A Review Towards the Design Optimization of High-Performance Additively Manufactured Rotating Detonation Rocket Engine Injectors

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Rotating Detonation Rocket Engines (RDRE) have been marketed primarily for their higher specific impulse potential over constant pressure (CP) liquid rocket engines. However, several other performance advantages exist such as heat transfer advantages for gas expander cycle, increased completeness of combustion at low chamber L*, compact engine design, reduced coolant channel pressure drop potential, and improved injector C* performance. NASA has paved the way for liquid engine system performance enhancement since the Apollo program and continues to do so with metal additive manufacturing (AM), new advanced materials, and advanced propulsion concepts. A team of propulsion development engineers at NASA are in the process of developing high-performance 7K lbf class RDRE hardware for their potential use in lander, upper stage, and even launch vehicle applications. Clear advantages have been demonstrated with AM including program cost and schedule reductions of up to 50%. It is well known that injector performance is integrally linked to the global performance of a combustion device. This is especially the case for RDREs since detonation stability is heavily dependent on the mixedness of propellants. A major goal of this work is to identify what has been done in the open experimental literature and what injectors design features are conducive to high performance in the detonation cycle. This paper reviews the available literature and reports the primary gaps in the knowledge base needed by the pressure gain combustion (PGC) community. Major conclusions are documented, and suggestions given towards the design of high-performance liquid RDRE injectors. In addition, the integration of metal AM into the design of liquid RDRE injectors is included.

I. Nomenclature

AFRL	=	Air Force Research Laboratory
AM	=	additive manufacturing
CP	=	constant pressure
CTAP	=	capillary tube attenuated pressure
*C**	=	characteristic exhaust velocity
Isp	=	specific impulse
L-PBF	=	laser powder bed fusion
LP-DED	=	laser powder directed energy deposition
*L**	=	chamber characteristic length (volume / throat area)
MSFC	=	Marshall Space Flight Center
NASA	=	National Aeronautics and Space Administration

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PGC	=	pressure gain combustion
RDRE	=	rotating detonation rocket engine
RDE	=	rotating detonation engine
TCA	=	thrust chamber assembly

II. Introduction

Detonations have been substantially researched in ducts, area contractions, and various combustor geometries. Their underlying physics are well understood with classical models that have been long since established. The rapid increase in pressure across a detonation front minimizes enthalpy generation causing high thermal efficiency when compared to the Brayton or Atkins cycles for constant pressure (CP) combustion. Harnessing this efficiency gain for practical use in modern combustor technology is another challenge altogether. The common solution for maintaining a stable detonation is by making use of an annular combustion chamber with axial or radial injection of propellants. Numerous experimental works have been conducted to date in an effort to understand the effects of geometry, scaling, and operability of this type of detonative combustor configuration. Not to mention the sheer magnitude of modeling efforts conducted to date. Regardless, these works have substantially researched the efficacy of using detonations to increase engine efficiency and are currently at a crossroads in terms of their development in pushing eventually towards flight scale engines.

It is well known that the predominant theoretical benefit to detonative combustion devices is the promised ~10-15% boost in specific impulse (Isp) compared to the maximum theoretical Isp in an equivalent CP combustor. However, other major performance characteristics lend advantages with these engines rather than solely quantities such as Isp performance. For one, these engines are often far more compact allowing for reduced hardware mass. The completion of combustion has been shown to occur much faster than in CP combustors and thus drastically reduces L* requirements while still maximizing characteristic velocity (C*) efficiency, though this quantity is much more difficult to measure in this engine scheme. Other works have suggested that these combustion devices could produce lower overall heat fluxes and lower total heat load to the actively cooled geometries. In addition, several of the literature sources reviewed indicate that the heat flux curve in a detonative combustor may decrease steadily as the flow gets closer to the chamber exit. This would allow for the design of ultra-low pressure drop integrated coolant channels, which is another major performance advantage for this engine type. Each of these performance criteria make a strong case for investment into detonative combustion device technologies. With that said, no single literature source summarizes the design optimization procedure for a detonative combustor or detonative liquid engine system. There are only design rules of thumb available that have been described as minimum geometry constraints for chamber design. It is the goal of this work to review, discuss, and summarize major findings in the available literature for;

1. The optimization of an annular type rotating detonation rocket engine utilizing cryogenic liquid/gas propellants.
2. Discuss the desirable performance criteria for thermal steady state RDRE liquid injector operation.
3. Characterize the feasibility and advantages of using additive manufacturing techniques.

The term rotating detonation engine (RDE) or even rotating detonation rocket engine (RDRE) are general terms used interchangeably in the literature that refers to any combustion device utilizing a continuous detonation to achieve combustion. The RDE configuration typically uses free available air and RDRE a stored oxidizer such as liquid oxygen (LOX). These terms are used interchangeably in this work since design constraints apply universally to both configurations and are dependent on propellant type utilized rather than injector configuration.

The majority of the experimental work conducted to date on RDE's utilize short duration heat sink type continuous detonation. Several examples of which are the works of [1]–[8] and many others. In addition, most of these efforts utilize air as the oxidizer including [9]–[12] and many others. One focus of this work is to identify hardware design concepts that can successfully achieve flight practical detonative burns while maintaining reasonable hardware lifecycle performances. The means by which this will be achieved is the incorporation of radially running square coolant channels directly integrated into a laser- powder bed fusion built (L-PBF) GRCop-alloy annular chamber.

NASA's Marshall Space Flight Center (MSFC) has actively utilizes regeneratively cooled metal additive manufactured (AM) thrust chamber hardware and has successfully demonstrated this hardware at stoichiometric conditions for cumulatively tens of thousands of seconds and hundreds of starts [13], [14]. Several images of these hot fire tests, in most cases use fully AM thrust chamber assemblies (TCA), are presented below.

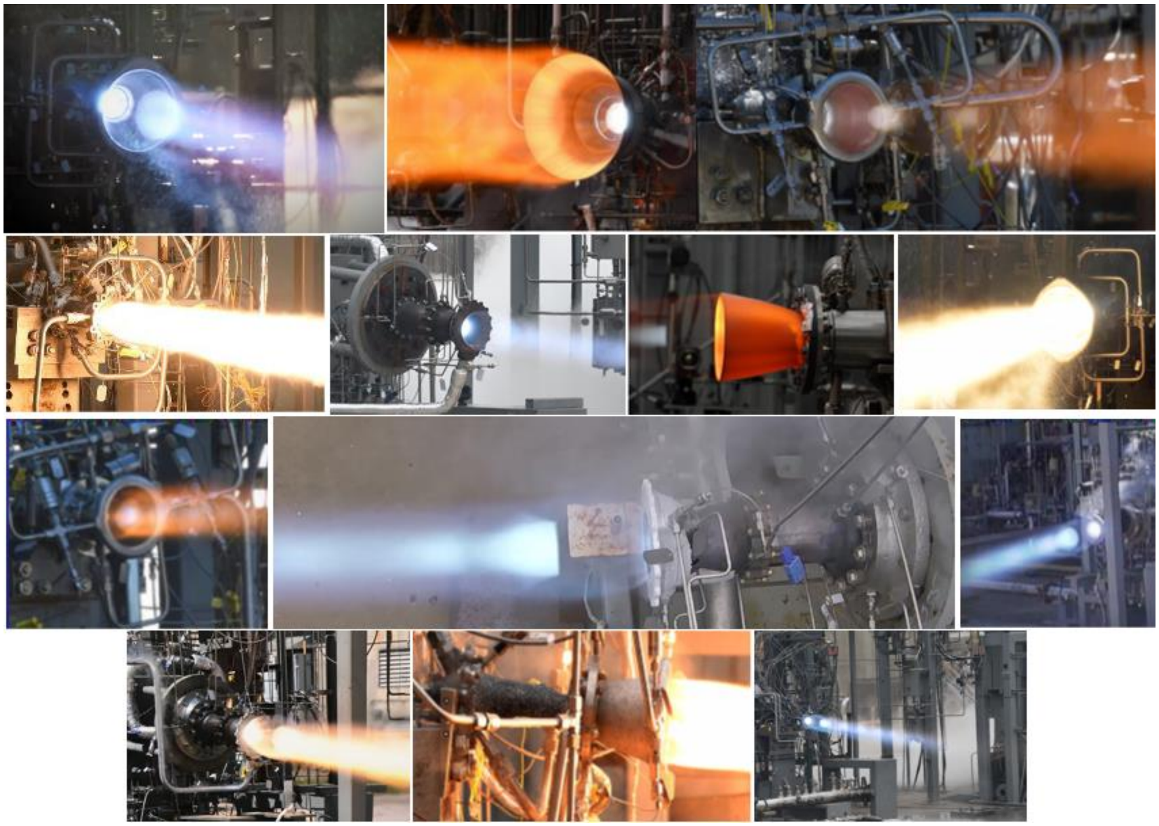

Fig. 1 Hot fire testing of metal additively manufactured thrust chamber assemblies including additive injectors, chambers, igniters, and nozzles in various propellant combinations and thrust classes.

Several of these metal AM components are operated in extreme environments for hundreds of seconds at a time. The highly conductive material GRCop-84/42, a NASA invented high conductance alloy, has been the basis for this success. Other materials that are easily used in AM include Inconel 625/718, JBK-75, Monel K500, NASA HR-1, and numerous other refractory and super-alloy materials. A list of metal additive materials commonly used in industry and grouped by their base material are shown in the figure below.

Industry Materials developed for L-PBF, E-PBF, and DED processes *(not fully inclusive)*

Ni-Base
Inconel 625
Inconel 718
Hastelloy-X
Haynes 230
Haynes 282
Haynes 188
Monel K-500
C276
Rene 80
Waspalloy

Cu-Base
GRCop-84
GRCop-42
C-18150
C-18200
Glidcop
CU110

Al-Base
AlSi10mg
A205
F357
6061 / 4047

Fe-Base
SS 17-4PH
SS 15-5 GP1
SS 304
SS 316L
SS 420
Tool Steel (4140/4340)
Invar 36
SS347
JBK-75
NASA HR-1

Ti-Base
Ti6Al4V
γ-TiAl
Ti-6-2-4-2

Co-Base
CoCr
Stellite 6, 21, 31

Refractory
W
W-25Re
Mo
Mo-41Re
Mo-47.5Re
C-103
Ta

Bimetallic
GRCop-84/IN625
C-18150/IN625

MMC
Al-base
Fe-base
Ni-base

Fig. 2 Metal additive materials commonly used in industry.

Of particular interest for use in RDRE injector design are the high-temperature nickel alloy material Inconel and iron alloy materials NASA HR-1 / JBK-75. These have been used in AM injectors and nozzle hardware under numerous test projects at MSFC. Several metal additive injectors have also been produced and tested and are shown in the figure below.

Fig. 3 Examples of various thrust class and material metal additively manufactured liquid rocket injectors hot fire tested at NASA MSFC.

All tested injectors shown above demonstrated exceptional life cycle performances with cumulatively hundreds of starts and tens of thousands of seconds of mainstage duration. For this reason alone, these materials would be an ideal candidate for use in RDRE injectors. The reader is referred to [15] which gives a comprehensive summary of AM injectors hot fire tested at NASA MSFC up until 2018. A more recent work is currently being prepared. Since reasonable life cycle performance is desired during a long duration burn of an RDRE, these materials would stand the best chance for demonstrating hardware survivability along with reduced risk of build failure. Several images of example nickel-alloy injector designs that have been tested at NASA MSFC and demonstrate exceptional life cycle performances are shown in the figures below.

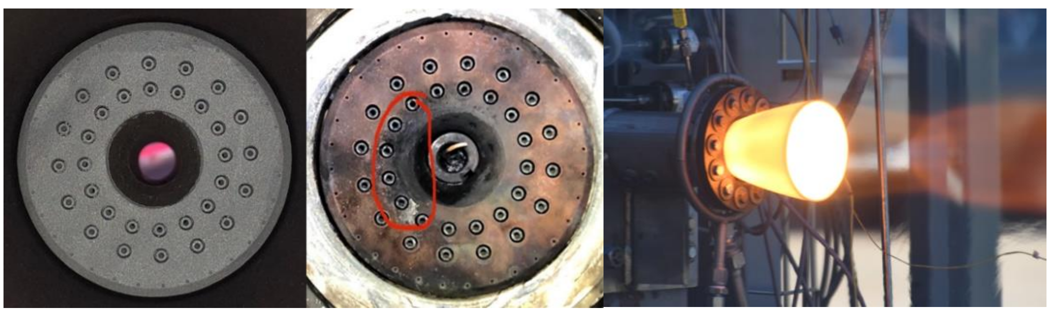

Fig. 4 JBK-75 additively manufactured shear coaxial injector. (Left) Prior to testing under NASA programs PJ141, PK020, and PK111. (Middle) Post-testing with More than 4130 seconds of cumulative duration and 36 starts over 3 test series in LOX/GH2 at a nominal Pc of 1000 psi. Slight face erosion shown in the red circle but not deleterious to injector operation. (Right) Hot fire test demonstration with NASA carbon composite nozzle.

The injector shown above used a JBK-75 additive material produced using the L-PBF process with a 32-element shear coaxial heritage design. This injector demonstrated great life cycle performance with no element erosion and minimal face erosion circled in red. This injector produced a very high C* performance of nearly 100% theoretical in some test cases. This is, however, typical of LOX/GH2 injectors when coupled with high L* chambers. This was also a lower thrust class injector, relatively speaking, producing thrusts upwards of 2500 lbf calculated. An example of a 7K lbf class lander scale Inconel 625 pentad impingement injector is shown in the figure below.

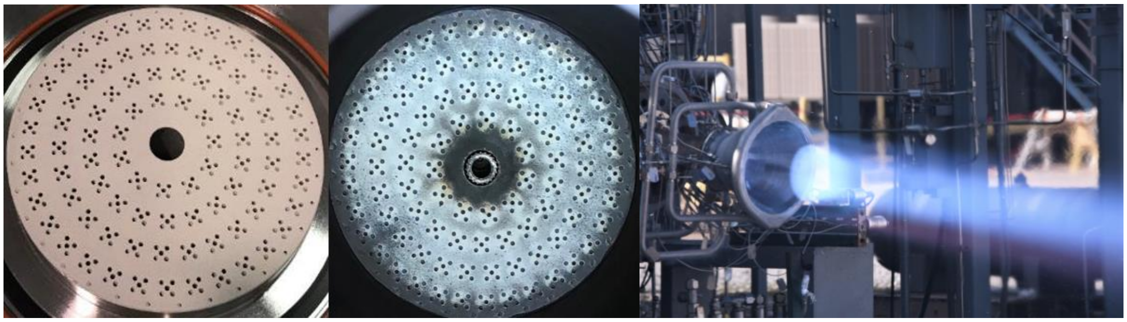

Fig. 5 Inconel 625 L-PBF 7K lbf class pentad impinging injector face. (Left) pre-hot fire testing with NiCrAlY thermal barrier. (Middle) More than 1100 seconds of cumulative hot fire duration and greater than 52 starts in LOX/GCH4 at a nominal Pc of 750 psi and mixture ratios up to 3.6. (Right) Hot fire test demonstration of AM pentad impinging injector, L-PBF GRCop-42 chamber, and LP-DED NASA HR-1 regenerative nozzle.

This injector design produced high C* efficiencies upwards of 99% theoretical at specific test conditions and was throttled between 114 psi and 760 psi. These two previously tested injector configurations show reasonably high C* performance and lifecycle performances that can be achieved with additive materials under extreme hot fire conditions. An RDRE variant AM injector will need to have similar mixing, atomization, and life cycle performances to successfully demonstrate flight practical hot fire test durations as well as demonstrating equivalent or higher engine specific impulse.

It is not well understood or even completely known if modern combustion device materials and cooling schemes will be able to handle the extreme environments that an RDRE is expected to produce. This is especially the case for flight practical operating durations needed during space missions and on launch vehicles. Current NASA efforts are underway to demonstrate that flight practical durations are possible in RDREs. These efforts will characterize the life cycle performances of a 7K lbf class AM chamber geometries, injectors, and nozzles. There are a substantial number of considerations for hardware development of high thrust regeneratively cooled and additively manufactured hardware as illustrated in the considerations process diagram figure below.

