

1 **Simulation and scaling of the turbulent vertical heat transport and**
2 **deep-cycle turbulence across the equatorial Pacific cold tongue**

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ABSTRACT

22 Microstructure observations in the Pacific cold tongue reveal that turbulence often penetrates into
23 the thermocline producing hundreds of W/m^2 of downward heat transport during nighttime and
24 early morning. However, virtually all observations of this deep-cycle turbulence (DCT) are from
25 $0^\circ\text{N}, 140^\circ\text{W}$. Here, a hierarchy of ocean process simulations including submesoscale-permitting
26 regional models and turbulence-permitting large eddy simulations (LES) embedded in a regional
27 model provide insight into mixing and DCT at and beyond $0^\circ\text{N}, 140^\circ\text{W}$. A regional hindcast
28 quantifies the spatio-temporal variability of subsurface turbulent heat fluxes throughout the cold
29 tongue from 1999-2016. Mean subsurface turbulent fluxes are strongest ($\sim 100 \text{ W/m}^2$) within 2° of
30 the equator, slightly ($\sim 10 \text{ W/m}^2$) stronger in the northern than southern hemisphere throughout the
31 cold tongue, and correlated with surface heat fluxes ($r^2 = 0.7$). The seasonal cycle of the subsurface
32 heat flux, which does not covary with the surface heat flux, ranges from 150 W/m^2 near the equator
33 to 30 W/m^2 and 10 W/m^2 at 4°N and S respectively. Aseasonal variability of the subsurface heat
34 flux is logarithmically distributed, covaries spatially with the time-mean flux, and is highlighted
35 in 34-day LES of boreal autumn at 0°N and $3^\circ\text{N}, 140^\circ\text{W}$. Intense DCT occurs frequently above the
36 undercurrent at 0°N and intermittently at 3°N . Daily-mean heat fluxes scale with the bulk vertical
37 shear and the wind stress, which together explain $\sim 90\%$ of the daily variance across both LES.
38 Observational validation of the scaling at $0^\circ\text{N}, 140^\circ\text{W}$ is encouraging, but observations beyond
39 $0^\circ\text{N}, 140^\circ\text{W}$ are needed to facilitate refinement of mixing parameterization in ocean models.

40 **Significance Statement**

41 This work is a fundamental contribution to a broad community effort to improve global long-
42 range weather and climate forecast models used for seasonal to longer-term prediction. Much of
43 the predictability on seasonal timescales is derived from the slow evolution of the upper eastern
44 equatorial Pacific Ocean as it varies between El Niño and La Niña conditions. This study presents
45 state-of-the-art high-resolution regional numerical simulations of ocean turbulence and mixing in
46 the eastern equatorial Pacific. The results inform future planning for field work as well as future
47 efforts to refine the representation of ocean mixing in global forecast models.

48 **1. Introduction**

49 Over the last several decades, multiple field campaigns have observed strong turbulence above
50 the equatorial undercurrent in the eastern Pacific Ocean (Gregg et al. 1985; Moum and Caldwell
51 1985; Peters et al. 1988; Lien et al. 1995; Moum et al. 2009, 2013; Warner and Moum 2019; Smyth
52 et al. 2021). Like upper-ocean turbulence elsewhere in the tropics and subtropics, the diurnal cycle
53 is a dominant mode of variability, but turbulence in the eastern equatorial Pacific is unusual in that
54 it penetrates tens of meters below the base of the surface mixed layer and into the thermocline.
55 This turbulence produces exceptionally strong heat fluxes of $O(100)$ W/m² on average and up to
56 1000 W/m² during occasional bursts of intense turbulence in the nighttime and early morning in
57 a stratified layer tens of meters thick (Moum et al. 2013, 2009; Smyth et al. 2021). Hence, this
58 “deep-cycle turbulence” (DCT) drives stronger cooling of the near surface and warming of the
59 thermocline compared to diurnal surface boundary layer turbulence in other areas of the global
60 oceans. DCT thus contributes to sustaining the relatively cool sea surface and net ocean heat
61 uptake in the eastern equatorial Pacific Ocean cold tongue on average (Wang and McPhaden 1999;
62 Moum et al. 2013). DCT also varies with and influences the regional sea-surface temperature

63 (SST) dynamics on multiple timescales beyond diurnal, including interannual (Warner and Moum
64 2019), seasonal (Wang and McPhaden 1999; Moum et al. 2013), and subseasonal (Lien et al. 2008;
65 Moum et al. 2009), although these variations are not as well understood as the diurnal cycle.

66 If the available data from 0° N, 140° W are representative, then turbulent mixing is an important
67 participant in the SST budget and air-sea interaction in the Pacific Ocean cold tongue. However,
68 neither the spatio-temporal variability of ocean mixing nor the physical drivers of variability on
69 timescales beyond diurnal are well observed or understood. In particular, our knowledge of the
70 area and vertical extent of strong turbulent heat fluxes is based almost entirely on extrapolation
71 using parameterizations beyond 0° N, 140° W (e.g., Pacanowski and Philander 1981; Holmes and
72 Thomas 2015; Holmes et al. 2019a; Pei et al. 2020; Deppenmeier et al. 2021; Cherian et al. 2021).
73 In addition, none of these parameterized modelling studies present results over a sufficient duration
74 to provide a climatological perspective from a model with sufficiently fine horizontal grid spacing
75 (< 10 km horizontal (Marchesiello et al. 2011), and < 5 m vertical (Jia et al. 2021)) to fully resolve
76 the mesoscale variations in vertical shear, which significantly modulate mixing (Moum et al. 2009;
77 Inoue et al. 2012; Holmes and Thomas 2015; Cherian et al. 2021). Hence, the broader implications
78 of downward turbulent heat transport and specifically DCT in the cold tongue for global ocean,
79 climate and Earth system dynamics are not well understood (but see Meehl et al. 2001; Richards
80 et al. 2009; Danabasoglu et al. 2006; Newsom and Thompson 2018; Holmes et al. 2019b,a; Zhu
81 and Zhang 2019; Huguenin et al. 2020; Deppenmeier et al. 2021). In addition, climate models
82 suffer from long-standing and significant biases in their simulation of the SST, thermocline, and
83 circulation in the eastern equatorial Pacific (Li and Xie 2014; Li et al. 2015). Since some biases
84 persist with refinements in model horizontal grid resolution and the mean ocean circulation (Small
85 et al. 2014) and are sensitive to the formulation of the mixing scheme (Meehl et al. 2001; Richards
86 et al. 2009; Zhu and Zhang 2019), it seems plausible if not likely that poor performance of

87 parameterizations of ocean mixing physics (Zaron and Moum 2009) is at least partially responsible
88 for equatorial Pacific biases in climate and Earth system models. Hence, we conducted a regional
89 process modelling study of turbulent heat transport and DCT in the equatorial Pacific Ocean cold
90 tongue as a contribution to a broader effort to conduct a pre-field process modeling study of Pacific
91 equatorial upwelling and mixing physics.

92 In this manuscript, we present new state-of-the-art simulations and new metrics to characterize
93 turbulent vertical heat transport in the Pacific Ocean cold tongue. First, we examine the clima-
94 tological (1999-2016) spatio-temporal variability of the turbulent vertical heat flux, including the
95 time-mean, seasonal cycle, and aseasonal variability (i.e., all deviations from the mean seasonal
96 cycle) of the daily-mean flux, in a relatively fine ($1/20^\circ$ horizontal, 2.5 m vertical) resolution
97 regional hindcast simulation of the eastern equatorial Pacific Ocean with parameterized vertical
98 mixing. The results provide a climatological perspective on the recent finding that global ocean
99 models can simulate DCT (Pei et al. 2020), as well as the finding of and explanation for DCT off
100 the equator in a regional ocean model (Cherian et al. 2021), and complement other climatological
101 studies of mixing in the equatorial Pacific cold tongue focused on different questions, different
102 metrics, and different models with coarser resolution (e.g., Ray et al. 2018; Holmes et al. 2019a;
103 Huguenin et al. 2020; Deppenmeier et al. 2021). The analysis of the regional model also shows that
104 the daily-mean turbulent heat transport is logarithmically distributed, thus relatively rare events
105 associated with aseasonal variability on timescales of days to weeks have a strong influence on and
106 spatially covary with the time-mean transport.

107 We build understanding of the subseasonal part of aseasonal variability in mixing via large eddy
108 simulations (LES) that are embedded in a regional ocean model so that the simulated turbulence
109 varies in the context of realistic variations in horizontal currents and temperature and atmospheric
110 forcing over timescales from hours to more than a month. These LES address a key source

111 of uncertainty in our regional model and all prior studies of ocean mixing on timescales from
112 weeks to months using models: our regional models and all prior models are based on uncertain
113 mixing parameterizations. Here, the LES are used to study the variability of explicit (rather than
114 parameterized) turbulent mixing and DCT on timescales from days to a month for the first time.
115 Our LES build on prior shorter simulations of diurnal cycles and shorter variability with idealized
116 boundary conditions and forcing (Skylingstad and Denbo 1994; Wang et al. 1996, 1998; Large
117 and Gent 1999; Wang and Müller 2002; Pham et al. 2013) as well as how the diurnal cycles vary
118 between the four seasons at 0° N, 140° W (Pham et al. 2017; Sarkar and Pham 2019). Through
119 both the analysis of the regional model and the LES, we confront the simulations of turbulence
120 with observations and critically evaluate the model representations, albeit only at 0° N, 140° W.
121 Future observations are needed to evaluate and constrain modeled turbulence beyond 0° N, 140°
122 W in the Pacific cold tongue.

123 **2. Methods**

124 *a. Ocean hindcast of the eastern equatorial Pacific, 1999-2016*

125 Climatological statistics of vertical mixing throughout the equatorial Pacific cold tongue are
126 derived from an ocean hindcast of the period 1999 through 2016 in the region from 170 to 95° W
127 and from 12° S to 12° N in a submesoscale-permitting 1/20° configuration (Cherian et al. 2021)
128 of the MITgcm (Adcroft et al. 2004; Marshall et al. 1997). As described previously (Cherian
129 et al. 2021), the model is forced at the surface by fluxes derived from bulk flux algorithms and
130 the JRA55do atmospheric reanalysis (Tsujino et al. 2018) and at side boundaries by daily-mean
131 horizontal velocity, temperature and salinity from the Mercator GLORYS 1/12° ocean reanalysis.
132 Solar radiation penetrates and warms the water below the surface, and there are no tides. Vertical

133 mixing is represented by the K profile parameterization (KPP) (Large et al. 1994), which was
134 compared against and tuned to match LES of partially resolved DCT at 0°N,140°W (Large and
135 Gent 1999). This hindcast is very similar to that of Cherian et al. (2021), where some observational
136 validations are presented. The main technical difference between the two hindcasts, in addition to
137 the different and longer simulated time interval, is that the model grid has a slightly coarser vertical
138 resolution (2.5 m versus 1 m over the top 250 m), because the reduced vertical resolution had a
139 negligible impact on the solutions in short tests and reduced the computational cost. The analysis
140 is conducted on the saved daily-mean temperature, salinity, and heat budget diagnostics. See Table
141 1 for a list of several of the most commonly used metrics to quantify and describe vertical mixing
142 as well as the sections in which they are defined and discussed.

143 *b. Large eddy simulation hindcasts of turbulence over 34 days*

144 To better understand and validate the subseasonal spatio-temporal variability in turbulent mixing
145 on and off the equator, we report results from two 34-day LES that are hindcasts of upper-ocean
146 turbulence in a small 306 m by 306 m by 108 m deep domain during the period from October 2 to
147 November 5, 1985 at 0° N and 3° N along 140° W in the equatorial Pacific cold tongue. Unlike
148 the regional ocean hindcast and most other ocean models, the LES explicitly simulates rather than
149 parameterizes the outer scales $\mathcal{O}(1)$ m of the turbulence and thus can provide insight into the
150 physics of ocean mixing and DCT. However, the LES has a computational cost that is many orders
151 of magnitude greater than the regional ocean model per unit simulated time and volume, hence
152 the LES must be run for much shorter time intervals and in much smaller domains (Skylingstad
153 and Denbo 1994; Wang et al. 1996, 1998; Wang and Müller 2002; Pham et al. 2013, 2017; Sarkar
154 and Pham 2019). A detailed description of the LES model is given in the Appendix. In short,
155 the LES is forced by variable six-hourly air-sea fluxes (including a diurnal cycle of penetrating

156 shortwave radiation) and larger-scale ($\gtrsim 15$ km) oceanic tendencies, such as advection and the
157 pressure gradient force, derived from a regional ocean hindcast simulation of the entire Pacific cold
158 tongue. The LES forcing is from the parent ocean model ROMS, not MITgcm, because ROMS
159 solutions (based on earlier work of Holmes and Thomas 2015) were available earlier with all
160 the necessary outputs. However, the domain, the horizontal resolution $1/20^\circ$, the vertical mixing
161 scheme KPP, the 3-hourly surface forcing (including diurnal cycle of penetrating solar radiation)
162 from JRA55do are all the same in ROMS and MITgcm, and the mesoscale fields and parameterized
163 mixing dynamics of interest are qualitatively similar (see the Appendix for details and compare the
164 results reported in Holmes and Thomas (2015) and Cherian et al. (2021)).

165 The inclusion of larger-scale oceanic tendencies of temperature and momentum from ROMS are
166 an important novelty in these LES and crucial for sustaining realistic temperature and horizontal
167 velocity profiles over timescales longer than a few days (Qiao and Weisberg 1997). These tendencies
168 also provide a source of subseasonal variability on timescales from days to a month (Holmes and
169 Thomas 2015; Cherian et al. 2021). Hence, an important point of reference is the one previous
170 LES study of the eastern equatorial Pacific that incorporated large-scale tendencies (Wang et al.
171 1998). In addition to finer grid resolution, comparisons with an off-equatorial domain, and longer
172 (34 day vs. 6 day) simulations than in Wang et al. (1998), the ocean tendencies used here also differ
173 from those in Wang et al. (1998) in that they are derived from a realistic regional ocean model
174 rather than idealized mathematical formulas. Thus, the large-scale oceanic conditions and related
175 large-scale tendencies (as well as the air-sea fluxes) evolve on timescales from 6 hours to 1 month
176 during the simulations, in conjunction with the passage of a tropical instability wave and other
177 mesoscale ocean variability. In addition, there is approximate dynamical consistency between the
178 initial conditions, surface fluxes and interior tendencies, as well as between the LES at 0° and 3°
179 N across this range of timescales. Hence, despite some broken feedbacks between the limited LES

180 domain and the larger-scale ocean and atmosphere, the differences between the LES and the ocean
181 model mean profiles of temperature and zonal momentum are always less than 0.5°C and 0.25 m/s.
182 That is, the turbulence simulated by LES, the surface fluxes, and the interior tendencies remain
183 approximately consistent as if the LES was part of a two-way coupled regional system rather than
184 an isolated domain throughout the 34-day simulations.

