

Practical Aspects of Multiple-Input Design for Aircraft System Identification Flight Tests

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Practical aspects of multiple-input design methods for aircraft system identification flight tests are discussed. The orthogonal optimized multisine multiple-input design technique is explained and demonstrated, along with useful variations. Practical implementation and application issues associated with applying orthogonal optimized multisine inputs in flight are explained, and solutions are demonstrated using flight data from full-scale and subscale aircraft, as well as simulated flight data from an F-16 nonlinear simulation. Topics include truncated and extended excitation inputs, variations for control effectors with limited movement capabilities, aircraft dynamic response to excitation inputs, pilot implementation, statistical uncertainty quantification, multiple-input efficiency and effectiveness, flight testing with active feedback control, frequency-response estimation, control actuator modeling, and control interaction effects. Examples demonstrate that the orthogonal optimized multisine input design method is a practical, versatile, and effective approach for designing multiple excitation inputs for aircraft system identification flight tests.

Nomenclature

a_x, a_y, a_z	=	body-axis translational accelerometer measurements, g
k	=	integer harmonic number
M	=	total number of harmonic frequencies
N	=	number of data points
p, q, r	=	body-axis roll, pitch, and yaw rates, rad/s or deg/s
rms	=	root mean square
$\text{Re}()$	=	real part
RPF	=	relative peak factor
T	=	excitation time length, s
α	=	angle of attack, rad or deg
β	=	sideslip angle, rad or deg
$\delta_r, \delta_e, \delta_{a_l}, \delta_{a_r}$	=	rudder, elevator, left aileron, and right aileron deflections, rad or deg
ϕ	=	phase angle, rad or deg
ω	=	frequency, rad/s
ξ	=	scaled and centered time series

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superscripts

$-$	= mean
$\sim$	= Fourier transform
T	= transpose
†	= complex conjugate transpose

I. Introduction

FLIGHT testing is expensive in both time and money. It is therefore important that flight test data be collected in an efficient manner. Inputs traditionally used for flight test maneuvers to collect modeling data include doublets, impulses (stick raps), multisteps, and frequency sweeps [1-5]. All of these input types are single-input designs, although they are often applied sequentially to different controls one at a time to collect multi-axis data. In contrast, inputs designed to be applied to multiple controls simultaneously are called multiple-input designs.

Three important reasons for using multiple-input designs for aircraft system identification flight testing are: 1) to improve flight test efficiency, 2) to improve excitation of the aircraft dynamics, and 3) to identify control interaction effects. As aircraft are designed and built with more control effectors, often in close proximity to other control effectors and/or lifting surfaces, important interaction effects can arise. A good example is the use of propellers mounted on wings as control effectors for Urban Air Mobility (UAM) vehicles [6,7], which introduces significant control interaction effects between the propellers and conventional aerodynamic control effectors on the wing, such as ailerons and flaps. As the number and proximity of control effectors increases, this issue becomes increasingly important, and the conventional strategy of applying single-input designs sequentially to many control effectors becomes more expensive and less effective.

Previous work on multiple-input design for aircraft system identification mainly involved optimizations that depended on a prior model [8]. The resulting input designs often could not be implemented on a real aircraft because a pilot could not execute them accurately, or because practical implementation constraints were not considered. Consequently, only a few inputs of this type have been applied in flight tests. As an alternative, an effective and practical multiple-input design technique that does not require a prior model was developed, called the orthogonal optimized multisine input design [1,8-10]. This input design combines the efficiency of multiple-input excitation with optimized (minimum) input amplitude excursions, selectable frequency content, and mutual orthogonality in both the time domain and the frequency domain simultaneously. In addition to improved flight test efficiency and effectiveness, this development has enabled new approaches to aircraft system identification flight testing that were previously not possible. The orthogonal optimized multisine input design technique has been successfully applied to stability and control flight testing for hypersonic vehicles [8,10-12], real-time modeling in nominal and unusual flight conditions for experimental fighter aircraft and subscale aircraft [1,9,13-18], simulation validation and flight envelope expansion using real-time modeling results [15], real-time global aerodynamic modeling [17,18], frequency response estimation for rigid and flexible subscale aircraft [19-22], flight envelope protection for a commuter aircraft in icing conditions [23,24], and dynamic modeling for a launch vehicle and spacecraft [25-28].

This work focuses on the orthogonal optimized multisine multiple-input design approach, along with useful variations and information on how this input design can be applied effectively in practice for a wide variety of flight testing. The issues are explained and demonstrated using flight test data from full-scale and subscale aircraft, as well as simulated flight data from a nonlinear simulation. Because the orthogonalized optimized multisine input design is inherently a multiple-input design and does not depend on any prior model, the approach differs from either sequentially applying heuristic single-input designs or optimization based on a prior model [8]. Combining these traits with optimized (minimum) input amplitude excursions, simultaneous orthogonality in the time and frequency domains, extensibility to many inputs, and customizable frequency content using perturbation inputs makes this input design very practical, versatile, and effective.

The next section describes the orthogonal optimized multisine input design technique, including insights and details that are important for practical applications. In Section III, useful variations of the multiple-input design are explained and demonstrated. Metrics and criteria are presented for evaluating flight test data for dynamic modeling prior to carrying out actual modeling calculations are presented in Section IV. This is useful for evaluating input designs prior to flight testing, as well as for evaluating flight data for modeling purposes after the flight data have been obtained but before any modeling begins. Section V is composed of a series of topics related to flight test

implementation and applications of the orthogonal optimized multisine multiple-input design technique. Section VI contains a summary and conclusions.

All of the flight test maneuver design, data analysis, and modeling tasks for this work were done using system identification software written in MATLAB®, called System IDentification Programs for AirCRAFT, or SIDPAC [1,29]. The SIDPAC software toolbox was developed at NASA Langley, and is continually expanded and improved. SIDPAC has been applied successfully to a wide variety of flight and wind tunnel experiments at NASA Langley, and is used at more than 100 institutions worldwide [30].

II. Orthogonal Optimized Multisine Input Design

The general idea for aircraft system identification flight testing is to move the aircraft controls in an uncorrelated way, using perturbation inputs with wideband frequency content encompassing the expected modal frequencies of the aircraft dynamic response, and eliciting aircraft response with sufficient signal-to-noise ratio without inducing a significant departure from the desired flight condition for the flight test. Effective excitations can be implemented by summing designed perturbation inputs with the actuator commands from the pilot and/or guidance/control system, just before the actuator limiting on command rate and position, regardless of whether or not the aircraft has active feedback control. This implementation, which will be discussed in more detail later, is important for achieving the required excitation and decorrelation in the explanatory variables that enables effective characterization of the dependent variables, such as nondimensional aerodynamic forces and moments.

For the orthogonal optimized multisine input design, each designed perturbation input is a sum of sinusoids with unique harmonic frequencies, optimized phase angles, and specified power distribution. The harmonic frequencies are selected to cover a frequency range of interest. The wideband frequency content of the inputs is important because there is naturally significant uncertainty in prior knowledge of the modal frequencies for the aircraft in flight, and wideband inputs provide robustness to that uncertainty. Phase shifts for the sinusoidal components of each input are optimized to achieve low peak-to-peak amplitude for the sum of sinusoids used for each input. Amplitudes of the individual sinusoidal components for each input can be selected to achieve a specific power distribution with frequency.

Multiple inputs are designed to be mutually orthogonal in both the time domain and the frequency domain simultaneously, and to produce high data information content in all axes, while minimizing excursions from the reference flight condition. The mutual orthogonality of the inputs allows simultaneous application of multiple inputs, which reduces the required excitation time for a given amount of input energy, or equivalently, increases the amount of input energy injected into the dynamic system over a given time period. The inputs provide continuous, effective, and efficient multi-axis excitation to collect informative flight data for dynamic modeling.

Each perturbation input $\mathbf{u}$ applied to a particular control effector is a sum of harmonic sinusoids with individual phase shifts ϕ_k ,

$$\mathbf{u} = \sum_{k \in \{1, 2, \dots, M\}} A \sqrt{P_k} \sin\left(\frac{2\pi k \mathbf{t}}{T} + \phi_k\right) \quad (1)$$

where M is the total number of available harmonic frequencies, T is the time length of the excitation, A is the amplitude for the multisine input, P_k is the power fraction for the k th sinusoidal component, and $\mathbf{t}$ is the time vector. Each of the generally multiple inputs is the sum of selected components from the pool of M harmonic sinusoids with frequencies

$$\omega_k = 2\pi k/T \quad k = 1, 2, \dots, M \quad (2)$$

The frequency $\omega_M = 2\pi M/T$ represents the upper limit of the frequency band for the excitation inputs, and $\omega_1 = 2\pi/T$ represents the lower limit. The interval $[\omega_1, \omega_M]$ rad/s specifies the range of frequencies where the aircraft dynamics are expected to lie. Note that these frequency limits are selectable, so that the excitation can be implemented on a wide variety of frequency bands.

A. Frequency Selection

The integers k specifying the harmonic frequencies for each multisine input $\mathbf{u}$ are selected to be unique to that input, but are not necessarily consecutive. A good approach for multiple inputs is to assign consecutive integer harmonic numbers to each input alternately. This is illustrated in Fig. 1 for an example multiple-input design with uniform power spectra. There are 4 inputs in this case: elevator, rudder, left aileron, and right aileron, and a total of 32 frequencies ($M = 32$) with an excitation time period $T = 20$ s in the frequency band $[0.10, 1.65]$ Hz, corresponding to $k = 2, 3, \dots, 33$. The harmonic frequencies were interleaved among the four inputs to achieve wideband frequency content in each input, provide robustness in the excitation, and enable accurate individual control effectiveness estimates.

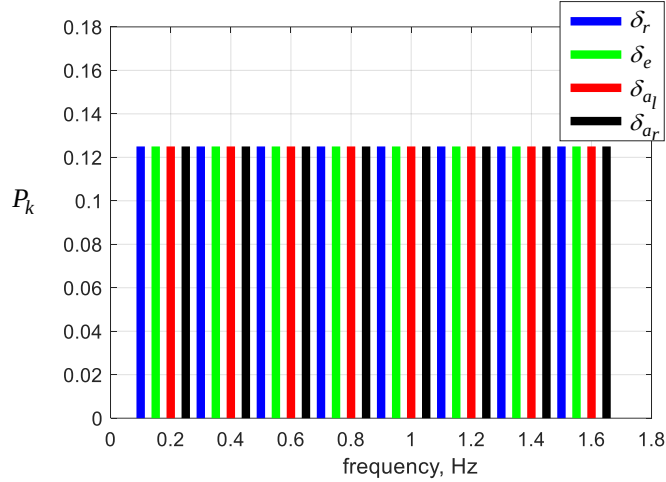


Figure 1. Orthogonal optimized multisine input spectra

Different assignments of the frequency indices could be made for other reasons. For example, lower frequency indices might be assigned to an input for a control effector that is known to effectively excite a low-frequency mode, or particular frequency indices might be omitted to avoid exciting an undesirable structural response or fuel slosh mode, or to accommodate actuator bandwidth limitations, or to simplify the input design. Each frequency index can be assigned to only one of the inputs, to preserve mutual orthogonality of the inputs in both the time and frequency domains, as will be described in the next subsection.

For a given selected frequency band for the excitation, a longer excitation time T will result in more harmonic frequencies with finer

frequency resolution $1/T$ Hz. These harmonic frequencies can be distributed among the existing multiple inputs to enhance their frequency content, or can be used to extend the input design to more inputs. More harmonic frequencies included in any multisine input will make the excitation more rich and varied, and generally improves the data information content [14]. The design trade-offs involve the excitation time T , the number of inputs, and the desired frequency content and frequency resolution for the multisine inputs.

B. Orthogonality

When the harmonic frequency indices k selected for each input $\mathbf{u}_j$ in Eq. (1) are distinct from those chosen for the other inputs, then the frequency content of each $\mathbf{u}_j$ consists of distinct spectral lines in the frequency domain, as can be seen in Fig. 1. Therefore, the vectors of Fourier transforms for the inputs as a function of frequency have inner products equal to zero. In this sense, the inputs are mutually orthogonal in the frequency domain, because each input contains frequencies that no other input has.

