

Fuel-Optimal Trajectory Optimization of Mach 0.745 Transonic Truss-Braced Wing with Variable Camber Continuous Trailing Edge Flap

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This paper describes a fuel-optimal trajectory optimization of the Mach 0.745 Transonic Truss-Braced Wing with the variable camber continuous trailing edge flaps (VCCTEF) capable of changing the chordwise pressure distribution and spanwise lift distribution to minimize drag. The trajectory optimization is posed as a minimization of the fuel burn over a fixed cruise range. The trajectory optimization is formulated by the adjoint method which results in a two-point boundary value problem. A numerical shooting method is implemented to solve the minimum-fuel trajectory optimization. The results indicates a fuel burn reduction of 793 lbs or 5.5% for a cruise range of 3000 miles. An approximate trajectory optimization is proposed using a single adjoint variable. This leads to a greatly simplified analytical solution of the adjoint solution. The results of the approximate trajectory optimization are in excellent agreement with those obtained by the shooting method. The approximate trajectory optimization produces a fuel burn reduction of 615 lbs or 4.3%.

I. Introduction

Air travel is an energy-intensive mode of transportation. Global airline fleets consume about 2.89 billion barrels of oil in 2019 and the global airline industry spends about \$188 billion in 2019 on the fuel bill.¹ Fuel economy is a driving factor in aircraft design and marketing. The modern transport aircraft design has evolved over time. In recent years, the latest design trend adopts the use of lightweight composite material and high aspect ratio wing design. As a result, the structural flexibility of the modern aircraft high aspect ratio wing becomes a significant factor in an aircraft aerodynamic design. As the structural flexibility increases, the changes in the lift distribution throughout a flight profile can result in reduced aerodynamic performance, hence increased fuel consumption.

Reducing fuel consumption is a focus of NASA aeronautics research and the NASA Advanced Air Transport Technology project (AATT). The Subsonic Ultra Green Aircraft Research (SUGAR) Transonic Truss-Braced Wing (TTBW) aircraft concept is a Boeing-developed N+3 aircraft configuration funded by the NASA ARMD AATT project.^{2,3} The TTBW aircraft concept is designed to be aerodynamically efficient by employing a wing aspect ratio of about 19.55 which is significantly greater than those of cantilever wing transport configurations. The increase in the aerodynamic efficiency would translate into a reduction in the fuel burn.

In the SUGAR configuration, the trade between the aerodynamic performance and structural design results in a truss-braced configuration with the wings braced at approximately mid-span by two main struts. In addition, two jury struts, one on each wing, provide the additional structural reinforcement. Two versions of the TTBW configurations are currently being developed by Boeing; a Mach 0.745 version and a Mach 0.8 version. Figure 1 is an illustration of the Mach 0.745 TTBW aircraft.

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Figure 1. Boeing SUGAR Transonic Truss-Braced Wing (TTBW) Aircraft Concept

Active control for drag minimization has become a viable option for aircraft design and in fact has been deployed on modern aircraft. The variable camber continuous trailing edge flap (VCCTEF) is an active control concept for drag minimization developed by NASA in 2010.⁴⁻⁶ Drag optimization studies^{7,8} and experimental investigations⁹⁻¹¹ have confirmed the effectiveness of the VCCTEF as a drag reduction control device.

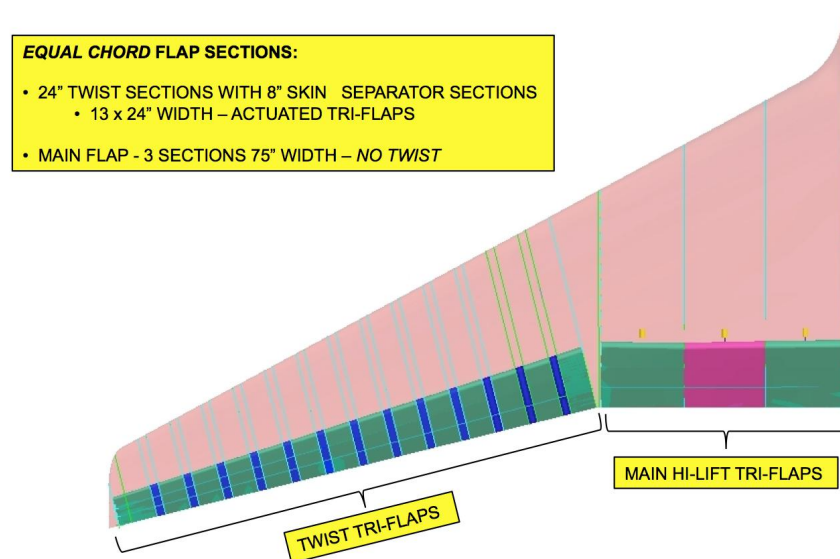


Figure 2. Typical Transport Wing Configured with the Variable Camber Continuous Trailing Edge Flap

The original VCCTEF system,^{4,5} shown in Fig. 2, employs a light-weight shaped memory alloy (SMA) technology for actuation^{12,13} and three individual chordwise segments,^{4,5,13} shown in Fig. 3, to provide a variable camber aerodynamic surface to shape the chordwise pressure distribution for improved aerodynamic performance. The VCCTEF system is divided into multiple sections attached to the outer wing and the inner wing. Each spanwise flap section has three cambered flap segments that can be individually commanded. These cambered flaps are joined to the next section by a flexible transition material installed with the same shape as the camber and thus providing a continuous trailing edge flap throughout the wing span with no drag-producing gaps.¹³ This continuous trailing edge flap design combined with the flap camber result in lower drag increase during flap deflections.

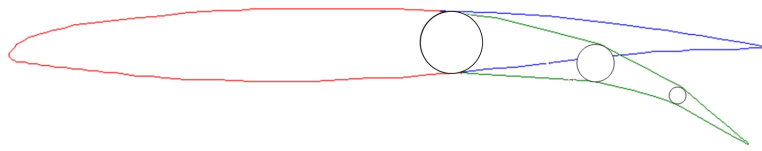


Figure 3. Three-Segment Variable Camber Flap Illustrated on Generic Supercritical Airfoil

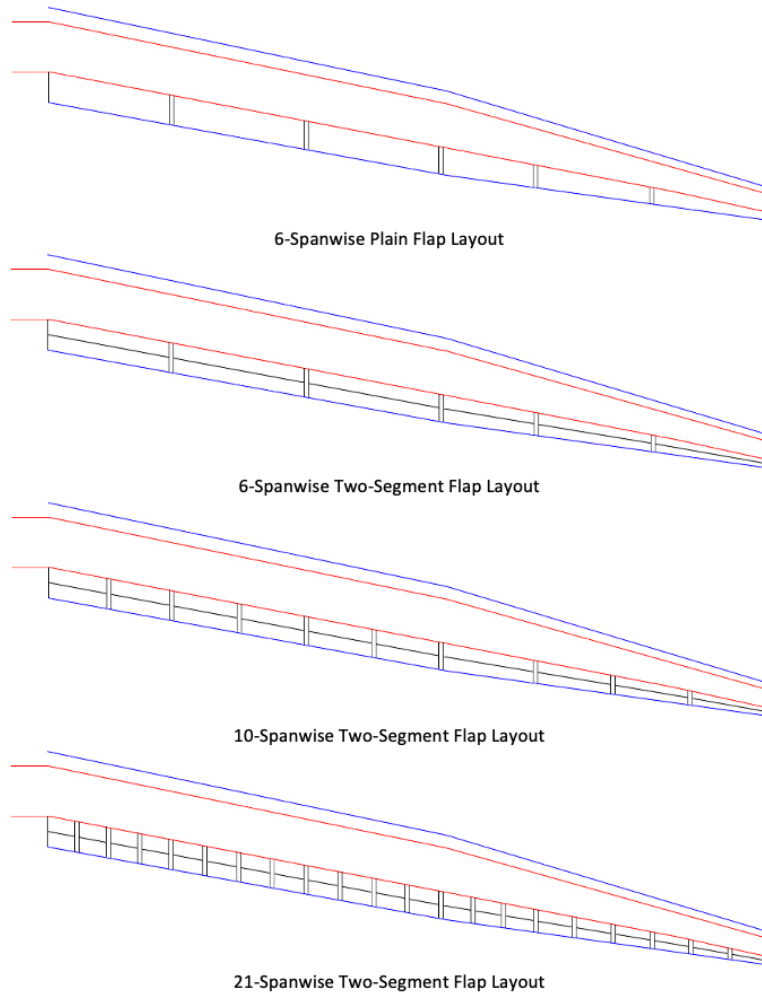


Figure 4. VCCTEF Layout Candidates for Mach 0.745 Transonic Truss-Braced Wing Aircraft

Pointwise drag optimization of the Mach 0.745 TTBW has been conducted for three different VCCTEF layouts ranging from a simple design to a complex design: 6 spanwise plain flaps, 6 spanwise two-segment flaps, 10 spanwise two-segment flaps, and 22 spanwise two-segment flaps.¹⁴ The drag optimization is conducted for the design C_L of 0.73 as well as two different off-design C_L of 0.68 and 0.78 representing 20% fuel and 80% fuel cases. Weight and drag trade analysis is performed to account for the added actuator weight of the four flap configurations. The optimization indicates that the 6-spanwise two-segment flap layout produces the most net gain in drag reduction.¹⁴ This flap layout therefore is the preferred flap layout that will be used in this study.

