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Symmetry and Error Mitigation in Quantum Alternating Operator Ansatz

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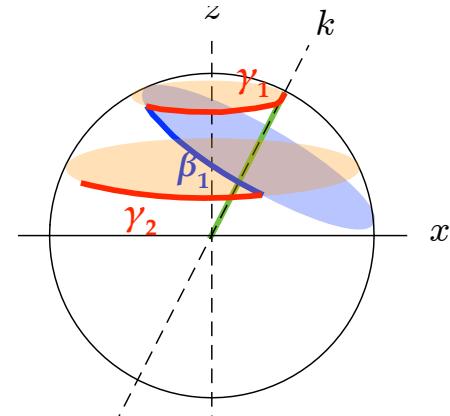
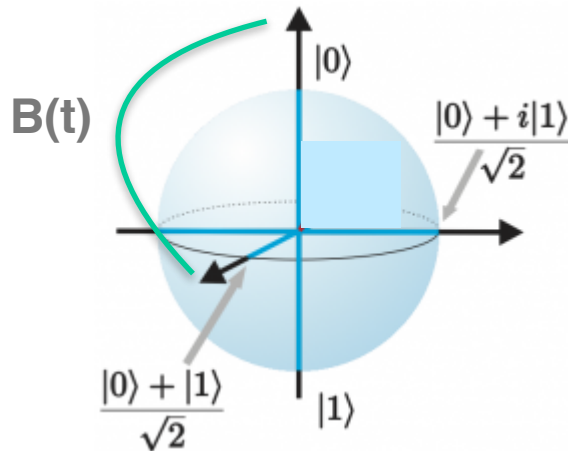


QUANTUM
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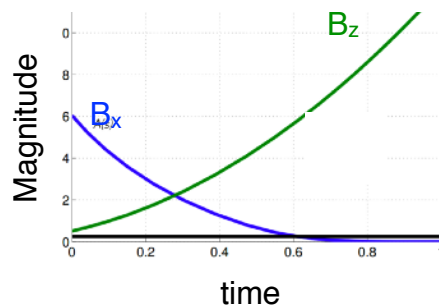
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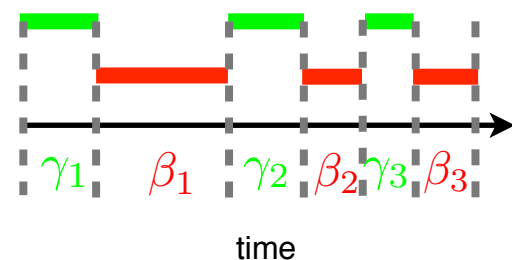
Single-Qubit “quantum computing”



Adiabatic QC / Anneal: slowly varying $B(t)$ from x to z ; At each moment, the qubit is aligned along the instantaneous field $B(t)$



Alternatively, the evolution between qubit states can be achieved by alternating between two controls along x and z . — **QAOA** or Bang-Bang control.





A Quantum Variational Algorithm: Quantum Alternating Operator Ansatz (QAOA) Quantum Approximate Optimization Algorithms (vanilla QAOA)

[Farhi, Goldstone, Gutmann, arXiv:1411.4028]

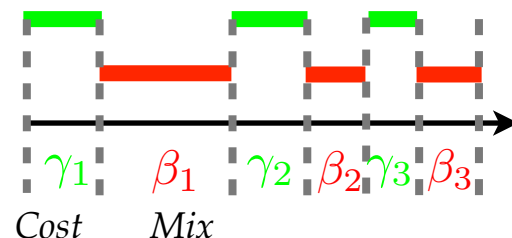
[Hadfield et. al., arXiv 1709.03489]

One formula summary of the algorithm/ansatz

$$U = \underbrace{e^{-i\beta_p H_M}}_{\text{Mixing}} e^{-i\gamma_p H_C} \dots e^{-i\beta_2 H_M} \underbrace{e^{-i\gamma_1 H_C}}_{\text{Cost}}$$

Mixing: explore the solution space

Cost: associate with problem
characteristics



- Suitable for near-term gate-model quantum hardware
- Variational (allows exploiting of classical methods for parameter searching)
- Output distribution hard to sample classically [Farhi, Harrow, arXiv 1602.07674]
- Universal [Seth Lloyd, arXiv1812.11075]
- Quantum speedup for Grover's problem [Jiang, Rieffel, Wang, PRA 2017]
- Challenges:
 - Efficiently acquire a good set of parameters
 - Performance under noise
 - Identify problem sets where quantum advantage might reside



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Outline

- Quickly review two QAOA studies using standard mixer $\sum_j X_j$
 - Grover's search problem
 - 1D XY chain: an analytical solvable case
- Mixers preserving symmetry: XY mixer for equality constraints
 - Performance, comparison with vanilla QAOA
 - Effect of noise and error mitigation

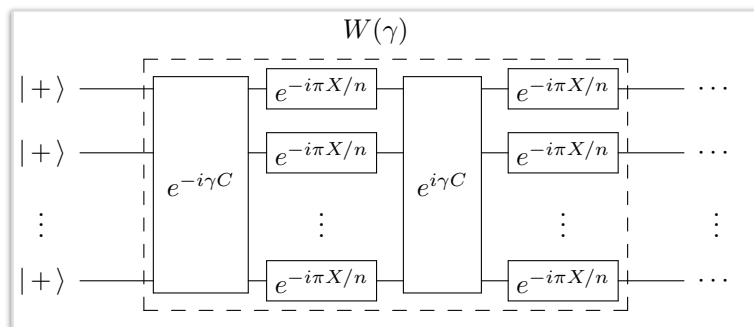
Working in (N+1)-dimensional subspace: Grover's in QAOA

QAOA for Grover's problem

Deterministic parameters:

$$\beta = \pi/n, \quad \gamma \text{ arbitrary}$$

$$W(\gamma) = e^{-i\pi B/n} e^{i\gamma C} e^{-i\pi B/n} e^{-i\gamma C}$$



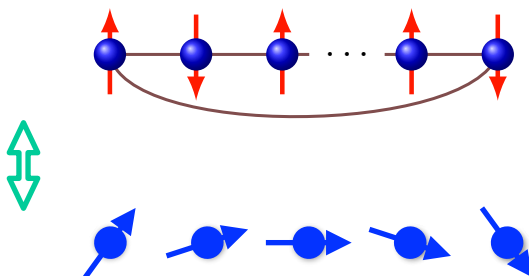
the state $|0\rangle$ and the initial state $|+\rangle^{\otimes n}$ are in the $(n+1)$ -dimensional symmetric subspace (under permutations of qubits), and the evolution under both B and C_0 preserves this subspace, so we need to consider only that $(n+1)$ -dimensional subspace instead of the whole Hilbert space of dimension 2^n .