185 LES outputs include instantaneous statistics, such as the horizontally-averaged turbulent vertical
186 fluxes of heat and momentum among others, which are saved irregularly about every 2-5 simulated
187 minutes and additionally binned into daily-mean statistics for some analyses (to obtain the data and
188 source code, see Whitt 2022). Note that all times are in UTC, and the local solar time is about
189 9 hours behind UTC, so solar noon occurs at about 21 UTC. All daily-mean LES statistics, such
190 as daily mean flux profiles, are calculated from 21 UTC so that the 34 daily means begin and end
191 at about solar noon, beginning on 21:00 UTC on October 2, 1985 and ending at 21:00 UTC on
192 November 5, 1985.

193 *c. Evaluation of the LES zonal velocity and temperature by comparison with observations*

194 Comparisons with observations suggest that the LES yield plausibly realistic zonal velocity and
195 temperature simulations with a few exceptions. Mean vertical profiles of temperature and zonal
196 velocity are generally within observed ranges at 0° N, 140° W where mooring observations from
197 the Tropical Atmosphere Ocean (TAO) array (McPhaden et al. 2010) are available (Figs. 1- 2).
198 At 0° N, 140° W, there is a clear depth range between about 10 m and 75 m where the gradient
199 Richardson number of the horizontally-averaged profile, that is the vertical gradient of buoyancy
200 over the squared vertical gradient of horizontal velocity

$$Ri_g = \frac{N^2}{S^2} = \frac{\partial b / \partial z}{|\partial \mathbf{u}_h / \partial z|^2} \approx 1/4, \quad (1)$$

201 is in a state of marginal instability as observed by Smyth and Moum (2013) (see Fig. 3). The
202 LES results are presented at 3° N for comparison in Figs. 1-3, although mooring observations are
203 not available at 3° N for validation. The observed annual mean climatology of zonal currents and
204 temperature (Johnson et al. 2002) is plotted for comparison with the LES at 3° N, 140° W, but the
205 observed annual climatology is insufficient to validate October mean profiles in the LES at 3° N
206 because there is significant seasonal, interannual, and subseasonal variability. Perhaps the most
207 notable difference between the two latitudes is that the shear is weaker on average at 3° N than at
208 0° N, and $Ri_g > 1/4$ most of the time at 3° N. Hence, marginal instability $Ri_g \approx 0.25$ is intermittent
209 (about 25% of the time) from 20 m to 70 m depth at 3° N rather than persistent as at 0° N.

210 The diurnal cycle in temperature and zonal velocity is plausible but on the weaker side of the
211 observed diurnal cycles at 0° N, 140° W, for example as shown at 25 m in Fig. 2. Consistent with
212 observations, the modeled diurnal cycle is stronger at shallower depths (e.g., shallower than 15 m),
213 weak but with a notable peak in the frequency spectra at intermediate depths (e.g., between 15 and
214 45 m), and difficult to discern from other nearby frequencies in the spectra at deeper depths (not
215 shown). A detailed investigation of the mechanisms controlling the amplitude of the diurnal cycle
216 of the horizontally-averaged current and temperature profiles (and all other variables) is left for
217 future work (for prior studies of the diurnal cycle and DCT at 0° N, 140° W in LES, see e.g. Wang
218 et al. 1998; Pham et al. 2013, 2017). This study instead focuses on variability in daily-averaged
219 quantities.

220 The simulated temperature and velocity variance at timescales from days to weeks is generally
221 realistic at 0° N, 140° W. For example, the power spectra of temperature and zonal velocity at 25
222 m depth (Fig. 2) show that variance at periods from a few days to a month is reasonably realistic,
223 but variability at internal wave timescales ranging from a few days to a few hours is consistently
224 weak in the LES relative to the TAO mooring observations (as shown at 25 m). The weakness of

225 internal wave activity at these frequencies is expected (qualitatively) in the LES since the parent
226 ROMS model does not have tides or grid resolution at horizontal scales from 5.5 km to 0.3 km
227 (and only 8 m vertical resolution in the upper ocean) where much internal wave activity occurs
228 and from which it cascades down to smaller scales (Gregg et al. 2003). That is, the embedded
229 LES represents only a limited subset of interactions between internal waves, shear instabilities, and
230 turbulence. First, the LES represents the response of small-scale shear instabilities, internal waves
231 and turbulence at horizontal wavelengths smaller than 300 m to large-scale internal waves (among
232 other processes) at horizontal wavelengths $\gtrsim 15$ km that are resolved by the parent model. Second,
233 the LES represents some interactions between internal waves, shear instabilities, and turbulence at
234 scales from about 1 to 300 m that are generated locally in the domain. In particular, the periodic
235 horizontal boundary conditions allow internal waves to persist in the model domain and propagate
236 vertically through the stratification. However, going beyond the comparison between the simulated
237 (black dotted) and observed (light blue) temperature spectra in Fig. 2a to a detailed investigation
238 of the internal waves and instabilities in the LES and observations (Lien et al. 1996; Smyth et al.
239 2011; Moum et al. 2011) is left for future work (for some analysis of these topics in other LES, see
240 Pham et al. 2013, 2017).

241 Finally, the turbulence simulated by the LES is difficult to validate directly since direct obser-
242 vations of the turbulence are so limited in space and time. That said, the simulated turbulence
243 is qualitatively and quantitatively similar to the turbulence observed by Lien et al. (1995) from
244 November 4 - December 12 1991 (as discussed in more detail below). And, previous studies in
245 simpler model configurations show that the model simulates idealized test cases and turbulent flows
246 with statistics that are consistent with basic conservation constraints (Watkins and Whitt 2020).

3. Spatial patterns, seasonal cycle, and aseasonal variability in the regional hindcast

Our analysis of the regional ocean model begins with the definition of the metrics to be used throughout the results (3.a), then provides a description of the climatological time-mean spatial patterns (3.b), seasonal cycles (3.c), and aseasonal variability (3.d) of ocean mixing in the model as well as comparisons to observations at 0° N, 140° W.

a. Metrics of ocean mixing

We quantify and compare the downward heat flux due to ocean mixing $F_Q(z)$, which tends to cool the upper ocean on average, with the net downward surface heat flux $Q_0^{net} = F_Q(z=0) + P_Q(z=0)$ (including turbulent fluxes F and penetrative fluxes P due to solar radiation), which tends to warm the upper ocean on average (Fig. 4). With regard to ocean mixing, we focus on the maximum over depth z of the daily-mean downward turbulent heat flux $\langle F_Q \rangle^{max} = \max_z \langle F_Q(z) \rangle$ where $\langle \rangle$ denotes a daily mean (and a horizontal average is implicit, over a single grid cell in the MITgcm and the entire domain in LES). Since the depth z_{max} at which $\langle F_Q \rangle^{max}$ occurs varies in time and space, we also quantify z_{max} and compare it with the mixed layer depth (MLD, defined by the first depth 0.015 kg/m³ denser than the top 10 m) for reference (Fig 5).

The maximum daily-mean turbulent heat flux $\langle F_Q \rangle^{max}$, the daily net surface heat flux $\langle Q_0^{net} \rangle$, and their difference $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max}$ provide useful measures of the significance of ocean mixing relative to the net surface heat flux in the upper-ocean heat and SST dynamics throughout the cold tongue. This is a simplified view because other terms also contribute to the heat budget above $\langle F_Q \rangle^{max}$ in addition to $\langle F_Q \rangle^{max}$ and $\langle Q_0^{net} \rangle$, including penetration of radiative heat fluxes $\langle P_Q \rangle^{max}$ below the depth z_{max} and advection (e.g., Moum et al. 2013). In addition, the precise role of ocean mixing in the heat budget depends on the depth to which the budget is integrated. Vertical mixing is generally significant if the heat budget is integrated vertically over a layer that is closely

270 correlated with SST (Ray et al. 2018). Having stated the caveats, there are two main reasons we
271 focus on $\langle F_Q \rangle^{max}$. First, it is intrinsically interesting because it essentially quantifies and bounds
272 the maximum impact that mixing could have on the upper ocean heat budget. Second, we aim
273 to use $\langle F_Q \rangle^{max}$ to model the whole vertical profile $\langle F_Q \rangle(z)$ in the upper ocean (see section 4.g).
274 The a priori motivation to focus on $\langle F_Q \rangle^{max}$ in modelling $\langle F_Q \rangle(z)$ is based on a hypothesis that
275 $\langle F_Q \rangle(z)$ can be approximately reconstructed as an interpolation of three points: the surface flux
276 $\langle F_Q \rangle(z = 0)$, a positive subsurface $\langle F_Q \rangle^{max}$ if it exists, and a point of nearly zero flux at some depth
277 deeper than z_{max} . In this manuscript, we quantify and parameterize $\langle F_Q \rangle^{max}$ and then demonstrate
278 that $\langle F_Q \rangle^{max}$ can be used to predict $\langle F_Q \rangle(z)$, leaving an exposition of the relationships between
279 $\langle F_Q \rangle^{max}$ and the upper-ocean heat budget formalism to future work (but see Ray et al. 2018).

280 Although DCT is characterized by strong $\langle F_Q \rangle^{max}$ and may contribute significantly to the clima-
281 tological $\langle F_Q \rangle^{max}$, we choose not to distinguish DCT from other causes of $\langle F_Q \rangle^{max}$ via a formal
282 quantitative metric in this manuscript. This is because we want to characterize $\langle F_Q \rangle^{max}$ across the
283 cold tongue without assumption about the driving mechanisms, and DCT is not ubiquitous across
284 the cold tongue (Cherian et al. 2021). In addition, even though DCT tends to be associated with
285 strong $\langle F_Q \rangle^{max}$, it is not known if strong $\langle F_Q \rangle^{max}$ is always indicative of DCT or why and to what
286 degree $\langle F_Q \rangle^{max}$ varies from day to day in DCT or otherwise. However, we refer to the turbulence
287 driving the mixing descriptively as DCT where and when we feel the subjective criteria (based
288 on prior studies) are met. In particular, prior studies have identified DCT as strong diurnally-
289 modulated turbulence in a marginally unstable stratified shear layer ($Ri_g \approx 1/4$) just below the
290 deepest nighttime MLD (for a recent review, see Cherian et al. (2021)).

291 *b. Time-mean spatial patterns*

292 We begin by characterizing the time-mean $\langle F_Q \rangle^{max}$, which contributes to sustaining relatively
293 cool time-mean SSTs and net ocean heat uptake $\langle Q_0^{net} \rangle$ in the cold tongue by transporting heat
294 downwards from the mixed layer to the thermocline (Ray et al. 2018; Holmes et al. 2019a).
295 Consistent with that interpretation, the comparisons between $\langle Q_0^{net} \rangle$ and $\langle F_Q \rangle^{max}$ demonstrate that
296 the time-mean surface flux and ocean mixing have similar spatial patterns ($r^2 = 0.7$; Figs. 4e-f).
297 Both $\langle Q_0^{net} \rangle$ and $\langle F_Q \rangle^{max}$ are broadly elevated throughout the cold tongue relative to other areas
298 and take similar area-average values between 6° S and 6° N from 95° to 170° W (77 W/m² for
299 $\langle F_Q \rangle^{max}$ and 59 W/m² for $\langle Q_0^{net} \rangle$). In addition, both $\langle Q_0^{net} \rangle$ and $\langle F_Q \rangle^{max}$ are enhanced by more than
300 a factor of two near the equator (e.g., between $\pm 2^\circ$) compared to the area means between 6° S and
301 6° N (Fig. 4e-f; see also Fig. 2 of Cherian et al. (2021) for snapshot plan views).

302 Closer inspection highlights several important differences in the climatological spatial structure
303 of $\langle F_Q \rangle^{max}$ and $\langle Q_0^{net} \rangle$. First, $\langle F_Q \rangle^{max}$ is significantly stronger than $\langle Q_0^{net} \rangle$ on average in an equatorial
304 mixing band about 2° wide and centered slightly north of the equator that extends zonally through
305 the entire domain (170 to 95° W; see Fig. 4d). In this equatorial mixing band, the annual mean
306 surface heat flux $\langle Q_0^{net} \rangle$ reaches a peak at just over 120 W/m² at about 110° W and just south of the
307 equator, whereas the downward heat flux due to ocean mixing $\langle F_Q \rangle^{max}$ reaches a peak of just over
308 240 W/m² at 130° W just north of the equator (cf. Figs. 4e-f). In addition, there is net cooling
309 $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$ over a greater fraction of the year and over more of the zonal distance in the
310 equatorial mixing band, where $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$ between 50-75% of the time (Fig. 4d). In
311 the equatorial mixing band, the depth of the peak daily-mean turbulent heat flux z_{max} ranges from
312 about 90 m at 170° W to 30 m at 95° W (Fig. 5f). In addition, z_{max} is virtually always deeper
313 than the MLD and ranges from about 20-60 m below the base of the mixed layer in the equatorial

314 mixing band (cf. Figs. 5d-f). The deep z_{max} in the equatorial mixing band is consistent with prior
 315 studies showing that mixing is particularly strong and extends to particularly cold isotherms in this
 316 band (Holmes et al. 2019a; Deppenmeier et al. 2021). These results are all consistent with the
 317 established results that: 1) ocean mixing is uniquely strong in the cold tongue near the equator and
 318 plays a leading role in the upper ocean heat budget, 2) the turbulent heat flux peaks in the stratified
 319 ocean below the mixed layer, and 3) the intensity of ocean mixing is sensitive to the strong mean
 320 vertical shear in the horizontal velocity (e.g., Figs. 1, 3) that arises from the eastward equatorial
 321 undercurrent at depth and westward south equatorial current at the surface.

