ns}} \frac{\partial p_{ns}}{\partial Q} \frac{dQ}{d\delta_f} \quad (1)$$

where p_{gs} is the ground-level pressure signature, p_{ns} is the near-field pressure signature, Q is the **Cart3D** flow solution, and the last derivative term is evaluated via the adjoint method in the optimization framework [33]. The gradient of *ASEL* with respect to the ground-level signature is provided by **sBOOM**, as is the gradient of the ground-level signature with respect to the near-field signature. While *ASEL* is the objective function, often constraints on the forces and moments acting on the aircraft are necessary for a realistic design problem. For example, if a constraint on the lift coefficient is enforced, that gradient with respect to a flap deflection can be computed using Eq. (2):

$$\frac{dC_L}{d\delta_f} = \frac{\partial C_L}{\partial Q} \frac{dQ}{d\delta_f} \quad (2)$$

where all of the terms are computed by **Cart3D**. For design problems involving boom propagation, exploiting cruise altitude as a design variable is often advantageous. Since **Cart3D** is an Euler code, altitude actually only affects atmospheric propagation results. Theoretically, the required engine thrust should also be affected by changes in altitude, but for this work, the plume flow into the computational domain is assumed to be constant. Doing otherwise would require inserting a propulsion analysis into the problem which is currently beyond the scope of this work. Thus, **sBOOM** must provide gradients of the noise metric with respect to altitude as in Eq. (3):

$$\frac{d(ASEL)}{dh} = \frac{\partial(ASEL)}{\partial p_{gs}} \frac{dp_{gs}}{dh} \quad (3)$$

Computing Eq. (1) represents a fully-coupled, multidisciplinary sensitivity calculation that combines near-field CFD, atmospheric propagation, and acoustic noise evaluation. Employing this approach with the tools already described, a fully-coupled, ground-level noise driven, design optimization method can be built using the architecture shown in Fig. 2. A typical design iteration begins with predicted values for control surface deflections and angle of attack provided by **SNOPT** to **trix**. The control surfaces are deflected accordingly to produce a new triangulation. **Cart3D** takes that new triangulation and generates a new volume mesh and solution. Forces and moments acting on the aircraft along with their gradients with respect to the design variables are fed back to **SNOPT**. The near-field pressure signature

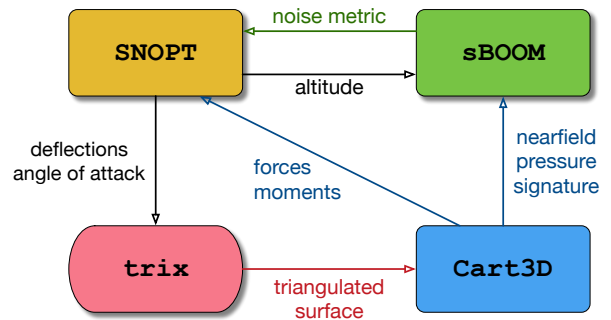


Fig. 2. Architecture of design optimization method.

is extracted from the solution and passed to **sBOOM**. Also, the altitude predicted by **SNOPT** is passed to **sBOOM**. The ground pressure signature and corresponding ground-level noise metric are computed along with gradients and are passed to **SNOPT**. At that point, **SNOPT** uses the objective and constraint values and gradients to determine new values for the design variables that attempt to improve the ground noise while satisfying the constraints. Using the SQP algorithm, the process continues until a local optimum is discovered.

IV. Applications

The methods described in the previous section were applied to the Low-Boom Flight Demonstrator (X-59) aircraft design [20]. This aircraft is shown in Fig. 3 with all the control surfaces highlighted and labeled in color. The surfaces include the flaps, ailerons, stabilator, and T-tail. This section presents three examples of optimizing the deflections of these control surfaces to achieve different combinations of ground-level noise objectives and flight constraints. Note the ailerons here are always deflected symmetrically.

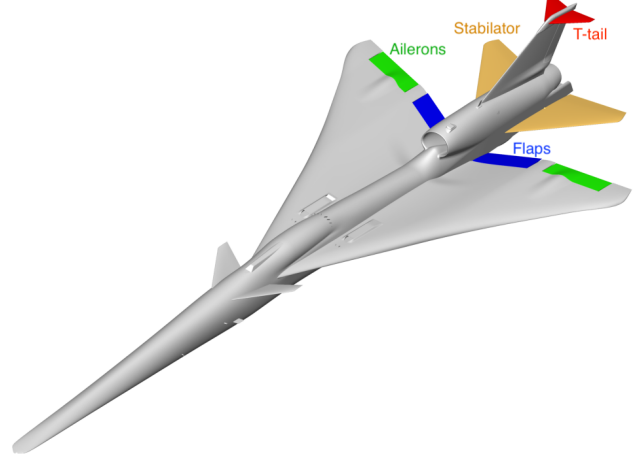


Fig. 3. X-59 aircraft with highlighted control surfaces.