Reproduces the $\sqrt{N}$ quantum speedup

$$\langle \mathbf{u} | \psi_{\text{out}} \rangle \simeq 1/\sqrt{2}, \quad p \simeq \frac{\pi}{4\sqrt{2}} \sqrt{N}$$

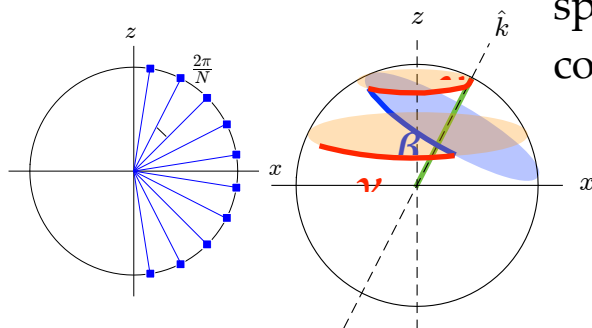
A benchmarking problem: QAOA for anti-ferromagnetic ring (MaxCut on a ring)

- Anti-Ferromagnetic Chain: $H_C = \sum_j \sigma_j^z \sigma_{j+1}^z$
- Analysis: Jordan-Wigner transformation for 1D spin chain with n.n. couplings



Coupled spin chain of size N

2-axis control of $N/2$ independent spins with collective angles



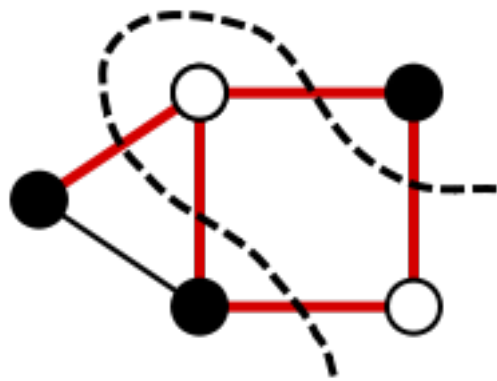


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Canonical example : MaxCut

Mapping to QAOA



One wants a subset S of the vertex set such that the number of edges between S and the complementary subset is as large as possible. [Wikipedia](#)

For each node v , assign variable $z_v \in \{\pm 1\}$ to indicate whether $v \in S$ or $v \in \bar{S}$

Then the number of edges between the two subsets, maps to the Cost Hamiltonian:

$$H_C = - \sum_{\langle u,v \rangle \in E} Z_u Z_v$$

Standard Mixing Hamiltonian:

$$H_X = \sum_{v \in V} X_v$$

In this problem, any solution bit strings are *feasible*: it is a cut

Advanced mixer: enforce constraints by exploiting symmetry in the system

Example: constant Hamming weight (total spin-z) for a subset of qubits

$$\begin{aligned} \sum_j x_j &= C & x_j &\in \{0,1\} & \text{Equality constraint in classical optimization} \\ \sum_j z_j &= C' & z_j &\in \{-1,1\} & \text{Preserved particle numbers in quantum spin system} \end{aligned}$$

- Only states satisfying the constraint $\sum_j z_j = C'$ is a feasible solution
- Standard mixer X_j explores the whole Hilbert space, including infeasible subspace
- (Some) operators that preserve such symmetry

$$\begin{aligned} \text{XY-model} \quad O &= X_j X_{j'} + Y_j Y_{j'} \propto |\uparrow\downarrow\rangle\langle\downarrow\uparrow| + |\downarrow\uparrow\rangle\langle\uparrow\downarrow| \\ [O, Z_j + Z_{j'}] &= 0 \end{aligned}$$

O keeps the quantum evolution in the feasible subspace defined by $\sum_j z_j = C'$

[From QAOA to QAOA, Hadfield, Wang, O'Gorman, Rieffel, Venturelli, Biswas, arXiv 1709.03489];

[XY mixers: Analytical and numerical results for QAOA, Wang, Rubin, Dominy, Rieffel, arXiv:1904.09314]

Mixer to preserve symmetry in the system

Example: Graph Coloring

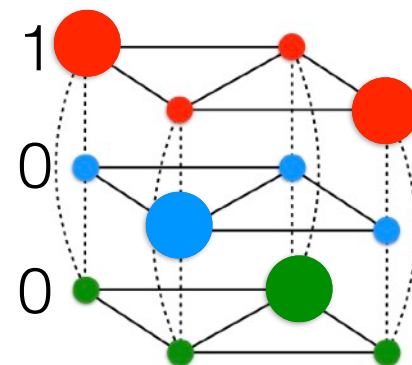
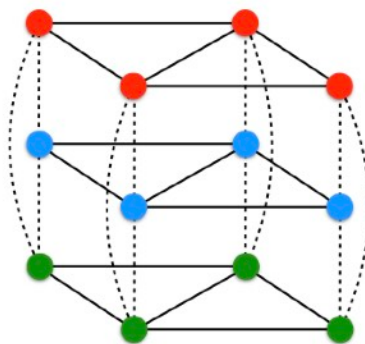
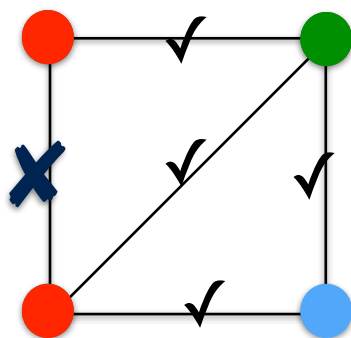
- *MaxColorableSubgraph* Problem: Assign colors to vertices to maximize properly-colored edges (connecting two vertices of different color) — NP hard