322 At latitudes between 2° - 6° , both $\langle Q_0^{net} \rangle$ and $\langle F_Q \rangle^{max}$ range from about 80 to 0 W/m² (Fig. 4e-f).
 323 The depth z_{max} is closer to the base of the MLD than in the equatorial mixing band and just
 324 10-30 m deeper than the MLD on average (cf. Figs. 5e-f). There is also a notable meridional
 325 asymmetry in net cooling $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$; ocean mixing is stronger relative to the surface
 326 flux more frequently and over a significantly greater area to the north of the equator (50-70%) than
 327 to the south (30-40%; see Fig. 4d). This meridional asymmetry arises partly because $\langle F_Q \rangle^{max}$
 328 is stronger, by $O(10)$ W/m², between about 2 - 5° N than between 2 - 5° S, but also partly because
 329 $\langle Q_0^{net} \rangle$ is stronger by $O(10)$ W/m² between 2 - 5° S than between 2 - 5° N. The weaker downward
 330 surface heat fluxes $\langle Q_0^{net} \rangle$ to the north are consistent with warmer SSTs to the north (through
 331 their impact on sensible, latent, and longwave surface heat fluxes). In addition, the asymmetry in
 332 time-mean mixing $\langle F_Q \rangle^{max}$ is qualitatively consistent with (but does not prove) the hypothesis that
 333 DCT and stronger ocean mixing events north of the equator arise due to stronger vertical shear in
 334 intermittent tropical instability waves and vortices that are also more energetic north of the equator
 335 as proposed by Cherian et al. (2021) (see Fig. 6b). The meridional asymmetry in mixing may
 336 also be a manifestation of a meridional asymmetry in SST in that warmer SSTs to the north may
 337 contribute to stronger upper-ocean temperature stratification that facilitates enhanced $\langle F_Q \rangle^{max}$.

338 The model results can be validated using multi-year microstructure observations that are available
 339 from chipods on moorings at 0°N, 140°W, from which an average annual cycle of the turbulent
 340 heat flux between 20-60 m has been estimated from deployments between 2008 and 2012 (Moum
 341 et al. 2013) (see also Smyth et al. 2021). Although the observed and modeled time intervals are
 342 not identical, we average the model heat fluxes over the same depth range $\langle F_Q \rangle^{20-60}$ and compare
 343 them with the observations of Moum et al. (2013) in Fig. 7. We find that the modeled annual
 344 mean $\langle F_Q \rangle^{20-60}$ is somewhat more than a factor of two larger than observed (150 W/m² vs 66
 345 W/m²). Restricting the model averaging to the observed years (2008-2012) does not change this
 346 discrepancy. The maximum flux $\langle F_Q \rangle^{max}$ is another 80 W/m² higher than $\langle F_Q \rangle^{20-60}$, because
 347 $z_{max} \approx 70$ m is below the 20-60 m averaging range and the modeled fluxes depend strongly on
 348 depth (Fig. 5f). Although it is not fully understood how the time-mean surface heat flux $\langle Q_0^{net} \rangle$ is
 349 mechanistically coupled to the time-mean subsurface flux $\langle F_Q \rangle^{max}$, it is interesting in light of their
 350 high degree of spatial correlation and similar magnitudes that $\langle Q_0^{net} \rangle$ is substantially stronger in the
 351 model than reported in Moum et al. (2013): Moum et al. (2013) report 55 W/m² while the modeled
 352 mean is twice as large at 110 W/m². This may indicate that the modeled heat uptake is biased
 353 high; this would be consistent with too-strong mixing assuming incomplete compensation for the
 354 too-strong mixing by other terms in the heat budget. However, other observational estimates of
 355 $\langle Q_0^{net} \rangle$ are higher than those reported by Moum et al. (2013). For example, Trenberth and Fasullo
 356 (2018) report an estimate of about 90 W/m² for the 2000-2016 period, and the model seems to be
 357 within the range of various estimates from 2001-2010 reported by Liang and Yu (2016) (roughly
 358 60-120 W/m² at 0°N, 140°W; see their Fig. 2). Hence, we do not conclude that the modeled time-
 359 mean surface heat flux $\langle Q_0^{net} \rangle$ in MITgcm is biased, although it is on the higher end of available
 360 estimates.

361 *c. Seasonal cycle*

362 The climatological seasonal cycle is another metric by which $\langle F_Q \rangle^{max}$ and $\langle Q_0^{net} \rangle$ are similar at first
363 glance but exhibit notable differences on closer inspection (Figs. 4b-c). Both the seasonal cycles
364 of $\langle F_Q \rangle^{max}$ and $\langle Q_0^{net} \rangle$ exhibit significant diversity. Four different varieties are present between 6°
365 S and 6° N: one-peak-one-trough, two-peaks-one-trough, two-troughs-one-peak, and two-peaks-
366 two-troughs, and there are variations in the timing, duration and amplitude of the peaks and troughs
367 (peaks are red and troughs are blue in Figs. 4b,c). In addition, these spatio-temporal structures
368 of the seasonal cycles in $\langle F_Q \rangle^{max}$ and $\langle Q_0^{net} \rangle$ are uncorrelated (pattern correlation $r^2 < 0.01$ for
369 zonal-mean seasonal anomalies, i.e. between the fields in Figs. 4b-c).

370 The phase and amplitude of the seasonal cycle of mixing in the equatorial mixing band is similar
371 to observations at 0° N, 140° W, even though the modeled time-mean $\langle F_Q \rangle^{20-60}$ is about a factor
372 of two higher than observed (see Fig. 7 and Moum et al. (2013)). In this equatorial band (see
373 Fig. 4c), the seasonal cycle of mixing $\langle F_Q \rangle^{max}$ is not in phase with and has a larger peak-to-trough
374 amplitude than the surface fluxes $\langle Q_0^{net} \rangle$ (Figs. 4a-c). In particular, the peak-to-trough amplitudes
375 are about 70 W/m² and 140 W/m² for $\langle Q_0^{net} \rangle$ and $\langle F_Q \rangle^{max}$, respectively. It is notable that the
376 observations reported by Moum et al. (2013) show a somewhat smaller peak-to-trough seasonal
377 cycle in $\langle Q_0^{net} \rangle \approx 50$ W/m², although the phasing is similar to the model. In particular, $\langle Q_0^{net} \rangle$
378 is minimum at about yearday 190 and maximum at about yearday 80, whereas mixing reaches a
379 minimum at about yearday 90 and a maximum at about yearday 215. There is also a secondary
380 peak in mixing at about the new year. Hence, there is a strong seasonal cycle in $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max}$,
381 which is negative (net cooling) at 0° N along more than 80% of longitudes between 170° and 95°
382 W in the boreal summer and early autumn (Fig. 4a), when the SST cools in the equatorial mixing
383 band (Moum et al. 2013). Conversely, $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$ at only about 20% of longitudes in

384 boreal spring (Fig. 4a), when the SST warms (Moum et al. 2013). These results highlight again the
385 importance of seasonal variations in ocean mixing for the seasonal cycle of cold tongue SST. The
386 seasonal cycle of the MLD and the depth z_{max} are highly correlated throughout the cold tongue.
387 In the equatorial mixing band, minima are achieved at about yearday 90 and local maxima at about
388 yearday 210 (Figs. 5b-c; $r^2 = 0.76$). But, the amplitude of the seasonal cycles are relatively modest
389 with peak-to-trough amplitudes of only about 15 m and 25 m for the MLD and z_{max} , respectively.

390 A qualitatively similar seasonal cycle is found off the equator in $\langle Q_0^{net} \rangle$ (Fig. 4b), but the off-
391 equatorial seasonal cycle in $\langle F_Q \rangle^{max}$ (Fig. 4c) is much weaker and has a different phase relative
392 to the equator. In addition, the amplitude of the seasonal cycle in $\langle F_Q \rangle^{max}$ is notably asymmetric
393 across the equator. There is a much stronger seasonal cycle to the north than the south; for example,
394 the peak-to-trough seasonal cycle amplitude is about 30 W/m² at 4° N but only 10 W/m² 4° S
395 (Fig. 4c). The stronger seasonal cycle in ocean mixing to the north of the equator is qualitatively
396 consistent with (but does not prove) the hypothesis that the seasonal cycle is due at least partially
397 to tropical instability waves, which have greatest variance from boreal summer to winter (Cherian
398 et al. 2021), although precisely quantifying and even determining the sign of the rectified effect of
399 tropical instability waves on ocean mixing is difficult (Holmes and Thomas 2015).

400 *d. Aseasonal variability*

401 Like the dissipation of turbulent kinetic energy (Crawford 1982; Moum et al. 1989; Smyth et al.
402 2021), the maximum daily-mean turbulent heat flux $\langle F_Q \rangle^{max}$ is highly variable and logarithmically
403 distributed (Fig. 8). Thus, the arithmetic averages of $\langle F_Q \rangle^{max}$ are significantly influenced by
404 relatively infrequent strong mixing events (in contrast to $\langle Q_0^{net} \rangle$). It follows that the processes
405 underpinning the aseasional variability in general and infrequent strong mixing events in particular
406 are significant for climatological statistics including the time mean. Hence, we conclude this

407 section on the regional climatological statistics by quantifying the aseasonal variability in $\langle F_Q \rangle^{max}$
408 and $\langle Q_0^{net} \rangle$, both to provide climatological context for and motivate a more detailed discussion of
409 subseasonal variability in $\langle F_Q \rangle^{max}$ simulated in LES (for discussion of the physics of subseasonal
410 variability in ocean models, see e.g. Holmes and Thomas 2015, 2016; Inoue et al. 2019; Liu et al.
411 2019b,a, 2020; Cherian et al. 2021). When plotting (in Fig. 6) and reporting the statistics from
412 the MITgcm results in this section, the aseasonal variability is separated from the full signal (i.e.,
413 defined) by subtracting a daily climatology, which is first averaged over 18 years and then smoothed
414 by applying a 15-day moving-average. Hence, aseasonal variability includes both inter-annual and
415 intra-annual timescales.

416 First, it may be noted that the minimum and maximum monthly means $\langle F_Q \rangle^{max}$ across the 18
417 simulated years (thin red lines in Fig. 7) span a factor of 3-8 or roughly 50 to 250 W/m². So,
418 any given monthly mean is reasonably likely to differ from the corresponding monthly climatology
419 by a factor of two. In addition, time series of aseasonal $\langle F_Q \rangle^{max}$ along 140° W in Fig. 6b reveal
420 variability in $\langle F_Q \rangle^{max}$ of hundreds of W/m² on timescales from days to months in 2012-2013. A
421 qualitative comparison of the modeled distribution of $\langle F_Q \rangle^{max}$ at 0° N, 140° W (Fig. 8a) to the
422 spread of observed daily-mean dissipation from chipods in Fig. B1 of Smyth et al. (2021) suggests
423 that there are fewer instances of weak mixing and a narrower distribution of mixing values in the
424 model compared to observations at 0° N, 140° W. But, the different vertical averaging precludes
425 a quantitative comparison (see Fig. 7). Aseasonal variability in mixing exhibits a spatial pattern
426 that is similar to the mean (cf. Fig. 6e and Fig. 4f), consistent with a logarithmic distribution.
427 In particular, the interquartile range (IQR) of aseasonal $\langle F_Q \rangle^{max}$ variability reaches 150 W/m² in
428 the strong equatorial mixing band but drops from 60 to 20 W/m² at latitudes from 2°-6°. There
429 is also a notable seasonal cycle to aseasonal variability, which is stronger in boreal autumn than
430 boreal spring (Fig. 6b; cf. Fig. 4c), as well as meridional asymmetry across the equator with

431 larger aseasonal variability to the north than to the south (Fig. 6e). Both the seasonal cycle and
 432 meridional asymmetry of aseasonal variability are consistent with tropical instability wave activity
 433 (Halpern et al. 1988; Moum et al. 2009; Cherian et al. 2021). There is also notable aseasonal
 434 variability in the depth at which maximum ocean mixing occurs z_{max} (Figs. 8e and 6c,f). The
 435 aseasonal variability in z_{max} has a similar spatial pattern as the time-mean z_{max} (cf. Fig. 5f and
 436 Fig. 6f). The IQR of aseasonal z_{max} variability is about 40 m at 170° W and 10 m at 95° W. This
 437 zonal gradient in the aseasonal IQR of z_{max} is qualitatively similar at all latitudes from 6° S to 6°
 438 N, but the IQR is elevated by 10-20 m in the equatorial mixing band relative to other latitudes (Fig.
 439 6f).

440 Aseasonal variability in $\langle Q_0^{net} \rangle$ is qualitatively different from aseasonal variability in $\langle F_Q \rangle^{max}$ (cf.
 441 Figs. 6a-b and cf. Figs. 8a,c). First, $\langle Q_0^{net} \rangle$ is more nearly normally distributed (Fig. 8c), and the
 442 IQR varies relatively little across the cold tongue from about 45-70 W/m² (Fig. 6d). In addition,
 443 the maximum Pearson's r^2 between aseasonal anomalies in $\langle Q_0^{net} \rangle$ and $\langle F_Q \rangle^{max}$ is only 0.15 (at
 444 about 2.5° S, 110° W) and the correlations are mostly much smaller (mean $r^2 = 0.02$ and median
 445 $r^2 = 0.01$). Hence, the aseasonal net surface heat flux $\langle Q_0^{net} \rangle$ anomalies do not covary with the
 446 aseasonal turbulent heat flux $\langle F_Q \rangle^{max}$ anomalies in the model (see Fig. 3i of Smyth et al. (2021)
 447 for a qualitatively similar observational result at 0° N, 140° W).

448 **4. Subseasonal variability on and off the equator in the LES**

449 To build further understanding of the subseasonal variability in ocean mixing and DCT, both on
 450 and off the equator, we turn to the LES (see the Methods and Appendix for details). First, section
 451 4.a describes how the metrics of ocean mixing (originally defined in section 3.a) are applied to the
 452 LES and in observational comparisons to the Tropical Instability Wave Experiment (TIWE, Lien
 453 et al. (1995)). Section 4.b summarizes and contextualizes these LES via comparisons with prior

454 results. Then, sections 4.c-4.g quantify the daily-mean turbulent buoyancy flux $\langle F_b \rangle$, including
 455 the vertical extent of strong mixing (4.c), the energetics of mixing (4.d), and the covariability of
 456 mixing with non-turbulent variables that may facilitate mixing parameterization (4.e-4.g).

