

1 **Simulation and scaling of the turbulent vertical heat transport and**  
2 **deep-cycle turbulence across the equatorial Pacific cold tongue**

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## ABSTRACT

22 Microstructure observations in the Pacific cold tongue reveal that turbulence often penetrates into  
23 the thermocline producing hundreds of  $\text{W/m}^2$  of downward heat transport during nighttime and  
24 early morning. However, virtually all observations of this deep-cycle turbulence (DCT) are from  
25  $0^\circ\text{N}, 140^\circ\text{W}$ . Here, a hierarchy of ocean process simulations including submesoscale-permitting  
26 regional models and turbulence-permitting large eddy simulations (LES) embedded in a regional  
27 model provide insight into mixing and DCT at and beyond  $0^\circ\text{N}, 140^\circ\text{W}$ . A regional hindcast  
28 quantifies the spatio-temporal variability of subsurface turbulent heat fluxes throughout the cold  
29 tongue from 1999-2016. Mean subsurface turbulent fluxes are strongest ( $\sim 100 \text{ W/m}^2$ ) within  $2^\circ$  of  
30 the equator, slightly ( $\sim 10 \text{ W/m}^2$ ) stronger in the northern than southern hemisphere throughout the  
31 cold tongue, and correlated with surface heat fluxes ( $r^2 = 0.7$ ). The seasonal cycle of the subsurface  
32 heat flux, which does not covary with the surface heat flux, ranges from  $150 \text{ W/m}^2$  near the equator  
33 to  $30 \text{ W/m}^2$  and  $10 \text{ W/m}^2$  at  $4^\circ\text{N}$  and  $\text{S}$  respectively. Aseasonal variability of the subsurface heat  
34 flux is logarithmically distributed, covaries spatially with the time-mean flux, and is highlighted  
35 in 34-day LES of boreal autumn at  $0^\circ\text{N}$  and  $3^\circ\text{N}, 140^\circ\text{W}$ . Intense DCT occurs frequently above the  
36 undercurrent at  $0^\circ\text{N}$  and intermittently at  $3^\circ\text{N}$ . Daily-mean heat fluxes scale with the bulk vertical  
37 shear and the wind stress, which together explain  $\sim 90\%$  of the daily variance across both LES.  
38 Observational validation of the scaling at  $0^\circ\text{N}, 140^\circ\text{W}$  is encouraging, but observations beyond  
39  $0^\circ\text{N}, 140^\circ\text{W}$  are needed to facilitate refinement of mixing parameterization in ocean models.

## 40 **Significance Statement**

41 This work is a fundamental contribution to a broad community effort to improve global long-  
42 range weather and climate forecast models used for seasonal to longer-term prediction. Much of  
43 the predictability on seasonal timescales is derived from the slow evolution of the upper eastern  
44 equatorial Pacific Ocean as it varies between El Niño and La Niña conditions. This study presents  
45 state-of-the-art high-resolution regional numerical simulations of ocean turbulence and mixing in  
46 the eastern equatorial Pacific. The results inform future planning for field work as well as future  
47 efforts to refine the representation of ocean mixing in global forecast models.

## 48 **1. Introduction**

49 Over the last several decades, multiple field campaigns have observed strong turbulence above  
50 the equatorial undercurrent in the eastern Pacific Ocean (Gregg et al. 1985; Moum and Caldwell  
51 1985; Peters et al. 1988; Lien et al. 1995; Moum et al. 2009, 2013; Warner and Moum 2019; Smyth  
52 et al. 2021). Like upper-ocean turbulence elsewhere in the tropics and subtropics, the diurnal cycle  
53 is a dominant mode of variability, but turbulence in the eastern equatorial Pacific is unusual in that  
54 it penetrates tens of meters below the base of the surface mixed layer and into the thermocline.  
55 This turbulence produces exceptionally strong heat fluxes of  $O(100)$  W/m<sup>2</sup> on average and up to  
56 1000 W/m<sup>2</sup> during occasional bursts of intense turbulence in the nighttime and early morning in  
57 a stratified layer tens of meters thick (Moum et al. 2013, 2009; Smyth et al. 2021). Hence, this  
58 “deep-cycle turbulence” (DCT) drives stronger cooling of the near surface and warming of the  
59 thermocline compared to diurnal surface boundary layer turbulence in other areas of the global  
60 oceans. DCT thus contributes to sustaining the relatively cool sea surface and net ocean heat  
61 uptake in the eastern equatorial Pacific Ocean cold tongue on average (Wang and McPhaden 1999;  
62 Moum et al. 2013). DCT also varies with and influences the regional sea-surface temperature

63 (SST) dynamics on multiple timescales beyond diurnal, including interannual (Warner and Moum  
64 2019), seasonal (Wang and McPhaden 1999; Moum et al. 2013), and subseasonal (Lien et al. 2008;  
65 Moum et al. 2009), although these variations are not as well understood as the diurnal cycle.

66 If the available data from  $0^\circ$  N,  $140^\circ$  W are representative, then turbulent mixing is an important  
67 participant in the SST budget and air-sea interaction in the Pacific Ocean cold tongue. However,  
68 neither the spatio-temporal variability of ocean mixing nor the physical drivers of variability on  
69 timescales beyond diurnal are well observed or understood. In particular, our knowledge of the  
70 area and vertical extent of strong turbulent heat fluxes is based almost entirely on extrapolation  
71 using parameterizations beyond  $0^\circ$  N,  $140^\circ$  W (e.g., Pacanowski and Philander 1981; Holmes and  
72 Thomas 2015; Holmes et al. 2019a; Pei et al. 2020; Deppenmeier et al. 2021; Cherian et al. 2021).  
73 In addition, none of these parameterized modelling studies present results over a sufficient duration  
74 to provide a climatological perspective from a model with sufficiently fine horizontal grid spacing  
75 ( $< 10$  km horizontal (Marchesiello et al. 2011), and  $< 5$  m vertical (Jia et al. 2021)) to fully resolve  
76 the mesoscale variations in vertical shear, which significantly modulate mixing (Moum et al. 2009;  
77 Inoue et al. 2012; Holmes and Thomas 2015; Cherian et al. 2021). Hence, the broader implications  
78 of downward turbulent heat transport and specifically DCT in the cold tongue for global ocean,  
79 climate and Earth system dynamics are not well understood (but see Meehl et al. 2001; Richards  
80 et al. 2009; Danabasoglu et al. 2006; Newsom and Thompson 2018; Holmes et al. 2019b,a; Zhu  
81 and Zhang 2019; Huguenin et al. 2020; Deppenmeier et al. 2021). In addition, climate models  
82 suffer from long-standing and significant biases in their simulation of the SST, thermocline, and  
83 circulation in the eastern equatorial Pacific (Li and Xie 2014; Li et al. 2015). Since some biases  
84 persist with refinements in model horizontal grid resolution and the mean ocean circulation (Small  
85 et al. 2014) and are sensitive to the formulation of the mixing scheme (Meehl et al. 2001; Richards  
86 et al. 2009; Zhu and Zhang 2019), it seems plausible if not likely that poor performance of

87 parameterizations of ocean mixing physics (Zaron and Moum 2009) is at least partially responsible  
88 for equatorial Pacific biases in climate and Earth system models. Hence, we conducted a regional  
89 process modelling study of turbulent heat transport and DCT in the equatorial Pacific Ocean cold  
90 tongue as a contribution to a broader effort to conduct a pre-field process modeling study of Pacific  
91 equatorial upwelling and mixing physics.

92 In this manuscript, we present new state-of-the-art simulations and new metrics to characterize  
93 turbulent vertical heat transport in the Pacific Ocean cold tongue. First, we examine the clima-  
94 tological (1999-2016) spatio-temporal variability of the turbulent vertical heat flux, including the  
95 time-mean, seasonal cycle, and aseasonal variability (i.e., all deviations from the mean seasonal  
96 cycle) of the daily-mean flux, in a relatively fine ( $1/20^\circ$  horizontal, 2.5 m vertical) resolution  
97 regional hindcast simulation of the eastern equatorial Pacific Ocean with parameterized vertical  
98 mixing. The results provide a climatological perspective on the recent finding that global ocean  
99 models can simulate DCT (Pei et al. 2020), as well as the finding of and explanation for DCT off  
100 the equator in a regional ocean model (Cherian et al. 2021), and complement other climatological  
101 studies of mixing in the equatorial Pacific cold tongue focused on different questions, different  
102 metrics, and different models with coarser resolution (e.g., Ray et al. 2018; Holmes et al. 2019a;  
103 Huguenin et al. 2020; Deppenmeier et al. 2021). The analysis of the regional model also shows that  
104 the daily-mean turbulent heat transport is logarithmically distributed, thus relatively rare events  
105 associated with aseasonal variability on timescales of days to weeks have a strong influence on and  
106 spatially covary with the time-mean transport.

107 We build understanding of the subseasonal part of aseasonal variability in mixing via large eddy  
108 simulations (LES) that are embedded in a regional ocean model so that the simulated turbulence  
109 varies in the context of realistic variations in horizontal currents and temperature and atmospheric  
110 forcing over timescales from hours to more than a month. These LES address a key source

111 of uncertainty in our regional model and all prior studies of ocean mixing on timescales from  
112 weeks to months using models: our regional models and all prior models are based on uncertain  
113 mixing parameterizations. Here, the LES are used to study the variability of explicit (rather than  
114 parameterized) turbulent mixing and DCT on timescales from days to a month for the first time.  
115 Our LES build on prior shorter simulations of diurnal cycles and shorter variability with idealized  
116 boundary conditions and forcing (Skylingstad and Denbo 1994; Wang et al. 1996, 1998; Large  
117 and Gent 1999; Wang and Müller 2002; Pham et al. 2013) as well as how the diurnal cycles vary  
118 between the four seasons at  $0^\circ$  N,  $140^\circ$  W (Pham et al. 2017; Sarkar and Pham 2019). Through  
119 both the analysis of the regional model and the LES, we confront the simulations of turbulence  
120 with observations and critically evaluate the model representations, albeit only at  $0^\circ$  N,  $140^\circ$  W.  
121 Future observations are needed to evaluate and constrain modeled turbulence beyond  $0^\circ$  N,  $140^\circ$   
122 W in the Pacific cold tongue.

## 123 **2. Methods**

### 124 *a. Ocean hindcast of the eastern equatorial Pacific, 1999-2016*

125 Climatological statistics of vertical mixing throughout the equatorial Pacific cold tongue are  
126 derived from an ocean hindcast of the period 1999 through 2016 in the region from  $170^\circ$  to  $95^\circ$  W  
127 and from  $12^\circ$  S to  $12^\circ$  N in a submesoscale-permitting  $1/20^\circ$  configuration (Cherian et al. 2021)  
128 of the MITgcm (Adcroft et al. 2004; Marshall et al. 1997). As described previously (Cherian  
129 et al. 2021), the model is forced at the surface by fluxes derived from bulk flux algorithms and  
130 the JRA55do atmospheric reanalysis (Tsujino et al. 2018) and at side boundaries by daily-mean  
131 horizontal velocity, temperature and salinity from the Mercator GLORYS  $1/12^\circ$  ocean reanalysis.  
132 Solar radiation penetrates and warms the water below the surface, and there are no tides. Vertical

133 mixing is represented by the K profile parameterization (KPP) (Large et al. 1994), which was  
134 compared against and tuned to match LES of partially resolved DCT at 0°N,140°W (Large and  
135 Gent 1999). This hindcast is very similar to that of Cherian et al. (2021), where some observational  
136 validations are presented. The main technical difference between the two hindcasts, in addition to  
137 the different and longer simulated time interval, is that the model grid has a slightly coarser vertical  
138 resolution (2.5 m versus 1 m over the top 250 m), because the reduced vertical resolution had a  
139 negligible impact on the solutions in short tests and reduced the computational cost. The analysis  
140 is conducted on the saved daily-mean temperature, salinity, and heat budget diagnostics. See Table  
141 1 for a list of several of the most commonly used metrics to quantify and describe vertical mixing  
142 as well as the sections in which they are defined and discussed.

#### 143 *b. Large eddy simulation hindcasts of turbulence over 34 days*

144 To better understand and validate the subseasonal spatio-temporal variability in turbulent mixing  
145 on and off the equator, we report results from two 34-day LES that are hindcasts of upper-ocean  
146 turbulence in a small 306 m by 306 m by 108 m deep domain during the period from October 2 to  
147 November 5, 1985 at 0° N and 3° N along 140° W in the equatorial Pacific cold tongue. Unlike  
148 the regional ocean hindcast and most other ocean models, the LES explicitly simulates rather than  
149 parameterizes the outer scales  $\mathcal{O}(1)$  m of the turbulence and thus can provide insight into the  
150 physics of ocean mixing and DCT. However, the LES has a computational cost that is many orders  
151 of magnitude greater than the regional ocean model per unit simulated time and volume, hence  
152 the LES must be run for much shorter time intervals and in much smaller domains (Skylingstad  
153 and Denbo 1994; Wang et al. 1996, 1998; Wang and Müller 2002; Pham et al. 2013, 2017; Sarkar  
154 and Pham 2019). A detailed description of the LES model is given in the Appendix. In short,  
155 the LES is forced by variable six-hourly air-sea fluxes (including a diurnal cycle of penetrating

156 shortwave radiation) and larger-scale ( $\gtrsim 15$  km) oceanic tendencies, such as advection and the  
157 pressure gradient force, derived from a regional ocean hindcast simulation of the entire Pacific cold  
158 tongue. The LES forcing is from the parent ocean model ROMS, not MITgcm, because ROMS  
159 solutions (based on earlier work of Holmes and Thomas 2015) were available earlier with all  
160 the necessary outputs. However, the domain, the horizontal resolution  $1/20^\circ$ , the vertical mixing  
161 scheme KPP, the 3-hourly surface forcing (including diurnal cycle of penetrating solar radiation)  
162 from JRA55do are all the same in ROMS and MITgcm, and the mesoscale fields and parameterized  
163 mixing dynamics of interest are qualitatively similar (see the Appendix for details and compare the  
164 results reported in Holmes and Thomas (2015) and Cherian et al. (2021)).

165 The inclusion of larger-scale oceanic tendencies of temperature and momentum from ROMS are  
166 an important novelty in these LES and crucial for sustaining realistic temperature and horizontal  
167 velocity profiles over timescales longer than a few days (Qiao and Weisberg 1997). These tendencies  
168 also provide a source of subseasonal variability on timescales from days to a month (Holmes and  
169 Thomas 2015; Cherian et al. 2021). Hence, an important point of reference is the one previous  
170 LES study of the eastern equatorial Pacific that incorporated large-scale tendencies (Wang et al.  
171 1998). In addition to finer grid resolution, comparisons with an off-equatorial domain, and longer  
172 (34 day vs. 6 day) simulations than in Wang et al. (1998), the ocean tendencies used here also differ  
173 from those in Wang et al. (1998) in that they are derived from a realistic regional ocean model  
174 rather than idealized mathematical formulas. Thus, the large-scale oceanic conditions and related  
175 large-scale tendencies (as well as the air-sea fluxes) evolve on timescales from 6 hours to 1 month  
176 during the simulations, in conjunction with the passage of a tropical instability wave and other  
177 mesoscale ocean variability. In addition, there is approximate dynamical consistency between the  
178 initial conditions, surface fluxes and interior tendencies, as well as between the LES at  $0^\circ$  and  $3^\circ$   
179 N across this range of timescales. Hence, despite some broken feedbacks between the limited LES

180 domain and the larger-scale ocean and atmosphere, the differences between the LES and the ocean  
181 model mean profiles of temperature and zonal momentum are always less than 0.5°C and 0.25 m/s.  
182 That is, the turbulence simulated by LES, the surface fluxes, and the interior tendencies remain  
183 approximately consistent as if the LES was part of a two-way coupled regional system rather than  
184 an isolated domain throughout the 34-day simulations.

185 LES outputs include instantaneous statistics, such as the horizontally-averaged turbulent vertical  
186 fluxes of heat and momentum among others, which are saved irregularly about every 2-5 simulated  
187 minutes and additionally binned into daily-mean statistics for some analyses (to obtain the data and  
188 source code, see Whitt 2022). Note that all times are in UTC, and the local solar time is about  
189 9 hours behind UTC, so solar noon occurs at about 21 UTC. All daily-mean LES statistics, such  
190 as daily mean flux profiles, are calculated from 21 UTC so that the 34 daily means begin and end  
191 at about solar noon, beginning on 21:00 UTC on October 2, 1985 and ending at 21:00 UTC on  
192 November 5, 1985.

### 193 *c. Evaluation of the LES zonal velocity and temperature by comparison with observations*

194 Comparisons with observations suggest that the LES yield plausibly realistic zonal velocity and  
195 temperature simulations with a few exceptions. Mean vertical profiles of temperature and zonal  
196 velocity are generally within observed ranges at 0° N, 140° W where mooring observations from  
197 the Tropical Atmosphere Ocean (TAO) array (McPhaden et al. 2010) are available (Figs. 1- 2).  
198 At 0° N, 140° W, there is a clear depth range between about 10 m and 75 m where the gradient  
199 Richardson number of the horizontally-averaged profile, that is the vertical gradient of buoyancy  
200 over the squared vertical gradient of horizontal velocity

$$Ri_g = \frac{N^2}{S^2} = \frac{\partial b / \partial z}{|\partial \mathbf{u}_h / \partial z|^2} \approx 1/4, \quad (1)$$

201 is in a state of marginal instability as observed by Smyth and Moum (2013) (see Fig. 3). The  
202 LES results are presented at 3° N for comparison in Figs. 1-3, although mooring observations are  
203 not available at 3° N for validation. The observed annual mean climatology of zonal currents and  
204 temperature (Johnson et al. 2002) is plotted for comparison with the LES at 3° N, 140° W, but the  
205 observed annual climatology is insufficient to validate October mean profiles in the LES at 3° N  
206 because there is significant seasonal, interannual, and subseasonal variability. Perhaps the most  
207 notable difference between the two latitudes is that the shear is weaker on average at 3° N than at  
208 0° N, and  $Ri_g > 1/4$  most of the time at 3° N. Hence, marginal instability  $Ri_g \approx 0.25$  is intermittent  
209 (about 25% of the time) from 20 m to 70 m depth at 3° N rather than persistent as at 0° N.

210 The diurnal cycle in temperature and zonal velocity is plausible but on the weaker side of the  
211 observed diurnal cycles at 0° N, 140° W, for example as shown at 25 m in Fig. 2. Consistent with  
212 observations, the modeled diurnal cycle is stronger at shallower depths (e.g., shallower than 15 m),  
213 weak but with a notable peak in the frequency spectra at intermediate depths (e.g., between 15 and  
214 45 m), and difficult to discern from other nearby frequencies in the spectra at deeper depths (not  
215 shown). A detailed investigation of the mechanisms controlling the amplitude of the diurnal cycle  
216 of the horizontally-averaged current and temperature profiles (and all other variables) is left for  
217 future work (for prior studies of the diurnal cycle and DCT at 0° N, 140° W in LES, see e.g. Wang  
218 et al. 1998; Pham et al. 2013, 2017). This study instead focuses on variability in daily-averaged  
219 quantities.

220 The simulated temperature and velocity variance at timescales from days to weeks is generally  
221 realistic at 0° N, 140° W. For example, the power spectra of temperature and zonal velocity at 25  
222 m depth (Fig. 2) show that variance at periods from a few days to a month is reasonably realistic,  
223 but variability at internal wave timescales ranging from a few days to a few hours is consistently  
224 weak in the LES relative to the TAO mooring observations (as shown at 25 m). The weakness of

225 internal wave activity at these frequencies is expected (qualitatively) in the LES since the parent  
226 ROMS model does not have tides or grid resolution at horizontal scales from 5.5 km to 0.3 km  
227 (and only 8 m vertical resolution in the upper ocean) where much internal wave activity occurs  
228 and from which it cascades down to smaller scales (Gregg et al. 2003). That is, the embedded  
229 LES represents only a limited subset of interactions between internal waves, shear instabilities, and  
230 turbulence. First, the LES represents the response of small-scale shear instabilities, internal waves  
231 and turbulence at horizontal wavelengths smaller than 300 m to large-scale internal waves (among  
232 other processes) at horizontal wavelengths  $\gtrsim 15$  km that are resolved by the parent model. Second,  
233 the LES represents some interactions between internal waves, shear instabilities, and turbulence at  
234 scales from about 1 to 300 m that are generated locally in the domain. In particular, the periodic  
235 horizontal boundary conditions allow internal waves to persist in the model domain and propagate  
236 vertically through the stratification. However, going beyond the comparison between the simulated  
237 (black dotted) and observed (light blue) temperature spectra in Fig. 2a to a detailed investigation  
238 of the internal waves and instabilities in the LES and observations (Lien et al. 1996; Smyth et al.  
239 2011; Moum et al. 2011) is left for future work (for some analysis of these topics in other LES, see  
240 Pham et al. 2013, 2017).

241 Finally, the turbulence simulated by the LES is difficult to validate directly since direct obser-  
242 vations of the turbulence are so limited in space and time. That said, the simulated turbulence  
243 is qualitatively and quantitatively similar to the turbulence observed by Lien et al. (1995) from  
244 November 4 - December 12 1991 (as discussed in more detail below). And, previous studies in  
245 simpler model configurations show that the model simulates idealized test cases and turbulent flows  
246 with statistics that are consistent with basic conservation constraints (Watkins and Whitt 2020).

### 3. Spatial patterns, seasonal cycle, and aseasonal variability in the regional hindcast

Our analysis of the regional ocean model begins with the definition of the metrics to be used throughout the results (3.a), then provides a description of the climatological time-mean spatial patterns (3.b), seasonal cycles (3.c), and aseasonal variability (3.d) of ocean mixing in the model as well as comparisons to observations at 0° N, 140° W.

#### a. Metrics of ocean mixing

We quantify and compare the downward heat flux due to ocean mixing  $F_Q(z)$ , which tends to cool the upper ocean on average, with the net downward surface heat flux  $Q_0^{net} = F_Q(z=0) + P_Q(z=0)$  (including turbulent fluxes  $F$  and penetrative fluxes  $P$  due to solar radiation), which tends to warm the upper ocean on average (Fig. 4). With regard to ocean mixing, we focus on the maximum over depth  $z$  of the daily-mean downward turbulent heat flux  $\langle F_Q \rangle^{max} = \max_z \langle F_Q(z) \rangle$  where  $\langle \rangle$  denotes a daily mean (and a horizontal average is implicit, over a single grid cell in the MITgcm and the entire domain in LES). Since the depth  $z_{max}$  at which  $\langle F_Q \rangle^{max}$  occurs varies in time and space, we also quantify  $z_{max}$  and compare it with the mixed layer depth (MLD, defined by the first depth 0.015 kg/m<sup>3</sup> denser than the top 10 m) for reference (Fig 5).

The maximum daily-mean turbulent heat flux  $\langle F_Q \rangle^{max}$ , the daily net surface heat flux  $\langle Q_0^{net} \rangle$ , and their difference  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max}$  provide useful measures of the significance of ocean mixing relative to the net surface heat flux in the upper-ocean heat and SST dynamics throughout the cold tongue. This is a simplified view because other terms also contribute to the heat budget above  $\langle F_Q \rangle^{max}$  in addition to  $\langle F_Q \rangle^{max}$  and  $\langle Q_0^{net} \rangle$ , including penetration of radiative heat fluxes  $\langle P_Q \rangle^{max}$  below the depth  $z_{max}$  and advection (e.g., Moum et al. 2013). In addition, the precise role of ocean mixing in the heat budget depends on the depth to which the budget is integrated. Vertical mixing is generally significant if the heat budget is integrated vertically over a layer that is closely

270 correlated with SST (Ray et al. 2018). Having stated the caveats, there are two main reasons we  
271 focus on  $\langle F_Q \rangle^{max}$ . First, it is intrinsically interesting because it essentially quantifies and bounds  
272 the maximum impact that mixing could have on the upper ocean heat budget. Second, we aim  
273 to use  $\langle F_Q \rangle^{max}$  to model the whole vertical profile  $\langle F_Q \rangle(z)$  in the upper ocean (see section 4.g).  
274 The a priori motivation to focus on  $\langle F_Q \rangle^{max}$  in modelling  $\langle F_Q \rangle(z)$  is based on a hypothesis that  
275  $\langle F_Q \rangle(z)$  can be approximately reconstructed as an interpolation of three points: the surface flux  
276  $\langle F_Q \rangle(z = 0)$ , a positive subsurface  $\langle F_Q \rangle^{max}$  if it exists, and a point of nearly zero flux at some depth  
277 deeper than  $z_{max}$ . In this manuscript, we quantify and parameterize  $\langle F_Q \rangle^{max}$  and then demonstrate  
278 that  $\langle F_Q \rangle^{max}$  can be used to predict  $\langle F_Q \rangle(z)$ , leaving an exposition of the relationships between  
279  $\langle F_Q \rangle^{max}$  and the upper-ocean heat budget formalism to future work (but see Ray et al. 2018).

280 Although DCT is characterized by strong  $\langle F_Q \rangle^{max}$  and may contribute significantly to the clima-  
281 tological  $\langle F_Q \rangle^{max}$ , we choose not to distinguish DCT from other causes of  $\langle F_Q \rangle^{max}$  via a formal  
282 quantitative metric in this manuscript. This is because we want to characterize  $\langle F_Q \rangle^{max}$  across the  
283 cold tongue without assumption about the driving mechanisms, and DCT is not ubiquitous across  
284 the cold tongue (Cherian et al. 2021). In addition, even though DCT tends to be associated with  
285 strong  $\langle F_Q \rangle^{max}$ , it is not known if strong  $\langle F_Q \rangle^{max}$  is always indicative of DCT or why and to what  
286 degree  $\langle F_Q \rangle^{max}$  varies from day to day in DCT or otherwise. However, we refer to the turbulence  
287 driving the mixing descriptively as DCT where and when we feel the subjective criteria (based  
288 on prior studies) are met. In particular, prior studies have identified DCT as strong diurnally-  
289 modulated turbulence in a marginally unstable stratified shear layer ( $Ri_g \approx 1/4$ ) just below the  
290 deepest nighttime MLD (for a recent review, see Cherian et al. (2021)).

291 *b. Time-mean spatial patterns*

292 We begin by characterizing the time-mean  $\langle F_Q \rangle^{max}$ , which contributes to sustaining relatively  
293 cool time-mean SSTs and net ocean heat uptake  $\langle Q_0^{net} \rangle$  in the cold tongue by transporting heat  
294 downwards from the mixed layer to the thermocline (Ray et al. 2018; Holmes et al. 2019a).  
295 Consistent with that interpretation, the comparisons between  $\langle Q_0^{net} \rangle$  and  $\langle F_Q \rangle^{max}$  demonstrate that  
296 the time-mean surface flux and ocean mixing have similar spatial patterns ( $r^2 = 0.7$ ; Figs. 4e-f).  
297 Both  $\langle Q_0^{net} \rangle$  and  $\langle F_Q \rangle^{max}$  are broadly elevated throughout the cold tongue relative to other areas  
298 and take similar area-average values between  $6^\circ$  S and  $6^\circ$  N from  $95^\circ$  to  $170^\circ$ W ( $77$  W/m<sup>2</sup> for  
299  $\langle F_Q \rangle^{max}$  and  $59$  W/m<sup>2</sup> for  $\langle Q_0^{net} \rangle$ ). In addition, both  $\langle Q_0^{net} \rangle$  and  $\langle F_Q \rangle^{max}$  are enhanced by more than  
300 a factor of two near the equator (e.g., between  $\pm 2^\circ$ ) compared to the area means between  $6^\circ$  S and  
301  $6^\circ$  N (Fig. 4e-f; see also Fig. 2 of Cherian et al. (2021) for snapshot plan views).

302 Closer inspection highlights several important differences in the climatological spatial structure  
303 of  $\langle F_Q \rangle^{max}$  and  $\langle Q_0^{net} \rangle$ . First,  $\langle F_Q \rangle^{max}$  is significantly stronger than  $\langle Q_0^{net} \rangle$  on average in an equatorial  
304 mixing band about  $2^\circ$  wide and centered slightly north of the equator that extends zonally through  
305 the entire domain ( $170$  to  $95^\circ$  W; see Fig. 4d). In this equatorial mixing band, the annual mean  
306 surface heat flux  $\langle Q_0^{net} \rangle$  reaches a peak at just over  $120$  W/m<sup>2</sup> at about  $110^\circ$  W and just south of the  
307 equator, whereas the downward heat flux due to ocean mixing  $\langle F_Q \rangle^{max}$  reaches a peak of just over  
308  $240$  W/m<sup>2</sup> at  $130^\circ$  W just north of the equator (cf. Figs. 4e-f). In addition, there is net cooling  
309  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$  over a greater fraction of the year and over more of the zonal distance in the  
310 equatorial mixing band, where  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$  between 50-75% of the time (Fig. 4d). In  
311 the equatorial mixing band, the depth of the peak daily-mean turbulent heat flux  $z_{max}$  ranges from  
312 about  $90$  m at  $170^\circ$  W to  $30$  m at  $95^\circ$  W (Fig. 5f). In addition,  $z_{max}$  is virtually always deeper  
313 than the MLD and ranges from about 20-60 m below the base of the mixed layer in the equatorial

314 mixing band (cf. Figs. 5d-f). The deep  $z_{max}$  in the equatorial mixing band is consistent with prior  
 315 studies showing that mixing is particularly strong and extends to particularly cold isotherms in this  
 316 band (Holmes et al. 2019a; Deppenmeier et al. 2021). These results are all consistent with the  
 317 established results that: 1) ocean mixing is uniquely strong in the cold tongue near the equator and  
 318 plays a leading role in the upper ocean heat budget, 2) the turbulent heat flux peaks in the stratified  
 319 ocean below the mixed layer, and 3) the intensity of ocean mixing is sensitive to the strong mean  
 320 vertical shear in the horizontal velocity (e.g., Figs. 1, 3) that arises from the eastward equatorial  
 321 undercurrent at depth and westward south equatorial current at the surface.