Encoding (one-hot):

Binary: $x_{v,c} = 1$ whether vertex v is assigned color- c

Constraints:

Each vertex should have exactly one color: $\sum_{c=1}^k x_{v,c} = 1 \iff \sum_{c=1}^k \sigma_{v,c}^z = k - 2$



Mixer to preserve symmetry in the system

Example: Graph Coloring

Assuming the constraints are satisfied

Each vertex should have exactly one color: $\sum_{c=1}^k x_{v,c} = 1 \iff \sum_{c=1}^k \sigma_{v,c}^z = k - 2$

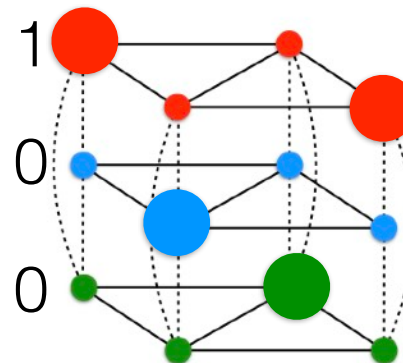
Cost function

$$f_C = m - \sum_{c=1}^k \sum_{\{v,v'\} \in E} x_{v,c} x_{v',c}$$

counts number of properly colored edges

Cost Hamiltonian

$$H_C = \sum_{c=1}^k \sum_{\{v,v'\} \in E} \sigma_{v,c}^z \sigma_{v',c}^z$$



Mixer to preserve symmetry in the system

Example: Graph Coloring

Implement Constraints:

Each vertex should have exactly one color:

$$\sum_{c=1}^k x_{v,c} = 1 \iff \sum_{c=1}^k \sigma_{v,c}^z = k - 2$$

How constraints
are implemented

By mixer that preserves the symmetry

Cost Hamiltonian

$$H_C = \sum_{c=1}^k \sum_{\{v,v'\} \in E} \sigma_{v,c}^z \sigma_{v',c}^z$$

Mixing Hamiltonian

XY-mixer

$$H_v^{(mix)} = \sum_{c \neq c'} X_{v,c} X_{v,c'} + Y_{v,c} Y_{v,c'}$$

Performance (simu)

Mixer to preserve symmetry in the system

Example: Graph Coloring

Implement Constraints:

Each vertex should have exactly one color:

$$\sum_{c=1}^k x_{v,c} = 1 \iff \sum_{c=1}^k \sigma_{v,c}^z = k - 2$$

How constraints are implemented	As penalty in cost	By mixer that preserves the symmetry
Cost Hamiltonian	$H_{\text{penalty}} = \left[\sum_{c=1}^k \sigma_{v,c}^z - (k - 2) \right]^2$ $H'_c = H_c + H_{\text{penalty}}$	$H_C = \sum_{c=1}^k \sum_{\{v,v'\} \in E} \sigma_{v,c}^z \sigma_{v',c}^z$
Mixing Hamiltonian	X-mixer $H_v^{(mix)} = \sum_c X_{v,c}$	XY-mixer $H_v^{(mix)} = \sum_{c \neq c'} X_{v,c} X_{v,c'} + Y_{v,c} Y_{v,c'}$
Performance (simu)		

Mixer to preserve symmetry in the system

Example: Graph Coloring

Implement Constraints:

Each vertex should have exactly one color:

$$\sum_{c=1}^k x_{v,c} = 1 \iff \sum_{c=1}^k \sigma_{v,c}^z = k - 2$$

How constraints
are implemented

As penalty in cost

By mixer that preserves the symmetry

Cost Hamiltonian

$$H_{\text{penalty}} = \left[\sum_{c=1}^k \sigma_{v,c}^z - (k - 2) \right]^2$$

$$H'_c = H_c + H_{\text{penalty}}$$

$$H_C = \sum_{c=1}^k \sum_{\{v,v'\} \in E} \sigma_{v,c}^z \sigma_{v',c}^z$$

Mixing Hamiltonian

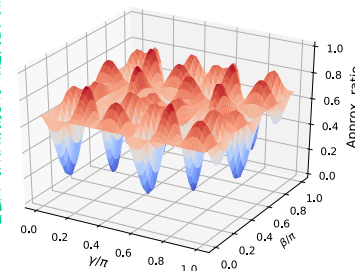
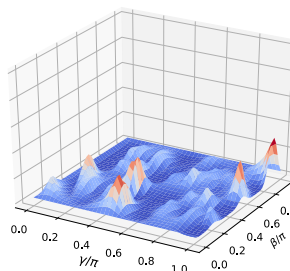
X-mixer

$$H_v^{(mix)} = \sum_c X_{v,c}$$

XY-mixer

$$H_v^{(mix)} = \sum_{c \neq c'} X_{v,c} X_{v,c'} + Y_{v,c} Y_{v,c'}$$

Performance (simu)



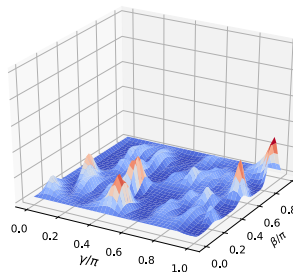
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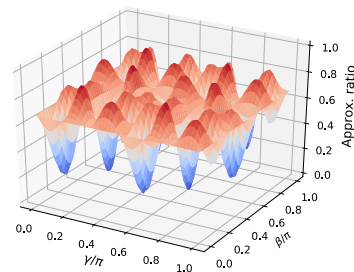
Penalty & X-mixer

XY-mixer

Performance (simu)



(a)



(b)

Figure 3. Numerical results for level 1 *QAOA* on the problem of 3-coloring of a triangle graph. (a) using *X* mixer along with phase-separating Hamiltonian, Eq. (8) where the penalty weight is taken to be the numerically determined optimal value $\alpha^* = 1.7$. (b) using the *XY* mixer with W-state being the initial state.