A. Fixed Altitude with No Trim Constraint

The first and simplest problem to which the above design method was applied is the direct minimization of ground-level noise at fixed altitude and lift. No trim constraints were enforced. For this design problem, the C612 revision of the X-59 was used. The optimizer varied angle of attack and all control surface deflections. Since the altitude, Mach number, and lift are constant, the lift coefficient was held fixed as a constraint. Eq. (4) summarizes the optimization problem:

$$\begin{aligned}
 &\text{minimize: } ASEL(\alpha, \delta_f, \delta_a, \delta_s, \delta_t) \\
 &\text{subject to: } C_L(\alpha, \delta_f, \delta_a, \delta_s, \delta_t) = C_{L,cruise} \\
 &\quad \delta_{f,min} \leq \delta_f \leq \delta_{f,max} \\
 &\quad \delta_{a,min} \leq \delta_a \leq \delta_{a,max} \\
 &\quad \delta_{s,min} \leq \delta_s \leq \delta_{s,max} \\
 &\quad \delta_{t,min} \leq \delta_t \leq \delta_{t,max}
 \end{aligned} \tag{4}$$

The bounds listed in Eq. (4) are specified in Table 1. Also listed are the initial values of the control surface deflections which are already somewhat optimized based on previous unpublished trade studies.

Table 1. Control surface deflection bounds and initial values in design optimization.

Control Surface	Minimum	Maximum	Initial
Ailerons	-3.0°	3.0°	1.1°
Flaps	-3.0°	3.0°	-1.8°
Stabilator	-15.0°	15.0°	3.2°
T-tail	0.0°	10.0°	4.3°

The optimization problem was run using the coupled design method. The computational meshes employed had roughly 140 million cells (half body) and were generated using a re-meshing capability of **Cart3D** to bypass mesh adaptation and speed up design cycle time. This process starts by generating an original mesh for the problem using output-based mesh adaptation. The functional for this adapted mesh is given in Eq. (5):

$$J_{ns} = \int_0^{l_{ns}} \left(\frac{p - p_\infty}{p_\infty} \right)^2 dl_{ns} \tag{5}$$

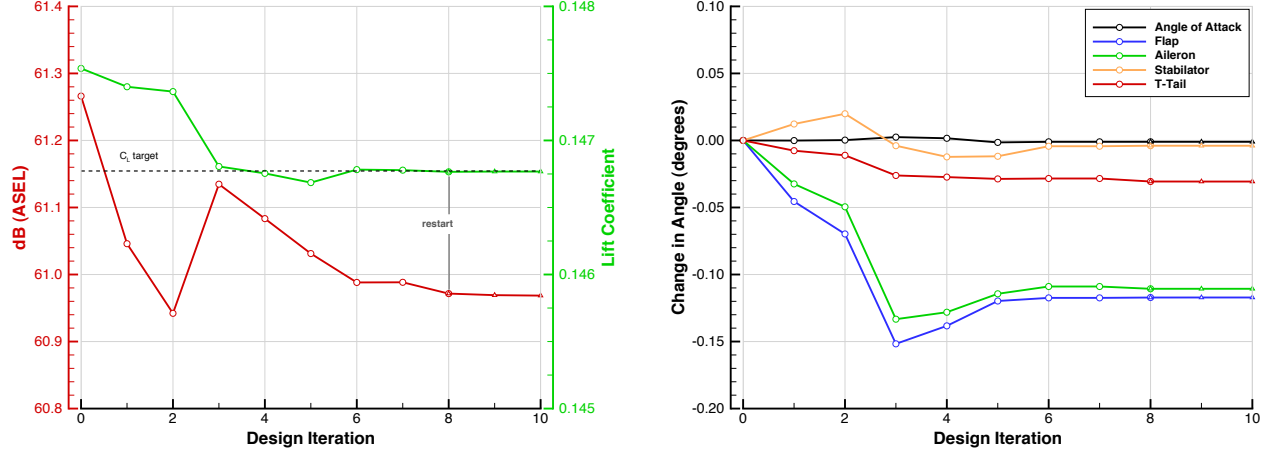


Fig. 4. Optimization history of objective (ASEL) and constraint (lift coefficient) in the left plot with the change in the control surface deflections in the right plot.

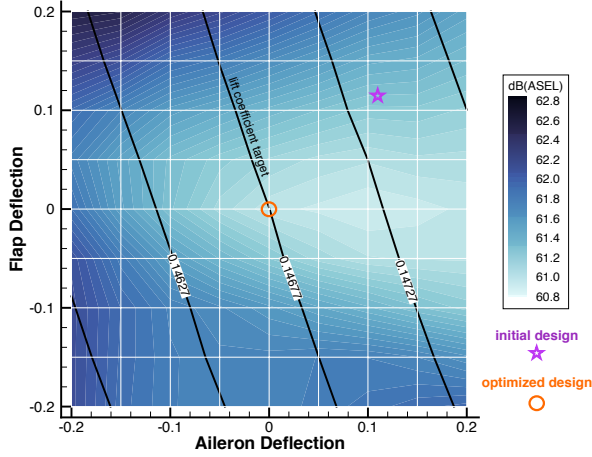


Fig. 5. Flood contours of ASEL with overlaid contour lines of lift coefficient in the flap-aileron design subspace. Note deflections are deviations from the optimized design which is now at the origin.