322 At latitudes between  $2^\circ$ - $6^\circ$ , both  $\langle Q_0^{net} \rangle$  and  $\langle F_Q \rangle^{max}$  range from about 80 to 0 W/m<sup>2</sup> (Fig. 4e-f).  
 323 The depth  $z_{max}$  is closer to the base of the MLD than in the equatorial mixing band and just  
 324 10-30 m deeper than the MLD on average (cf. Figs. 5e-f). There is also a notable meridional  
 325 asymmetry in net cooling  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$ ; ocean mixing is stronger relative to the surface  
 326 flux more frequently and over a significantly greater area to the north of the equator (50-70%) than  
 327 to the south (30-40%; see Fig. 4d). This meridional asymmetry arises partly because  $\langle F_Q \rangle^{max}$   
 328 is stronger, by  $O(10)$  W/m<sup>2</sup>, between about  $2$ - $5^\circ$  N than between  $2$ - $5^\circ$  S, but also partly because  
 329  $\langle Q_0^{net} \rangle$  is stronger by  $O(10)$  W/m<sup>2</sup> between  $2$ - $5^\circ$  S than between  $2$ - $5^\circ$  N. The weaker downward  
 330 surface heat fluxes  $\langle Q_0^{net} \rangle$  to the north are consistent with warmer SSTs to the north (through  
 331 their impact on sensible, latent, and longwave surface heat fluxes). In addition, the asymmetry in  
 332 time-mean mixing  $\langle F_Q \rangle^{max}$  is qualitatively consistent with (but does not prove) the hypothesis that  
 333 DCT and stronger ocean mixing events north of the equator arise due to stronger vertical shear in  
 334 intermittent tropical instability waves and vortices that are also more energetic north of the equator  
 335 as proposed by Cherian et al. (2021) (see Fig. 6b). The meridional asymmetry in mixing may  
 336 also be a manifestation of a meridional asymmetry in SST in that warmer SSTs to the north may  
 337 contribute to stronger upper-ocean temperature stratification that facilitates enhanced  $\langle F_Q \rangle^{max}$ .

338 The model results can be validated using multi-year microstructure observations that are available  
339 from chipods on moorings at 0°N, 140°W, from which an average annual cycle of the turbulent  
340 heat flux between 20-60 m has been estimated from deployments between 2008 and 2012 (Moum  
341 et al. 2013) (see also Smyth et al. 2021). Although the observed and modeled time intervals are  
342 not identical, we average the model heat fluxes over the same depth range  $\langle F_Q \rangle^{20-60}$  and compare  
343 them with the observations of Moum et al. (2013) in Fig. 7. We find that the modeled annual  
344 mean  $\langle F_Q \rangle^{20-60}$  is somewhat more than a factor of two larger than observed (150 W/m<sup>2</sup> vs 66  
345 W/m<sup>2</sup>). Restricting the model averaging to the observed years (2008-2012) does not change this  
346 discrepancy. The maximum flux  $\langle F_Q \rangle^{max}$  is another 80 W/m<sup>2</sup> higher than  $\langle F_Q \rangle^{20-60}$ , because  
347  $z_{max} \approx 70$  m is below the 20-60 m averaging range and the modeled fluxes depend strongly on  
348 depth (Fig. 5f). Although it is not fully understood how the time-mean surface heat flux  $\langle Q_0^{net} \rangle$  is  
349 mechanistically coupled to the time-mean subsurface flux  $\langle F_Q \rangle^{max}$ , it is interesting in light of their  
350 high degree of spatial correlation and similar magnitudes that  $\langle Q_0^{net} \rangle$  is substantially stronger in the  
351 model than reported in Moum et al. (2013): Moum et al. (2013) report 55 W/m<sup>2</sup> while the modeled  
352 mean is twice as large at 110 W/m<sup>2</sup>. This may indicate that the modeled heat uptake is biased  
353 high; this would be consistent with too-strong mixing assuming incomplete compensation for the  
354 too-strong mixing by other terms in the heat budget. However, other observational estimates of  
355  $\langle Q_0^{net} \rangle$  are higher than those reported by Moum et al. (2013). For example, Trenberth and Fasullo  
356 (2018) report an estimate of about 90 W/m<sup>2</sup> for the 2000-2016 period, and the model seems to be  
357 within the range of various estimates from 2001-2010 reported by Liang and Yu (2016) (roughly  
358 60-120 W/m<sup>2</sup> at 0°N, 140°W; see their Fig. 2). Hence, we do not conclude that the modeled time-  
359 mean surface heat flux  $\langle Q_0^{net} \rangle$  in MITgcm is biased, although it is on the higher end of available  
360 estimates.

361 *c. Seasonal cycle*

362 The climatological seasonal cycle is another metric by which  $\langle F_Q \rangle^{max}$  and  $\langle Q_0^{net} \rangle$  are similar at first  
363 glance but exhibit notable differences on closer inspection (Figs. 4b-c). Both the seasonal cycles  
364 of  $\langle F_Q \rangle^{max}$  and  $\langle Q_0^{net} \rangle$  exhibit significant diversity. Four different varieties are present between 6°  
365 S and 6° N: one-peak-one-trough, two-peaks-one-trough, two-troughs-one-peak, and two-peaks-  
366 two-troughs, and there are variations in the timing, duration and amplitude of the peaks and troughs  
367 (peaks are red and troughs are blue in Figs. 4b,c). In addition, these spatio-temporal structures  
368 of the seasonal cycles in  $\langle F_Q \rangle^{max}$  and  $\langle Q_0^{net} \rangle$  are uncorrelated (pattern correlation  $r^2 < 0.01$  for  
369 zonal-mean seasonal anomalies, i.e. between the fields in Figs. 4b-c).

370 The phase and amplitude of the seasonal cycle of mixing in the equatorial mixing band is similar  
371 to observations at 0° N, 140° W, even though the modeled time-mean  $\langle F_Q \rangle^{20-60}$  is about a factor  
372 of two higher than observed (see Fig. 7 and Moum et al. (2013)). In this equatorial band (see  
373 Fig. 4c), the seasonal cycle of mixing  $\langle F_Q \rangle^{max}$  is not in phase with and has a larger peak-to-trough  
374 amplitude than the surface fluxes  $\langle Q_0^{net} \rangle$  (Figs. 4a-c). In particular, the peak-to-trough amplitudes  
375 are about 70 W/m<sup>2</sup> and 140 W/m<sup>2</sup> for  $\langle Q_0^{net} \rangle$  and  $\langle F_Q \rangle^{max}$ , respectively. It is notable that the  
376 observations reported by Moum et al. (2013) show a somewhat smaller peak-to-trough seasonal  
377 cycle in  $\langle Q_0^{net} \rangle \approx 50$  W/m<sup>2</sup>, although the phasing is similar to the model. In particular,  $\langle Q_0^{net} \rangle$   
378 is minimum at about yearday 190 and maximum at about yearday 80, whereas mixing reaches a  
379 minimum at about yearday 90 and a maximum at about yearday 215. There is also a secondary  
380 peak in mixing at about the new year. Hence, there is a strong seasonal cycle in  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max}$ ,  
381 which is negative (net cooling) at 0° N along more than 80% of longitudes between 170° and 95°  
382 W in the boreal summer and early autumn (Fig. 4a), when the SST cools in the equatorial mixing  
383 band (Moum et al. 2013). Conversely,  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$  at only about 20% of longitudes in

384 boreal spring (Fig. 4a), when the SST warms (Moum et al. 2013). These results highlight again the  
385 importance of seasonal variations in ocean mixing for the seasonal cycle of cold tongue SST. The  
386 seasonal cycle of the MLD and the depth  $z_{max}$  are highly correlated throughout the cold tongue.  
387 In the equatorial mixing band, minima are achieved at about yearday 90 and local maxima at about  
388 yearday 210 (Figs. 5b-c;  $r^2 = 0.76$ ). But, the amplitude of the seasonal cycles are relatively modest  
389 with peak-to-trough amplitudes of only about 15 m and 25 m for the MLD and  $z_{max}$ , respectively.

390 A qualitatively similar seasonal cycle is found off the equator in  $\langle Q_0^{net} \rangle$  (Fig. 4b), but the off-  
391 equatorial seasonal cycle in  $\langle F_Q \rangle^{max}$  (Fig. 4c) is much weaker and has a different phase relative  
392 to the equator. In addition, the amplitude of the seasonal cycle in  $\langle F_Q \rangle^{max}$  is notably asymmetric  
393 across the equator. There is a much stronger seasonal cycle to the north than the south; for example,  
394 the peak-to-trough seasonal cycle amplitude is about 30 W/m<sup>2</sup> at 4° N but only 10 W/m<sup>2</sup> 4° S  
395 (Fig. 4c). The stronger seasonal cycle in ocean mixing to the north of the equator is qualitatively  
396 consistent with (but does not prove) the hypothesis that the seasonal cycle is due at least partially  
397 to tropical instability waves, which have greatest variance from boreal summer to winter (Cherian  
398 et al. 2021), although precisely quantifying and even determining the sign of the rectified effect of  
399 tropical instability waves on ocean mixing is difficult (Holmes and Thomas 2015).

#### 400 *d. Aseasonal variability*

401 Like the dissipation of turbulent kinetic energy (Crawford 1982; Moum et al. 1989; Smyth et al.  
402 2021), the maximum daily-mean turbulent heat flux  $\langle F_Q \rangle^{max}$  is highly variable and logarithmically  
403 distributed (Fig. 8). Thus, the arithmetic averages of  $\langle F_Q \rangle^{max}$  are significantly influenced by  
404 relatively infrequent strong mixing events (in contrast to  $\langle Q_0^{net} \rangle$ ). It follows that the processes  
405 underpinning the aseasonal variability in general and infrequent strong mixing events in particular  
406 are significant for climatological statistics including the time mean. Hence, we conclude this

407 section on the regional climatological statistics by quantifying the aseasonal variability in  $\langle F_Q \rangle^{max}$   
408 and  $\langle Q_0^{net} \rangle$ , both to provide climatological context for and motivate a more detailed discussion of  
409 subseasonal variability in  $\langle F_Q \rangle^{max}$  simulated in LES (for discussion of the physics of subseasonal  
410 variability in ocean models, see e.g. Holmes and Thomas 2015, 2016; Inoue et al. 2019; Liu et al.  
411 2019b,a, 2020; Cherian et al. 2021). When plotting (in Fig. 6) and reporting the statistics from  
412 the MITgcm results in this section, the aseasonal variability is separated from the full signal (i.e.,  
413 defined) by subtracting a daily climatology, which is first averaged over 18 years and then smoothed  
414 by applying a 15-day moving-average. Hence, aseasonal variability includes both inter-annual and  
415 intra-annual timescales.

416 First, it may be noted that the minimum and maximum monthly means  $\langle F_Q \rangle^{max}$  across the 18  
417 simulated years (thin red lines in Fig. 7) span a factor of 3-8 or roughly 50 to 250 W/m<sup>2</sup>. So,  
418 any given monthly mean is reasonably likely to differ from the corresponding monthly climatology  
419 by a factor of two. In addition, time series of aseasonal  $\langle F_Q \rangle^{max}$  along 140° W in Fig. 6b reveal  
420 variability in  $\langle F_Q \rangle^{max}$  of hundreds of W/m<sup>2</sup> on timescales from days to months in 2012-2013. A  
421 qualitative comparison of the modeled distribution of  $\langle F_Q \rangle^{max}$  at 0° N, 140° W (Fig. 8a) to the  
422 spread of observed daily-mean dissipation from chipods in Fig. B1 of Smyth et al. (2021) suggests  
423 that there are fewer instances of weak mixing and a narrower distribution of mixing values in the  
424 model compared to observations at 0° N, 140° W. But, the different vertical averaging precludes  
425 a quantitative comparison (see Fig. 7). Aseasonal variability in mixing exhibits a spatial pattern  
426 that is similar to the mean (cf. Fig. 6e and Fig. 4f), consistent with a logarithmic distribution.  
427 In particular, the interquartile range (IQR) of aseasonal  $\langle F_Q \rangle^{max}$  variability reaches 150 W/m<sup>2</sup> in  
428 the strong equatorial mixing band but drops from 60 to 20 W/m<sup>2</sup> at latitudes from 2°-6°. There  
429 is also a notable seasonal cycle to aseasonal variability, which is stronger in boreal autumn than  
430 boreal spring (Fig. 6b; cf. Fig. 4c), as well as meridional asymmetry across the equator with

431 larger aseasonal variability to the north than to the south (Fig. 6e). Both the seasonal cycle and  
 432 meridional asymmetry of aseasonal variability are consistent with tropical instability wave activity  
 433 (Halpern et al. 1988; Moum et al. 2009; Cherian et al. 2021). There is also notable aseasonal  
 434 variability in the depth at which maximum ocean mixing occurs  $z_{max}$  (Figs. 8e and 6c,f). The  
 435 aseasonal variability in  $z_{max}$  has a similar spatial pattern as the time-mean  $z_{max}$  (cf. Fig. 5f and  
 436 Fig. 6f). The IQR of aseasonal  $z_{max}$  variability is about 40 m at 170° W and 10 m at 95° W. This  
 437 zonal gradient in the aseasonal IQR of  $z_{max}$  is qualitatively similar at all latitudes from 6° S to 6°  
 438 N, but the IQR is elevated by 10-20 m in the equatorial mixing band relative to other latitudes (Fig.  
 439 6f).

440 Aseasonal variability in  $\langle Q_0^{net} \rangle$  is qualitatively different from aseasonal variability in  $\langle F_Q \rangle^{max}$  (cf.  
 441 Figs. 6a-b and cf. Figs. 8a,c). First,  $\langle Q_0^{net} \rangle$  is more nearly normally distributed (Fig. 8c), and the  
 442 IQR varies relatively little across the cold tongue from about 45-70 W/m<sup>2</sup> (Fig. 6d). In addition,  
 443 the maximum Pearson's  $r^2$  between aseasonal anomalies in  $\langle Q_0^{net} \rangle$  and  $\langle F_Q \rangle^{max}$  is only 0.15 (at  
 444 about 2.5° S, 110° W) and the correlations are mostly much smaller (mean  $r^2 = 0.02$  and median  
 445  $r^2 = 0.01$ ). Hence, the aseasonal net surface heat flux  $\langle Q_0^{net} \rangle$  anomalies do not covary with the  
 446 aseasonal turbulent heat flux  $\langle F_Q \rangle^{max}$  anomalies in the model (see Fig. 3i of Smyth et al. (2021)  
 447 for a qualitatively similar observational result at 0° N, 140° W).

#### 448 **4. Subseasonal variability on and off the equator in the LES**

449 To build further understanding of the subseasonal variability in ocean mixing and DCT, both on  
 450 and off the equator, we turn to the LES (see the Methods and Appendix for details). First, section  
 451 4.a describes how the metrics of ocean mixing (originally defined in section 3.a) are applied to the  
 452 LES and in observational comparisons to the Tropical Instability Wave Experiment (TIWE, Lien  
 453 et al. (1995)). Section 4.b summarizes and contextualizes these LES via comparisons with prior

454 results. Then, sections 4.c-4.g quantify the daily-mean turbulent buoyancy flux  $\langle F_b \rangle$ , including  
 455 the vertical extent of strong mixing (4.c), the energetics of mixing (4.d), and the covariability of  
 456 mixing with non-turbulent variables that may facilitate mixing parameterization (4.e-4.g).

457 *a. Metrics of mixing and observational comparisons*

458 Throughout the analysis of the LES we continue to focus on the maximum of the daily-mean  
 459 flux profile, but we shift our focus from the turbulent heat flux  $\langle F_Q \rangle^{max}$  to the turbulent buoyancy  
 460 flux  $\langle F_b \rangle^{max}$  to leverage links with turbulence energetics, in which  $F_b$  appears but not  $F_Q$  (see the  
 461 Appendix for the relevant equations). However, to facilitate comparisons between the LES and the  
 462 MITgcm simulations and the chipod observations (Fig. 7), we often report

$$\frac{\rho c_p}{g \alpha} F_b \approx F_Q, \quad (2)$$

463 where  $\rho$  is the reference density of seawater,  $c_p$  is the specific heat of seawater,  $g$  is the acceleration  
 464 due to gravity, and  $\alpha$  is the thermal expansion coefficient of seawater. In the LES, the coefficient  
 465 fraction is constant  $1.37 \times 10^9$  [ $\text{Wm}^{-2} \text{s}^3 \text{m}^{-2}$ ] (see the Appendix for details) and we apply the same  
 466 constant scaling to produce  $F_Q$  from the TIWE data in Fig. 7 and 8 . At  $z_{max}$ , the relative error in  
 467 approximating a constant ratio  $F_Q/F_b$  is roughly

$$\frac{N_T^2}{N^2} - 1, \quad (3)$$

468 assuming the turbulent vertical fluxes of temperature and buoyancy can be approximated using local  
 469 flux-gradient relationships (i.e., down-gradient diffusion) and have the same turbulent diffusivity  
 470 such that

$$\frac{F_Q}{F_b} \approx \frac{\rho c_p \partial T / \partial z}{\partial b / \partial z} = \frac{\rho c_p}{g \alpha} \frac{N_T^2}{N^2}, \quad (4)$$

471 where  $N_T^2 = g\alpha\partial T/\partial z$ . The errors from this approximation are small; the 68 days of LES estimates  
472 of  $\langle F_Q \rangle^{max}$  yields an estimate for the mean bias of +6% (-7% and +20% at 0° N and 3° N,  
473 respectively) and a standard deviation of 26% (10% and 30% at 0° N and 3° N, respectively).

474 We explicitly compare the LES results to 38 days of observations of DCT from the TIWE at  
475 0° N, 140° W in November-December 1991 (Lien et al. 1995). The TIWE dataset is a uniquely  
476 good point of comparison in that it includes a similarly long 38 days of hourly-averaged turbulence  
477 profiles based on thousands of microstructure casts (roughly 6-7 per hour) as well as relevant ocean  
478 velocity and density profiles and surface flux information derived from continuous occupation of a  
479 station at 0° N, 140° W by two ships. Although turbulent heat and buoyancy fluxes are not directly  
480 measured, they are inferred to within about a factor of two using the relationship  $F_b = \Gamma\epsilon$  where  $\epsilon$  is  
481 the observed dissipation rate of turbulent kinetic energy and a mixing efficiency factor is assumed  
482 to be a constant  $\Gamma = 0.2$  at depths below 20 m for simplicity (Osborn 1980; Gregg et al. 2018).  
483 The maximum of the daily mean turbulent buoyancy flux  $\langle F_b \rangle^{max}$  is calculated after first binning  
484 hourly mean  $F_b$  profiles into daily means  $\langle F_b \rangle$  at 1 m vertical resolution and then smoothing  $\langle F_b \rangle$   
485 with a 10 m moving average. The resulting 38-day mean  $\langle F_b \rangle^{max}(\rho c_p)/(g\alpha) \approx \langle F_Q \rangle^{max}$  based on  
486 the TIWE data is plotted in Fig. 7 and the distribution of the daily means is shown in Fig. 8 for  
487 context. As in the analysis of the LES, we apply the assumption of constant  $\langle F_Q \rangle^{max}/\langle F_b \rangle^{max}$  to  
488 the TIWE observations (in Fig. 7). We estimate that this assumption yields larger but still modest  
489 high bias in the  $\langle F_Q \rangle^{max}$  of up to about +30%, which is smaller than the factor of 2 observational  
490 uncertainty. Hourly mean velocity and density from the ADCP and CTD respectively are extended  
491 to the surface by replicating the top reliable value before calculating vertical gradients in horizontal  
492 velocity and buoyancy and related derived quantities.

493 *b. Summary and context*

494 We chose to run LES at 0° N and 3° N along 140° W in October 1985, which was characterized  
495 by neutral Oceanic Niño Index, so mixing is expected to be reasonably strong but not maximal  
496 both at and north of the equator (Figs. 4 and 7; see also Warner and Moum (2019); Huguenin et al.  
497 (2020); Deppenmeier et al. (2021)). Tropical instability waves are a dominant cause of subseasonal  
498 variability in currents and density in the LES and are also an important driver of aseasonal variations  
499 in mixing (e.g. Moum et al. 2009; Cherian et al. 2021). The 34 day simulations are just long enough  
500 to span one full tropical instability wave period, but the tropical instability wave spanned by these  
501 LES is not especially strong. The peak-to-trough amplitude of the meridional velocity averaged  
502 from 25 to 75 m is only 45 cm/s at 0°N and 88 cm/s 3°N (Fig. 9). For comparison, the peak-to-  
503 trough amplitude of the meridional velocity variability during the TIWE is about 50 cm/s (plate 3  
504 in Lien et al. 1995) and quite similar to the LES at the same site, even though tropical instability  
505 waves were weak during the TIWE due to the onset of El Niño conditions. In contrast, Moum  
506 et al. (2009) observed strong turbulent mixing in the presence of a strong tropical instability  
507 wave with peak-to-trough meridional velocity amplitude of about 1.5 m/s at 0°N, 140°W during  
508 October-November 2008 in La Niña conditions (see also Inoue et al. 2012, 2019).

509 We find that the mixing in the LES qualitatively reflects the seasonal, interannual and mesoscale  
510 context. The 34-day mean  $\langle F_Q \rangle^{max}$  in the LES at 0° N (about 110 W/m<sup>2</sup>) is just above the  
511 minimum of the eighteen October means simulated from 1999-2016 in the MITgcm. In addition,  
512 the LES parent ROMS simulation with the same KPP mixing scheme as the MITgcm also has  
513 a rather low mean  $\langle F_Q \rangle^{max} \approx 140$  W/m<sup>2</sup> (compared to an October mean of about 275 W/m<sup>2</sup> in  
514 the MITgcm), suggesting that the large-scale conditions (e.g., shear, stratification, and air-sea  
515 fluxes) in the simulated October 1985 are not exceptional but not as conducive to strong mixing

516 as is typically the case from 1999-2016. However, the 34-day mean  $\langle F_Q \rangle^{max}$  is still larger than  
 517 the 38-day mean  $\langle F_Q \rangle^{max}$  from the TIWE observations (77 W/m<sup>2</sup>) and about 50% above the  
 518 climatological  $\langle F_Q \rangle^{20-60}$  (averaged from 20-60 m depth) from chipod observations in October.  
 519 Noting that  $\langle F_Q \rangle^{max} / \langle F_Q \rangle^{20-60} \approx 1.5 - 2$  in the MITgcm, these results suggest that the mixing in  
 520 the LES is fairly typical for October. Consistent with this conclusion, the mixing in our LES is  
 521 also stronger than that simulated in the LES of Sarkar and Pham (2019) (see also Pham et al.  
 522 2017), in which the resolved turbulent heat flux was about 60 W/m<sup>2</sup> and  $\epsilon \approx 10^{-7}$  m<sup>2</sup>/s<sup>3</sup> at the  
 523 maximum MLD over three days in October at 0° N, 140° W (compared to  $\langle F_Q \rangle^{max} \approx 110$  W/m<sup>2</sup>  
 524 and  $\langle \epsilon \rangle^{max} \approx 3 \times 10^{-7}$  m<sup>2</sup>/s<sup>3</sup> here). Conversely, the mixing in our LES is substantially weaker  
 525 than the especially strong mixing (with time-mean  $F_Q \approx 400$  W/m<sup>2</sup> and  $\epsilon \approx 10^{-6}$  m<sup>2</sup>/s<sup>3</sup>) observed  
 526 by Moum et al. (2009) at 0° N, 140° W in the midst of a strong tropical instability wave during  
 527 October-November 2008 in La Niña conditions. Finally, the time-averaged  $\langle F_Q \rangle^{max}$  in the LES at  
 528 3° N, 140° W is about 30 W/m<sup>2</sup>, that is 1/4 to 1/3 of the magnitude in the LES at 0° N, 140° W.  
 529 This ratio of  $\langle F_Q \rangle^{max}$  at 3° N to 0° N is approximately consistent with the climatological ratio from  
 530 1999-2016 found in the MITgcm even though the mixing in the LES is weaker at both latitudes  
 531 (Fig. 4f) .

532 Consistent with earlier studies, we find that the diurnal cycle is the dominant mode of temporal  
 533 variability in the turbulence near the surface, and the simulated diurnal cycles at 0° N, 140° W  
 534 exhibit many of the previously observed and simulated features of DCT at that location (Gregg  
 535 et al. 1985; Moum et al. 1989; Schudlich and Price 1992; Peters et al. 1994; Lien et al. 1995;  
 536 Wang et al. 1998; Large and Gent 1999; Danabasoglu et al. 2006; Smyth et al. 2013; Pham et al.  
 537 2013, 2017; Smyth et al. 2017; Sarkar and Pham 2019; Pei et al. 2020; Cherian et al. 2021).  
 538 For example,  $F_Q$  is shown in Figs. 10a and 11a and can be compared to the time series of  
 539 the dissipation rate of turbulent kinetic energy  $\epsilon$  observed during the TIWE in Plate 7 of Lien

540 et al. (1995) ( $\epsilon \approx 5F_b \approx 4F_Q/10^9 \text{ m}^2/\text{s}^3$  below the MLD; see also Fig. 12). During the daytime,  
541 shortwave radiation stratifies a shallow near-surface layer where wind-driven turbulence is confined  
542 and accelerates a near-surface current with strong vertical shear. During the afternoon and early  
543 evening, the stabilizing net surface buoyancy flux weakens and eventually becomes destabilizing.  
544 The near-surface shear and stratification descend downward toward the highly sheared and stratified  
545 but marginally unstable layer below, where  $Ri_g \approx 1/4$  (Fig. 12). At the same time, strong turbulent  
546 heat and momentum fluxes  $F_Q$  and  $\mathbf{F}_m$  as well as dissipation rates  $\epsilon$  descend downward as well  
547 (Figs. 11-13; see the Appendix for definitions). During nighttime and early morning, turbulence  
548 penetrates deeply below the MLD and into the stratified thermocline (i.e., between about 30 and  
549 90 m depth), where downward turbulent heat fluxes  $F_Q$  reach a subsurface maximum of hundreds  
550 of  $\text{W}/\text{m}^2$ . Strong turbulent momentum fluxes extract kinetic energy from the shear to drive strong  
551 heat fluxes and dissipation rates in the thermocline (Figs. 11-13; the energetics is quantified in  
552 section 4.d). The strong turbulence that is energized locally below the MLD often persists there  
553 for hours while the extent and intensity of the near-surface turbulence decline with increasing  
554 solar radiation in the morning. In addition, on many nights and mornings there are 2-4 bursts of  
555 particularly strong turbulence that cause the heat flux to be elevated by up to hundreds of  $\text{W}/\text{m}^2$  for  
556 hours (Fig. 11a) as observed (Smyth et al. 2017).

557 At first glance, the diurnal cycles of turbulent heat fluxes  $F_Q$  at  $3^\circ \text{ N}$  in Fig. 10b seem to  
558 differ qualitatively from those at  $0^\circ \text{ N}$ , consistent with the hypothesis that equatorial turbulence is  
559 enhanced relative to turbulence at higher latitudes due to DCT associated with the strong mean  
560 shear between the eastward undercurrent and the westward surface south equatorial current (Figs.  
561 1a and 9a). However, DCT and strong heat and momentum fluxes do occur at  $3^\circ \text{ N}$  in conjunction  
562 with strong vertical shear of horizontal velocity (Figs. 9b,d), most prominently on November 3, 4,  
563 and 5 when the subsurface turbulence at  $3^\circ \text{ N}$  exhibits all of the qualitative features described in the

564 previous paragraph in reference to the DCT at  $0^\circ$  N (Figs. 11-13). In addition, some days in early  
565 and mid October exhibit downward turbulent heat fluxes  $F_Q$  below the MLD, although the intensity  
566 of these subsurface heat fluxes is weaker than most days at  $0^\circ$  N and there are no obvious nighttime  
567 turbulent bursts. These results add significant new support to the hypothesis that DCT occurs off  
568 the equator. Off-equatorial DCT has previously been hypothesized based on ocean model results  
569 with fully parameterized DCT (Pei et al. 2020; Cherian et al. 2021) but has not been previously  
570 simulated in LES or observed in microstructure. Although the diurnal cycle of DCT remains a  
571 topic of interest for future analysis of our LES, this topic has received substantial attention in prior  
572 LES studies (Wang et al. 1998; Large and Gent 1999; Pham et al. 2013, 2017; Sarkar and Pham  
573 2019) and we leave further analysis of the diurnal cycle in these LES to future work.

574 The objective of this analysis of the LES is to build understanding of the subseasonal variability  
575 of the daily-mean  $\langle F_Q \rangle$  on timescales from days to weeks, building on our analysis of the regional  
576 MITgcm. The distributions of  $\langle F_Q \rangle^{max}$ ,  $z_{max}$ , and  $\langle Q_0^{net} \rangle$  in Fig. 8 show how this variability  
577 simulated in the LES compares to the variability in the MITgcm and observed in the TIWE data  
578 and generally support the suggestion that the LES are representative of fairly typical conditions  
579 in October. As explored in more detail in the subsequent sections, a motivating hypothesis (e.g.,  
580 Cherian et al. 2021; Smyth et al. 2021) is that the spatio-temporal variability in the vertical shear  
581 in the upper ocean (which is defined more precisely later, but see Figs. 9 and 12c-d) is perhaps the  
582 most important driver of the day-to-day and spatial variability in DCT and  $\langle F_Q \rangle^{max}$  (e.g., in Fig. 8b  
583 and Fig. 10). This vertical shear is strong on average above the equatorial undercurrent along the  
584 equator, but the shear is also highly variable and intermittently strong throughout the cold tongue  
585 (e.g., as shown in Fig. 9) due to a variety of interacting equatorial waves and instabilities (Moum  
586 et al. 2009; Inoue et al. 2012; Jing et al. 2014; Tanaka et al. 2015; Holmes and Thomas 2015,  
587 2016; Inoue et al. 2019; Liu et al. 2019b,a; Pei et al. 2020; Liu et al. 2020; Cherian et al. 2021).

588 Hence, strong DCT and  $\langle F_Q \rangle^{max}$  vary in time and space and occur intermittently throughout the  
589 cold tongue (and at  $3^\circ$  N specifically) when the shear is strong. Over the next few sections, we  
590 explore the hypothesis that shear covaries with  $\langle F_Q \rangle^{max}$  on and off the equator and more generally  
591 seek to identify covariates that provide information about  $\langle F_Q \rangle^{max}$  without direct simulations or  
592 observations of turbulence.