This study investigates a trajectory optimization to minimize the fuel burn of the Mach 0.745 TTBW over a fixed cruise range. An adjoint method is developed for the trajectory optimization. The aeroelastic effect is accounted for in the trajectory optimization as reflected in the aerodynamic analysis.

II. Aeroelastic Modeling

The wing structural deformation can change the aerodynamic performance of high aspect ratio wing aircraft such as the TTBW. To account for the structural deformation, an aeroelastic model of the Mach 0.745 TBW is developed. The aerodynamic model is developed using the VSPAERO solver coupled to a two-dimensional transonic small disturbance code TSFOIL and an in-house integral boundary layer code. The VSPAERO solver is coupled to an in-house nonlinear finite-element code BEAM3D.

VSPAERO is a solver that includes both the vortex lattice method and the full panel method based on generalized vortex rings. The core VSPAERO solver is based on an agglomerated multi-pole approach, coupled with a pre-conditioned linear solver, to reduce solution times. Adaptive wakes, time-accurate, unsteady analyses, and propeller modeling are all supported. The VSPAERO model is implemented with TSFOL, a transonic small disturbance code, coupled to an integral boundary layer code. In addition, a transonic wing-strut interference correction is implemented using high-fidelity FUN3D solver. More details about the tools developed can be found in Ref.^{15–18}

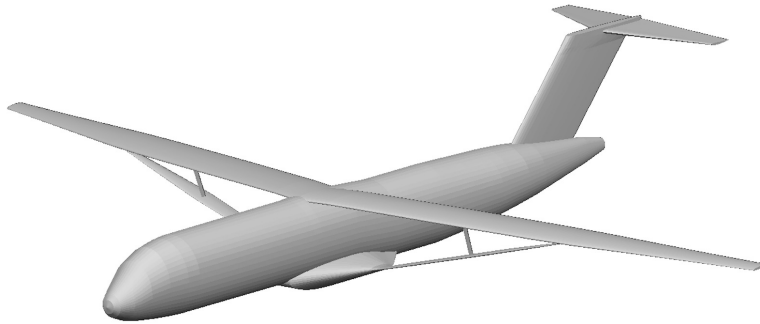


Figure 5. VSPAERO Model of Mach 0.745 Transonic Truss-Braced Wing Aircraft

BEAM3D is an in-house finite-element code that is used to model a general wing structure using beam, rod, and rigid elements. The code contains a number of analysis modules: a static deflection analysis module, an eigenvalue analysis module, a flutter analysis module, and an aeroservoelastic flight dynamic simulation module. The static deflection analysis module can be coupled to a flow solver such as VORLAX, VSPAERO, and CART3D to perform aero-structural and static aeroelastic deflection analyses. The eigenvalue analysis module computes the structural dynamic frequencies and the normal mode shapes of a wing structure. The flutter analysis module computes the frequencies and damping of the wing aeroelastic modes and performs a flutter speed search in the frequency domain using the Theodorsen's theory for unsteady aerodynamics or in the time domain using the R. T. Jones approximation to generate the aerodynamic mass, damping, and stiffness. The aeroservoelastic flight dynamic simulation module provides the capability to couple the dynamic aeroservoelastic state-space model of a wing structure to a nonlinear 6-degree-of-freedom flight dynamic model of an aircraft to perform time simulations of the aircraft response to atmospheric gust inputs and flight control surface inputs.

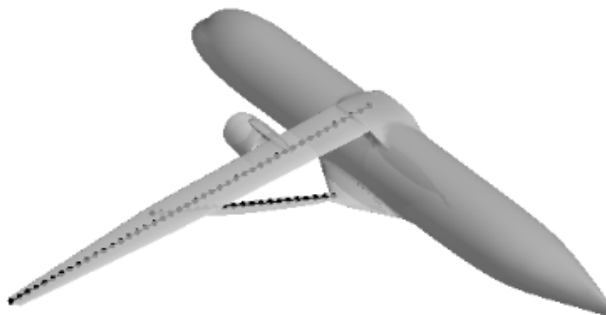


Figure 6. BEAM3D Finite Element Model of Mach 0.745 Transonic Truss-Braced Wing Aircraft

Figure 6 illustrates the TTBW finite-element model in BEAM3D. The left wing and main strut are modeled as the major structural elements. The jury strut is neglected in the model. The mass and structural properties of the finite-

element model are made available to NASA by Boeing.¹⁹ The stiffness properties and the elastic definition of the wing and strut are included in the structural properties. Fuel weight is also modeled in the BEAM3D model. The full fuel weight corresponds to a take-off gross weight (TOGW) of 138,000 lbs. The inertias of the fuselage and tails are estimated in OpenVSP. BEAM3D also includes a nonlinear capability to model the tension-induced stiffening effect of the struts which contributes to the overall stiffness matrix of the finite-element model.²⁰

For static aeroelastic analysis, an iterative solution method is implemented for the nonlinear tension-induced stiffness. At each iteration between the VSPAERO model and BEAM3D model, the iterative solution method iterates on the static deflection solution until it converges. The aeroelastic modeling framework is illustrated in Fig. 7.

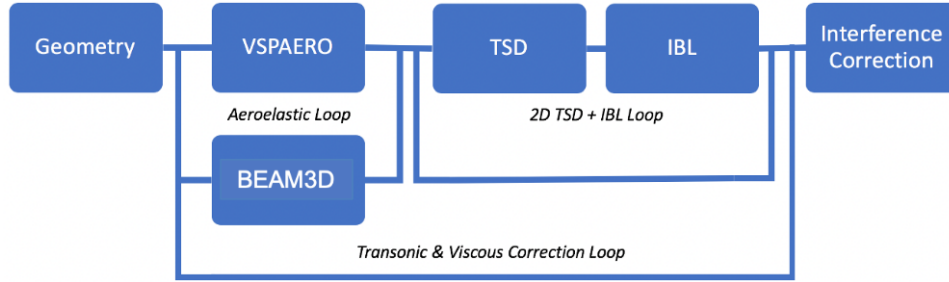


Figure 7. VSPAERO/TSD/IBL-BEAM3D Aeroelastic Modeling Framework

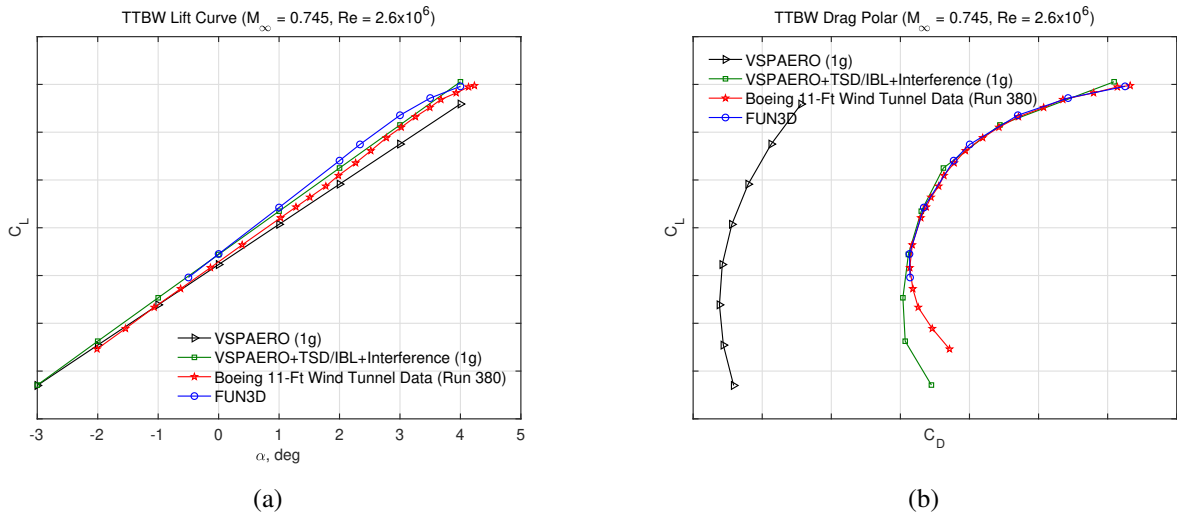


Figure 8. Lift Curve C_L vs. α and Drag Polar C_L vs. C_D of Mach 0.745 Transonic Truss-Braced Wing

The VSPAERO model of the Mach 0.745 TTBW has been validated with wind tunnel experimental data. It is also compared to a high-fidelity FUN3D CFD model. Figures 8(a) and (b) show the lift curve and drag polar, respectively, of the VSPAERO model compared to wind tunnel experimental data of the Mach 0.745 TTBW in the NASA Ames 11-Ft-By-11-Ft Unitary Plan Transonic Wind Tunnel and the FUN3D CFD results. The VSPAERO results are in very good agreement with the wind tunnel experimental data and FUN3D results.