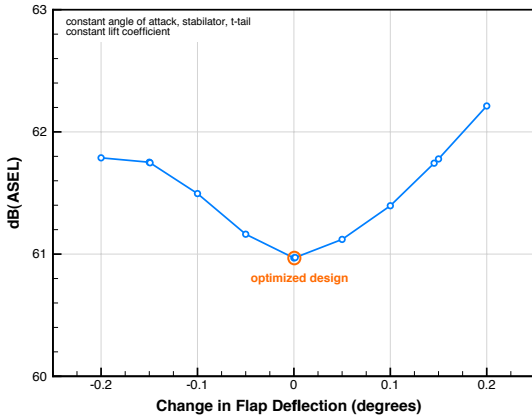


Fig. 6. ASEL variation at constant lift coefficient target in flap-aileron design subspace.

This adapted mesh is then used as guidance for generating all other meshes, thus matching spatial mesh refinement patterns in any new meshes. This process was appropriate for this problem since the lift coefficient is fixed and only small changes in control surface deflections were anticipated. Tiny angle of attack changes were consequently expected and thus no significant changes in the mesh density were necessary.

The results from this optimization are shown in Fig. 4. The objective function (ASEL) is shown to converge to a value about 0.3 dB lower than the initial value. The lift constraint also converges to the target value ($C_{L, cruise}$) designated by the dotted line. The design variable history shows that while no change in control surface deflection was substantial (all under 0.15°), the flaps and ailerons deflected the most.

This relatively simple problem presented the opportunity to conduct a detailed verification of the optimization results. Thusly, the design space centered on the new optimized design was surveyed by running 80 additional analyses (9 by 9 mesh). Since the flaps and ailerons deflected the most from the baseline, the first survey was conducted on the subspace where only the flaps and ailerons vary. The results of this survey are shown in Fig. 5. Note that the new optimized design is located at the origin of this plot.

At first glance, the color contours in Fig. 5 do not seem to indicate that the new design point is indeed an optimum in this subspace. An even better solution appears to exist with just a small increase in aileron deflection (to the right in Fig. 5). However, that evaluation does not account for the lift constraint. Fig. 5 also shows overlaid black contour lines of constant lift. When taking into account that the optimal solution must lie on the proper lift constraint contour line, it becomes clear that the optimum of the problem has been located, at least in this subspace.

This is further established in Fig. 6, where $ASEL$ is plotted along this lift coefficient contour as a function of flap deflection. This plot confirms that the optimized point is indeed a local minimum in the flap-aileron design subspace.

The stabilator is the largest control surface and therefore should have a significant effect on the aircraft sonic boom. Accordingly, design space surveys were also conducted in the flap-stabilator and aileron-stabilator subspaces. These design subspaces are presented in Fig. 7. Plots of $ASEL$ along the cruise lift coefficient contour are presented in Fig. 8. These plots verify that the optimized design point is also a local minimum in the constrained flap-stabilator and aileron-stabilator design subspaces.

To assess the robustness of the design method, the optimization process was repeated with a different initial starting point in the design space. Based on the variations of the design variables as shown in Fig. 4, these repeat applications of the design method were conducted in reduced design spaces. In the first repeat optimization, the stabilator, T-tail, and angle of attack were held fixed to their optimal values. The second repeat run is identical except the angle of attack was allowed to vary. For both problems, the starting values of the aileron and flap deflections were quite different than for the original values as shown in Table 2.

Table 2. Initial deflections of control surfaces for repeat runs in reduced design spaces.

Control Surface	Minimum	Maximum	Initial
Ailerons	-3.0°	3.0°	0.79°
Flaps	-3.0°	3.0°	-1.72°