In the time domain, a sum of sinusoids is orthogonal to any other sum of sinusoids with harmonically-related frequencies for the excitation time period T , regardless of the constant phase shift of each sinusoidal component. For example, consider two inputs, each containing a single harmonic sinusoidal component with distinct integer harmonic numbers k_1 and k_2 ,

$$\mathbf{u}_1 = \sin\left(\frac{2\pi k_1}{T}t + \phi_1\right) \quad \mathbf{u}_2 = \sin\left(\frac{2\pi k_2}{T}t + \phi_2\right) \quad k_1 \neq k_2 \quad (3)$$

Using the discrete-time notation $t_i = i\Delta t$ and $T = (N-1)\Delta t$, where Δt is a constant time step, the inner product of these inputs is

$$\mathbf{u}_1^T \mathbf{u}_2 = \sum_{i=0}^{N-1} \sin \left[\frac{2\pi k_1}{(N-1)} i + \phi_1 \right] \sin \left[\frac{2\pi k_2}{(N-1)} i + \phi_2 \right] \quad (4)$$

Using the trigonometric identity

$$\sin(u) \sin(v) = \frac{1}{2} [\cos(u-v) - \cos(u+v)] \quad (5)$$

Equation (4) becomes

$$\mathbf{u}_1^T \mathbf{u}_2 = \frac{1}{2} \sum_{i=0}^{N-1} \cos \left[\frac{2\pi(k_1 - k_2)}{(N-1)} i + \phi_1 - \phi_2 \right] - \frac{1}{2} \sum_{i=0}^{N-1} \cos \left[\frac{2\pi(k_1 + k_2)}{(N-1)} i + \phi_1 + \phi_2 \right] = 0 \quad (6)$$

Each summation in Eq. (6) is zero because the sum of a harmonic sinusoid over the base period T is zero,

$\sum_{i=0}^{N-1} \cos \left[\frac{2\pi k}{(N-1)} i + \phi \right] = 0$, for any integer k and any constant phase angle ϕ . This shows that harmonic sinusoids

are orthogonal in the time domain. For more than one sinusoidal component in each input, the analysis is similar, so that multisine inputs assembled according to Eq. (1) are mutually orthogonal in the time domain, regardless of the individual phase angles ϕ_k .

The mutual orthogonality of the inputs in the time domain comes from the fact that each multisine input is a sum of harmonic sinusoids with the same base period T , but with unique harmonic frequencies. Orthogonality in the frequency domain comes from using unique frequencies for the component sinusoids in each multisine input. Both orthogonality properties exist simultaneously for all inputs designed in this way.

Mutually orthogonal inputs have waveforms that mathematically are as unlike each other as they could be. Such inputs are sometimes described as having zero pairwise correlation, a concept that can be quantified in the time domain by the pairwise correlation coefficient. For two time series, such as excitation inputs $\mathbf{u}_1$ and $\mathbf{u}_2$, the pairwise correlation coefficient r is computed as [1]:

$$r = \xi_1^T \xi_2 \quad (7a)$$

$$\xi_j \equiv (\mathbf{u}_j - \bar{\mathbf{u}}_j) / \sqrt{(\mathbf{u}_j - \bar{\mathbf{u}}_j)^T (\mathbf{u}_j - \bar{\mathbf{u}}_j)} \quad (7b)$$

The pairwise correlation coefficient quantifies the similarity of two time series with their mean values and scaling removed, which is equivalent to comparing the normalized waveforms. Two time series with the same normalized waveform are correlated and have pairwise correlation $r=1.0$. If the normalized waveforms are the same except for a sign change, then $r=-1.0$. A correlation coefficient $r=0.0$ means that the two time series are orthogonal, or completely uncorrelated mathematically.

Uncorrelated excitation inputs are necessary both for accurate estimation of each individual control effectiveness and for flight test efficiency that results from the capability to apply multiple inputs simultaneously. Excitation inputs with low correlations also indirectly reduce correlations with and among other explanatory variables, such as angle of attack, sideslip angle, and body-axis angular rates.

Correlations among more than two time series can be evaluated using a correlation matrix of inner products of multiple normalized times series computed according to Eqs. (7). For a matrix of normalized column vector time series $\mathbf{X}^* \equiv [\xi_1 \ \xi_2 \ \dots \ \xi_n]$, the correlation matrix is

$$\mathbf{X}^{*T} \mathbf{X}^* = \begin{bmatrix} 1 & r_{12} & r_{13} & \dots & r_{1n} \\ r_{12} & 1 & r_{23} & \dots & r_{2n} \\ r_{13} & r_{23} & 1 & \dots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{1n} & r_{2n} & r_{3n} & \dots & 1 \end{bmatrix} \quad (8)$$

where r_{12} denotes the pairwise correlation coefficient for ξ_1 and ξ_2 , and similarly for the others. Pairwise correlation coefficients are commutative according to Eq. (7a), so that the correlation matrix is symmetric. Because models typically include a bias term and each model term is multiplied by a parameter to quantify the influence of that term, normalized waveforms for the model terms contain the relevant correlation information for the input design [1].

Multiple correlation metrics that can be computed from the correlation matrix include the rank, condition number, variance inflation factors, variance decomposition proportions, eigenvalues, eigenvectors, singular values, and orthogonalization constants [1]. In most practical cases, examining pairwise correlations is sufficient, but a thorough analysis would include checks for multiple correlations as well.

Correlation metrics and analysis should be applied more generally to all explanatory variables, e.g., including angle of attack, sideslip angle, Mach number, and angular rates, as well as the excitation inputs. However, doing so involves the use of a prior model or simulation, which will be in error to some extent. Consequently, for evaluating a multiple-input design, input correlations should be the focus, and the goal is to make the excitation inputs as uncorrelated as possible and preferably mutually orthogonal. Adverse effects on modeling results due to high correlations among the explanatory variables are discussed in detail in Ref. [1].

In the frequency domain, the pairwise correlation coefficient for Fourier transform data at selected analysis frequencies [1,16] is computed using a variation of Eqs. (7) to account for both the real and imaginary parts of the complex frequency-domain data,

$$r = \text{Re} \left(\tilde{\xi}_1^* \tilde{\xi}_2 \right) \quad (9a)$$

$$\tilde{\xi}_j \equiv \tilde{\mathbf{u}}_j / \sqrt{\text{Re} \left(\tilde{\mathbf{u}}_j^* \tilde{\mathbf{u}}_j \right)} \quad (9b)$$

Pairwise correlation coefficients in the frequency domain and the associated correlation matrix are interpreted in the same way as in the time domain when the frequency-domain modeling is done using Fourier transform data [1,16]. No mean values are removed from the frequency-domain data because the time-domain data are detrended prior to applying the Fourier transform [1,16]. Note that when the frequency-domain data for two inputs $\mathbf{u}_1$ and $\mathbf{u}_2$ are nonzero only at different frequencies, then the vector inner products over the analysis frequencies in Eq. (9a) are zero, and the inputs are orthogonal in the frequency domain.

Orthogonal excitation inputs improve modeling accuracy because control effectiveness estimates are more accurate as the controls are moved in more uncorrelated ways, and because orthogonal inputs help to decorrelate the other explanatory variables, such as angle of attack, sideslip angle, and the body-axis angular rates. Mutual orthogonality of multiple inputs in both the time and frequency domains simultaneously, which is a characteristic of the orthogonal optimized input design, represents the best that can be done in terms of decorrelating the excitation inputs.

Orthogonal inputs improve flight test efficiency because all inputs can be applied simultaneously throughout the maneuver, as opposed to applying a single-input design sequentially to one control at a time. Note that single-input designs applied sequentially to each control are orthogonal because only one input is moved at a time, whereas the orthogonal optimized multisine input design achieves input orthogonality mathematically, so that all excitation inputs can be applied at the same time. Furthermore, because the orthogonal optimized multisine input design moves all controls simultaneously, but in different ways, rich data information is produced for identifying coupled dynamics and control interaction effects. In contrast, applying a single-input design sequentially to one control at a time provides little information on coupled dynamics and no information on control interaction effects, rendering any control interaction effects unobservable and unidentifiable. For the orthogonal optimized multisine input design, the practical advantages of uncorrelated inputs remain intact regardless of whether the modeling is done in the time domain or in the frequency domain.

C. Phase Optimization

The previous subsection showed that multisine inputs with unique harmonic frequencies are mutually orthogonal regardless of the phase angles ϕ_k . This means the phase angles are free parameters that can be chosen without affecting the mutual orthogonality of the inputs. If the phase angles ϕ_k in Eq. (1) were chosen at random on the interval $(-\pi, \pi]$ rad, then in general, the various harmonic components would add together at some points to produce a multisine input $\mathbf{u}_j$ with relatively large amplitude excursions. This is undesirable because such inputs can move the aircraft too far from the reference flight condition for the test. To prevent this, the phase angles ϕ_k for the harmonic components are optimized to minimize the relative peak factor RPF, defined by

$$\text{RPF}(\mathbf{u}) = \frac{[\max(\mathbf{u}) - \min(\mathbf{u})]/2}{\sqrt{2(\mathbf{u}^T \mathbf{u})/N}} = \frac{[\max(\mathbf{u}) - \min(\mathbf{u})]}{2\sqrt{2} \text{rms}(\mathbf{u})} \quad (10)$$

Relative peak factor is a measure of the efficiency of an input for dynamic modeling purposes, and is computed as the amplitude range of the input divided by a measure of the input energy. Low values of relative peak factor are desirable for efficient and effective dynamic modeling because the objective is to excite the aircraft with good input energy over a variety of frequencies while minimizing the input amplitudes in the time domain, to avoid driving the aircraft too far away from the reference flight condition.

For each multisine input $\mathbf{u}$ in Eq. (1), minimum RPF is achieved by adjusting the phase angles ϕ_k for the individual sinusoidal components of the input. The resulting optimization problem is non-convex; however, a simplex algorithm can be applied to find a solution [1,8-10,31]. The RPF is 1 for a single sinusoid with unit amplitude, regardless of frequency or phase shift. Consequently, the target minimum RPF for a multisine input is 1.

After the phase angles ϕ_k are optimized for minimum RPF, the start and end points of the multisine input will generally not be zero. However, each multisine input must be periodic with period T , because all of its component harmonic sinusoids have the same base period T . For each multisine input, a search along the time axis can be done to find a constant time offset so that the input begins at zero amplitude, as required for a perturbation input, which also makes the input zero at time T . This is equivalent to sliding the multisine input along the time axis until a zero crossing is placed at the origin of the time axis. The appropriate phase shift is added to each component phase angle ϕ_k determined previously from the RPF optimization. Note that to implement a constant time shift for a multisine input, the phase offset for each sinusoidal component will be different, because each has a different frequency. To slide the multisine input by τ seconds, the phase angle change $\Delta\phi_k$ in radians for each component frequency ω_k is

$$\Delta\phi_k = \frac{\tau}{\omega_k} = \frac{\tau T}{2\pi k} \quad (11)$$

If all component phase angles ϕ_k of each multisine input are shifted so that the initial value of the input is zero, then the final value of the multisine input at time T will also be zero. The resulting perturbation input can then be added to whatever value the control may have for another reason, e.g., trim or maneuvering. The mutual orthogonality and relative peak factors of the multisine inputs are unaffected by these phase angle shifts.

Figures 2a-2d show the process for an example 20-second multisine input composed of four harmonic sinusoid components with frequency indices 3, 6, 9, and 12, uniform power distribution (all P_k in Eq. (1) equal to $1/4$), and unit amplitude $A=1$. Figure 2a shows the four component sinusoids, and Fig. 2b shows the result of adding those component sinusoids with all phase angles set equal to zero. The RPF for this input is high at 1.62, and the input shows undesirable large peaks. Figure 2c shows the result of optimizing the sinusoidal component phase angles for minimum RPF, and Fig. 2d shows the same input with the phase angles adjusted to achieve a perturbation input. The RPF of the optimized multisine is 1.01, which is a significant improvement compared to using zero phase angles, and is close to the RPF target value of 1 associated with a single sinusoid. A similar process is applied for each individual multisine input to minimize the RPF and convert the resulting time series to a perturbation input.

Even with phase shifts applied, orthogonal optimized multisines are balanced perturbations about zero amplitude, and have zero mean value. This is important for staying near the nominal flight trajectory or reference

flight condition while the perturbation inputs are applied, and contrasts with unbalanced single inputs such as multistep 3-2-1-1 or 2-1-1 inputs and most piloted frequency sweeps.