For this study, a series of aeroelastic simulations are conducted using the jig twist outer mold line (OML) of the Mach 0.745 TTBW to compute the wing structural deformation under a 1-g load. The aerodynamic coefficients of the deformed Mach 0.745 TTBW are then obtained to create an aerodynamic database for the trajectory optimization. Figures 79 (a), (b), and (c) show the lift coefficient C_L , drag coefficient C_D , and pitching moment coefficient C_m for the clean-wing configuration, respectively, at three different altitudes of 35,000 ft, 40,000 ft, and 45,000 ft. Figures (a), (b), and (c) show the incremental lift coefficient ΔC_L , drag coefficient ΔC_D , and pitching moment coefficient ΔC_m for the 12 flap segments of the VCCTEF and the horizontal tail, respectively, at an altitude of 40,000 ft and an angle of attack of 0° . It is observed that the incremental aerodynamic coefficients due to the elevator exhibit a linear relationship.

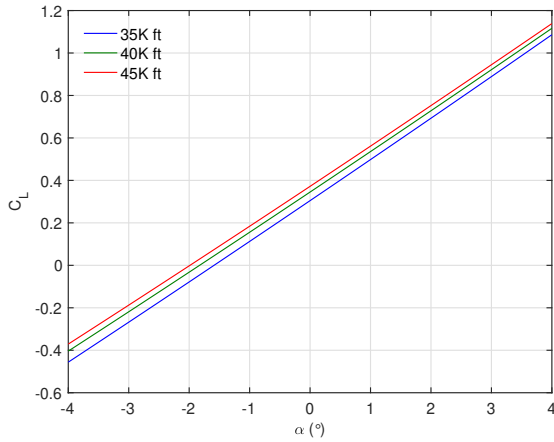
The aerodynamic coefficients can be modeled as

$$C_L = \sum_{i=0}^2 a_i(h) \alpha^i + \sum_{i=1}^{n_f} \sum_{j=1}^2 \sum_{k=0}^2 b_{ijk}(h) \alpha^k \delta_i^j + \sum_{i=0}^2 c_i(h) \alpha^i \delta_h \quad (1)$$

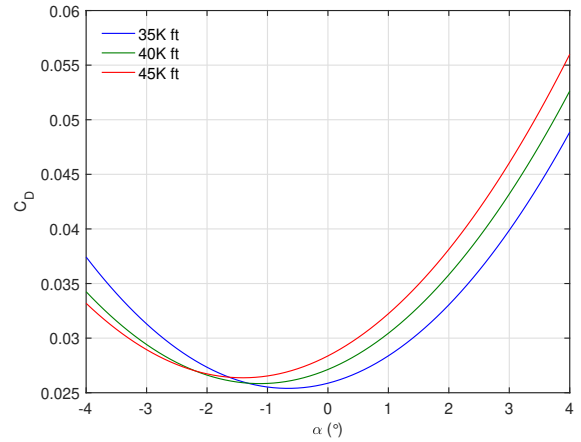
$$C_D = \sum_{i=0}^2 d_i(h) \alpha^i + \sum_{i=1}^{n_f} \sum_{j=1}^2 \sum_{k=0}^2 e_{ijk}(h) \alpha^k \delta_i^j + \sum_{i=0}^2 f_i(h) \alpha^i \delta_h \quad (2)$$

$$C_m = \sum_{i=0}^2 u_i(h) \alpha^i + \sum_{i=1}^{n_f} \sum_{j=1}^2 \sum_{k=0}^2 v_{ijk}(h) \alpha^k \delta_i^j + \sum_{i=0}^2 w_i(h) \alpha^i \delta_h \quad (3)$$

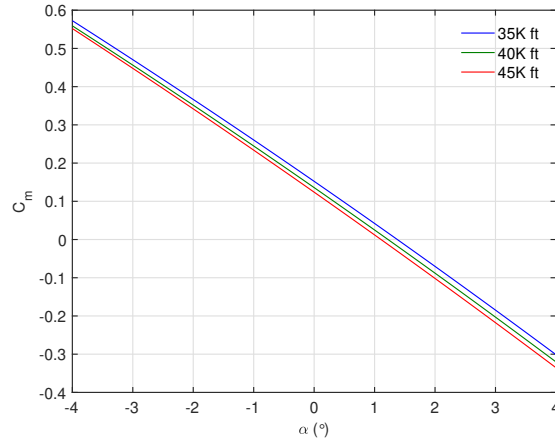
where δ_h is the horizontal tail incidence angle and δ_i is the deflection of the i^{th} flap segment with δ_1 as the deflection of the inner flap segment at the wing root and δ_7 as the deflection of the outer flap segment at the wing root.



(a)



(b)



(c)

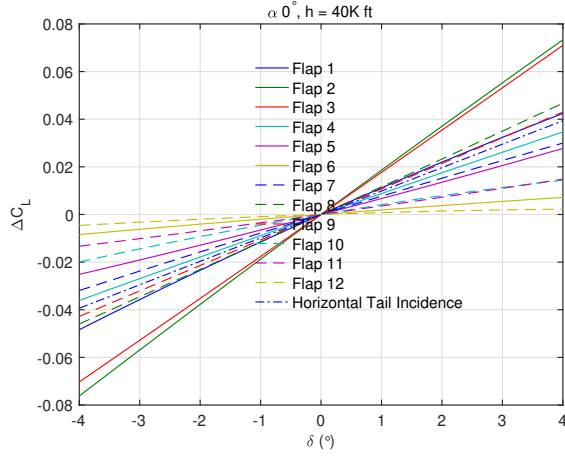
Figure 9. Aerodynamic Coefficients of Mach 0.745 Transonic Truss-Braced Wing Clean-Wing Configuration

In general, we express the aerodynamic coefficients as

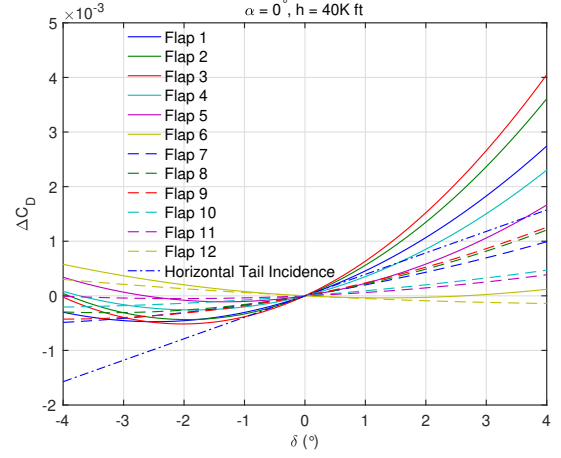
$$C_L = C_{L_0}(h) + C_{L_\alpha}(h) \alpha + C_{L_{\alpha^2}}(h) \alpha^2 + \sum_{i=1}^{n_f} \left[C_{L_{\delta_i}}(h, \alpha) \delta_i + C_{L_{\delta_i^2}}(h, \alpha) \delta_i^2 \right] + C_{L_{\delta_h}}(h, \alpha) \delta_h \quad (4)$$

$$C_D = C_{D_0}(h) + C_{D_\alpha}(h) \alpha + C_{D_{\alpha^2}}(h) \alpha^2 + \sum_{i=1}^{n_f} \left[C_{D_{\delta_i}}(h, \alpha) \delta_i + C_{D_{\delta_i^2}}(h, \alpha) \delta_i^2 \right] + C_{D_{\delta_h}}(h, \alpha) \delta_h \quad (5)$$

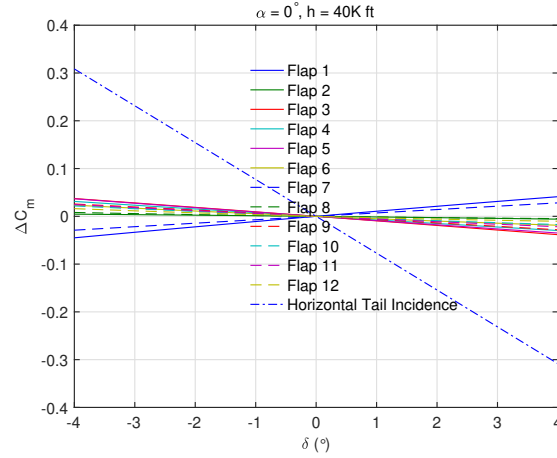
$$C_m = C_{m_0}(h) + C_{m_\alpha}(h) + C_{m_{\alpha^2}}(h) \alpha^2 + \sum_{i=1}^{n_f} \left[C_{m_{\delta_i}}(h, \alpha) \delta_i + C_{m_{\delta_i^2}}(h, \alpha) \delta_i^2 \right] + C_{m_{\delta_h}}(h, \alpha) \delta_h \quad (6)$$



(a)



(b)



(c)

Figure 10. Incremental Aerodynamic Coefficients of Mach 0.745 Transonic Truss-Braced Wing Clean-Wing

III. Trajectory Optimization

A. Equations of Motion

The equations of motion of an aircraft in symmetric flight are given by

$$\dot{x} = V \cos \gamma \quad (7)$$

$$\dot{h} = V \sin \gamma \quad (8)$$

$$\dot{V} = \frac{T - D - W \sin \gamma}{m} \quad (9)$$

$$\dot{\gamma} = \frac{L - W \cos \gamma}{mV} \quad (10)$$

$$\dot{W} = -cT \quad (11)$$

where x is the range distance variable, h is the altitude, V is the airspeed, γ is the flight path angle, W is the aircraft weight, D is the drag force, L is the lift force, m is the aircraft mass, T is the engine thrust, and c is the thrust specific fuel consumption.