457 *a. Metrics of mixing and observational comparisons*

458 Throughout the analysis of the LES we continue to focus on the maximum of the daily-mean
 459 flux profile, but we shift our focus from the turbulent heat flux $\langle F_Q \rangle^{max}$ to the turbulent buoyancy
 460 flux $\langle F_b \rangle^{max}$ to leverage links with turbulence energetics, in which F_b appears but not F_Q (see the
 461 Appendix for the relevant equations). However, to facilitate comparisons between the LES and the
 462 MITgcm simulations and the chipod observations (Fig. 7), we often report

$$\frac{\rho c_p}{g \alpha} F_b \approx F_Q, \quad (2)$$

463 where ρ is the reference density of seawater, c_p is the specific heat of seawater, g is the acceleration
 464 due to gravity, and α is the thermal expansion coefficient of seawater. In the LES, the coefficient
 465 fraction is constant 1.37×10^9 [$\text{Wm}^{-2} \text{s}^3 \text{m}^{-2}$] (see the Appendix for details) and we apply the same
 466 constant scaling to produce F_Q from the TIWE data in Fig. 7 and 8 . At z_{max} , the relative error in
 467 approximating a constant ratio F_Q/F_b is roughly

$$\frac{N_T^2}{N^2} - 1, \quad (3)$$

468 assuming the turbulent vertical fluxes of temperature and buoyancy can be approximated using local
 469 flux-gradient relationships (i.e., down-gradient diffusion) and have the same turbulent diffusivity
 470 such that

$$\frac{F_Q}{F_b} \approx \frac{\rho c_p \partial T / \partial z}{\partial b / \partial z} = \frac{\rho c_p}{g \alpha} \frac{N_T^2}{N^2}, \quad (4)$$

471 where $N_T^2 = g\alpha\partial T/\partial z$. The errors from this approximation are small; the 68 days of LES estimates
472 of $\langle F_Q \rangle^{max}$ yields an estimate for the mean bias of +6% (-7% and +20% at 0° N and 3° N,
473 respectively) and a standard deviation of 26% (10% and 30% at 0° N and 3° N, respectively).

474 We explicitly compare the LES results to 38 days of observations of DCT from the TIWE at
475 0° N, 140° W in November-December 1991 (Lien et al. 1995). The TIWE dataset is a uniquely
476 good point of comparison in that it includes a similarly long 38 days of hourly-averaged turbulence
477 profiles based on thousands of microstructure casts (roughly 6-7 per hour) as well as relevant ocean
478 velocity and density profiles and surface flux information derived from continuous occupation of a
479 station at 0° N, 140° W by two ships. Although turbulent heat and buoyancy fluxes are not directly
480 measured, they are inferred to within about a factor of two using the relationship $F_b = \Gamma\epsilon$ where ϵ is
481 the observed dissipation rate of turbulent kinetic energy and a mixing efficiency factor is assumed
482 to be a constant $\Gamma = 0.2$ at depths below 20 m for simplicity (Osborn 1980; Gregg et al. 2018).
483 The maximum of the daily mean turbulent buoyancy flux $\langle F_b \rangle^{max}$ is calculated after first binning
484 hourly mean F_b profiles into daily means $\langle F_b \rangle$ at 1 m vertical resolution and then smoothing $\langle F_b \rangle$
485 with a 10 m moving average. The resulting 38-day mean $\langle F_b \rangle^{max}(\rho c_p)/(g\alpha) \approx \langle F_Q \rangle^{max}$ based on
486 the TIWE data is plotted in Fig. 7 and the distribution of the daily means is shown in Fig. 8 for
487 context. As in the analysis of the LES, we apply the assumption of constant $\langle F_Q \rangle^{max}/\langle F_b \rangle^{max}$ to
488 the TIWE observations (in Fig. 7). We estimate that this assumption yields larger but still modest
489 high bias in the $\langle F_Q \rangle^{max}$ of up to about +30%, which is smaller than the factor of 2 observational
490 uncertainty. Hourly mean velocity and density from the ADCP and CTD respectively are extended
491 to the surface by replicating the top reliable value before calculating vertical gradients in horizontal
492 velocity and buoyancy and related derived quantities.

493 *b. Summary and context*

494 We chose to run LES at 0° N and 3° N along 140° W in October 1985, which was characterized
495 by neutral Oceanic Niño Index, so mixing is expected to be reasonably strong but not maximal
496 both at and north of the equator (Figs. 4 and 7; see also Warner and Moum (2019); Huguenin et al.
497 (2020); Deppenmeier et al. (2021)). Tropical instability waves are a dominant cause of subseasonal
498 variability in currents and density in the LES and are also an important driver of aseasonal variations
499 in mixing (e.g. Moum et al. 2009; Cherian et al. 2021). The 34 day simulations are just long enough
500 to span one full tropical instability wave period, but the tropical instability wave spanned by these
501 LES is not especially strong. The peak-to-trough amplitude of the meridional velocity averaged
502 from 25 to 75 m is only 45 cm/s at 0°N and 88 cm/s 3°N (Fig. 9). For comparison, the peak-to-
503 trough amplitude of the meridional velocity variability during the TIWE is about 50 cm/s (plate 3
504 in Lien et al. 1995) and quite similar to the LES at the same site, even though tropical instability
505 waves were weak during the TIWE due to the onset of El Niño conditions. In contrast, Moum
506 et al. (2009) observed strong turbulent mixing in the presence of a strong tropical instability
507 wave with peak-to-trough meridional velocity amplitude of about 1.5 m/s at 0°N, 140°W during
508 October-November 2008 in La Niña conditions (see also Inoue et al. 2012, 2019).

509 We find that the mixing in the LES qualitatively reflects the seasonal, interannual and mesoscale
510 context. The 34-day mean $\langle F_Q \rangle^{max}$ in the LES at 0° N (about 110 W/m²) is just above the
511 minimum of the eighteen October means simulated from 1999-2016 in the MITgcm. In addition,
512 the LES parent ROMS simulation with the same KPP mixing scheme as the MITgcm also has
513 a rather low mean $\langle F_Q \rangle^{max} \approx 140$ W/m² (compared to an October mean of about 275 W/m² in
514 the MITgcm), suggesting that the large-scale conditions (e.g., shear, stratification, and air-sea
515 fluxes) in the simulated October 1985 are not exceptional but not as conducive to strong mixing

516 as is typically the case from 1999-2016. However, the 34-day mean $\langle F_Q \rangle^{max}$ is still larger than
 517 the 38-day mean $\langle F_Q \rangle^{max}$ from the TIWE observations (77 W/m²) and about 50% above the
 518 climatological $\langle F_Q \rangle^{20-60}$ (averaged from 20-60 m depth) from chipod observations in October.
 519 Noting that $\langle F_Q \rangle^{max} / \langle F_Q \rangle^{20-60} \approx 1.5 - 2$ in the MITgcm, these results suggest that the mixing in
 520 the LES is fairly typical for October. Consistent with this conclusion, the mixing in our LES is
 521 also stronger than that simulated in the LES of Sarkar and Pham (2019) (see also Pham et al.
 522 2017), in which the resolved turbulent heat flux was about 60 W/m² and $\epsilon \approx 10^{-7}$ m²/s³ at the
 523 maximum MLD over three days in October at 0° N, 140° W (compared to $\langle F_Q \rangle^{max} \approx 110$ W/m²
 524 and $\langle \epsilon \rangle^{max} \approx 3 \times 10^{-7}$ m²/s³ here). Conversely, the mixing in our LES is substantially weaker
 525 than the especially strong mixing (with time-mean $F_Q \approx 400$ W/m² and $\epsilon \approx 10^{-6}$ m²/s³) observed
 526 by Moum et al. (2009) at 0° N, 140° W in the midst of a strong tropical instability wave during
 527 October-November 2008 in La Niña conditions. Finally, the time-averaged $\langle F_Q \rangle^{max}$ in the LES at
 528 3° N, 140° W is about 30 W/m², that is 1/4 to 1/3 of the magnitude in the LES at 0° N, 140° W.
 529 This ratio of $\langle F_Q \rangle^{max}$ at 3° N to 0° N is approximately consistent with the climatological ratio from
 530 1999-2016 found in the MITgcm even though the mixing in the LES is weaker at both latitudes
 531 (Fig. 4f) .

532 Consistent with earlier studies, we find that the diurnal cycle is the dominant mode of temporal
 533 variability in the turbulence near the surface, and the simulated diurnal cycles at 0° N, 140° W
 534 exhibit many of the previously observed and simulated features of DCT at that location (Gregg
 535 et al. 1985; Moum et al. 1989; Schudlich and Price 1992; Peters et al. 1994; Lien et al. 1995;
 536 Wang et al. 1998; Large and Gent 1999; Danabasoglu et al. 2006; Smyth et al. 2013; Pham et al.
 537 2013, 2017; Smyth et al. 2017; Sarkar and Pham 2019; Pei et al. 2020; Cherian et al. 2021).
 538 For example, F_Q is shown in Figs. 10a and 11a and can be compared to the time series of
 539 the dissipation rate of turbulent kinetic energy ϵ observed during the TIWE in Plate 7 of Lien

540 et al. (1995) ($\epsilon \approx 5F_b \approx 4F_Q/10^9 \text{ m}^2/\text{s}^3$ below the MLD; see also Fig. 12). During the daytime,
541 shortwave radiation stratifies a shallow near-surface layer where wind-driven turbulence is confined
542 and accelerates a near-surface current with strong vertical shear. During the afternoon and early
543 evening, the stabilizing net surface buoyancy flux weakens and eventually becomes destabilizing.
544 The near-surface shear and stratification descend downward toward the highly sheared and stratified
545 but marginally unstable layer below, where $Ri_g \approx 1/4$ (Fig. 12). At the same time, strong turbulent
546 heat and momentum fluxes F_Q and \mathbf{F}_m as well as dissipation rates ϵ descend downward as well
547 (Figs. 11-13; see the Appendix for definitions). During nighttime and early morning, turbulence
548 penetrates deeply below the MLD and into the stratified thermocline (i.e., between about 30 and
549 90 m depth), where downward turbulent heat fluxes F_Q reach a subsurface maximum of hundreds
550 of W/m^2 . Strong turbulent momentum fluxes extract kinetic energy from the shear to drive strong
551 heat fluxes and dissipation rates in the thermocline (Figs. 11-13; the energetics is quantified in
552 section 4.d). The strong turbulence that is energized locally below the MLD often persists there
553 for hours while the extent and intensity of the near-surface turbulence decline with increasing
554 solar radiation in the morning. In addition, on many nights and mornings there are 2-4 bursts of
555 particularly strong turbulence that cause the heat flux to be elevated by up to hundreds of W/m^2 for
556 hours (Fig. 11a) as observed (Smyth et al. 2017).

557 At first glance, the diurnal cycles of turbulent heat fluxes F_Q at 3° N in Fig. 10b seem to
558 differ qualitatively from those at 0° N , consistent with the hypothesis that equatorial turbulence is
559 enhanced relative to turbulence at higher latitudes due to DCT associated with the strong mean
560 shear between the eastward undercurrent and the westward surface south equatorial current (Figs.
561 1a and 9a). However, DCT and strong heat and momentum fluxes do occur at 3° N in conjunction
562 with strong vertical shear of horizontal velocity (Figs. 9b,d), most prominently on November 3, 4,
563 and 5 when the subsurface turbulence at 3° N exhibits all of the qualitative features described in the

564 previous paragraph in reference to the DCT at 0° N (Figs. 11-13). In addition, some days in early
565 and mid October exhibit downward turbulent heat fluxes F_Q below the MLD, although the intensity
566 of these subsurface heat fluxes is weaker than most days at 0° N and there are no obvious nighttime
567 turbulent bursts. These results add significant new support to the hypothesis that DCT occurs off
568 the equator. Off-equatorial DCT has previously been hypothesized based on ocean model results
569 with fully parameterized DCT (Pei et al. 2020; Cherian et al. 2021) but has not been previously
570 simulated in LES or observed in microstructure. Although the diurnal cycle of DCT remains a
571 topic of interest for future analysis of our LES, this topic has received substantial attention in prior
572 LES studies (Wang et al. 1998; Large and Gent 1999; Pham et al. 2013, 2017; Sarkar and Pham
573 2019) and we leave further analysis of the diurnal cycle in these LES to future work.

574 The objective of this analysis of the LES is to build understanding of the subseasonal variability
575 of the daily-mean $\langle F_Q \rangle$ on timescales from days to weeks, building on our analysis of the regional
576 MITgcm. The distributions of $\langle F_Q \rangle^{max}$, z_{max} , and $\langle Q_0^{net} \rangle$ in Fig. 8 show how this variability
577 simulated in the LES compares to the variability in the MITgcm and observed in the TIWE data
578 and generally support the suggestion that the LES are representative of fairly typical conditions
579 in October. As explored in more detail in the subsequent sections, a motivating hypothesis (e.g.,
580 Cherian et al. 2021; Smyth et al. 2021) is that the spatio-temporal variability in the vertical shear
581 in the upper ocean (which is defined more precisely later, but see Figs. 9 and 12c-d) is perhaps the
582 most important driver of the day-to-day and spatial variability in DCT and $\langle F_Q \rangle^{max}$ (e.g., in Fig. 8b
583 and Fig. 10). This vertical shear is strong on average above the equatorial undercurrent along the
584 equator, but the shear is also highly variable and intermittently strong throughout the cold tongue
585 (e.g., as shown in Fig. 9) due to a variety of interacting equatorial waves and instabilities (Moum
586 et al. 2009; Inoue et al. 2012; Jing et al. 2014; Tanaka et al. 2015; Holmes and Thomas 2015,
587 2016; Inoue et al. 2019; Liu et al. 2019b,a; Pei et al. 2020; Liu et al. 2020; Cherian et al. 2021).

588 Hence, strong DCT and $\langle F_Q \rangle^{max}$ vary in time and space and occur intermittently throughout the
589 cold tongue (and at 3° N specifically) when the shear is strong. Over the next few sections, we
590 explore the hypothesis that shear covaries with $\langle F_Q \rangle^{max}$ on and off the equator and more generally
591 seek to identify covariates that provide information about $\langle F_Q \rangle^{max}$ without direct simulations or
592 observations of turbulence.