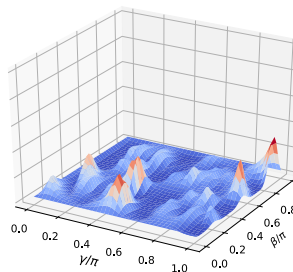
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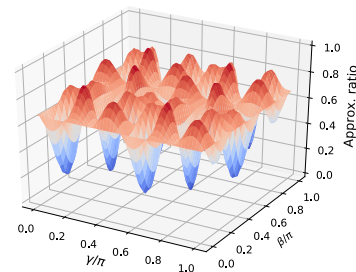
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Full Hilbert space

Size of search space

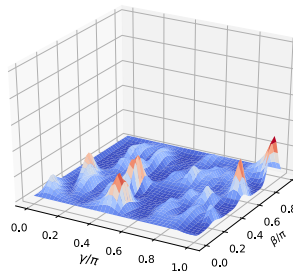
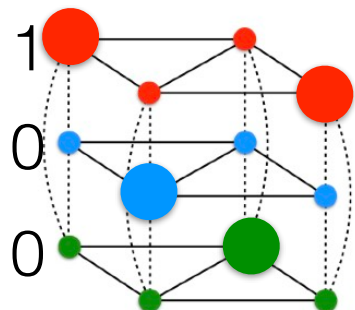
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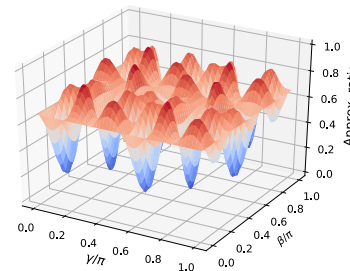
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XY-mixer

Performance (simu)



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Full Hilbert space 2^{nk}

Size of search space

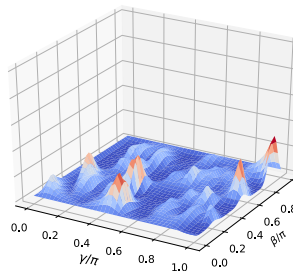
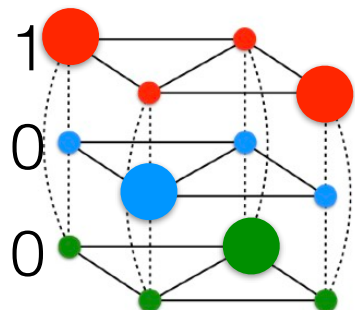
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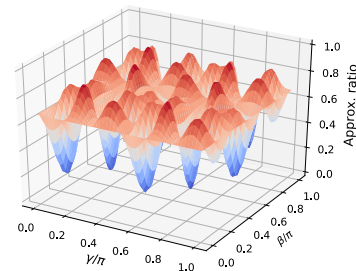
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Performance (simu)



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Feasible subspace k^n

Size of search space

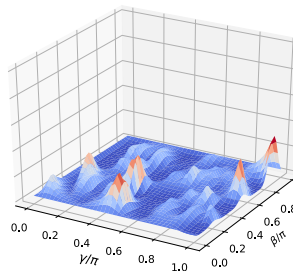
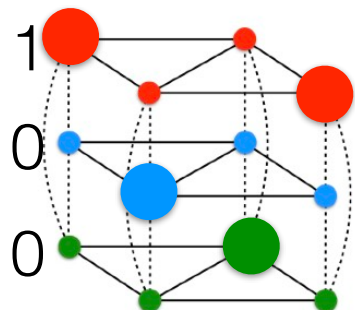
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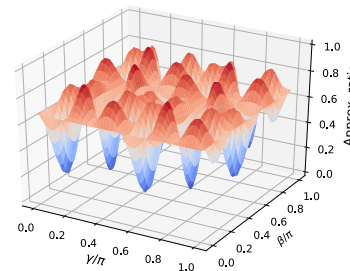
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(a)



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Full Hilbert space 2^{nk}

Feasible subspace k^n

Size of search space

Ratio: $\left(\frac{k}{2^k}\right)^n$

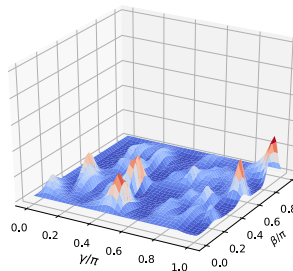
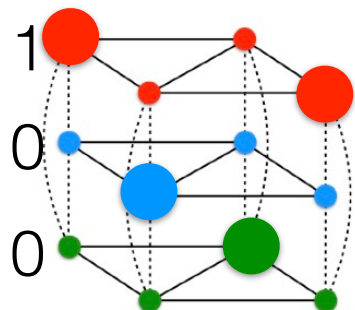
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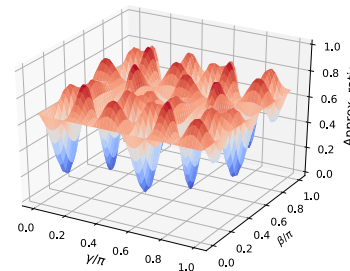
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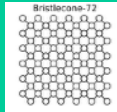
Size of search space

Ratio: $\left(\frac{k}{2^k}\right)^n$ The feasible space **shrinks exponentially** with n .