593 *c. Shear, stratification, Richardson numbers, and the vertical extent of strong turbulence*

594 Previous studies have identified the gradient Richardson number of the horizontally-averaged  
595 profile  $Ri_g$  (defined in (1)) as an important indicator of the occurrence of DCT and strong ocean  
596 mixing in the equatorial Pacific (Pacanowski and Philander 1981; Peters et al. 1988; Large et al.  
597 1994; Smyth and Moum 2013). Consistent with these previous studies, we find that Richardson  
598 numbers provide some useful information about the spatio-temporal structure and in particular the  
599 vertical extent of strong mixing in the LES and the TIWE observations. Below, we show that two  
600 Richardson numbers, both of which are based on the horizontally-averaged velocity and density  
601 profiles, can be used to model the depth  $z_{max}$  where daily mean turbulent vertical heat fluxes  $\langle F_Q \rangle$   
602 are maximum as well as the daily maximum depth  $z_{pen}$  to which strong turbulence penetrates. We  
603 define  $z_{pen}$  based on a constant threshold in the dissipation rate of turbulent kinetic energy  $\epsilon$ . It  
604 is reasonably straightforward to identify a depth  $z_{pen}$  from inspection of time-depth series of  $\epsilon$  or  
605  $F_Q$  profiles (as in the mid-latitudes, see Brainerd and Gregg 1995). After brief trial and error, we  
606 identify the shallowest depth where  $\epsilon < 2 \times 10^{-8} \text{ m}^2/\text{s}^3$  to be a useful threshold applicable to both  
607 of the LES (Figs. 11-13) and the TIWE observations. For reference, this  $\epsilon$  threshold corresponds  
608 to a turbulent heat flux of roughly  $7 \text{ W/m}^2$ , which is an order of magnitude smaller than typical  
609  $\langle F_Q \rangle^{max}$  and about two orders of magnitude smaller than peak nighttime heat fluxes  $F_Q$  during  
610 turbulent bursts.

611 The depth  $z_{max}$  varies from about 10 to 70 m at  $0^\circ$  N and from 20 to 60 m at  $3^\circ$  N over timescales  
 612 ranging from days to weeks (black plus symbols in Fig. 10; see also Fig. 8f). The occurrence of  
 613  $z_{max}$  deeper than the nighttime MLD is hypothesized to be an indicator of DCT and strong heat  
 614 fluxes. Consistent with this suggestion, the nighttime maximum MLD is shallower than  $z_{max}$  at  $0^\circ$   
 615 N on 29 of 34 days and 9 m shallower on average, but the nighttime MLD is deeper than  $z_{max}$  at  
 616  $3^\circ$  N on 32 of 34 days and 9 m deeper on average. Qualitatively, we interpret these results as an  
 617 indication that DCT occurs about 85% of the time at  $0^\circ$  N and about 5% of the time at  $3^\circ$  N, but  
 618 there is not a one-to-one correspondence between DCT and  $z_{max}$  deeper than MLD as demonstrated  
 619 on 11/03-11/04 at  $0^\circ$  N,  $140^\circ$  W in Fig. 11. Although the nighttime maximum MLD is somewhat  
 620 correlated with the depth  $z_{max}$ , the relationship is in fact fairly scattered and the nighttime MLD  
 621 can only explain about 30% of the variance in  $z_{max}$  across both LES. On the other hand, about half  
 622 of the simulated variance in the depth  $z_{max}$  can be explained by  $H_{Rib}$  ( $r^2 = 0.5$ ), the depth at which  
 623 the mean-profile bulk Richardson number  $Ri_b = 0.2$ . Here,

$$Ri_b = \frac{\Delta b H_{Rib}}{\Delta u^2 + v_t^2}, \quad (5)$$

624 where  $\Delta b$  and  $\Delta u$  are the bulk buoyancy and velocity differences between the depth  $H_{Rib}$  and the  
 625 top  $0.1H_{Rib}$ ,  $v_t$  is a turbulent velocity scale that depends on the surface forcing as in Large et al.  
 626 (1994), and the depth  $H_{Rib}$  is identified iteratively using the default parameters of Large et al.  
 627 (1994) in an implementation of KPP by Smyth et al. (2002) (Fig. 14a). The inclusion of  $v_t$  in  $Ri_b$   
 628 systematically deepens  $H_{Rib}$  by 6 m on average, but has marginal and probably insignificant benefit  
 629 on the best linear model or correlation with  $z_{max}$  (increasing  $r^2$  by 15%). The specific threshold  
 630  $Ri_b = 0.2$  was chosen via trial and error. Larger and smaller thresholds for  $Ri_b$  were not as useful  
 631 for identifying  $z_{max}$ , but there may be room for future refinement of the model for  $z_{max}$ , because  
 632 half of the variance in  $z_{max}$  is not explained by  $H_{Rib}$ .

633 The deepest depth to which DCT penetrates each day  $z_{pen}$  also varies significantly from about 40  
 634 to 90 m at  $0^\circ$  N and from about 35 to 85 m at  $3^\circ$  N (Fig. 10). And again, the Richardson number—in  
 635 this case the local gradient Richardson number  $Ri_g$  of the horizontally-averaged profiles— provides  
 636 useful information about  $z_{pen}$  each day. In particular, we define  $H_{Rig}$  as the base of the deep-cycle  
 637 layer, which is defined by a low gradient Richardson number  $Ri_g < 0.35$ . In practical applications  
 638 (e.g., to the TIWE data),  $Ri_g$  is noisy and the definition of  $H_{Rig}$  requires some additional logic  
 639 and filtering. In particular, the deep-cycle layer is defined by applying a rectangular filter of about  
 640 35 hours and 35 meters depth to a logical field that equals one where  $Ri_g < 0.35$  and the depth  
 641 is below the daily maximum  $H_{Rib}$ . The second threshold based on  $H_{Rib}$  is necessary because  $Ri_g$   
 642 sometimes rises to high values within the weakly stratified turbulent boundary layer above  $H_{Rib}$ ,  
 643 particularly at  $3^\circ$  N and even fairly deep within  $H_{Rib}$  during nighttime (Fig. 12e-f). With regard  
 644 to  $Ri_g$ , a threshold  $Ri_g = 0.25$  has a theoretical basis that makes it appealing (Miles 1961; Howard  
 645 1961; Holt et al. 1992; Rohr et al. 1988), and  $Ri_g = 0.25$  has been used previously for identifying  
 646 the base of the deep-cycle layer in observations at  $0^\circ$  N,  $140^\circ$  W (Lien et al. 1995; Smyth et al.  
 647 2021). However, we found via trial and error that a somewhat larger threshold  $Ri_g = 0.35$  is more  
 648 useful across the LES at  $0^\circ$  N and  $3^\circ$  N as well as the TIWE observations. Our approach is also  
 649 supported by the LES of Pham et al. (2017), in which simulated turbulent bursts penetrate below  
 650 the layer defined by a threshold  $Ri_g = 0.25$  in DCT as in our LES. A linear regression on  $H_{Rig}$ ,  
 651  $-6 + 1.1H_{Rig}$  has slope near one, intercept near zero, and explains 80% of the variance in the  
 652 daily-maximum  $z_{pen}$  (see Fig. 14b).

653 Finally, it may be noted that these relationships between  $z_{max}$ ,  $H_{Rib}$ ,  $z_{pen}$  and  $H_{Rig}$  are useful  
 654 beyond the LES. For example, the TIWE observations reveal similar variability and relationships  
 655 between  $z_{max}$ ,  $H_{Rib}$ ,  $z_{pen}$ , and  $H_{Rig}$  as the LES at  $0^\circ$  N (cf. blue stars and black + symbols in  
 656 Figs. 14a-b). And,  $H_{Rig}$  is also a useful lower boundary for the deep-cycle layer in the MITgcm

657 regional model with DCT parameterized by KPP (section 3), but the threshold has to be increased  
 658 to  $Ri_g = 0.5$  (Cherian et al. 2021).

659 *d. (Non)local energetics of  $\langle F_b \rangle^{max}$*

660 To begin to understand why the intensity of  $\langle F_b \rangle^{max}$  varies in time and space, it is useful to  
 661 consider these variations in the context of the daily mean turbulent kinetic energy budget under  
 662 the premise that some of the variability in  $\langle F_b \rangle^{max}$  is related to variations in the kinetic energy  
 663 available to drive turbulent mixing (see the Appendix for details). In this kinetic energy budget,  
 664 the tendency or rate of change of turbulent kinetic energy is driven by vertical transport  $\langle T \rangle^{max}$ ,  
 665 shear production  $\langle SP \rangle^{max} = \langle \mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z \rangle^{max}$ , dissipation  $\langle \epsilon \rangle^{max}$ , and buoyancy flux  $\langle F_b \rangle^{max}$  (Fig.  
 666 15a-b). Integrated over a full day, the budget is dominated by a net source due to shear production  
 667 and net sinks due to buoyancy flux and dissipation at  $z_{max}$ . That is, all other terms (tendency and  
 668 vertical transport) are sub-dominant in all but one day and contribute less than 20% of the energy  
 669 for dissipation and buoyancy flux  $\langle \epsilon \rangle^{max} + \langle F_b \rangle^{max}$  when mixing is strong (roughly  $\langle F_b \rangle^{max} > 10^{-7.5}$   
 670  $\text{m}^2/\text{s}^3$ ; see Fig. 15a). Hence, the shear production of turbulent kinetic energy at  $z_{max}$   $\langle SP \rangle^{max}$   
 671 is highly correlated with  $\langle \epsilon \rangle^{max} + \langle F_b \rangle^{max}$  ( $r^2 = 0.98$ ; Fig. 15a). In addition, when mixing is  
 672 strong,  $\langle F_b \rangle^{max}$  is in approximately constant proportion to  $\langle SP \rangle^{max}$  (about 0.2) and to  $\langle \epsilon \rangle^{max}$   
 673 (about 0.25) (Figs. 15a-b). When the buoyancy flux is weaker  $\langle F_b \rangle^{max} < 10^{-7.5} \text{m}^2/\text{s}^3$ , the ratio  
 674  $Ri_f^{-1} = \langle SP \rangle^{max} / \langle F_b \rangle^{max}$  declines from 5 to  $\sim 2$  as  $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$  decreases from 5 to  
 675 0.5 and  $\langle F_b \rangle^{max}$  weakens to  $10^{-8.5} \text{m}^2/\text{s}^3$  (Figs. 15b-c). Here,  $Ri_f$  is the flux Richardson number  
 676 (e.g., Osborn 1980; Venayagamoorthy and Koseff 2016). In addition, the relationship between  
 677  $Ri_f^{-1}$  and  $Ri_g^{-1}$  is associated with a relationship between  $Ri_g^{-1}$  and the turbulent Prandtl number  
 678  $Pr_t^{-1} = Ri_f / Ri_g$ , which quantifies how the turbulent diffusivity of buoyancy declines relative to  
 679 the turbulent viscosity as  $Ri_g^{-1}$  decreases (Fig. 15d). Finally, it is notable that the turbulent kinetic

energy budget contains significant non-local (transport) contributions at low  $\langle F_b \rangle^{max} < 10^{-7.5}$   $m^2/s^3$ . In particular, transport  $\langle T \rangle^{max} \approx \langle F_b \rangle^{max} + \langle \epsilon \rangle^{max} - \langle SP \rangle^{max}$  becomes a more significant and scattered contributor to the dissipation and buoyancy flux, as  $\langle T \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$  reaches values of 40% and takes both signs (Fig. 15a).

In summary, when mixing is strong ( $\langle F_b \rangle^{max} > 10^{-7.5} m^2/s^3$ ), the energetics are dominantly local to the depth  $z_{max}$  with shear production balanced by dissipation plus buoyancy flux and nearly constant  $Ri_f \approx 0.2$  and  $Ri_g \approx 0.25$  both on and off the equator. However, the energetics of  $\langle F_b \rangle^{max}$  in general (including weaker values) are more complex: the energetics are approximately local on average, but non-local (transport) contributes 10-40% to the energetics on many days and takes both signs. In addition,  $Ri_f$  systematically varies with  $Ri_g$ , both of which take values substantially higher than the canonical values ( $Ri_f \approx 0.2$  and  $Ri_g \approx 0.25$ ) on most days at  $3^\circ N$ . At  $0^\circ N$ , the canonical DCT and local dynamics are the norm, but at  $3^\circ N$  the canonical DCT and local dynamics are the exception rather than the norm. The simulated energetic relationships encapsulated in relationships between  $Ri_f$ ,  $Pr_t$  and  $Ri_g$  (Fig. 15c-d) are qualitatively consistent with observations in the atmospheric boundary layer (Anderson 2009), a previous LES of ocean turbulence under a hurricane in the coastal mid-latitudes reported by Watkins and Whitt (2020), and direct numerical simulations (Venayagamoorthy and Koseff 2016). However, it still remains somewhat uncertain whether the relationships modeled here in the LES are in any sense universal, especially given the significance of non-local (transport) dynamics at weak  $\langle F_b \rangle^{max}$ .

*e. Scaling  $\langle F_b \rangle^{max}$  based on the horizontally-averaged velocity and buoyancy profiles*

Building on the result that  $\langle F_b \rangle^{max}$  varies in concert with other metrics of the turbulence energetics such as the shear production and dissipation rate, this section demonstrates how the intensity of  $\langle F_b \rangle^{max}$  covaries with readily measured or simulated non-turbulent variables such as horizontally-

703 averaged velocity and buoyancy profiles as well as the surface momentum and buoyancy fluxes. In  
 704 a second step, we evaluate scaled predictions of  $\langle F_b \rangle^{max}$  derived from the LES results by applying  
 705 the scaling to the independent TIWE observations.

706 We begin by quantifying the relationship between the mean profile  $Ri_g$  and the intensity of  
 707 mixing at  $z_{max}$  motivated by popular existing parameterizations of the local intensity of turbulent  
 708 diffusion as a function of  $Ri_g$  (Pacanowski and Philander 1981; Peters et al. 1988; Large et al.  
 709 1994). We find that the simulated inverse Richardson number  $Ri_g^{-1}$  at  $z_{max}$  can explain most  
 710 of the simulated variability in  $\langle F_b \rangle^{max}$  across the LES at both  $0^\circ$  and  $3^\circ$  N (Fig. 16a;  $r^2 = 0.6$   
 711 for the regression  $\log_{10}(\langle F_b \rangle^{max}) \sim \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ ). On the other hand,  $Ri_g^{-1}$  on its own does  
 712 not explain the temporal variability in  $\langle F_b \rangle^{max}$  very well at  $0^\circ$  N in either the LES ( $r^2 = 0.2$ ) or  
 713 the TIWE observations ( $r^2 = 0.0$ ). These results are consistent with the hypothesis that  $Ri_g$  is a  
 714 useful predictor of the intensity of mixing across a range of  $Ri_g$  that includes marginal instability  
 715 ( $1 \gtrsim Ri_g \gtrsim 0.25$ , as at  $3^\circ$  N) but a poor predictor of the intensity of mixing when marginal instability  
 716 is either persistent ( $Ri_g \approx 0.25$ , as at  $0^\circ$  N) or marginal instability never occurs and  $Ri_g \gg 0.25$   
 717 is always very large (for background on marginal instability, see Thorpe and Liu 2009; Smyth  
 718 and Moum 2013; Smyth 2020). For better comparison with previous studies, we also show that  
 719 variations in the effective turbulent diffusivity of buoyancy at  $z_{max}$  ( $K_b = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$ ) are  
 720 more weakly correlated with  $Ri_g^{-1}$  ( $r^2 = 0.2$  for  $\log_{10}(K_b) \sim \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$  in LES;  $r^2 = 0.0$  in  
 721 TIWE) and thus not well-explained by  $Ri_g^{-1}$  (Fig. 16b) or  $Ri_g$ -based parameterizations (Pacanowski  
 722 and Philander 1981; Peters et al. 1988; Large et al. 1994). However, it may be noted that the  
 723 underlying variables in the regressions for  $K_b$  and  $\langle F_b \rangle^{max}$  are actually the same,  $\langle S^2 \rangle^{max}$ ,  $\langle N^2 \rangle^{max}$   
 724 and  $\langle F_b \rangle^{max}$ , which suggests that the relatively poor correlation between  $\log_{10}(K_b)$  and  $Ri_g^{-1}$  may  
 725 be improved by simply reformulating the predictor function of  $\langle S^2 \rangle^{max}$  and  $\langle N^2 \rangle^{max}$ . Indeed, a  
 726 general two-variable linear regression of  $\log_{10} K_b$  on  $\log_{10} \langle S^2 \rangle^{max}$  and  $\log_{10} \langle N^2 \rangle^{max}$  yields an

727  $r^2 = 0.6$  for  $\log_{10}(K_b) \sim \log_{10}(\langle S^2 \rangle^{max} (\langle N^2 \rangle^{max})^{-3/2})$ . In summary, although the LES yield results  
728 that are loosely consistent with previous studies (e.g., Fig. 16b), there is significant room to  
729 improve parameterizations of ocean mixing in the cold tongue. That is,  $Ri_g$  is useful but certainly  
730 not sufficient to explain all of the spatio-temporal variability in  $\langle \epsilon \rangle^{max}$  or  $\langle F_b \rangle^{max}$  in the eastern  
731 equatorial Pacific (Moum et al. 1989; Zaron and Moum 2009). Other variables and combinations  
732 of variables likely contain valuable information about  $\langle F_b \rangle^{max}$  in DCT and in general across the  
733 cold tongue.

734 In an attempt to refine our understanding of the mean-profile properties that drive temporal vari-  
735 ations in  $\langle F_Q \rangle^{max} \sim \langle F_b \rangle^{max}$ , we conduct a more general multi-variable linear regression analysis  
736 with the aim of identifying an optimal power law product (e.g., a product of the generic form  
737  $c x^a y^b z^d \dots$ , with variables  $x, y, z, \dots$  and constants  $a, b, c, d, \dots$  to be determined) to model the max-  
738 imum buoyancy flux  $\langle F_b \rangle^{max}$  as a function of horizontally-averaged and readily-measured (and  
739 modeled) properties, including surface fluxes and the horizontally-averaged profiles of velocity  
740 and density but without a priori knowledge of the depth  $z_{max}$  at which  $\langle F_b \rangle^{max}$  occurs. Although a  
741 formulation as a power law may seem arbitrary, this choice is motivated by two factors. First, many  
742 familiar mixing models are expressed as a product of terms (e.g., a diffusivity times a gradient, or  
743 a mixing efficiency times a momentum flux times a shear; e.g., Fig. 16b) and are therefore power  
744 laws. In addition,  $\langle F_b \rangle^{max}$  is thought to be logarithmically distributed (see section 3.d and Fig. 8),  
745 and power laws are readily amenable to linear regression after applying a log-transform.

746 Numerous variables were considered in the regressions, but we only highlight two low-complexity  
747 models that we identified. First, the most useful variable that we identified for modelling the  
748 combined LES output from  $0^\circ$  N and  $3^\circ$  N is the vertical shear  $S$ . In particular, if  $S_b$  is a bulk shear  
749 defined by a least-squares linear fit to the daily-mean and horizontally-averaged velocity profile  
750 from  $H_{Rig}$  to 5 m depth, then we find that  $S_b$  alone can explain about 70% of the daily variance

751 in  $\langle F_b \rangle^{max}$  from both the LES at  $0^\circ$  N and  $3^\circ$  N (Fig. 17a;  $\langle F_b \rangle^{max} \approx 3 \times 10^{-6} |S_b|^{0.9}$ ;  $r^2 = 0.7$  in  
 752  $\log_{10}$  space ignoring the TIWE data). In an encouraging result, independent validation of the  $S_b$   
 753 scaling of  $\langle F_b \rangle^{max}$  on the TIWE data is quite good ( $r^2 = 0.5$  with little mean bias) and even better  
 754 than the LES at  $0^\circ$  N alone ( $r^2 = 0.2$ ). In addition, including the TIWE data in the regression in  
 755 Fig. 17c has little impact on the optimal linear model, which seems fairly robust with relatively  
 756 narrow confidence intervals on the parameters (cf. Figs. 17a,c). However, the model fit to the LES  
 757  $\langle F_b \rangle^{max}$  can be improved substantially by adding the surface friction velocity due to the wind stress  
 758  $u_* = \sqrt{|\tau|/\rho}$  as a variable ( $\tau$  is the wind stress vector). The optimal linear model based on these  
 759 two variables  $\langle F_b \rangle^{max} \approx 0.16 |S_b|^{0.98} u_*^{2.1}$  explains about 90% of the LES variance and 70% at  $0^\circ$   
 760 N or  $3^\circ$  N alone (Fig. 17b). In independent validation on the TIWE data, the two-variable model  
 761 explains only 40% of the TIWE variance and also has a slight mean bias (Fig. 17b). Including the  
 762 TIWE observations in the two-variable regression in Fig. 17d leads to a fairly substantial change  
 763 in the optimal two-variable model  $0.0017 |S_b|^{0.92} u_*^{1.2}$  and somewhat reduces the correlation at  $3^\circ$   
 764 N in the LES but reduces the mean bias in the TIWE data and slightly improves the corresponding  
 765 correlation (cf. Figs. 17b,d). These results suggest that although wind stress certainly provides  
 766 useful information about  $\langle F_b \rangle^{max}$ , the available data (including 108 days spanned by the LES and  
 767 TIWE) is only marginally sufficient to provide a robust linear model based on both  $S_b$  and  $u_*$ .

768 For reference, the 95% confidence intervals for various optimal power laws stated in the previous  
 769 paragraph and obtained via regression are given in the caption of Fig. 17. Consistent with the  
 770 above discussion, only the power on  $S_b$  is tightly constrained to be within 0.7 and 1.1. There is  
 771 substantial joint uncertainty in the power on  $u_*$  (which may range from 0.9-2.5) and the magnitude  
 772 of the constant coefficient (which may range from  $3 \times 10^{-4}$  to 2). The coefficient, which in general  
 773 has units, is smaller if the power on  $u_*$  is lower, and conversely the coefficient is larger if the power  
 774 on  $u_*$  is larger. Assuming a fixed relationship  $\langle F_b \rangle^{max} \sim u_*^2 S_b$  and regressing  $\langle F_b \rangle^{max}$  on  $u_*^2 S_b$  (the

775 exponents of which yield an appealingly unitless coefficient) yields 95% confidence intervals on  
776 the slope of 0.15 to 0.19, an intercept indistinguishable from zero, and  $r^2 = 0.82$ . Applying  $\log_{10}$  to  
777 both sides before regressing puts more weight on accuracy at weaker  $\langle F_b \rangle^{max}$  and yields confidence  
778 intervals on the intercept of [-1.63,-0.33], which corresponds to a coefficient ranging from 0.02  
779 to 0.47 in the power law. Either way, it seems that  $\langle F_b \rangle^{max} \approx 0.2u_*^2 S_b$  is a plausible model with  
780 roughly a factor of 3 uncertainty. Although many other variables were considered, we found at best  
781 marginal improvements in the correlations (e.g., when adding a measure of stratification and/or the  
782 net surface buoyancy flux to create multi-variate linear regressions) and many lower correlations  
783 if shear and/or wind stress is omitted or the definition of the shear is changed. Hence, we do not  
784 report any further results of our statistical modelling.

785 *f. Discussion of the empirical power law scaling of  $\langle F_b \rangle^{max}$  in light of prior results*

786 In the context of DCT on the equator, it is neither surprising nor novel that shear and wind  
787 stress are correlated with the intensity of mixing. Several previous studies have identified such  
788 relationships using observations and theory (Moum and Caldwell 1985; Pham et al. 2017; Smyth  
789 et al. 2017, 2021). In addition, we reanalyzed the results from the LES of Wang et al. (1998) (see  
790 also Large and Gent 1999), nominally at  $0^\circ$  N,  $140^\circ$  W, and found that those results are consistent  
791 with the  $\langle F_b \rangle^{max} \sim 0.2u_*^2 S_b$  scaling identified empirically here to the degree that it is reasonable to  
792 make claims of consistency, which is only within a factor of 3. However, the application of such  
793 a relation beyond  $0^\circ$  N,  $140^\circ$  W and in situations without DCT as well as the precise formulation  
794 of the statistical models proposed in section 4.e and Fig. 17 are new and somewhat unintuitive  
795 in light of the energetics of  $\langle F_b \rangle^{max}$ , which indicate dominantly local dynamics remote from the  
796 surface forcing. Hence, we find it useful to see how the empirical scalings in section 4.e relate to  
797 the turbulent energetics discussed in section 4.d. In addition, we briefly discuss how the scalings

798 relate to a theory previously developed by Smyth et al. (2017) to model DCT at 0° N, 140° W and  
 799 compare the results from LES with analogous results derived from the KPP scheme (Large et al.  
 800 1994) in the parent ocean model ROMS.

801 To reveal how the energetics at  $z_{max}$  (e.g., Fig. 15) relates to the scaling derived via linear  
 802 regression and shown in Fig. 17b, we write:

$$\frac{\langle F_b \rangle^{max}}{0.2u_*^2|S_b|} = \frac{Ri_f \Theta}{0.2} \frac{|\langle \mathbf{F}_m \rangle^{max}|}{u_*^2} \frac{|\langle S \rangle^{max}|}{|S_b|} \quad (6)$$

803 and quantify how the local turbulent momentum flux  $\langle \mathbf{F}_m \rangle^{max}$  and vertical shear  $\langle S \rangle^{max}$  at  $z_{max}$  relate  
 804 to the bulk shear  $S_b$  and friction velocity squared  $u_*^2$  in the scaling. Here,  $Ri_f = \langle F_b \rangle^{max} / \langle SP \rangle^{max}$   
 805 is the flux Richardson number at  $z_{max}$ ,  $\langle SP \rangle = \langle \mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z \rangle$  is the daily-mean shear production,  
 806 and  $\Theta = \langle \mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z \rangle^{max} / (|\langle \mathbf{F}_m \rangle^{max}| |\langle \partial \mathbf{u}_h / \partial z \rangle^{max}|)$  is a dimensionless measure of the combined  
 807 effects of misalignment and sub-daily correlations between shear and momentum flux on shear  
 808 production at  $z_{max}$ . Various ratios of terms in this expression are plotted in Fig. 18. We interpret  
 809 these results in two parts, focusing first on instances of strong mixing and DCT and then on  
 810 instances of weaker mixing.

811 First, we recall that strong mixing (roughly  $\langle F_b \rangle^{max} > 10^{-7.5} \text{ m}^2/\text{s}^3$ ) tends to be in a state of  
 812 marginal instability with fairly uniform  $Ri_g^{-1} \approx 4$  (Fig. 15), i.e. only the yellow, orange and red  
 813 colored points are associated with strong mixing in Fig. 18. For these points, the ratios on the  
 814 right side of (6) are fairly simple:  $\Theta \approx 1$  (Fig. 18a),  $Ri_f \approx 0.2$  (Fig. 18b,d),  $|\langle S \rangle^{max}| / |S_b| \approx 1$   
 815 (Fig. 18b-c), and  $|\langle \mathbf{F}_m \rangle^{max}| / u_*^2$  ranges from about 0.3 to 1.1 (Fig. 18c-d). That is, our empirical  
 816  $0.2u_*^2|S_b|$  scaling derived via regression can reasonably be interpreted as a proxy for local dynamics  
 817 at  $z_{max}$  with 0.2 a proxy for  $Ri_f$  at  $z_{max}$ ,  $u_*^2$  a proxy for the momentum flux at  $z_{max}$ , and  $|S_b|$  a  
 818 proxy for the shear at  $z_{max}$ .

819 In the presence of strong mixing and DCT at 0° N, 140° W ( $\langle F_b \rangle^{max} > 10^{-7.5} \text{ m}^2/\text{s}^3$ ), the empirical  
 820  $0.2u_*^2|S_b|$  scaling from the LES is also consistent with the theory of Smyth et al. (2017), which  
 821 yields  $F_b \approx 0.2\epsilon$  where  $\epsilon \approx u_*^2|S_b|$  in steady state. To briefly summarize Smyth et al. (2017), the  
 822 theory explicitly models the shear and turbulent kinetic energy in the deep-cycle layer, which is  
 823 defined to be a layer of thickness  $H$  with homogeneous shear  $S_b$  and turbulent kinetic energy  $k$  from  
 824 the base of the mixed layer to the top of the undercurrent core. The shear  $S_b$  evolves due to changes  
 825 in the surface mixed layer velocity, which in turn evolves due to any convergence between the  
 826 downward momentum flux at the surface ( $u_*^2 = F_m(0)$ ) and the MLD ( $F_m(h)$  where  $h$  is the MLD).  
 827 The momentum flux is assumed to be dominated by the zonal component, which is about 3 times  
 828 stronger than the meridional component at the surface in our LES at 0° N, 140° W. The turbulent  
 829 kinetic energy  $k$  evolves in the theory due to shear production and dissipation plus buoyancy flux  
 830 in the shear layer. That is,

$$\frac{\partial S_b}{\partial t} = \frac{1}{Hh} (u_*^2 - F_m(h)), \quad (7)$$

$$\frac{\partial k}{\partial t} = F_m S_b - \epsilon - F_b, \quad (8)$$

831 following their Eqns. 3.2-3.3. Closure of turbulent fluxes in terms of turbulent kinetic energy  
 832 is discussed in Smyth et al. (2017). But, the expressions (7)-(8) suggest that if the shear  $S_b$  and  
 833 turbulent kinetic energy  $k$  are in a steady state then  $F_m(h)/u_*^2 \approx 1$ , as in the strong DCT simulated  
 834 by LES (Fig. 18). In addition,  $\epsilon + F_b \approx u_*^2 S_b$ . With the additional assumption that  $F_b/SP \approx 0.2$ ,  
 835 then  $F_b \approx 0.2u_*^2 S_b$ .

836 That is, the theory of Smyth et al. (2017) suggests essentially the same mathematical form as the  
 837 empirical linear model derived from the LES, although the definition of  $S_b$  differs. In their theory,  
 838  $S_b$  is interpreted as an average over the deep cycle layer, from  $H_{Rig}$  to the daily maximum MLD,  
 839 whereas in our empirical model  $S_b$  is fit to the velocity profile from  $H_{Rig}$  to 5 m depth. However,

840 the different definitions of  $S_b$  turn out to have only a small impact on the prediction of  $\langle F_b \rangle^{max}$  at  
 841  $0^\circ$  N because the two definitions of  $S_b$  turn out to be highly correlated and similar in magnitude;  
 842 both are also good proxies for the shear at  $z_{max}$ . Hence,  $r^2$  is only reduced from 0.8 to 0.7 if  $S_b$  is  
 843 calculated only in the deep cycle layer, i.e. from  $H_{Rig}$  to the deepest MLD during a given day rather  
 844 than to 5 m if the data is restricted to the LES at  $0^\circ$  N. This property of the velocity profile may  
 845 contribute to the success of our empirical scaling in predicting  $\langle F_b \rangle^{max}$  in the TIWE observations,  
 846 in which we had to extrapolate the velocity profiles to the surface to define  $S_b$ , as well as the relative  
 847 success of Smyth et al. (2021) in modeling  $\epsilon$  from chipods at  $0^\circ$  defining  $S_b$  as an average over the  
 848 deep cycle layer.