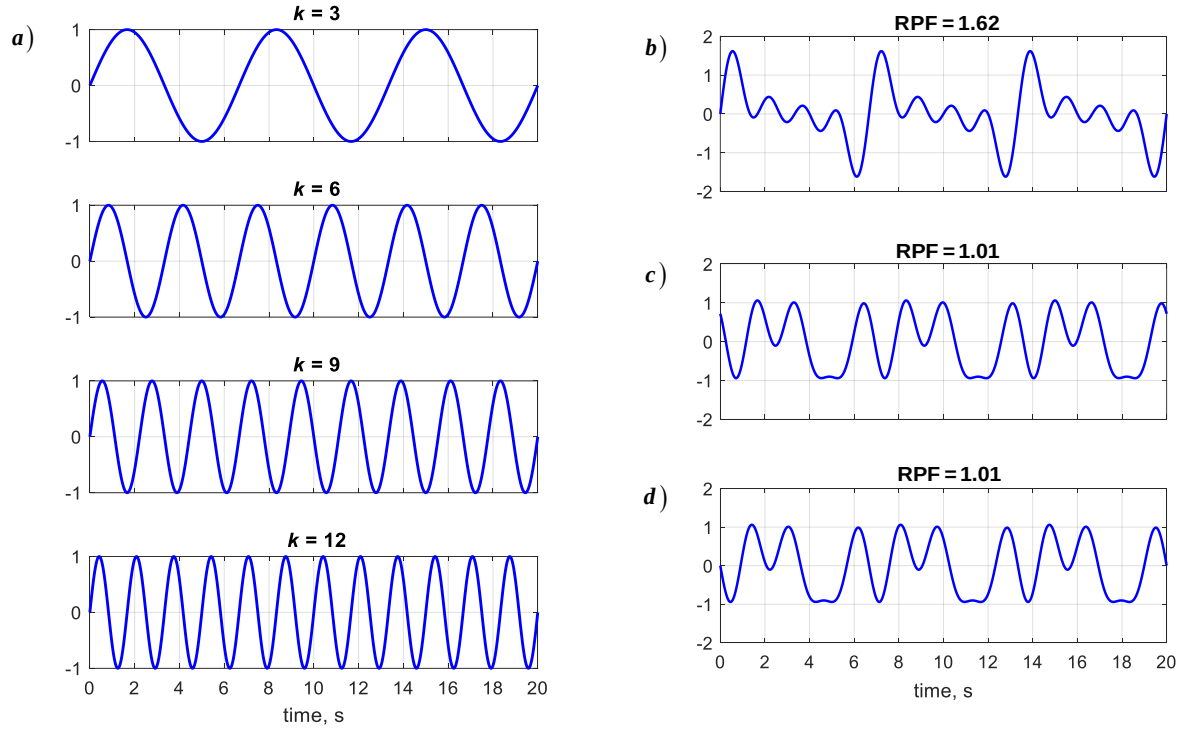


Figure 2. Multisine phase optimization: *a*) sinusoidal components; *b*) multisine with zero phase angles; *c*) multisine with phase angles optimized for minimum RPF; *d*) phase-optimized multisine with phase shifts to achieve a perturbation input

D. Power Spectra

The sinusoidal components in Eq. (1) can be assigned arbitrary fractions of the total power in the multisine input by choosing the power fractions P_k , independently of the multisine amplitude A . The power fractions for the component sinusoids in the multisine input must sum to 1,

$$\sum_k P_k = 1 \quad (12a)$$

To achieve a uniform power distribution, P_k are selected as

$$P_k = \frac{1}{n} \quad (12b)$$

where n is the number of sinusoidal components included in the summation of Eq. (1) for $\mathbf{u}$. Each input $\mathbf{u}$ can have arbitrary amplitude A , subject to practical flight test and modeling considerations, such as load limits and collecting data appropriate for a linear model structure, among others. Power fractions for the individual sinusoidal components of a multisine input can be changed to concentrate input excitation power at frequencies where dynamic modes are likely or expected, or to avoid exciting unwanted dynamic response such as fuel slosh or structural modes, or to improve dynamic excitation by assigning frequency content according to capabilities of individual control effectors or actuators. In the absence of such knowledge, the best approach is to implement uniform power distribution according to Eq. (12b). The multisine input amplitude and power distribution across the harmonic components can be specified independently using A and P_k , respectively, and neither has any effect on the mutual orthogonality of the designed inputs. The amplitude selection is simply a scaling of the input waveform design, and therefore has no effect on the input design process. The optimal phase angles depend on the selected power spectra,

which means that the phase optimization must follow the selection of the input power spectra. However, regardless of the selected power spectra, the input design proceeds in the same way, producing different phase optimization solutions for different power spectra.

Although there are methods for optimizing the frequency spectra of inputs for parameter estimation, all of them require some knowledge of the aircraft dynamics, usually in the form of a prior dynamic model. For most flight tests, there is questionable value to such methods, because of significant uncertainty associated with preflight predictions of the aircraft dynamics. Under these circumstances, it is good practice to define the frequency spectra for all inputs to be relatively flat across the selected frequency band, so that the aircraft dynamics will be sufficiently excited regardless of where the modal frequencies are located within the frequency band. However, the multisine input power fractions can be modified as desired, e.g. based on prior information or previous flight test results, subject to the power fraction constraint in Eq. (12a) for each individual multisine input.

E. Example 20-Second, 4-Input Design

Figure 3 shows time series for orthogonal optimized multisine inputs designed using the frequency content depicted in Fig. 1. Table 1 contains the information needed to assemble the inputs using Eq. (1). This is a 20-second, 4-input design with wideband frequency content and phase angles optimized for minimum RPF and adjusted to implement perturbations from zero.

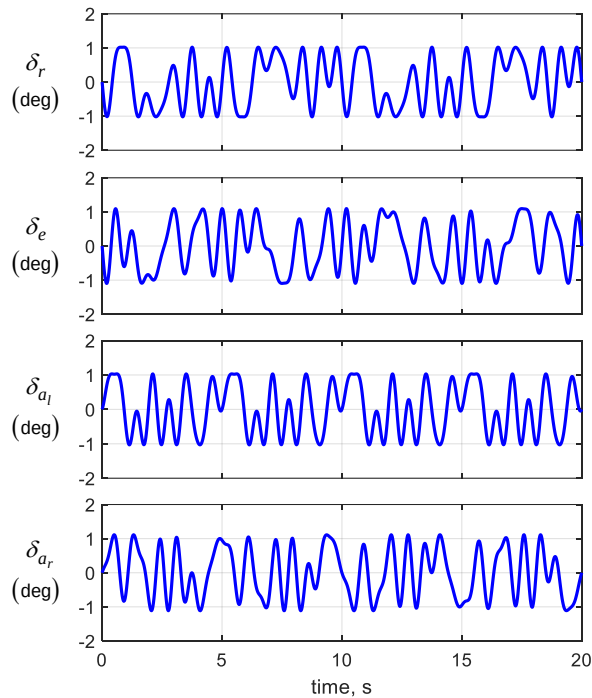


Figure 3. Orthogonal optimized multisine inputs

Table 1 Orthogonal optimized multisine input design, $T = 20$ s

Control Effector	A (deg)	P_k	k	ϕ_k (rad)	RPF
δ_r	1.0	0.125	2	2.6402	1.02
		0.125	6	0.7017	
		0.125	10	-1.7211	
		0.125	14	-2.5863	
		0.125	18	-2.6017	
		0.125	22	1.2960	
		0.125	26	2.7835	
		0.125	30	-2.7187	
δ_e	1.0	0.125	3	-2.9563	1.10
		0.125	7	0.3267	
		0.125	11	-1.4788	
		0.125	15	1.1339	
		0.125	19	2.9157	
		0.125	23	-2.3217	
		0.125	27	3.0634	
		0.125	31	2.7503	
δ_{a_l}	1.0	0.125	4	1.4582	1.03
		0.125	8	0.9160	
		0.125	12	0.3831	
		0.125	16	-2.6551	
		0.125	20	-1.9295	
		0.125	24	-1.2881	
		0.125	28	1.7400	
		0.125	32	-0.8976	
δ_{a_r}	1.0	0.125	5	-0.4304	1.12
		0.125	9	-0.3606	
		0.125	13	1.0070	
		0.125	17	0.4070	
		0.125	21	-1.3131	
		0.125	25	2.6196	
		0.125	29	-1.8821	
		0.125	33	1.2515	

The inner product of any two of the time series shown in Fig. 3 is zero, indicating that these perturbation inputs are mutually orthogonal (completely uncorrelated mathematically) in the time domain. The unique frequencies assigned to each input, shown by the interleaved bars in Fig. 1 and the unique harmonic frequency indices in Table 1, make the inputs also mutually orthogonal in the frequency domain.

Because the orthogonal optimized multisine inputs are sums of harmonic sinusoids with a common base period T , they are periodic for the excitation period T , so that the inputs can be applied repeatedly without any discontinuities in magnitude or slope. The inputs have minimum RPF and various frequencies and phase angles, which keeps the aircraft response close to the nominal flight trajectory or reference flight condition, with dynamic response similar to flight in light to moderate turbulence. The aircraft response also stays near the nominal flight trajectory or reference flight condition because each multisine perturbation input is a sum of harmonic sinusoids, which are all balanced about zero (equal area above and below zero), regardless of the phase angles. The result is rich multi-axis dynamic response without moving the aircraft away from the nominal flight trajectory or reference flight condition.

Orthogonal optimized multisine inputs can be designed in a straightforward fashion using the `mkmsswp.m` program in SIDPAC. This program requires the excitation time T , the number of inputs to be designed, the assignments of the frequency indices to each input (which implicitly defines the excitation frequency band), and the selected power fractions for the frequencies included in each input, then conducts the phase angle optimization for minimum RPF and adjusts the phase angles to achieve perturbation inputs.

A summary of the multiple-input design procedure for orthogonal optimized multisines is as follows:

- 1) Select the time period T for the excitation, which determines the harmonic frequency resolution $\Delta f = 1/T$ Hz. Sometimes the choice of T is dictated by practical considerations such as length of time available at a particular flight condition, flight safety limits, or test range constraints.
- 2) Select the frequency band $[f_{min}, f_{max}]$ Hz for the excitation. This corresponds to the frequency band where the dynamic response of the system is expected. The input frequencies are equally spaced by Δf on the interval $[f_{min}, f_{max}]$. The total number of frequencies is $M = \text{fix}[(f_{max} - f_{min})/\Delta f] + 1$, where $\text{fix}[\cdot]$ indicates rounding to the nearest integer toward zero. Often the minimum frequency is chosen as $f_{min} \geq 2/T$ Hz. This ensures that at least two complete cycles at each frequency are observed in the data.
- 3) Assign indices k from the set $\{1, 2, \dots, M\}$ to each input by alternating each consecutive frequency among the multiple inputs. This approach produces lower relative peak factors for the individual inputs, and also ensures that each input has unique frequency content distributed evenly across the frequency band $[f_{min}, f_{max}]$. This frequency assignment strategy can be modified as necessary, for a variety of purposes, with the constraint that any frequency can only be included in one multisine input.
- 4) Choose the input power fraction by specifying P_k for each harmonic component in each multisine input, subject to the constraint of Eq. (12a), or use Eq. (12b) to implement a uniform power distribution.
- 5) Optimize the ϕ_k for each multisine input to achieve minimum relative peak factor for that input. The simplex optimization technique works well and does not require gradients. Starting values for the phase angles ϕ_k can be set at random from a uniform distribution on the interval $(-\pi, \pi]$ rad.
- 6) For each input, conduct a one-dimensional search along the time axis to find a constant time offset τ so that the input begins and ends at zero amplitude. This is equivalent to sliding the input along the time axis until a zero crossing is placed at the origin of the time axis. The appropriate phase shift is added to each harmonic component phase angle ϕ_k using Eq. (11). The power spectrum, relative peak factor, and input orthogonality are unaffected by these phase shifts.

Steps 5) and 6) can be carried out using SIDPAC program `mkmsswp.m`. Because the phase angle optimization for minimum relative peak factor is a non-convex optimization problem, sometimes improved relative peak factor values can be obtained by repeating steps 5 and 6, which repeats the optimization with different random starting values for the phase angles ϕ_k .

III. Useful Variations

Because of the wideband frequency content of orthogonal optimized multisine inputs, it is possible to use the same multiple-input design for different flight conditions and even for different aircraft, with only changes in the input amplitudes A to account for changes in nominal dynamic pressure and/or control effectiveness. This generally simplifies the excitation strategy and reduces flight computer memory requirements.