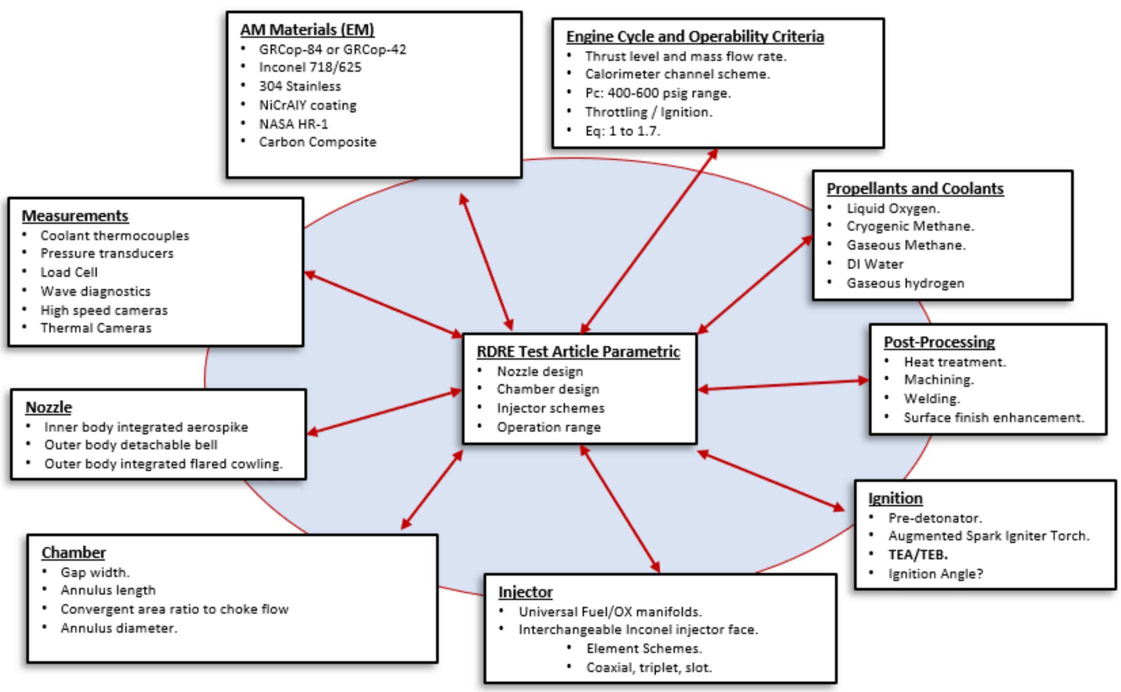

Fig. 6 Consideration diagram for development of an optimized performance RDRE.

This work primarily focuses on RDRE injector hardware design development in preparation for upcoming experimental work at NASA. Several topics and considerations are discussed in reference to optimizing performance for the detonative combustion cycle. However, a great deal is still unknown to the community at large and significant gaps in the literature still exist.

III. Background and Literature Review

All The available literature was reviewed on annular continuous detonation cycle injectors experimentally tested to date. In addition, an in-depth review on the available CP combustion device injector design was also conducted to identify attributes of high performing element schemes. These reviews are presented below.

A. RDE/RDRE Injector Research to Date

A limited amount of work has been conducted on detonation combustion device injection. Several articles are in the available literature that describe how injection properties, injector design, and injection mixing effects RDE operation. It was found that air breathing injection schemes dominate the experimental literature in terms of most tests conducted. The typical scheme used is the annular slot injector or annular cross flow orifice injector. It is also very common to see variations in radial and axial injection schemes. Two examples of injection feed systems are shown in the figure below from [1], [16].

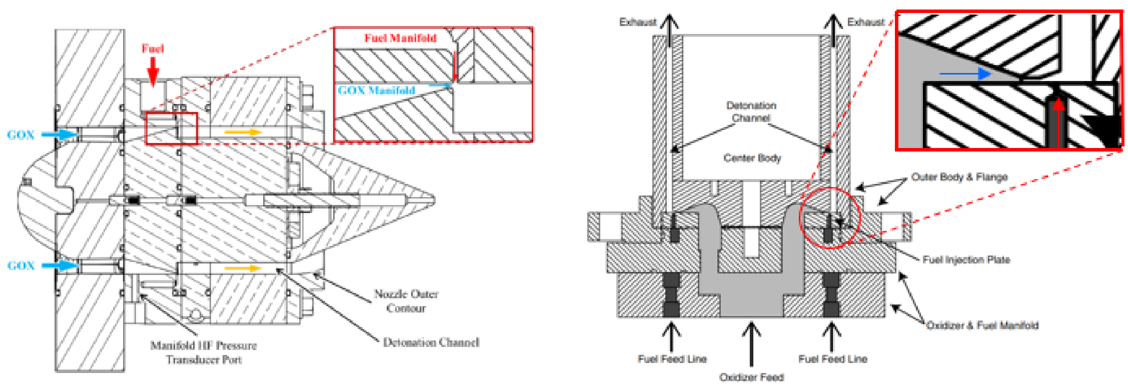

Fig. 7 Injection feed systems tested and reported in (left) [16] **and (right)** [1]**.**

The propellant phase commonly employed is the gas/gas bi-propellant combination with hydrogen, methane, or natural gas (NG) commonly used as fuel and air or gaseous oxygen as oxidizer is typical. Other works have utilized propane, ethane, ethylene, acetylene, and other long chain hydrocarbon fuels including various grades of refined petroleum (RP).

Injector scheme aside, several articles were available that looked at variable flow area at constant mass flow rate. Bennewitz et al. 2018 [17], conducted an experimental test series on an uncooled rotating detonation engine at AFRL. The injector flow area and thus pressure drop was varied as well as the mass flow rate across a broad range of mixture ratios using GCH4/GOX as the propellant. Key results noted were a maximum of thrust and specific impulse at or near stoichiometric mixture ratios and a linear increase in thrust with increasing injection mass flow rate. Another interesting feature to note, though not explicitly stated, the number of wave modes present in the combustor for a given test was dependent upon the injection mass flow rate or unburned injected flow velocity.

It was apparent that the number of wave modes present balanced with the propagation velocity of the detonation wave and the annular geometry of the combustor. This lends credibility to the notion that if detonation is achieved, sustainability of the wave is not so sensitive to injection properties and channel geometry that the wave will simply be quenched. Instead, the wave will divide or merge the balancing number of wave modes to become anchored in the annulus. Further observations of the experimental data include the following; pressure tap data suggests anchoring of the detonation wave occurred very close to the injector face, specific impulse is maximized when wave speed is the highest and thus when the fewest number of waves are present, a larger injection flow area seems to yield higher wave speeds and higher specific impulse but may be predominantly due to constant mass flow rate error. Finally, the authors note that the engine tended to operate in a specific frequency range which may be caused by a coupling between the chamber's natural acoustic modes and the detonation modes.

Anand and Gutmark 2018 [18], summarized the work conducted at the University of Cincinnati. This paper focuses on the experimental results of both an annular RDE and a hollow variation. It was found that lower injection areas and higher-pressure ratios contribute to an optimal RDE injector design. Both of these variables contribute to a net higher Injector fluidic impedance thus reducing the backflow potential commonly encountered in these combustion devices. Furthermore, a high number of injector ports with small diameters compared to a low number of injector ports with larger flow area, thus holding total injection area constant, was better for sustaining detonation. The cause of this is primarily due to refresh rates and mixing efficiencies. A higher number of ports increases the mixing while simultaneously reducing backflow potential thus improving fresh propellant injection timing. Interestingly, the authors also found that the wave mode operation of detonations to be dependent on the pressure ratio across the injector area. This could be due to a change in mixedness of the propellants which has been previously found to cause wave mode shifts [16].

Full propagation of rotating detonation was observed for pressure ratios that were greater than the choked criteria pressure ratio. At lower pressure ratios, azimuthal pulsed detonations were observed. It was determined that this was caused by a "chugging" type phenomenon where combustion products back flowed through the injector and out again. Finally, this work summarized the wave mode operation space of an annular RDE. In general, stable wave mode operation occurs at equivalence ratios between 0.8-1.75 and from mass flow rates of approximately 0.2 kg/s and up. These values are likely specific to a certain propellant combination and chamber geometry but nevertheless demonstrates that a minimum mass flow rate and a bounded equivalence ratio band allows for steady and stable detonation to precess.

Since it has been inferred that propellant mixing is of such high importance to wave mode operation, [19] conducted an experimental study on the effects of three different injector configurations in an RDRE. These element configurations from [19] are shown in the figure below.

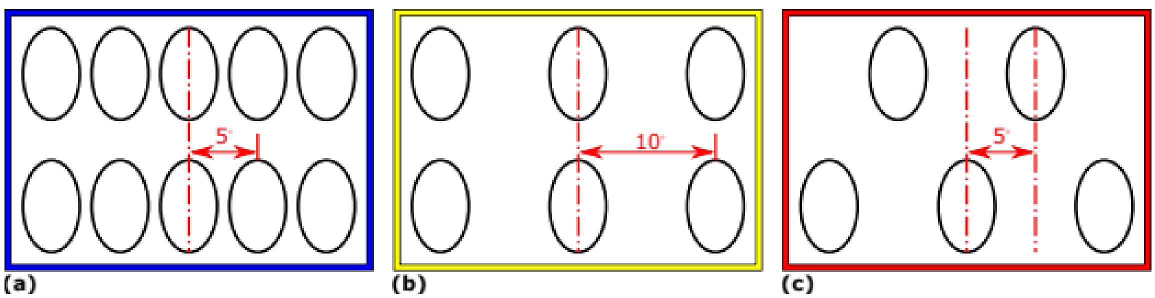

Fig. 8 Impinging injector element schemes a) aligned 72 element pairs ~0.131" spacing, b) 36 element pairs ~0.262" spacing, c) misaligned 36 element pairs ~0.131" spacing [19]**.**

The authors found that poor mixing sustained fewer waves in the annulus and causes a breakdown of detonation mode structure, less well-defined waves, and counter propagating waves. In addition, both thrust and wave speed were reduced significantly with lower mixing quality. As the mass flow rate and thus mass flux increased through the annulus, the number of waves present in the annulus increased linearly to seven waves at a mass flow of ~1.3 lbm/s. It is not discussed if the injectors were choked or not at any given mass flow rate. Some inferences can be made in the data presented in this work to that end. The wave mode operation does increase linearly until about 0.75 lbm/s for the 72-element injector. The wave mode operation appeared to remain constant at five waves until the mass flow rate increases to ~1.3 lbm/s. The tests conducted at this mass flow shows operation at both six waves and seven waves for two different tests. From these observations, it could be that the injector was choked at and above the 0.75 lbm/s mass flow rate. Since the injection area is not known, this is not certain. The authors did give measured CTAP pressure data for increasing mass flow rates. As expected, the averaged chamber pressure did increase linearly with increasing mass flow rate. Since a single RDRE geometry is presented in this work and as discussed above the wave mode operation ceases to increase linearly at about 0.75 lbm/s, the wave mode operation may plateau and then vary between six and seven waves for the last two cases. At the mass flow rate of 0.75 lbm/s, the measured CTAP pressure is above ~30 psig, which is greater than ~2X atmospheric pressure. This would likely mean that annulus exit was choked and thus the injector outlet would also be choked since the injection area is very likely less than the annulus exit area. It is theorized from these results that wave mode operation is not as strongly influenced by the chamber pressure as it is by the mixing efficiency, injection velocity, and exit choked condition.

The last interesting observation made by the authors was that degraded mixing resulted in higher average chamber pressure measurements due to shifting of the reaction zone closer to the measurement location. It can be concluded from this work that injector design for high mixing efficiency is of extreme importance for RDE operation. A similar finding was made by [20] via simulated results.

Walters et al. 2019 [11], conducted an experimental investigation using natural gas and pre-heated air as well as two different injector configurations. The first was an axial slot injector and the second was an dubbed a sting injector. Both configurations are shown in the image below from [11].

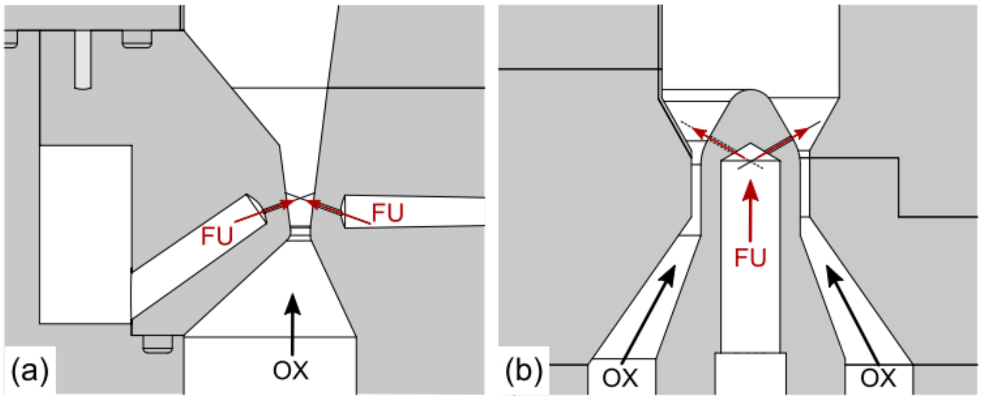

Fig. 9 a) Axial slot injector and b) axial sting injector [11]**.**

It was found that injector configuration (b) produced somewhat higher performance metrics than configuration (a). The thrust efficiency was higher for this configuration and was approximately the same for larger cell sizes. It is unclear to the authors why this was the case. Similarly, injector (b) produced higher C* efficiency band than injector (a).

Fotia et al. 2016 [1], examined the performance gains of axial fed injection versus radially fed injection. Typical injection systems of modern rocket engines employ axial feed systems that inject directly along the fluid flow path through the main combustor. To date, very little work has been done to quantify the effects of axial feed injection in RDREs. This study compared several configurations with JP-8 fuel and hydrogen fuel. It was determined that an increase of ~15% in the corrected thrust was attained with axial fed propellants in similar channel gap configurations when compared to radial feed systems.

Mizener and Lu 2016 [21], built an analytical model of an RDE to determine performance trends with a parametric analysis. The model was designed based on integrated conservation equations and varied several key annular channel designs and operational parameters. It was found that for optimal performance the geometry should have a high injection pressure, low propellant temperature, and positive injection swirl. It was chosen by the authors to model injection swirl likely due to the increased mixing effects. Positive injection swirl likely increased the residence time and thus the mixing efficiency, though it is uncertain if this level of detail was captured in the presented model.

Li et al. 2018 [22], investigated injection strategies for liquid-fuel RDEs. A premixed and non-premixed combination of liquid jet A-1 fuel and air was injected into the chamber via a radial slot inlet scheme. It was found that a rotating mode detonation formed in the startup and transitioned to an axial pulsating mode using premixed propellants. In the non-premixed cases, similar results were observed. In all cases, the detonation front was fairly weak with a low wave pressure gradient.

Stechmann et al. 2017 [16], conducted a high-pressure experimental investigation of an RDRE using O2/methane, O2/NG, and O2/H2. The facility and test article utilized an oxygen pre-burner setup and annular slot with various upstream and downstream fuel orifices in a crossflow arrangement to the oxidizer slot. An image of the injection element arrangement is shown in the figure below from [16].

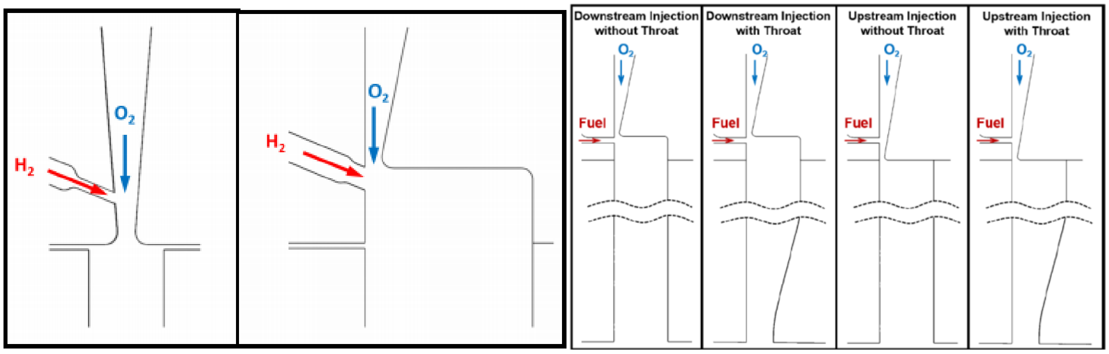

Fig. 10 Arrangement of injector and channel schemes experimentally investigated in [16]**. On top left is V1.0 configuration, top right is the V1.1 configuration, and on the bottom are various V1.2 configurations** [16]**.**

This work is the most applicable to the injector operating conditions expected during this program. The injector orifice and slot sizing were altered as shown in the figure above to gauge the detonation stability and engine performances. The element scheme shown on the top left (V1.0) is more similar to CP liquid rocket engine elements. Detonation in the V1.0 and V1.1 configurations only occurred near shutdown when the injector plenum pressures were low. It must be noted that a significant analysis was given on flame holding and parasitic deflagration through short ignition delay. This was likely a problem experienced due to the hot oxygen from the pre-burner greatly increasing the chance for parasitic deflagration. Other works such as [23], were able to achieve rotating detonation in O2/H2 using much lower injection temperature thus increasing the ignition delay. The V1.2 injector did successfully yield rotating detonation in all configurations. It was concluded that the detonation behavior was highly sensitive to the injector and throat configuration. The work of [16] and other works by this author are further reviewed in a later section.