The aircraft is trimmed so that

$$I_{yy}\dot{q} = C_m\bar{q}S\bar{c} + z_e T = 0 \quad (12)$$

where q is the pitch rate, I_{yy} is the pitch inertia, C_m is the aerodynamic pitching moment, $\bar{q}$ is the dynamic pressure, S is the wing reference area, $\bar{c}$ is the mean aerodynamic chord, z_e is the engine thrust centerline offset from the aircraft center of gravity (CG) and is positive if the thrust centerline is below the aircraft CG.

The pitch attitude angle θ is related to the pitch rate as $\dot{\theta} = q$. For trimmed flight, the pitch rate is zero. Therefore, the pitch attitude angle θ is constant. The flight path angle is related to the pitch attitude angle θ and the angle of attack α as

$$\gamma = \theta - \alpha \quad (13)$$

Making a small angle assumption and transforming the time variable t into the range distance variable x , we express the equations of motion as

$$h' = \gamma \quad (14)$$

$$V' = \frac{T - D - W\gamma}{mV} \quad (15)$$

$$\alpha' = \frac{W - L}{mV^2} = \frac{g}{V^2} \left(1 - \frac{L}{W}\right) \quad (16)$$

$$W' = -\frac{cT}{V} \quad (17)$$

the prime denotes the derivatives with respect to x .

If we assume a constant Mach cruise, the this type of cruise introduces a Mach number constraint

$$V(M, h) = Ma(h) \quad (18)$$

where $a(h)$ is the speed of sound.

Taking the derivative of Eq. (18) yields

$$V' = M \frac{da}{dh} h' = M \frac{da}{dh} \gamma \quad (19)$$

Substituting Eq. (19) into Eq. (9), we obtain the thrust relationship

$$T = D + W \left(1 + \frac{M^2 a}{g} \frac{da}{dh}\right) \gamma \quad (20)$$

Therefore, the engine thrust is an explicit function of the drag force, aircraft gross weight, flight path angle, and altitude. The trim condition allows us to solve for the horizontal tail incidence angle explicitly as

$$\delta_h = -\frac{\Delta C_m + \frac{z_e}{\bar{c}} \left[\Delta C_D + \frac{W}{\bar{q}S} \left(1 + \frac{M^2 a}{g} \frac{da}{dh}\right) \gamma \right]}{C_{m\delta_h} + \frac{z_e}{\bar{c}} C_{D\delta_h}} \quad (21)$$

where ΔC_m and ΔC_D are the pitching moment and drag coefficients contributions by the angle of attack and the flap deflections as functions of the altitude.

B. Aircraft Engine Performance Model

The fuel burn is a function of the engine thrust and the thrust specific fuel consumption. For an turbofan engine, the similitude laws for the corrected thrust and corrected thrust specific fuel consumption are generally functions of the Mach number and corrected engine speed

$$\frac{T}{\delta_t(h, M)} = f\left(M, \frac{N}{\sqrt{\theta_t}}\right) \quad (22)$$

$$\frac{c}{\sqrt{\theta_t(h, M)}} = f(M) \quad (23)$$

where M is the aircraft Mach number, N is the engine RPM, $\delta_t = \frac{p_t(h, M)}{p_{SL}}$ is ratio of the stagnation pressure $p_t(h, M)$ to the sea-level pressure p_{SL} , and $\theta_t = \frac{\tau_t(h, M)}{\tau_{SL}}$ is the ratio of the stagnation temperature $\tau_t(h, M)$ to the sea-level temperature τ_{SL} . The use of the stagnation pressure and temperature for the corrected quantities accounts for the “ram” effect on aircraft engine performance which can be significant when an aircraft flies at high speed.

In lieu of the engine speed, the corrected thrust may also be modeled as a function of the engine throttle as

$$T(h, M, \delta_T) = T_{max}(h, M) \delta_T \quad (24)$$

where δ_T is the engine throttle position between 0 and 1 and T_{max} is the engine maximum thrust which could be estimated from the static thrust and net thrust as a function of the Mach number. For simplicity, we assume the maximum thrust varies linearly with the Mach number as

$$T_{max}(h, M) = \delta(h) (T_0 + T_M M) \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} \quad (25)$$

where $\delta(h) = \frac{p(h)}{p_{SL}}$ is the ratio of the pressure $p(h)$ at altitude to the sea-level pressure, T_0 is the static thrust and T_M is a parameter such that the net thrust at a given Mach number at sea level is equal to $T_0 + T_M M$.

The thrust specific fuel consumption may be assumed to be a linear function of the Mach number. Then, the thrust specific fuel consumption is expressed as

$$c(h, M) = (c_0 + c_M M) \sqrt{\delta(h) \left(1 + \frac{k-1}{2} M^2\right)} \quad (26)$$

where $\delta(h) = \frac{\tau(h)}{\tau_{SL}}$ is the ratio of the temperature $\tau(h)$ at altitude to the sea-level temperature, c_0 is static thrust specific fuel consumption and c_M is a parameter such that the net thrust specific fuel consumption at a given Mach number at sea level is equal to $c_0 + c_M M$.

IV. Minimum-Fuel Trajectory Optimization

The minimum-fuel trajectory optimization is posed as an optimal control problem of minimizing the fuel burn

$$\min J = \Delta W_f = \int_0^r \frac{cT}{V} dx \quad (27)$$

where ΔW_f is the burned fuel weight and r is the cruise range. The objective is to determine the optimal angle of attack α , flap deflection δ , and engine thrust T that minimize the fuel consumption.

A. Adjoint Method

We formulate the optimization using the adjoint method. Toward this end, Hamiltonian function of the trajectory optimization problem is defined as

$$\begin{aligned} H &= \frac{cT}{V} + \lambda_h h' + \lambda_\alpha \alpha' + \lambda_W W' \\ &= (1 - \lambda_W) \frac{c}{V} \left[D + W \left(1 + \frac{M^2 a}{g} \frac{da}{dh} \right) \gamma \right] + \lambda_h \gamma + \frac{\lambda_\alpha g}{V^2} \left(1 - \frac{L}{W} \right) \end{aligned} \quad (28)$$

where $\lambda_h(x)$, $\lambda_\alpha(x)$, and $\lambda_W(x)$ are the adjoint variables for the altitude, angle of attack, and aircraft weight.

The adjoint equations are obtained as

$$\begin{aligned} \lambda_h' &= -\frac{\partial H}{\partial h} = -(1 - \lambda_W) \frac{\partial c}{\partial h} \left[\frac{D}{Ma} + W \left(\frac{1}{Ma} + \frac{M}{g} \frac{da}{dh} \right) \gamma \right] \\ &\quad - (1 - \lambda_W) c \left[\frac{1}{Ma} \frac{\partial D}{\partial h} - \frac{D}{Ma^2} \frac{da}{dh} + W \left(-\frac{1}{Ma^2} \frac{da}{dh} + \frac{M}{g} \frac{d^2 a}{dh^2} \right) \gamma \right] \\ &\quad - \frac{2\lambda_\alpha g}{M^2 a^3} \frac{da}{dh} \left(1 - \frac{L}{W} \right) + \frac{\lambda_\alpha g}{WM^2 a^2} \frac{\partial L}{\partial h} \end{aligned} \quad (29)$$

$$\lambda'_\alpha = -\frac{\partial H}{\partial \alpha} = -(1-\lambda_W)c \left[\frac{1}{Ma} \frac{\partial D}{\partial \alpha} - W \left(\frac{1}{Ma} + \frac{M}{g} \frac{da}{dh} \right) \right] + \lambda_h + \frac{\lambda_\alpha g}{WM^2 a^2} \frac{\partial L}{\partial \alpha} \quad (30)$$

$$\lambda'_W = -\frac{\partial H}{\partial W} = -(1-\lambda_W)c \left(\frac{1}{Ma} + \frac{M}{g} \frac{da}{dh} \right) \gamma - \frac{\lambda_\alpha g L}{W^2 M^2 a^2} \quad (31)$$

subject to the transversality conditions $\lambda_h(r) = 0$, $\lambda_\alpha(r) = 0$, and $\lambda_W(r) = 0$.