593 *c. Shear, stratification, Richardson numbers, and the vertical extent of strong turbulence*

594 Previous studies have identified the gradient Richardson number of the horizontally-averaged
595 profile Ri_g (defined in (1)) as an important indicator of the occurrence of DCT and strong ocean
596 mixing in the equatorial Pacific (Pacanowski and Philander 1981; Peters et al. 1988; Large et al.
597 1994; Smyth and Moum 2013). Consistent with these previous studies, we find that Richardson
598 numbers provide some useful information about the spatio-temporal structure and in particular the
599 vertical extent of strong mixing in the LES and the TIWE observations. Below, we show that two
600 Richardson numbers, both of which are based on the horizontally-averaged velocity and density
601 profiles, can be used to model the depth z_{max} where daily mean turbulent vertical heat fluxes $\langle F_Q \rangle$
602 are maximum as well as the daily maximum depth z_{pen} to which strong turbulence penetrates. We
603 define z_{pen} based on a constant threshold in the dissipation rate of turbulent kinetic energy ϵ . It
604 is reasonably straightforward to identify a depth z_{pen} from inspection of time-depth series of ϵ or
605 F_Q profiles (as in the mid-latitudes, see Brainerd and Gregg 1995). After brief trial and error, we
606 identify the shallowest depth where $\epsilon < 2 \times 10^{-8} \text{ m}^2/\text{s}^3$ to be a useful threshold applicable to both
607 of the LES (Figs. 11-13) and the TIWE observations. For reference, this ϵ threshold corresponds
608 to a turbulent heat flux of roughly 7 W/m^2 , which is an order of magnitude smaller than typical
609 $\langle F_Q \rangle^{max}$ and about two orders of magnitude smaller than peak nighttime heat fluxes F_Q during
610 turbulent bursts.

611 The depth z_{max} varies from about 10 to 70 m at 0° N and from 20 to 60 m at 3° N over timescales
 612 ranging from days to weeks (black plus symbols in Fig. 10; see also Fig. 8f). The occurrence of
 613 z_{max} deeper than the nighttime MLD is hypothesized to be an indicator of DCT and strong heat
 614 fluxes. Consistent with this suggestion, the nighttime maximum MLD is shallower than z_{max} at 0°
 615 N on 29 of 34 days and 9 m shallower on average, but the nighttime MLD is deeper than z_{max} at
 616 3° N on 32 of 34 days and 9 m deeper on average. Qualitatively, we interpret these results as an
 617 indication that DCT occurs about 85% of the time at 0° N and about 5% of the time at 3° N, but
 618 there is not a one-to-one correspondence between DCT and z_{max} deeper than MLD as demonstrated
 619 on 11/03-11/04 at 0° N, 140° W in Fig. 11. Although the nighttime maximum MLD is somewhat
 620 correlated with the depth z_{max} , the relationship is in fact fairly scattered and the nighttime MLD
 621 can only explain about 30% of the variance in z_{max} across both LES. On the other hand, about half
 622 of the simulated variance in the depth z_{max} can be explained by H_{Rib} ($r^2 = 0.5$), the depth at which
 623 the mean-profile bulk Richardson number $Ri_b = 0.2$. Here,

$$Ri_b = \frac{\Delta b H_{Rib}}{\Delta u^2 + v_t^2}, \quad (5)$$

624 where Δb and Δu are the bulk buoyancy and velocity differences between the depth H_{Rib} and the
 625 top $0.1H_{Rib}$, v_t is a turbulent velocity scale that depends on the surface forcing as in Large et al.
 626 (1994), and the depth H_{Rib} is identified iteratively using the default parameters of Large et al.
 627 (1994) in an implementation of KPP by Smyth et al. (2002) (Fig. 14a). The inclusion of v_t in Ri_b
 628 systematically deepens H_{Rib} by 6 m on average, but has marginal and probably insignificant benefit
 629 on the best linear model or correlation with z_{max} (increasing r^2 by 15%). The specific threshold
 630 $Ri_b = 0.2$ was chosen via trial and error. Larger and smaller thresholds for Ri_b were not as useful
 631 for identifying z_{max} , but there may be room for future refinement of the model for z_{max} , because
 632 half of the variance in z_{max} is not explained by H_{Rib} .

633 The deepest depth to which DCT penetrates each day z_{pen} also varies significantly from about 40
 634 to 90 m at 0° N and from about 35 to 85 m at 3° N (Fig. 10). And again, the Richardson number—in
 635 this case the local gradient Richardson number Ri_g of the horizontally-averaged profiles— provides
 636 useful information about z_{pen} each day. In particular, we define H_{Rig} as the base of the deep-cycle
 637 layer, which is defined by a low gradient Richardson number $Ri_g < 0.35$. In practical applications
 638 (e.g., to the TIWE data), Ri_g is noisy and the definition of H_{Rig} requires some additional logic
 639 and filtering. In particular, the deep-cycle layer is defined by applying a rectangular filter of about
 640 35 hours and 35 meters depth to a logical field that equals one where $Ri_g < 0.35$ and the depth
 641 is below the daily maximum H_{Rib} . The second threshold based on H_{Rib} is necessary because Ri_g
 642 sometimes rises to high values within the weakly stratified turbulent boundary layer above H_{Rib} ,
 643 particularly at 3° N and even fairly deep within H_{Rib} during nighttime (Fig. 12e-f). With regard
 644 to Ri_g , a threshold $Ri_g = 0.25$ has a theoretical basis that makes it appealing (Miles 1961; Howard
 645 1961; Holt et al. 1992; Rohr et al. 1988), and $Ri_g = 0.25$ has been used previously for identifying
 646 the base of the deep-cycle layer in observations at 0° N, 140° W (Lien et al. 1995; Smyth et al.
 647 2021). However, we found via trial and error that a somewhat larger threshold $Ri_g = 0.35$ is more
 648 useful across the LES at 0° N and 3° N as well as the TIWE observations. Our approach is also
 649 supported by the LES of Pham et al. (2017), in which simulated turbulent bursts penetrate below
 650 the layer defined by a threshold $Ri_g = 0.25$ in DCT as in our LES. A linear regression on H_{Rig} ,
 651 $-6 + 1.1H_{Rig}$ has slope near one, intercept near zero, and explains 80% of the variance in the
 652 daily-maximum z_{pen} (see Fig. 14b).

653 Finally, it may be noted that these relationships between z_{max} , H_{Rib} , z_{pen} and H_{Rig} are useful
 654 beyond the LES. For example, the TIWE observations reveal similar variability and relationships
 655 between z_{max} , H_{Rib} , z_{pen} , and H_{Rig} as the LES at 0° N (cf. blue stars and black + symbols in
 656 Figs. 14a-b). And, H_{Rig} is also a useful lower boundary for the deep-cycle layer in the MITgcm

657 regional model with DCT parameterized by KPP (section 3), but the threshold has to be increased
 658 to $Ri_g = 0.5$ (Cherian et al. 2021).

659 *d. (Non)local energetics of $\langle F_b \rangle^{max}$*

660 To begin to understand why the intensity of $\langle F_b \rangle^{max}$ varies in time and space, it is useful to
 661 consider these variations in the context of the daily mean turbulent kinetic energy budget under
 662 the premise that some of the variability in $\langle F_b \rangle^{max}$ is related to variations in the kinetic energy
 663 available to drive turbulent mixing (see the Appendix for details). In this kinetic energy budget,
 664 the tendency or rate of change of turbulent kinetic energy is driven by vertical transport $\langle T \rangle^{max}$,
 665 shear production $\langle SP \rangle^{max} = \langle \mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z \rangle^{max}$, dissipation $\langle \epsilon \rangle^{max}$, and buoyancy flux $\langle F_b \rangle^{max}$ (Fig.
 666 15a-b). Integrated over a full day, the budget is dominated by a net source due to shear production
 667 and net sinks due to buoyancy flux and dissipation at z_{max} . That is, all other terms (tendency and
 668 vertical transport) are sub-dominant in all but one day and contribute less than 20% of the energy
 669 for dissipation and buoyancy flux $\langle \epsilon \rangle^{max} + \langle F_b \rangle^{max}$ when mixing is strong (roughly $\langle F_b \rangle^{max} > 10^{-7.5}$
 670 m^2/s^3 ; see Fig. 15a). Hence, the shear production of turbulent kinetic energy at z_{max} $\langle SP \rangle^{max}$
 671 is highly correlated with $\langle \epsilon \rangle^{max} + \langle F_b \rangle^{max}$ ($r^2 = 0.98$; Fig. 15a). In addition, when mixing is
 672 strong, $\langle F_b \rangle^{max}$ is in approximately constant proportion to $\langle SP \rangle^{max}$ (about 0.2) and to $\langle \epsilon \rangle^{max}$
 673 (about 0.25) (Figs. 15a-b). When the buoyancy flux is weaker $\langle F_b \rangle^{max} < 10^{-7.5} \text{m}^2/\text{s}^3$, the ratio
 674 $Ri_f^{-1} = \langle SP \rangle^{max} / \langle F_b \rangle^{max}$ declines from 5 to ~ 2 as $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ decreases from 5 to
 675 0.5 and $\langle F_b \rangle^{max}$ weakens to $10^{-8.5} \text{m}^2/\text{s}^3$ (Figs. 15b-c). Here, Ri_f is the flux Richardson number
 676 (e.g., Osborn 1980; Venayagamoorthy and Koseff 2016). In addition, the relationship between
 677 Ri_f^{-1} and Ri_g^{-1} is associated with a relationship between Ri_g^{-1} and the turbulent Prandtl number
 678 $Pr_t^{-1} = Ri_f / Ri_g$, which quantifies how the turbulent diffusivity of buoyancy declines relative to
 679 the turbulent viscosity as Ri_g^{-1} decreases (Fig. 15d). Finally, it is notable that the turbulent kinetic

energy budget contains significant non-local (transport) contributions at low $\langle F_b \rangle^{max} < 10^{-7.5}$ m^2/s^3 . In particular, transport $\langle T \rangle^{max} \approx \langle F_b \rangle^{max} + \langle \epsilon \rangle^{max} - \langle SP \rangle^{max}$ becomes a more significant and scattered contributor to the dissipation and buoyancy flux, as $\langle T \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$ reaches values of 40% and takes both signs (Fig. 15a).

In summary, when mixing is strong ($\langle F_b \rangle^{max} > 10^{-7.5} m^2/s^3$), the energetics are dominantly local to the depth z_{max} with shear production balanced by dissipation plus buoyancy flux and nearly constant $Ri_f \approx 0.2$ and $Ri_g \approx 0.25$ both on and off the equator. However, the energetics of $\langle F_b \rangle^{max}$ in general (including weaker values) are more complex: the energetics are approximately local on average, but non-local (transport) contributes 10-40% to the energetics on many days and takes both signs. In addition, Ri_f systematically varies with Ri_g , both of which take values substantially higher than the canonical values ($Ri_f \approx 0.2$ and $Ri_g \approx 0.25$) on most days at $3^\circ N$. At $0^\circ N$, the canonical DCT and local dynamics are the norm, but at $3^\circ N$ the canonical DCT and local dynamics are the exception rather than the norm. The simulated energetic relationships encapsulated in relationships between Ri_f , Pr_t and Ri_g (Fig. 15c-d) are qualitatively consistent with observations in the atmospheric boundary layer (Anderson 2009), a previous LES of ocean turbulence under a hurricane in the coastal mid-latitudes reported by Watkins and Whitt (2020), and direct numerical simulations (Venayagamoorthy and Koseff 2016). However, it still remains somewhat uncertain whether the relationships modeled here in the LES are in any sense universal, especially given the significance of non-local (transport) dynamics at weak $\langle F_b \rangle^{max}$.

e. Scaling $\langle F_b \rangle^{max}$ based on the horizontally-averaged velocity and buoyancy profiles

Building on the result that $\langle F_b \rangle^{max}$ varies in concert with other metrics of the turbulence energetics such as the shear production and dissipation rate, this section demonstrates how the intensity of $\langle F_b \rangle^{max}$ covaries with readily measured or simulated non-turbulent variables such as horizontally-

703 averaged velocity and buoyancy profiles as well as the surface momentum and buoyancy fluxes. In
 704 a second step, we evaluate scaled predictions of $\langle F_b \rangle^{max}$ derived from the LES results by applying
 705 the scaling to the independent TIWE observations.

706 We begin by quantifying the relationship between the mean profile Ri_g and the intensity of
 707 mixing at z_{max} motivated by popular existing parameterizations of the local intensity of turbulent
 708 diffusion as a function of Ri_g (Pacanowski and Philander 1981; Peters et al. 1988; Large et al.
 709 1994). We find that the simulated inverse Richardson number Ri_g^{-1} at z_{max} can explain most
 710 of the simulated variability in $\langle F_b \rangle^{max}$ across the LES at both 0° and 3° N (Fig. 16a; $r^2 = 0.6$
 711 for the regression $\log_{10}(\langle F_b \rangle^{max}) \sim \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$). On the other hand, Ri_g^{-1} on its own does
 712 not explain the temporal variability in $\langle F_b \rangle^{max}$ very well at 0° N in either the LES ($r^2 = 0.2$) or
 713 the TIWE observations ($r^2 = 0.0$). These results are consistent with the hypothesis that Ri_g is a
 714 useful predictor of the intensity of mixing across a range of Ri_g that includes marginal instability
 715 ($1 \gtrsim Ri_g \gtrsim 0.25$, as at 3° N) but a poor predictor of the intensity of mixing when marginal instability
 716 is either persistent ($Ri_g \approx 0.25$, as at 0° N) or marginal instability never occurs and $Ri_g \gg 0.25$
 717 is always very large (for background on marginal instability, see Thorpe and Liu 2009; Smyth
 718 and Moum 2013; Smyth 2020). For better comparison with previous studies, we also show that
 719 variations in the effective turbulent diffusivity of buoyancy at z_{max} ($K_b = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$) are
 720 more weakly correlated with Ri_g^{-1} ($r^2 = 0.2$ for $\log_{10}(K_b) \sim \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ in LES; $r^2 = 0.0$ in
 721 TIWE) and thus not well-explained by Ri_g^{-1} (Fig. 16b) or Ri_g -based parameterizations (Pacanowski
 722 and Philander 1981; Peters et al. 1988; Large et al. 1994). However, it may be noted that the
 723 underlying variables in the regressions for K_b and $\langle F_b \rangle^{max}$ are actually the same, $\langle S^2 \rangle^{max}$, $\langle N^2 \rangle^{max}$
 724 and $\langle F_b \rangle^{max}$, which suggests that the relatively poor correlation between $\log_{10}(K_b)$ and Ri_g^{-1} may
 725 be improved by simply reformulating the predictor function of $\langle S^2 \rangle^{max}$ and $\langle N^2 \rangle^{max}$. Indeed, a
 726 general two-variable linear regression of $\log_{10} K_b$ on $\log_{10} \langle S^2 \rangle^{max}$ and $\log_{10} \langle N^2 \rangle^{max}$ yields an

727 $r^2 = 0.6$ for $\log_{10}(K_b) \sim \log_{10}(\langle S^2 \rangle^{max} (\langle N^2 \rangle^{max})^{-3/2})$. In summary, although the LES yield results
728 that are loosely consistent with previous studies (e.g., Fig. 16b), there is significant room to
729 improve parameterizations of ocean mixing in the cold tongue. That is, Ri_g is useful but certainly
730 not sufficient to explain all of the spatio-temporal variability in $\langle \epsilon \rangle^{max}$ or $\langle F_b \rangle^{max}$ in the eastern
731 equatorial Pacific (Moum et al. 1989; Zaron and Moum 2009). Other variables and combinations
732 of variables likely contain valuable information about $\langle F_b \rangle^{max}$ in DCT and in general across the
733 cold tongue.