Sosa et al. 2020 [23], conducted an experimental investigation with gaseous O2/H2 in an uncooled annular RDRE. The injector configuration incorporated 72 aligned impingement ox and fuel jets with orifice sizes of 0.9 mm and 1.1 mm in diameter. Consequently, this element scheme is identical to a configuration described in [24]. An image of the test article and injection scheme is shown in the figure below.

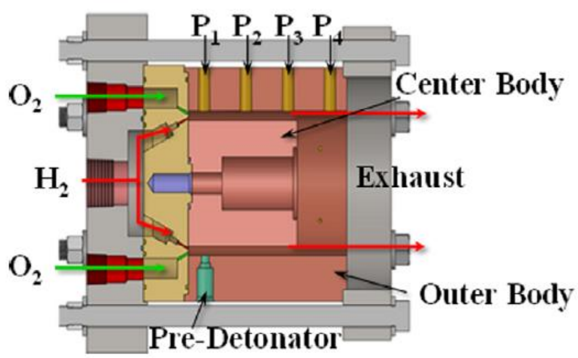

Fig. 11 RDRE test article hot fire tested in [23]**.**

This injector configuration was tested up to just under a total mass flow rate of 1.2 lbm/s and produced 5 rotating detonation waves in all cases shown. CTAP pressures measured reached up to 153 psia. All cases tested utilized some percentage of methane as a tracer to visualize the rotating detonation wave front.

As mentioned above, a similar experimental investigation by Hargus et al. 2018 [24], was conducted at the Air Force Research Laboratory (AFRL) using an uncooled annular combustor with typical hot fire tests lasting 1.25 seconds. In addition to the injector geometry presented in Sosa et al. 2020, another misaligned geometry was considered with a 2.5 degree spacing which translates to roughly 0.065 inches. These two element schemes are shown in the figure below from [24].

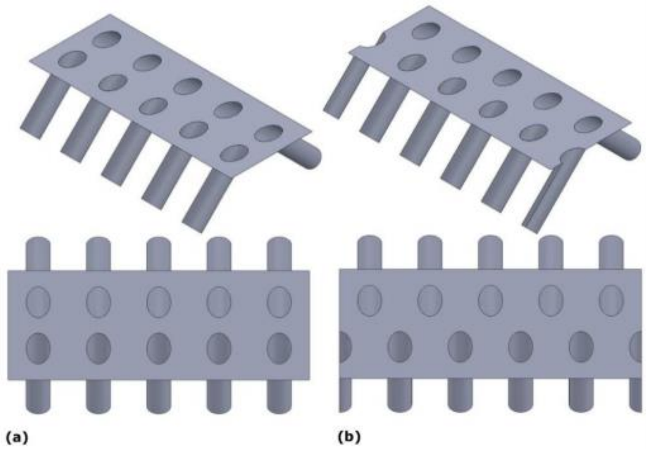

Fig. 12 a) Aligned impingement injector element scheme, b) non-impinging and misaligned injector element scheme [24]**.**

The misaligned element scheme solely relies either on wall impingement and/or secondary detonation front mixing while the aligned injector scheme relied on jet impingement in addition to secondary mixing processes. This investigation was also capable of altering the element flow areas from 1, 1.5, 2, and 2.5 times the smallest area. A suite of tests were conducted across equivalence ratios and mass flow rates. The authors note that the performance of the engine varied only slightly with altered orifice area and equivalence ratio. It was also noted that substantial drops in plenum pressure did not appear to be detrimental to engine performance. A closer look at the data suggests that the 2X area elements performed the highest in the range of typical liquid rocket engine equivalence ratios but this may not be statistically significant. This may be due to some optimization between element spacing causing increased mixedness as the elements encompassed a large amount of the injector faces "dead space". Once the orifice area became large enough, atomization may have been adversely affected. As for the injector element type, impingement or misaligned, there is a notable drop in engine performances with the misaligned elements. As tests moved away from stoichiometric conditions, the performance of both element schemes showed similar performances. The performance metrics mainly assessed included thrust and specific impulse. The authors also note that mixedness may compete with equivalence ratio for direct impacts on the detonation cell width. An image during hot fire test of this RDRE configuration is shown in the figure below from [24].

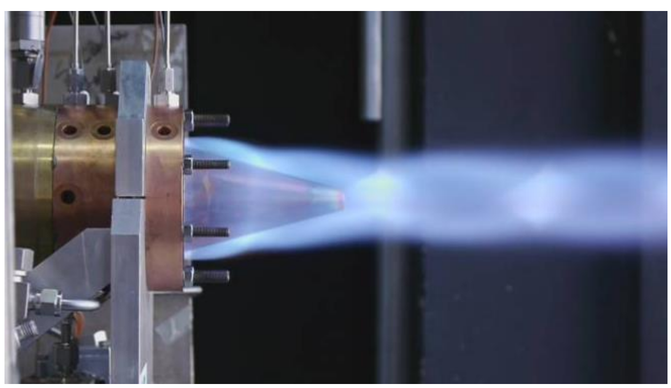
Fig. 13 AFRL GHKN RDRE during hot fire test [24]**.**

Goto et al. 2019 [25], conducted an experimental investigation of an RDE with different nozzle and injector configuration to asses performance. Two injector types were used including a triplet and doublet element scheme. These schemes from [25] are shown in the figure below.

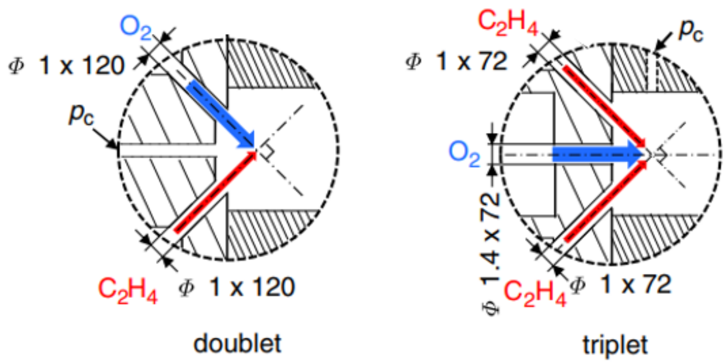

Fig. 14 Injector element configurations tested under the experimental investigation in [25]**.**

The doublet scheme used 120 sets of 1mm diameter fuel and oxidizer orifices. The triplet scheme used 72 sets of 1mm fuel orifices and 1.4mm oxidizer orifices. A slot injector configuration was also included in the study, likely as a control. Both the triplet and doublet injector schemes showed clear performance advantages over the slot injector. Both schemes trended closely with the optimally calculated specific impulse. As the pressure ratio increased, the specific impulse increased as expected but with the injector and chamber configuration producing nearly 100% theoretical specific impulse in some cases. Not surprisingly, cases that employed a throat constriction performed very well. It was interesting to note that a single case of the triplet injector with open throat produced higher specific impulse than the highly constricted throat configuration at nearly the same operating conditions. These results were, however, within measurement error. The authors go on to note that characteristic velocity was nearly identical for all cases of the triplet and doublet schemes at elevated conditions. The slot injector on the other hand was consistently in the range of 15-40% lower in normalized chamber pressure at the same conditions. Finally, the authors note that the doublet configuration cause damage to the combustor wall likely due oxidizer rich environment near the chamber wall causing substantial erosion. The authors thus recommend a triplet scheme due to improved wall compatibility.

The study above as well as the work of Ishihara et al. 2017 [26] utilized the same triplet injector configuration in addition to a slot injector shown in the figure below from [26].

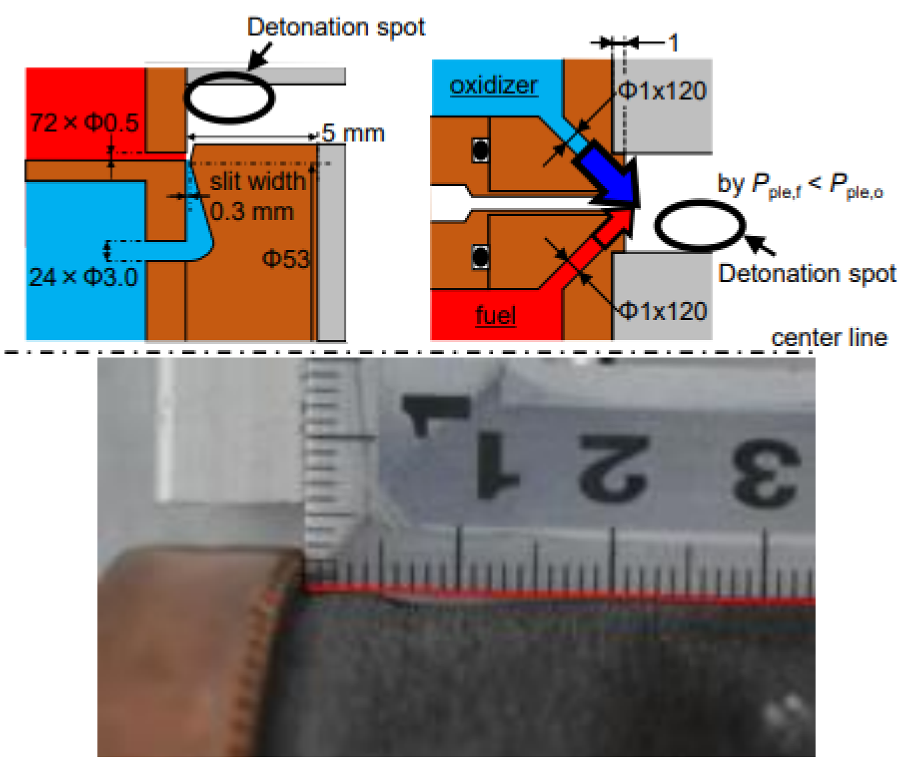

Fig. 15 Injector element schemes tested under [26] (top) and region of anchored detonation causing damage to the chamber carbon composite wall using the triplet configuration.

Consequently, the figure below also reveals the location of where the detonation was anchored confirmed by an indentation from where the detonation resided causing damage to the combustor wall. This piece of information is invaluable as it suggests that the detonation can be stood off from the injector face by means of moving the impingement location similarly to altering the location of the anchored flame in a CP combustion device. This work also reveals the length of the detonation front which is not typically reported in the experimental literature. In this case, the detonation "groove" in the chamber wall appears to be about 0.4 inches in length. Of course, this must be confirmed prior to making significant assumptions.

Lim 2019 [27], conducted an exhaustive study (as is typical with a PhD dissertation) on the responds of liquid injector orifices to transverse detonation waves at elevated pressures and hot fire tested an RP-2/gaseous oxygen RDE. Several performance parameters of the RDE were quantified including heat flux, multiple injector element schemes, thrust and Isp, and visual confirmation of detonation stability. Tests were conducted up to 7.7 lbm/s with static chamber pressures up to 258 psia. A significant study on liquid injector response to a passing transverse detonation wave was given. Summarizing the findings, the author found a correlation of backflow potential and orifice geometry. Several different injectors were hot fire tested and their performances compared. The thrust was found to be consistently between 85% and 95% that of a CP equivalent engine operating at 100% efficiency.

Many more articles are available on various RDE injector topics but not included in this work, which focused primarily on experimental and application-based research. A few more notable articles include; Suzuki et al 2020 [28], who simulated the pressure loss in RDE injectors. The work of Goto et al. 2020 [29], investigated a novel cooling and injection concept for an RDE by direct wall injection rather than axial injection. Bach et al. 2019 [30], investigated the performance and operation of a generic RDE with various outlet boundary conditions. While no positive pressure gain was measured, it was found that specific design configurations created a positive impact on heat release in the combustor. Ross et al. 2020 [31], numerically simulated an RDRE with an abrupt outlet restriction. The authors found evidence of counter propagating wave behavior at a detriment to detonation performance.

Several articles have been reviewed in an attempt to understand how to optimally design an injector for operation in an RDRE. Several observations were noted on design schemes that appeared to yield higher overall performances. These observations are documented below.

- Higher thrust and stronger wave fronts are observed for well mixed propellants.
- Stronger wave fronts are associated with higher pressure gradients and thus produce higher overall engine performance.
- High element density with a large number of elements in a close spacing produced higher overall engine performance and detonation stability.
- Small inlet orifices yield higher fluidic impedances reducing the likelihood of back flow and increasing the refresh rate for identical back pressures.
- Axial fed propellants produce higher overall engine performances than radial fed propellants.
- Low feed pressures don't appear to be detrimental to engine performance.
- A choked annulus exit and choked flow at the injection orifices may isolate the detonation front from upstream and downstream perturbations and increase detonation stability.

The findings listed above appear to suggest that detonation cycle injectors are not so dissimilar from CP combustor injectors. They do also rely on highly mixed and well atomized propellants to improve overall combustion performance as well as maintain engine stability. The injector configurations experimented with in the available literature are typically annular slots, doublet impingers, or liquid centered triplets. In most cases, the propellant phase types were gas/gas or liquid fuel hydrocarbon / gaseous oxidizer. No works in the available literature were found by the authors utilizing liquid oxygen, a typical liquid rocket engine (LRE) oxidizer.

B. Inferences from the Available Literature

Several inferences can be made, in some cases without quantitative verification, but can still be useful towards the design optimization process. As is shown in [26], it may be possible and likely that stand off of the detonation from the injector face occurs. This would have the implication of reducing overall backflow potential by removing the sharp pressure gradient from directly above the injector orifices. Injector face life-cycle performance could be extended by a reduction in face heat transfer and shifting of combustion products further downstream. However, it is not known if standing off the detonation from the injector face would cause any detriment to global engine or detonation performance, which would be counterproductive to the goal of this work. A diagram of this concept is shown in the figure below using a gas center triplet, or O-F-O configuration.

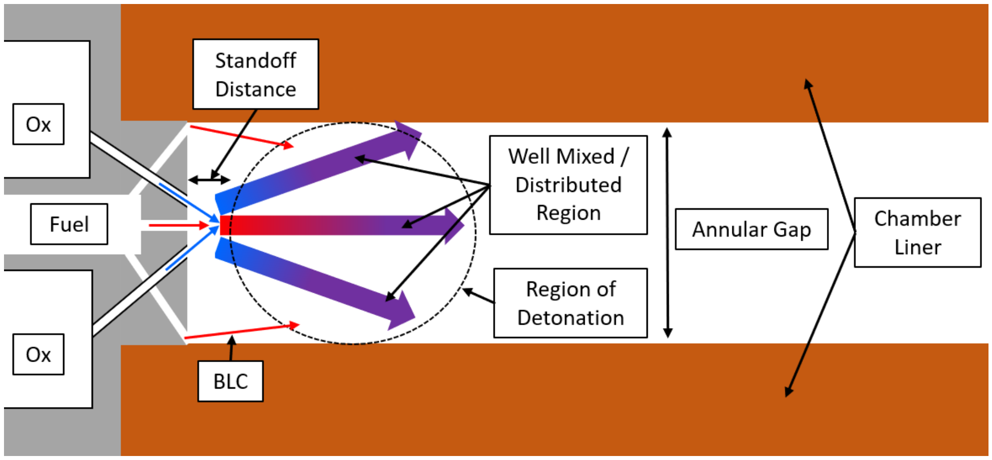

Fig. 16 Diagram of hypothetical triplet injector scheme standing off the detonation wave from the face of the injector.

Another important injection metric is the complete filling of the annular gap to maximize the detonation crossection and total consumption of propellant. Theoretically, if the gap is not completely filled then the detonation may have detached interfaces, a reduced overall crossection, regions of recirculation and deflagration, and a weaker overall pressure gradient across the detonation front. Each of these could be potential sources of performance detriment. Thus, the injector would need to be designed to well atomize, mix, and distribute the propellants throughout the annulus gap prior to detonation arrival. In Fig. **16** above, boundary layer coolant (BLC) orifices are shown in addition to the triplet element configuration main propellant. It was shown in [11] that one injector configuration produced slightly higher performance than the baseline injector. This could have been due to increased number of ox/fuel interfaces and thus a qualitatively better mixed or distributed region of unburned propellant. Furthermore, this configuration may aid in the complete filling of the annular gap than the baseline element configuration could provide.