The partial derivatives of the thrust specific fuel consumption, lift, and drag with respect to the altitude are computed as

$$\frac{\partial c}{\partial h} = \frac{c \partial \tau}{2 \tau \partial h} \quad (32)$$

$$\frac{\partial L}{\partial h} = \frac{\partial C_L}{\partial h} \bar{q} S + C_L \frac{\partial \bar{q}}{\partial h} S = \left(\frac{\partial C_L}{\partial h} + \frac{C_L}{p} \frac{\partial p}{\partial h} \right) \bar{q} S \quad (33)$$

$$\frac{\partial D}{\partial h} = \frac{\partial C_D}{\partial h} \bar{q} S + \bar{C}_D \frac{\partial \bar{q}}{\partial h} S = \left(\frac{\partial C_D}{\partial h} + \frac{C_D}{p} \frac{\partial p}{\partial h} \right) \bar{q} S \quad (34)$$

The optimal flap deflection are obtained from the optimality condition as

$$\frac{\partial H}{\partial \delta_i} = (1-\lambda_W) \frac{c}{Ma} \frac{\partial D}{\partial \delta_i} - \frac{\lambda_\alpha g}{WM^2 a^2} \frac{\partial L}{\partial \delta_i} = 0 \quad (35)$$

B. Aerodynamic Partial Derivatives

To reduce the cost of numerical evaluations of the partial derivatives by the finite-difference method, the various partial derivatives of the aerodynamic coefficients are evaluated analytically as

$$\frac{\partial C_L}{\partial h} = \frac{\partial C_{L_0}}{\partial h} + \frac{\partial C_{L_\alpha}}{\partial h} \alpha + \frac{\partial C_{L_{\alpha^2}}}{\partial h} \alpha^2 + \sum_{i=1}^{n_f} \left(\frac{\partial C_{L_{\delta_i}}}{\partial h} \delta_i + \frac{\partial C_{L_{\delta_i^2}}}{\partial h} \delta_i^2 \right) + \frac{\partial C_{L_{\delta_h}}}{\partial h} \delta_h + C_{L_{\delta_h}} \frac{\partial \delta_h}{\partial h} \quad (36)$$

$$\frac{\partial C_D}{\partial h} = \frac{\partial C_{D_0}}{\partial h} + \frac{\partial C_{D_\alpha}}{\partial h} \alpha + \frac{\partial C_{D_{\alpha^2}}}{\partial h} \alpha^2 + \sum_{i=1}^{n_f} \left(\frac{\partial C_{D_{\delta_i}}}{\partial h} \delta_i + \frac{\partial C_{D_{\delta_i^2}}}{\partial h} \delta_i^2 \right) + \frac{\partial C_{D_{\delta_h}}}{\partial h} \delta_h + C_{D_{\delta_h}} \frac{\partial \delta_h}{\partial h} \quad (37)$$

$$\frac{\partial C_L}{\partial \alpha} = C_{L_\alpha} + 2C_{L_{\alpha^2}} \alpha + \sum_{i=1}^{n_f} \left(\frac{\partial C_{L_{\delta_i}}}{\partial \alpha} \delta_i + \frac{\partial C_{L_{\delta_i^2}}}{\partial \alpha} \delta_i^2 \right) + \frac{\partial C_{L_{\delta_h}}}{\partial \alpha} \delta_h + C_{L_{\delta_h}} \frac{\partial \delta_h}{\partial \alpha} \quad (38)$$

$$\frac{\partial C_D}{\partial \alpha} = C_{D_\alpha} + 2C_{D_{\alpha^2}} \alpha + \sum_{i=1}^{n_f} \left(\frac{\partial C_{D_{\delta_i}}}{\partial \alpha} \delta_i + \frac{\partial C_{D_{\delta_i^2}}}{\partial \alpha} \delta_i^2 \right) + \frac{\partial C_{D_{\delta_h}}}{\partial \alpha} \delta_h + C_{D_{\delta_h}} \frac{\partial \delta_h}{\partial \alpha} \quad (39)$$

$$\frac{\partial C_L}{\partial \delta_i} = C_{L_{\delta_i}} + 2C_{L_{\delta_i^2}} \delta_i + C_{L_{\delta_h}} \frac{\partial \delta_h}{\partial \delta_i} \quad (40)$$

$$\frac{\partial C_D}{\partial \delta_i} = C_{D_{\delta_i}} + 2C_{D_{\delta_i^2}} \delta_i + C_{D_{\delta_h}} \frac{\partial \delta_h}{\partial \delta_i} \quad (41)$$

$$\frac{\partial \delta_h}{\partial h} = - \frac{\frac{\partial \Delta C_m}{\partial h} + \frac{z_e}{c} \left\{ \frac{\partial \Delta C_D}{\partial h} + \frac{W}{\bar{q} S} \left[\frac{M^2}{g} \left(\frac{da}{dh} \right)^2 + \frac{M^2 a}{g} \frac{d^2 a}{dh^2} - \frac{dp}{pdh} \left(1 + \frac{M^2 a}{g} \frac{da}{dh} \right) \right] \gamma \right\}}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} \quad (42)$$

$$\frac{\partial \delta_h}{\partial \alpha} = - \frac{\frac{\partial \Delta C_m}{\partial \alpha} + \frac{z_e}{c} \left[\frac{\partial \Delta C_D}{\partial \alpha} - \frac{W}{\bar{q} S} \left(1 + \frac{M^2 a}{g} \frac{da}{dh} \right) \right]}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} - \frac{\delta_h \left(\frac{\partial C_{m_{\delta_h}}}{\partial \alpha} + \frac{z_e}{c} \frac{\partial C_{D_{\delta_h}}}{\partial \alpha} \right)}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} \quad (43)$$

$$\frac{\partial \delta_h}{\partial \delta_i} = - \frac{\frac{\partial \Delta C_m}{\partial \delta_i} + \frac{z_e}{c} \frac{\partial \Delta C_D}{\partial \delta_i}}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} \quad (44)$$

$$\frac{\partial \Delta C_D}{\partial h} = \frac{\partial C_{D_0}}{\partial h} + \frac{\partial C_{D_\alpha}}{\partial h} \alpha + \frac{\partial C_{D_{\alpha^2}}}{\partial h} \alpha^2 + \sum_{i=1}^{n_f} \left(\frac{\partial C_{D_{\delta_i}}}{\partial h} \delta_i + \frac{\partial C_{D_{\delta_i^2}}}{\partial h} \delta_i^2 \right) \quad (45)$$

$$\frac{\partial \Delta C_m}{\partial h} = \frac{\partial C_{m_0}}{\partial h} + \frac{\partial C_{m_\alpha}}{\partial h} \alpha + \frac{\partial C_{m_{\alpha^2}}}{\partial h} \alpha^2 + \sum_{i=1}^{n_f} \left(\frac{\partial C_{m_{\delta_i}}}{\partial h} \delta_i + \frac{\partial C_{m_{\delta_i^2}}}{\partial h} \delta_i^2 \right) \quad (46)$$

$$\frac{\partial \Delta C_D}{\partial \alpha} = C_{D_\alpha} + 2C_{D_{\alpha^2}} \alpha + \sum_{i=1}^{n_f} \left(\frac{\partial C_{D_{\delta_i}}}{\partial \alpha} \delta_i + \frac{\partial C_{D_{\delta_i^2}}}{\partial \alpha} \delta_i^2 \right) \quad (47)$$

$$\frac{\partial \Delta C_m}{\partial \alpha} = C_{m_\alpha} + 2C_{m_{\alpha^2}} \alpha + \sum_{i=1}^{n_f} \left(\frac{\partial C_{m_{\delta_i}}}{\partial \alpha} \delta_i + \frac{\partial C_{m_{\delta_i^2}}}{\partial \alpha} \delta_i^2 \right) \quad (48)$$

$$\frac{\partial \Delta C_D}{\partial \delta_i} = C_{D_{\delta_i}} + 2C_{D_{\delta_i^2}} \delta_i \quad (49)$$

$$\frac{\partial \Delta C_m}{\partial \delta_i} = C_{m_{\delta_i}} + 2C_{m_{\delta_i^2}} \delta_i \quad (50)$$

V. Implementation

The optimality condition is expressed as

$$(1 - \lambda_W) c W M a \frac{\partial C_D}{\partial \delta_i} - \lambda_\alpha g \frac{\partial C_L}{\partial \delta_i} = 0 \quad (51)$$

where

$$\frac{\partial C_L}{\partial \delta_i} = \bar{C}_{L_{\delta_i}} + 2\bar{C}_{L_{\delta_i^2}} \delta_i \quad (52)$$

$$\frac{\partial C_D}{\partial \delta_i} = \bar{C}_{D_{\delta_i}} + 2\bar{C}_{D_{\delta_i^2}} \delta_i \quad (53)$$

with

$$\bar{C}_{L_{\delta_i}} = C_{L_{\delta_i}} - \frac{C_{L_{\delta_h}} \left(C_{m_{\delta_i}} + \frac{z_e}{c} C_{D_{\delta_i}} \right)}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} \quad (54)$$

$$\bar{C}_{L_{\delta_i^2}} = C_{L_{\delta_i^2}} - \frac{C_{L_{\delta_h}} \left(C_{m_{\delta_i^2}} + \frac{z_e}{c} C_{D_{\delta_i^2}} \right) \delta_i}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} \quad (55)$$

$$\bar{C}_{D_{\delta_i}} = C_{D_{\delta_i}} - \frac{C_{D_{\delta_h}} \left(C_{m_{\delta_i}} + \frac{z_e}{c} C_{D_{\delta_i}} \right)}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} \quad (56)$$

$$\bar{C}_{D_{\delta_i^2}} = C_{D_{\delta_i^2}} - \frac{C_{D_{\delta_h}} \left(C_{m_{\delta_i^2}} + \frac{z_e}{c} C_{D_{\delta_i^2}} \right) \delta_i}{C_{m_{\delta_h}} + \frac{z_e}{c} C_{D_{\delta_h}}} \quad (57)$$