Circuit depth for various XY mixers

$$U_M = \exp[-i\beta \sum_{\langle uv \rangle \in G_M} X_u X_v + Y_u Y_v]$$

Hardware graph Mixer graph G_M	all-to-all 	chain 	Realistic Hardware 
ring 	$O(\log(\kappa))$ <i>Fast-Fermionic- Fourier transform</i>	$O(\kappa)$ <i>Givens rotation network</i>	
Complete-graph Simultaneous 	$K-1$ <i>One special partition is exact</i>	?? — <i>Classical scheduling problem</i>	
Complete-graph Partitioned 	K	K <i>Bubble-Sort style; $XY(\beta) + \text{SWAP} = XY(\beta')$</i>	



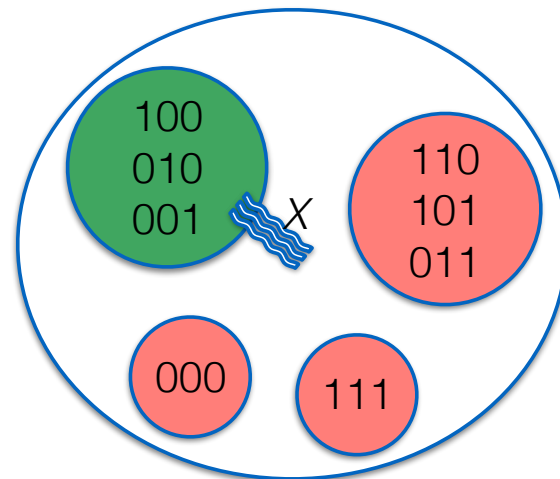
In presence of symmetry-breaking noise

**Probability of staying in the desired subspace:
an upper bound to performance**

Local depolarizing channel

$$\mathcal{E}(\rho) : \rho \rightarrow \sum_i K_i \rho K_i^\dagger$$

$$K_0 = \sqrt{1 - \eta} \mathbb{1}, \quad K_{1,2,3} = \sqrt{\frac{\eta}{3}} \sigma_{1,2,3}$$





In presence of symmetry-breaking noise

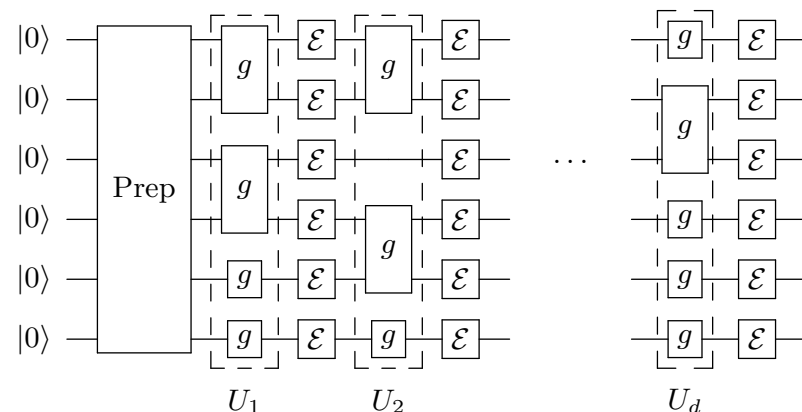
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$$\langle \mathcal{P}_H^i \rangle = \text{Tr}[\rho_0 U_1^\dagger \mathcal{E}(U_2^\dagger \mathcal{E}(U_3^\dagger \cdots \mathcal{E}(U_d^\dagger \mathcal{E}(\mathcal{P}_H^i) U_d) \cdots U_3) U_2) U_1]$$





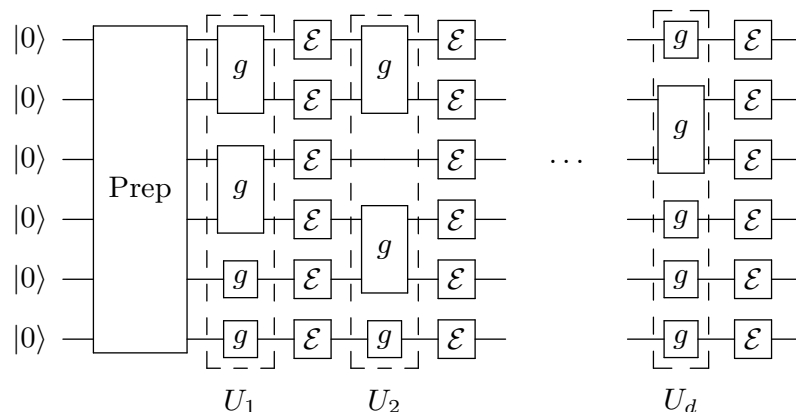
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We derived **Exact** formula for the probability of preserving Z_{tot} under **locally homogeneous noise** (e.g., depolarizing channel) for any circuit that preserves Z_{tot} .



In presence of symmetry-breaking noise

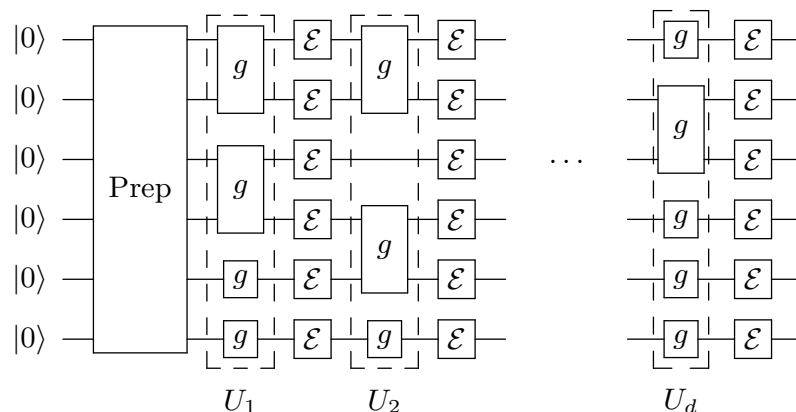
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We derived **Exact** formula for the probability of preserving Z_{tot} under **locally homogeneous noise** (e.g., depolarizing channel) for any circuit that preserves Z_{tot} .