A common notion for achieving high performance in RDREs is to improve the overall fill height of propellants into the annulus. This is fairly easy to do with gas/gas injection but could be a challenge with liquid/gas and especially liquid/liquid injection. The maximum liquid injection velocity is substantially slower than gaseous injection velocity. The maximum fill height would then be solely dependent on the liquid phase propellant. Regardless, it is thought that the fill height may play a role in the number of waves present in an RDRE, but this may not dictate the global performance. The higher the fill height of propellants, the more waves appear. If the energy density of propellants were to increase, the detonation strength would increase but at a detriment to fill height due to increased wave numbers of waves present. This is further supported by the general design recommendations for liquid RDEs in Bykovskii et al. 2006 [5]. The existence of wave multiplicity is a highly complex process dependent on a number of variables including propellant properties, chamber geometry, and injection mixedness. Assuming the propellants are perfectly mixed, and the chamber geometry does not hinder the detonation in any way, the maximum number of waves possible with the given condition may appear. If the mass flow rate were to increase from there, only the density of injected propellants would increase and not the fill height or injection velocity. This is in-part supported by observations and hypotheses made in the works of [3], [32]–[34], however not completely verified by experimental observations. Thus, if the fill height were low enough, a single wave could be sustained but at diminished global engine performance [35] if injector recovery and parasitic deflagration are not ideal.

In reality, injected propellants are not perfectly distributed, perfectly mixed, and these characteristics alter based on the mixture ratio and throttle level. Many experimentalists have found that a specific number of waves produce slightly higher engine performance than a single wave or numerous waves. In addition, mechanisms not fully understood in RDREs can produce high overall thrust at a detriment to detonation CJ velocity. This was a major finding of Bigler et al. 2020 [35]. Lastly, the injection temperature appears to be just as important as the fill height. Generally speaking, as the temperature of injected propellants drops so does the number of waves in the chamber [3], [16]. It is not certain if detonation performance would be substantially hindered by a decrease in injection temperature, but it stands to reason that it would not since the detonation pressure ratio tends to rise with decreasing temperatures

[36]. In addition to data provided by [36], this is verified using NASAs online Chemical Equilibrium with Applications (CEA) code.

The available literature reviewed above lacks specifics of element design geometry and permutations therein. Thus, the literature is also reviewed for the design optimization of CP combustion device injectors to gain better insight into element design specificity optimization.

C. Review of Constant Pressure Combustion Device Injector Design

In an effort to better understand how to optimize the performance of an RDRE injector, and since RDREs are not so dissimilar from CP combustion devices, the available literature was reviewed on the design of traditional liquid rocket injector elements. Several of these traditional element schemes are shown in the figure below.

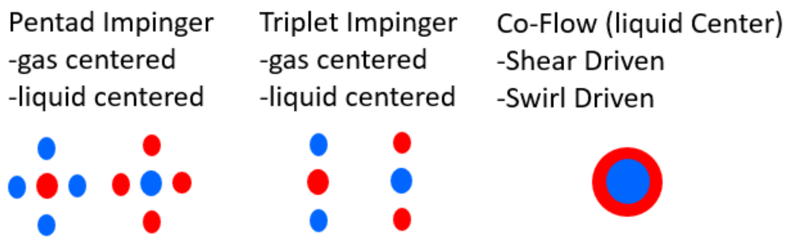

Fig. 17 Traditionally manufactured and most common liquid bi-propellant injector elements schemes. Blue represents liquid phase propellants and red represents gas phase propellants.

A particular focus was placed on the review of co-flow or coaxial elements as they were the most prevalent in the literature and in many articles, the highest performing. Impingement type elements were also reviewed but to a lesser extent.

D. Coaxial Type Injector Elements

The coaxial injector configuration is a common type of non-impinging injector element, well known for its use on the Space Shuttle Main Engine. It is noted for excellent mixing and atomization, low pressure drop, and proven dependability [37]. The element typically consists of a central low velocity liquid oxidizer surrounded by a high velocity gaseous fuel. Mixing, atomization, and mass distribution rely upon shearing or swirled crossflow of the liquid with the gaseous exterior. Maintaining a high velocity gradient between the gaseous fuel and liquid oxidizer is crucial to ensuring that proper breakup from shearing occurs [38]. Mixing plays a significant role in determining the performance and wave propagation characteristics of RDREs [19]. While research on RDRE coaxial injectors is sparse, coaxial injectors for constant pressure combustors have been studied in great depths. An image representation of a typical coaxial injector element is shown in the figure below.

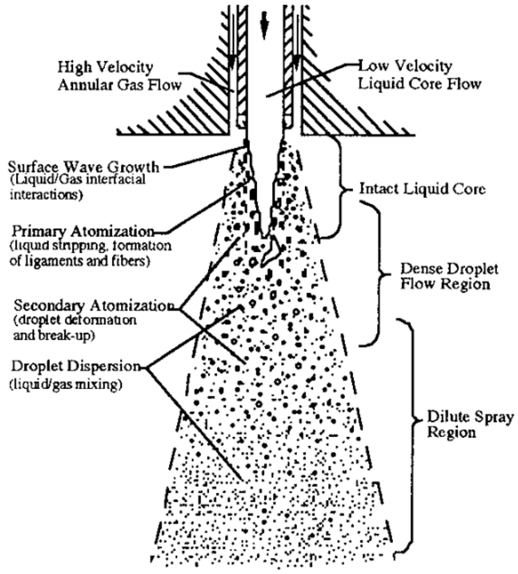

Fig. 18 Coaxial injector element and atomization spray field [39]**.**

The state of available co-flow injector literature was reviewed for ways to optimize performance, specifically C* efficiency, which is a confluence of atomization and mixing efficiency [40]. Several articles reviewed provided a fundamental background for coaxial injectors [37], [38], [41]–[45], with articles [19], [45], [46] specifically detailing the state of injectors in RDRE's. Additive manufacturing methods and applications were reviewed in articles [15], [46]–[51]. Aspects of atomization were investigated in articles [19], [39], [40], [52]–[67], while flame dynamics were reviewed in [68]–[77] . Ignition dynamics literature was found to be less prevalent but reported in [78], [79] as well as injector face heat flux in [80]. Finally, supercritical propellant conditions are examined in [44], [57], [60], [74], [75], [81]–[84].

Atomization describes the ability of the injector to break down the propellants into small droplets, where the droplets mix in the spray field, vaporize, and begin to combust. The atomization process strongly depends upon injection conditions, propellant properties, and the local flow of the spray field. The ability for a coaxial injector to atomize liquid propellants is essential to high-performance. The complexity of the atomization process means that coaxial injector design must be based on both basic principles and empirical rules, leading to numerous experimental and theoretical investigations.

Maintaining a high velocity ratio between the gaseous fuel and liquid oxidizer is critical for overall performance by helping to properly atomize the propellants [37]. LOX and fuel velocities can be influenced by several factors. These include injector geometry such as the LOX or fuel post diameters, or system parameters such as propellant density and mass flow rate. Adding a tapered angle to the inside of the LOX post has consistently been found to increase the mixing quality. Mixing can be further improved by recessing the distance of the exit of the LOX post from the exit of the fuel post. Ideal performance occurs when the recess length is approximately the same distance as the LOX post diameter [61]. Other studies have been conducted to determine a specific model for predicting the diameter of atomized droplets. It was found that droplet diameters of 1000 microns or less should be required, with an ideal injector creating droplets smaller than 10 microns [55]. A test campaign completed by R.J. Burick of the Rocketdyne Corporation found, for example, that a reduction in droplet diameter by a factor of 2 would increase efficiency enough to allow a reduction of L* by a factor of 4 [40]. It should be noted that there are many other non-dimensional numbers or ratios besides the velocity ratio that are commonly used to evaluate injector performance in coaxial injector studies. The momentum flux ratio and Weber number can play an important part in characterizing an injector [64].

Investigations of reactive sprays have shown a notable interaction of combustion and atomization. This means that aerodynamic forces between the propellants are not the only thing controlling atomization. The release of heat and combustion reaction products change the conditions for atomization and droplet vaporization [70]. Flame dynamics and the type of fuel can have a further effect on atomization. Different atomization regimes can occur depending on whether the flame is attached or detached from the LOX post. Flame blowout and liftoff can initiate by a high velocity ratio and small LOX post diameter. Tapered LOX posts have been shown to improve atomization by inducing an oscillation in the liquid jet core [40]. Consequently, this also reduces the vorticity region in the gap between the LOX and fuel annulus where a flame would typically anchor [68]. Another study found that the high-pressure flame emission spectra of LOX/CH4 is similar to that of LOX/H2. They found that as soon as the oxidizer was supplied, a flame anchored in the wake of the LOX post at all observable operating conditions. The flame shape itself was found to be influenced by the injection velocity ratio and momentum flux. Sonic injection of both Ch4 and H2, can result in low velocity ratios and produce stretched flames, while a larger momentum flux can cause a more confined spreading angle and constricted flame [69]. Another experiment comparing the flames of H2 to CH4, found that under the same chamber pressure, the liquid intact core length was shorter for CH4 than H2. This is attributed to the higher density of CH4. Higher momentum flux ratio tends to promote jet instability and result in an earlier onset of jet disintegration. Key findings in this work showed that as the weber number decreased, the flame lift off distance and the intact core length decreased [70]. Flame-acoustic interaction was found to be most sensitive to changes in fuel-oxidizer density ratio [72]. In a test of supercritical hydrogen-oxygen with a chamber pressure of 80-bar, dynamic mode decomposition results showed that the flame dynamics were strongly influenced by the LOX injector acoustics, whereas no flame response to injector acoustics was observed for a 50-bar chamber pressure [75].

As for the optimized design of a coaxial element, a few articles stand out. The work of Oschwald et al. 2008 [70], describes a number of advantageous design characteristics for coaxial injectors. First, the atomization process is highly complex and can be described by the momentum flux ratio. For large values, the liquid jet tends to result in earlier onset of disintegration due to imparted instability at the surface of the jet. Similarly, the Weber number represents the relative disturbance of the gaseous co-flow to the liquid jet surface. It has been observed that as Weber number increases, ligament and droplet dimensions become smaller and secondary atomization is encouraged. Droplets have also been observed being transported to a larger radial distance. Key findings in this work showed that as the weber number decreased, the flame lift off distance decreased and the intact core length also decreased. Finally, it was found that the intact LOX core length decreased when the LOX post thickness was decreased. This also resulted in a thinner anchored flame at the post and in the stagnation region downstream.

Next, the work of Burick 1972 [40], gave a comprehensive experimental review of co-flow type injector design optimization. Several key takeaways can be gathered from this work. First, as the gas density increased, so did the mixing efficiency and overall C* performance. A LOX post recess of approximately one LOX post diameter resulted in higher overall efficiencies as described previously. Element spacing plays a key role in the mixing process and generally speaking, a well distributed field of elements produces higher efficiencies. Smaller LOX jet diameters produce smaller droplet diameters. As the liquid jet velocity increases, the average droplet diameter decreases. High gas injection velocity allows for improved mixing and secondary atomization of the liquid jet. Finally, small droplet diameters are essential for high efficiency combustion.

Finally, several secondary observations have been made from the available literature. Mixing quality increased with an increase in LOX post inner taper and injection gas density [61]. Nondimensional numbers characterizing fluid-dynamic interactions are not sufficient to scale coaxial injector performance in hot fire tests from one fuel to another [64]. If the liquid jet has a much higher velocity than the flame speed, flame holding can only occur when the post thickness is greater than or on the order of the flame thickness [68]. Under the same chamber pressure, the liquid intact core length was shorter for CH_4 than H_2 which is attributed to the higher density of CH_4 [70]. The combustion efficiency of the subcritical oxygen case had the highest performance with a slightly increasing efficiency as the injection velocity ratio increased [81]. Higher chamber pressures correlate to higher efficiencies at the same droplet sizes [61].

In summary, there are two types of control that can be used to alter the performance of coaxial injectors; outlet geometry and flow parameters.

- A beveled internal LOX post allows the liquid jet to disperse and impinge on the high velocity gas sooner, thus reducing the in-tact core length and increasing combustion performance.
- A recessed LOX post of 1X post diameter is found to be optimal in imparting an oscillation in the liquid jet which allows for the jet to fan out and break up sooner.
- High velocity gas and low relative velocity liquid allows for quicker breakup of the liquid core.
- A higher velocity liquid core produces better atomization but reduces the mixedness between injector elements.
- Swirling of the liquid core, rather than relying on shearing actions alone, radically increases the mixedness and atomization and thus drastically improves combustion performance.

In the case of the swirl coaxial element, put simply, swirling allows for the effective even distribution “thinning” of the liquid core into an annular sheet which is then effectively atomized in the crossflow of the high velocity gas annulus. Consequently, this swirling also rapidly disperses the propellants between elements so that mixedness is maximized. An image of the swirl coaxial AMDE injector from [15] demonstrating this high degree of atomization and mixedness in cold flow testing is shown in the bottom right image of Fig. **27**.

E. Impingement Type Injector Elements

This section focuses predominantly on the operations, variations, advantages, and optimization of impinging injectors with some information on pintle type injectors. A handful of articles discussed the most influential types of injectors [37], [85]–[88]. Meanwhile, other articles primarily focused on the variations and performances of impinging injectors [15], [89]–[93]. Finally, several others described the design of impinging injectors [19], [94]–[106]. Most of the reviewed articles are interrelated and, thus, overlap on these subtopics. First, a quick overview of pintle type injectors.

A common injector type used in the application of throttling is the pintle, or variable area, injector. Pintle injectors offer reasonably high combustion efficiencies, enable deep throttling, injector face shutoff, and can be optimized by only adjusting two parts [87]. These injectors have also never had a reported incidence of flight failure or combustion instability during any engine operations [87]. This is, however, according to works produced around the Apollo program era. Pintles do possess wall compatibility issues and do not have a correlation between mixing level and on average droplet size [37]. This means that the mixing efficiency is difficult to adjust and thus performance detriments are often observed to significant proportion. However, some works have demonstrated effective control of the atomization process by altering the spray angle and thus holding droplet size roughly constant [107]. This may yield an effective recovery in combustion performance at low throttle points.

According to one paper, impingers are the preferred LRE element scheme, especially when it comes to storable propellants [89]. Impinging injectors possess a relatively simplistic design, meaning they are cheaper and easier to produce, maintain, and inspect than coaxial and pintle injectors [90]. There are two main types of impinging injectors: unlike-impinging and like-impinging. Like-impingers achieve atomization by striking two jets of propellant (i.e. fuel *or* oxidizer) against one another at a point called the jet impingement point. Mixing then occurs downstream as adjacent fuel and oxidizer spray fans interact [37]. Efficient mixing is accomplished by precise geometric arrangement of the fuel impinging elements relative to the oxidizer impinging elements. A high quality element arrangement yields uniform mixing in the core elements along with a mixing gradient in the outer elements near the chamber wall [37]. This arrangement minimizes mixing losses in comparison to normal-boundary layer cooling techniques [37]. On the other hand, unlike-impingers achieve atomization and mixing at the impingement point simultaneously. This is due to direct impingement of two or more jets of an unlike substance at high velocities [91]. Upon impingement, the liquid and/or gaseous jets atomize into small droplets and mix. The atomized droplets transform into a perpendicular sheet that disintegrates into droplets and propagate as wave or streamlets [91]. A visual depiction of this phenomenon is shown in Fig. **19**. Based on the reviewed material, it would appear that all impinged propellants, regardless of chemical composition or phase state, display similar behavior. A 2015 study that looked at the behavior of green storable propellants found that all spray shapes appeared to be a wavy liquid sheets following impingement and then primary breakup created ligaments and small droplets formed at secondary breakup [92].