Subsets of the inputs from a multiple-input design can be applied without modification, because the members of any subset of an orthogonal optimized multiple-input design are still mutually orthogonal. However, that will result in some gaps in the frequency band coverage, because the frequency content associated with the omitted inputs will be absent from the excitation. A better approach is to design for the required number of inputs from the start, so that the frequency range can be covered as completely as possible, which results in the most robust and effective excitation. Along that line, a single input can be generated using the same design approach, by simply including all of the available harmonic frequencies in a single input design. This results in a single input with high-density frequency content.

A. Frequency Content Modification

For a given excitation frequency band, a longer excitation time T will provide more available harmonic frequencies (higher value of M) by providing a finer frequency resolution, as shown in Eq. (2). These additional frequencies can be applied to enrich the frequency content of each input or to include more inputs in the design.

The time length of an orthogonal optimized input design can be increased or decreased without affecting the orthogonality properties or the relative peak factors, by simply recomputing each input from Eq. (1) using the same optimized phase angles ϕ_k and amplitudes $A\sqrt{P_k}$, but a different value of T and corresponding time vector t . This is equivalent to stretching or shrinking all of the harmonic components. Of course, when T is changed, the frequency content of the multisine input changes.

Figure 4 shows the effect of changing the excitation time T for the elevator input design in Fig. 3 and Table 1, starting with the original value of $T = 20$ s, and changing to the values $T = 25$ s and $T = 16$ s. Frequency spectra are shown in the plots on the right for the time series shown in the corresponding plots on the left. As T is increased, the frequencies in the multisine input shift lower with tighter frequency spacing. Conversely, as T is decreased, the frequencies shift higher with wider frequency spacing. Each of the inputs shown in Fig. 4 has RPF=1.10.

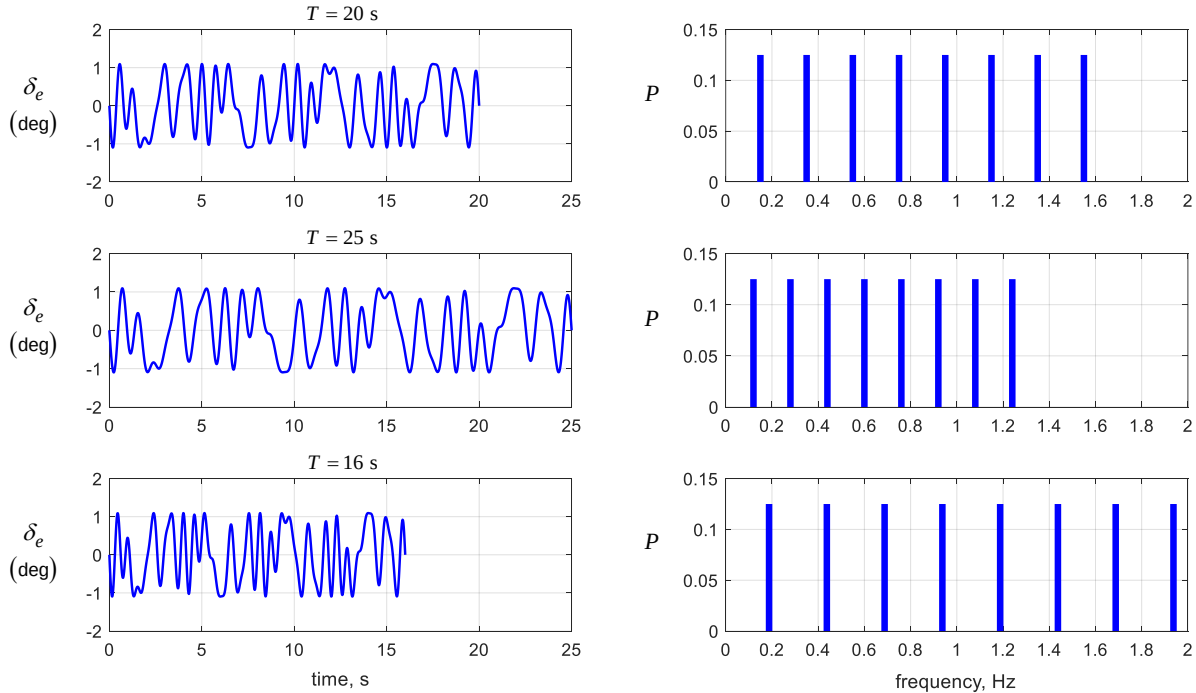


Figure 4. Multisine input with various excitation time T

Changes to the excitation time T could be made for individual multisine inputs in a multiple-input design, but all of the multisine inputs must have the same value of T to be mutually orthogonal in the time domain. Mutual orthogonality in the frequency domain remains intact even if some inputs have different T than others, as long as the component frequencies are different for each input. As noted earlier, any change in the input power spectra implemented by P_k requires a new phase angle optimization for minimum RPF and new phase angle adjustments to implement a perturbation input.

B. Linear Combinations of Mutually Orthogonal Inputs

In Fig. 3 and Table 1, separate inputs with unique frequencies were designed for the left and right ailerons. This allows the modeling to be done considering either the effects of the left and right aileron controls individually, or any linear combination of those controls, such as the traditional asymmetric aileron deflection defined by $\delta_a = (\delta_{a_r} - \delta_{a_l})/2$. This is possible because any linear combination of mutually orthogonal multisine inputs is also orthogonal to all of the other orthogonal optimized multisine inputs, due to the unique harmonic frequency assignments and because Eq. (1) is a sum of harmonic sinusoids. In this example, the δ_a multisine input would have the combined frequency content of both the δ_{a_r} and δ_{a_l} multisine inputs.

A good practice is to design and apply an orthogonal optimized multisine input for every individual control, so that the modeling can be done with either individual controls or any linear combinations of the individual controls. This can be useful in situations where the linear control mixing is not yet finalized or might be changed over the course of the flight testing. On the other hand, if the control mixing is known and fixed, then an orthogonal optimized input should be designed and applied to each control as it will be used, to achieve the best modeling accuracy by testing in the same manner as the control will be used. However, in that case, the ability to identify individual control effectiveness for controls that are moved proportionally will be forfeited.

C. Simple Ways to Vary the Excitation

Because the input amplitude A has no effect on the orthogonality or phase optimization of orthogonal optimized multisine inputs, it is possible to change the sign of a particular input in order to vary the excitation. This variation can be very useful for collecting data to evaluate model prediction accuracy. Another simple variation is to reassign excitation inputs to different control effectors, with adjustments made for the amplitudes appropriate for each control effector. These two variations can also be used in combination, and require no updates to the input design or phase optimization other than amplitude adjustments. All of these simple changes provide rich and varied excitation very efficiently, which can improve confidence in the models identified from the data.

D. Amplitude Selection

The amplitudes of orthogonal optimized multisine inputs are design parameters that must be selected. This task is difficult because control effectiveness and the aircraft dynamic response as a function of input frequency content are not well known prior to flight testing. Acquiring accurate knowledge of these characteristics is in fact the purpose of stability and control flight testing.

A good practical approach is to design the inputs with uniform power spectra, set the amplitudes very low initially, then increase the amplitudes slowly with repeated maneuvers. This is a conservative buildup approach, and is possible because only the input amplitudes need to be modified, not the input waveforms which embody the wideband frequency content, phase optimality, and mutual orthogonality. Real-time modeling approaches [1,9,13-20,32,33] can be used to adjust input amplitudes, but it is also acceptable to simply monitor signal-to-noise ratios of the aircraft dynamic responses, which can be done visually from real-time plots of measured response data or with simple high-pass filtering or data smoothing [1,32]. Excellent modeling results can be obtained with signal-to-noise ratios for the measured responses equal to at least 10, with 20 as a good target value. Input amplitudes can be adjusted to induce large response amplitudes for investigating nonlinearities and/or unsteady aerodynamic effects, or to keep response amplitudes relatively small for linear modeling or practical reasons.

Individual input amplitudes can be modified one at a time without affecting the input orthogonality or phase angle optimizations, but in practice it is usually possible to slowly increase all of the input amplitudes simultaneously. In general, this approach is more practical and simpler than attempting to modify the input power spectra to concentrate input excitation power closer to the natural frequencies of the dynamic system, which typically requires information from a prior model or from real-time modeling results.

E. Phase Angles for Many Harmonic Components

For orthogonal optimized multisine input designs with many harmonic components (e.g., more than 50) for each input, the non-convex phase optimization problem can be very difficult and time-consuming to solve. This situation can occur for long excitation times T and/or when the frequency range for the excitation is large. In such cases, because the RPF calculation in Eq. (10) is simple and fast, the phase angles can be determined by simply trying random phase angles repeatedly to find a good solution with low (but not optimized) RPF. After a low RPF is achieved, the phase angles can be adjusted using Eq. (11) to achieve perturbation inputs, as described earlier.

F. Controls Moved with Step Changes

Some controls are typically commanded using step changes, such as flaps with discrete settings, speed brakes, and the throttle or speed control for propulsion. Other controls on the aircraft, such as aerodynamic control surfaces, can be moved in a continuous fashion. Mutually orthogonal excitations for both types of control can be designed by combining orthogonal optimized multisines with square-wave analogs of harmonic sinusoids, which might be called harmonic square waves. Harmonic square wave inputs can be made orthogonal to each other, and to orthogonal optimized multisine inputs, to generate an orthogonal multiple-input design that can be applied to both types of control simultaneously.

Figure 5 demonstrates the idea for harmonic sinusoids and corresponding harmonic square waves with the same base period and analogous frequency indicated by the harmonic number k , which is the number of cycles over the input time length $T=20$ s. The plots on the left side of Fig. 5 show harmonic sinusoids with phase angles set to 0, and the plots on the right show equivalent harmonic square waves, generated as simply the sign of the values in the corresponding harmonic sinusoid time series shown on the left.

Harmonic square waves have the same mutual orthogonality properties as harmonic sine functions, but only if the ratio of their harmonic numbers is not equal to an odd integer. The reason for this can be seen by examining the harmonic square wave plots on the right side of Fig. 5. For harmonic square waves with harmonic number k equal to 2 and 4 (harmonic number ratio = 2), the inner product is 0, because an equal number of positive and negative pulses occur for the $k=4$ input during the time of either positive or negative pulses for the $k=2$ input. Conversely, for the $k=6$ input, an unequal number of positive and negative pulses occur during the time of either positive or negative pulses for the $k=2$ input (harmonic number ratio = 3), so that the inner product of the $k=2$ and $k=6$ harmonic square wave inputs is not 0. This limitation does not apply to the mutual orthogonality of harmonic sinusoids, but does apply to any pairwise correlation between two harmonic square waves or between a harmonic square wave and a harmonic sinusoid. All of these properties are unaffected by the phase angles selected for either the harmonic sinusoids or the harmonic square waves.

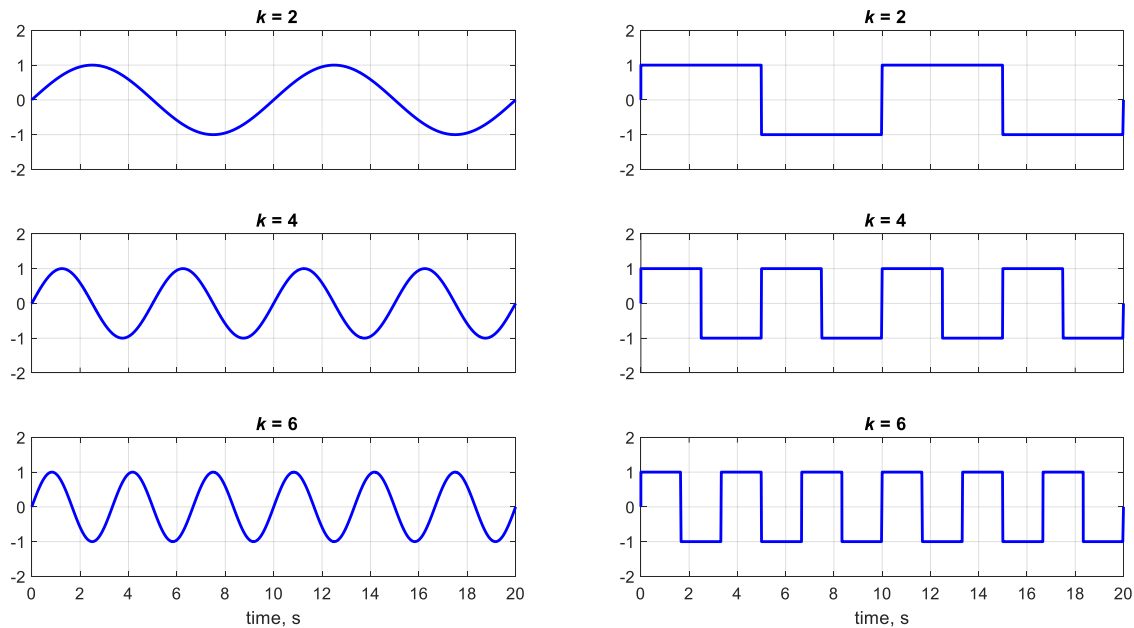


Figure 5. Harmonic sinusoids and corresponding harmonic square waves

Harmonic square waves can be applied in a manner similar to harmonic sinusoids, except for the limitation that the ratio of the harmonic number for a harmonic square wave to any other harmonic number used in the input design cannot equal an odd integer. Note that harmonic square waves are related to Walsh functions, which are mutually orthogonal harmonic square waves with specific harmonic numbers and phase angles equal to 0 deg or ± 90 deg. Multi-harmonic square waves can be combined in a single input, similar to multisine inputs assembled according to Eq. (1), with the same limitation on the harmonic numbers that can be used in the multi-harmonic square wave inputs. The restriction on the harmonic numbers of multi-harmonic square wave inputs can be violated and still achieve a multiple-input design with low pairwise correlations, but the inputs will not be mutually orthogonal.