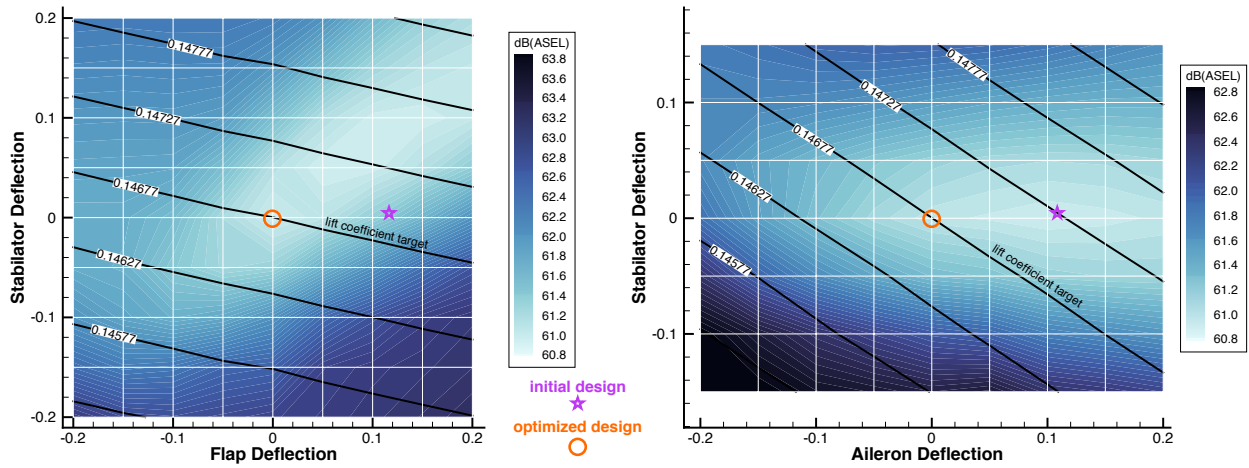


Fig. 7. Flood contours of $ASEL$ with overlaid contour lines of lift coefficient in the flap-stabilator and aileron-stabilator design spaces subspace centered about the optimized design.

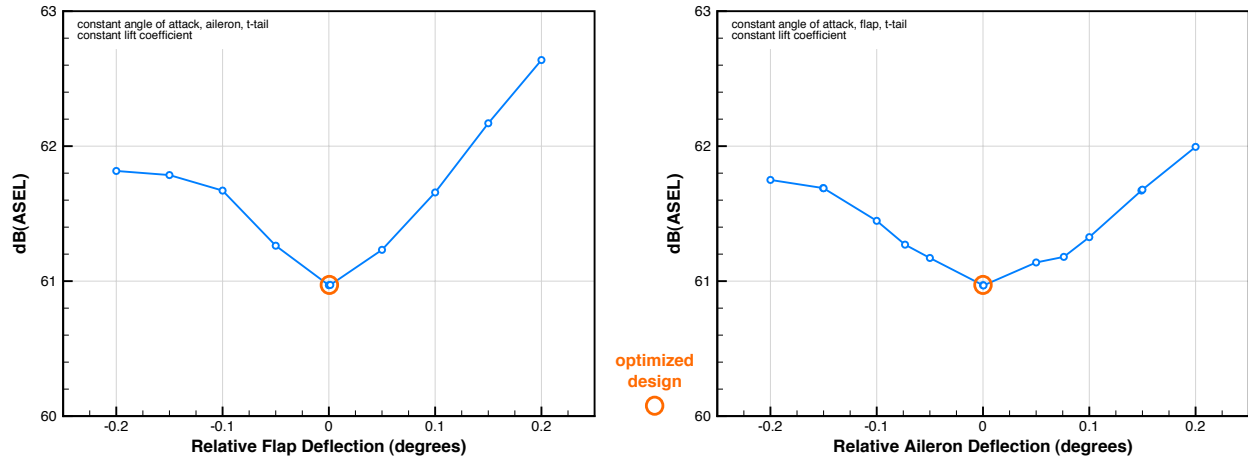


Fig. 8. $ASEL$ variation at constant lift in flap-stabilator and aileron-stabilator design subspaces.

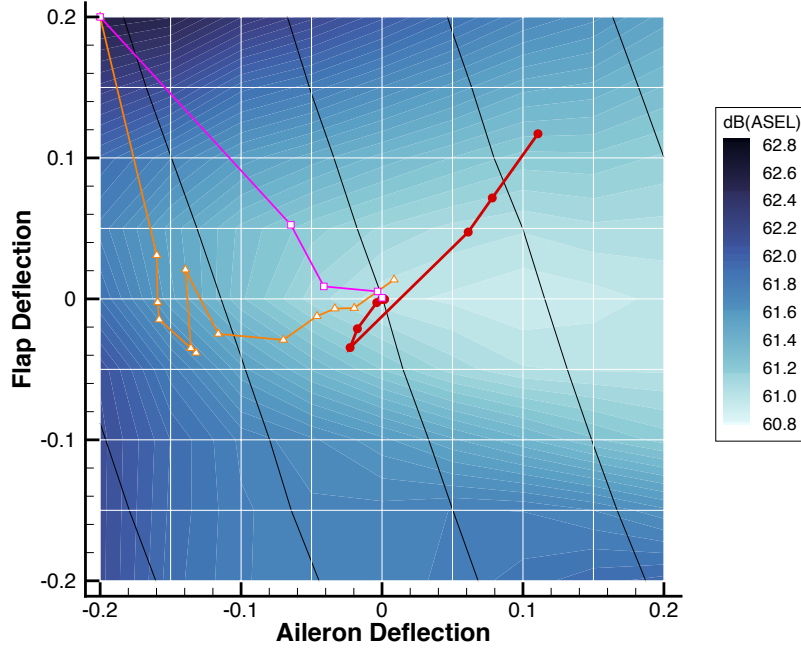


Fig. 9. Optimization trajectories in the aileron-flap subspace. Red curve is original optimization with all variables active. Magenta curve is repeat optimization but only aileron and flap variables active. Orange curve had aileron, flap, and angle of attack variables active.