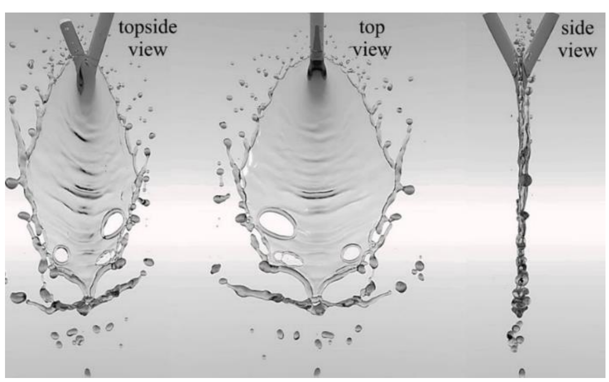

Fig. 19 The results of two liquid jets impinging on one another [108]**.**

Unlike-impinging injectors also come in a wide variety of sub-types such as doublets, triplets, quadlets, and pentads. Essentially, the names are indicators of how many jet streams an injector has impinging on one another. So, a pentad generates propellant atomization and mixing by impinging five jet streams on each other. Each sub-type can also have variations in oxidizer streaming elements and fuel streaming elements. For example, one triplet design might have a fuel-oxidizer-fuel (F-O-F) element scheme where fuel flows from the outer elements while oxidizer flows from the central element. On the other hand, another triplet design may possess an oxidizer-fuel-oxidizer (O-F-O) scheme where the element flows are inverted. Figure 5 displays a few examples of different element schemes that impinging injectors may possess. Doublets are most commonly used for storable propellants and offer easy manifolding, simple design, and potential chamber walls compatibility issues [37]. O-F-O triplets offer better mixing uniformity than doublets, but they also have more serious wall compatibility issues. F-O-F triplets, however, reduce heat flux to the walls and have better compatibility at a detriment to performance [37]. Triplets have been used for liquid/liquid and gas/liquid propellant combinations. In reference to Fig. **20** below, the quadlet version (a) performs similarly to the doublet while version (b) creates an axial resultant spray fan allowing the flow to be insensitive to operating conditions and thus yield better performance over a broad range of mixture ratios [37]. Pentads have not yet been used in a production engine, but they are highly appealing for scenarios with propellant mixture ratios much greater than one, or close to stoichiometric. This element scheme offers the most uniform mixing, but it also produces very high heat flux to the injector face [37]. Like the triplet, the pentad has more recently been studied for use with gas/liquid propellants.

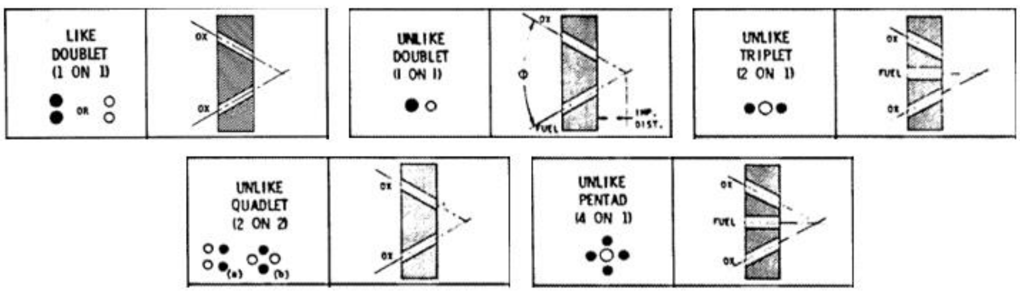

Fig. 20 Different element schemes available to impinging injectors [37]**.**

To expand on triplets, according to a 2005 study, the O-F-O element scheme is regarded as the best impinging injector when it comes to combustion performance and stability [93]. Regrettably, this scheme also has poor chamber wall compatibility. Therefore, F-O-F triplets are typically used when wall compatibility is valued over performance. However, F-O-F schemes employ oxidizer elements that are approximately three times as big as the fuel elements which reduces atomization efficiency substantially [93]. To combat this issue, a previous study found that splitting the central oxidizer element into two separate elements produced combustion performances that were nearly as high as the O-F-O scheme [93]. The 2005 study did find that the F-O-O-F scheme had lower mixing efficiency than F-O-F, but that may be a small price to pay for enhanced atomization and maintaining wall compatibility [93]. For a visualization, Fig. **21** depicts a couple triplet injectors. The left image graphically illustrates the F-O-O-F and F-O-F unlike triplet injector element schemes. The right image portrays an additively manufactured impinging triplet injector with an F-O-F element scheme undergoing cold flow water testing. Even at this low pressure, cold-flow condition, the atomization and mixing appear to be of high quality. Injectors in general, but specifically impinging injectors, are often tested under cold flow conditions using low pressure water to verify mixing and atomization.

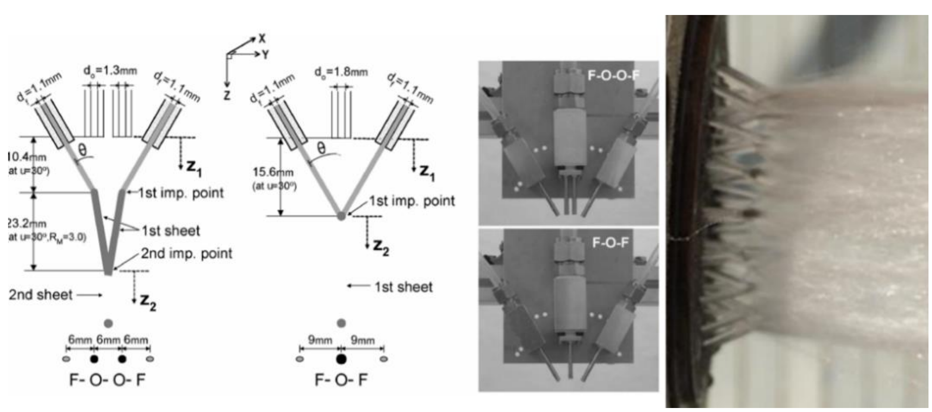

Fig. 21 FOOF & FOF Triplet Element Scheme (Left) Impinger Water Flow Testing [15] **(Right).**

Overall, impinging injectors have proven dependability, exceptional atomization, axial resultant spray, and good mixing. Unfortunately, they also present wall compatibility issues, design tolerance sensitivity, blow-apart with hypergolic fuels, and throttling sensitivity [37]. These sensitivities have been found to cause detriments in performance when off nominal operating conditions are attempted. Another concern for impingers is that higher performance often means sacrificing combustion stability. For example, in order to increase stability margin, impingers typically need to have fewer, larger elements. This decreases vaporization and mixing efficiency, thus reducing performance [90]. With these attractive benefits and high performance available, it is vastly important to optimize impinging injector designs in order to balance potential issues with potential gains in performance.

Now that the importance of design optimization has been established, the question then becomes: how are impinging injector designs actually optimized? Before this question is explored in depth, it needs to be pointed out that research on impinger optimization is lacking. Primarily due to a dearth of knowledge on processes that control propellant atomization, mixing, and spray fan formation [94]. It is difficult to study flow characteristics near the injector face. This is predominantly where atomization and mixing occur and is so hot and dynamic that it is nearly impossible to collect data due to the physical limitations of measurement instrumentation. In addition, computer modeling is difficult due to the complexity and non-linearity of the region [89]. However, this area is the most influential determining factor for spray characteristics and overall injector performance [89]. Despite this insufficiency in research, it is widely believed that injector performance and combustion stability are determined by four main factors: propellant atomization, propellant mixing, spray fan formation, and the physical injector geometry.

Knowledge of the effects that injector geometry has on performance and combustion stability has largely been determined experimentally via trial and error. Experimentation has found that smaller injector elements reduce weight and propellant “dribble”, as well as allow higher flow velocities and faster engine starts. Meanwhile, larger elements possess low flow velocities and poor atomization [99]. High pressure drops across the injection orifice decrease combustion instability and enhance atomization. On the other hand, low pressure drops minimize feed system mass and pumping power which increases engine efficiency [99]. The combination of orifice type and pressure drop across the injection orifice determines injection flow velocities [99]. Higher flow velocities permit local flow composition variations and poor flow distribution. Low flow velocities lead to proper flow distribution but poor atomization as well [99]. Experimentation has also yielded some knowledge that is a bit more obvious. For instance, injection holes need to be machined properly and devoid of impurities to ensure jet uniformity. Also, seals are required between fuel and oxidizer manifolds and cannot fail or else risk premature combustion. Support structures that may be required to brace injector faces also cannot obstruct manifold passages. Another note is that injector performance is highly dependent upon propellant combination. A specific geometry might have high performance with LOX and LH2, but it may have lower performance with a LOX and RP1 combination [99]. Overall injector performance is partially dependent on the configuration of geometry parameters such as orifice size, impingement angle, resultant momentum angle, impingement point distance, number of injection orifices, flow per injection element, and the orifice distribution over the injector face. The ideal configuration of these parameters is dependent upon the individual engine and is usually determined experimentally [99].

As mentioned above, atomization strongly influences overall combustor performance. Atomization occurs when liquid propellant jets are broken down and separated into finer, smaller droplets. This process is crucial to performance as finer particles vaporize and combust more rapidly and produce higher mixture uniformity. A 2001 report studied the mixing of impinging jets based on variations in atomization and found that mixing is largely controlled by the pre-atomization and post-atomization processes [100]. The pre-atomization process dictates resultant directional flow of the droplets and can result in two different types of atomization: reflective and transmitive. Reflective atomization transpires when the impinging fluids “bounce” off each other and cause each fluid to remain on the same side of the jet [100]. This typically occurs at low flow velocities when jets have a steady, smooth surface. However, if jet velocity increases, droplets from each impinging stream cross to the opposite side of the impingement plane and transmitive atomization arises [100]. If the surface of the jet is non-smooth, a greater number of droplets tend to cross the impingement plane which increases transmitive atomization. Increasing impingement angle, jet velocity, and/or jet diameter also increases transmitive atomization by reducing the amount of time needed to redirect jet momentum and increasing surface disturbance amplitude which, in turn, increases the extent of jet crossing [100]. In the post-atomization region, turbulent dispersion of fluid droplets creates a mixing layer along the axis of the spray which enhances propellant mixing. However, mixing may decrease if flow separation, which has been caused by the initial atomization process, is present in the post-atomization region [100]. Overall, the study concluded that changes that result in a more rapid redirection of jet momentum may improve mixing of the propellant fluids.

Another study, completed in 1995, looked at the atomization characteristics of sheets that were formed by impinging jets [101]. The study found that turbulent jets created fluid sheets that possessed different atomization characteristics than sheets formed by laminar jets. Specifically, sheet breakup lengths were much longer for laminar jets and the lengths increased with an increase in impingement angle and jet velocity. For turbulent jets, sheet breakup length decreased with an increase in these parameters. It was also determined that sheet wavelengths were directly proportional to orifice diameter and independent of jet velocity and impingement angle [101]. These results suggest that initial jet conditions, such as velocity profile and turbulence characteristics, strongly influence the sheet breakup mechanism and have a dramatic effect on the atomization process.

The third main factor influencing combustion stability and injector performance is propellant mixing. Mixing is a relatively simple concept, but it occurs when atomized fuel and oxidizer droplets combine to form a homogeneous mixture, ideally. This mechanism can heavily impact global engine performances. Mixing efficiency is regulated by injector parameters such as element scheme and orifice size and several studies have been conducted in order to better understand how these parameters alter mixing. A 2002 study found that the unlike split triplet element possessed superior mixing to unlike doublets and typical unlike triplet elements [102]. The study also found that secondary impingement plays a significant role in mixing and that mixing in split triplet elements increased as the secondary impingement angle decreased. Most importantly, this study concluded that mixing efficiency was maximized when the jet momentum ratio between the fuel and oxidizer was at unity [102]. A second study, published in 2019, examined the effect that three different injection configurations had on the performance parameters of a modular rotating detonation engine [19]. The baseline configuration was a seventy-two element unlike impinging doublet and the other two configurations possessed a reduced number of elements and misaligned spacing. Naturally, the baseline configuration proved to have the superior mixing and the configurations with poor mixing were associated with lower performance, lower thrust, longer mixing times, increased counter-propagating behavior, and fewer waves at lower CJ velocities [19]. The study concluded that mixing was a key parameter that controlled detonative efficiency and global performance.

Another study, completed in 1959, examined the correlation between mixture-ratio distributions and various impinging injector configurations [103]. The memorandum opened by stating that the optimum propellant distribution produced by each element is one, i.e. a homogeneous mixture, and that optimizing this ratio will attain highest engine performance. It also stated that flow in the manifold, which feeds injector orifices, must be as uniform as possible to ensure consistent mixture ratio distribution in the spray fan. The study further determined that doublets possessed the best mixture ratio when stream-momentum ratio was equal to stream-diameter ratio. The quadlet element had large mixture ratio deviations in the outer spray fan regions and had about 10% of the fluid back spray from the impingement point [103]. Mix ratio distributions for the triplet were very sensitive to changes in element distribution and volume-flow ratios. For a given mixture ratio, the quadlet and doublet were both optimized at the same oxidizer-to-fuel velocity ratio [103]. However, the triplet needed a lower oxidizer-to-fuel velocity ratio than the doublet and the pentad required a higher outer stream-to-center stream velocity ratio. Essentially, the triplet needed the highest inner-jet momentum to penetrate its outer streams in the case of a gas centered design [103].

Finally, spray fan formation is also a main factor that influences global performance and combustion stability. Propellant flow in the combustion chamber and nozzle is composed of four main physical elements: the impinging jets, the spray sheet, propellant droplets, and the spray fan. The spray sheet materializes after the oxidizer and fuel jets impinge on one another. Propellant droplets form following atomization and breakup of the spray sheet and these droplets form the spray fan. The geometry and behavior of the spray sheet and droplets largely determine the efficiency of combustion. For droplets, smaller sizes typically lead to better mixing and atomization of propellants. Smaller sizes also require shorter characteristic length to mix adequately [85]. Droplet size tends to decrease with increasing jet velocity and increasing impingement angle [94]. As they move in the axial direction of the combustion chamber, droplets will vaporize and decrease in diameter. The distribution of the droplets downstream is strongly determined by the dynamic behavior of the spray sheet [89]. The breakup length of the sheet tends to increase as jet velocity increases and impingement angle decreases [94]. As for propellant mixture distributions in spray fans, flows with a fuel-rich mixture in their outer regions tend to have low heat transfer rates to the chamber walls, nozzle, and injector face. However, flows with an oxidizer-rich mixture have high heat transfer rates but also tend to produce higher performance [99]. Fig. **22** provides a visualization of a cone-type spray. As the momentum ratio of oxidizer-to-fuel increases, the gradient of the radius of this cone shape decreases [104].

Fig. 22 Visualization of flow from an impinging injector following spray fan breakup [109]**.**

Furthermore, there have been several studies over the years that have looked at flow characteristics of injectors. One dissertation, published in 2016, detailed the ratio of jet breakup length-to-impingement distance imparted on the spray characteristics of like-doublet injectors [110]. When the ratio was greater than one, the spray sheet was found to be steady and had strong impingement spray. When the ratio was equal to one, the sheet was unsteady due to intermittent jet breakup at the point of impingement. As soon as the ratio was less than one, no sheets formed at all [110]. The study also found that droplet diameter and droplet distribution width were inversely proportional to impingement angle and Weber number. For reference, Weber number is a measure of a fluid's inertial force to its surface tension force. In addition, it was determined that local momentum imbalances between the two jets at the point of impingement created disturbances and impact waves on the resultant spray sheet and ligaments [110].

Another study from 1967 looked at the effects of injector element size on mixing [105]. The researchers observed that some of the fluid jets experienced blow-apart due to hypergolic combustion at the point of impingement, which surprisingly, prevented efficient mixing. This issue was exacerbated when element size was increased [105]. Blow-apart is essentially the de-mixing and stream separation of propellants that occurs when chemical reactions take place simultaneously with atomization. This is a common occurrence among hypergolic propellants. The study also stated that flows often separate into fuel-rich and oxidizer-rich combustion zones due to secondary combustion effects on mixing. This separation degrades mixing efficiency and engine performance, but impingers that enable secondary mixing can help alleviate this issue [105].

A 1992 investigation looked into the effects of flow rate on impinger spray characteristics [91]. The study found that increasing the flow rate of the propellants decreased the radius of the cone-shaped flow following sheet breakup. It also discovered that the droplet number density within the spray fan was highest at the edges of the fan and decreased moving inwards to the spray fan center. In contrast, if volume flux were maximized, the jet velocity at the center of the fan would increase and decrease towards the edges of the fan [91]. The researchers mentioned several conclusions from the work of Heidmann et al. 1951 [111], in which found wave frequency in the spray sheet increased as impingement angle decreased, and as jet velocity increased. The wave pattern was more pronounced in liquids with high viscosity. The researchers also concluded that spray velocity varied from about 72-99% of initial jet velocity [91]. Lastly, it was found that about 50% of the mass in the spray fan region was concentrated within forty degrees of the expected spray axis [91].

Several summarizing observations can be made from the reviewed literature that could aid in creating high-performance impinging injector designs.

- High performance for impinging injectors often comes at the cost of combustion stability and wall compatibility.
- To maximize atomization, flow through injector orifices should be as turbulent as possible and jet momentum should be redirected as rapidly as possible. This may be achieved by increasing impingement angle and jet velocity.
- To maximize mixing, the fuel-to-oxidizer jet momentum ratio should be roughly at unity and there should be uniform flow in the manifold with a maximized number of properly spaced elements.
- Secondary impingement is highly beneficial for mixing as well and is offered by unlike split triplet injectors. Thus, an increasing number of mixing and atomization actions will greatly improve performances.