An important practical variation is that harmonic or multi-harmonic square waves can be shifted by a constant bias and scaled without impacting the time-domain orthogonality properties. This can be understood from the calculation of the pairwise correlation coefficient in Eqs. (9a)-(9b), which removes both the bias and scaling from each input time series. Consequently, the same orthogonality properties apply if the harmonic or multi-harmonic square wave ranges between 0 and 1, corresponding to an on/off control or a control with only discrete positive or negative settings. As with harmonic sinusoids, scaling harmonic square wave inputs has no effect on orthogonality in the time domain.

In the frequency domain, leakage from harmonic square waves spoils the input orthogonality [1,16,31], but the inputs will still have low correlations as long as the harmonic numbers for each input are different and no ratio of the harmonic numbers is an odd integer.

One approach for designing multiple inputs that are a mix of harmonic sinusoids and harmonic square waves (because of different control types) is to design multiple orthogonal optimized multisine inputs in the manner described earlier, but making sure that the harmonic numbers for the inputs that will become square waves are not odd multiples of any of the other harmonic numbers used in the input design. Then the orthogonal square wave inputs can be obtained from

$$\mathbf{u}_{sw} = \text{Asign}(\mathbf{u}) \quad (13)$$

where $\mathbf{u}_{sw}$ is the square wave equivalent of an orthogonal optimized multisine input $\mathbf{u}$ from Eq. (1). If more than one amplitude is of interest for the harmonic square wave excitation, which might happen, for example, because of discrete flap settings, then the same maneuver can be repeated with simply a change in the harmonic square wave amplitude and/or offset from zero. Nothing else would need to be changed for the multiple-input design, and the mutual orthogonality of the inputs would not be affected.

Harmonic square waves have abrupt transitions that can be troublesome for aerodynamic control surfaces, because the abrupt changes can result in control surface rate limiting and/or unsteady aerodynamic effects. In these cases, it is better to use multisine inputs for the excitation. Otherwise, if the input amplitude transition rates must be limited, the resulting input will have ramp transitions between amplitudes and nonzero correlations with the other excitation inputs.

V. Flight Test Implementation and Applications

This section explains important flight test implementation issues for multiple-input flight testing using orthogonal optimized multisine inputs, and discusses practical flight test applications.

A. Truncated and Extended Inputs

The time-domain orthogonality of the multiple-input design described earlier applies at times that are integer multiples of the excitation time length T . In practice, it is not always possible for the flight test maneuver length to be exactly an integer multiple of the excitation time T , because of practical issues such as test range operational limitations or inability to maintain a flight condition. Furthermore, if the inputs are applied for real-time modeling or during time-varying flight conditions, then the applicable parts of the excitation input time series will usually be different than an integer multiple of the excitation time T used to design the inputs. Consequently, it is of interest to investigate multiple-input pairwise correlations as a function of time.

Using the orthogonal optimized multisine input design example described earlier, Fig. 6 shows how pairwise correlation coefficients for the inputs vary from $t=0$ s to $t=T=20$ s. All inputs are mutually orthogonal with $r=0$ at $t=T=20$ s. Absolute values of all pairwise correlation coefficients are less than 0.4 after approximately 5 s, and less than 0.2 after 8 s.

Inputs with only even integer harmonics (u_2 and u_4) are also orthogonal at the halfway point $t = T/2 = 10$ s, and similarly for inputs with only odd integer harmonics (u_1 and u_3). This can be understood by recognizing that the harmonic frequencies for the inputs with only even integer harmonics for $T = 20$ s could also be considered harmonic frequencies for an excitation time $T = 10$ s. For example, the first 10 s of an input designed for $T = 20$ s with harmonic numbers 2, 4, 6 is equivalent to an input with harmonic numbers 1, 2, 3 for $T = 10$ s. Inputs with only odd harmonic numbers for $T = 20$ s have harmonic numbers that are not integers for $T = 10$ s, but those harmonic numbers still differ by an integer, which makes the inputs also orthogonal at $t = 10$ s, cf. Eq. (6). In general, two multisine inputs will be orthogonal at any time when their equivalent harmonic numbers differ by an integer, by Eq. (6). For example, if one multisine input has harmonic numbers 4, 12, 20 and a second input has harmonic numbers 8, 16, 24 for $T = 20$ s, then the inputs will be orthogonal at times 20 s, 10 s, and 5 s, because their equivalent harmonic numbers at those times differ by an integer.

If an input designed for an excitation time T is continued, the correlations remain very low (absolute values less than approximately 0.1), and return to all zero (mutual orthogonality) at integer multiples of T , as shown in Fig. 7 for inputs designed using excitation time $T = 20$ s. Although inputs with absolute pairwise correlations below 0.9 can be used to obtain adequate modeling results [1], in general, lower absolute pairwise correlations for the inputs lead to more accurate modeling results.

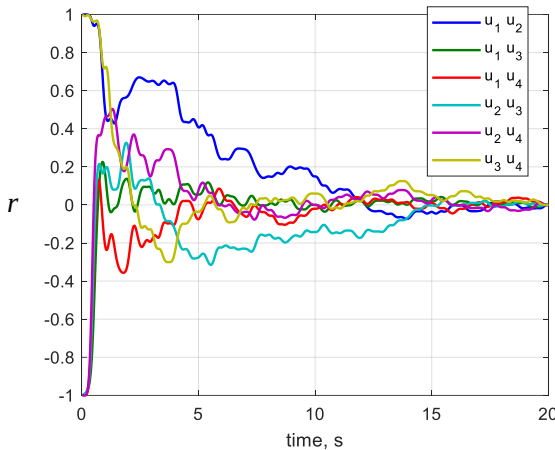


Figure 6. Orthogonal optimized input correlation coefficients for excitation time $T = 20$ s

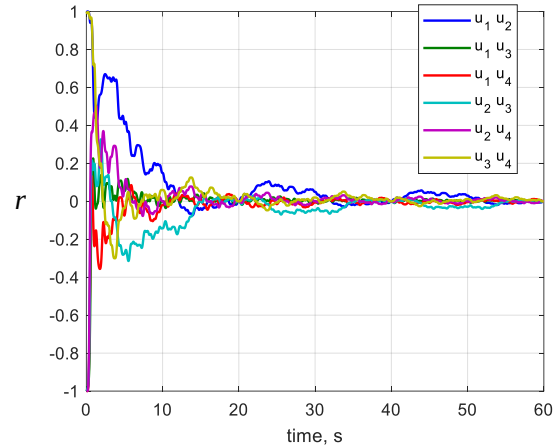


Figure 7. Orthogonal optimized input correlation coefficients for excitation time $T = 20$ s for continued applications of the periodic inputs

Inputs designed for a shorter excitation time T will have low correlations at an earlier time than a truncated input designed for a longer excitation time. The trade-off is that an input designed for a shorter excitation time T will have coarser frequency resolution and fewer available harmonic frequencies in a given frequency range. In general, inputs with richer frequency content in a given frequency range produce better modeling results [1,14]. As explained earlier, input frequency content for the orthogonal multisine input design can be altered by changing the excitation time T , changing how the available frequencies are assigned to each multisine input, or changing the assignments of excitation inputs to control effectors.

In the frequency domain, the orthogonality of the inputs has a similar character, in that orthogonality only occurs at multiples of the excitation time T . At other times, leakage causes the pairwise correlation coefficients for the inputs to be nonzero, but progressively lower in absolute value as time increases, similar to the time-domain case illustrated in Figs. 6 and 7.

B. Controls Moved Slowly or Infrequently

Some controls are typically moved relatively slowly and/or infrequently, for example, wing flap deflections or engine power levels. For these controls, any input waveform that is sufficiently different (has low pairwise correlation) relative to the orthogonal optimized multisine inputs applied to the other controls will result in an effective multiple-input maneuver design. This concept was applied during flight testing of the T-2 subscale aircraft [34-36] shown in Fig. 8, by applying a slow ramp input to the (relatively slow-moving) flaps while the other

(relatively fast) aerodynamic control surfaces were moved using orthogonal optimized multisine inputs [15]. Although the ramp input on the flaps was not orthogonal to the multisine inputs, the ramp input was sufficiently different from the multisine inputs that the flap effects on the body-axis nondimensional longitudinal aerodynamic forces were estimated from the flight data with standard errors less than 5 percent of the estimated value [15].



Figure 8. T-2 subscale jet transport aircraft
Credit: NASA Langley Research Center

C. Dynamic Response

Aircraft dynamic response to orthogonal optimized multisine excitation inputs is similar to flying in light to moderate turbulence. Figure 9 shows body-axis accelerometer outputs when orthogonal optimized multisine input excitations are applied to an F-16 nonlinear simulation [1] at a flight condition of 5 deg angle of attack and 10,000 ft

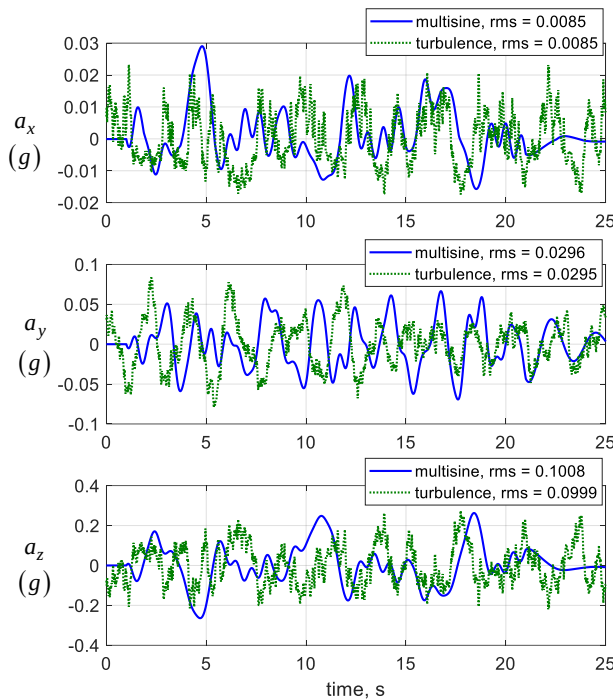


Figure 9. F-16 accelerometer outputs for orthogonal optimized multisine input excitations compared to flight in light-to-moderate von Karman turbulence

altitude. Amplitudes for the orthogonal optimized multisine inputs were set at 1 deg for the stabilator and ailerons, and 2 deg for the rudder, with all excitation inputs applied simultaneously for 20 s, starting at $t=1$ s. These input amplitudes were known to be sufficient for accurate dynamic modeling results. Sensor noise was turned off in the nonlinear simulation to make the comparison valid and clear.