The optimal flap deflections are obtained explicitly as

$$\delta_i = - \frac{(1 - \lambda_W) c W M a \bar{C}_{D_{\delta_i}} - \lambda_\alpha g \bar{C}_{L_{\delta_i}}}{2(1 - \lambda_W) c W M a \bar{C}_{D_{\delta_i^2}} - 2\lambda_\alpha g \bar{C}_{L_{\delta_i^2}}} \quad (58)$$

The trajectory optimization problem is characterized as a two-point boundary value problem. The equations of motion are specified by the initial conditions $h(0) = h_0$, $\alpha(0) = \alpha_0$, and $W(0) = W_0$. On the other hand, the adjoint

equations are specified by the transversality conditions $\lambda_h(r) = 0$, $\lambda_\alpha(r) = 0$, and $\lambda_W(r) = 0$. We introduce the distance-to-go variable $x_r = r - x$, then the adjoint equations are transformed as

$$\frac{d\lambda_h}{dx_r} = -\lambda'_h \quad (59)$$

$$\frac{d\lambda_\alpha}{dx_r} = -\lambda'_\alpha \quad (60)$$

$$\frac{d\lambda_W}{dx_r} = -\lambda'_W \quad (61)$$

where the transversality conditions are transformed into the initial conditions in the distance-to-go variable as $\lambda_h(x_r = 0) = 0$, $\lambda_\alpha(x_r = 0) = 0$, and $\lambda_W(x_r = 0) = 0$.

A shooting method is implemented for the two-point boundary value problem. The flap deflections $\delta_i^{(k)}(x)$ and the initial adjoint solutions of $\lambda_h^{(k)}(x_r)$, $\lambda_\alpha^{(k)}(x_r)$, and $\lambda_W^{(k)}(x_r)$ are initialized with zero values.

The equations of motion are then integrated numerically to obtain the trajectory solutions of $h^{(k+1)}(x)$, $\alpha^{(k+1)}(x)$, and $W^{(k+1)}(x)$. With the trajectory solutions and the optimal flap deflections, the adjoint equations are integrated in the distance-to-go variables to obtain the adjoint solutions of $\lambda_h^{(k+1)}(x_r)$, $\lambda_\alpha^{(k+1)}(x_r)$, and $\lambda_W^{(k+1)}(x_r)$. With the new adjoint solutions, the optimal flap deflections $\delta_i^{(k+1)}(x)$ are updated with

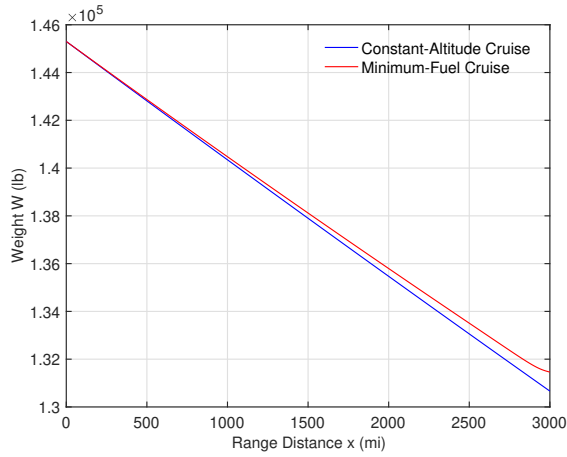
$$\delta_i^{(k+1)} = -\frac{\left(1 - \lambda_W^{(k+1)}\right) c^{k+1} W^{(k+1)} M a^{(k+1)} \bar{C}_{D\delta_i}^{(k+1)} - \lambda_\alpha^{(k+1)} g \bar{C}_{L\delta_i}^{(k+1)}}{2 \left(1 - \lambda_W^{(k+1)}\right) c^{(k+1)} W^{(k+1)} M a^{(k+1)} \bar{C}_{D\delta_i^2}^{(k+1)} - 2 \lambda_\alpha^{(k+1)} g \bar{C}_{L\delta_i^2}^{(k+1)}} \quad (62)$$

The optimal flap deflections $\delta_i^{(k+1)}(x)$ are also limited to within $\pm 5^\circ$ which are slightly outside the aerodynamic database. The solution process is repeated over several iterations until the trajectory solutions converge.

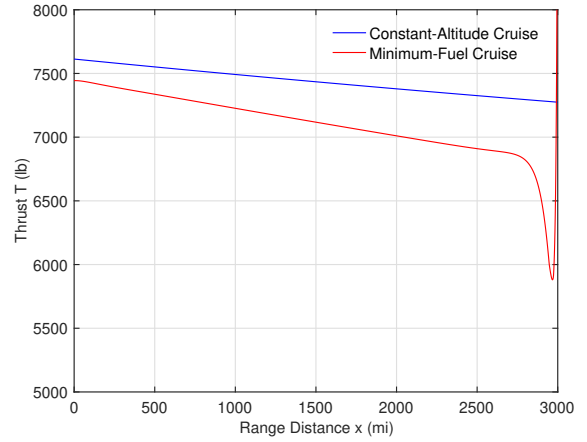
VI. Results

The initial altitude is set at 38,700 ft and the initial gross weight of the Mach 0.745 TTBW is 145,300 lbs. The trajectory is optimized over a cruise range of 3,000 miles. The thrust specific fuel consumption is assumed to be 0.3225 /hr at Mach 0.745 and an altitude of 38,700 ft. The initial altitude and gross weight are selected such that the mid-cruise gross weight is reduced to about 138,000 lbs at a mid-cruise altitude of about 40,000 ft. The mid-cruise point is defined to be where the distance traveled is half way through the cruise. The results of the trajectory optimization are presented in the following figures. For comparison, the baseline trajectory for a constant-altitude cruise at 38,700 ft for the Mach 0.745 TTBW without the VCCTEF is also presented.

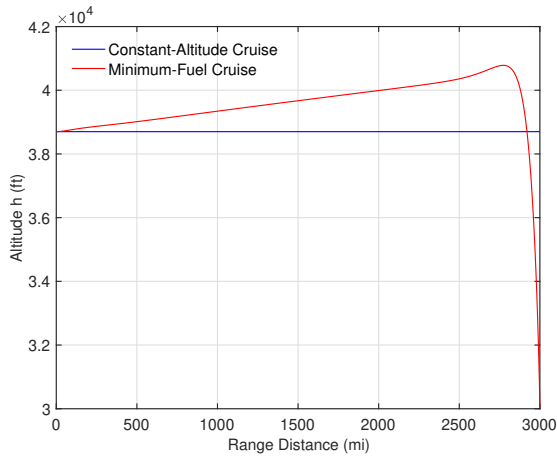
Figures 11(a)-(f) show solutions of the minimum-fuel trajectory optimization and the baseline constant-altitude cruise for comparison. As can be seen in Fig. 11(a), the final weight of the aircraft with the minimum-fuel cruise is greater than that with the constant-altitude cruise. The fuel burn with the constant-altitude cruise is 14,360 lbs. A fuel burn reduction of 793 lbs is achieved. This represents a 5.5% fuel burn reduction. The reduction in the thrust requirement for the minimum-fuel cruise varies from about 2.2% at the start of cruise to 6.0% near the end of cruise where the range distance is 2740 mi as shown in Fig. 11(b). The optimal control action near the end of cruise causes a significant reduction in the thrust requirement followed by a sharp rise in the thrust requirement. This is due to the enforcement of the transversality conditions which require the optimal flap deflections to be specified with the zero values of the adjoint variable $\lambda_W(x)$ and $\lambda_\alpha(x)$. The minimum-fuel cruise is a climb cruise as the altitude increases with the range distance as shown in Fig. 11(c). The altitude increase results in a cruise at a nearly constant maximum aerodynamic efficiency measured by the maximum lift-to-drag ratio L/D which is shown to be about 19.3 in Fig. 11(d). Figures 11(e) and (f) show the trajectories of the angle of attack and the horizontal tail incidence angle. The angle of attack for the constant-altitude cruise steadily decreases as the range distance increases while the horizontal tail incidence angle increases steadily. On the other hand, both the angle of attack and the horizontal tail incidence angle for the minimum-fuel cruise are nearly constant. Near the end of cruise, the angle of attack increases rapidly. This is due to the enforcement of the transversality conditions.