$$\mathcal{E}(\mathcal{P}_N) = \sum_{h=0}^{\kappa} c_h^{(1)} \mathcal{P}_h$$

Important feature:

$$\mathcal{E}(\mathcal{E}(\cdots \mathcal{E}(\mathcal{P}_N))) = \sum_{h=0}^{\kappa} c_h^{(d)} \mathcal{P}_h$$

$$U_l^\dagger \mathcal{E}(\mathcal{P}_h) U_l = \mathcal{E}(\mathcal{P}_h)$$



In presence of symmetry-breaking noise

**Probability of staying in the desired subspace:
an upper bound to performance**

Local depolarizing channel

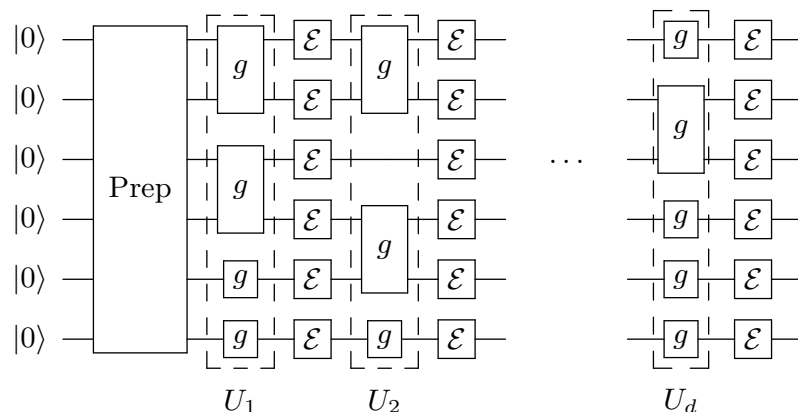
$$\mathcal{E}(\rho) : \rho \rightarrow \sum_i K_i \rho K_i^\dagger$$

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$$\langle \mathcal{P}_H^i \rangle = \text{Tr}(\rho_0 \mathcal{E}(\mathcal{E}(\cdots \mathcal{E}(\mathcal{P}_H^i) \cdots)))$$

$$= \text{Tr} \left(\sum_h c_h^{(d)} \mathcal{P}_h^i \rho_0 \right) = \text{Tr} \left(c_H^{(d)} \mathcal{P}_H^i \right) = c_H^{(d)}$$



We derived **Exact** formula for the probability of preserving Z_{tot} under **locally homogeneous noise** (e.g., depolarizing channel) for any circuit that preserves Z_{tot} .

$$\mathcal{E}(\mathcal{P}_N) = \sum_{h=0}^{\kappa} c_h^{(1)} \mathcal{P}_h$$

Important feature:

$$\mathcal{E}(\mathcal{E}(\cdots \mathcal{E}(\mathcal{P}_N))) = \sum_{h=0}^{\kappa} c_h^{(d)} \mathcal{P}_h$$

$$U_l^\dagger \mathcal{E}(\mathcal{P}_h) U_l = \mathcal{E}(\mathcal{P}_h)$$



In presence of symmetry-breaking noise

**Probability of staying in the desired subspace:
an upper bound to performance**

Local depolarizing channel

$$\mathcal{E}(\rho) : \rho \rightarrow \sum_i K_i \rho K_i^\dagger$$

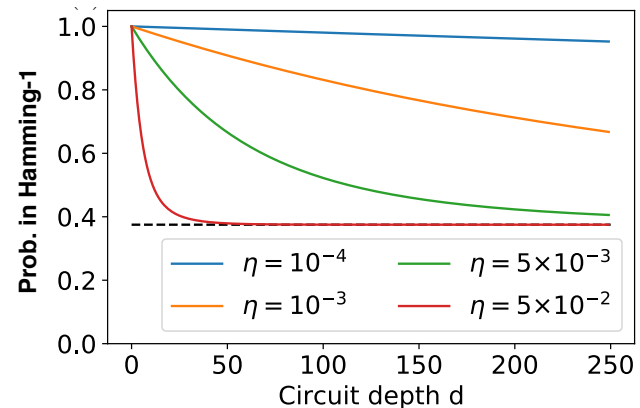
$$K_0 = \sqrt{1 - \eta} \mathbb{1}, \quad K_{1,2,3} = \sqrt{\frac{\eta}{3}} \sigma_{1,2,3}$$

$$\langle \mathcal{P}_H^i \rangle = \text{Tr}[\rho_0 U_1^\dagger \mathcal{E}(U_2^\dagger \mathcal{E}(U_3^\dagger \cdots \mathcal{E}(U_d^\dagger \mathcal{E}(\mathcal{P}_H^i) U_d) \cdots U_3) U_2) U_1]$$

$$\langle \mathcal{P}_H^i \rangle = \text{Tr}(\rho_0 \mathcal{E}(\mathcal{E}(\cdots \mathcal{E}(\mathcal{P}_H^i) \cdots)))$$

$$= \text{Tr} \left(\sum_h c_h^{(d)} \mathcal{P}_h^i \rho_0 \right) = \text{Tr} \left(c_H^{(d)} \mathcal{P}_H^i \right) = c_H^{(d)}$$

Fixed system size: 3 qubits





In presence of symmetry-breaking noise

**Probability of staying in the desired subspace:
an upper bound to performance**

Local depolarizing channel

$$\mathcal{E}(\rho) : \rho \rightarrow \sum_i K_i \rho K_i^\dagger$$

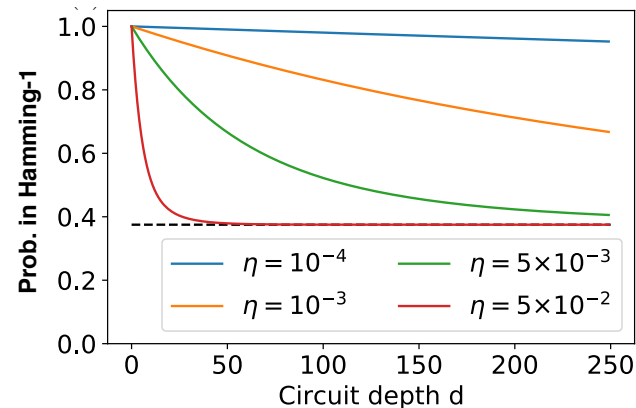
$$K_0 = \sqrt{1 - \eta} \mathbb{1}, \quad K_{1,2,3} = \sqrt{\frac{\eta}{3}} \sigma_{1,2,3}$$

$$\langle \mathcal{P}_H^i \rangle = \text{Tr}[\rho_0 U_1^\dagger \mathcal{E}(U_2^\dagger \mathcal{E}(U_3^\dagger \cdots \mathcal{E}(U_d^\dagger \mathcal{E}(\mathcal{P}_H^i) U_d) \cdots U_3) U_2) U_1]$$