The plots in Fig. 9 also show accelerometer outputs at the same flight condition for the F-16 flying through turbulence with von Karman power spectra implemented using the technique described in Ref. [37]. For the maneuver flown in turbulence, no control excitations were applied, and the rms body-axis turbulence velocity components were selected as $\sigma_u = 8$ ft/s, $\sigma_v = 5$ ft/s, and $\sigma_w = 8$ ft/s, to match the rms accelerometer outputs obtained using the orthogonal optimized multisine inputs. According to MIL-F-8785C [38], these values for the body-axis turbulence velocity components correspond to light-to-moderate turbulence at this flight condition. This demonstrates that aircraft response to orthogonal optimized multisine inputs is similar to the dynamic response for flight in light-to-moderate turbulence. Because the orthogonal optimized multisine inputs are composed of sinusoidal components that are

balanced about zero, but generally out of phase with each other, the aircraft stays on its nominal trajectory with dynamic loading similar to flight in light-to-moderate turbulence, while the dynamic response produces highly informative multi-axis data with low correlations among the explanatory variables.

D. Uncorrelated Pilot Inputs (UPI)

The typical implementation of orthogonal optimized multisine excitation inputs is to use an automated system to sum the inputs with control commands from the pilot, guidance system, and/or feedback control system, just before the limiters on the command rates and positions. This arrangement provides the most effective flight test capability and is required when there is active feedback control, as will be discussed later. When there is no automated system to implement orthogonal optimized multisine inputs, a pilot can approximate the inputs by implementing uncorrelated pilot inputs (UPI) [32]. This generally applies when there is no active feedback control, because typical

feedback control correlates the control deflections and the aircraft states, no matter what inputs the pilot implements with stick and rudder pedals.

To implement UPI, the pilot applies uncorrelated 3-axis (roll, pitch, yaw) inputs with varying amplitudes and phasing, while also varying the frequency of the inputs and covering a desired range of control surface deflections. This was demonstrated successfully by several test pilots during flight testing of the Aermacchi MB-326M Impala jet aircraft at the National Test Pilot School in Mojave, CA. Figure 10 shows the Impala jet aircraft. The aircraft elevator, aileron, and rudder control surface deflections were commanded directly by pilot stick and rudder pedal inputs, with no feedback control. In most cases, the test pilots flew UPI maneuvers very well after receiving only a briefing on the ground explaining UPI and how to execute them in flight.



Figure 10. Aermacchi MB-326M Impala jet aircraft

Credit: NASA Langley Research Center

Cross plots of the control surface deflections during a 27-second flight test maneuver flown on the Impala aircraft using UPI are shown in Fig. 11. The pilot moved the controls in an uncorrelated fashion over the desired range of control surface deflections. Pairwise correlations for the control surface deflections were less than 0.2 in absolute value, as indicated on the plots. Although these inputs are not orthogonal (pairwise correlations are not equal to zero), the pairwise correlations achieved by the pilot were very low and easily low enough for excellent local and global modeling results.

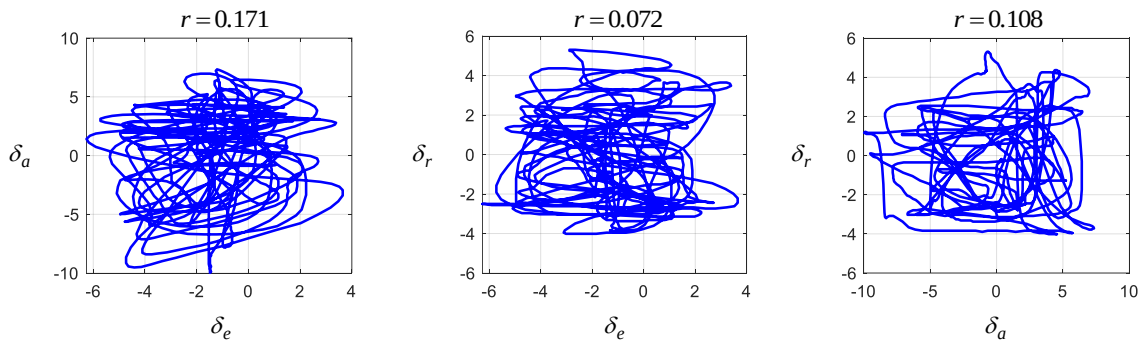


Figure 11. Cross plots of Impala control surface deflection data for uncorrelated pilot inputs

Figure 12 shows that the pairwise correlation coefficients for the inputs during the UPI maneuver were similar to those shown in Fig. 6 for automated orthogonal optimized multisine inputs. Three or four inputs are probably the practical limit for a pilot implementing UPI, whereas an automated system can implement an arbitrary number of excitation inputs accurately and simultaneously. The pilot also made occasional random step changes to the power level during the UPI excitations to collect informative data for modeling propulsion effects. These changes in the power level were made in a manner uncorrelated with the UPI, similar to the approach taken with the flaps on the T-2 aircraft.

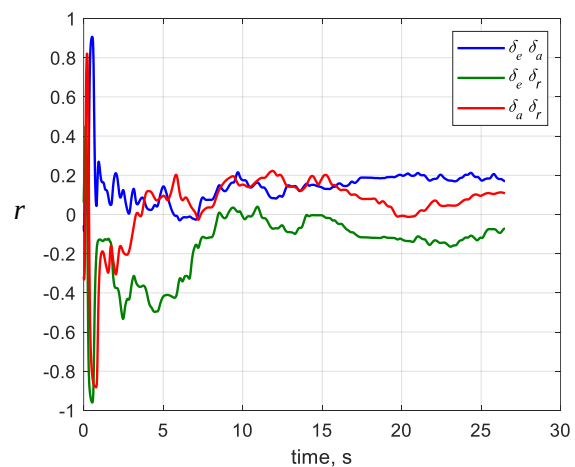


Figure 12. Uncorrelated pilot input correlation coefficients for a 27 s maneuver

E. Statistical Uncertainty Quantification

When excitation inputs are applied by the flight computer, statistical modeling uncertainty can be determined based on modeling results from repeated maneuvers. This approach practically eliminates variation from differences in the inputs for each maneuver, which would occur if a pilot were implementing the excitation inputs. Furthermore, the flight data can be collected very efficiently by flying the aircraft to the desired flight test condition, then repeatedly activating the automated excitation inputs.

This idea was applied during flight testing of the subscale T-2 aircraft to investigate the uncertainty associated with modeling results computed from flight test maneuvers flown at 5 deg nominal angle of attack. Figure 13 shows the excellent repeatability for the angular rate response to multi-axis excitation using automated orthogonal optimized multisine inputs applied repeatedly at the same flight condition. Random scatter in model parameter estimates from repeated maneuvers can be compared to statistical model parameter uncertainties computed from individual maneuvers. This quantifies the uncertainty in the experimental apparatus interpreted broadly as the entire flight test operation, and validates statistical model parameter uncertainty metrics calculated from individual maneuvers [1]. The validation is important, because in practice, it is not economically feasible to fly repeated maneuvers at every flight condition to quantify the statistical model parameter uncertainty. The effective and repeatable multi-axis excitation from orthogonal optimized multisine inputs is critical for collecting this important flight data properly and efficiently.

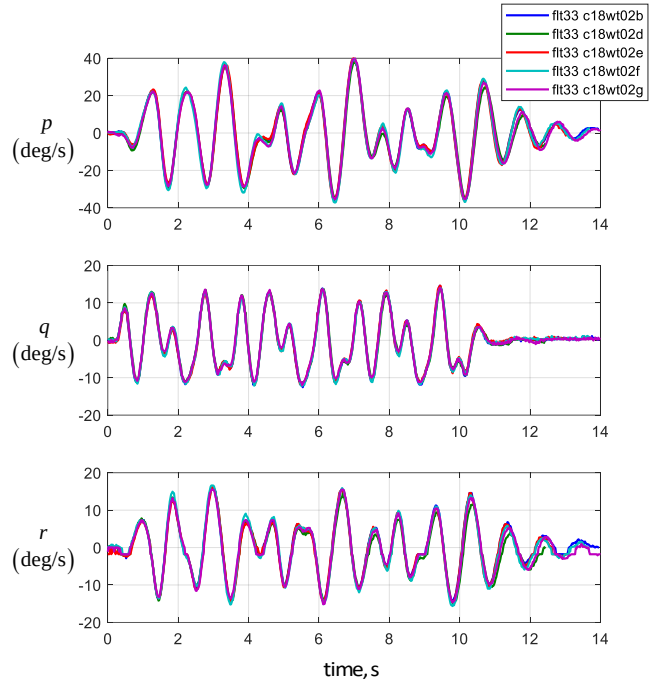


Figure 13. Flight data from repeated maneuvers flown on the T-2 subscale jet transport aircraft

F. Multiple-Input Efficiency and Effectiveness

Orthogonal optimized multisine inputs can be applied to achieve efficient and effective multi-axis excitation in many flight test situations, including stability and control flight testing in normal or unusual flight conditions, time-varying flight conditions, and flight conditions that cannot be maintained for very long. This multiple-input design has been applied successfully for local and global real-time modeling, including both parametric modeling and nonparametric frequency response estimation [1,8-28].

In terms of flight test time, applying simultaneous multiple inputs is more efficient than applying single inputs sequentially, by a factor of the number of inputs. Furthermore, the orthogonal optimized multisine input design moves all inputs actively using many frequencies in parallel throughout the excitation time, whereas single inputs move only one input at a time, and typically move sequentially from low frequency to high frequency, as in frequency sweep inputs and 3-2-1-1 multi-step inputs [1-5]. The parallel application of diverse excitation frequencies on multiple inputs simultaneously in the orthogonal optimized multisine input design results in more diverse and effective excitation of the aircraft dynamics, which further improves flight test efficiency and effectiveness [1,8-28]. The concept is illustrated in Fig. 14, which shows time-frequency plots of the Fourier transform magnitude (the square root of input energy) for a frequency sweep input on the left, and for an orthogonal optimized multisine input with the same amplitude and covering the same frequency range on the right.

Figure 14 shows increased input energy and more complete coverage in time and frequency for the orthogonal optimized multisine input in Fig. 14b, compared to the frequency sweep input in Fig. 14a. These characteristics are directly related to the effectiveness and efficiency of the excitation. Input energy for the multisine is applied over all frequencies in parallel continuously throughout the maneuver, whereas input energy is applied sequentially in time and frequency, and only during limited time periods for the frequency sweep. The differences become more pronounced as the number of inputs increases. Furthermore, multiple orthogonal optimized multisine inputs can be applied simultaneously to different control effectors, whereas frequency sweep inputs are applied sequentially to one

control effector at a time. These characteristics of the orthogonal optimized multisine input design are ideal for flight test situations requiring highly efficient injection of input energy on multiple control effectors in an uncorrelated fashion, which is required for real-time modeling, efficient flight testing, and flight testing in time-varying flight conditions, or flight conditions that cannot be maintained for a long time.

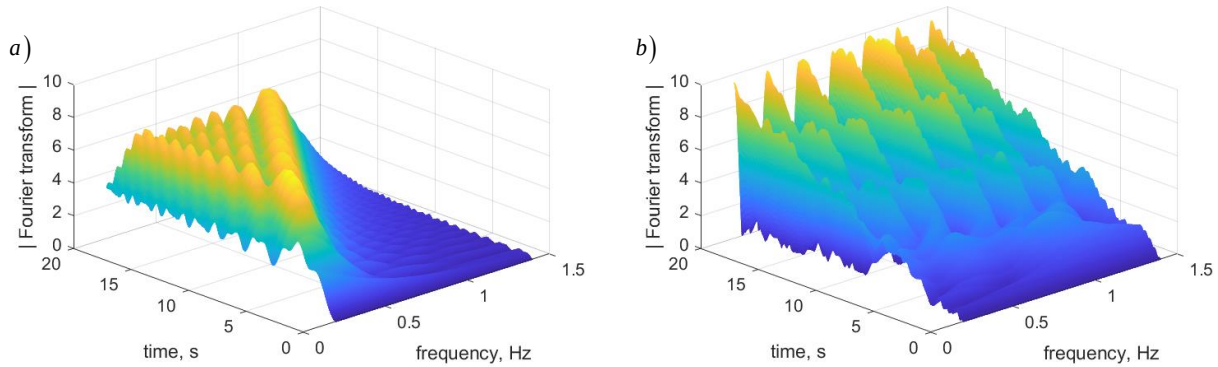


Figure 14. Fourier transform magnitudes: a) frequency sweep, b) orthogonal optimized multisine

Considering the overall flight test program, there is overhead involved in getting an aircraft to a specific flight condition in preparation for a flight test maneuver. Single-input designs require more of this by roughly a factor of the number inputs, compared to a multiple simultaneous input design like the orthogonal optimized multisine input design. In some practical flight test situations, there simply is not enough time at the desired flight condition to apply sequential single-input excitations. Some examples are flight at high nominal angles of attack, unpowered drop tests, subscale aircraft tests within visual range of a remote-control pilot on the ground, and flight tests with rapidly-changing flight conditions, such as hypersonic flight tests.