849 So, why are we introducing a new definition of  $S_b$ ? The answer is that the new definition turns out  
 850 to be crucial off the equator and in instances of weaker mixing ( $\langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$ ), as discussed  
 851 in the next paragraph. However, there are also some practical advantages and disadvantages to the  
 852 new definition. First, it is more difficult to observe ocean currents above 25 m, and hence more  
 853 difficult to calculate our  $S_b$  with observations, although ADCPs on modern autonomous platforms  
 854 (Shcherbina et al. 2019; Gentemann et al. 2020) and moored ADCPs (Masich et al. 2021) can  
 855 sample to 10 m depth or less and current meters can be deployed at shallower depths on moorings  
 856 to mitigate issues particular to upward looking ADCPs on moorings. On the other hand, it is  
 857 advantageous to define  $S_b$  as we do for application in ocean model parameterizations, since it does  
 858 not depend on the extra diagnosis and definition of the daily-maximum MLD and our approach  
 859 works even when the deep cycle layer has zero thickness.

860 However, the main motivation for the new definition of  $S_b$  is that it substantially improves the  
 861 predictions of  $\langle F_b \rangle^{max}$  off the equator at  $3^\circ$  N and when mixing is weak (roughly,  $\langle F_b \rangle^{max} < 10^{-7.5}$   
 862  $\text{m}^2/\text{s}^3$ ). The reasons for this improvement are highlighted via the terms in (6): the LES results in Fig.  
 863 18 show that  $|\langle \mathbf{F}_m \rangle^{max}|/u_*^2$  and  $|\langle S \rangle^{max}|/|S_b|$  deviate systematically from 1 and  $\langle F_b \rangle^{max}/\langle SP \rangle^{max}$

864 deviates systematically from 0.2 in many instances of weaker mixing at 3° N. In conjunction with  
 865 these deviations,  $Ri_g^{-1} < 4$  deviates toward stability (i.e., points are colored blue to yellow in Fig.  
 866 18; see also Fig. 15). The deviation of  $|\langle S \rangle^{max}|/|S_b|$  is indicative of a divergence between our bulk  
 867 shear  $S_b$  and the shear in the deep cycle layer (used in the theory of Smyth et al. 2017), which has zero  
 868 thickness on 3 out of 34 days at 3°N. A practical consequence of this divergence in  $|\langle S \rangle^{max}|/|S_b|$  is  
 869 that replacing  $S_b$  with the shear in the deep cycle layer in the linear model  $u_*^2 S_b$  leads to a reduction  
 870 in the correlation from  $r^2 = 0.7$  to  $r^2 = 0.4$  when the data are from only the LES at 3° N. Specifically,  
 871 these deviations indicate that the shear is more concentrated at the base of the mixed layer, the wind  
 872 contributes more to accelerating the mixed layer than below, and the shear at the base of the mixed  
 873 layer is weaker than necessary for marginal instability. All of these features are consistent with a  
 874 transition to a mid-latitude inertial regime when the shear, wind stress, and hence turbulent heat  
 875 fluxes are sufficiently weak (e.g., Pollard and Millard 1970). In this regime, strong turbulent heat  
 876 fluxes like those in strong equatorial DCT only occur intermittently under the right conditions, such  
 877 as when the shear and wind are sufficiently strong and well aligned and the system is near a state of  
 878 marginal instability (e.g., Pollard et al. 1972; Burchard and Rippeth 2009; Brannigan et al. 2013;  
 879 Watkins and Whitt 2020). Yet, the scaling  $F_b \approx 0.2u_*^2 S_b$  in combination still approximately holds  
 880 when mixing is weaker  $10^{-8.5} < \langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$  at 3° N. In addition, it is interesting to  
 881 note that a reanalysis of the LES of Watkins and Whitt (2020) showed that  $F_b^{max} \sim 0.2u_*^2 S_b$  within  
 882 a factor of 3 in hurricane-driven entrainment in the coastal mid-latitude ocean, from  $F_b^{max} = 10^{-8}$   
 883 to  $10^{-5} \text{ m}^2/\text{s}^3$  without time averaging (only horizontal averaging). Analysis of the co-variability  
 884 of the ratios in (6) in Fig. 18 shows that the empirical  $F_b \approx 0.2u_*^2 S_b$  scaling continues to perform  
 885 reasonably well for  $\langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$  at 3° N because the changes in the ratios  $|\langle F_m \rangle^{max}|/u_*^2$ ,  
 886  $|\langle S \rangle^{max}|/|S_b|$ , and  $Ri_f$  compensate for each other (Fig. 18). Thus, as the turbulence weakens such  
 887 that  $\langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$ , it is more difficult to interpret the empirical scalings from section

888 4.e as proxies for local dynamics at  $z_{max}$  or using the theory for homogeneous DCT of Smyth  
889 et al. (2017). That is, the empirical scaling  $F_b \approx 0.2u_*^2 S_b$  can be explained by theory for DCT at  
890  $0^\circ, 140^\circ\text{W}$  (Smyth et al. 2017), but the theory cannot explain the applicability of the scaling at  
891  $3^\circ, 140^\circ\text{W}$  in the LES.

892 A question that arises at this point is how the results from LES and the parameterization for  
893  $\langle F_b \rangle^{max}$  compare with existing mixing parameterizations that are designed to be applicable both on  
894 and off the equator (unlike the theory of Smyth et al. 2017), such as the KPP scheme (building on  
895 Figs. 7, 8, and 16b). Properly addressing this question is beyond the scope of this paper and a subject  
896 of interest for future work, but a comparison between the simulations of  $\rho c_p / (g\alpha) \langle F_b \rangle^{max} \approx \langle F_Q \rangle^{max}$   
897 in the LES and  $\langle F_Q \rangle^{max}$  in ROMS (KPP) highlights substantial differences despite the similar large-  
898 scale oceanic and atmospheric forcing. However, minor differences in these large-scale forcings  
899 mean that the comparisons should be treated as qualitative rather than quantitative (see section 4.a  
900 and the Appendix for further details). In any case, we find that the maximum of the daily-averaged  
901 turbulent heat flux  $\langle F_Q \rangle^{max}$  ranges over a similar set of values from about 10 to 300 W/m<sup>2</sup> and the  
902 daily variability in the LES and ROMS is correlated in space and time at both  $0^\circ$  and  $3^\circ$  N ( $r^2 = 0.5$ ;  
903 see Fig. 19a). However, most of the correlation comes from  $3^\circ$  N, where  $r^2 = 0.3$ . There is no  
904 temporal correlation in  $\langle F_Q \rangle^{max}$  between the LES and ROMS at  $0^\circ$  N. The turbulent diffusivity  
905 at  $z_{max}$  is more scattered than the heat flux with a marginally significant correlation across space  
906 and time, and no temporal correlation at either latitude individually (Fig. 19b). Similarly, the  
907 depth  $z_{max}$  is similar in the LES and ROMS on many days, but there are numerous outliers with  
908 much deeper  $z_{max}$  in ROMS, such that the spatio-temporal correlation between ROMS and the  
909 LES is marginal to insignificant (Fig. 19c). These results support earlier indications that the  
910 mixing produced by KPP and the LES differ. Yet, these large day-to-day differences in the flux  
911 and diffusivity shown in Figs. 19a,b tend to take both signs and add up to fairly subtle impacts on

912 the time-mean temperature and horizontal velocity tendencies due to vertical mixing (see Fig. A1)  
913 and therefore the mean velocity and temperature profiles over 34 days, as discussed in section 2.b.

914 *g. Parameterization of  $\langle F_b \rangle$  profiles*

915 Finally, it is desirable to incorporate the information gleaned about  $\langle F_b \rangle^{max}$  from the LES into  
916 parameterizations of flux profiles for application in ocean models or in estimating turbulent fluxes  
917 from observations without turbulence data. Although it is beyond the scope of this paper to  
918 incorporate the scaling for  $\langle F_b \rangle^{max}$  into a complete mixing parameterization, we conclude the  
919 paper and motivate future work toward refined mixing parameterizations by presenting the results  
920 of a preliminary effort to parameterize the daily-mean buoyancy flux profiles  $\langle F_b \rangle$  simulated in  
921 the LES based on  $\langle F_b \rangle^{max}$ . It is important to emphasize that this effort involves a non-exhaustive  
922 exploration of a wide range of possible choices and thus is likely sub-optimal. Nevertheless, we  
923 find that the results are valuable motivation and guidance for future work and thus worth presenting.

924 More precisely, the objective of this section is to model the daily-averaged net buoyancy flux  
925 profile  $\langle B \rangle(z)$  from the surface  $z = 0$  to the base of the low-Richardson layer  $z = H_{Rig}$ , below which  
926 turbulent mixing is typically much weaker since  $H_{Rig}$  is highly correlated with  $z_{pen}$  (Fig. 14). That  
927 is, we seek to model

$$\langle B \rangle = \langle P_b \rangle + \langle F_b \rangle, \quad (9)$$

928 the sum of the daily-averaged penetrative and turbulent buoyancy fluxes. We model  $\langle B \rangle$  rather than  
929  $\langle F_b \rangle$  because  $\langle B \rangle$  profiles do not exhibit the exponential structure characteristic of  $\langle P_b \rangle$ , whereas  
930  $\langle F_b \rangle$  profiles do (compare Figs. 10a-b and Figs. 20a-b). Thus, we interpret  $\langle B \rangle$  as the residual  
931 turbulent flux, after subtracting the part of  $\langle F_b \rangle$  that is equal and opposite to the penetrating solar  
932 radiative flux  $\langle P_b \rangle$  (see the Appendix for details on  $P_b$ ).

933 Rather than parameterize  $\langle B \rangle$  at each depth based on the local properties (as in several previous  
 934 studies, e.g., Pacanowski and Philander 1981; Peters et al. 1988; Zaron and Moum 2009), the entire  
 935  $\langle B \rangle$  profile on a given day, from the surface to the base of the low- $Ri_g$  layer  $z = H_{Ri_g}$ , is modeled  
 936 from a shape function and three bulk parameters: the net air-sea fluxes of buoyancy  $\langle B(z=0) \rangle$  and  
 937 momentum  $\langle |\tau|/\rho \rangle$  and the bulk vertical shear of horizontal currents  $S_b$  from  $H_{Ri_g}$  to 5 m depth.  
 938 We take this bulk parameterization approach because we find that knowing  $\langle B \rangle$  at just  $z = 0$  and  
 939  $z = z_{max}$  is sufficient to explain about 90% of the simulated variance in  $\langle B \rangle$  at all depths above  
 940  $H_{Ri_g}$  in the LES at both  $0^\circ$  N and  $3^\circ$  N. In particular, we find that a linear combination

$$\langle B \rangle(z) = w_1(z)\langle B \rangle(z=0) + w_2(z)\langle B \rangle(z=z_{max}) \quad (10)$$

941 explains about 90% of the variance in  $\langle B \rangle$  for all depths above  $H_{Ri_g}$  (compare Figs. 20a-b and  
 942 Figs. 20e-f), where

$$\langle B \rangle(z=0) = \frac{g\alpha}{\rho c_p} \langle Q_0 \rangle^{net} - g\beta \langle VSF_0^{net} \rangle, \quad (11)$$

$$\langle B \rangle(z=z_{max}) = \langle F_b \rangle(z=z_{max}) + \langle P_b \rangle(z=z_{max}), \quad (12)$$

943 and  $Q_0^{net}$  and  $VSF_0^{net}$  are given net surface heat and virtual salt fluxes across the air-sea interface  
 944 and  $\langle P_b \rangle$  is a given penetrative buoyancy flux profile associated with shortwave radiation (see  
 945 the Appendix for details). The depth-dependent weights  $w_1$  and  $w_2$  in (10) are piecewise linear  
 946 functions of depth, that is

$$w_1 = \frac{z_{max} - z}{z_{max}} \quad \text{for } z \leq z_{max}, \quad (13)$$

$$w_1 = 0 \quad \text{for } z > z_{max}, \quad (14)$$

$$w_2 = 1 - w_1 \quad \text{for } z \leq z_{max}, \quad (15)$$

$$w_2 = \frac{H_{Ri_g} - z}{H_{Ri_g} - z_{max}} \quad \text{for } H_{Ri_g} > z > z_{max}, \quad (16)$$

947 where  $z$ ,  $z_{max}$  and  $H_{Rib}$  are all positive depths by definition in the expressions above. It may be  
 948 noted that our approach results in piecewise constant heat flux convergence with one value below  
 949  $z_{max}$  (with sign of  $\langle F_Q \rangle^{max}$ ) and another above  $z_{max}$  (with sign of  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max}$ ).

950 Having chosen to represent the vertical structure of  $\langle B \rangle$  as a piecewise linear function that  
 951 depends on  $\langle B \rangle$  at just the surface and  $z_{max}$  and taking the surface flux as given, the stated objective  
 952 of this section is reduced to specifying  $\langle F_b \rangle^{max}$  and  $z_{max}$ . To recapitulate previous sections, we use  
 953 linear regression to model  $z_{max}$ ,

$$z_{max} \approx 0.6H_{Rib} + 14, \quad (17)$$

954 since we found that  $H_{Rib}$  could explain about half of the variance in  $z_{max}$  (Fig. 14a). In addition,  
 955 we use the scaling developed and discussed in sections 4.e-4.f (specifically, the one plotted in Fig.  
 956 17b) to model

$$\langle F_b \rangle^{max} \approx 0.16u_*^{2.1} S_b^{0.98}. \quad (18)$$

957 With these parameterized representations of  $\langle F_b \rangle^{max}$  and  $z_{max}$  in (10), we find that this linear  
 958 combination explains 75% of the variance in simulated  $\langle B \rangle$  above  $H_{Rib}$  across both LES (cf. Figs.  
 959 20a-b and Figs. 20c-d;  $r^2 = 0.6$  and  $r^2 = 0.7$  at  $0^\circ$  and  $3^\circ$  N, respectively).

960 To put the flux profiles from the LES and the parameterization (10) in context, compare the  
 961 results to those from the KPP output in parent regional ocean model (ROMS) at the LES locations  
 962 (Fig. 20). Consistent with earlier results, there are qualitative differences between the flux profiles  
 963 in the LES and ROMS (KPP). Perhaps most notably, strong mixing extends to deeper depths in  
 964 ROMS (KPP), consistent with many instances of deeper  $z_{max}$  (Fig. 19c). The ROMS (KPP)  
 965 solution also seems to have a more distinct modulation of mixing on the timescale of the tropical  
 966 instability wave than in the LES. However, given the previously identified differences, including  
 967 the absence of correlation between the LES and KPP in the depth  $z_{max}$  or the turbulent diffusivity

968 at  $z_{max}$  in Fig. 19, it is perhaps remarkable how similar the KPP and LES solutions are (see also  
969 Fig. A1). In any case, the results of our preliminary effort to parameterize flux profiles suggest  
970 that future work is both justified and needed to incorporate information about  $\langle F_b \rangle^{max}$  into a more  
971 general mixing parameterization that handles momentum and tracer fluxes as well as an explicit  
972 diurnal cycle.

## 973 5. Conclusions

974 This manuscript synthesizes results from submesoscale-permitting regional ocean models and  
975 large eddy simulations of turbulence embedded in a regional model to build understanding of  
976 deep-cycle turbulence and upper-ocean mixing more generally in the equatorial Pacific Ocean cold  
977 tongue at and beyond  $0^\circ$  N,  $140^\circ$  W.

978 First, a submesoscale-permitting regional hindcast simulation of the period 1999-2016 in the  
979 MITgcm is used to quantify the climatological mean, seasonal cycle, and aseasonal variability of  
980 ocean mixing as measured by the maximum over depth of the daily-mean turbulent vertical heat  
981 flux  $\langle F_Q \rangle^{max}$ . We found that there is a good spatial correlation ( $r^2 = 0.7$ ) between  $\langle F_Q \rangle^{max}$  and the  
982 time-mean net ocean surface heat flux  $\langle Q_0^{net} \rangle$ . Although both  $\langle F_Q \rangle^{max}$  and  $\langle Q_0^{net} \rangle$  are stronger in  
983 the cold tongue relative to other areas, there is a prominent equatorial mixing band within about  
984  $1-2^\circ$  of the equator where the time-mean, seasonal-cycle amplitude, and aseasonal variability are  
985 much larger in  $\langle F_Q \rangle^{max}$  than  $\langle Q_0^{net} \rangle$ . Aseasonal anomalies in  $\langle F_Q \rangle^{max}$  (i.e., all deviations from the  
986 climatological seasonal cycle) are uncorrelated with aseasonal anomalies in  $\langle Q_0^{net} \rangle$ , which suggests  
987 that the surface heat flux exerts little control on the aseasonal variability of ocean mixing in the cold  
988 tongue. In addition,  $\langle F_Q \rangle^{max}$  is logarithmically distributed and exhibits rare but intense mixing  
989 events as far as  $5^\circ$  from the equator driven by mesoscale oceanic variability. Thus, strong mixing  
990 and DCT are not limited to the equatorial mixing band above the undercurrent, and aseasonal

991 variability in general and infrequent strong mixing events in particular have substantial impacts on  
992 the climatologies of mixing across the cold tongue. As a result, the spatial patterns of aseasonal  
993 variance and time-mean  $\langle F_Q \rangle^{max}$  are very similar. However, it is not clear if the parameterized  
994 mechanisms that control mixing in the regional model are realistic. Comparisons between modeled  
995 and measured turbulence at  $0^\circ$  N,  $140^\circ$  W suggest that the mixing has a realistic seasonal cycle in  
996 the regional model but the time-mean turbulent heat fluxes may be too strong and there may be too  
997 few instances of weak mixing at this location.

998 State-of-the-art large eddy simulations embedded in a regional model simulate the outer scales  
999 of turbulence  $O(1)$  m as it evolves over 34 days in response to changing atmospheric and oceanic  
1000 forcing at both  $0^\circ$ N,  $140^\circ$ W and  $3^\circ$ N,  $140^\circ$ W in October. The time-averaged LES results are  
1001 consistent with the spatial pattern of mixing simulated in the regional model. In particular, mixing  
1002 is elevated below the surface both on and off the equator, but the time-mean  $\langle F_Q \rangle^{max}$  in the LES  
1003 is about 3-4 times stronger at  $0^\circ$  N ( $110$  W/m<sup>2</sup>) than at  $3^\circ$  N ( $30$  W/m<sup>2</sup>) along  $140^\circ$  W. However,  
1004 mixing in the LES is about a factor of two weaker than on average in all Octobers from 1999-  
1005 2016 in the MITgcm. More direct comparisons between the mixing in the LES and its parent  
1006 regional model ROMS, in which mixing occurs under essentially the same day-to-day oceanic  
1007 and atmospheric conditions as in the LES but via the KPP scheme (Large et al. 1994) as in the  
1008 MITgcm, also suggest that parameterized mixing in the regional model is stronger and more deeply  
1009 penetrating than in the LES, but the time-mean  $\langle F_Q \rangle^{max}$  is only 20% lower in the LES. Individually,  
1010 these LES results may not be sufficient to conclude that the KPP mixing scheme yields too-strong  
1011 mixing in the regional models, but taken with similar conclusions derived from comparisons to  
1012 chipod microstructure observations, it seems likely that the time-mean mixing in the cold tongue  
1013 is too strong in the regional models and the mixing scheme needs to be modified.

1014 The LES results also provide important insight into the aseasonal variability of mixing and its  
1015 covariates on timescales from days to a month and thus facilitate the identification and evaluation of  
1016 empirical scalings for ocean mixing that might be applicable across a range of different atmospheric  
1017 and oceanic conditions throughout the Pacific Ocean cold tongue and possibly beyond. A highlight  
1018 is the finding that a relatively simple two-variable linear model approximately proportional to  $u_*^2 S_b$   
1019 can explain about 90% of this daily variance in  $\langle F_Q \rangle^{max}$  across both LES locations, where  $u_*$  is  
1020 the surface friction velocity,  $S_b$  is the bulk vertical shear of the ocean currents averaged from 5  
1021 m depth to  $H_{Rig}$ , below which  $Ri_g > 0.35$ . In an independent validation, this scaling explains  
1022 40% of the observed variance in the TIWE observations of Lien et al. (1995), which exhibit a  
1023 similar distribution of  $\langle F_Q \rangle^{max}$  as the LES at 0°N, 140°W with mean bias that is smaller than the  
1024 measurement uncertainty of a factor of two. Even more encouraging is that the empirical scaling  
1025 can be interpreted with prior theory by Smyth et al. (2017) at 0°N, 140°W. However, while the  
1026 scaling is successful off the equator at 3°N, 140°W, its applicability beyond 0°N, 140°W cannot  
1027 be interpreted with the theory of Smyth et al. (2017), nor has it been validated with observations.  
1028 Nevertheless, the finding that LES simulates strong DCT at 3°N, 140°W away from the undercurrent  
1029 adds significant new evidence in support of these hypotheses that strong DCT, marginal instability,  
1030 and intense mixing can occur both with and without the undercurrent, as long as the vertical shear  
1031 of upper-ocean currents and (to a lesser degree) the wind stress are sufficiently strong (building on  
1032 Pei et al. 2020; Cherian et al. 2021). However, future observational process studies are needed to  
1033 refine and likely modify these hypotheses and scalings of ocean mixing throughout the cold tongue  
1034 and particularly off the equator. In addition, these results are both a motivation and a promising  
1035 foundation for needed refinement of the parameterizations of equatorial mixing in ocean models.

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1055 *Data availability statement.* Data and software necessary to reproduce the figures are published  
1056 and links are provided in the references (Whitt 2022).

## 1057 APPENDIX

### 1058 **Large eddy simulation methods**

1059 The LES (Taylor 2008; Whitt and Taylor 2017; Watkins and Whitt 2020) solves a filtered version  
 1060 of the Navier Stokes equations under the Boussinesq approximation on a traditional  $f$  plane along  
 1061 with evolution equations for temperature and salinity,

$$\frac{D\mathbf{u}}{\partial t} + \mathbf{f} \times \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{b} + \nabla \cdot (\nu_{sgs} \nabla \mathbf{u}) + \mathcal{F}_{\mathbf{u}} + \mathcal{R}_{\mathbf{u}} + \mathcal{D}_{\mathbf{u}}, \quad (\text{A1})$$

$$\nabla \cdot \mathbf{u} = 0, \quad (\text{A2})$$

$$\frac{DT}{Dt} = \nabla \cdot (\kappa_{sgs} \nabla T) + I + \mathcal{F}_T + \mathcal{R}_T + \mathcal{D}_T, \quad (\text{A3})$$

$$\frac{DS}{Dt} = \nabla \cdot (\kappa_{sgs} \nabla S) + \mathcal{F}_S + \mathcal{R}_S + \mathcal{D}_S, \quad (\text{A4})$$

$$b = -g(1 - \alpha(T - T_0) + \beta(S - S_0)), \quad (\text{A5})$$

1062 where  $\mathbf{f} = (0, 0, f)$ ,  $f = 14.6 \times 10^{-5} \sin(\text{latitude}) \text{ s}^{-1}$  is the traditional Coriolis frequency, the  
 1063 buoyancy force is  $\mathbf{b} = (0, 0, b)$ , the density of the seawater is  $-\rho b/g$ , where the constant reference  
 1064 density of seawater  $\rho = 1023.5 \text{ kg/m}^3$ ,  $g = 9.81 \text{ m/s}^2$ , and the density and buoyancy vary linearly  
 1065 with temperature  $T$  and salinity  $S$ ;  $\alpha = 2.96 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ ,  $T_0 = 25.0^\circ\text{C}$ ,  $\beta = 7.38 \times 10^{-4} \text{ psu}^{-1}$ , and  
 1066  $S_0 = 35.25 \text{ psu}$ . The equations are solved in a horizontally periodic domain that is 108 m deep and  
 1067 306 by 306 m square and discretized on a mesh with 216 by 360 by 360 points with a resolution of  
 1068 0.5 m vertically by 0.85 m horizontally.

1069 The vertical profiles of temperature, salinity, and horizontal momentum are initialized in the  
 1070 LES on October 2, 1985 at 6:00 UTC by interpolating the output of a hindcast from the Regional  
 1071 Ocean Modelling System (ROMS) (Shchepetkin and McWilliams 2005; Haidvogel et al. 2008),  
 1072 which simulates the period August 1984-February 1986 in a regional ocean domain spanning  $95^\circ$   
 1073 to  $170^\circ$  W and  $12^\circ$  S to  $12^\circ$  N at  $1/20^\circ$  (5.5 km) horizontal resolution on 50 terrain-following  
 1074 sigma levels (spaced about every 8 m in the top 100 m) as in Holmes and Thomas (2015). The  
 1075 interpolation procedure involves first averaging the 6-hour averaged ROMS output horizontally over  
 1076 a 3-by-3 array of grid cells (about 16.5 km square) around the LES location and then interpolating

1077 vertically to the LES grid using cubic splines. The ROMS hindcast was used as the parent model  
 1078 instead of the MITgcm hindcast described above mainly because it was available with all relevant  
 1079 outputs before the MITgcm run was completed. In ROMS, the initial conditions and daily ocean  
 1080 side boundary conditions are from the global mesoscale-resolving ocean/sea-ice hindcast used by  
 1081 Deppenmeier et al. (2021). Neither model has tides. In both of these regional and global ocean  
 1082 models, the surface fluxes are calculated using the JRA55do atmospheric reanalysis (Tsujino et al.  
 1083 2018) and the same bulk flux algorithms (Large and Yeager (2004, 2009), see also Small et al.  
 1084 (2015); Whitt et al. (2019)). In particular, the ROMS hindcast is forced by a diurnal cycle of  
 1085 shortwave radiation (3-hourly) and vertical mixing is parameterized with the KPP scheme of Large  
 1086 et al. (1994) with the same parameters as in the Parallel Ocean Program used by Deppenmeier  
 1087 et al. (2021) (as in Whitt et al. 2019). The resulting diurnal cycles of upper-ocean turbulence look  
 1088 qualitatively similar to those reported in Cherian et al. (2021) and simulated in MITgcm with the  
 1089 same mixing parameterization, surface forcing, and horizontal grid resolution.

1090 The subgrid-scale viscosity in the LES  $\nu_{sgs} = \nu_0 + \nu_t$  includes small and constant “molecular”  
 1091 viscosity  $\nu_0 = 10^{-6}$  m<sup>2</sup>/s. The much larger and variable turbulent viscosity is modeled after  
 1092 Kaltenbach et al. (1994), that is

$$\nu_t = C_s^2 \Delta^2 (2S'_{ij}S'_{ji})^{1/2} \quad (\text{A6})$$

1093 where the Smagorinski coefficient  $C_s = 0.13$ , the grid scale  $\Delta = (2\delta x\delta y\delta z)^{1/3}$  (where  $\delta x$ ,  $\delta y$ ,  
 1094 and  $\delta z$  are grid spacings in the x, y and z dimensions), the resolved deformation tensor is  $S_{ij} =$   
 1095  $1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$  and  $i, j = 1, 2, 3$  correspond to x, y, z dimensions and summation over  
 1096 repeated indices is implied and the horizontally averaged shear is subtracted from the deformation  
 1097 tensor  $S_{ij}$  in  $S'_{ij}$ . The diffusivity  $\kappa_{sgs} = \kappa_0 + \nu_t/Pr_t$ , where the turbulent Prandtl number is as in

1098 Whitt and Taylor (2017) based on Anderson (2009),

$$Pr_t = \left(1 + \frac{Ri_{GS}}{0.94}\right)^{1.5}, \quad (A7)$$

1099 and the grid-scale gradient Richardson number is

$$Ri_{GS} = \frac{\delta b \delta z}{\delta u^2 + \delta v^2}, \quad (A8)$$

1100 where  $\delta b$ ,  $\delta z$ ,  $\delta u$ , and  $\delta v$  are the vertical differences in buoyancy, depth, and horizontal velocity  
1101 between two adjacent grid cells.

1102 At the top surface  $z = 0$ , the horizontally-uniform vertical fluxes are specified via time-evolving  
1103 gradient boundary conditions:

$$\frac{\partial \mathbf{u}_h}{\partial z} = \frac{\boldsymbol{\tau}}{\rho \nu_{SGS}}, \quad (A9)$$

$$\frac{\partial T}{\partial z} = \frac{Q_0^{net} - P_Q(0)}{\rho c_p \kappa_{SGS}}, \quad (A10)$$

$$\frac{\partial S}{\partial z} = \frac{VSF_0^{net}}{\kappa_{SGS}}, \quad (A11)$$

1104 where  $\nu_{sgs} = \nu_0$ ,  $\kappa_{sgs} = \kappa_0$  are constant,  $c_p = 4000 \text{ J/(kg } ^\circ\text{C)}$  is the specific heat of the seawater, and  
1105 the net virtual salt flux  $VSF_0^{net}$ , the net surface heat flux  $Q_0^{net}$ , the net surface shortwave heat flux  
1106  $P_Q(0)$ , and the surface wind stress  $\boldsymbol{\tau}$  are linearly interpolated from the 6-hourly-averaged ROMS  
1107 fluxes, averaged over a 16.5 km square around the LES location, and shown in Fig. 10. Thus,  
1108 the fluxes do not depend on the LES state. There is a diurnal cycle of shortwave solar radiation  
1109  $P_Q(0)$ , which penetrates and warms the interior of the LES during daytime as described below.  
1110 The top is rigid, so the vertical velocity  $w = 0$  at  $z = 0$  (see Fig. 10). The LES domain bottom is  
1111 rigid,  $w = 0$ , with  $u = 0.865 \text{ m/s}$  and  $0.465 \text{ m/s}$  at  $0^\circ$  and  $3^\circ \text{ N}$  respectively,  $v = 0 \text{ m/s}$ ,  $T = 22.3^\circ\text{C}$ ,  
1112 and  $S = 35.28 \text{ psu}$  are held constant. Although a variable bottom boundary to match the parent  
1113 model solution would be preferred, the constant bottom boundary is thought to have little impact  
1114 on the results in this study, because we set  $\nu_{sgs} = \nu_0 = 10^{-6} \text{ m}^2/\text{s}$  at the interface between the

1115 bottom boundary velocity and the first interior point. Thus, the horizontally-averaged velocity and  
 1116 temperature profiles evolve to remain approximately consistent with ROMS and are as shown in  
 1117 Fig. 9, and the resulting artificially strong vertical gradients at the domain bottom do not result in  
 1118 strong vertical fluxes of momentum, temperature, and salinity that significantly modify the interior  
 1119 evolution. Yet, extra caution should be exercised when interpreting the turbulent statistics near the  
 1120 bottom of the LES domain (e.g., Fig. 12h).