734 In an attempt to refine our understanding of the mean-profile properties that drive temporal vari-
735 ations in $\langle F_Q \rangle^{max} \sim \langle F_b \rangle^{max}$, we conduct a more general multi-variable linear regression analysis
736 with the aim of identifying an optimal power law product (e.g., a product of the generic form
737 $c x^a y^b z^d \dots$, with variables x, y, z, \dots and constants a, b, c, d, \dots to be determined) to model the max-
738 imum buoyancy flux $\langle F_b \rangle^{max}$ as a function of horizontally-averaged and readily-measured (and
739 modeled) properties, including surface fluxes and the horizontally-averaged profiles of velocity
740 and density but without a priori knowledge of the depth z_{max} at which $\langle F_b \rangle^{max}$ occurs. Although a
741 formulation as a power law may seem arbitrary, this choice is motivated by two factors. First, many
742 familiar mixing models are expressed as a product of terms (e.g., a diffusivity times a gradient, or
743 a mixing efficiency times a momentum flux times a shear; e.g., Fig. 16b) and are therefore power
744 laws. In addition, $\langle F_b \rangle^{max}$ is thought to be logarithmically distributed (see section 3.d and Fig. 8),
745 and power laws are readily amenable to linear regression after applying a log-transform.

746 Numerous variables were considered in the regressions, but we only highlight two low-complexity
747 models that we identified. First, the most useful variable that we identified for modelling the
748 combined LES output from 0° N and 3° N is the vertical shear S . In particular, if S_b is a bulk shear
749 defined by a least-squares linear fit to the daily-mean and horizontally-averaged velocity profile
750 from H_{Rig} to 5 m depth, then we find that S_b alone can explain about 70% of the daily variance

751 in $\langle F_b \rangle^{max}$ from both the LES at 0° N and 3° N (Fig. 17a; $\langle F_b \rangle^{max} \approx 3 \times 10^{-6} |S_b|^{0.9}$; $r^2 = 0.7$ in
 752 \log_{10} space ignoring the TIWE data). In an encouraging result, independent validation of the S_b
 753 scaling of $\langle F_b \rangle^{max}$ on the TIWE data is quite good ($r^2 = 0.5$ with little mean bias) and even better
 754 than the LES at 0° N alone ($r^2 = 0.2$). In addition, including the TIWE data in the regression in
 755 Fig. 17c has little impact on the optimal linear model, which seems fairly robust with relatively
 756 narrow confidence intervals on the parameters (cf. Figs. 17a,c). However, the model fit to the LES
 757 $\langle F_b \rangle^{max}$ can be improved substantially by adding the surface friction velocity due to the wind stress
 758 $u_* = \sqrt{|\tau|/\rho}$ as a variable (τ is the wind stress vector). The optimal linear model based on these
 759 two variables $\langle F_b \rangle^{max} \approx 0.16 |S_b|^{0.98} u_*^{2.1}$ explains about 90% of the LES variance and 70% at 0°
 760 N or 3° N alone (Fig. 17b). In independent validation on the TIWE data, the two-variable model
 761 explains only 40% of the TIWE variance and also has a slight mean bias (Fig. 17b). Including the
 762 TIWE observations in the two-variable regression in Fig. 17d leads to a fairly substantial change
 763 in the optimal two-variable model $0.0017 |S_b|^{0.92} u_*^{1.2}$ and somewhat reduces the correlation at 3°
 764 N in the LES but reduces the mean bias in the TIWE data and slightly improves the corresponding
 765 correlation (cf. Figs. 17b,d). These results suggest that although wind stress certainly provides
 766 useful information about $\langle F_b \rangle^{max}$, the available data (including 108 days spanned by the LES and
 767 TIWE) is only marginally sufficient to provide a robust linear model based on both S_b and u_* .

768 For reference, the 95% confidence intervals for various optimal power laws stated in the previous
 769 paragraph and obtained via regression are given in the caption of Fig. 17. Consistent with the
 770 above discussion, only the power on S_b is tightly constrained to be within 0.7 and 1.1. There is
 771 substantial joint uncertainty in the power on u_* (which may range from 0.9-2.5) and the magnitude
 772 of the constant coefficient (which may range from 3×10^{-4} to 2). The coefficient, which in general
 773 has units, is smaller if the power on u_* is lower, and conversely the coefficient is larger if the power
 774 on u_* is larger. Assuming a fixed relationship $\langle F_b \rangle^{max} \sim u_*^2 S_b$ and regressing $\langle F_b \rangle^{max}$ on $u_*^2 S_b$ (the

775 exponents of which yield an appealingly unitless coefficient) yields 95% confidence intervals on
 776 the slope of 0.15 to 0.19, an intercept indistinguishable from zero, and $r^2 = 0.82$. Applying \log_{10} to
 777 both sides before regressing puts more weight on accuracy at weaker $\langle F_b \rangle^{max}$ and yields confidence
 778 intervals on the intercept of $[-1.63, -0.33]$, which corresponds to a coefficient ranging from 0.02
 779 to 0.47 in the power law. Either way, it seems that $\langle F_b \rangle^{max} \approx 0.2u_*^2 S_b$ is a plausible model with
 780 roughly a factor of 3 uncertainty. Although many other variables were considered, we found at best
 781 marginal improvements in the correlations (e.g., when adding a measure of stratification and/or the
 782 net surface buoyancy flux to create multi-variate linear regressions) and many lower correlations
 783 if shear and/or wind stress is omitted or the definition of the shear is changed. Hence, we do not
 784 report any further results of our statistical modelling.

785 *f. Discussion of the empirical power law scaling of $\langle F_b \rangle^{max}$ in light of prior results*

786 In the context of DCT on the equator, it is neither surprising nor novel that shear and wind
 787 stress are correlated with the intensity of mixing. Several previous studies have identified such
 788 relationships using observations and theory (Moum and Caldwell 1985; Pham et al. 2017; Smyth
 789 et al. 2017, 2021). In addition, we reanalyzed the results from the LES of Wang et al. (1998) (see
 790 also Large and Gent 1999), nominally at 0° N, 140° W, and found that those results are consistent
 791 with the $\langle F_b \rangle^{max} \sim 0.2u_*^2 S_b$ scaling identified empirically here to the degree that it is reasonable to
 792 make claims of consistency, which is only within a factor of 3. However, the application of such
 793 a relation beyond 0° N, 140° W and in situations without DCT as well as the precise formulation
 794 of the statistical models proposed in section 4.e and Fig. 17 are new and somewhat unintuitive
 795 in light of the energetics of $\langle F_b \rangle^{max}$, which indicate dominantly local dynamics remote from the
 796 surface forcing. Hence, we find it useful to see how the empirical scalings in section 4.e relate to
 797 the turbulent energetics discussed in section 4.d. In addition, we briefly discuss how the scalings

798 relate to a theory previously developed by Smyth et al. (2017) to model DCT at 0° N, 140° W and
 799 compare the results from LES with analogous results derived from the KPP scheme (Large et al.
 800 1994) in the parent ocean model ROMS.

801 To reveal how the energetics at z_{max} (e.g., Fig. 15) relates to the scaling derived via linear
 802 regression and shown in Fig. 17b, we write:

$$\frac{\langle F_b \rangle^{max}}{0.2u_*^2|S_b|} = \frac{Ri_f \Theta}{0.2} \frac{|\langle \mathbf{F}_m \rangle^{max}|}{u_*^2} \frac{|\langle S \rangle^{max}|}{|S_b|} \quad (6)$$

803 and quantify how the local turbulent momentum flux $\langle \mathbf{F}_m \rangle^{max}$ and vertical shear $\langle S \rangle^{max}$ at z_{max} relate
 804 to the bulk shear S_b and friction velocity squared u_*^2 in the scaling. Here, $Ri_f = \langle F_b \rangle^{max} / \langle SP \rangle^{max}$
 805 is the flux Richardson number at z_{max} , $\langle SP \rangle = \langle \mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z \rangle$ is the daily-mean shear production,
 806 and $\Theta = \langle \mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z \rangle^{max} / (|\langle \mathbf{F}_m \rangle^{max}| |\langle \partial \mathbf{u}_h / \partial z \rangle^{max}|)$ is a dimensionless measure of the combined
 807 effects of misalignment and sub-daily correlations between shear and momentum flux on shear
 808 production at z_{max} . Various ratios of terms in this expression are plotted in Fig. 18. We interpret
 809 these results in two parts, focusing first on instances of strong mixing and DCT and then on
 810 instances of weaker mixing.

811 First, we recall that strong mixing (roughly $\langle F_b \rangle^{max} > 10^{-7.5} \text{ m}^2/\text{s}^3$) tends to be in a state of
 812 marginal instability with fairly uniform $Ri_g^{-1} \approx 4$ (Fig. 15), i.e. only the yellow, orange and red
 813 colored points are associated with strong mixing in Fig. 18. For these points, the ratios on the
 814 right side of (6) are fairly simple: $\Theta \approx 1$ (Fig. 18a), $Ri_f \approx 0.2$ (Fig. 18b,d), $|\langle S \rangle^{max}| / |S_b| \approx 1$
 815 (Fig. 18b-c), and $|\langle \mathbf{F}_m \rangle^{max}| / u_*^2$ ranges from about 0.3 to 1.1 (Fig. 18c-d). That is, our empirical
 816 $0.2u_*^2|S_b|$ scaling derived via regression can reasonably be interpreted as a proxy for local dynamics
 817 at z_{max} with 0.2 a proxy for Ri_f at z_{max} , u_*^2 a proxy for the momentum flux at z_{max} , and $|S_b|$ a
 818 proxy for the shear at z_{max} .

819 In the presence of strong mixing and DCT at 0° N, 140° W ($\langle F_b \rangle^{max} > 10^{-7.5} \text{ m}^2/\text{s}^3$), the empirical
 820 $0.2u_*^2|S_b|$ scaling from the LES is also consistent with the theory of Smyth et al. (2017), which
 821 yields $F_b \approx 0.2\epsilon$ where $\epsilon \approx u_*^2|S_b|$ in steady state. To briefly summarize Smyth et al. (2017), the
 822 theory explicitly models the shear and turbulent kinetic energy in the deep-cycle layer, which is
 823 defined to be a layer of thickness H with homogeneous shear S_b and turbulent kinetic energy k from
 824 the base of the mixed layer to the top of the undercurrent core. The shear S_b evolves due to changes
 825 in the surface mixed layer velocity, which in turn evolves due to any convergence between the
 826 downward momentum flux at the surface ($u_*^2 = F_m(0)$) and the MLD ($F_m(h)$ where h is the MLD).
 827 The momentum flux is assumed to be dominated by the zonal component, which is about 3 times
 828 stronger than the meridional component at the surface in our LES at 0° N, 140° W. The turbulent
 829 kinetic energy k evolves in the theory due to shear production and dissipation plus buoyancy flux
 830 in the shear layer. That is,

$$\frac{\partial S_b}{\partial t} = \frac{1}{Hh} (u_*^2 - F_m(h)), \quad (7)$$

$$\frac{\partial k}{\partial t} = F_m S_b - \epsilon - F_b, \quad (8)$$

831 following their Eqns. 3.2-3.3. Closure of turbulent fluxes in terms of turbulent kinetic energy
 832 is discussed in Smyth et al. (2017). But, the expressions (7)-(8) suggest that if the shear S_b and
 833 turbulent kinetic energy k are in a steady state then $F_m(h)/u_*^2 \approx 1$, as in the strong DCT simulated
 834 by LES (Fig. 18). In addition, $\epsilon + F_b \approx u_*^2 S_b$. With the additional assumption that $F_b/SP \approx 0.2$,
 835 then $F_b \approx 0.2u_*^2 S_b$.

836 That is, the theory of Smyth et al. (2017) suggests essentially the same mathematical form as the
 837 empirical linear model derived from the LES, although the definition of S_b differs. In their theory,
 838 S_b is interpreted as an average over the deep cycle layer, from H_{Rig} to the daily maximum MLD,
 839 whereas in our empirical model S_b is fit to the velocity profile from H_{Rig} to 5 m depth. However,

840 the different definitions of S_b turn out to have only a small impact on the prediction of $\langle F_b \rangle^{max}$ at
 841 0° N because the two definitions of S_b turn out to be highly correlated and similar in magnitude;
 842 both are also good proxies for the shear at z_{max} . Hence, r^2 is only reduced from 0.8 to 0.7 if S_b is
 843 calculated only in the deep cycle layer, i.e. from H_{Rig} to the deepest MLD during a given day rather
 844 than to 5 m if the data is restricted to the LES at 0° N. This property of the velocity profile may
 845 contribute to the success of our empirical scaling in predicting $\langle F_b \rangle^{max}$ in the TIWE observations,
 846 in which we had to extrapolate the velocity profiles to the surface to define S_b , as well as the relative
 847 success of Smyth et al. (2021) in modeling ϵ from chipods at 0° defining S_b as an average over the
 848 deep cycle layer.

849 So, why are we introducing a new definition of S_b ? The answer is that the new definition turns out
 850 to be crucial off the equator and in instances of weaker mixing ($\langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$), as discussed
 851 in the next paragraph. However, there are also some practical advantages and disadvantages to the
 852 new definition. First, it is more difficult to observe ocean currents above 25 m, and hence more
 853 difficult to calculate our S_b with observations, although ADCPs on modern autonomous platforms
 854 (Shcherbina et al. 2019; Gentemann et al. 2020) and moored ADCPs (Masich et al. 2021) can
 855 sample to 10 m depth or less and current meters can be deployed at shallower depths on moorings
 856 to mitigate issues particular to upward looking ADCPs on moorings. On the other hand, it is
 857 advantageous to define S_b as we do for application in ocean model parameterizations, since it does
 858 not depend on the extra diagnosis and definition of the daily-maximum MLD and our approach
 859 works even when the deep cycle layer has zero thickness.