- Based on the reviewed articles, the O-F-O unlike split triplet impinging injector appears to possess the highest performance capabilities. However, the O-F-O scheme has poor chamber wall compatibility.
- The most common type of unlike impinging injector appears to be the triplet. Doublets have been used in the past but seem to have been phased out in priority of other element schemes. Quadlets provide similar results to doublets but don't appear to have been utilized in many liquid rocket engine applications.
- The exit condition of liquid propellants as well as the atomization action within proximity to the injector face greatly affects the global performance of the thrust chamber assembly. Thus, care should be taken during the design process to maximize the effectiveness of atomization and mixing within this region.

Coaxial, impingement, and pintle type element schemes all provide a mechanism by which liquid propellants can breakup, disperse, mix, vaporize, and combust efficiently. In most cases achieving greater than 90% combustion efficiency with common bipropellants. The available literature has been reviewed towards the design optimization of CP combustion device injectors. In general, the processes by which high performance is achieved are shown in the diagrams below.

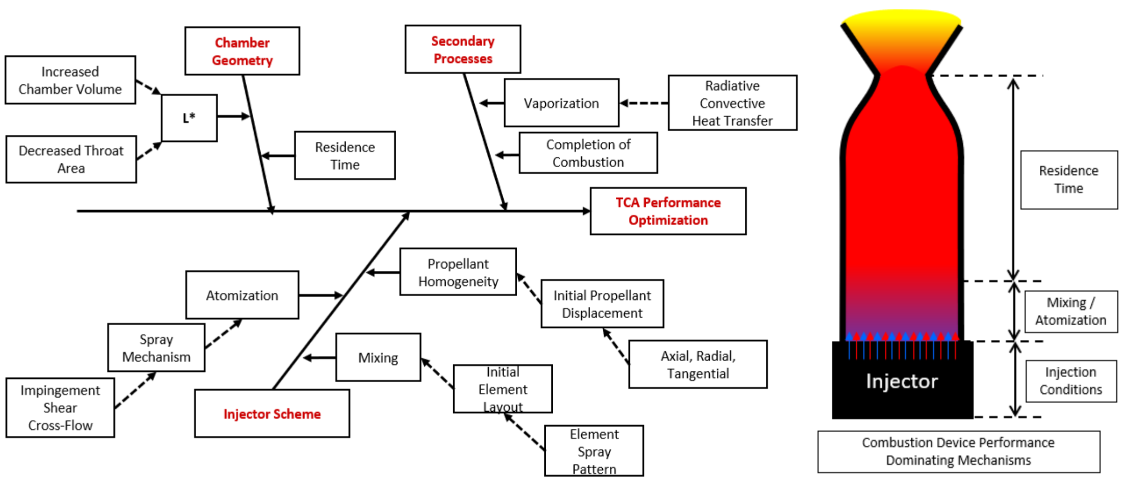

Fig. 23 (left) Fish bone diagram for constant pressure combustor performance considerations and (right) constant pressure combustion device axial reference frame.

While rotating detonation cycle injectors are similar to CP cycle injectors, there are still major differences that need to be considered. For one, the extremely short cycle periods that an injector would experience is likely to require rapid mixing actions as well as immediate recovery and refresh rates. In addition, the axial reference frame alone gives an incomplete picture of the high-performance processes by which these combustors exploit. The radial reference frame would need to be considered in addition to the axial. A performance dominating mechanism schematic and process flow diagram similar to the ones in Fig. **23**, but for an RDRE injector and chamber assembly, are shown below.

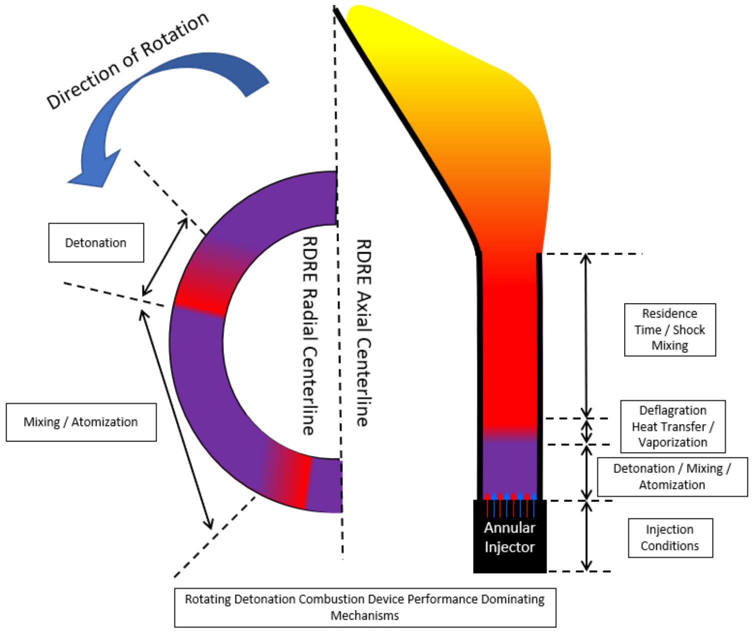

Fig. 24 Schematic of an RDE in the axial and radial reference frames.

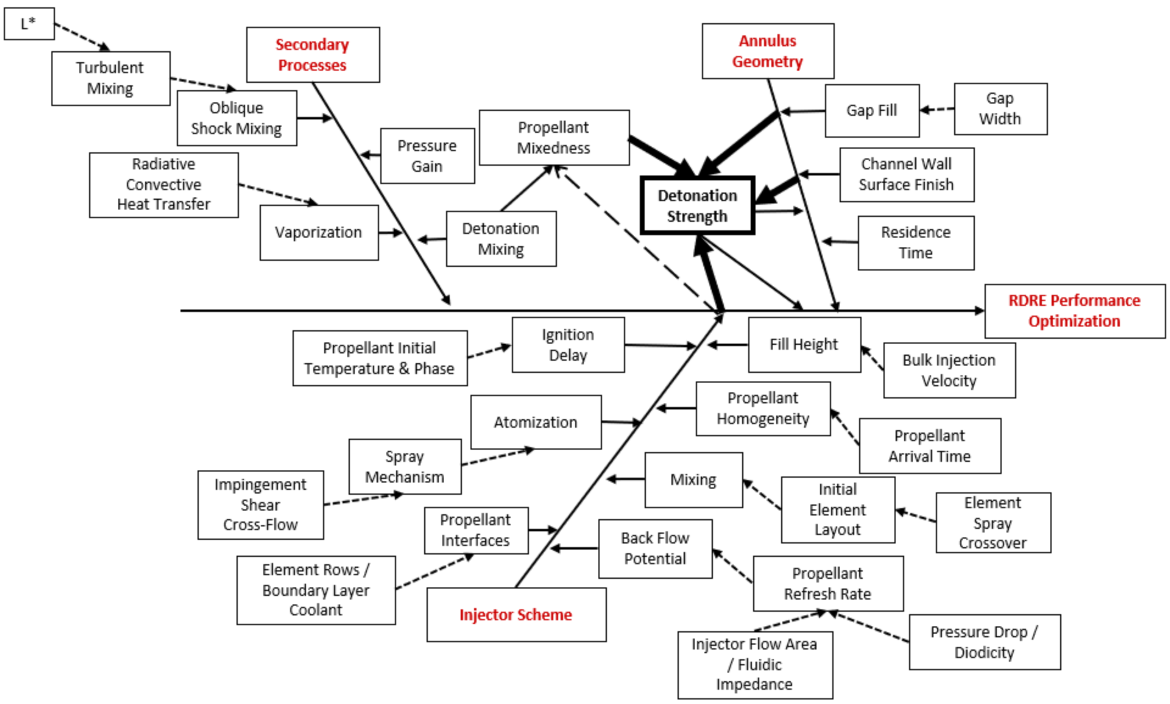

Fig. 25 RDRE performance mechanism fish bone diagram.

The figures above are representations that give the reader an idea of what mechanisms to consider in the design optimization of an RDRE injector and TCA. RDREs have a far more complex web of interlinked dependencies that, for the most part, center around the detonation "strength" itself. In addition, there are likely mechanisms and considerations that this work has simply missed or are not yet known. Regardless, the key to higher performances seems to be maximizing detonation strength. But that leaves the question, what defines strength? As mentioned above, % of CJ velocity has traditionally been used to qualitatively determine the detonation strength and subsequent engine global performance. However, only a slight, if any, correlation has been shown with global engine performance and % CJ velocity, but in the opposite of what is traditionally thought to be the case. In many cases as % CJ velocity decreases, and the number of waves increase, as the engines thrust and Isp increase. The work of Bigler et al. 2020 [35], gives a good analysis on this topic.

The following sections explore the integration of AM technology into the design of liquid rocket injectors and the inherent advantages of different element schemes towards their use in RDREs.

F. Integration of Additive Manufacturing into Injector Design

Before getting into the specifics of AM RDRE injector design, a discussion on the development of AM CP injectors is needed to lend credibility to their life-cycle and combustion performance metrics. Several different coaxial type injectors have been cold flow and hot fire tested at NASA MSFC as outlined in [15]. Several different thrust class, bi-propellant, and material AM co-flow injectors from [15] are shown in the figure below.

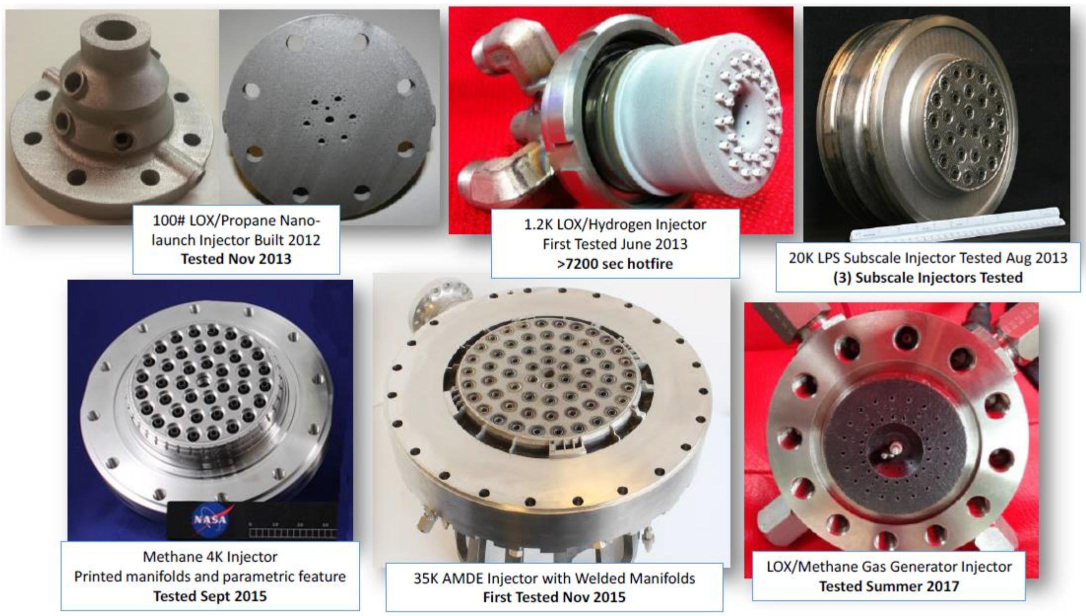

Fig. 26 Various L-PBF Injectors manufactured and tested at NASA MSFC.

Clear advantages are outline for the use of AM to produce injector hardware including reduced number of parts, rapid manufacturing, equivalent performance to traditionally manufactured injectors, and reduced overall costs of manufacturing [15]. In addition, AM allows for the production of components with complex internal structures not previously possible with single piece traditional manufacturing. Even large scale AM injectors can be produced where the work of [112] designed and hot fire tested a 35K lbf class upper stage AM injector.

Water flow testing to characterize the mixing and atomization has been a benchmark for injector design. This is particularly the case for AM produced injectors since validation of free and clear channels is vital to their operation. Often powder can remain in the elements, thus water flow testing can be used to confirm their flow uniformity. Several images of AM injector water flow testing are shown below.

Fig. 27 Water flow testing of various thrust class AM injectors.

Water flow testing can also give an engine designer a qualitative idea of its effective atomization and mixing since a high degree of which would translate to high performance. The common notion for injector design would be to simply print thousands of extremely small orifices that effectively atomize and disperse propellants into the chamber to optimize combustion performance. However, there are still limitations with metal AM that the design must be aware of. First, there is a lower limit of orifice size that can be effectively printed. Laser powder bed fusion (L-PBF) would yield the lowest orifice size possible of all AM build methodologies. This limit is further compounded by the requirement of the orifice being cleared of powder in post processing. All powder must be cleared from the orifice prior to heat treatments. In the case of materials that require stress relieving and solutioning heat treatments, the designer must also consider the build orientation. For Inconel components, residual stresses must be relieved prior to removal from the build plate. Thus, the engineer must be able to remove powder from all orifices while the part remains on the build plate. An example image of an Inconel 718 L-PBF produced NASA impingement pentad injector on the build plate and off the build plate is shown below.

Fig. 28 NASA 7K lbf LOX/methane pentad impingement Inconel 718 injector.

Both NASA and industry have been developing AM injector elements. Several works are available that describe AM injector performances under cold flow and hot fire conditions, their development process, and empirical models for design iteration. NASA, in particular, has had a high degree of success with AM injectors and documented their hot fire test performances and major findings. The major observations and conclusions from these programs are documented below.

- AM allows for the rapid and cost-effective production of monolithic liquid rocket injectors for multiple thrust classes.
- AM injectors to date have demonstrate excellent life-cycle performances often lasting through multiple hot fire test programs with no signs of imminent failure.
- Combustion performance has been shown to be equivalent to that of traditionally manufactured counterparts with measured C* efficiencies up to 100%.
- AM injectors are uniquely capable of being in-situ post-processed to alter their performances even during a hot fire test program.
- Unique element schemes not limited to traditional circular orifice configurations are now possible and may be the key to optimal transient mixing and atomization processes just outside of the orifice exit.

G. Traditional Injector Element Flow Properties

The reviews provided above give indications of what the ideal element design trade space could be for rotating detonation cycle injectors. The tradeoffs from one injector type to another are discussed in this section.

First, traditionally manufactured injectors typically rely on circular ports to transport propellants into the combustion chamber. The flow vector of traditional element schemes often incorporates axial injection except for the like or unlike doublet element scheme. A representation of some common element orifice flow vectors relative to the axial chamber geometry are shown below.

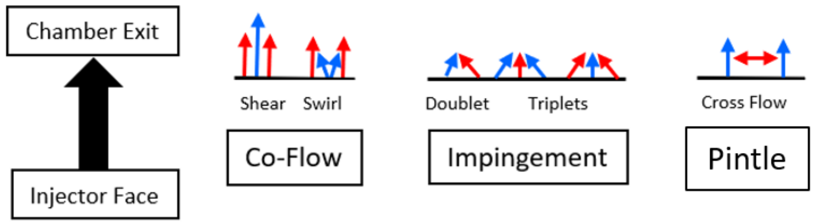

Fig. 29 Traditional injector element orifice flow vector relative to the axial chamber coordinate system.

These element schemes often rely on high gaseous fuel velocity or rapid chemical kinetic rates to effectively breakup the liquid oxidizer jets. It has been observed during hot fire testing at NASA MSFC that low injector pressure drop conditions on the gas fuel and LOX sides can produce low C* performance due to ineffective breakup of the liquid jet. This has generally been observed to be the case for all element types that employ axial liquid injection. With slower chemical kinetic propellants, these performance detriments are only further amplified. The performance of axial injected gas elements is affected to a lesser extent and typically produce slightly higher on average performances but with identical peak performances.

Some, if not all, traditional injector types can produce higher performances with low engine throttling. This has been observed to be caused by slower axial injection of liquid propellants which increases their residence time in the chamber and decreases the jet stiffness allowing for more effective breakup by secondary mechanisms such as attached flames. This also assumes equivalent effectiveness of atomization. Another important observation is that the optimum injector performance may not align with the design performance of the engine. This is due to a number of challenges including but not limited to off nominal injector flow area and reduced effectiveness of secondary mixing processes. This is depicted in the figure above. In addition, some element schemes as previously discussed may not be ideal for detonative cycle engines though are high performers for CP cycle engines, such as the swirl coaxial element.