The efficient multi-axis excitation provided by orthogonal optimized multisine inputs means that highly informative flight data can be collected over a wide range of flight conditions in a single maneuver. This idea was demonstrated during T-2 flight testing, where the pilot flew the aircraft slowly through a range of nominal angle of attack while orthogonal optimized multisine inputs were applied to the control surfaces simultaneously and autonomously by the flight computer.

Figure 15 shows flight data from a single maneuver of this type, where the pilot flew the aircraft through nearly the entire range of nominal angle of attack with the multi-axis excitation inputs active. Figure 15a shows that the multi-axis excitations were continued through stall and recovery, with informative data collected throughout the maneuver. The plot in Fig. 15b shows rich but not excessive variation in the sideslip angle data, and the same applies for other flight variables. The result is highly-efficient collection of informative flight data covering a wide swath through the multidimensional space of relevant flight data, including control surface deflections, body-axis angular rates, translational accelerations, angle of attack, and sideslip angle.

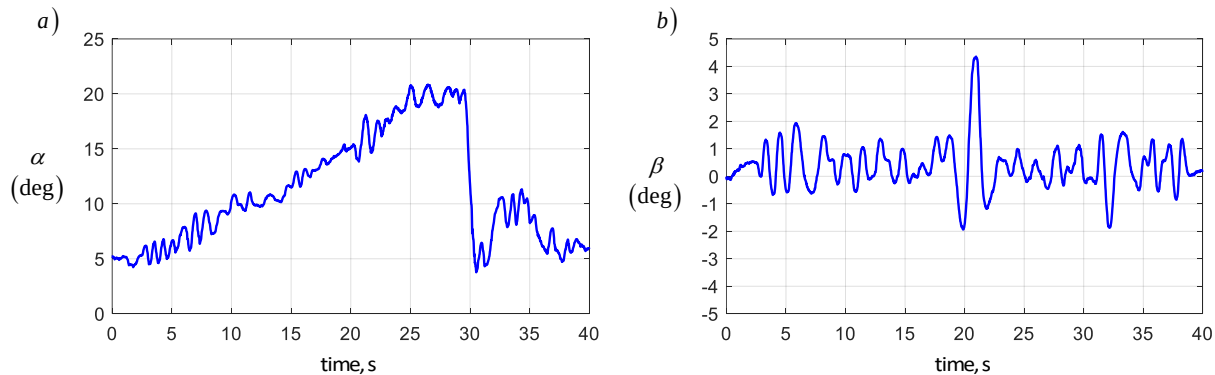


Figure 15. Flight data for orthogonal optimized multisine inputs applied during a slow approach to stall and recovery on the T-2 subscale jet transport aircraft

This data can be used for accurate global modeling, or can be partitioned for localized linear modeling [1,15,17,18,32]. The balanced characteristic and minimum RPF of orthogonal optimized multisine inputs help enforce input/output amplitude and rate constraints, and maintain the target flight conditions for the flight testing. When the excitation inputs are automated, the pilot workload is reduced to flying the aircraft to the target flight test condition, then activating the automated excitation inputs. This enables efficient flight testing that would be difficult or impossible otherwise, including flight testing in unusual, precarious, and time-limited situations. A trained test pilot can effectively implement approximations to orthogonal optimized multisine inputs while simultaneously and slowly changing the nominal flight condition [32], thus approximately duplicating the type of maneuver used to collect the data shown in Fig. 15.

Orthogonal optimized multisine excitations are also advantageous for flight test situations that require maximum input energy applied to multiple controls as quickly as possible, to rapidly interrogate the aircraft. Some examples are hypersonic stability and control flight testing with limited flight time at specific Mach numbers, time-varying flight conditions such as the variation in nominal angle of attack shown in Fig. 15, and real-time modeling. Excitation inputs with mathematical orthogonality and parallel application of many frequencies provides this capability. Balanced inputs optimized for minimum RPF and balanced makes the multi-axis excitation practical and keeps the aircraft dynamic response centered around the desired flight test condition.

G. Flight Testing with Active Feedback Control

In practice, pilot inputs, automated guidance, and/or active feedback control can act to spoil the orthogonality (zero pairwise correlations) of a multiple-input design. However, good modeling results require only low correlations, not zero correlations, so that slightly correlated inputs still work well in practice. The arrangement shown in Fig. 16 has been used very successfully in a wide variety of flight test applications with relatively high-gain active feedback control, including hypersonic vehicles stability and control [10-12], launch vehicle thrust effects [25-27], stability and control for an open-loop unstable fighter aircraft [9,14], real-time global modeling [17,18], and real-time modeling for an aeroelastic aircraft [39].

The injection point for multiple excitation inputs is important when flight testing aircraft with active feedback control. For bare airframe modeling, the injection point must be after the control system and before the control command rate and position limiters, as shown in Fig. 16. Using this approach, the control system only senses the effects of the excitation inputs on the aircraft response through the feedback of measured outputs, and has no other knowledge of the excitation inputs. Because the effects of the excitation inputs on the aircraft measured outputs are delayed by the aircraft dynamics, the control system cannot completely extinguish the excitation inputs, and some of the excitation input energy gets through to the aircraft. The general idea is to inject perfectly uncorrelated (orthogonal) excitation inputs, so that any correlation caused by the pilot, guidance system, and/or feedback control will only partially correlate the control effector deflections to levels usually much lower than the practical upper limit of 0.9 in absolute value [1]. Accurate bare-airframe modeling results can then be obtained using a variety of methods [1], based on flight data collected with feedback control active.

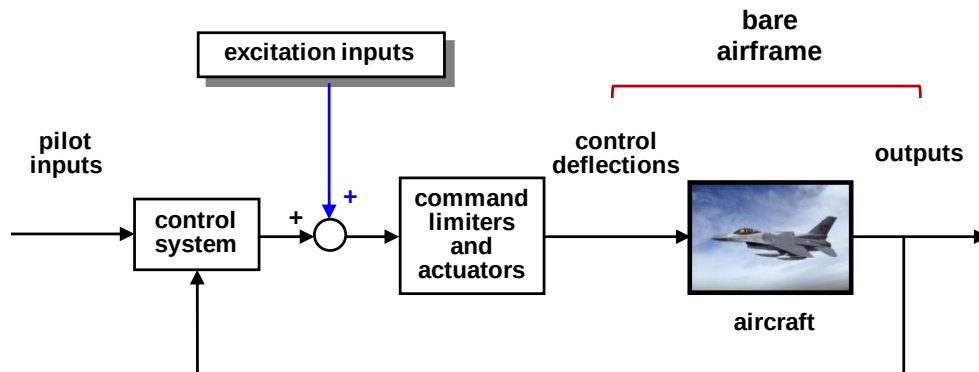


Figure 16. Injecting excitation inputs for an aircraft flying with active feedback control

H. Frequency Response Estimation

Orthogonal optimized multisines can be used to accurately and efficiently estimate multiple frequency responses simultaneously [1,16,21,22], even in flight testing with active feedback control [39], and also in real time [16,19,20,39]. The basic idea is that orthogonal optimized multisine inputs have frequency content that is well-defined, discrete, and known, so that the frequency response at each discrete multisine frequency can be computed simply by the ratio of the Fourier transforms of the output to the Fourier transform of the input:

$$\hat{H}(\omega_k) = \frac{\tilde{y}(\omega_k)}{\tilde{u}(\omega_k)} \quad k = 1, 2, \dots, M \quad (14)$$

This simple calculation can be done for each frequency in each of the multisine inputs, even when all of the multisine inputs are applied simultaneously, because of the frequency-domain orthogonality that comes from using unique frequencies for each multisine input. With this approach, frequency responses for multiple inputs and multiple outputs can be estimated accurately from a single maneuver [1,16,21,22,39].

The approach can be considered a parallel application of sine dwells at the input frequencies, with frequency response computed at the input frequencies using the definition of frequency response in Eq. (14). Furthermore, the parallel application of frequency content in the multisine inputs induces steady-state response at each frequency early in the maneuver. This is consistent with the definition of frequency response as the steady-state response to a single input as a function of frequency, and improves the accuracy of the frequency response estimates.

Because there is no uncertainty as to the frequency content of the orthogonal optimized multisine inputs, there is also no need for spectral estimation techniques such as windowing, averaging, or binning [1,5,16,31]. Such techniques are appropriate when the input has frequency content that is not precisely known prior to the analysis, such as when piloted frequency sweeps are used for aircraft flight testing. Results from Eq. (14) are estimates of the frequency response at each multisine frequency, with uncertainties calculated using the method described in Ref. [20]. Each frequency response can be completed by interpolating between the point estimates at the multisine frequencies, or by flying additional maneuvers with different excitation frequencies. This can be done by simply changing T or changing the assignments of multisine inputs to control effectors, as described earlier, or by re-designing the orthogonal optimized multisine inputs using different harmonic frequencies.

To estimate bare-airframe frequency responses from flight test data collected with active feedback control, orthogonal optimized multisine inputs can be injected as described earlier, but the analysis must be modified from the simple calculation given in Eq. (14), as described in Refs. [22,39].

I. Control Actuator Modeling

If data for both the control surface actuator commands and the control surface positions are collected during a flight test using orthogonal optimized multisine excitations, it is possible to identify a model for the loaded actuator dynamics, even when the inputs were designed for aircraft dynamic modeling. This can be done in real time using frequency-domain methods [1,13] with a fixed model structure, e.g., a first-order transfer function, and also nonparametrically by estimating frequency response at the known frequencies of the orthogonal optimized multisine inputs, as described earlier. The results are useful for practical control system design, adaptive control, fault detection, and control surface actuator performance evaluation. Accurate modeling for the loaded control actuator dynamics is important because the excitation input implementation can be compromised due to actuator bandwidth or linkage slop, and because accurate characterization of loaded actuator dynamics is difficult to achieve on the ground but relatively straightforward in flight. This task can be done in a very efficient and straightforward way for many actuators simultaneously using orthogonal optimized multisine inputs.

J. Control Interaction Effects

Element-wise multiplication of two orthogonal optimized multisine input time series produces an input with harmonic frequencies that are the sum and difference of the harmonic frequencies of the multisine inputs being multiplied. For example, multiplying two harmonic sinusoid inputs $\mathbf{u}_1$ and $\mathbf{u}_2$ from Eq. (3) produces

$$\mathbf{u}_1 \mathbf{u}_2 = \sin\left(\frac{2\pi k_1 t}{T} + \phi_1\right) \sin\left(\frac{2\pi k_2 t}{T} + \phi_2\right) \quad (15)$$

Using the trigonometric identity in Eq. (5),

$$\mathbf{u}_1 \mathbf{u}_2 = \frac{1}{2} \cos\left(\frac{2\pi(k_1 - k_2)t}{T} + \phi_1 - \phi_2\right) - \frac{1}{2} \cos\left(\frac{2\pi(k_1 + k_2)t}{T} + \phi_1 + \phi_2\right) \quad (16)$$

For multisine inputs with multiple harmonic sinusoid components, the result would be a sum of terms like those shown on the right side of Eq. (16). Both sine and cosine harmonic sinusoids are mutually orthogonal if their harmonic frequencies are different by an integer, regardless of the phase angles.

If the sums and differences of the harmonic numbers for individual multiplied multisines are not the same as the harmonic numbers in the remaining multisine inputs, then the multiplied multisine input is also mutually orthogonal to the remaining multisine inputs.

For example, if the interaction effect between the rudder δ_r and elevator δ_e were of interest, then the multisine inputs in Table 1 could be reassigned so that inputs with even harmonic numbers were applied to δ_r and δ_e , and inputs with odd harmonic numbers were assigned to the left aileron δ_{a_l} and right aileron δ_{a_r} . This could be done by swapping the excitation inputs for δ_e and δ_{a_l} in Table 1. Then, because the sum or difference of even integers can never be an odd integer, the multiplied excitation input $\delta_e \delta_r$ would be orthogonal to the excitation inputs for δ_{a_l} and δ_{a_r} . This simple variation in the multiple input design can enable accurate estimation of selected control interaction effects. In this example, the multiplied multisine excitation is no longer orthogonal to the δ_r excitation, but the correlation is very low, because only a few of the harmonic numbers are the same. None of the harmonic numbers for the multiplied excitation would be the same as those for δ_e , so that the orthogonality of $\delta_e \delta_r$ and δ_e would remain intact.