1121 Interior warming due to solar radiation is represented as the convergence of a two-component  
 1122 exponential:

$$I(t, z) = \frac{I_0(t)}{\rho c_p} \frac{\partial}{\partial z} \left( a_I e^{-z/\zeta_{I1}} + (1 - a_I) e^{-z/\zeta_{I2}} \right) \quad (\text{A12})$$

1123 where  $a_I = 0.58$ ,  $\zeta_{I1} = 2.0$  m,  $\zeta_{I2} = 23$  m and net incoming shortwave radiation  $I_0$  ( $\text{W/m}^2$ ) has  
 1124 a diurnal cycle and is linearly interpolated from 6-hour-average ROMS output. We call the total  
 1125 penetrative heat flux from solar radiation

$$P_Q(t, z) = I_0(t) \left( a_I e^{-z/\zeta_{I1}} + (1 - a_I) e^{-z/\zeta_{I2}} \right), \quad (\text{A13})$$

1126 and the analogous penetrative buoyancy flux is  $P_b = P_Q g \alpha / (\rho c_p)$ . The chosen profile  $P_Q(z)$  is a  
 1127 modified Jerlov type I profile (Paulson and Simpson 1977), such that the first e-folding scale is  
 1128 increased from 0.35 to 2.0 m in an ad hoc attempt to compensate for missing near-surface mixing  
 1129 due to surface gravity waves as in Watkins and Whitt (2020).

1130 There are three new terms on the right side of the equations that are new implementations specific  
 1131 for this study and discussed briefly in the main methods section of the manuscript. These terms,  
 1132 large-scale tendencies  $\mathcal{F}$ , restoring  $\mathcal{R}$  and damping  $\mathcal{D}$ , are included to make the solution more  
 1133 realistic given the limited domain size. First,  $\mathcal{F}(t, z)$  includes horizontally-uniform (in the LES)

1134 large-scale tendencies, that is

$$\mathcal{F}_u(z, t) = -\mathbf{u}_{ROMS} \cdot \nabla u_{ROMS} - \frac{1}{\rho} \frac{\partial p_{ROMS}}{\partial x} + D_{u_{ROMS}}, \quad (\text{A14})$$

$$\mathcal{F}_v(z, t) = -\mathbf{u}_{ROMS} \cdot \nabla v_{ROMS} - \frac{1}{\rho} \frac{\partial p_{ROMS}}{\partial y} + D_{v_{ROMS}}, \quad (\text{A15})$$

$$\mathcal{F}_w(z, t) = 0, \quad (\text{A16})$$

$$\mathcal{F}_T(z, t) = -\mathbf{u}_{ROMS} \cdot \nabla T_{ROMS} + D_{T_{ROMS}}, \quad (\text{A17})$$

$$\mathcal{F}_S(z, t) = 0, \quad (\text{A18})$$

1135 where  $D$  represents the explicit lateral mixing from ROMS. The restoring  $\mathcal{R}$  operates throughout  
1136 the entire depth of the LES domain but operates only on the horizontal average:

$$\mathcal{R}_u(z, t) = -(\bar{u} - u_{ROMS})/t_r, \quad (\text{A19})$$

$$\mathcal{R}_v(z, t) = -(\bar{v} - v_{ROMS})/t_r, \quad (\text{A20})$$

$$\mathcal{R}_w(z, t) = 0, \quad (\text{A21})$$

$$\mathcal{R}_T(z, t) = -(\bar{T} - T_{ROMS})/t_r, \quad (\text{A22})$$

$$\mathcal{R}_S(z, t) = 0, \quad (\text{A23})$$

1137 where the over-bar denotes the lateral average and the restoring timescale  $t_r = 11.6$  days ( $10^6$  s).

1138 In general,  $\mathcal{F} \gg \mathcal{R}$  because  $t_r$  is so long.

1139 Since the LES only simulates a small domain, the tendencies associated with larger scales, namely  
1140  $\mathcal{F}$  which includes three-dimensional advection, horizontal mixing, and the pressure gradient force  
1141 but excludes the Coriolis force and vertical mixing because they are simulated in LES, are obtained  
1142 from the 6-hourly-averaged budget diagnostic output of ROMS and are independent of the LES  
1143 state. These large-scale tendencies are first averaged over a 3 by 3 array of ROMS grid cells (about  
1144 a 16.5 km square) centered on the LES locations, then interpolated using cubic splines from the  
1145 ROMS sigma levels (about every 8 m) to the LES vertical levels, and finally linearly interpolated

1146 in time and added as a tendency to the horizontally-averaged components of the LES momentum  
 1147 and tracer equations as the LES runs (as expressed in equations above). The analogous large-scale  
 1148 interior salinity tendencies are set to zero in the LES for simplicity. Although the omission of  
 1149 interior salinity tendencies may complicate the interpretation, temperature is highly correlated with  
 1150 buoyancy (initial  $r^2 = 0.99$  at both  $0^\circ$  and  $3^\circ\text{N}, 140^\circ\text{W}$ ) and has a three-fold stronger influence on  
 1151 buoyancy than salinity in the region. Specifically, the initial bulk 108 m buoyancy differences  
 1152 are  $0.0029 \text{ m/s}^2$  (for a 0.4 psu salinity difference) and  $0.0080 \text{ m/s}^2$  (for a  $2.74^\circ \text{C}$  temperature  
 1153 difference) at  $0^\circ \text{N}, 140^\circ \text{W}$ . Thus, the results are expected to be qualitatively unaffected by the  
 1154 omission of interior salinity tendencies, but future simulations are required to precisely quantify  
 1155 the turbulent response to salinity advection.

1156 Finally, the fluctuations below 84 m depth are slowly damped toward zero:

$$\mathcal{D}_u(z, t) = -\sigma(u - \bar{u})/t_r, \quad (\text{A24})$$

$$\mathcal{D}_v(z, t) = -\sigma(v - \bar{v})/t_r, \quad (\text{A25})$$

$$\mathcal{D}_w(z, t) = -\sigma w/t_r, \quad (\text{A26})$$

$$\mathcal{D}_T(z, t) = -\sigma(T - \bar{T})/t_r, \quad (\text{A27})$$

$$\mathcal{D}_S(z, t) = -\sigma(S - \bar{S})/t_r, \quad (\text{A28})$$

1157 where

$$\sigma(z) = \left( \frac{z + H - L_s}{L_s} \right)^2 \quad \text{for } z < (L_s - H) \text{ and} \quad (\text{A29})$$

$$\sigma(z) = 0 \quad \text{for } z \geq (L_s - H), \quad (\text{A30})$$

1158 where  $z$  is the depth from 0 to  $-H$ ,  $H = 108 \text{ m}$  is the domain height,  $L_s = 24 \text{ m}$  is the thickness  
 1159 of the damping layer. It is notable that the timescale  $t_r$  is very long; it is about 66 days at 94 m,  
 1160 17 days at 104 m, and 12 days at the bottom 108 m. These timescales are much longer than the

1161 time scale of the relevant stratified shear instabilities or internal waves (Smyth et al. 2011; Moum  
1162 et al. 2011) and thus the damping has a negligible influence on shear instabilities, internal waves  
1163 and turbulence at essentially all depths (the damping is of the order  $10^{-12}$  to  $10^{-10}$   $\text{m}^2/\text{s}^3$ ), and the  
1164 DCT never gets within 15 m of the bottom in any case. Despite the slow damping rate and shallow  
1165 domain bottom, the bottom 20 m remains strongly stratified with internal wave fluctuations that  
1166 are weak compared to DCT. Short tests with a deeper 144 m domain suggested that the shallow  
1167 domain bottom does not qualitatively impact the results. The stability analysis of Smyth et al.  
1168 (2011) also suggests that the shallow domain depth is unlikely to impact the results since all of the  
1169 shear instabilities they identify occur at depths shallower than 100 m and have a thickness of 20-40  
1170 m.

1171 This manuscript focuses on the horizontally-averaged dynamics in the LES,

$$\frac{\partial \bar{\mathbf{u}}_h}{\partial t} = -\mathbf{f} \times \bar{\mathbf{u}}_h + \frac{\partial}{\partial z} \overline{\left( \nu_{sgs} \frac{\partial \mathbf{u}_h}{\partial z} - w \mathbf{u}_h \right)} + \mathcal{F}_{\mathbf{u}} + \mathcal{R}_{\mathbf{u}}, \quad (\text{A31})$$

$$\frac{\partial \bar{T}}{\partial t} = \frac{\partial}{\partial z} \overline{\left( \kappa_{sgs} \frac{\partial T}{\partial z} - w T \right)} + I + \mathcal{F}_T + \mathcal{R}_T, \quad (\text{A32})$$

$$\frac{\partial \bar{S}}{\partial t} = \frac{\partial}{\partial z} \overline{\left( \kappa_{sgs} \frac{\partial S}{\partial z} - w S \right)}, \quad (\text{A33})$$

$$\bar{b} = -g(1 - \alpha(\bar{T} - T_0) + \beta(\bar{S} - S_0)). \quad (\text{A34})$$

1172 The right-hand side terms in these budgets are averaged over the duration of the LES simulations  
1173 and plotted in Fig. A1 and compared with output from ROMS (the subscript  $h$  indicates horizontal,  
1174 e.g. the horizontal velocity vector  $(\mathbf{u}, \mathbf{v}, 0)$ ). We define

$$\mathbf{F}_{\mathbf{m}} = \overline{\left( \nu_{sgs} \frac{\partial \mathbf{u}_h}{\partial z} - w \mathbf{u}_h \right)}, \text{ and} \quad (\text{A35})$$

$$F_b = \overline{\left( \kappa_{sgs} \frac{\partial b}{\partial z} - w b \right)} = g(\alpha F_T - \beta F_S), \quad (\text{A36})$$

1175 where  $F_T$  and  $F_S$  have the same functional form as  $F_b$  but operate on temperature (A32) and salinity  
1176 (A33). The kinetic and potential energy equations for the horizontally-averaged state are then given

1177 by:

$$\frac{\partial |\bar{\mathbf{u}}_h|^2/2}{\partial t} = \frac{\partial}{\partial z} (\bar{\mathbf{u}}_h \cdot \mathbf{F}_m) - \mathbf{F}_m \cdot \frac{\partial \bar{\mathbf{u}}_h}{\partial z} + \bar{\mathbf{u}}_h \cdot \mathcal{F}_u + \bar{\mathbf{u}}_h \cdot \mathcal{R}_u, \quad (\text{A37})$$

$$\frac{\partial \bar{b}_z}{\partial t} = \frac{\partial}{\partial z} (zF_b) - F_b + z\mathcal{F}_b + z\mathcal{R}_b, \quad (\text{A38})$$

1178 and  $\mathcal{F}_b = g(\alpha\mathcal{F}_T - \beta\mathcal{F}_S)$  and similarly for  $\mathcal{R}_b$ . On the right hand side, the first terms represent  
 1179 vertical redistribution or transport in the interior and sources and sinks at the surface boundary  
 1180 (e.g., wind work on the mean flow). The third and fourth terms are interior sources and sinks  
 1181 related to the larger-scale dynamics inherited from ROMS (e.g., advection, pressure work, etc.).  
 1182 The second term is the sink of mean kinetic energy to turbulence usually referred to as shear  
 1183 production  $-\mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z$  and the source of potential energy due to turbulent vertical mixing or  
 1184 buoyancy flux  $-F_b$ .

1185 The governing equation for the horizontally-averaged turbulent kinetic energy (i.e., for  $k = \overline{|\mathbf{u}'|^2}/2$   
 1186 where  $\mathbf{u}' = \mathbf{u} - \bar{\mathbf{u}}$ ) is given by

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial z} \left( \overline{wp/\rho} + \overline{wk} - \overline{\nu_{SGS} \frac{\partial k}{\partial z}} - \overline{\mathbf{u}' \nu_{SGS} \frac{\partial \bar{\mathbf{u}}}{\partial z}} \right) = -\overline{w\mathbf{u}'_h} \cdot \frac{\partial \bar{\mathbf{u}}_h}{\partial z} + \overline{wb} - \bar{\epsilon} + \overline{\mathbf{u}' \cdot \mathcal{D}_u} \quad (\text{A39})$$

1187 where the dissipation of turbulent kinetic energy is  $\bar{\epsilon} = \overline{\nu_{SGS} S'_{ij} S'_{ji}} + \overline{\nu_{SGS} \partial \mathbf{u}'_h / \partial z \cdot \partial \mathbf{u}_h / \partial z}$ . In the  
 1188 limit that  $\nu_t \rightarrow 0$ , the shear production and buoyancy flux terms in the turbulent kinetic energy  
 1189 equation and the mean kinetic energy/potential energy equations become effectively identical.  
 1190 However, because the LES is a filtered approximation of high-Reynolds number flow with finite  
 1191  $\nu_t \gg \nu_0$ , a finite amount of mean-profile buoyancy flux, shear production and total dissipation  
 1192 occur via the subgrid-scales without passing through  $k$ . Hence, we plot the total dissipation  
 1193  $\overline{\nu_{SGS} S'_{ij} S'_{ji}}$ , shear production  $\mathbf{F}_m \cdot \partial \bar{\mathbf{u}}_h / \partial z$ , and buoyancy flux  $F_b$  throughout the manuscript and  
 1194 define the deviations from this balance to be transport and transience, i.e.:

$$T = \mathbf{F}_m \cdot \frac{\partial \bar{\mathbf{u}}_h}{\partial z} - F_b - \bar{\epsilon}, \quad (\text{A40})$$

1195 where  $\epsilon$  is total dissipation. Consistent with the discussion in Osborn (1980), the left hand side is  
1196 generally small when averaged horizontally and over a full day at  $z_{max}$  in the LES. For reference,  
1197 the subgrid-scale parts of  $F_b$  and  $F_m$  are small relative to the resolved parts where  $F_b$  and  $\epsilon$  are  
1198 strong and  $Ri_g$  is low, e.g. above  $H_{Rig}$  or shallower than about 75 m depth on average. The  
1199 subgrid-scale fluxes become relatively large deeper in the thermocline, where  $Ri_g > 1$  is relatively  
1200 high and  $F_b$  and  $\epsilon$  are relatively weak, e.g. below  $H_{Rig}$  or below 75 m on average; results from  
1201 these depths should be interpreted more cautiously.

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1445 21–37.

1446 **LIST OF TABLES**

1447 **Table 1.** A glossary table with definitions and sections where key metrics are defined. . . . 70

Metric	Definition (key defining sections)
$Q_0^{net}$	net surface heat flux (3.a)
$\langle F_Q \rangle^{max}$	maximum (over depth) of the daily-mean downward turbulent heat flux (3.a, 4.a)
$F_b$	downward turbulent buoyancy flux; roughly proportional to $F_Q$ (4.a, 4.c, Appendix)
$\mathbf{F}_m$	downward turbulent momentum flux (4.b, Appendix)
$\epsilon$	dissipation of turbulent kinetic energy (4.b, Appendix)
$SP$	shear production of turbulent kinetic energy $\mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z$ (4.d, Appendix)
$T$	convergence of the vertical transport of turbulent kinetic energy (4.d, Appendix)
$z_{max}$	depth at which the maximum $\langle F_Q \rangle^{max}$ or $\langle F_b \rangle^{max}$ occurs (3.a, 4.c)
$z_{pen}$	depth to which DCT penetrates; shallowest depth $\epsilon \leq 2 \times 10^{-8} \text{ m}^2/\text{s}^3$ (4.c)
MLD	mixed layer depth, first depth $0.015 \text{ kg/m}^3$ denser than 0-10 m mean (3.a and 4.c)
$H_{Rib}$	thickness of the surface layer with bulk $Ri_b = 0.2$ (4.c)
$Ri_b$	bulk Richardson number of a surface layer (4.c)
$H_{Rig}$	thickness of the low $Ri_g$ layer, $Ri_g < 0.35$ (4.c)
$Ri_g$	gradient Richardson number, $Ri_g = \partial b / \partial z /  \partial \mathbf{u}_h / \partial z ^2 = N^2 / S^2$ (2.c)
$Ri_f$	flux Richardson number, $Ri_f = F_b / SP$ (4.d)
$Pr_t$	turbulent Prandtl number, $Pr_t = Ri_g / Ri_f$ (4.d)
$S_b$	bulk vertical shear from least-squares fit to the horizontal velocity from $H_{Rig}$ to 5 m depth (4.e, 4.f)

TABLE 1. A glossary table with definitions and sections where key metrics are defined.

## LIST OF FIGURES

- 1449 **Fig. 1.** A comparison between the simulated (LES, solid lines) and observed mean temperature  
 1450 (a) and zonal velocity (b) profiles at 0° (blue) and 3° N (red) along 140° W. At 0° N, 140°  
 1451 W, the observations (horizontal bars) span the inter-quartile ranges of all monthly means  
 1452 (September-October-November only) from the TAO mooring (1988-2018). At 3° N, 140°  
 1453 W, a ship-based annual climatology is plotted (Johnson et al. 2002), but these are more for  
 1454 reference than for validation since there is significant seasonal and inter-annual variability. . . . 75
- 1455 **Fig. 2.** Simulated (LES) and observed frequency spectra of temperature (a) and zonal velocity (b) at  
 1456 25 m depth at 0° (blue) and 3° N (red) along 140° W. Observed spectra are calculated from  
 1457 the moored temperature sensor (10 minute instantaneous sampling) and current meter (1 hour  
 1458 average sampling) from the months September-October-November on the TAO mooring at 0°  
 1459 N, 140° W for comparison (1988-2018). The observed spectra are calculated in overlapping  
 1460 time windows that are the same length as the LES simulations (with 17% of points overlapped  
 1461 in each window). The 10% and 90% quantile at each frequency (across all of the spectra  
 1462 windows) is plotted in light blue. The black dotted and blue lines are derived from LES:  
 1463 the sampling is instantaneous (averaged over a single time step) every 10 min (a) or 1 hour  
 1464 (b) and averaged spatially over a single grid cell/virtual mooring (black dotted) or the entire  
 1465 horizontal extent of the domain (blue). The spectrum from the virtual mooring (black)  
 1466 flattens similarly to the observations from the TAO mooring at frequencies higher than 3  
 1467 cyc/day due to aliasing in (a). . . . . 76
- 1468 **Fig. 3.** Profiles of the median (thick lines) and inter-quartile range (iqr, thin lines) of the squared  
 1469 vertical shear of horizontal velocity  $S^2$ , the vertical buoyancy gradient  $N^2$ , and the gradient  
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- 1473 **Fig. 4.** Climatological spatial structure and seasonal cycle of downward heat fluxes in a regional  
 1474 ocean model of the equatorial Pacific Ocean cold tongue forced by atmospheric reanalysis  
 1475 from 1999-2016. The net air-sea flux  $\langle Q_0^{net} \rangle$  is in (b) and (e), and the maximum flux due to  
 1476 ocean mixing  $\langle F_Q \rangle^{max}$  is in (c) and (f). b-c are the zonal means from 95-170° W with the  
 1477 time-mean subtracted, and e-f are the time-means. In addition, we quantify the fraction of  
 1478 the zonal distance (a) and time (d) over which there is net cooling of the surface ocean due  
 1479 to air-sea exchange and ocean mixing, that is  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$ . The flux due to ocean  
 1480 mixing  $\langle F_Q \rangle^{max}$  (c,f) is defined as the maximum (over depth) of the daily-mean downward  
 1481 turbulent heat flux, so the zonal and time means are calculated at a depth that varies in time  
 1482 and space that is plotted in Fig. 5. . . . . 78
- 1483 **Fig. 5.** Climatological comparisons between mixed layer depth (MLD, b,e) and the depth  $z_{max}$  (c,f)  
 1484 where the downward turbulent heat flux is maximum (i.e., the depth where  $\langle F_Q \rangle^{max}$  plotted  
 1485 in Fig. 4 occurs). As in Fig. 4, b-c are the zonal mean anomalies from the time mean, and e-f  
 1486 are the time-means. In addition, we quantify the fraction of the zonal distance (a) and time  
 1487 (d) over which the the MLD is deeper than  $z_{max}$ . The MLD is defined to be the shallowest  
 1488 depth where water is 0.015 kg/m<sup>3</sup> denser than the top 10 m in the daily-mean density profile  
 1489 (since higher-frequency output is not available). . . . . 79
- 1490 **Fig. 6.** The top row shows the hindcast aseasonal daily-mean vertical heat fluxes during 2012 and  
 1491 2013 along the 140° W meridian (a: net surface flux  $\langle Q_0^{net} \rangle$ , b: ocean mixing  $\langle F_Q \rangle^{max}$   
 1492 and c: the depth where strongest mixing occurs  $z_{max}$ ). Maps (d-f) quantify the respective  
 1493 aseasonal inter-quartile ranges over all latitudes and years 1999-2016. Aseasonal variability  
 1494 is defined by subtracting the mean seasonal cycle (i.e., a daily annual climatology, which is

1495 averaged over 18 years and then smoothed with a 15-day moving average), from the total  
 1496 signal at each grid point. . . . . 80

1497 **Fig. 7.** Climatological annual cycle of the downward turbulent heat flux at 0° N, 140° W in the  
 1498 MITgcm regional ocean model, including monthly means at  $z_{max}$  ( $\langle F_Q \rangle^{max}$ , thick red) as  
 1499 well as monthly means from 20-60 m depth  $\langle F_Q \rangle^{20-60}$  (thick gray). Corresponding minima  
 1500 and maxima of monthly  $\langle F_Q \rangle^{20-60}$  (thin gray) and  $\langle F_Q \rangle^{max}$  (thin red) from 1999-2016 are  
 1501 included. For comparison, the observational climatology of  $\langle F_Q \rangle^{20-60}$  from chipods (Moum  
 1502 et al. 2013) is plotted in black circles. The 95% confidence intervals for the monthly mean  
 1503  $\langle F_Q \rangle^{max}$  from ROMS and LES (roughly October 1985) as well as the TIWE observations  
 1504 (roughly November 1991) are in magenta, green and blue respectively. Note, however, that  
 1505 the LES and TIWE are computed as  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$  [W/m<sup>2</sup>] where  $\rho$ ,  $c_p$ ,  
 1506 and  $\alpha$  are the reference density, specific heat, and thermal expansion coefficient of seawater,  
 1507 respectively,  $g$  is the acceleration due to gravity, and  $F_b$  is the downward turbulent buoyancy  
 1508 flux. Data from two other shorter field experiments (not shown) resulted in means of roughly  
 1509 400 W/m<sup>2</sup> in Oct/Nov 2008 (Moum et al. 2009) and 100 W/m<sup>2</sup> in Nov 1984 (Gregg et al.  
 1510 1985; Moum and Caldwell 1985) (see Fig. 2d of Moum et al. 2009). . . . . 81

1511 **Fig. 8.** Relative probability distributions of the maximum daily-mean turbulent heat flux due to ocean  
 1512 mixing  $\langle F_Q \rangle^{max}$  (a-b), the daily-mean net surface heat flux  $\langle Q_0^{net} \rangle$  (c-d), and the depth  $z_{max}$   
 1513 at which  $\langle F_Q \rangle^{max}$  occurs (e-f). Histograms are included for both 0° N, 140° W (blue) and 3°  
 1514 N, 140° W (red) for the 18-year MITgcm simulation (left column) as well as the 34-day LES  
 1515 in October 1985 (red and blue histograms) and the 38-day TIWE experiment at 0° N, 140° W  
 1516 in November 1991 (right column, dark-blue edged bars). Note that the data from LES and  
 1517 TIWE are computed based on buoyancy fluxes, e.g.  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$ . . . . . 82

1518 **Fig. 9.** Time series of zonal and meridional velocity (color), temperature (white contours in °C),  
 1519 mixed layer depth (MLD, dashed magenta), the depth where the bulk Richardson number  
 1520  $Ri_b = 0.2$  ( $H_{Rib}$ , thin black), and the base of the low-gradient Richardson number layer  
 1521  $Ri_g < 0.35$  ( $H_{Rig}$ , thick black) in the LES at 0° N and 3° N along 140° W. All fields are  
 1522 defined from horizontally-averaged profiles. The MLD is defined to be the shallowest depth  
 1523 where water is 0.015 kg/m<sup>3</sup> denser than the top 10 m in the instantaneous but horizontally-  
 1524 averaged density profile. All time tick marks are at 0 UTC; local solar time at 0°N, 140° W  
 1525 is about 9 hours behind UTC, so local solar noon is at about 21 UTC. . . . . 83

1526 **Fig. 10.** Time series of the net surface heat flux  $Q_0^{net}$  (left axis, blue), the magnitude of the wind  
 1527 stress  $|\tau|$  (right axis, red), and the subsurface downward turbulent heat flux  $F_Q$  profiles from  
 1528 October-November 1985 in the LES at 0° N (a) and 3° N (b) along 140° W. Overlaid on  $F_Q$   
 1529 are the depth at which the bulk Richardson number  $Ri_b = 0.2$  ( $H_{Rib}$ , thin black line), the  
 1530 depth of the maximum daily-mean downward heat flux  $z_{max}$  (+ symbols), the daily maximum  
 1531 MLD (defined from the horizontally averaged LES density profiles; magenta circles), and  
 1532 the base of the low gradient Richardson layer  $Ri_g < 0.35$  ( $H_{Rig}$ , thick black line). The  
 1533 daily-mean meridional velocity averaged from 25 to 75 m depth is in blue; the origin is at  
 1534 a depth of 100 m, a 1m spacing corresponds to 10 cm/s, and the peak-to-trough amplitudes  
 1535 are about 40 cm/s at 0° N and 90 cm/s at 3° N. For consistency with other results in section  
 1536 4, we plot  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$  [W/m<sup>2</sup>] where  $\rho$ ,  $c_p$ , and  $\alpha$  are the reference  
 1537 density, specific heat, and thermal expansion coefficient of seawater, respectively,  $g$  is the  
 1538 acceleration due to gravity, and  $F_b$  is the downward turbulent buoyancy flux. All time tick  
 1539 marks are at 0 UTC, but local solar time at 0°N, 140° W is about 9 hours behind UTC, so  
 1540 local solar noon is at about 21 UTC. Daily mean statistics (e.g.,  $z_{max}$  indicated by + symbols)  
 1541 are calculated from 21 UTC so that the averages begin and end near solar noon. . . . . 85

- 1542 **Fig. 11.** As in Fig. 10, but zoomed in on a few days in November and with the addition of the MLD  
1543 (dashed magenta) and the DCT penetration depth  $z_{pen}$  ( $\epsilon \geq 2 \times 10^{-8} \text{ m}^2/\text{s}^3$ ; thin green). The  
1544 MLD is defined to be the shallowest depth where water is  $0.015 \text{ kg}/\text{m}^3$  denser than the top  
1545 10 m in the instantaneous but horizontally-averaged density profile. . . . . 86
- 1546 **Fig. 12.** As in Fig. 11, but plots show (a)-(b) the vertical buoyancy gradient  $N^2$ , (c)-(d) the squared  
1547 vertical shear  $S^2$ , (e)-(f)  $Ri_g = N^2/S^2$ , and (g)-(h) the rate of dissipation of kinetic energy  
1548  $\epsilon$ . It may be noted that there are a few instances of elevated dissipation  $10^{-8} < \epsilon < 10^{-7}$   
1549  $\text{m}^2/\text{s}^3$  below the deepest depths of DCT ( $z_{pen}$ , green line) in (h) where  $Ri_g > 1$ . However,  
1550 these instances of elevated dissipation near the bottom are dominated by dissipation of the  
1551 mean-flow kinetic energy, and the turbulent fluxes and energetics depend strongly on the  
1552 subgrid-scale parameterization in the LES (A6)-(A7), may be influenced by the bottom  
1553 boundary, and should be interpreted with caution. . . . . 87
- 1554 **Fig. 13.** As in Fig. 11, but turbulent vertical momentum fluxes projected onto the shear, i.e.  $(\mathbf{F}_m \cdot$   
1555  $\partial \mathbf{u}_h / \partial z) / |\partial \mathbf{u}_h / \partial z|$ . . . . . 88
- 1556 **Fig. 14.** In (a), the depth  $z_{max}$  of maximum daily mean turbulent heat flux is related to the depth  
1557  $H_{Rib}$  at which the bulk Richardson number is 0.2. And in (b), the daily maximum depth  
1558  $z_{pen}$  to which DCT penetrates ( $\epsilon > 2 \times 10^{-8} \text{ m}^2/\text{s}^3$ ) is related to the low-gradient Richardson  
1559 layer depth  $H_{Rig}$  (above which  $Ri_g < 0.35$ ). . . . . 89
- 1560 **Fig. 15.** Relationships between various terms in the daily mean turbulent kinetic energy budget  
1561 at the depth  $z_{max}$  where the downward turbulent buoyancy flux is maximum ( $\langle SP \rangle^{max}$   
1562  $+ \langle T \rangle^{max} \approx \langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}$ ; see the Appendix for details). The depths  $z_{max}$  are plotted  
1563 as + symbols in Fig. 10. Buoyancy flux  $\langle F_b \rangle^{max}$  is plotted against (a) shear production  
1564 over buoyancy flux plus dissipation  $\langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$  and (b) shear production  
1565 over buoyancy flux (i.e., the inverse flux Richardson number  $Ri_f^{-1} = \langle SP \rangle^{max} / \langle F_b \rangle^{max}$ ).  
1566 The inverse gradient Richardson number of the horizontally-averaged profile  $Ri_g^{-1} =$   
1567  $\langle |\partial \mathbf{u}_h / \partial z|^2 \rangle^{max} / \langle \partial b / \partial z \rangle^{max} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$  is shown in color on all four panels and  
1568 on the y axes in (c)-(d) against  $Ri_f^{-1}$  (c) and  $Pr_t^{-1} = Ri_f / Ri_g$  (d) (the inverse turbulent Prandtl  
1569 number  $Pr_t^{-1}$  is the ratio of the turbulent diffusivity of buoyancy over the turbulent viscosity of  
1570 momentum). The thick black line (c) is the 1-1 line, the thin solid line is a fit to LES of a coastal  
1571 boundary layer under a hurricane by Watkins and Whitt (2020), and the thin dashed line is a  
1572 fit to atmospheric boundary layer observations by Anderson (2009), which parameterizes the  
1573 subgrid-scale  $Pr_t^{-1}$  in the LES. The two days with most anomalously low  $Ri_f^{-1}$  (b-c;  $Ri_f^{-1} = 0.9$   
1574 and 1.6) and high  $Pr_t^{-1}$  (d;  $Pr_t^{-1} = 0.4$  and 1.8) also have the largest relative non-local sources  
1575 of turbulent kinetic energy  $\langle T \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}) \approx 1 - \langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$   
1576 (i.e., the points with lowest values in a; 0.3 and 0.6). Plus (+) symbols are from LES at  $0^\circ \text{ N}$   
1577 and circles (o) from  $3^\circ \text{ N}$ . . . . . 90
- 1578 **Fig. 16.** Relationship between  $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$  and  $\langle F_b \rangle^{max}$  (a) and  $K_b = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$   
1579 (b) at  $z_{max}$  (i.e., at the depths indicated by the + symbols in Fig. 10). Averaging diffusivity  
1580 directly in (b) yields quantitatively different results but qualitatively the same conclusion  
1581 that  $K_b$  is at best weakly related to  $Ri_g$ . Overlaid in (b) are parameterizations of turbulent  
1582 diffusivity as a function of Richardson number from Pacanowski and Philander (1981) (PP)  
1583 Peters et al. (1988) (PGT), and Large and Gent (1999) (KPP). . . . . 91
- 1584 **Fig. 17.** Maximum daily mean turbulent buoyancy flux  $\langle F_b \rangle^{max}$  scales with oceanic bulk vertical  
1585 shear  $S_b$  (a,c) and even more closely with a product of  $S_b$  and the magnitude of the surface  
1586 wind stress  $|\tau| = u_*^2 \rho$  (b,d). The scalings are obtained via linear regression on the LES output