RDRE injectors tested in the literature almost solely rely on traditional element schemes, use differing propellants, tested with varying chamber design schemes and at different throttle conditions. This makes it very difficult to identify key features that would maximize performance on the element scale. However, a combination of observations from CP combustors and RDREs lend suggestion to what may work better. As discussed above, some works are suggestive that atomization and mixing on the time scales needed are a major challenge for the technology development. Some efforts have suggested that the detonation itself may be able to produce the majority of mixing. Thus, an increased number of propellant interfaces and high element density may improve performance. For example, several closely spaced coaxial elements could produce better performance than impingement elements in liquid/gas RDREs since both the fuel and oxidizer are introduced practically in the same location. The detonation front itself would then do the majority of the mixing, theoretically.

In addition to this speculation, the spacing between elements, regardless of element scheme, would be rather large relatively speaking and could cause discontinuities in propellant at the detriment to detonation performance. Counter propagating modes, slapping modes, or longitudinal mode shapes could dominate. The traditional thought is that these modes would cause detriments in performance, though this is not known for certain. It would then be prudent to design an RDRE injectors individual element in a way that allows for rapid expansion of the propellants into a spray fan to crossover between other element spray fans. A diagram of this concept is shown in the figure below.

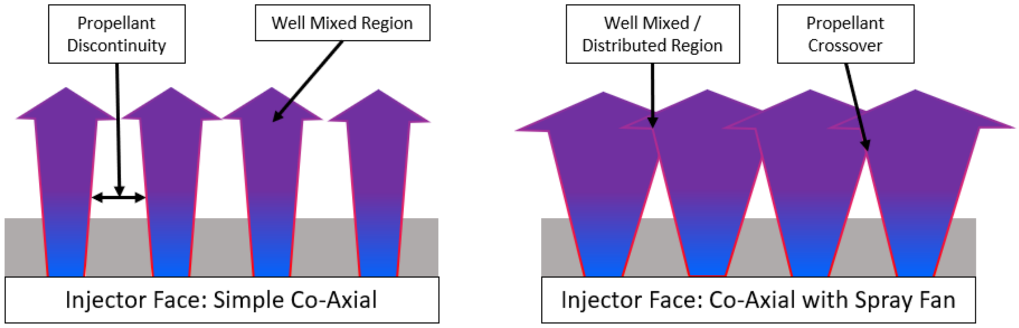

Fig. 30 Diagram of injector element layouts with no propellant crossover and with propellant spray crossover.

Many CP combustion device injectors are designed in a way that maximizes propellant distribution into a cylindrical chamber, thus improving combustion performance. Shear coaxial elements and LOX centered pentad elements cause a similar distribution effect as the left diagram in Fig. **30**. Generally speaking, with larger hydrocarbon fuels these element schemes produce somewhat lower performances than other schemes. Swirl coaxial and gas centered triplets, on the other hand, are able to produce wide spray fans that crossover to other elements in a short distance from the injector face, but at the cost of wall compatibility, in some cases. These schemes in turn yield very high performance over a broad range of propellant equivalence ratios. They achieve better performance by also increasing the number of mixing and atomization interaction. First the liquid phase propellant is atomized via the elements primary mixing action; in a swirl coaxial element the liquid is swirled which decreases the liquids injection stiffness by thinning it into a sheet and effectively atomized by the crossflow of high velocity gaseous fuel. Second, due to the wide spray angle the droplets can then interact between other element spray fields which further mixes and potentially atomizes the propellants.

Other than the element scheme itself, certain modifications can be made that would increase the spray angle without substantial alteration of the element itself. For example, the work of [83] explored geometric alterations to the LOX post of a shear coaxial element by beveling the internal edge with a 10 degree tapper and recessing the LOX post by one post diameter. These modifications increased the spray fan angle and substantially reduced the in-tact LOX core length compared to the baseline. Simple modifications such as orifice beveling could easily provide improve spray field distribution and breakup of high injection stiffness liquid jets. To lend credibility to this, the combustion performance has been shown to improve in CP combustors by upwards of 5-10% by incorporating these element modifications [61].

Topics discussed and observations are summarized below.

- The injection vector of gas/liquid and liquid/liquid bipropellants does appear to be important for overall combustion performance in CP combustion devices. Stiff liquid jet cores could similarly be detrimental to detonation strength.
- CJ velocity may not be as important as previously thought in relation to the global performance of an RDRE. However, the strength or peak pressure and temperature gradient of the detonation front may be a more relevant.
- The spray fan angle and subsequent crossover between elements in an annular combustor may be a necessary design feature to maximize the detonations strength.
- A wide spray fan angle has been shown to improve combustion performance by beveling injector elements. This design feature may also lead to improvement of detonation performance.

In addition to the beveling of element orifices, the orifice discharge coefficient as well as its acoustic impedance may play a key role in improving detonation performance. These topics are further discussed in a later section.

IV. Considerations for AM Detonation Cycle Injectors

Several attributes that may resolve major challenges experienced by detonation cycle engines are discussed in this section. Topics such as backflow, parasitic deflagration, vaporization and ignition delay, as well as diodicity and discharge coefficient are all discussed. Many of these topics are discussed with informed observations made from the available literature and the authors CP hot fire test experience. Ultimately, several gaps currently exist towards the development of rotating detonation cycle injectors which are outlined at the end of this section.

At a high level, a rotating detonation cycle injector element can be viewed in three parts; the metering inlet, the flow altering exit, and the fluidic diode intermediary. In addition, consideration of the general cycle of detonation passage, back flow or flow halting, and propellant refresh are valuable to consider. A diagram of this for a gas centered triplet configuration is shown in the figure below along with depictions of the recovery process in a rotating detonation cycle engine.

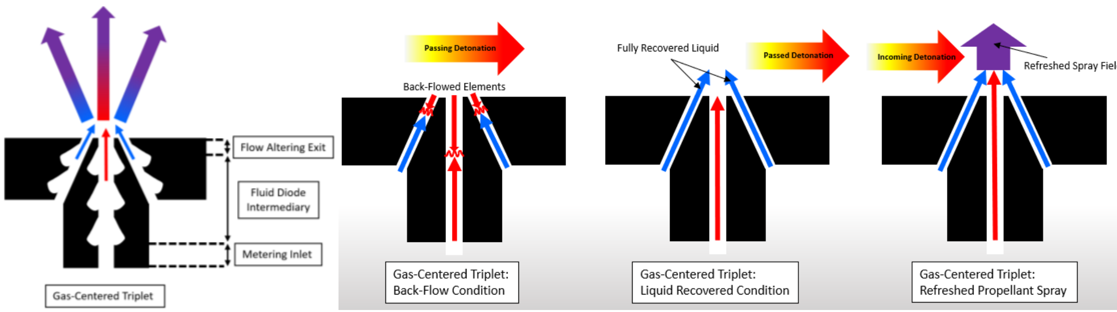

Fig. 31 (left) Diagram of three-part AM injector element, gas-centered triplet, (right) back-flow, refresh, and re-mixed spray process for liquid/gas injector example.

The figure above gives the reader an idea of what a detonative injector likely experiences, though on the order of 10K to 100K cycles per second. Sectioning of an injector element can also be valuable for the conceptual design process. The following discusses flow area and discharge coefficient assumptions towards reduction of back flow potential and ultimately increased recovery performance.

A. Flow Area and Discharge Coefficient for AM Injectors

The flow area is comprised of the discharge or flow coefficient, C_d, multiplied by the physical area of an orifice or set of orifices. This significantly impacts the upstream injector manifold pressure and thus global injector performances. The C_d, is a simple experimental determined quantity and is directly related to the fluid density ρ, and pressure drop ΔP. A major consideration with small AM injector orifices is the accuracy in production of design flow area. It will later be shown that not only will the C_d need to be accounted for but also the accuracy of build. For now, this section considers a few articles that experimentally measured the C_d for different traditionally manufactured orifice configurations as well as the observed trends in flow area from various NASA AM injectors.

The work of Spaur 2011 [113], investigates exactly the question this section aims to answer; what is a reasonable assumption for the C_d of an irregular geometry orifice? Several irregular orifices and uniform geometry orifices were water flow tested with varying l/d values and Reynolds numbers. It was found that on average tube orifices with l/d values of 2-10 were in agreement with values from established literature. Values of $l/d < 2$ yielded similar results to that of thin plate orifices where $C_d = 0.63$. Irregular plate and tube orifices produced at most 12.5% higher C_d than that of typical circular orifices at $l/d < 2$ and similar C_d values to that of circular orifices as l/d becomes greater than 2. As noted in this article, the work of [114] found that the peak C_d occurs at $l/d = 2$ with a trail off as l/d approaches greater values. If geometry were held constant but Reynolds number increased, significantly, it was found by [115] that the discharge coefficient would converge to the typical ~0.62 value that long orifices would converge to. Finally, similar to the conclusion of this work, irregular shaped but axisymmetric orifices produce higher C_d resulting in lower pressure drop than circular orifices [116]. Irregular orifice ΔP, when orifice area was equated, at the most was found to be %55 that of the triangular and circular orifices.

While there is a significant wealth of information on the topic of the discharge coefficient of orifices, there are really only a few cases that would apply to the ultimate goal of this work, designing an optimal detonative performance injection. For one, the orifice would likely be very small with a large wall thickness to manage expected high pressures, thus making the l/d very large. Second, the flow through the elements will be highly turbulent as is typical with liquid rocket engines and a veritable requirement with RDREs. Thus, the only applicable cases that apply are the “full flow” and extruded orifice cases presented in these works. Of particular interest are the circular, square, ellipse, and small orifices experimented with in [113]. Spaur 2008 found that for high Reynolds numbers, the C_d for a square tube was on average around 0.625. A similar result was found with elliptical tubes with an average C_d of 0.63.

A similar work by Brahma 2019 [117], found that certain orifices that have low flow rates and high pressures produced C_d values in the range of 0.25-0.4, which is radically different from the typical 0.6-0.75 range expected. This is consistent with cold flow experimental results observed with small shear coaxial element fuel annuli at NASA

MSFC. However, it must be noted that these C_d are more likely due to a reduction in build dimensions and not solely due to discharge losses.

This section on the elementary topic of discharge coefficient was included because a designer can leverage AM to produce simple orifice geometries with a high reverse flow pressure drop and low forward flow pressure drop, diodicity. For example, if the inlet of an injector orifice had a non-axisymmetric geometry, such as a star shape, which then transitioned into an oval or circular orifice then by the results presented above, the diodicity of the element would be at best about 1.82. This would need to be verified either through simulation or water flow testing but still demonstrates a major advantage to the use of AM. The topic of diodicity is expanded upon in the following section.

Last and most importantly, there are established systematic errors and uncertainties in the AM production of small features, such as orifices. It has been observed at NASA MSFC that the flow area of a specific injector compared with the identically designed injector from a different vendor, can vary by as much as 30-40%. This is heavily dependent on orifice scale, material, and vendor process parameters. In reality, this is not due to alteration of the C_d term but rather physical alteration of the area term and is often caused by build shrinkage or oversizing by the vendor. Initial builds would need to be produced to identify the flow area for a specific element design and then scaled to the desired flow area for the full-scale print. The identical vendor, machine, material, and process parameters would be required to produce the idealized full-scale build with minimal discrepancy from the design.

B. Fluidic Diode Check Valves

Diodicity is typically utilized as a measure of performance for fluid diodes such as tesla valves, which are passive check valves. This is formulated as the ratio of the pressure drop in the reverse direction of flow to the pressure drop in the forward direction of flow, $D_i = \Delta P_r / \Delta P_f$.

Several articles were reviewed for information on the design and integration of a simple fluidic diode structures [118]–[125]. Experimental values of D_i for optimized fluid diodes can be as high as 10 for low Reynolds numbers (~100) [118]. It is theoretically possible to achieve D_i values significantly higher for systems where the Reynolds number is very high. More applicable to the application of RDEs, the impulsive diodicity, may be substantially higher still. The impulsive diodicity is a transient measure of the diodicity where the reverse flow is achieved from a transient pressure gradient rather than steady state condition. An example of a fluid diode is shown in the figure below.

Fig. 32 Tesla type fluid diode with high resistance in the reverse direction and low resistance in the forward direction.

For a single injector element, the diodicity may not fully characterize the elements performance potential to mitigate back flow when a sharp pressure gradient passes. In this case, the impulsive or instantaneous diodicity is more a function of the fluid's inertia, duration of event, compressibility, and the original back pressure of the orifice. Understanding this complex system described above can only be fully realized through computational fluid dynamic modeling or experimental testing and can be provided as an effective recovery time. This would be the ratio of the recovery time for a fluid diode optimized orifice to the recovery time of a control orifice. This work is currently being conducted at NASA MSFC as well as Glenn Research Center to gain a more qualitative sense of how a specific injector element design may perform over another. Several simple examples of fluidic diode schemes under consideration are shown below.

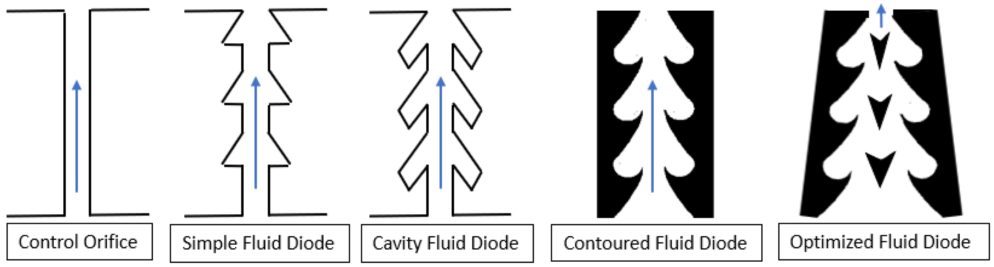

Fig. 33 Potential fluidic diode structures integrated into an injector orifice and control case for comparison.

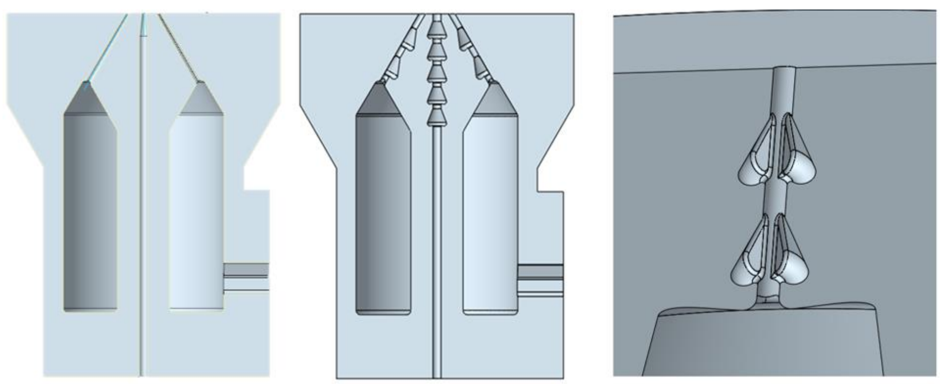

Fig. 34 Computer aided designs of a single injector element specimen with integrated fluid diode structures.

The injector element specimens shown in the figure above have been designed to specifically demonstrate the fluidic characteristics that may be ideal for RDE injector performance. These specimens are currently being AM L-PBF produced and will be water flow tested later in this year at NASA MSFC and collaborating partner facilities. In addition, CFD is being used to identify various performance advantages and disadvantages with the designs above. A simulated detonation front is propagated perpendicular to the injector face and area averaged quantities such as pressure, velocity, and temperature will be captured.

Not surprisingly, to achieve the mixedness and atomization desired for a high performance and stable detonation, small element orifices are practically a must. This is predominantly for a uniform and homogeneous mixture distributed into the annulus. As a general rule of thumb for maximizing mixedness, the smaller the element size and more uniformly distributed the elements are the better [40], [61]. This design practice for injector elements has also yielded higher on average C* efficiency for constant pressure TCAs. It stands to reason that this would similarly hold true for detonative combustors. This is however, only one side of the coin for consideration with injector design. The other consideration is high atomization of the mixture which can be achieved by optimizing the injector elements geometry and providing a similar pressure drop across both the fuel and oxidizer orifices [40]. Pressure drop is really a stand in for injection velocity and injection momentum of the fluid. In addition, the experimental literature often suggests that gas/gas and even liquid/gas RDE injectors are typically operated with high back pressures. This substantially raises the pressure drop across the injector and promotes high degrees of atomization. In a realistic flight engine solution, this method of operation will need to be reduced which will in-turn likely reduce combustion performance. Yet another challenge that will need to be addressed with detonative engines.