All optimization runs are mapped onto the aileron-flap subspace and presented in Fig. 9. The red curve is the projected trajectory of the original optimization as shown in Fig. 4. The magenta and orange curves show the trajectory of the repeat optimizations in the reduced design spaces. The magenta curve allowed only flap and aileron deflections to vary, while the orange did the same but added angle of attack as a variable. Note that the magenta and orange trajectories have a different initial starting point than the red trajectory. The orange trajectory reaches the optimum is a less direct path because angle of attack was a variable, though in the end it reached about the same value as all the other trajectories. Also note that all three optimizations terminate in the same small area surrounding the optimum. These results demonstrate the potential robustness of the optimization method.

B. Fixed Lift and Trimmed

The typical X-59 mission will not require fixed altitude in the cruise segment, which means the lift coefficient is not really a hard constraint. Of course, for level flight with a specific payload and fuel load, the dimensional value of lift is determined. Thus, to represent a more realistic design goal, a second optimization problem was set up where the altitude was allowed to float as summarized in Eq. (6):

$$\begin{aligned}
 &\text{minimize:} && ASEL(\alpha, \delta_f, \delta_a, \delta_s, \delta_t, h) \\
 &\text{subject to:} && L(\alpha, \delta_f, \delta_a, \delta_s, \delta_t, h) = L_{cruise} \\
 &&& C_M(\alpha, \delta_f, \delta_a, \delta_s, \delta_t, h) = 0 \\
 &&& \delta_{f,min} \leq \delta_f \leq \delta_{f,max} \\
 &&& \delta_{a,min} \leq \delta_a \leq \delta_{a,max} \\
 &&& \delta_{s,min} \leq \delta_s \leq \delta_{s,max} \\
 &&& \delta_{t,min} \leq \delta_t \leq \delta_{t,max} \\
 &&& h_{min} \leq h \leq h_{max}
 \end{aligned} \tag{6}$$

Note that in addition to the constraint on lift, a pitching moment constraint is enforced to ensure trim. Since the control surface deflections are free to vary, maintaining trim is critical to a realistic design problem.

The versions of the **sBOOM** tool used in this work (**v2.8.5** and **v2.8.6**) do not provide gradients of the noise metrics with respect to altitude. Such a feature is under development, but until it is available, a model of the effects of altitude on ground-level noise has been developed as a surrogate to close the design problem. Fig. 10 presents a plot of ground-level noise (*ASEL*) as a function of altitude for a fixed near-field pressure signature very similar to that produced by the X-59 at cruise. Note the *ASEL* curve is nearly linear. This greatly simplifies the problem of the missing gradient of noise with respect to altitude. For this work, this gradient is assumed to be constant and hence the slope of a line fitted through the curve in Fig. 10. In practice, the altitude provided to **sBOOM** is fixed and then the computed ground-level loudness is adjusted for actual altitude using this constant gradient. This simple model should work for small changes to the design and flight conditions. However, when large changes to the pressure signature are expected, it is preferable to obtain these gradients directly from the atmospheric propagation code.

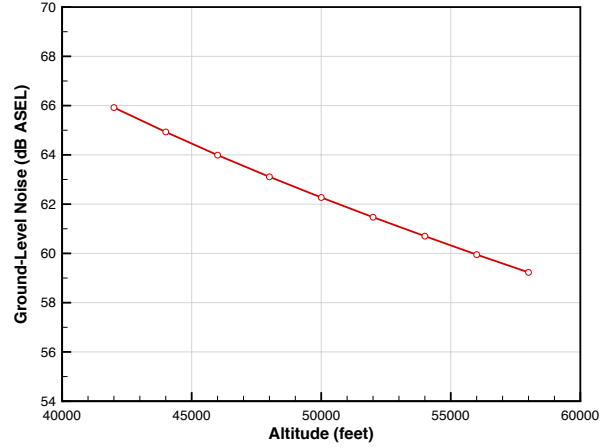


Fig. 10. Variations of ground-level noise metrics as a function of altitude for a fixed near-field pressure signature.

This optimization used the most current revision (C612A) of the X-59 geometry. The starting point for the optimization is what is considered to be the best flight condition by the program and is listed below in Table 3. Bounds for these design variables are also given, including bounds on altitude which stem from the vehicle's flight envelope.

Table 3. Control surface deflection bounds and initial values in design optimization.

Design Variable	Minimum	Maximum	Initial
Ailerons	-3.0°	3.0°	1.1°
Flaps	-3.0°	3.0°	-2.4°
Stabilator	-15.0°	15.0°	3.42°
T-tail	0.0°	10.0°	3.7°
Altitude (feet)	52,000	54,000	53,200

For this problem, output-based mesh adaptation was used for every mesh generated on the full aircraft. The remeshing feature was not appropriate for this problem since angle of attack was expected to vary significantly as altitude changes during the optimization. Body-axis-aligned volume meshes with roughly 23 million cells were used to compute forces and moments. The aircraft forces acted as the functional for adapting these meshes. Stretched, Mach-wave-aligned adapted meshes with about 84 million cells were used to compute the near-field pressure signature three body lengths below the aircraft. The integral of the square of this signature served as the functional as described in Eq. (5). Even with the output-based adaptation, using a series of totally independent and automatically generated meshes has the potential to introduce numerical artifacts into the design space, making this a good test of the optimizer's robustness.