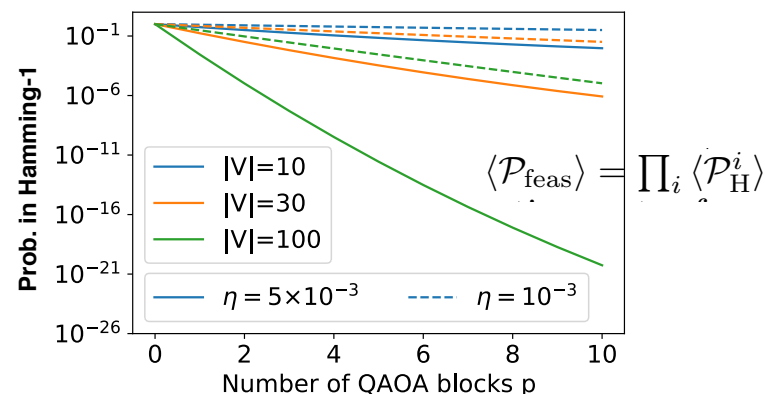
$$\langle \mathcal{P}_H^i \rangle = \text{Tr}(\rho_0 \mathcal{E}(\mathcal{E}(\cdots \mathcal{E}(\mathcal{P}_H^i)) \cdots))$$

$$= \text{Tr} \left(\sum_h c_h^{(d)} \mathcal{P}_h^i \rho_0 \right) = \text{Tr} \left(c_H^{(d)} \mathcal{P}_H^i \right) = c_H^{(d)}$$