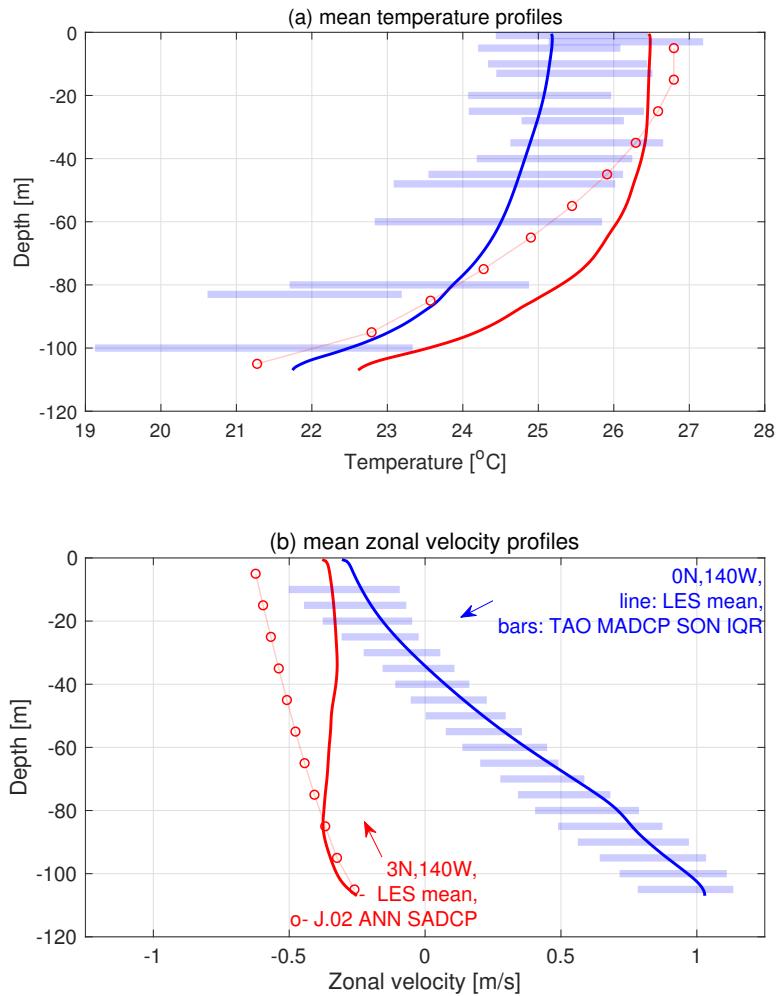
1587 in (a)-(b), which includes 34 days at 3° N (black ○) and 34 days at 0° N (black +), or on  
 1588 the 68 days of LES output plus 38 days of TIWE data (blue \*) in (c)-(d). Hence, the TIWE  
 1589 observations serve as an independent validation of the regressions in (a)-(b) and constrain  
 1590 the regressions in (c)-(d). The predictors include  $S_b$ , which is derived from a linear fit to  
 1591 the mean velocity from  $H_{Rig}$  to 5 m depth (thick black lines in Fig. 4), and the friction  
 1592 velocity  $u_* = \sqrt{|\tau|/\rho}$ . All variables are log-transformed and Pearson's  $r$  in the panel titles is  
 1593 calculated in log space. The various diagonal black lines indicate where the data are along  
 1594 the 1-1 line, within a factor of 2, and within a factor of 3. With 95% confidence intervals, the  
 1595 scalings are as follows:  $(2-6) \times 10^{-6} |S_b|^{(0.7-1.0)}$  (a),  $(1-200) \times 10^{-2} |S_b|^{(0.9-1.1)} u_*^{(1.6-2.5)}$   
 1596 (b),  $(2-6) \times 10^{-6} |S_b|^{(0.8-1.0)}$  (c), and  $(0.03-1.3) \times 10^{-2} |S_b|^{(0.8-1.0)} u_*^{(0.9-1.6)}$  (d). . . . . 92

1597 **Fig. 18.** Various ratios of terms in Eqn. (6) showing how the local energetics of the buoyancy flux  
 1598 at  $z_{max}$  (Fig. 15) relate to the bulk scalings derived via regression (Fig. 17). Circles (○)  
 1599 are from the LES at 3° N, and pluses (+) are from the LES at 0° N; the color indicates  
 1600  $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ . . . . . 93

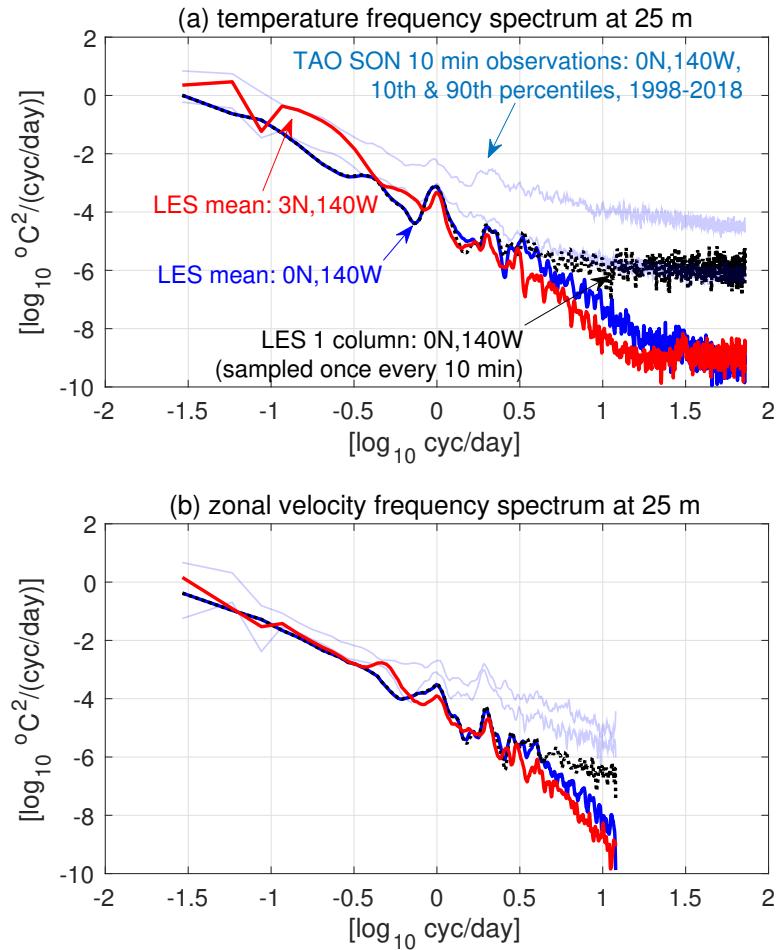
1601 **Fig. 19.** Comparisons between the LES and ROMS (KPP) at the LES locations (+ at 0° N and ○ at 3° N  
 1602 along 140° W): (a) The maximum turbulent heat flux  $\langle F_Q \rangle^{max}$ , (b) the turbulent diffusivity of  
 1603 heat  $K$  at  $z_{max}$ , and (c) the depth  $z_{max}$  at which  $\langle F_Q \rangle^{max}$  occurs. Note, however, that the LES  
 1604 results are derived from the buoyancy dynamics whereas the ROMS results are derived from  
 1605 the temperature dynamics. That is, the LES results are  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 \langle F_b \rangle \approx \langle F_Q \rangle$   
 1606 [W/m<sup>2</sup>] in (a) and  $K = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$  in (b), and  $z_{max}$  is calculated from from  $\langle F_b \rangle$   
 1607 profiles. . . . . 94

1608 **Fig. 20.** Daily averaged net vertical heat flux  $\langle Q \rangle$  (including turbulent  $F_Q$  as in Fig. 10 plus penetrative  
 1609 radiative  $P_Q$  components) at 0° N, 140° W (left column) and 3° N, 140° W (right column) as  
 1610 simulated by the LES [(a)-(b)] and as parameterized based on horizontally-averaged velocity  
 1611 and density profiles and net surface buoyancy and momentum fluxes [(c)-(d)]. For reference,  
 1612 the the piecewise linear flux profiles with  $\langle Q \rangle(z=0)$  and  $\langle Q \rangle(z=z_{max})$  from LES are  
 1613 shown in e-f. In addition, the vertical heat fluxes (penetrating shortwave plus turbulent)  
 1614 from the parent ROMS model are shown in the bottom row [(e)-(f)]. Note the different  
 1615 colorbar ranges in the left and right columns. For consistency with earlier results, (a)-(f)  
 1616 plot  $(\rho c_p / g \alpha) \langle B \rangle \approx \langle Q \rangle$  where  $\langle B \rangle$  is the daily-averaged vertical buoyancy flux including  
 1617 the parts due to turbulence and penetrative shortwave radiation. . . . . 95

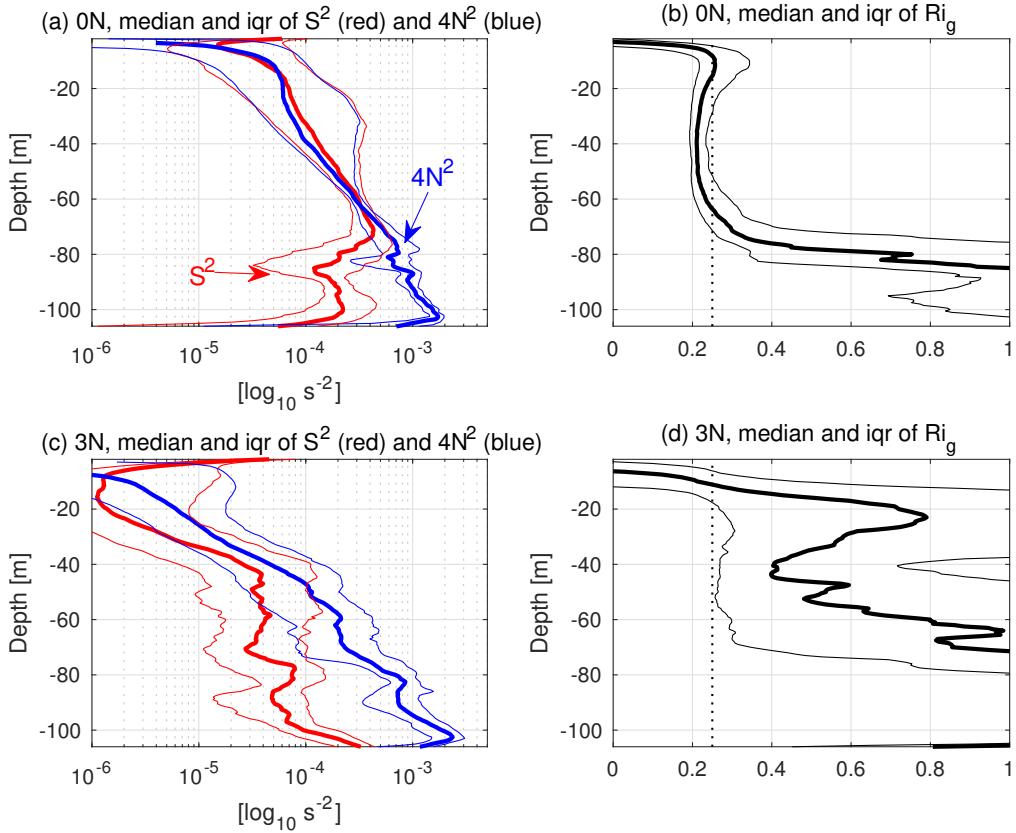
1618 **Fig. A1.** Time-means of various terms in the horizontally-averaged tracer and momentum budgets  
 1619 from ROMS (solid lines) and LES (dashed lines) at 0° N, 140° W (top) and 3° N, 140° W  
 1620 (bottom). The blue lines represent the time-mean convergence of vertical transport of (a,e)  
 1621 temperature, (b,f) zonal momentum and (c,g) meridional momentum and (d,h) salinity due  
 1622 to turbulence (and solar radiation in the case of temperature). The black lines represent all  
 1623 other tendencies of horizontally-averaged momentum and tracers as diagnosed from ROMS,  
 1624 i.e.  $\mathcal{F}$  (plus Coriolis in the case of momentum), and as diagnosed in LES, i.e.  $\mathcal{F} + \mathcal{R}$  (plus  
 1625 Coriolis in the case of momentum). See the Appendix for the budget formulas. . . . . 96



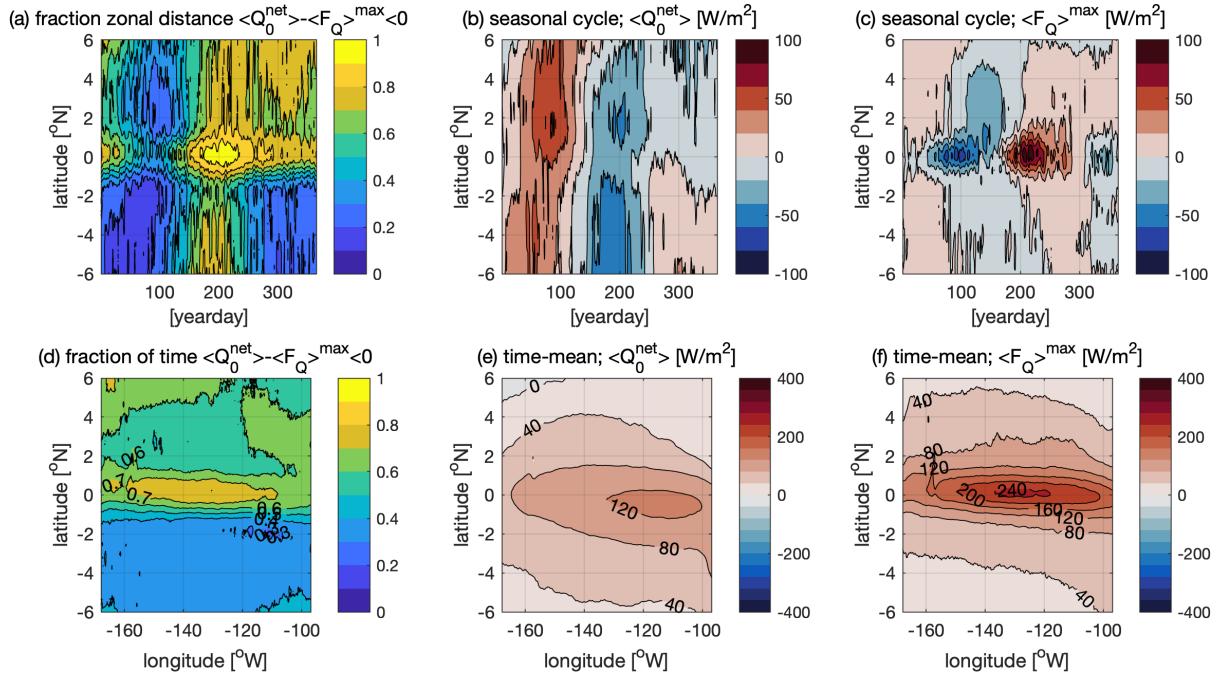
1626 FIG. 1. A comparison between the simulated (LES, solid lines) and observed mean temperature (a) and zonal  
 1627 velocity (b) profiles at 0° (blue) and 3° N (red) along 140° W. At 0° N, 140° W, the observations (horizontal bars)  
 1628 span the inter-quartile ranges of all monthly means (September-October-November only) from the TAO mooring  
 1629 (1988-2018). At 3° N, 140° W, a ship-based annual climatology is plotted (Johnson et al. 2002), but these are  
 1630 more for reference than for validation since there is significant seasonal and inter-annual variability.



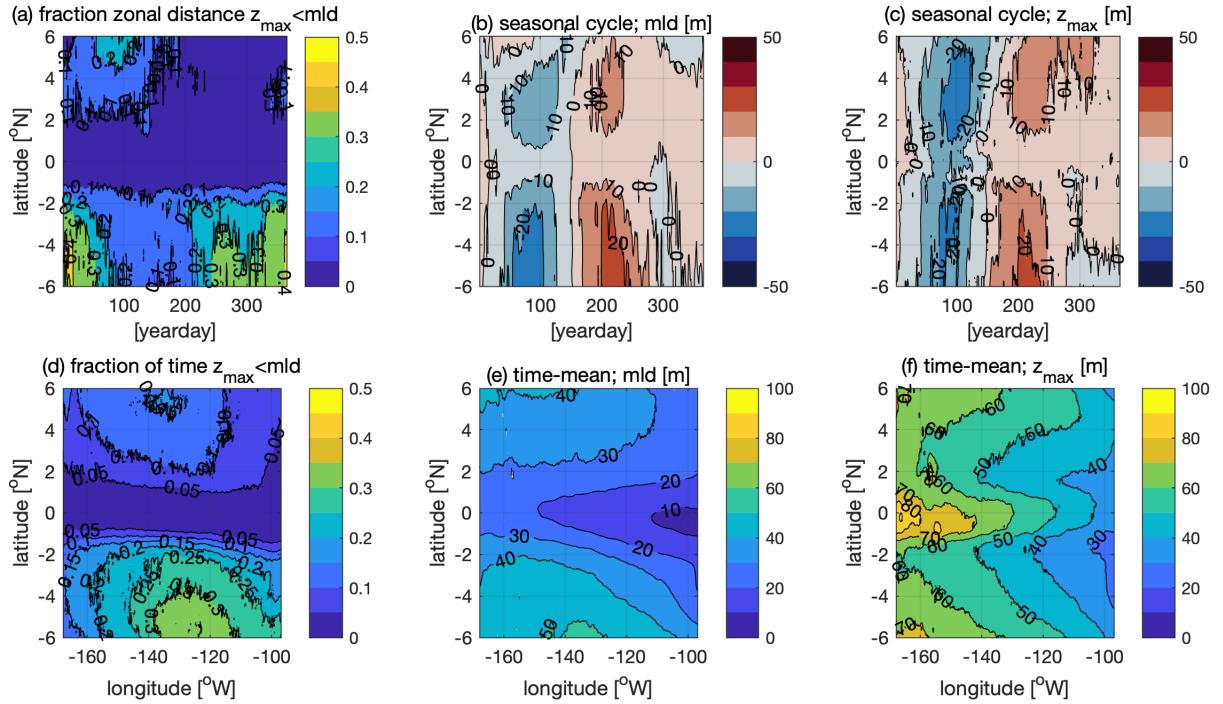
1631 FIG. 2. Simulated (LES) and observed frequency spectra of temperature (a) and zonal velocity (b) at 25 m depth  
 1632 at 0° (blue) and 3° N (red) along 140° W. Observed spectra are calculated from the moored temperature sensor  
 1633 (10 minute instantaneous sampling) and current meter (1 hour average sampling) from the months September-  
 1634 October-November on the TAO mooring at 0° N, 140° W for comparison (1988-2018). The observed spectra  
 1635 are calculated in overlapping time windows that are the same length as the LES simulations (with 17% of points  
 1636 overlapped in each window). The 10% and 90% quantile at each frequency (across all of the spectra windows)  
 1637 is plotted in light blue. The black dotted and blue lines are derived from LES: the sampling is instantaneous  
 1638 (averaged over a single time step) every 10 min (a) or 1 hour (b) and averaged spatially over a single grid  
 1639 cell/virtual mooring (black dotted) or the entire horizontal extent of the domain (blue). The spectrum from the  
 1640 virtual mooring (black) flattens similarly to the observations from the TAO mooring at frequencies higher than  
 1641 3 cyc/day due to aliasing in (a).



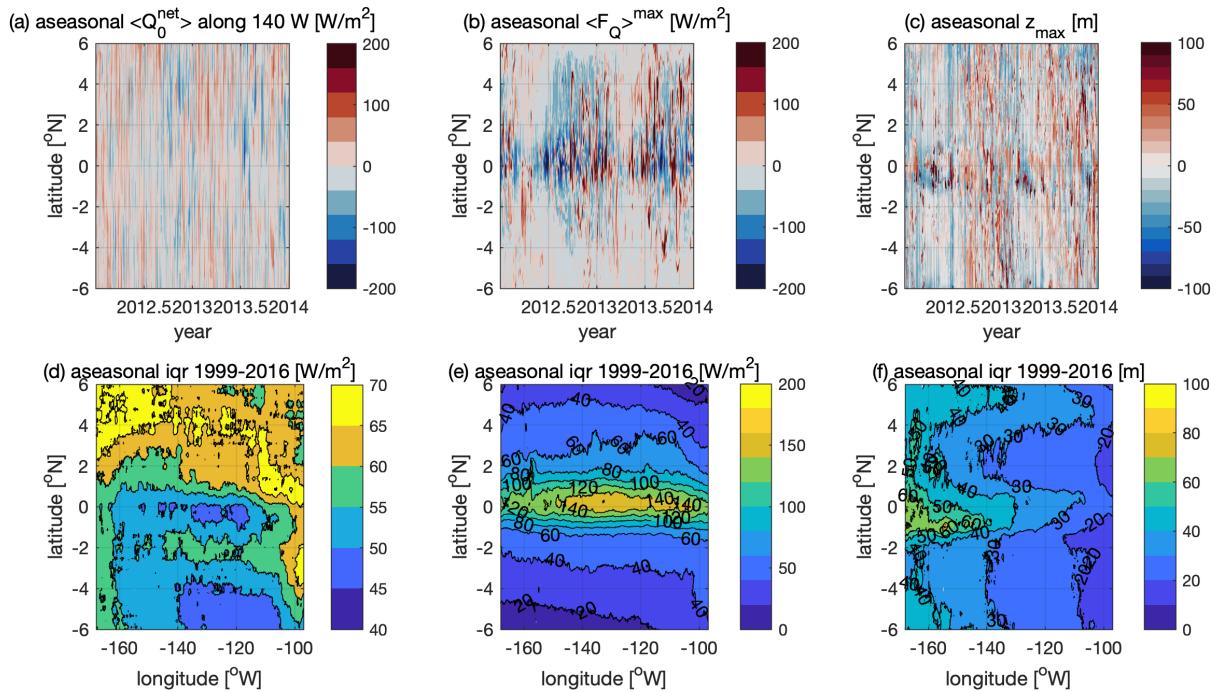
1642 FIG. 3. Profiles of the median (thick lines) and inter-quartile range (iqr, thin lines) of the squared vertical shear  
 1643 of horizontal velocity  $S^2$ , the vertical buoyancy gradient  $N^2$ , and the gradient Richardson number  $Ri_g = N^2/S^2$   
 1644 (all of the horizontally-averaged profiles). The top row show results from the LES at  $0^\circ\text{N}$  and the bottom row  
 1645 the results from the LES at  $3^\circ\text{N}$ . The dotted vertical line in (b) and (d) indicates  $Ri_g = 0.25$  for reference.



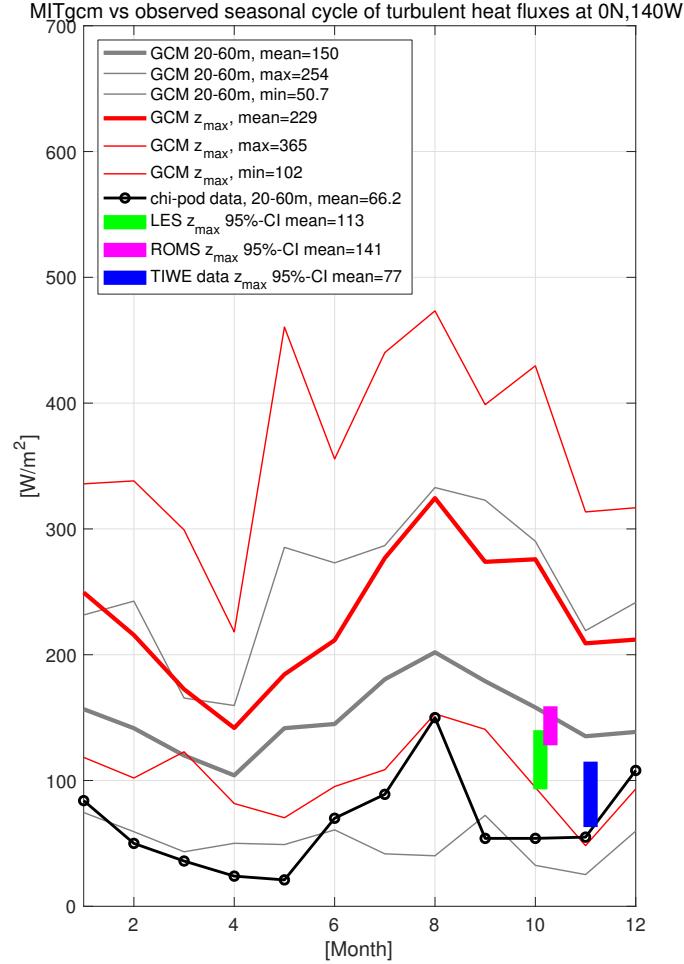
1646 FIG. 4. Climatological spatial structure and seasonal cycle of downward heat fluxes in a regional ocean model  
 1647 of the equatorial Pacific Ocean cold tongue forced by atmospheric reanalysis from 1999-2016. The net air-sea  
 1648 flux  $\langle Q_0^{net} \rangle$  is in (b) and (e), and the maximum flux due to ocean mixing  $\langle F_Q \rangle^{max}$  is in (c) and (f). b-c are  
 1649 the zonal means from 95-170° W with the time-mean subtracted, and e-f are the time-means. In addition, we  
 1650 quantify the fraction of the zonal distance (a) and time (d) over which there is net cooling of the surface ocean  
 1651 due to air-sea exchange and ocean mixing, that is  $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$ . The flux due to ocean mixing  $\langle F_Q \rangle^{max}$   
 1652 (c,f) is defined as the maximum (over depth) of the daily-mean downward turbulent heat flux, so the zonal and  
 1653 time means are calculated at a depth that varies in time and space that is plotted in Fig. 5.



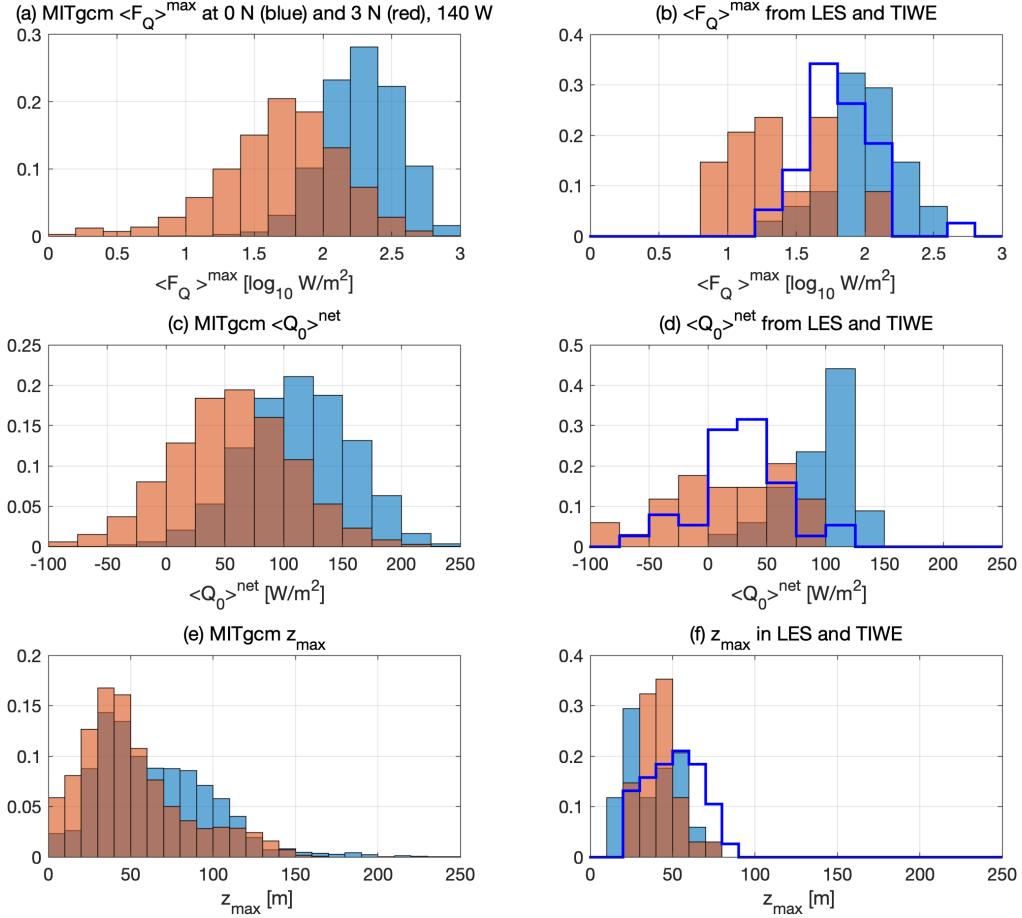
1654 FIG. 5. Climatological comparisons between mixed layer depth (MLD, b,e) and the depth  $z_{max}$  (c,f) where the  
 1655 downward turbulent heat flux is maximum (i.e., the depth where  $\langle F_Q \rangle^{max}$  plotted in Fig. 4 occurs). As in Fig. 4,  
 1656 b-c are the zonal mean anomalies from the time mean, and e-f are the time-means. In addition, we quantify the  
 1657 fraction of the zonal distance (a) and time (d) over which the the MLD is deeper than  $z_{max}$ . The MLD is defined  
 1658 to be the shallowest depth where water is  $0.015 \text{ kg/m}^3$  denser than the top 10 m in the daily-mean density profile  
 1659 (since higher-frequency output is not available).