The results of this design optimization are given in Fig. 11. Note the values of *ASEL* are somewhat high compared to typical analysis results for this aircraft because the pressure signature used for computing ground-level noise was truncated (and brought gently to freestream) further upstream than what is typically done. This currently seems to be the best way to mitigate sensitivity corruption from the plume flow behind the airplane. This means the presented results likely do not concur with the most accurate ground-level noise predictions. Nevertheless, the design process worked quite well and produced significant improvement in ground-level noise demonstrating again a most promising capability. The constraints were also essentially satisfied in the final solutions.

Referring back to Fig. 11, note the two non-coincident *ASEL* values at design iteration 9. This is actually a restart of the optimization starting with the same design variable values from the last successful line search of the previous

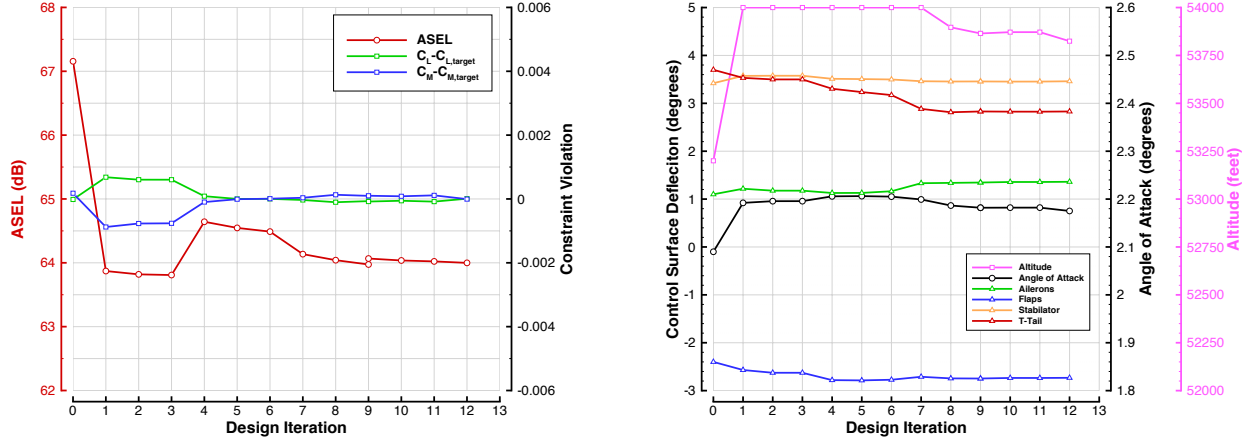


Fig. 11. Optimization history of objective (ASEL) and constraints (lift and pitching moment coefficients) in the left plot for ground-level noise minimization problem. Variation of control surface deflections, angle of attack, and altitude in the right plot.

9 iterations. Even though the same design variable values were used to restart the problem (as indicated by the right plot of Fig. 11), a slightly different value of the objective was computed. This computational artifact likely results from the adapted meshes and possibly computational inaccuracies from the propagation code. This lack of repeatability may pose a major problem for the entire design optimization process. The authors will continue to work to address this problem. Another observation during the set up of this problem is the immense difficulty in computing accurate sensitivities. Computational artifacts from the adapted meshes likely also produce significant inaccuracies in the gradients. The accuracy of the gradients of the loudness metrics from **sBOOM** also remains an open issue that is being addressed in the near term.

To determine if the predicted improvement in ground-level noise is realized with even finer meshes, two additional **Cart3D** solutions were generated on the initial and optimized designs (which both also satisfied the lift and trim constraints). These solutions rely on an experimental feature supporting near-field CFD mesh adaptation driven directly by the ground-level noise metric. In other words, the ground-level noise predicted by **sBOOM** when starting with the near-field pressure signature computed by **Cart3D** actually acted as the functional for adapting the meshes. So far this capability has proven to match noise levels as predicted when using best practices. The results for these solutions are shown in Fig. 12. The results strongly suggest that at least a 2 dB(ASEL) improvement in ground-level noise is indeed realized, despite being optimized on relatively coarse meshes. It will certainly be interesting to see what kind of improvement is possible when the design process is employed without a truncated near-field pressure signature.

The design trajectory and final design in this case are very interesting. In addition to keeping the aircraft trimmed, the optimizer navigated the design space trading off the benefit of weakening the ground signal (through increased altitude) with the potentially increased noise that comes from flying at a higher angle of attack to maintain dimensional lift. It did this while not only maintaining longitudinal trim, but also while coordinating the control surfaces to minimize noise along the way. While computational artifacts in the design space are unquestionably present, and further work remains to be done on the tools, the optimization was clearly very successful despite the problem complexity.