The phase state of the propellant, typically gas or liquid, is also an important consideration for reducing backflow potential. Liquids can theoretically reduce the bulk back flow potential due to their inherent incompressibility but with the drawback of reducing their effective fill height or refresh rate due to low injection velocity compared to gaseous phase propellants. Gasses, on the other hand, can recover much quicker but at the detriment of being forced further back into the plenum than liquids. It must also be noted that liquid cavitation within an injector element may prove to be deleterious to hardware life cycle performance. Implementation of fluidic diodes could theoretically accentuate the positive attributes of propellant injection and hinder the negative attributes. For example, an incompressible liquid would naturally act as a rigid column when experiencing a sharp pressure gradient. In a fluid diode structured element, the entire column would resist backflow and allow for quicker refresh once the detonation passes. Gaseous propellants will not behave this way due to their natural compressibility. In this case, the gases would compress and be forced upstream. Regardless, a fluid diode could in theory hinder back flow but would need to act on time scales similar to that of the rate of detonation passage. In the image of a fluid diodes presented in Fig. **33**, the first fluid junction would play the most important role in hindering back flow since it would experience the sharp pressure gradient the first. These short time scales would also require very short junctions that can delay, purge, and refresh the unburned propellants into the chamber. With the advent of additive manufacturing, these geometries could be feasibly printed directly into injector elements. Their lifecycle performance, however, would be unknown until after hot fire testing.

C. Other Considerations for AM Detonation Injector Design

Parasitic deflagration is a major concern for the detonative combustion cycle in general. However, it is not clear that it would be so much of a concern for liquid/liquid or even gas/liquid engines as opposed to gas/gas engines. A study conducted by [33] discussed the role of ignition delay on RDE performance and operability. Several important factors are discussed in detail including the challenge of overcoming back-flow through the injector, a topic previously discussed. It was determined that if the manifold to annulus pressure ratio was large enough, 1.5-2 times the annulus pressure, meeting the critical pressure criteria and choking the injector orifices then back-flow is unlikely to happen. This is attributed to the large pressure pulse meeting the momentum of the injection flow. While this is the case for gas/gas detonation engines, it is likely that the dynamics of liquid/gas and liquid/liquid detonation engines would respond differently.

In addition, the authors discuss how vaporization time and ignition delay may be enough to delay parasitic deflagration so that the detonation may consume the majority of fresh propellants. To further add to this, the average droplet size would need to be considered to gain insight into these time scales. The vaporization time alone would be on the order of magnitude of milli-seconds for typical LRE conditions which puts the droplets complete vaporization on the same order as typical rotating detonation cycle time [33], [126]. If the on average droplet size approached that of the lower limit of what has been observed in LREs, approximately 50 micro-meters in diameter, the vaporization efficiency and subsequent C* efficiency of a typical combustion device would be close to 100% [61] for even low L* CP engines. If this were the case for a detonative cycle engine, the droplets lifetime would be very long relative to the detonation cycle time. The mathematics for a cryogenic LOX droplets lifetime are laid out by [127]–[129]. If a 50-micron diameter on average droplet is assumed with a typical LRE combustion environment, the chamber length required for complete vaporization of the droplet would be in the range of 30 to 50 inches. This of course neglects secondary mixing from combustion and radiative heat transfer. For a typical RDRE configuration, this detonation region would be reasonably within the first inch of the injector face and the droplet would only have lost about 1-5% of its mass to vaporization at these time and length scales. In addition, at typical high pressure drop injection conditions the flame attachment point for shear coaxial and low angle impingement element schemes would be > 15-20 LOX core diameters from the injector face. This is assuming the use of LOX/methane propellants and is solely based on the authors previous hot fire test experience. For typical element diameters of 0.050", this turns out to be approximately 0.75"-1" from the injector face. This, of course, is heavily dependent on the injector element scheme and propellants. Nevertheless, this clearly shows that it is within design capability to produce injector element geometries that potentially minimize parasitic deflagration in preference for detonation performance.

To further expand on the concept of using liquid propellants, Borman and Raglands Combustion Engineering 2011 [130] text gives two sections on detonations. One in purely gaseous mixtures and one in liquid-gaseous mixtures. Of particular interest is the detonation of liquid fuel sprays and spray-initiated detonations. A clear correlation is laid out for the required on-average droplet size in a spray field for a detonation to achieve its full CJ-velocity. Where in this case, the CJ velocity is a stand in term for detonation strength. The text references the work of [131] that conducted detonation tube experiments of liquid fuel sprays dispersed in gaseous oxygen. It was found that the detonation structure was similar to that of a purely gaseous mixture detonation but with a much thicker reaction zone behind the detonation front. The authors point out that this is due to deformation, stripping, vaporization, and diffusion of the liquid fuel by the detonation. The breakup-combustion process for the droplets is further described as being synergistic for the wave front sustainability. Primary and secondary explosions were observed and described as a powerful means of accelerating the detonation front.

Furthermore, this section for which the work of Ragland et al. 1968 [131] is referenced, also discusses the effects of detonations in spray fields of various droplet sizes. To summarize, it was found that detonations occurred in a fuel-oxygen combination, controlled for minimizing initial fuel vapor in the system, in a wide range of on average droplet size spray fields of 2 μm – 2600 μm in diameter. It was found that the propagation velocity was significantly less than the theoretical CJ-velocity in a gaseous equivalent for larger average droplet diameter sprays but only above ~1000 μm in diameter. It was also found through modeling efforts that for droplets below 10 μm, the mechanism of vaporization was enough to sustain detonation. Droplets from 10 μm to 1000 μm would need an additional mechanism for creating micro sprays from the detonation front to further break up the droplets and then vaporize. Regardless, as long as the average droplet size is below 1000 μm on average, the detonation wave propagates with approximately a 2% detriment to the theoretical CJ-velocity of the gaseous equivalent mixture. It must also be noted that this work was conducted at ambient initial temperatures and pressures. It is likely that these general trends would change for spray fields of cryogenic liquid propellants. Since the droplet lifetime for cryogenic fluids is significantly shorter than ambient liquid fuels, it is feasible that the detonation limits would shift more conductively to higher on average initial droplet sizes. This only serves to expand the bounds of operability.

The implications of the works presented above is multi-faceted. First, high-pressure detonations can be achieved with liquid spray fields and rotating detonation modes are possible with liquid/gas injection [27]. Second, an injector can be designed so that it accounts for ignition and vaporization delays alone. Third, lower operating injection temperatures can be utilized with cryogenic fluids to enhance detonation properties. For these reasons, liquid propellant injection may produce better global engine performance compared with similar gas/gas configurations.

As mentioned in [16] on RDRE development and CP injector design [40], the propellants fluidic characteristics and resultant spray field are a critical design consideration. In addition, the fill height has been discussed as a potential important performance parameter in the literature. However, it is not clear if the fill height equates to better global engine performance or even detonation performance. At this point, the author would like to provide a more theoretical musing of potentially high performing characteristics of a detonative injection system.

For a liquid/gas injector, recovery of the liquid phase fluid as well as homogeneous distribution both propellants into the chamber is certainly an important consideration for injector design. An injector element that prioritizes these actions would stand to produce better overall engine performances. Velocity assisted atomization and liquid transport could be a method for achieving these ends. This is not a novel concept as some traditional injector element designs employ it. The idea here is to assist a low velocity liquid jet with a high velocity gaseous stream or streams. Injectors that exhibit this feature include the gas centered impinging element scheme and the swirl coaxial element scheme. In both cases, the streams of liquid oxidizer are better atomized as well as "velocity assisted" with high velocity gaseous fuel crossflow. A simplified schematic of these elements and flow paths are shown in the figure below.

Fig. 35 Water flow testing of (Left) swirl coaxial element scheme and (Right) impinging scheme showing impinged liquid oxidizer jet with gaseous center fuel.

In CP combustion devices these schemes have produced very high C* efficiencies in multiple bipropellants. However, in a detonative combustion device consideration needs to be taken for the refresh time and transient atomization that occurs during the interim between waves. In addition, it is not known what effect, if any, the detonation wave would have on a swirling liquid jet. If the liquid jet orifice is small enough, viscous forces would dominate and prevent swirl all together. The abrupt halting of flow through the orifice may negate swirl as well.

For any high-pressure detonative engine, the detonation height is likely to be small. On the order of magnitude of an inch or less and likely only fractions of an inch in height with numerous waves present. The injector element would need to be designed with consideration of where the detonation would reside in the annulus and how injection recovery dynamics would impact inevitable inefficiencies such as parasitic deflagration at specific propellant, detonation, and residually burning interfaces. For example, liquid rocket engines typically operate in fuel rich conditions, thus, it may be beneficial to have slight fuel lead during the recovery process. It may also be the case that a sufficiently "stood off" wave from the injector face may never induce back flow and thus continuous streams of propellant would be provided. This may create a region where the fresh propellants reside just upstream of the detonation critical region. It may be beneficial to design an injector element so that the oxidizer is shielded by surrounding fuel until the mixing action is initiated. This action could be achieved by impingement or shearing of propellants after entrance to the chamber. This is where specificity in injector element design scheme would play a major roll.

To further expand on element specificity, the outlet geometry may be important for rapid and even distribution of propellants. For example, an oval or even slot design rather than straight circular orifice may yield better atomization and diffusion of a liquid oxidizer. Abrupt changes in orifice area near the exit would achieve this effect. Examination of the experimental literature as well as observations made during hot fire testing at NASA MSFC suggest that element schemes with axial injection of liquid phase oxidizer requires long chambers for complete combustion. This is likely not advantageous for a detonative cycle engine and may cause one of three outcomes. First, abrupt changes in propellant homogeneity may cause odd mode shapes like counter propagating waves and axial pulse modes to dominate. This, however, says nothing about performance and some works have suggested that slapping modes and axial pulse modes have produced higher specific impulse than rotating modes. Second, the detonation may induce mixing between elements and remain unimpeded. Third, the detonation may quench and result in constant pressure combustion. Regardless, significant work is required to better understand how detonation performance and global engine performance are impacted by injector design.

The following list gives the reader and idea of current gaps in the available literature which are needed for full characterization of detonative engine injection systems. These topics need to be assessed experimentally and computationally to identify their impacts on detonation performance and global engine performance.

- The impact and capability of detonation standoff from the injector face with different standoff distances.
- Specificity in injector element design towards promoting a high degree of atomization, mixing, and distribution into the annulus.
- The roll of parasitic deflagration at specific interfaces and bipropellant injection recovery biases.
- Mitigation of back flow potential with element geometries that have high-impedance and high-diodicity.
- Characterization of true detonation wave structure with injection spray field mechanics and geometry.
- Hardware life cycle performance with long duration burns.

V. Summary and Conclusions

The available literature has been reviewed for common practices of RDRE injector design and experimental findings, constant pressure injector design optimization, liquid spray field detonability, and many other performance enhancing characteristics for detonation cycle injectors. Major conclusions from each review have been listed along with qualitative observations from experimental work, NASA CP hot fire test programs, and fundamental analysis of the mechanisms by which would afford high-performance. Lastly, design considerations are given towards the development of additively manufactured injector elements not previously investigated. The conclusions are summarized below.

1) Higher thrust and stronger wave fronts are observed for well mixed propellants. Stronger wave fronts are associated with higher pressure gradients and thus produce higher overall engine performance.
2) High element density with a large number of elements in a close spacing produced higher overall engine performance and detonation stability.
3) Axial fed propellants produce higher overall engine performances than radial fed propellants. This was found for gas/gas injection and is not clear if it can be translated to gas/liquid or liquid/liquid injection.
4) Low feed pressures don’t appear to be detrimental to engine performance.

The major conclusions in the available literature towards optimization of constant pressure combustion device injectors are below.

- For coaxial type elements:
    1) A beveled internal LOX post allows the liquid jet to more effectively breakup with the high velocity gas, thus reducing the in-tact core length and increasing combustion performance.
    2) A recessed LOX post of 1X post diameter is found to be optimal in imparting an oscillation in the liquid jet which allows for the jet to fan out and break up sooner.
    3) High velocity gas and low relative velocity liquid allows for quicker breakup of the liquid core.
    4) A higher velocity liquid core produces better atomization but reduces the mixedness between injector elements.

5) Swirling of the liquid core, rather than relying on shearing actions alone, increases the mixedness and atomization and thus drastically improves combustion performance.

- For impinging type element schemes:
    1) High performance for impinging injectors often comes at the cost of combustion stability and wall compatibility.
    2) To maximize atomization, flow through injector orifices should be as turbulent as possible and jet momentum should be redirected as rapidly as possible. This may be achieved by increasing impingement angle and jet velocity.
    3) To maximize mixing, the fuel-to-oxidizer jet momentum ratio should be roughly at unity and there should be uniform flow in the manifold with a maximized number of properly spaced elements.
    4) Secondary impingement is highly beneficial for mixing as well and is offered by unlike split triplet injectors. Thus, an increasing number of mixing and atomization actions will greatly improve performances.
    5) Based on the reviewed articles, the O-F-O unlike split triplet impinging injector appears to possess the highest performance capabilities. However, the O-F-O scheme has poor chamber wall compatibility.
    6) The most common type of unlike impinging injector appears to be the triplet. Doublets have been used in the past but seem to have been phased out in priority of other element schemes. Quadlets provide similar results to doublets but don't appear to have been utilized in many liquid rocket engine applications.
    7) The exit condition of liquid propellants as well as the atomization action within proximity to the injector face greatly affects the global performance of the thrust chamber assembly. Thus, care should be taken during the design process to maximize the effectiveness of atomization and mixing within this region.

Advantages of integrating additive manufacturing into the development of RDRE injectors.

1) AM allows for the rapid and cost-effective production of monolithic liquid rocket injectors for multiple thrust classes.
2) AM injectors to date have demonstrate excellent life-cycle performances often lasting through multiple hot fire test programs with no significant degradation.
3) Combustion performance has been shown to be equivalent to that of traditionally manufactured counterparts with measured C* efficiencies up to 100%.
4) AM injectors are capable of being in-situ post-processed to alter their performances even during a hot fire test program.
5) Unique element schemes not limited to traditional circular orifice configurations are now possible and may be the key to optimal transient mixing and atomization processes just outside of the orifice exit.

Recommendations towards future improvements in RDRE injector performances.

1) The injection vector of gas/liquid and liquid/liquid bipropellants does appear to be important for overall combustion performance in CP combustion devices. Stiff liquid jet cores could similarly be detrimental to detonation strength.
2) The spray fan angle and subsequent crossover between elements in an annular combustor may be a necessary design feature to maximize the detonations strength rather than solely relying on the detonation for mixing.
3) A wide spray fan angle has been shown to improve combustion performance by beveling injector elements. Similar design features may also lead to improvement of detonation performance.

The ultimate goal of this work is to direct future efforts towards investigation of major gaps in understanding of RDE and RDRE injectors. How the injector design impacts the global engine and detonation performance should be the primary focus. These gaps are summarized below.

1) The impact of performance with detonation standoff from the injector face.
2) Impact of injector element design specificity on performance and their ability towards promoting a high degree of atomization, mixing, and distribution into the annulus.
3) The roll of parasitic deflagration at specific interfaces and bipropellant injection recovery biases.
4) Mitigation of back flow potential with element geometries that have high-impedance and high-diodicity.
5) Characterization of true detonation wave structure with liquid injection spray fields.
6) Injector hardware life cycle performance with long duration burns.

Primary Conclusions and inferences from the available literature have been summarized and discussed towards their integration into high performance AM RDE/RDRE injectors. AM has the unique capability of producing new element schemes not previously possible with traditional manufacturing techniques. These include slots, ovals, mesh screens, and micro-elements. Geometries such as these have the potential to produce better atomization and mixing compared to traditionally manufactured element schemes, particularly for the rotating detonation cycle. While AM can produce new complex geometries, there are still inherent limitations that need to be considered prior to the design process.

Injector element schemes for the rotating detonation cycle should explore leveraging of primary and secondary mixing actions that promote a region of standoff from the injector face where propellants are not yet mixed. Otherwise, hardware may experience deleterious conditions and a short life cycle. In the case of typical LRE propellants, LOX and fuel would then impinge so that a high degree of atomization and distribution can be achieved. Modification to the orifice exit shape may allow for better dispersion between injector elements, thus creating a more continuous region of well mixed and distributed propellants around the annulus.

Once the conceptual design of the specific element scheme has been decided on, the reader can simply follow standard practices for the design of individual metering orifice geometry based on required flow area. Assumptions will have to be made as to what the flow area would be for a specific size and set of orifices prior to an initial build. Consultation with established AM vendors would ensure high tolerances for a full-scale build, and to an even greater level of certainty if initial builds were conducted and characterized. In addition, consideration will need to be given towards irregular or typical inlet geometry. The inlet geometry will impact diodicity, flow area, and subsequent recovery ability of the element scheme, each of which are very important for high performance.

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