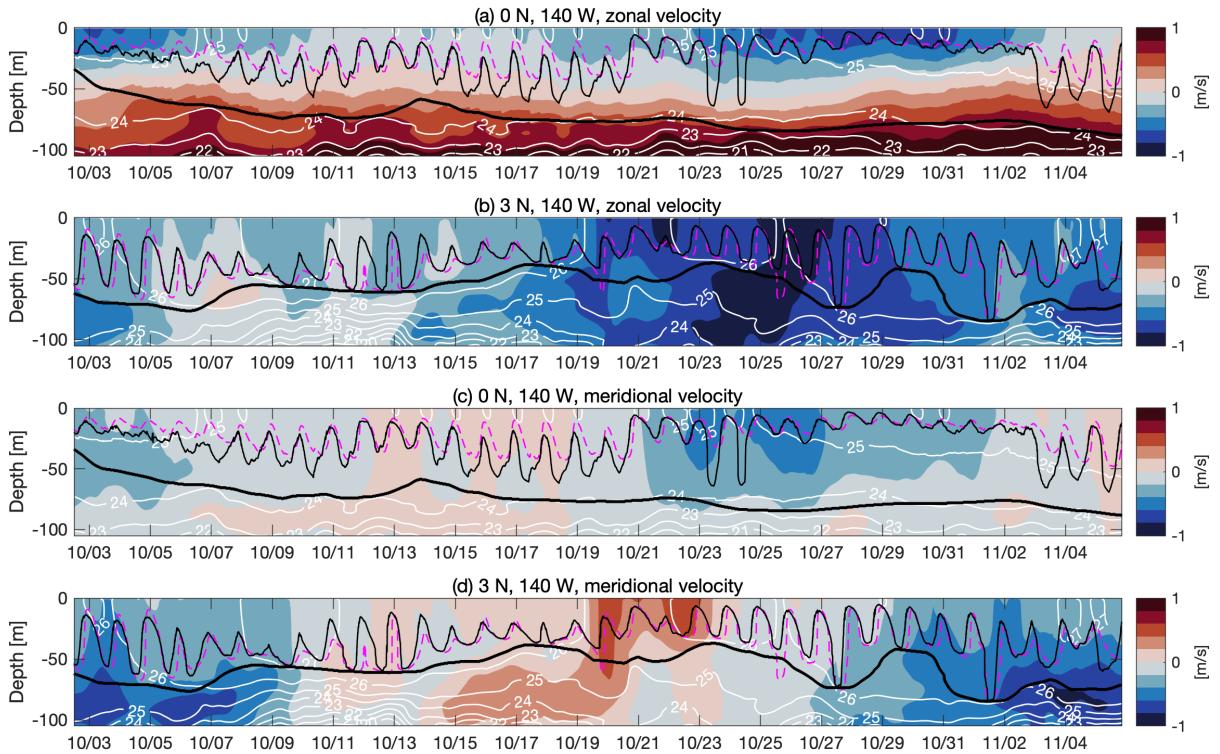
1660 FIG. 6. The top row shows the hindcast aseasonal daily-mean vertical heat fluxes during 2012 and 2013 along  
 1661 the 140° W meridian (a: net surface flux  $\langle Q_0^{net} \rangle$ , b: ocean mixing  $\langle F_Q \rangle^{max}$  and c: the depth where strongest  
 1662 mixing occurs  $z_{max}$ ). Maps (d-f) quantify the respective aseasonal inter-quartile ranges over all latitudes and  
 1663 years 1999-2016. Aseasonal variability is defined by subtracting the mean seasonal cycle (i.e., a daily annual  
 1664 climatology, which is averaged over 18 years and then smoothed with a 15-day moving average), from the total  
 1665 signal at each grid point.



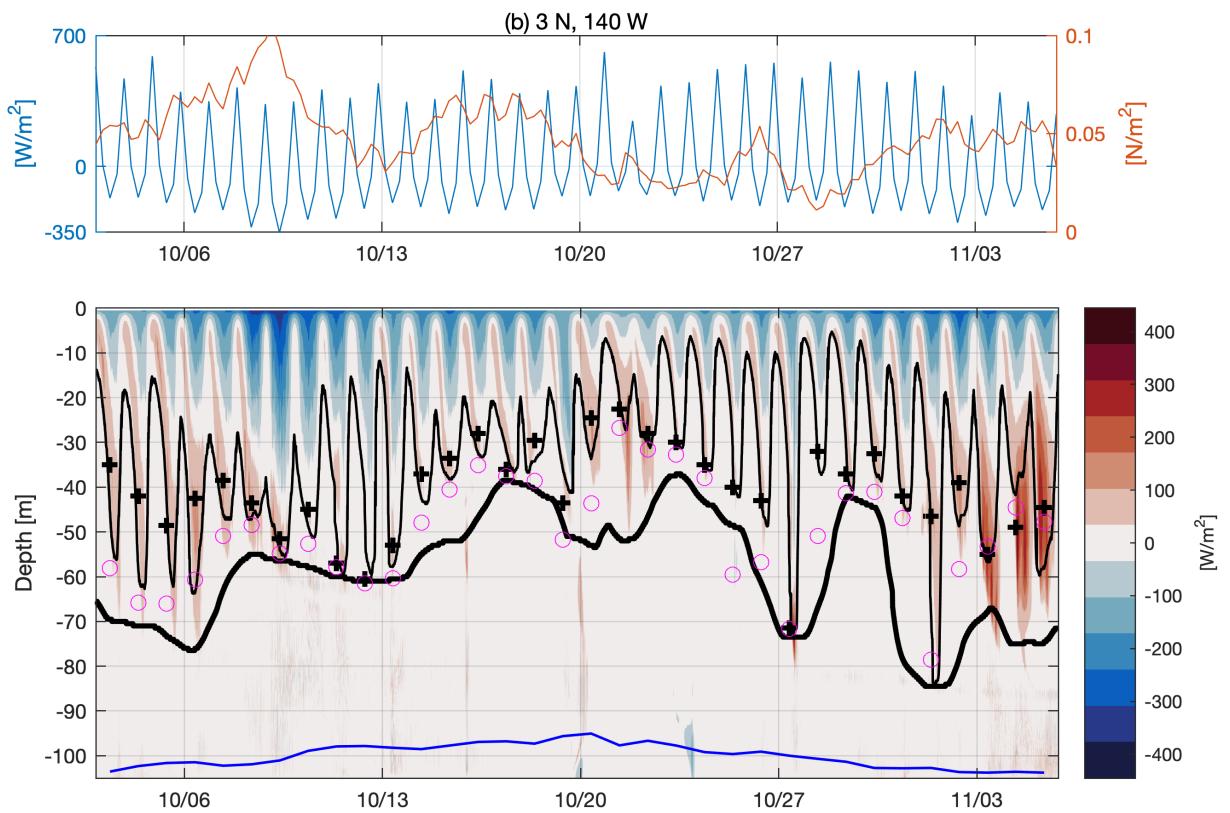
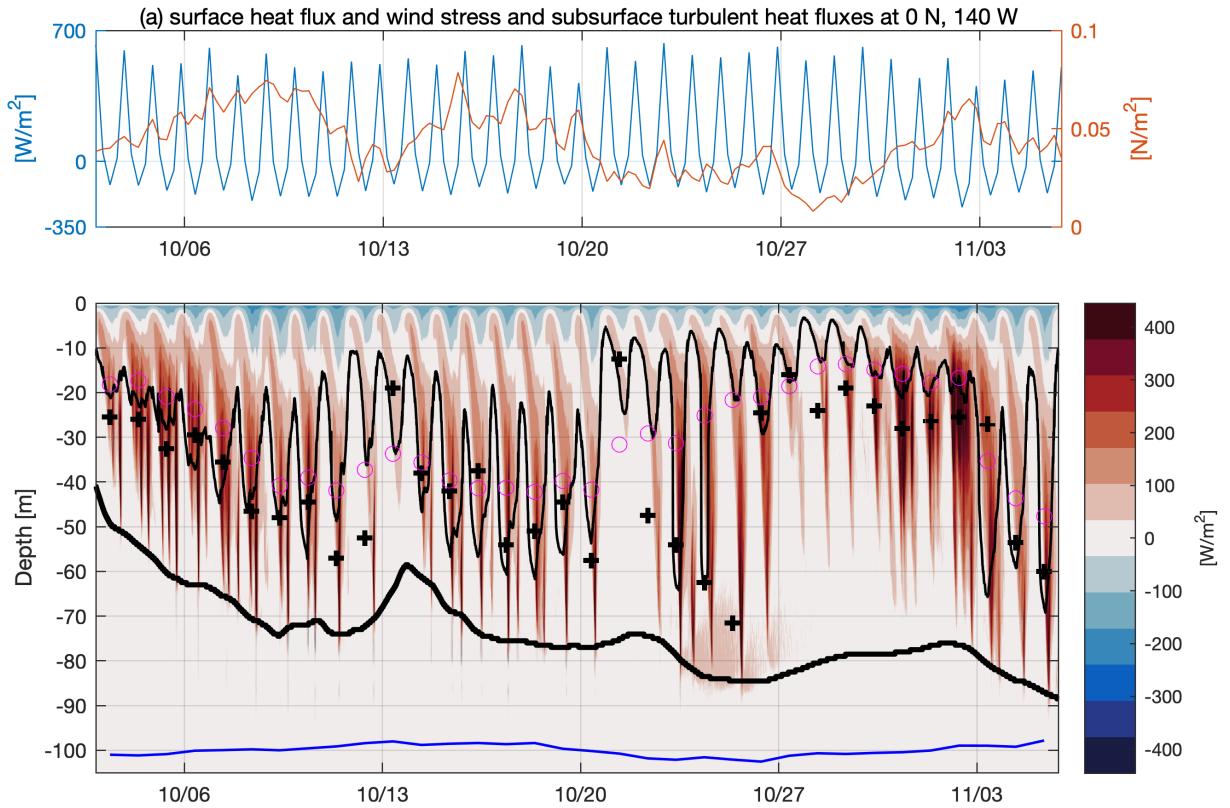
1666 FIG. 7. Climatological annual cycle of the downward turbulent heat flux at 0° N, 140° W in the MITgcm  
 1667 regional ocean model, including monthly means at  $z_{max}$  ( $\langle F_Q \rangle^{max}$ , thick red) as well as monthly means from  
 1668 20-60 m depth  $\langle F_Q \rangle^{20-60}$  (thick gray). Corresponding minima and maxima of monthly  $\langle F_Q \rangle^{20-60}$  (thin gray) and  
 1669  $\langle F_Q \rangle^{max}$  (thin red) from 1999-2016 are included. For comparison, the observational climatology of  $\langle F_Q \rangle^{20-60}$   
 1670 from chipods (Moum et al. 2013) is plotted in black circles. The 95% confidence intervals for the monthly mean  
 1671  $\langle F_Q \rangle^{max}$  from ROMS and LES (roughly October 1985) as well as the TIWE observations (roughly November  
 1672 1991) are in magenta, green and blue respectively. Note, however, that the LES and TIWE are computed  
 1673 as  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$  [W/m<sup>2</sup>] where  $\rho$ ,  $c_p$ , and  $\alpha$  are the reference density, specific heat, and  
 1674 thermal expansion coefficient of seawater, respectively,  $g$  is the acceleration due to gravity, and  $F_b$  is the downward  
 1675 turbulent buoyancy flux. Data from two other shorter field experiments (not shown) resulted in means of roughly  
 1676 400 W/m<sup>2</sup> in Oct/Nov 2008 (Moum et al. 2009) and 100 W/m<sup>2</sup> in Nov 1984 (Gregg et al. 1985; Moum and  
 1677 Caldwell 1985) (see Fig. 2d of Moum et al. 2009).



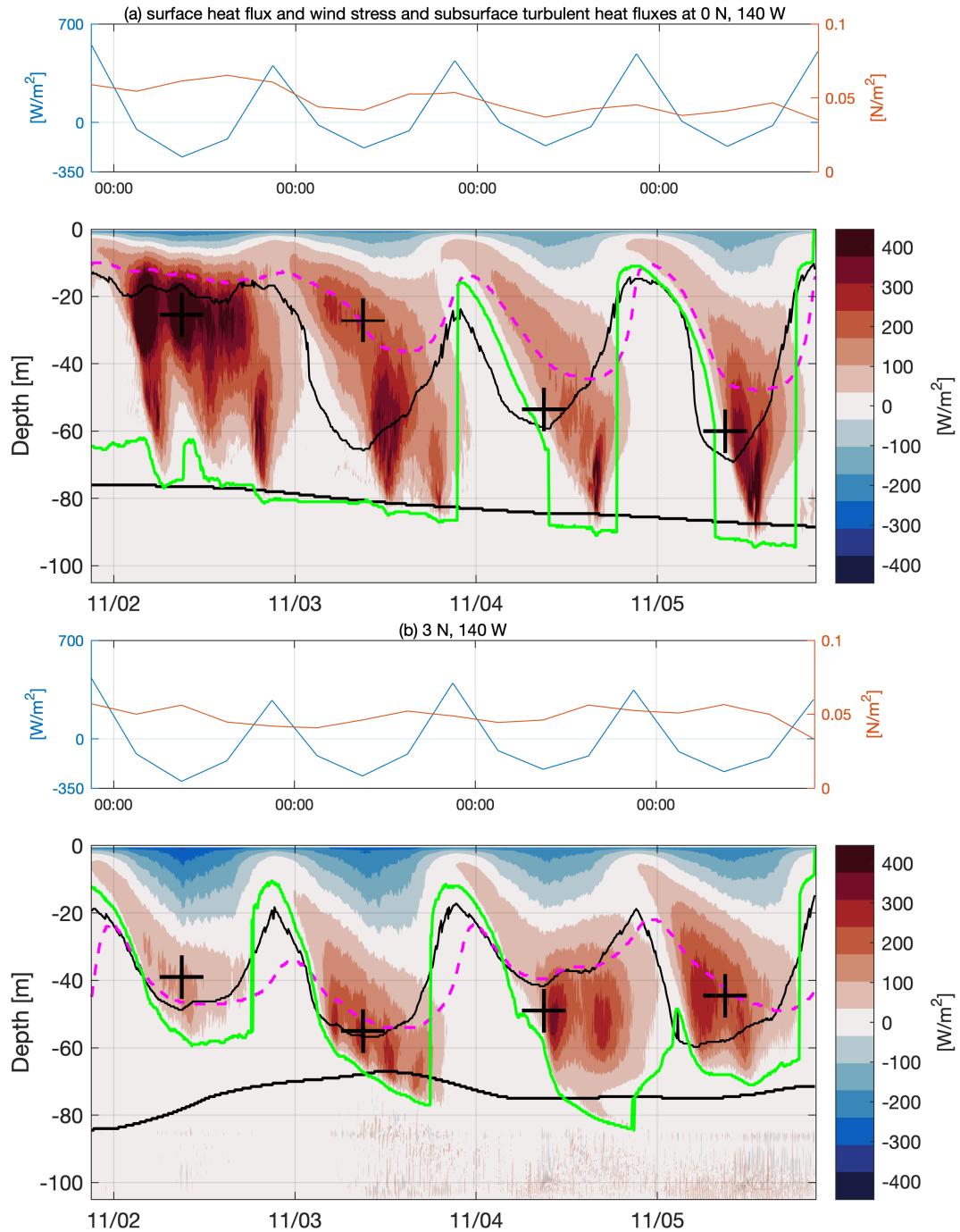
1678 FIG. 8. Relative probability distributions of the maximum daily-mean turbulent heat flux due to ocean mixing  
 1679  $\langle F_Q \rangle^{max}$  (a-b), the daily-mean net surface heat flux  $\langle Q_0^{net} \rangle$  (c-d), and the depth  $z_{max}$  at which  $\langle F_Q \rangle^{max}$   
 1680 occurs (e-f). Histograms are included for both  $0^\circ$  N,  $140^\circ$  W (blue) and  $3^\circ$  N,  $140^\circ$  W (red) for the 18-year MITgcm  
 1681 simulation (left column) as well as the 34-day LES in October 1985 (red and blue histograms) and the 38-day  
 1682 TIWE experiment at  $0^\circ$  N,  $140^\circ$  W in November 1991 (right column, dark-blue edged bars). Note that the data  
 1683 from LES and TIWE are computed based on buoyancy fluxes, e.g.  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$ .



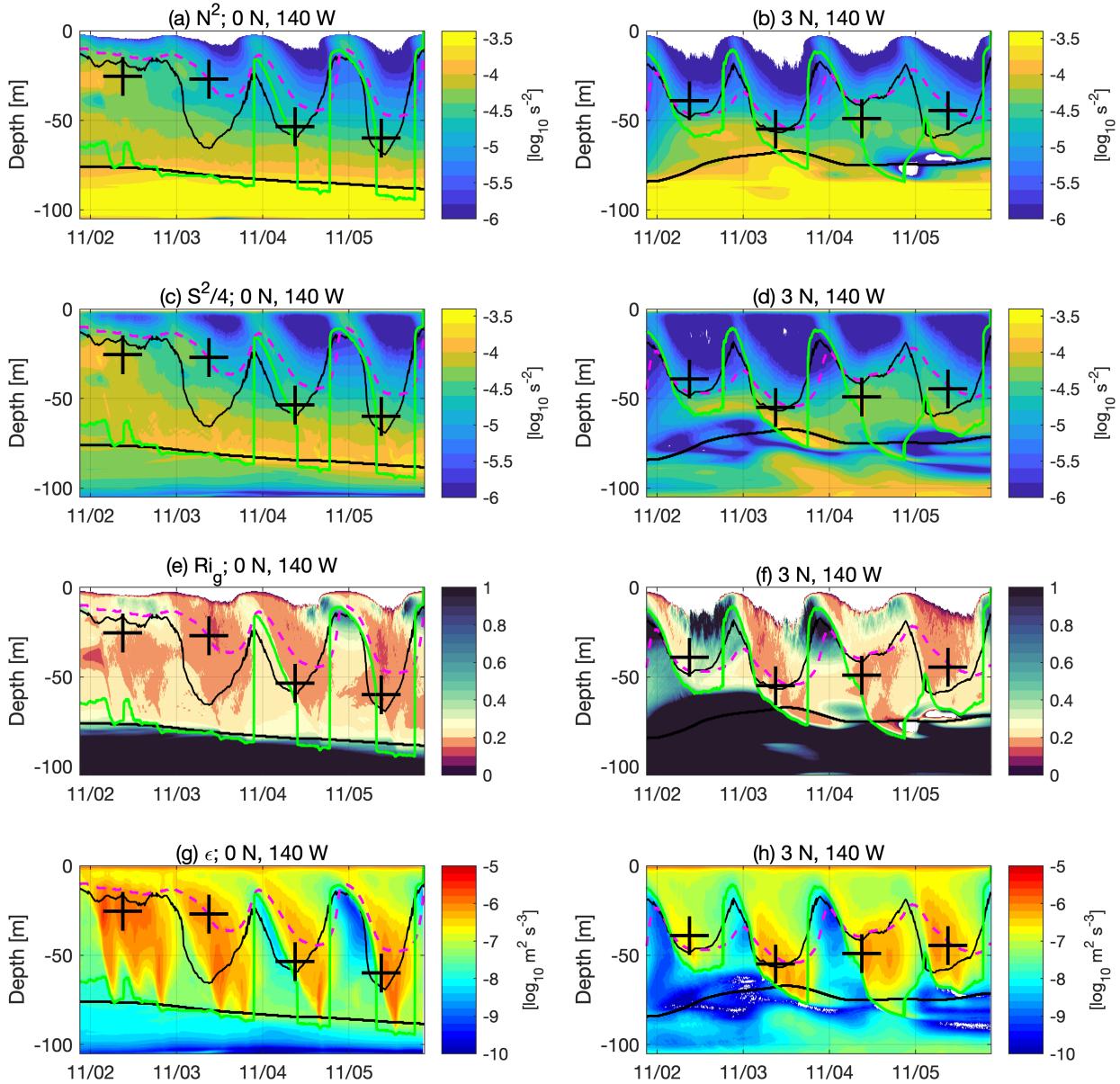
1684 FIG. 9. Time series of zonal and meridional velocity (color), temperature (white contours in  $^{\circ}\text{C}$ ), mixed layer  
 1685 depth (MLD, dashed magenta), the depth where the bulk Richardson number  $Ri_b = 0.2$  ( $H_{Rib}$ , thin black), and  
 1686 the base of the low-gradient Richardson number layer  $Ri_g < 0.35$  ( $H_{Rig}$ , thick black) in the LES at  $0^{\circ}$  N and  $3^{\circ}$  N  
 1687 along  $140^{\circ}$  W. All fields are defined from horizontally-averaged profiles. The MLD is defined to be the shallowest  
 1688 depth where water is  $0.015 \text{ kg/m}^3$  denser than the top 10 m in the instantaneous but horizontally-averaged density  
 1689 profile. All time tick marks are at 0 UTC; local solar time at  $0^{\circ}\text{N}$ ,  $140^{\circ}$  W is about 9 hours behind UTC, so local  
 1690 solar noon is at about 21 UTC.



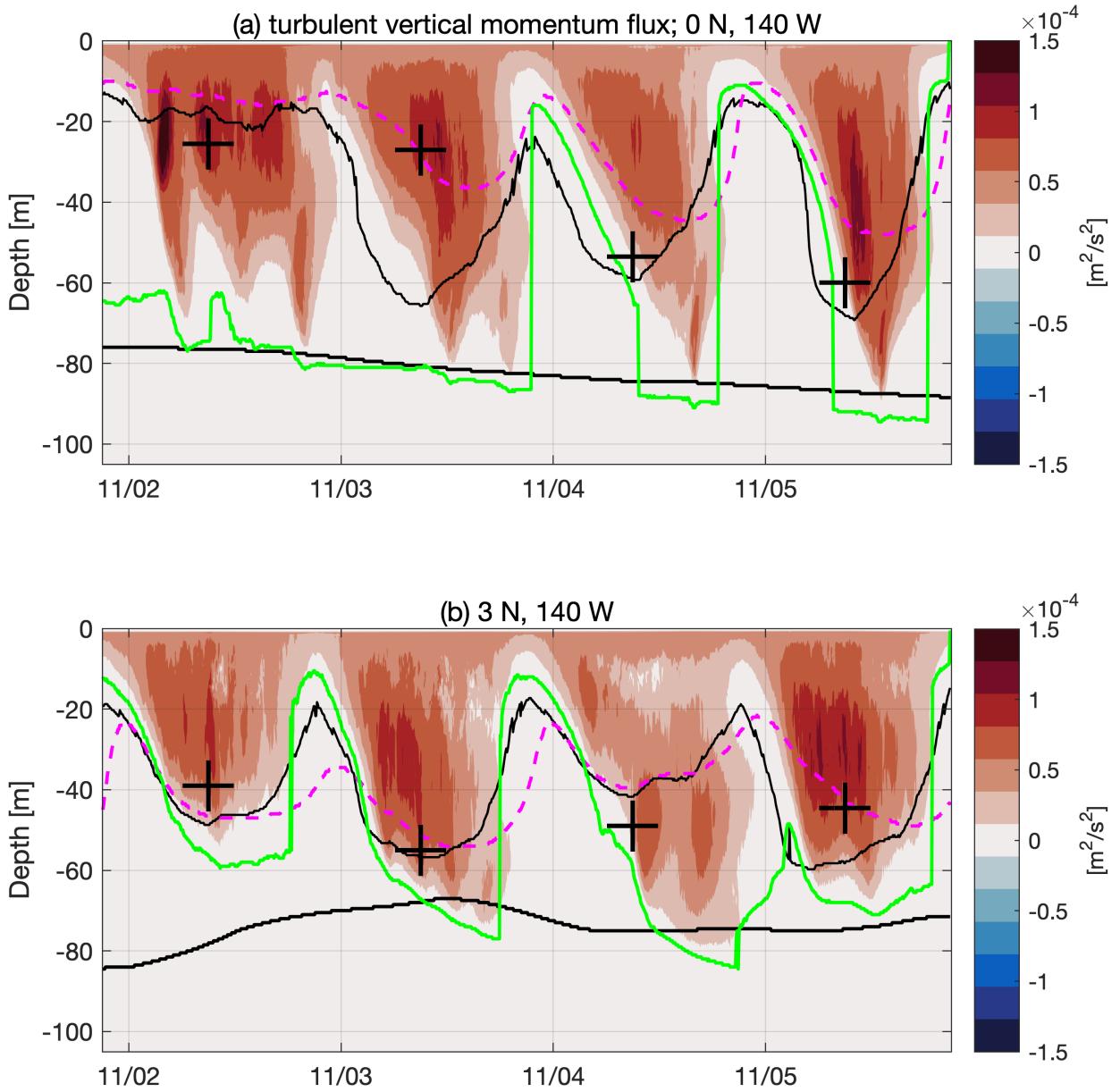
1691 FIG. 10. Time series of the net surface heat flux  $Q_0^{net}$  (left axis, blue), the magnitude of the wind stress  $|\tau|$   
 1692 (right axis, red), and the subsurface downward turbulent heat flux  $F_Q$  profiles from October-November 1985 in  
 1693 the LES at  $0^\circ$  N (a) and  $3^\circ$  N (b) along  $140^\circ$  W. Overlaid on  $F_Q$  are the depth at which the bulk Richardson number  
 1694  $Ri_b = 0.2$  ( $H_{Rib}$ , thin black line), the depth of the maximum daily-mean downward heat flux  $z_{max}$  (+ symbols),  
 1695 the daily maximum MLD (defined from the horizontally averaged LES density profiles; magenta circles), and  
 1696 the base of the low gradient Richardson layer  $Ri_g < 0.35$  ( $H_{Rig}$ , thick black line). The daily-mean meridional  
 1697 velocity averaged from 25 to 75 m depth is in blue; the origin is at a depth of 100 m, a 1 m spacing corresponds to  
 1698 10 cm/s, and the peak-to-trough amplitudes are about 40 cm/s at  $0^\circ$  N and 90 cm/s at  $3^\circ$  N. For consistency with  
 1699 other results in section 4, we plot  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$  [W/m<sup>2</sup>] where  $\rho$ ,  $c_p$ , and  $\alpha$  are the reference  
 1700 density, specific heat, and thermal expansion coefficient of seawater, respectively,  $g$  is the acceleration due to  
 1701 gravity, and  $F_b$  is the downward turbulent buoyancy flux. All time tick marks are at 0 UTC, but local solar time  
 1702 at  $0^\circ$ N,  $140^\circ$  W is about 9 hours behind UTC, so local solar noon is at about 21 UTC. Daily mean statistics (e.g.,  
 1703  $z_{max}$  indicated by + symbols) are calculated from 21 UTC so that the averages begin and end near solar noon.



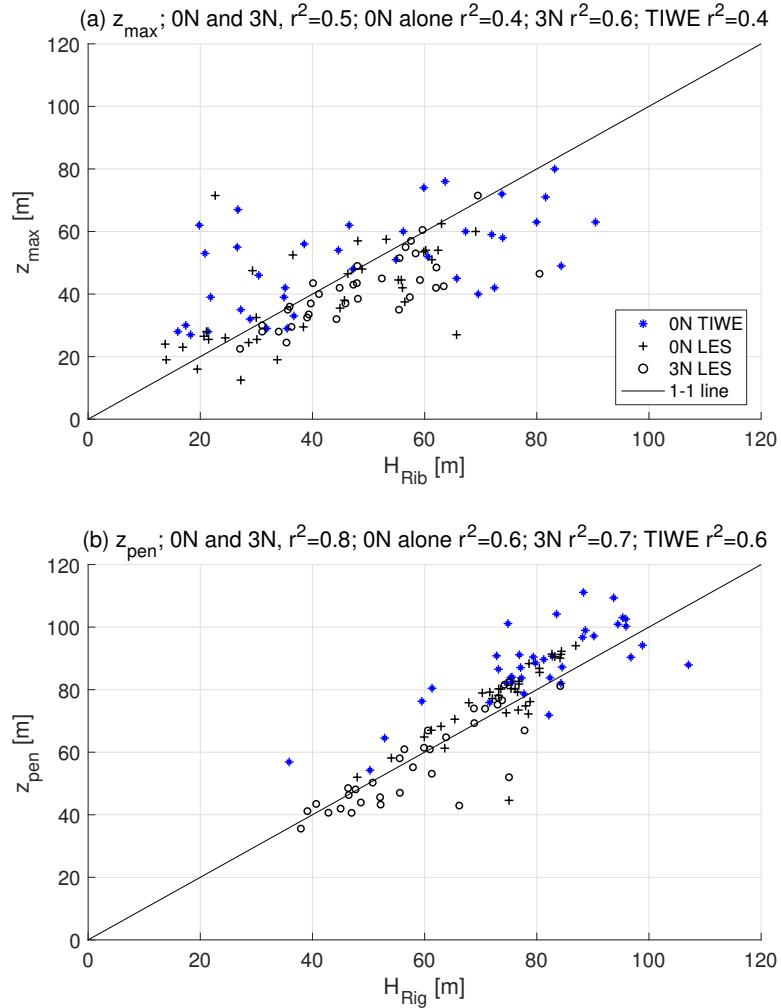
1704 FIG. 11. As in Fig. 10, but zoomed in on a few days in November and with the addition of the MLD  
 1705 (dashed magenta) and the DCT penetration depth  $z_{pen}$  ( $\epsilon \geq 2 \times 10^{-8} \text{ m}^2/\text{s}^3$ ; thin green). The MLD is defined  
 1706 to be the shallowest depth where water is  $0.015 \text{ kg/m}^3$  denser than the top 10 m in the instantaneous but  
 1707 horizontally-averaged density profile.