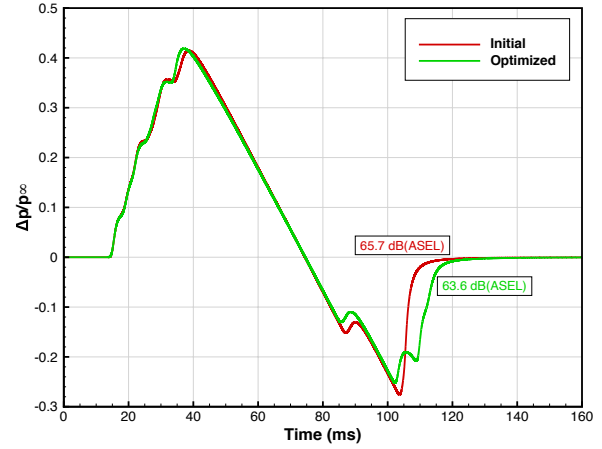


Fig. 12. Ground-level pressure signatures as computed using fine mesh solutions on initial and optimized X-59 configurations.

C. Maximize Noise at Fixed Altitude

The X-59 project presented the authors with an interesting problem in the past year. The flight test program wishes to fly a much louder aircraft over the test range for comparison with the quieter X-59 flights. Rather than fly a second (louder) airplane, the program sought to identify a feasible flight condition that would *maximize* the ground-level noise of the X-59. Limited time was given to solve the problem so a much louder flight condition was manually identified through trial and error and by using basic engineering knowledge. However, as the noise-driven design optimization process became available, it was used to verify that the maximum noise condition had indeed been discovered. The results of this work are presented here.

The manually identified condition for maximum noise is detailed in Table 4. This aircraft configuration and flight condition produces a ground-level noise of 77.9 dB (*ASEL*). To verify this was a maximum noise condition, the design optimization method was applied to the aircraft at fixed altitude and Mach number. All control surface deflections and the angle of attack were allowed to vary.

Table 4. Flight condition and control surface deflection values for maximum ground noise.

Ailerons	4.0°
Flaps	4.0°
Stabilator	0.82°
T-tail	3.7°
Mach number	1.3
Angle of attack	1.02°
Altitude (feet)	43,000

The control surface initial conditions of this optimization are given in Table 5 along with the bounds. Note that they are quite different than the deflections given in Table 4. The objective was to maximize *ASEL* while maintaining trimmed, cruise lift at Mach 1.3 at an altitude of 43,000 feet, which is near the bottom of the operating envelope at this speed. This optimization problem is summarized in Eq. (7):

$$\begin{aligned}
 &\text{maximize:} && ASEL(\alpha, \delta_f, \delta_a, \delta_s, \delta_t) \\
 &\text{subject to:} && C_L(\alpha, \delta_f, \delta_a, \delta_s, \delta_t) = C_{L,cruise} \\
 & && C_M(\alpha, \delta_f, \delta_a, \delta_s, \delta_t) = 0 \\
 & && \delta_{f,min} \leq \delta_f \leq \delta_{f,max} \\
 & && \delta_{a,min} \leq \delta_a \leq \delta_{a,max} \\
 & && \delta_{s,min} \leq \delta_s \leq \delta_{s,max} \\
 & && \delta_{t,min} \leq \delta_t \leq \delta_{t,max}
 \end{aligned} \tag{7}$$

Table 5. Control surface deflection bounds and initial values in design optimization.

Design Variable	Minimum	Maximum	Initial
Ailerons	-3.0°	4.0°	2.0°
Flaps	-3.0°	4.0°	2.0°
Stabilator	-15.0°	15.0°	0.0°
T-tail	0.0°	10.0°	5.0°

For this problem, output-based mesh adaptation was used for every mesh generated on the full aircraft since angle of attack was expected to vary significantly. A body-axis-aligned volume mesh with roughly 22 million cells was used to compute forces and moments. A stretched, Mach-angle-aligned mesh with about 33 million cells was used to compute the near-field pressure signature.