860 However, the main motivation for the new definition of S_b is that it substantially improves the
 861 predictions of $\langle F_b \rangle^{max}$ off the equator at 3° N and when mixing is weak (roughly, $\langle F_b \rangle^{max} < 10^{-7.5}$
 862 m^2/s^3). The reasons for this improvement are highlighted via the terms in (6): the LES results in Fig.
 863 18 show that $|\langle \mathbf{F}_m \rangle^{max}|/u_*^2$ and $|\langle S \rangle^{max}|/|S_b|$ deviate systematically from 1 and $\langle F_b \rangle^{max}/\langle SP \rangle^{max}$

864 deviates systematically from 0.2 in many instances of weaker mixing at 3° N. In conjunction with
 865 these deviations, $Ri_g^{-1} < 4$ deviates toward stability (i.e., points are colored blue to yellow in Fig.
 866 18; see also Fig. 15). The deviation of $|\langle S \rangle^{max}|/|S_b|$ is indicative of a divergence between our bulk
 867 shear S_b and the shear in the deep cycle layer (used in the theory of Smyth et al. 2017), which has zero
 868 thickness on 3 out of 34 days at 3°N. A practical consequence of this divergence in $|\langle S \rangle^{max}|/|S_b|$ is
 869 that replacing S_b with the shear in the deep cycle layer in the linear model $u_*^2 S_b$ leads to a reduction
 870 in the correlation from $r^2 = 0.7$ to $r^2 = 0.4$ when the data are from only the LES at 3° N. Specifically,
 871 these deviations indicate that the shear is more concentrated at the base of the mixed layer, the wind
 872 contributes more to accelerating the mixed layer than below, and the shear at the base of the mixed
 873 layer is weaker than necessary for marginal instability. All of these features are consistent with a
 874 transition to a mid-latitude inertial regime when the shear, wind stress, and hence turbulent heat
 875 fluxes are sufficiently weak (e.g., Pollard and Millard 1970). In this regime, strong turbulent heat
 876 fluxes like those in strong equatorial DCT only occur intermittently under the right conditions, such
 877 as when the shear and wind are sufficiently strong and well aligned and the system is near a state of
 878 marginal instability (e.g., Pollard et al. 1972; Burchard and Rippeth 2009; Brannigan et al. 2013;
 879 Watkins and Whitt 2020). Yet, the scaling $F_b \approx 0.2u_*^2 S_b$ in combination still approximately holds
 880 when mixing is weaker $10^{-8.5} < \langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$ at 3° N. In addition, it is interesting to
 881 note that a reanalysis of the LES of Watkins and Whitt (2020) showed that $F_b^{max} \sim 0.2u_*^2 S_b$ within
 882 a factor of 3 in hurricane-driven entrainment in the coastal mid-latitude ocean, from $F_b^{max} = 10^{-8}$
 883 to $10^{-5} \text{ m}^2/\text{s}^3$ without time averaging (only horizontal averaging). Analysis of the co-variability
 884 of the ratios in (6) in Fig. 18 shows that the empirical $F_b \approx 0.2u_*^2 S_b$ scaling continues to perform
 885 reasonably well for $\langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$ at 3° N because the changes in the ratios $|\langle F_m \rangle^{max}|/u_*^2$,
 886 $|\langle S \rangle^{max}|/|S_b|$, and Ri_f compensate for each other (Fig. 18). Thus, as the turbulence weakens such
 887 that $\langle F_b \rangle^{max} < 10^{-7.5} \text{ m}^2/\text{s}^3$, it is more difficult to interpret the empirical scalings from section

888 4.e as proxies for local dynamics at z_{max} or using the theory for homogeneous DCT of Smyth
889 et al. (2017). That is, the empirical scaling $F_b \approx 0.2u_*^2 S_b$ can be explained by theory for DCT at
890 $0^\circ, 140^\circ\text{W}$ (Smyth et al. 2017), but the theory cannot explain the applicability of the scaling at
891 $3^\circ, 140^\circ\text{W}$ in the LES.

892 A question that arises at this point is how the results from LES and the parameterization for
893 $\langle F_b \rangle^{max}$ compare with existing mixing parameterizations that are designed to be applicable both on
894 and off the equator (unlike the theory of Smyth et al. 2017), such as the KPP scheme (building on
895 Figs. 7, 8, and 16b). Properly addressing this question is beyond the scope of this paper and a subject
896 of interest for future work, but a comparison between the simulations of $\rho c_p / (g\alpha) \langle F_b \rangle^{max} \approx \langle F_Q \rangle^{max}$
897 in the LES and $\langle F_Q \rangle^{max}$ in ROMS (KPP) highlights substantial differences despite the similar large-
898 scale oceanic and atmospheric forcing. However, minor differences in these large-scale forcings
899 mean that the comparisons should be treated as qualitative rather than quantitative (see section 4.a
900 and the Appendix for further details). In any case, we find that the maximum of the daily-averaged
901 turbulent heat flux $\langle F_Q \rangle^{max}$ ranges over a similar set of values from about 10 to 300 W/m² and the
902 daily variability in the LES and ROMS is correlated in space and time at both 0° and 3° N ($r^2 = 0.5$;
903 see Fig. 19a). However, most of the correlation comes from 3° N, where $r^2 = 0.3$. There is no
904 temporal correlation in $\langle F_Q \rangle^{max}$ between the LES and ROMS at 0° N. The turbulent diffusivity
905 at z_{max} is more scattered than the heat flux with a marginally significant correlation across space
906 and time, and no temporal correlation at either latitude individually (Fig. 19b). Similarly, the
907 depth z_{max} is similar in the LES and ROMS on many days, but there are numerous outliers with
908 much deeper z_{max} in ROMS, such that the spatio-temporal correlation between ROMS and the
909 LES is marginal to insignificant (Fig. 19c). These results support earlier indications that the
910 mixing produced by KPP and the LES differ. Yet, these large day-to-day differences in the flux
911 and diffusivity shown in Figs. 19a,b tend to take both signs and add up to fairly subtle impacts on

912 the time-mean temperature and horizontal velocity tendencies due to vertical mixing (see Fig. A1)
913 and therefore the mean velocity and temperature profiles over 34 days, as discussed in section 2.b.

914 *g. Parameterization of $\langle F_b \rangle$ profiles*

915 Finally, it is desirable to incorporate the information gleaned about $\langle F_b \rangle^{max}$ from the LES into
916 parameterizations of flux profiles for application in ocean models or in estimating turbulent fluxes
917 from observations without turbulence data. Although it is beyond the scope of this paper to
918 incorporate the scaling for $\langle F_b \rangle^{max}$ into a complete mixing parameterization, we conclude the
919 paper and motivate future work toward refined mixing parameterizations by presenting the results
920 of a preliminary effort to parameterize the daily-mean buoyancy flux profiles $\langle F_b \rangle$ simulated in
921 the LES based on $\langle F_b \rangle^{max}$. It is important to emphasize that this effort involves a non-exhaustive
922 exploration of a wide range of possible choices and thus is likely sub-optimal. Nevertheless, we
923 find that the results are valuable motivation and guidance for future work and thus worth presenting.

924 More precisely, the objective of this section is to model the daily-averaged net buoyancy flux
925 profile $\langle B \rangle(z)$ from the surface $z = 0$ to the base of the low-Richardson layer $z = H_{Rig}$, below which
926 turbulent mixing is typically much weaker since H_{Rig} is highly correlated with z_{pen} (Fig. 14). That
927 is, we seek to model

$$\langle B \rangle = \langle P_b \rangle + \langle F_b \rangle, \quad (9)$$

928 the sum of the daily-averaged penetrative and turbulent buoyancy fluxes. We model $\langle B \rangle$ rather than
929 $\langle F_b \rangle$ because $\langle B \rangle$ profiles do not exhibit the exponential structure characteristic of $\langle P_b \rangle$, whereas
930 $\langle F_b \rangle$ profiles do (compare Figs. 10a-b and Figs. 20a-b). Thus, we interpret $\langle B \rangle$ as the residual
931 turbulent flux, after subtracting the part of $\langle F_b \rangle$ that is equal and opposite to the penetrating solar
932 radiative flux $\langle P_b \rangle$ (see the Appendix for details on P_b).

933 Rather than parameterize $\langle B \rangle$ at each depth based on the local properties (as in several previous
 934 studies, e.g., Pacanowski and Philander 1981; Peters et al. 1988; Zaron and Moum 2009), the entire
 935 $\langle B \rangle$ profile on a given day, from the surface to the base of the low- Ri_g layer $z = H_{Rig}$, is modeled
 936 from a shape function and three bulk parameters: the net air-sea fluxes of buoyancy $\langle B(z=0) \rangle$ and
 937 momentum $\langle |\tau|/\rho \rangle$ and the bulk vertical shear of horizontal currents S_b from H_{Rig} to 5 m depth.
 938 We take this bulk parameterization approach because we find that knowing $\langle B \rangle$ at just $z = 0$ and
 939 $z = z_{max}$ is sufficient to explain about 90% of the simulated variance in $\langle B \rangle$ at all depths above
 940 H_{Rig} in the LES at both 0° N and 3° N. In particular, we find that a linear combination

$$\langle B \rangle(z) = w_1(z)\langle B \rangle(z=0) + w_2(z)\langle B \rangle(z=z_{max}) \quad (10)$$

941 explains about 90% of the variance in $\langle B \rangle$ for all depths above H_{Rig} (compare Figs. 20a-b and
 942 Figs. 20e-f), where

$$\langle B \rangle(z=0) = \frac{g\alpha}{\rho c_p} \langle Q_0 \rangle^{net} - g\beta \langle VSF_0^{net} \rangle, \quad (11)$$

$$\langle B \rangle(z=z_{max}) = \langle F_b \rangle(z=z_{max}) + \langle P_b \rangle(z=z_{max}), \quad (12)$$

943 and Q_0^{net} and VSF_0^{net} are given net surface heat and virtual salt fluxes across the air-sea interface
 944 and $\langle P_b \rangle$ is a given penetrative buoyancy flux profile associated with shortwave radiation (see
 945 the Appendix for details). The depth-dependent weights w_1 and w_2 in (10) are piecewise linear
 946 functions of depth, that is

$$w_1 = \frac{z_{max} - z}{z_{max}} \quad \text{for } z \leq z_{max}, \quad (13)$$

$$w_1 = 0 \quad \text{for } z > z_{max}, \quad (14)$$

$$w_2 = 1 - w_1 \quad \text{for } z \leq z_{max}, \quad (15)$$

$$w_2 = \frac{H_{Rig} - z}{H_{Rig} - z_{max}} \quad \text{for } H_{Rig} > z > z_{max}, \quad (16)$$

947 where z , z_{max} and H_{Rib} are all positive depths by definition in the expressions above. It may be
 948 noted that our approach results in piecewise constant heat flux convergence with one value below
 949 z_{max} (with sign of $\langle F_Q \rangle^{max}$) and another above z_{max} (with sign of $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max}$).

950 Having chosen to represent the vertical structure of $\langle B \rangle$ as a piecewise linear function that
 951 depends on $\langle B \rangle$ at just the surface and z_{max} and taking the surface flux as given, the stated objective
 952 of this section is reduced to specifying $\langle F_b \rangle^{max}$ and z_{max} . To recapitulate previous sections, we use
 953 linear regression to model z_{max} ,

$$z_{max} \approx 0.6H_{Rib} + 14, \quad (17)$$

954 since we found that H_{Rib} could explain about half of the variance in z_{max} (Fig. 14a). In addition,
 955 we use the scaling developed and discussed in sections 4.e-4.f (specifically, the one plotted in Fig.
 956 17b) to model

$$\langle F_b \rangle^{max} \approx 0.16u_*^{2.1} S_b^{0.98}. \quad (18)$$

957 With these parameterized representations of $\langle F_b \rangle^{max}$ and z_{max} in (10), we find that this linear
 958 combination explains 75% of the variance in simulated $\langle B \rangle$ above H_{Rib} across both LES (cf. Figs.
 959 20a-b and Figs. 20c-d; $r^2 = 0.6$ and $r^2 = 0.7$ at 0° and 3° N, respectively).

960 To put the flux profiles from the LES and the parameterization (10) in context, compare the
 961 results to those from the KPP output in parent regional ocean model (ROMS) at the LES locations
 962 (Fig. 20). Consistent with earlier results, there are qualitative differences between the flux profiles
 963 in the LES and ROMS (KPP). Perhaps most notably, strong mixing extends to deeper depths in
 964 ROMS (KPP), consistent with many instances of deeper z_{max} (Fig. 19c). The ROMS (KPP)
 965 solution also seems to have a more distinct modulation of mixing on the timescale of the tropical
 966 instability wave than in the LES. However, given the previously identified differences, including
 967 the absence of correlation between the LES and KPP in the depth z_{max} or the turbulent diffusivity

968 at z_{max} in Fig. 19, it is perhaps remarkable how similar the KPP and LES solutions are (see also
969 Fig. A1). In any case, the results of our preliminary effort to parameterize flux profiles suggest
970 that future work is both justified and needed to incorporate information about $\langle F_b \rangle^{max}$ into a more
971 general mixing parameterization that handles momentum and tracer fluxes as well as an explicit
972 diurnal cycle.

973 5. Conclusions

974 This manuscript synthesizes results from submesoscale-permitting regional ocean models and
975 large eddy simulations of turbulence embedded in a regional model to build understanding of
976 deep-cycle turbulence and upper-ocean mixing more generally in the equatorial Pacific Ocean cold
977 tongue at and beyond 0° N, 140° W.