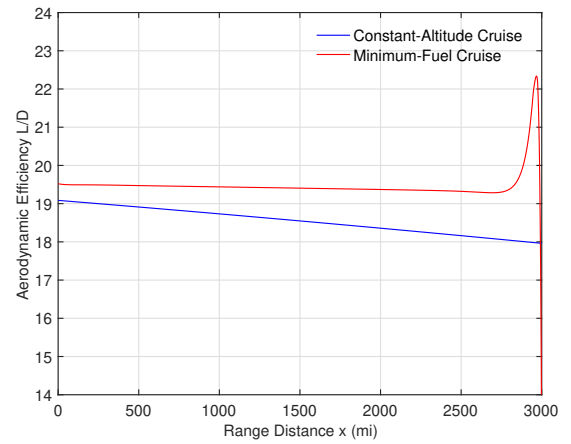
(a)



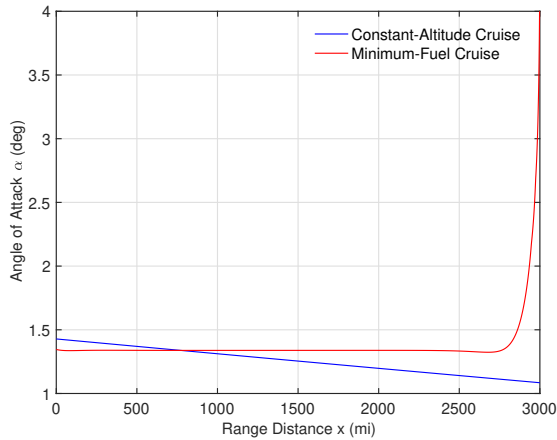
(b)



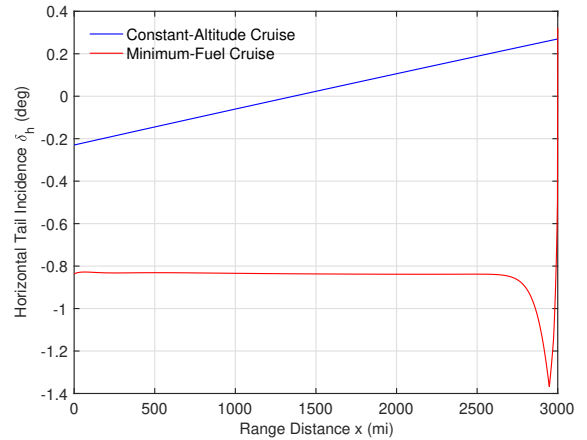
(c)



(d)



(e)



(f)

Figure 11. Solutions of Minimum-Fuel Trajectory Optimization and Constant-Altitude Cruise

Figure 12(a) shows the plots of the adjoint solutions. The solutions of $\lambda_h(x)$ and $\lambda_\alpha(x)$ are nearly constant. The solution of $\lambda_w(x)$ is nearly linear with the range distance. Figure 12(b) shows the optimal flap deflections computed from the adjoint solutions. The optimal flap deflections are nearly constant except near the end of cruise where they

decrease rapidly due to the enforcement of the transversality. All but one flap deflections range from -1° to 2° . The deflection of flap 12 is at the maximum flap deflection limit of 5° .

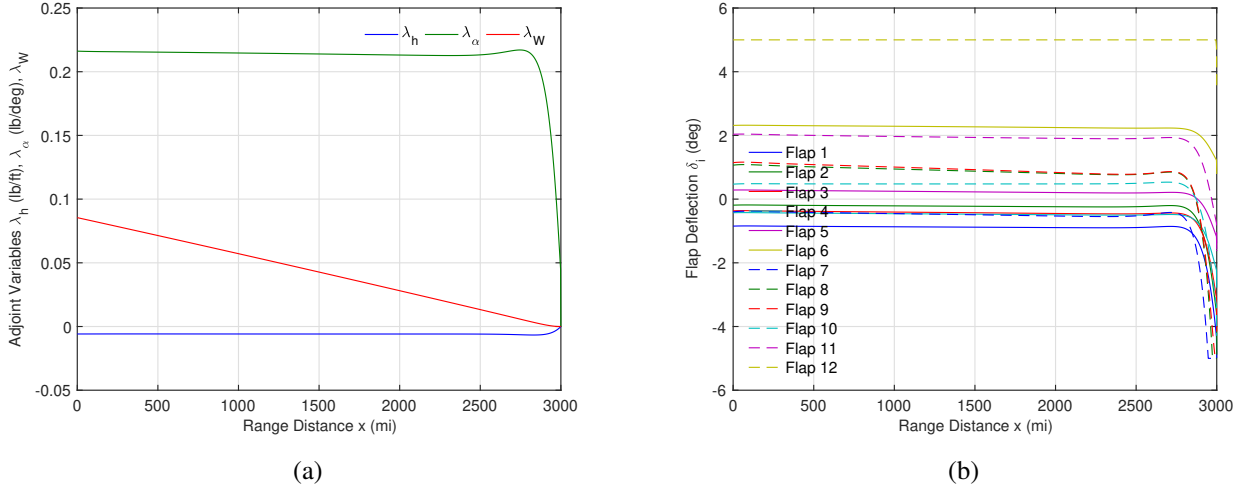


Figure 12. Adjoint Solutions and Optimal Flap Deflections

VII. Approximate Trajectory Optimization

Solving a two-point boundary value problem is usually a difficult endeavor. Approximate trajectory optimization can be highly desired as it can provide a rapid optimal trajectory optimization for a mission analysis. From the adjoint solutions, we see that $\lambda'_h(x) \approx 0$ and $\lambda_w(x) \approx 0$. Therefore, we can approximate the adjoint equation for $\lambda_\alpha(x)$ by setting $\lambda'_h(x) = 0$ and $\lambda_w(x) = 0$. This results in

$$0 = -\frac{\partial c}{\partial h} \left[\frac{D}{Ma} + W \left(\frac{1}{Ma} + \frac{M}{g} \frac{da}{dh} \right) \gamma \right] - c \left[\frac{1}{Ma} \frac{\partial D}{\partial h} - \frac{D}{Ma^2} \frac{da}{dh} + W \left(-\frac{1}{Ma^2} \frac{da}{dh} + \frac{M}{g} \frac{d^2a}{dh^2} \right) \gamma \right] - \frac{2\lambda_\alpha g}{M^2 a^3} \frac{da}{dh} \left(1 - \frac{L}{W} \right) + \frac{\lambda_\alpha g}{WM^2 a^2} \frac{\partial L}{\partial h} \quad (63)$$

Thus, $\lambda_\alpha(x)$ can be solved from Eq. (63). If an aircraft flies at an altitude above 36,089 ft where the temperature and the speed of sound are constant with the altitude. Then, Eq. (63) can be simplified as

$$0 = -\frac{c}{Ma} \frac{\partial D}{\partial h} + \frac{\lambda_\alpha g}{WM^2 a^2} \frac{\partial L}{\partial h} \quad (64)$$

which yields

$$\lambda_\alpha = \frac{cWMa}{g} \frac{\frac{\partial D}{\partial h}}{\frac{\partial L}{\partial h}} \quad (65)$$

The adjoint variable $\lambda_\alpha(x)$ thus is seen to account for the effect of the ratio of the lift variation with the altitude to the drag variation with the altitude.

The approximate optimality condition becomes

$$\frac{\partial C_D}{\partial \delta_i} - \frac{\frac{\partial D}{\partial h}}{\frac{\partial L}{\partial h}} \frac{\partial C_L}{\partial \delta_i} = 0 \quad (66)$$

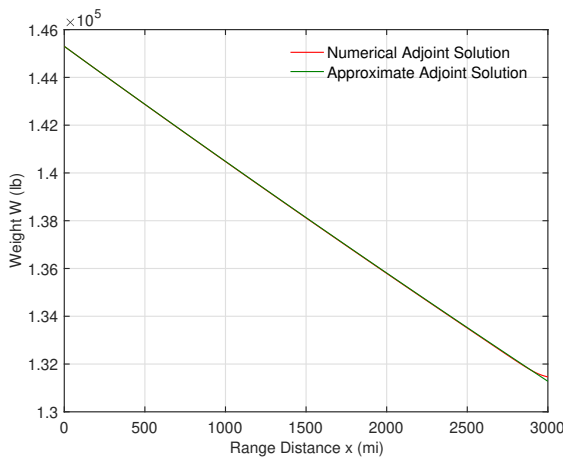
The approximate optimal flap deflection is obtained explicitly as

$$\delta_i = -\frac{\bar{C}_{D\delta_i} - \frac{\frac{\partial D}{\partial h}}{\frac{\partial L}{\partial h}} \bar{C}_{L\delta_i}}{2 \left(\bar{C}_{D\delta_i^2} - \frac{\frac{\partial D}{\partial h}}{\frac{\partial L}{\partial h}} \bar{C}_{L\delta_i^2} \right)} \quad (67)$$

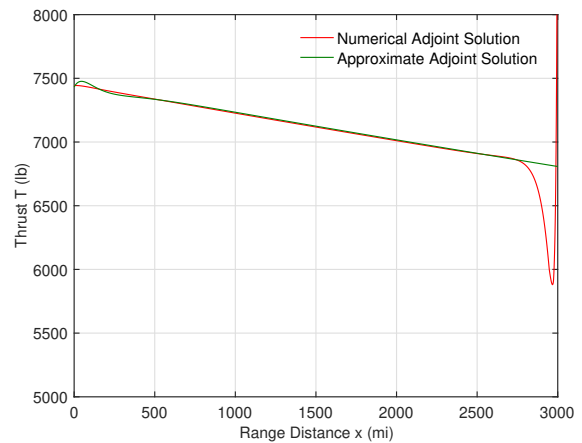
This expression is highly convenient because the adjoint variable $\lambda_\alpha(x)$ is completely eliminated from the optimal flap deflection, thereby enabling a rapid computation of a minimum-fuel trajectory without incurring the computational expense and the difficulty of setting up a numerical method for the trajectory optimization problem by solving the adjoint equations or by a direct numerical optimization method.