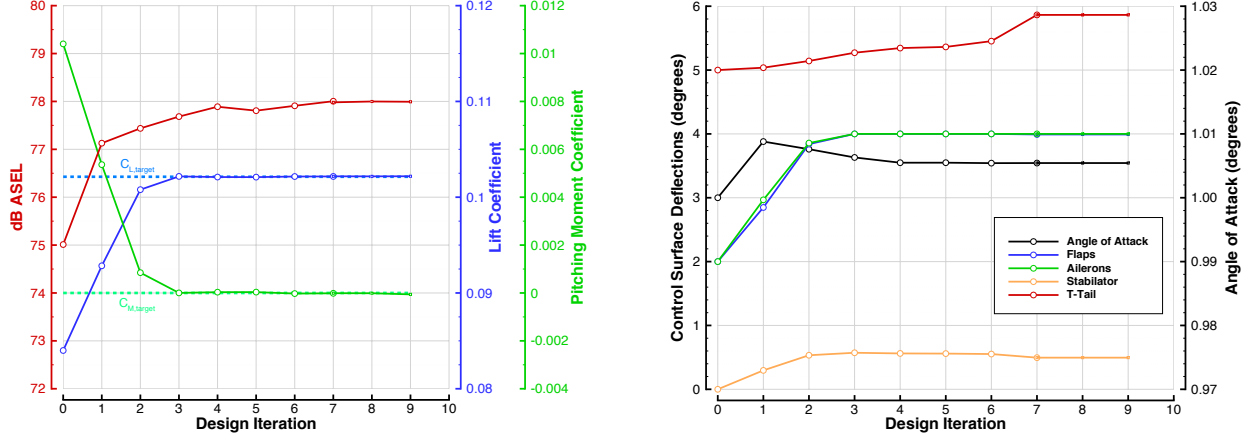


Fig. 13. Optimization history of objective (*ASEL*) and constraints (lift and pitching moment coefficients) in the left plot for ground-level noise maximization problem. Variation of control surface deflections and angle of attack in the right plot.

The results of this optimization are shown in Fig. 13. Note that the final solution is essentially the same as that in Table 4 except for a slight change in the stabilator and T-tail deflections. The optimizer was able to find a slightly higher noise (78.0 dB (*ASEL*)) configuration but mostly verified a maximum noise case had been identified. The flap and aileron deflections hit the maximum bound values. Angle of attack and stabilator values were mostly driven by the lift and moment constraints. The T-tail did deflect a bit more to relieve the stabilator's need to trim the aircraft, but overall the T-tail's influence was very small. Overall, the design optimization was impressive in this case, especially when compared to the previous two cases. The variations in the control surfaces from the starting point were significant; both the flaps and ailerons deflected two more degrees. Angle of attack varied over half a degree. The noise was increased by three full decibels while attaining level trimmed flight. This was definitely a successful application of the design method.

V. Conclusions

A multidisciplinary design methodology has been developed to allow for the direct optimization of ground-level noise of a supersonic aircraft. The method successfully couples a Cartesian Euler flow solver with an atmospheric propagation tool with built-in noise metric calculation. The adjoint solvers of both the flow solver and propagation tool are enabling technologies for this design process. This allows for gradient-based design optimization of the sonic boom noise level of a supersonic aircraft. When using fixed or even very similar flow solver meshes, the method has proven to be robust and repeatable. Design space surveys and repeated runs with different starting points were conducted that confirmed that a local minimum can be found using this process.

The method has been shown to be successful on three design problems that each differ in complexity and even objectives. Despite some apparent issues in terms of repeatability when using adapted meshes for every analysis, the process shows tremendous promise in terms of capability and efficiency, particularly in the design of low-boom aircraft. While the method has so far only been applied to the control surface deflections of an already finalized aircraft design, the process could prove to be extremely useful as early as in the conceptual design phase of a low-boom supersonic aircraft once the known issues impacting robustness have been resolved.

VI. Future Work

The design process presented in this work directly optimizes ground-level noise of a supersonic aircraft and exhibits tremendous promise. In addition, the work presented here has very clearly highlighted deficiencies which most urgently need addressing before this capability can mature from prototype to production capability. The second example problem demonstrated issues with repeatability when the mesh can vary significantly between nearby or even identical functional evaluations. In addition, gradients of the ground-level noise metrics from the propagation tool can be surprisingly inaccurate making navigating the flat design space near an optimum challenging. On the **Cart3D** side of the problem, the authors will continue to experiment and improve the process of computing accurate sensitivities of the near-field pressure signature to changes in the aircraft geometry and flight condition.

Generating more repeatable meshes to which the computed pressure signature is less sensitive will certainly be a goal. Improved convergence of the flow solver and thus the adjoint solver will also be a goal to ensure accurate gradients. In addition, direct adaptation of the near-field mesh to the ground-level noise metric (rather than pressure variation along the near-field sampling) will be researched. These are just a few of the ideas being considered. On the propagation side, mesh convergence of the noise metrics and their gradients are also potentially a source of computational uncertainty. The authors are working closely with the **sBOOM** developers to verify and, if necessary, address this issue. Higher-order methods with improved repeatability, accuracy, sensitivity accuracy, and even computational efficiency also offer an interesting avenue for exploration. While the current design process has proven to be capable of generating very good results, improved gradient accuracy and repeatability are expected to drastically improve the robustness and productivity of this multidisciplinary design optimization tool.

Acknowledgements

This research was supported by the NASA ARMD Commercial Supersonic Technology (CST) Project and by NASA Ames Research Center contract NNA16BD60C. Computing resources supporting this work were provided by the NASA High-End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center. The authors would also like to sincerely thank Sriram Rallabhandi of NASA Langley Research Center for not only providing extensive support with but also continued algorithmic improvements in **sBOOM**. Finally, the authors would also like to recognize that George Anderson performed some of the initial studies of this work.

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