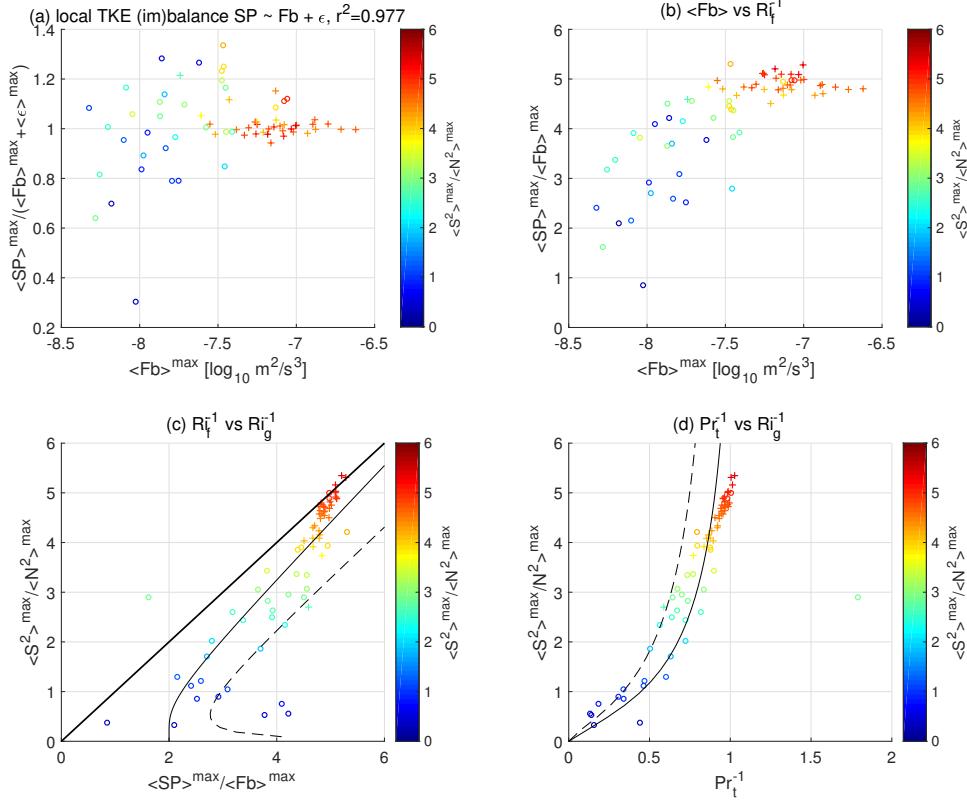
1708 FIG. 12. As in Fig. 11, but plots show (a)-(b) the vertical buoyancy gradient  $N^2$ , (c)-(d) the squared vertical  
 1709 shear  $S^2$ , (e)-(f)  $Ri_g = N^2/S^2$ , and (g)-(h) the rate of dissipation of kinetic energy  $\epsilon$ . It may be noted that there  
 1710 are a few instances of elevated dissipation  $10^{-8} < \epsilon < 10^{-7} \text{ m}^2/\text{s}^3$  below the deepest depths of DCT ( $z_{pen}$ , green  
 1711 line) in (h) where  $Ri_g > 1$ . However, these instances of elevated dissipation near the bottom are dominated  
 1712 by dissipation of the mean-flow kinetic energy, and the turbulent fluxes and energetics depend strongly on the  
 1713 subgrid-scale parameterization in the LES (A6)-(A7), may be influenced by the bottom boundary, and should be  
 1714 interpreted with caution.



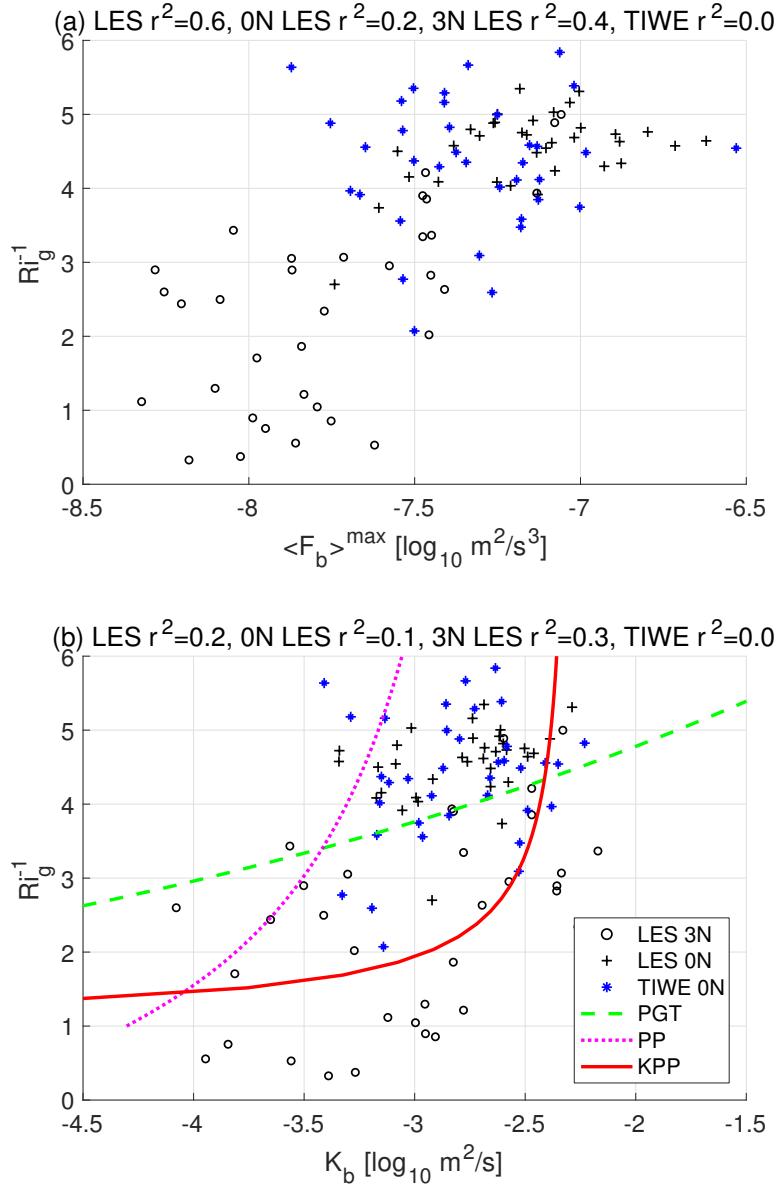
1715 FIG. 13. As in Fig. 11, but turbulent vertical momentum fluxes projected onto the shear, i.e.  $(\mathbf{F}_m \cdot$   
 1716  $\partial \mathbf{u}_h / \partial z) / |\partial \mathbf{u}_h / \partial z|$ .



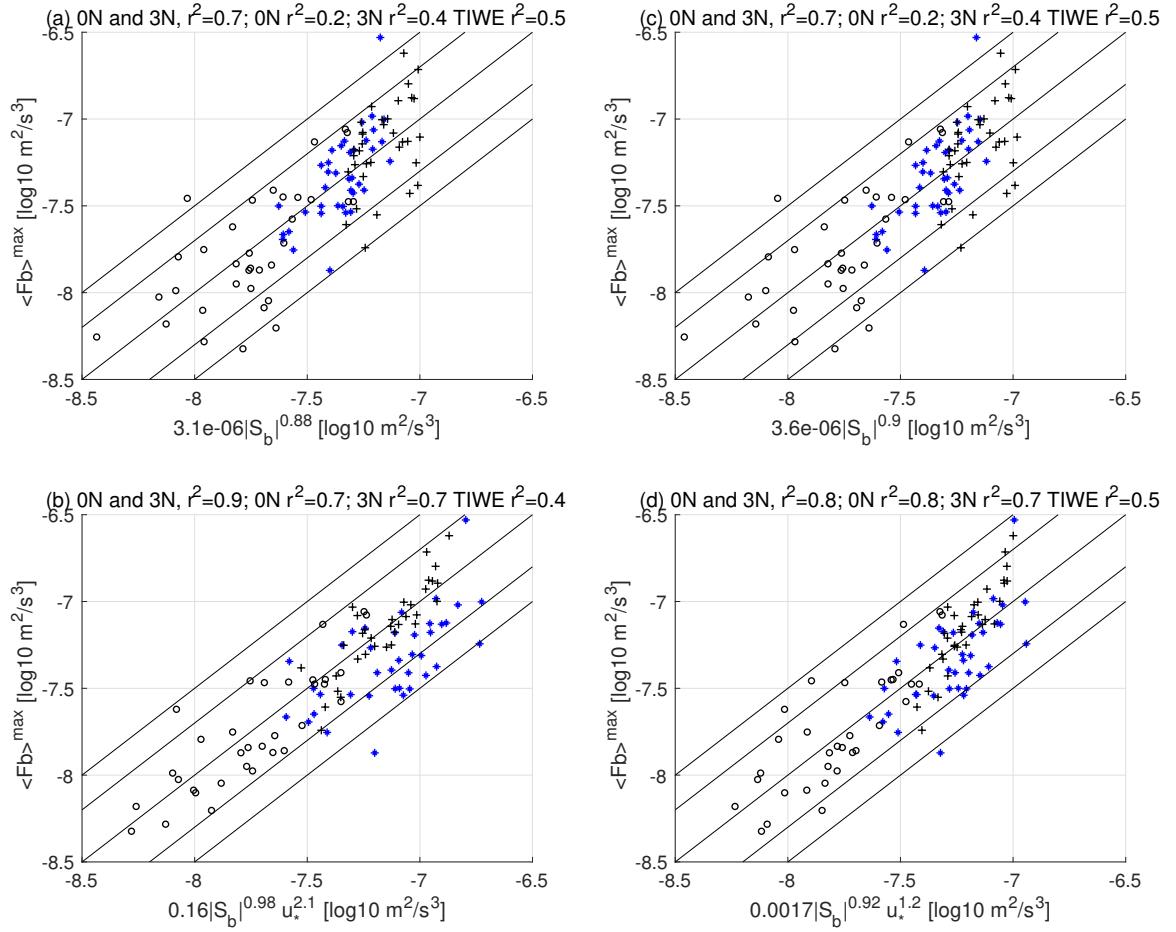
1717 FIG. 14. In (a), the depth  $z_{max}$  of maximum daily mean turbulent heat flux is related to the depth  $H_{Rib}$  at  
 1718 which the bulk Richardson number is 0.2. And in (b), the daily maximum depth  $z_{pen}$  to which DCT penetrates  
 1719 ( $\epsilon > 2 \times 10^{-8} \text{ m}^2/\text{s}^3$ ) is related to the low-gradient Richardson layer depth  $H_{Rig}$  (above which  $Ri_g < 0.35$ ).



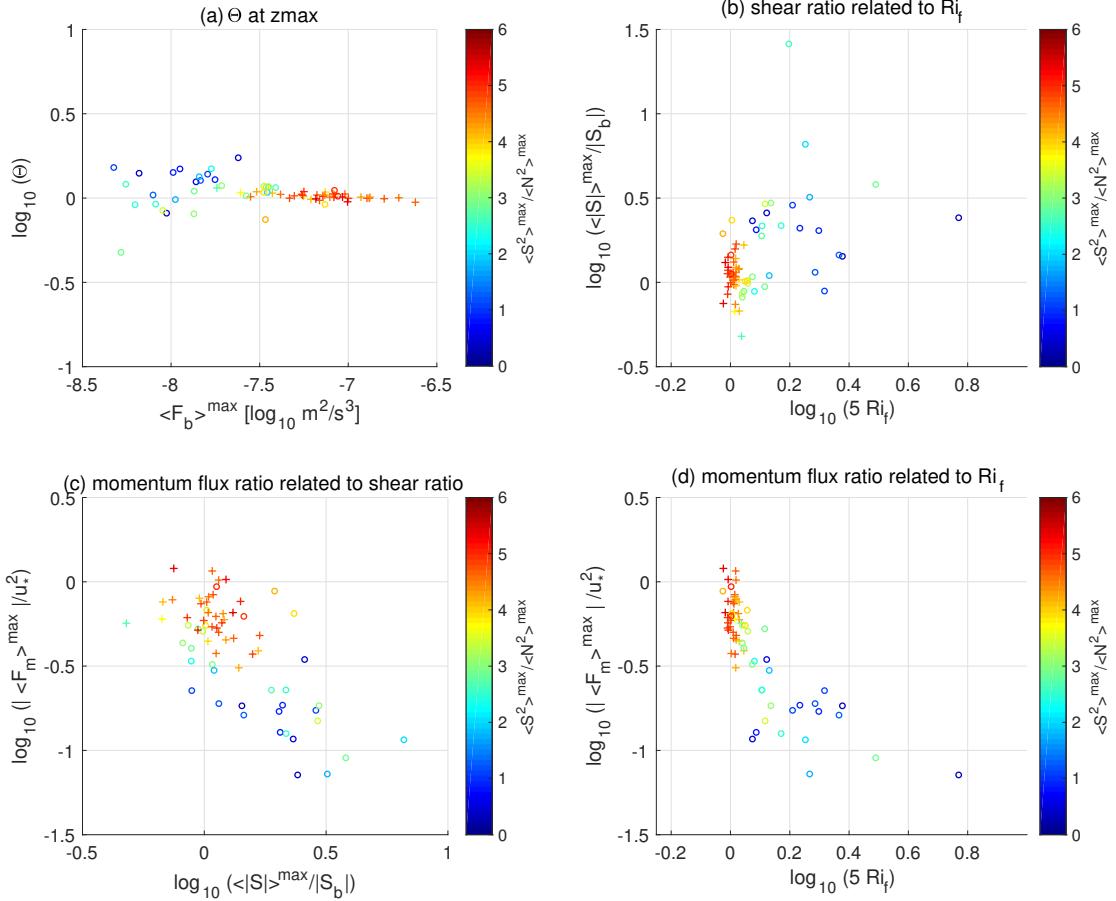
1720 FIG. 15. Relationships between various terms in the daily mean turbulent kinetic energy budget at the depth  
 1721  $z_{max}$  where the downward turbulent buoyancy flux is maximum ( $\langle SP \rangle^{max} + \langle T \rangle^{max} \approx \langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}$ ; see  
 1722 the Appendix for details). The depths  $z_{max}$  are plotted as + symbols in Fig. 10. Buoyancy flux  $\langle F_b \rangle^{max}$  is  
 1723 plotted against (a) shear production over buoyancy flux plus dissipation  $\langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$  and (b)  
 1724 shear production over buoyancy flux (i.e., the inverse flux Richardson number  $Ri_f^{-1} = \langle SP \rangle^{max} / \langle F_b \rangle^{max}$ ). The  
 1725 inverse gradient Richardson number of the horizontally-averaged profile  $Ri_g^{-1} = \langle |\partial \mathbf{u}_h / \partial z|^2 \rangle^{max} / \langle \partial b / \partial z \rangle^{max} =$   
 1726  $\langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$  is shown in color on all four panels and on the y axes in (c)-(d) against  $Ri_f^{-1}$  (c) and  $Pr_t^{-1} =$   
 1727  $Ri_f / Ri_g$  (d) (the inverse turbulent Prandtl number  $Pr_t^{-1}$  is the ratio of the turbulent diffusivity of buoyancy over  
 1728 the turbulent viscosity of momentum). The thick black line (c) is the 1-1 line, the thin solid line is a fit to LES  
 1729 of a coastal boundary layer under a hurricane by Watkins and Whitt (2020), and the thin dashed line is a fit to  
 1730 atmospheric boundary layer observations by Anderson (2009), which parameterizes the subgrid-scale  $Pr_t^{-1}$  in  
 1731 the LES. The two days with most anomalously low  $Ri_f^{-1}$  (b-c;  $Ri_f^{-1} = 0.9$  and  $1.6$ ) and high  $Pr_t^{-1}$  (d;  $Pr_t^{-1} = 0.4$   
 1732 and  $1.8$ ) also have the largest relative non-local sources of turbulent kinetic energy  $\langle T \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}) \approx$   
 1733  $1 - \langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$  (i.e., the points with lowest values in a; 0.3 and 0.6). Plus (+) symbols are from  
 1734 LES at  $0^\circ$  N and circles (o) from  $3^\circ$  N.



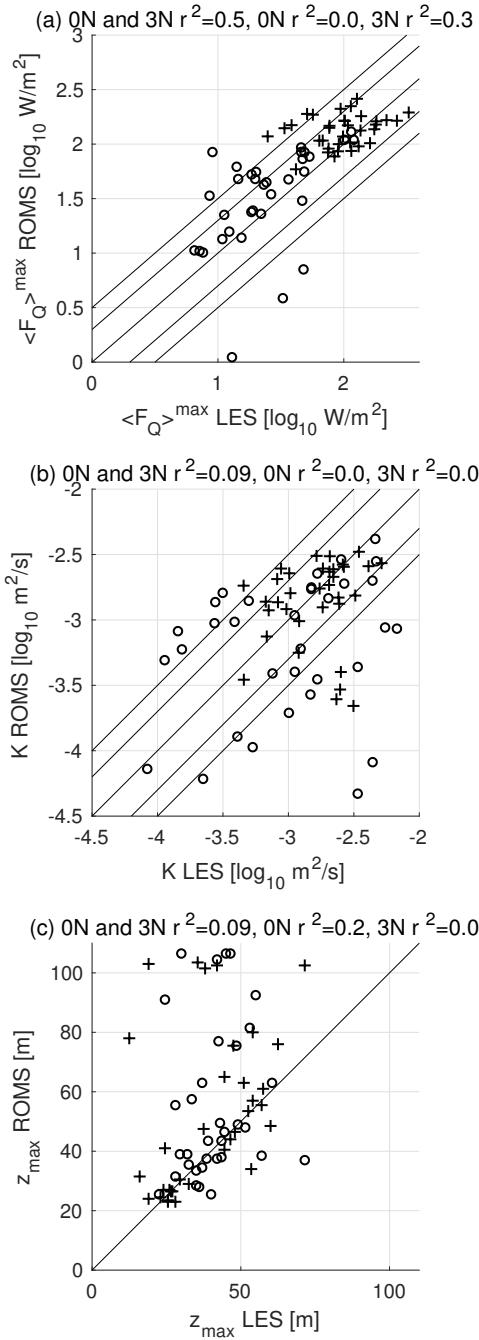
1735 FIG. 16. Relationship between  $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$  and  $\langle F_b \rangle^{max}$  (a) and  $K_b = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$  (b) at  
 1736  $z_{max}$  (i.e., at the depths indicated by the + symbols in Fig. 10). Averaging diffusivity directly in (b) yields  
 1737 quantitatively different results but qualitatively the same conclusion that  $K_b$  is at best weakly related to  $Ri_g$ .  
 1738 Overlaid in (b) are parameterizations of turbulent diffusivity as a function of Richardson number from Pacanowski  
 1739 and Philander (1981) (PP) Peters et al. (1988) (PGT), and Large and Gent (1999) (KPP).



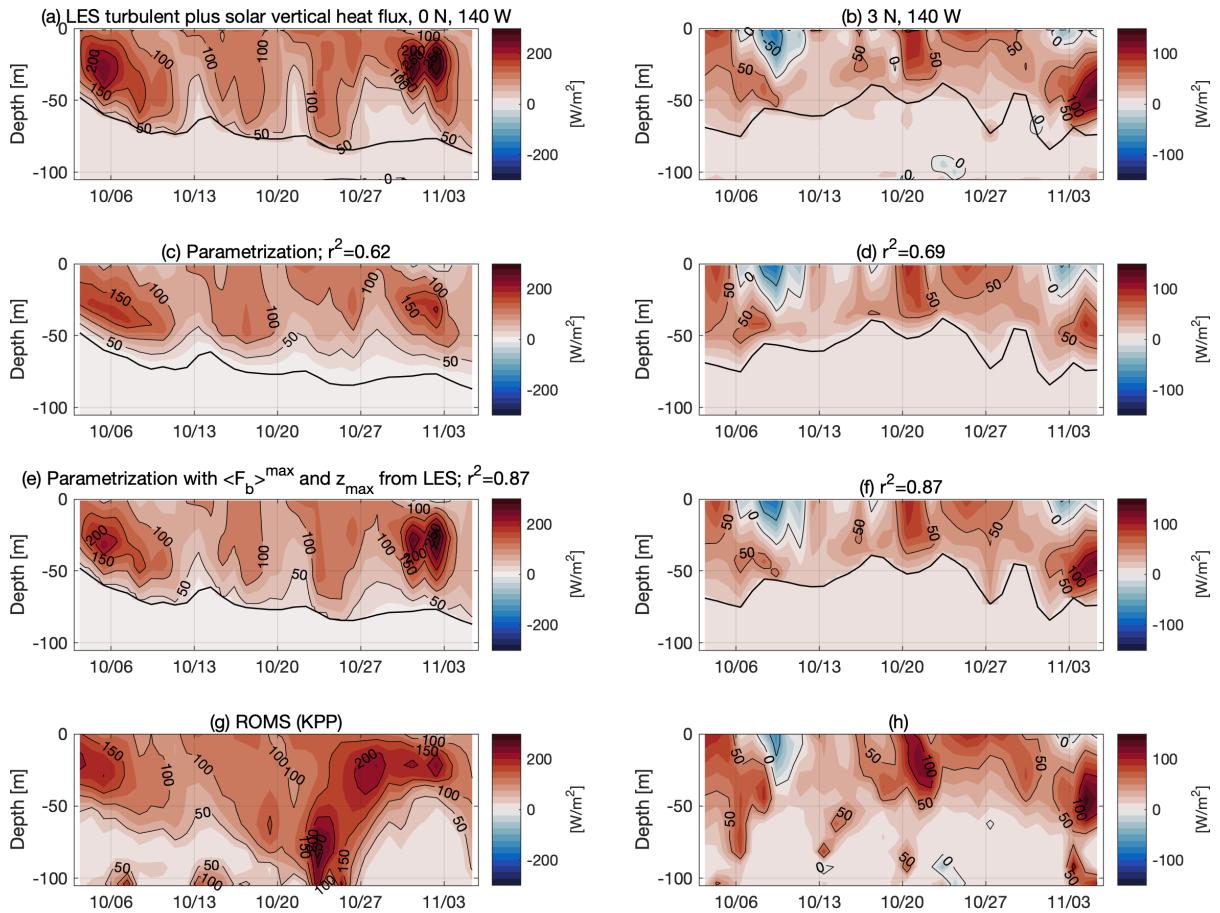
1740 FIG. 17. Maximum daily mean turbulent buoyancy flux  $\langle F_b \rangle^{\max}$  scales with oceanic bulk vertical shear  $S_b$   
 1741 (a,c) and even more closely with a product of  $S_b$  and the magnitude of the surface wind stress  $|\tau| = u_*^2$  (b,d).  
 1742 The scalings are obtained via linear regression on the LES output in (a)-(b), which includes 34 days at  $3^\circ$  N  
 1743 (black  $\circ$ ) and 34 days at  $0^\circ$  N (black  $+$ ), or on the 68 days of LES output plus 38 days of TIWE data (blue  $*$ )  
 1744 in (c)-(d). Hence, the TIWE observations serve as an independent validation of the regressions in (a)-(b) and  
 1745 constrain the regressions in (c)-(d). The predictors include  $S_b$ , which is derived from a linear fit to the mean  
 1746 velocity from  $H_{Rig}$  to 5 m depth (thick black lines in Fig. 4), and the friction velocity  $u_* = \sqrt{|\tau|/\rho}$ . All variables  
 1747 are log-transformed and Pearson's  $r$  in the panel titles is calculated in log space. The various diagonal black lines  
 1748 indicate where the data are along the 1-1 line, within a factor of 2, and within a factor of 3. With 95% confidence  
 1749 intervals, the scalings are as follows:  $(2-6) \times 10^{-6} |S_b|^{(0.7-1.0)}$  (a),  $(1-200) \times 10^{-2} |S_b|^{(0.9-1.1)} u_*^{(1.6-2.5)}$  (b),  
 1750  $(2-6) \times 10^{-6} |S_b|^{(0.8-1.0)}$  (c), and  $(0.03-1.3) \times 10^{-2} |S_b|^{(0.8-1.0)} u_*^{(0.9-1.6)}$  (d).



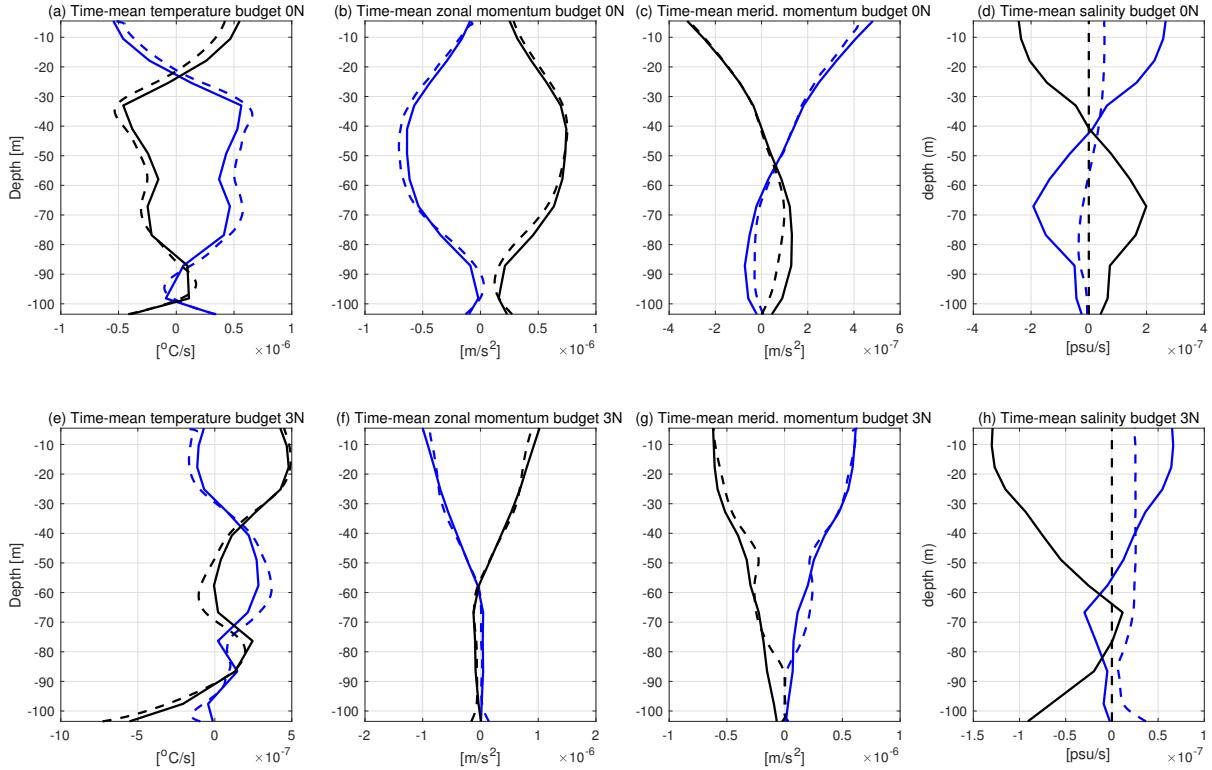
1751 FIG. 18. Various ratios of terms in Eqn. (6) showing how the local energetics of the buoyancy flux at  $z_{\max}$   
 1752 (Fig. 15) relate to the bulk scalings derived via regression (Fig. 17). Circles ( $\circ$ ) are from the LES at  $3^\circ$  N, and  
 1753 pluses ( $+$ ) are from the LES at  $0^\circ$  N; the color indicates  $Ri_g^{-1} = \langle S^2 \rangle_{\max} / \langle N^2 \rangle_{\max}$ .



1754 FIG. 19. Comparisons between the LES and ROMS (KPP) at the LES locations (+ at 0° N and ○ at 3° N along  
 1755 140° W): (a) The maximum turbulent heat flux  $\langle F_Q \rangle^{max}$ , (b) the turbulent diffusivity of heat  $K$  at  $z_{max}$ , and (c)  
 1756 the depth  $z_{max}$  at which  $\langle F_Q \rangle^{max}$  occurs. Note, however, that the LES results are derived from the buoyancy  
 1757 dynamics whereas the ROMS results are derived from the temperature dynamics. That is, the LES results are  
 1758  $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 \langle F_b \rangle \approx \langle F_Q \rangle$  [W/m<sup>2</sup>] in (a) and  $K = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$  in (b), and  $z_{max}$  is calculated  
 1759 from from  $\langle F_b \rangle$  profiles.



1760 FIG. 20. Daily averaged net vertical heat flux  $\langle Q \rangle$  (including turbulent  $F_Q$  as in Fig. 10 plus penetrative  
 1761 radiative  $P_Q$  components) at  $0^\circ$  N,  $140^\circ$  W (left column) and  $3^\circ$  N,  $140^\circ$  W (right column) as simulated by the  
 1762 LES [(a)-(b)] and as parameterized based on horizontally-averaged velocity and density profiles and net surface  
 1763 buoyancy and momentum fluxes [(c)-(d)]. For reference, the the piecewise linear flux profiles with  $\langle Q \rangle(z = 0)$   
 1764 and  $\langle Q \rangle(z = z_{max})$  from LES are shown in e-f. In addition, the vertical heat fluxes (penetrating shortwave plus  
 1765 turbulent) from the parent ROMS model are shown in the bottom row [(e)-(f)]. Note the different colorbar  
 1766 ranges in the left and right columns. For consistency with earlier results, (a)-(f) plot  $(\rho c_p / g \alpha) \langle B \rangle \approx \langle Q \rangle$  where  
 1767  $\langle B \rangle$  is the daily-averaged vertical buoyancy flux including the parts due to turbulence and penetrative shortwave  
 1768 radiation.



1769 Fig. A1. Time-means of various terms in the horizontally-averaged tracer and momentum budgets from  
 1770 ROMS (solid lines) and LES (dashed lines) at 0° N, 140° W (top) and 3° N, 140° W (bottom). The blue lines  
 1771 represent the time-mean convergence of vertical transport of (a,e) temperature, (b,f) zonal momentum and (c,g)  
 1772 meridional momentum and (d,h) salinity due to turbulence (and solar radiation in the case of temperature). The  
 1773 black lines represent all other tendencies of horizontally-averaged momentum and tracers as diagnosed from  
 1774 ROMS, i.e.  $\mathcal{F}$  (plus Coriolis in the case of momentum), and as diagnosed in LES, i.e.  $\mathcal{F} + \mathcal{R}$  (plus Coriolis in  
 1775 the case of momentum). See the Appendix for the budget formulas.