An approximate trajectory optimization is conducted and compared to the numerical solution obtained by the shooting method for the two-point boundary value problem in the previous section. Figures 13 (a)-(f) compare the approximate trajectory optimization to the numerical trajectory optimization presented in the previous section. As can be seen, the approximate trajectory optimization is in excellent agreement with the numerical trajectory optimization when the effect of the transversality condition near the end of cruise is neglected. The approximate trajectory optimization produces a fuel burn reduction of 615 lbs or 4.3% vs. a fuel burn reduction of 793 lbs or 5.5% obtained from the numerical trajectory optimization. It should be noted that the numerical trajectory optimization shows that the altitude is reduced near the end of cruise whereas the altitude with the approximate trajectory optimization steadily increases with the range distance. The difference in the altitude at the end of cruise is 10,517 ft. If the fuel burn during the descent is accounted for, the approximate trajectory optimization would result in a lower fuel burn reduction than as indicated.

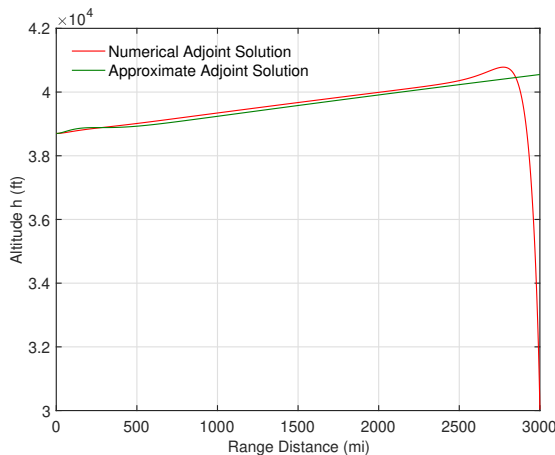
Figure 14(a) is the plot of the approximate adjoint solution of $\lambda_\alpha(x)$. The initial value of the approximate solution of $\lambda_\alpha(x)$ is slightly higher than that of the numerical solution, but the approximate solution of $\lambda_\alpha(x)$ converges to the numerical solution near the end of cruise. The difference is attributed to the assumption of $\lambda_W(x) = 0$ used in the approximate trajectory optimization. Figure 14(b) is the plot of the flap deflections. The flap deflections are in agreement with those obtained from the numerical trajectory optimization except near the end of cruise.



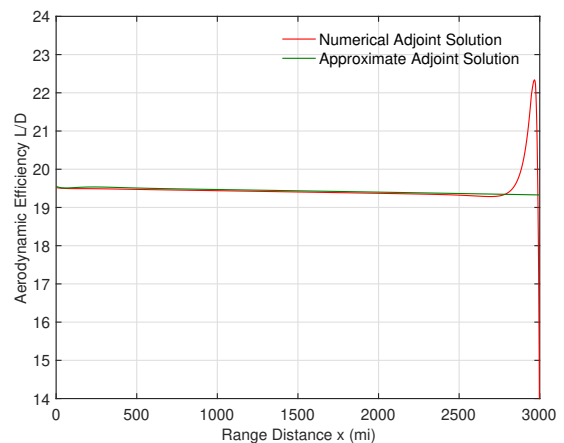
(a)



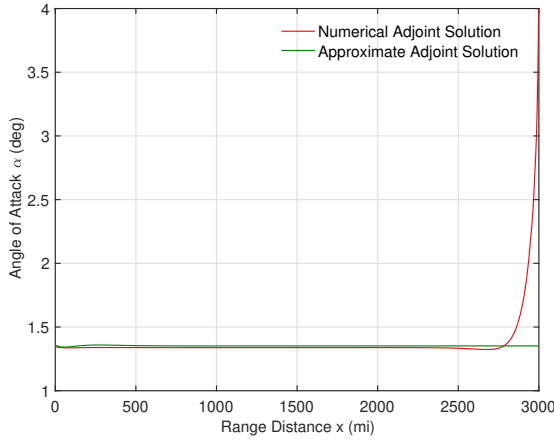
(b)



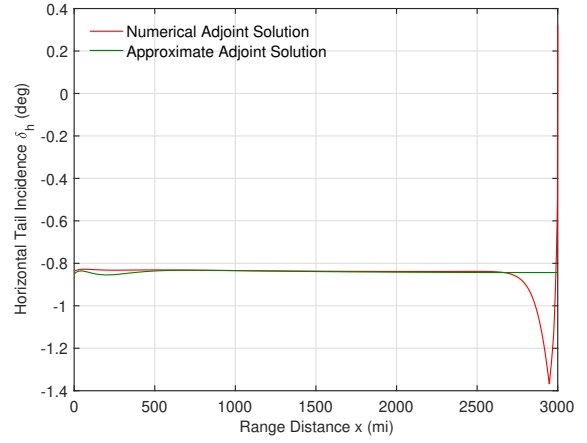
(c)



(d)

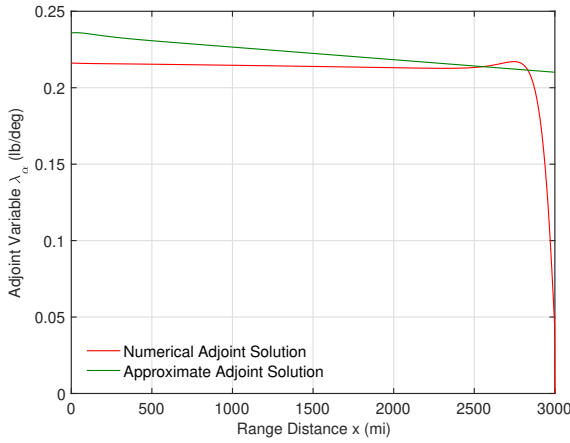


(e)

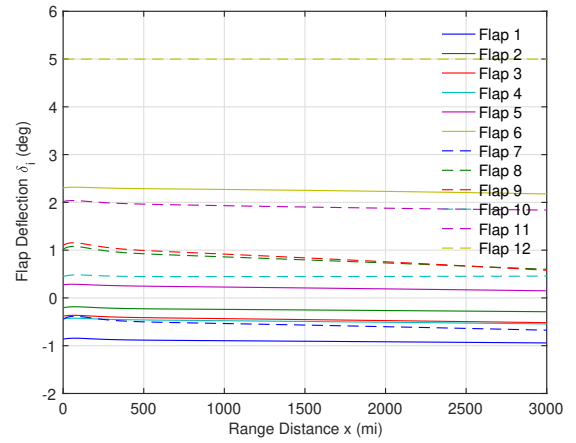


(f)

Figure 13. Approximate Solution vs. Numerical Solution of Minimum-Fuel Trajectory Optimization



(a)



(b)

Figure 14. Approximate Adjoint Solution and Optimal Flap Deflections

Thus, it can be seen that the approximate trajectory optimization provides an excellent approximate solution for a minimum-fuel trajectory optimization problem. The computational procedure for the approximate adjoint solution is much simpler than the procedure for solving a two-point boundary value problem or a direct numerical optimization method.

VIII. Conclusions

This paper presents a study of a minimum-fuel trajectory optimization of the Mach 0.745 Transonic Truss-Braced Wing aircraft. The trajectory optimization accounts for the aeroelastic effects on the aerodynamic performance of the aircraft. An aerodynamic database is computed by an aeroelastic model of the Mach 0.745 Transonic Truss-Braced Wing using the VSPAERO solver with a transonic and viscous correction coupled to an in-house finite-element code BEAM3D. A trim solution is implemented to trim out the pitching moment with the horizontal tail incidence angle. An adjoint method is formulated that results in a two-point boundary value problem. The solution of the two-point boundary value problem is obtained by a numerical shooting method. The numerical results of the trajectory optimization indicates a fuel burn reduction of 793 lbs or 5.5% for a range of 3000 miles.

An approximate trajectory optimization is proposed using a single adjoint variable. This leads to an analytical solution of the adjoint solution without solving a differential equation for the adjoint variable. The numerical results of the approximate trajectory optimization are in excellent agreement with the results of the numerical trajectory optimization obtained by the shooting method. The approximate trajectory optimization produces a fuel burn reduction of 615 lbs or 4.3%.

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