Fixed system size: 3 qubits



Decay with num. subsystems

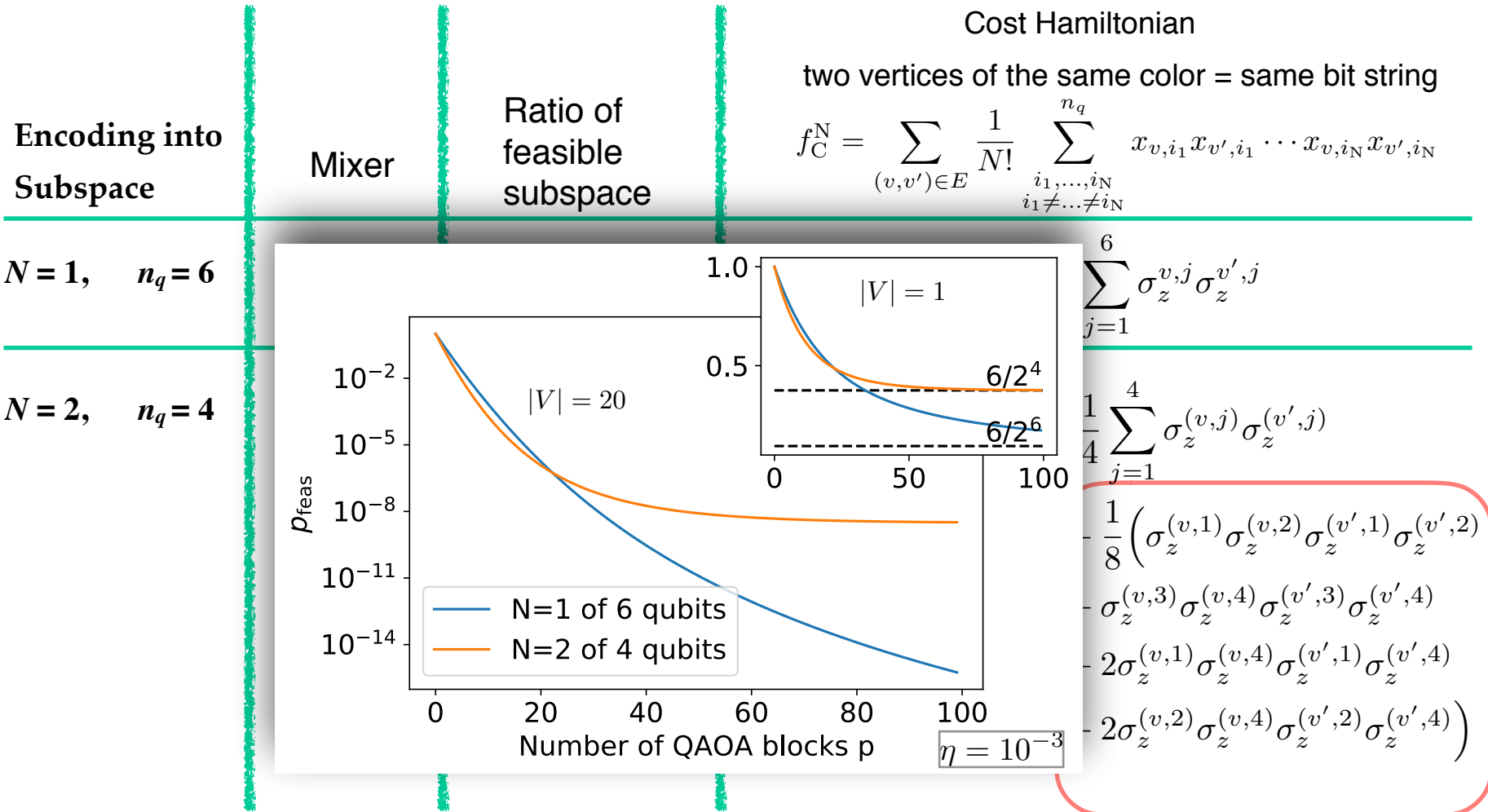




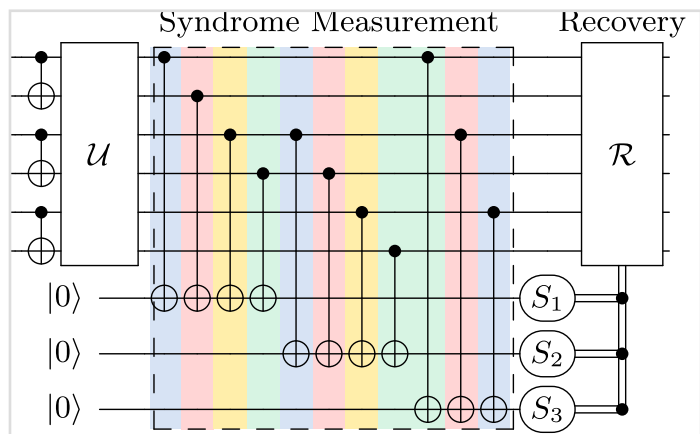
Encoding into a larger subspace?

Encoding into Subspace	Mixer	Ratio of feasible subspace	Cost Hamiltonian
$N = 1, \quad n_q = 6$	$XX + YY$	$6/2^6 \approx 9\%$	two vertices of the same color = same bit string $f_C^N = \sum_{(v,v') \in E} \frac{1}{N!} \sum_{\substack{i_1, \dots, i_N \\ i_1 \neq \dots \neq i_N}}^{n_q} x_{v,i_1} x_{v',i_1} \cdots x_{v,i_N} x_{v',i_N}$ $H_{(v,v')} = \sum_{j=1}^6 \sigma_z^{v,j} \sigma_z^{v',j}$
$N = 2, \quad n_q = 4$	$XX + YY$	$4/2^6 \approx 38\%$	$H_{(v,v')} = \frac{1}{4} \sum_{j=1}^4 \sigma_z^{(v,j)} \sigma_z^{(v',j)}$ Extra 4-local terms require depth-3 $\begin{aligned} &+ \frac{1}{8} \left(\sigma_z^{(v,1)} \sigma_z^{(v,2)} \sigma_z^{(v',1)} \sigma_z^{(v',2)} \right. \\ &+ \sigma_z^{(v,3)} \sigma_z^{(v,4)} \sigma_z^{(v',3)} \sigma_z^{(v',4)} \\ &+ 2\sigma_z^{(v,1)} \sigma_z^{(v,4)} \sigma_z^{(v',1)} \sigma_z^{(v',4)} \\ &\left. + 2\sigma_z^{(v,2)} \sigma_z^{(v,4)} \sigma_z^{(v',2)} \sigma_z^{(v',4)} \right) \end{aligned}$

Encoding into a larger subspace?



Error correction exploiting symmetry



On 3 data qubits

$$S_1 = Z_1 Z_2 Z_3 Z_4 \mathbb{1}_5 \mathbb{1}_6$$

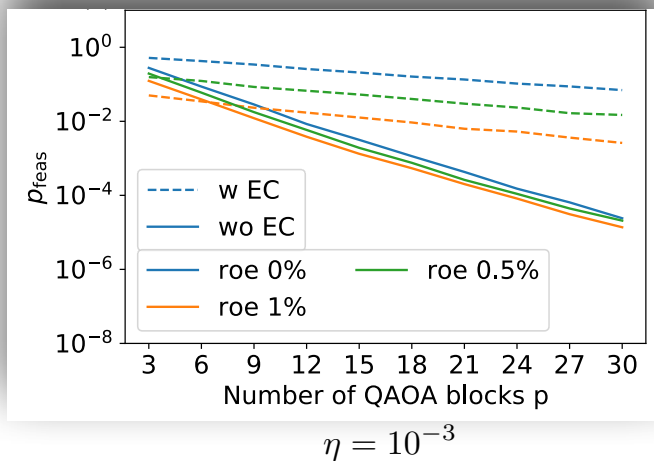
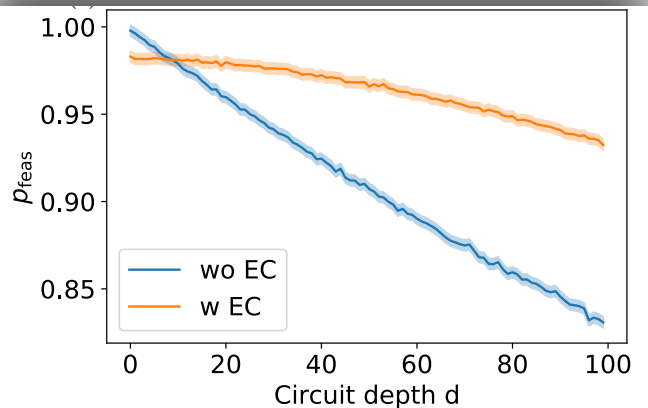
$$S_2 = \mathbb{1}_1 \mathbb{1}_2 Z_3 Z_4 Z_5 Z_6$$

$$S_3 = Z_1 \mathbb{1}_2 Z_3 \mathbb{1}_4 Z_5 \mathbb{1}_6$$

Resource efficiency due to symmetry

Original bit flip code: **two** copies of each qubit are needed

The present adaption: **one** copies of each qubit are needed (only correct for subspace $N=1$)



Quantum algorithms with local particle number conservation: noise effects and error correction

M Streif, M Leib, F Wudarski, E Rieffel, Z Wang, arXiv:2011.06873

Kaleidoscope of Open Problems and Research Topics

Choice of Mixer: mixing efficiency vs implementation complexity

Choice of “Cost Hamiltonian”: Role of energy in QAOA?

Sample vs expectation value: What can we infer from the expected value / average performance?

Choice of initial states

Parameter setting: what we learn from analysis

Landscape and search methods

Underlying physics: Role of entanglement

Benchmarking problem sets: where are small and hard problems?

Effect of noise, error mitigation/correction

Compilation and Routing

Quantum Advantage / Speedup



National Aeronautics and
Space Administration



Thank You!