Control interaction effects cannot be identified from flight data generated by applying single-input designs to one control at a time. A multiple-input design must be used for that purpose, and the orthogonal optimized multisine input design can be used to collect highly informative flight data for determining control surface interaction effects. Even if no control interaction effects exist, using the orthogonal optimized multisine input design will produce flight data to confirm the absence of control interaction effects, whereas the question of whether or not control interaction effects exist will be unanswerable using data from single-input designs applied one at a time.

This issue is illustrated with cross plots for elevator and rudder deflections using a single-input design, such as a frequency sweep, applied to one control at a time in Fig. 17a, and using simultaneous orthogonal optimized multisine inputs in Fig. 17b. The data collected using the orthogonal optimized multisine input shows many combinations of control settings, which provides rich data information content for determining control interaction effects, whereas moving one control at a time only provides information for a particular control deflection when other controls are held fixed at their trim values. Obviously, this issue becomes more important as the number and proximity of the controls increase.

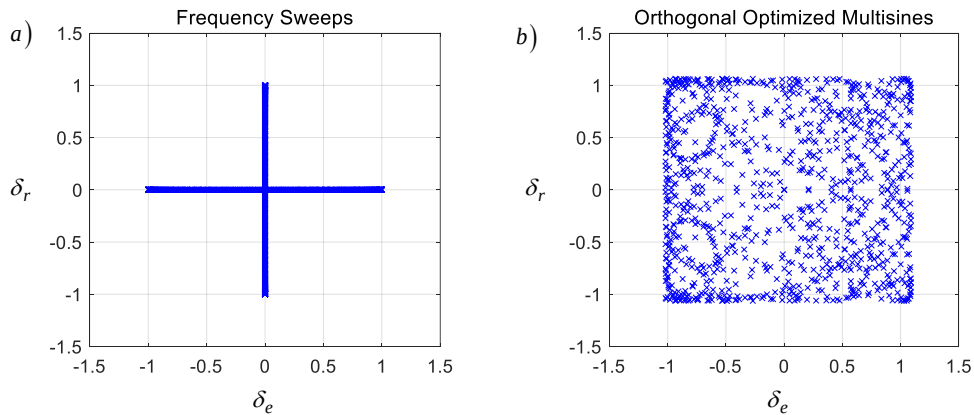


Figure 17. Control interaction effects data: a) frequency sweeps, b) orthogonal optimized multisines

VI. Conclusions

The method for designing multiple orthogonalized optimized inputs for aircraft system identification was explained in detail, including a step-by-step design process. Useful practical variations on the basic multiple-input design were discussed, along with the flight test situations where these variations can be applied effectively. Flight test implementation issues and applications were covered, demonstrating that the orthogonal optimized multisine input design is practical, versatile, time-efficient, robust, and very effective for a wide variety of flight test situations and aircraft system identification problems.

Modern aircraft designs for missions such as Urban Air Mobility are trending toward using both aerodynamic and propulsion control effectors in close proximity to each other and to the aircraft fuselage, wings, and empennage. Individual control effectors exhibit interaction effects with other control effectors and with quantities such as local angle of attack and dynamic pressure. In addition, these aircraft are intended to operate in multiple flight modes such as hover, transition, and cruise. Aircraft configuration changes such as tilt wings and propellers that operate in some flight modes but not others create a situation where many possible aircraft configurations must be tested. Efficient and effective multiple-input flight testing will be required for these and other modern aircraft, to collect the informative multi-axis flight data needed for good modeling results in various flight modes, and to accurately characterize complex, nonlinear, unsteady, and interaction effects for aircraft with many control effectors.

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References

- [1] Morelli, E.A. and Klein, V. *Aircraft System Identification – Theory and Practice*, 2nd Edition, Sunflyte Enterprises, Williamsburg, VA, 2016, Chapters 3,5-8,10-12.
- [2] Plaetschke, E. and Shulz, G. “Practical Input Signal Design. Parameter Identification,” Paper 3, AGARD-LS-104, 1979.
- [3] Maine, R.E. and Iliff, K.W. “Application of Parameter Estimation to Aircraft Stability and Control: The Output Error Approach,” NASA RP 1168, 1986.
- [4] Williams, J.N., Ham, J.A., and Tischler, M.B. “Flight Test Manual, Rotorcraft Frequency Domain Flight Testing,” AQTD Project No. 93–14, U.S. Army Aviation Technical Test Center, Edwards AFB, CA, 1995.
- [5] Tischler, M.B. and Remple, R.K. *Aircraft and Rotorcraft System Identification – Engineering Methods with Flight Test Examples*, 2nd Edition, AIAA Education Series, AIAA, Reston, VA, 2012.
- [6] <https://evtol.news/aircraft>, accessed 03 February 2021.
- [7] North, D.D., Busan, R.C., and Howland, G. “Design and Fabrication of the Langley Aerodrome No. 8 Distributed Electric Propulsion VTOL Testbed,” AIAA-2021-1188, *AIAA SciTech Forum*, January 2021.
- [8] Morelli, E.A. “Optimal Input Design for Aircraft Stability and Control Flight Testing,” *Journal of Optimization Theory and Applications*, Special Issue on Optimization Theory and Application to Aerospace Systems, accepted for publication.
- [9] Morelli, E.A. “Multiple Input Design for Real-Time Parameter Estimation in the Frequency Domain,” Paper REG-360, 13th IFAC Symposium on System Identification, Rotterdam, The Netherlands, August 2003.
- [10] Morelli, E.A. “Flight-Test Experiment Design for Characterizing Stability and Control of Hypersonic Vehicles,” *Journal of Guidance, Control, and Dynamics*, Vol. 32, No. 3, May-June 2009, pp. 949-959.
- [11] Morelli, E.A., Derry, S.D., and Smith, M.S. “Aerodynamic Parameter Estimation of the X-43A (Hyper-X) from Flight Test Data,” AIAA-2005-5921, *AIAA Atmospheric Flight Mechanics Conference*, San Francisco, CA, August 2005.
- [12] Morelli, E.A., Rexius, S.L., and Lechniak, J.A. “Flight Test Experiment Design and Aerodynamic Parameter Estimation for the X-51A Waverider,” AIAA-2014-2767 20th AIAA International Space Planes and Hypersonic Systems and Technologies Conference, Atlanta, GA, June 2014.
- [13] Morelli, E.A. “Real-Time Parameter Estimation in the Frequency Domain,” *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, September-October 2000, pp. 812-818.

- [14] Morelli, E.A. and Smith, M.S. "Real-Time Dynamic Modeling – Data Information Requirements and Flight Test Results," *Journal of Aircraft*, Vol. 46, No. 6, November-December 2009, pp. 1894-1905.
- [15] Morelli, E.A. "Flight Test Maneuvers for Efficient Aerodynamic Modeling," *Journal of Aircraft*, Vol. 49, No. 6, November-December 2012, pp. 1857-1867.
- [16] Morelli, E.A. and Grauer, J.A. "Practical Aspects of Frequency-Domain Approaches for Aircraft System Identification," *Journal of Aircraft*, Vol. 57, No. 2, March 2020, pp. 268-291.
- [17] Morelli, E.A. "Practical Aspects of Real-Time Modeling for the Learn-To-Fly Concept," AIAA-2018-3309, *Atmospheric Flight Mechanics Conference, AIAA Aviation Forum*, Atlanta, GA, June 2018.
- [18] Morelli, E.A. "Autonomous Real-Time Global Aerodynamic Modeling in the Frequency Domain," AIAA-2020-0761, *AIAA SciTech 2020 Conference*, Orlando, FL, January 2020.
- [19] Holzel, M.S. and Morelli, E.A. "Real-Time Frequency Response Estimation from Flight Data," *Journal of Guidance, Control, and Dynamics*, Vol. 35, No. 5, September-October 2012, pp. 1406-1417.
- [20] Grauer, J.A. and Morelli, E.A. "Method for Real-Time Frequency Response and Uncertainty Estimation," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 1, January-February 2014, pp. 336-343.
- [21] Grauer, J.A. "Dynamic Modeling using Output-Error Parameter Estimation based on Frequency Responses Estimated with Multisine Inputs," NASA/TM 2018-220108, November 2018.
- [22] Grauer, J.A. and Boucher, M.J. "Aircraft System Identification from Multisine Inputs and Frequency Responses," AIAA 2020-0287, *AIAA SciTech Forum*, Orlando, FL, January 2020.
- [23] Gingras, D.R., Barnhart, B., Ranaudo, R., Ratvasky, T.P., and Morelli, E.A. "Envelope Protection for In-Flight Ice Contamination," AIAA-2009-1458, *47th AIAA Aerospace Sciences Meeting*, Orlando, FL, January 2009; also published as NASA TM-2010-216072, February 2010.
- [24] Gingras, D.R., Barnhart, B., Ranaudo, R., Martos, B., Ratvasky, T.P., and Morelli, E.A. "Development and Implementation of a Model-Driven Envelope Protection System for In-Flight Ice Contamination," AIAA-2010-8141, *AIAA Guidance, Navigation, and Control Conference*, Toronto, Canada, August 2010; also published as NASA TM-2011-216960, October 2011.
- [25] Brandon, J.M., Derry, S.D., Heim, E.H., Hueschen, R.M., and Bacon, B.J. "Ares-I-X Stability and Control Flight Test: Analysis and Plans," AIAA 2008-7807, *AIAA SPACE 2008 Conference & Exposition*, San Diego, CA, September 2008.
- [26] Derry, S.D. "Ares I-X Flight Test Overview and System ID Results", *106th SAE / IEEE Aerospace Control and Guidance Systems Committee (ACGSC) Meeting*, La Jolla, CA., October 2010.
- [27] Derry, S.D. "Ares I-X Flight Test System Identification Results", *58th JANNAF Propulsion Meeting*, Arlington, VA., April 2011.
- [28] Morelli, E.A. "Subsonic Flight Test Maneuver Design and Aerodynamic Modeling for the Dream Chaser Space Vehicle," NASA/TM-2019-220261, March 2019.
- [29] <https://software.nasa.gov/software/LAR-16100-1>, accessed 03 February 2021.
- [30] http://www.sunflyte.com/SIDBook_SIDPAC.htm, accessed 03 February 2021.
- [31] Press, W.H., Teukolsky, S.A., Vetterling, W.T., and Flannery, B.R. *Numerical Recipes in FORTRAN: The Art of Scientific Computing*, 2nd Ed., Cambridge University Press, New York, NY, 1992, Chapter 10.
- [32] Brandon, J.M. and Morelli, E.A. "Real-Time Global Nonlinear Aerodynamic Modeling from Flight Data," *Journal of Aircraft*, Vol. 53, No. 5, September-October 2016, pp. 1261-1297.
- [33] Morelli, E.A. "In-Flight System Identification," AIAA 98-4261, *AIAA Atmospheric Flight Mechanics Conference*, Boston, MA, August 1998.
- [34] Jordan, T.L., Langford, W.M., and Hill, J.S. "Airborne Subscale Transport Aircraft Research Testbed: Aircraft Model Development", AIAA-2005-6432, *AIAA Guidance, Navigation, and Control Conference and Exhibit*, August 2005.
- [35] Jordan, T.L., Foster, J.V., Bailey, R.M., and Belcastro, C.M., "AirSTAR: A UAV Platform for Flight Dynamics and Control System Testing," AIAA-2006-3307, *25th AIAA Aerodynamic Measurement Technology and Ground Testing Conference*, San Francisco, CA, June 2006.
- [36] Murch, A.M. "A Flight Control System Architecture for the NASA AirSTAR Flight Test Facility," AIAA-2008-6990, *AIAA Guidance, Navigation, and Control Conference and Exhibit*, Honolulu, HI, August 2008.
- [37] Grauer, J.A. "Random Noise Generation Using Fourier Series," *Journal of Aircraft*, Vol. 55, No. 4, July-August 2018, pp. 1753-1759.

- [38] "Flying Qualities of Piloted Airplanes," Technical Report MIL-F-8785C, U.S. Air Force, Wright-Patterson Air Force Base, Dayton, OH, 1980.
- [39] Grauer, J.A. and Boucher, M.J. "Real-Time Estimation of Bare-Airframe Frequency Responses from Closed-Loop Data and Multisine Inputs," *Journal of Guidance, Control, and Dynamics*, Vol. 43, No. 2, February 2020, pp. 288-298.