978 First, a submesoscale-permitting regional hindcast simulation of the period 1999-2016 in the
979 MITgcm is used to quantify the climatological mean, seasonal cycle, and aseasonal variability of
980 ocean mixing as measured by the maximum over depth of the daily-mean turbulent vertical heat
981 flux $\langle F_Q \rangle^{max}$. We found that there is a good spatial correlation ($r^2 = 0.7$) between $\langle F_Q \rangle^{max}$ and the
982 time-mean net ocean surface heat flux $\langle Q_0^{net} \rangle$. Although both $\langle F_Q \rangle^{max}$ and $\langle Q_0^{net} \rangle$ are stronger in
983 the cold tongue relative to other areas, there is a prominent equatorial mixing band within about
984 $1\text{-}2^\circ$ of the equator where the time-mean, seasonal-cycle amplitude, and aseasonal variability are
985 much larger in $\langle F_Q \rangle^{max}$ than $\langle Q_0^{net} \rangle$. Aseasonal anomalies in $\langle F_Q \rangle^{max}$ (i.e., all deviations from the
986 climatological seasonal cycle) are uncorrelated with aseasonal anomalies in $\langle Q_0^{net} \rangle$, which suggests
987 that the surface heat flux exerts little control on the aseasonal variability of ocean mixing in the cold
988 tongue. In addition, $\langle F_Q \rangle^{max}$ is logarithmically distributed and exhibits rare but intense mixing
989 events as far as 5° from the equator driven by mesoscale oceanic variability. Thus, strong mixing
990 and DCT are not limited to the equatorial mixing band above the undercurrent, and aseasonal

991 variability in general and infrequent strong mixing events in particular have substantial impacts on
992 the climatologies of mixing across the cold tongue. As a result, the spatial patterns of aseasonal
993 variance and time-mean $\langle F_Q \rangle^{max}$ are very similar. However, it is not clear if the parameterized
994 mechanisms that control mixing in the regional model are realistic. Comparisons between modeled
995 and measured turbulence at 0° N, 140° W suggest that the mixing has a realistic seasonal cycle in
996 the regional model but the time-mean turbulent heat fluxes may be too strong and there may be too
997 few instances of weak mixing at this location.

998 State-of-the-art large eddy simulations embedded in a regional model simulate the outer scales
999 of turbulence $O(1)$ m as it evolves over 34 days in response to changing atmospheric and oceanic
1000 forcing at both 0° N, 140° W and 3° N, 140° W in October. The time-averaged LES results are
1001 consistent with the spatial pattern of mixing simulated in the regional model. In particular, mixing
1002 is elevated below the surface both on and off the equator, but the time-mean $\langle F_Q \rangle^{max}$ in the LES
1003 is about 3-4 times stronger at 0° N (110 W/m²) than at 3° N (30 W/m²) along 140° W. However,
1004 mixing in the LES is about a factor of two weaker than on average in all Octobers from 1999-
1005 2016 in the MITgcm. More direct comparisons between the mixing in the LES and its parent
1006 regional model ROMS, in which mixing occurs under essentially the same day-to-day oceanic
1007 and atmospheric conditions as in the LES but via the KPP scheme (Large et al. 1994) as in the
1008 MITgcm, also suggest that parameterized mixing in the regional model is stronger and more deeply
1009 penetrating than in the LES, but the time-mean $\langle F_Q \rangle^{max}$ is only 20% lower in the LES. Individually,
1010 these LES results may not be sufficient to conclude that the KPP mixing scheme yields too-strong
1011 mixing in the regional models, but taken with similar conclusions derived from comparisons to
1012 chipod microstructure observations, it seems likely that the time-mean mixing in the cold tongue
1013 is too strong in the regional models and the mixing scheme needs to be modified.

1014 The LES results also provide important insight into the aseasonal variability of mixing and its
1015 covariates on timescales from days to a month and thus facilitate the identification and evaluation of
1016 empirical scalings for ocean mixing that might be applicable across a range of different atmospheric
1017 and oceanic conditions throughout the Pacific Ocean cold tongue and possibly beyond. A highlight
1018 is the finding that a relatively simple two-variable linear model approximately proportional to $u_*^2 S_b$
1019 can explain about 90% of this daily variance in $\langle F_Q \rangle^{max}$ across both LES locations, where u_* is
1020 the surface friction velocity, S_b is the bulk vertical shear of the ocean currents averaged from 5
1021 m depth to H_{Rig} , below which $Ri_g > 0.35$. In an independent validation, this scaling explains
1022 40% of the observed variance in the TIWE observations of Lien et al. (1995), which exhibit a
1023 similar distribution of $\langle F_Q \rangle^{max}$ as the LES at 0°N, 140°W with mean bias that is smaller than the
1024 measurement uncertainty of a factor of two. Even more encouraging is that the empirical scaling
1025 can be interpreted with prior theory by Smyth et al. (2017) at 0°N, 140°W. However, while the
1026 scaling is successful off the equator at 3°N, 140°W, its applicability beyond 0°N, 140°W cannot
1027 be interpreted with the theory of Smyth et al. (2017), nor has it been validated with observations.
1028 Nevertheless, the finding that LES simulates strong DCT at 3°N, 140°W away from the undercurrent
1029 adds significant new evidence in support of these hypotheses that strong DCT, marginal instability,
1030 and intense mixing can occur both with and without the undercurrent, as long as the vertical shear
1031 of upper-ocean currents and (to a lesser degree) the wind stress are sufficiently strong (building on
1032 Pei et al. 2020; Cherian et al. 2021). However, future observational process studies are needed to
1033 refine and likely modify these hypotheses and scalings of ocean mixing throughout the cold tongue
1034 and particularly off the equator. In addition, these results are both a motivation and a promising
1035 foundation for needed refinement of the parameterizations of equatorial mixing in ocean models.

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1055 *Data availability statement.* Data and software necessary to reproduce the figures are published
1056 and links are provided in the references (Whitt 2022).

1057 APPENDIX

1058 **Large eddy simulation methods**

1059 The LES (Taylor 2008; Whitt and Taylor 2017; Watkins and Whitt 2020) solves a filtered version
 1060 of the Navier Stokes equations under the Boussinesq approximation on a traditional f plane along
 1061 with evolution equations for temperature and salinity,

$$\frac{D\mathbf{u}}{\partial t} + \mathbf{f} \times \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{b} + \nabla \cdot (\nu_{sgs} \nabla \mathbf{u}) + \mathcal{F}_{\mathbf{u}} + \mathcal{R}_{\mathbf{u}} + \mathcal{D}_{\mathbf{u}}, \quad (\text{A1})$$

$$\nabla \cdot \mathbf{u} = 0, \quad (\text{A2})$$

$$\frac{DT}{Dt} = \nabla \cdot (\kappa_{sgs} \nabla T) + I + \mathcal{F}_T + \mathcal{R}_T + \mathcal{D}_T, \quad (\text{A3})$$

$$\frac{DS}{Dt} = \nabla \cdot (\kappa_{sgs} \nabla S) + \mathcal{F}_S + \mathcal{R}_S + \mathcal{D}_S, \quad (\text{A4})$$

$$b = -g(1 - \alpha(T - T_0) + \beta(S - S_0)), \quad (\text{A5})$$

1062 where $\mathbf{f} = (0, 0, f)$, $f = 14.6 \times 10^{-5} \sin(\text{latitude}) \text{ s}^{-1}$ is the traditional Coriolis frequency, the
 1063 buoyancy force is $\mathbf{b} = (0, 0, b)$, the density of the seawater is $-\rho b/g$, where the constant reference
 1064 density of seawater $\rho = 1023.5 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$, and the density and buoyancy vary linearly
 1065 with temperature T and salinity S ; $\alpha = 2.96 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$, $T_0 = 25.0^\circ\text{C}$, $\beta = 7.38 \times 10^{-4} \text{ psu}^{-1}$, and
 1066 $S_0 = 35.25 \text{ psu}$. The equations are solved in a horizontally periodic domain that is 108 m deep and
 1067 306 by 306 m square and discretized on a mesh with 216 by 360 by 360 points with a resolution of
 1068 0.5 m vertically by 0.85 m horizontally.

1069 The vertical profiles of temperature, salinity, and horizontal momentum are initialized in the
 1070 LES on October 2, 1985 at 6:00 UTC by interpolating the output of a hindcast from the Regional
 1071 Ocean Modelling System (ROMS) (Shchepetkin and McWilliams 2005; Haidvogel et al. 2008),
 1072 which simulates the period August 1984-February 1986 in a regional ocean domain spanning 95°
 1073 to 170° W and 12° S to 12° N at $1/20^\circ$ (5.5 km) horizontal resolution on 50 terrain-following
 1074 sigma levels (spaced about every 8 m in the top 100 m) as in Holmes and Thomas (2015). The
 1075 interpolation procedure involves first averaging the 6-hour averaged ROMS output horizontally over
 1076 a 3-by-3 array of grid cells (about 16.5 km square) around the LES location and then interpolating

1077 vertically to the LES grid using cubic splines. The ROMS hindcast was used as the parent model
 1078 instead of the MITgcm hindcast described above mainly because it was available with all relevant
 1079 outputs before the MITgcm run was completed. In ROMS, the initial conditions and daily ocean
 1080 side boundary conditions are from the global mesoscale-resolving ocean/sea-ice hindcast used by
 1081 Deppenmeier et al. (2021). Neither model has tides. In both of these regional and global ocean
 1082 models, the surface fluxes are calculated using the JRA55do atmospheric reanalysis (Tsujino et al.
 1083 2018) and the same bulk flux algorithms (Large and Yeager (2004, 2009), see also Small et al.
 1084 (2015); Whitt et al. (2019)). In particular, the ROMS hindcast is forced by a diurnal cycle of
 1085 shortwave radiation (3-hourly) and vertical mixing is parameterized with the KPP scheme of Large
 1086 et al. (1994) with the same parameters as in the Parallel Ocean Program used by Deppenmeier
 1087 et al. (2021) (as in Whitt et al. 2019). The resulting diurnal cycles of upper-ocean turbulence look
 1088 qualitatively similar to those reported in Cherian et al. (2021) and simulated in MITgcm with the
 1089 same mixing parameterization, surface forcing, and horizontal grid resolution.

1090 The subgrid-scale viscosity in the LES $\nu_{sgs} = \nu_0 + \nu_t$ includes small and constant “molecular”
 1091 viscosity $\nu_0 = 10^{-6}$ m²/s. The much larger and variable turbulent viscosity is modeled after
 1092 Kaltenbach et al. (1994), that is

$$\nu_t = C_s^2 \Delta^2 (2S'_{ij}S'_{ji})^{1/2} \quad (\text{A6})$$

1093 where the Smagorinski coefficient $C_s = 0.13$, the grid scale $\Delta = (2\delta x\delta y\delta z)^{1/3}$ (where δx , δy ,
 1094 and δz are grid spacings in the x, y and z dimensions), the resolved deformation tensor is $S_{ij} =$
 1095 $1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ and $i, j = 1, 2, 3$ correspond to x, y, z dimensions and summation over
 1096 repeated indices is implied and the horizontally averaged shear is subtracted from the deformation
 1097 tensor S_{ij} in S'_{ij} . The diffusivity $\kappa_{sgs} = \kappa_0 + \nu_t/Pr_t$, where the turbulent Prandtl number is as in

1098 Whitt and Taylor (2017) based on Anderson (2009),

$$Pr_t = \left(1 + \frac{Ri_{GS}}{0.94}\right)^{1.5}, \quad (A7)$$

1099 and the grid-scale gradient Richardson number is

$$Ri_{GS} = \frac{\delta b \delta z}{\delta u^2 + \delta v^2}, \quad (A8)$$

1100 where δb , δz , δu , and δv are the vertical differences in buoyancy, depth, and horizontal velocity
1101 between two adjacent grid cells.

1102 At the top surface $z = 0$, the horizontally-uniform vertical fluxes are specified via time-evolving
1103 gradient boundary conditions:

$$\frac{\partial \mathbf{u}_h}{\partial z} = \frac{\boldsymbol{\tau}}{\rho \nu_{SGS}}, \quad (A9)$$

$$\frac{\partial T}{\partial z} = \frac{Q_0^{net} - P_Q(0)}{\rho c_p \kappa_{SGS}}, \quad (A10)$$

$$\frac{\partial S}{\partial z} = \frac{VSF_0^{net}}{\kappa_{SGS}}, \quad (A11)$$

1104 where $\nu_{sgs} = \nu_0$, $\kappa_{sgs} = \kappa_0$ are constant, $c_p = 4000 \text{ J/(kg } ^\circ\text{C)}$ is the specific heat of the seawater, and
1105 the net virtual salt flux VSF_0^{net} , the net surface heat flux Q_0^{net} , the net surface shortwave heat flux
1106 $P_Q(0)$, and the surface wind stress $\boldsymbol{\tau}$ are linearly interpolated from the 6-hourly-averaged ROMS
1107 fluxes, averaged over a 16.5 km square around the LES location, and shown in Fig. 10. Thus,
1108 the fluxes do not depend on the LES state. There is a diurnal cycle of shortwave solar radiation
1109 $P_Q(0)$, which penetrates and warms the interior of the LES during daytime as described below.
1110 The top is rigid, so the vertical velocity $w = 0$ at $z = 0$ (see Fig. 10). The LES domain bottom is
1111 rigid, $w = 0$, with $u = 0.865 \text{ m/s}$ and 0.465 m/s at 0° and 3° N respectively, $v = 0 \text{ m/s}$, $T = 22.3^\circ\text{C}$,
1112 and $S = 35.28 \text{ psu}$ are held constant. Although a variable bottom boundary to match the parent
1113 model solution would be preferred, the constant bottom boundary is thought to have little impact
1114 on the results in this study, because we set $\nu_{sgs} = \nu_0 = 10^{-6} \text{ m}^2/\text{s}$ at the interface between the

1115 bottom boundary velocity and the first interior point. Thus, the horizontally-averaged velocity and
 1116 temperature profiles evolve to remain approximately consistent with ROMS and are as shown in
 1117 Fig. 9, and the resulting artificially strong vertical gradients at the domain bottom do not result in
 1118 strong vertical fluxes of momentum, temperature, and salinity that significantly modify the interior
 1119 evolution. Yet, extra caution should be exercised when interpreting the turbulent statistics near the
 1120 bottom of the LES domain (e.g., Fig. 12h).

1121 Interior warming due to solar radiation is represented as the convergence of a two-component
 1122 exponential:

$$I(t, z) = \frac{I_0(t)}{\rho c_p} \frac{\partial}{\partial z} \left(a_I e^{-z/\zeta_{I1}} + (1 - a_I) e^{-z/\zeta_{I2}} \right) \quad (\text{A12})$$

1123 where $a_I = 0.58$, $\zeta_{I1} = 2.0$ m, $\zeta_{I2} = 23$ m and net incoming shortwave radiation I_0 (W/m^2) has
 1124 a diurnal cycle and is linearly interpolated from 6-hour-average ROMS output. We call the total
 1125 penetrative heat flux from solar radiation

$$P_Q(t, z) = I_0(t) \left(a_I e^{-z/\zeta_{I1}} + (1 - a_I) e^{-z/\zeta_{I2}} \right), \quad (\text{A13})$$

1126 and the analogous penetrative buoyancy flux is $P_b = P_Q g \alpha / (\rho c_p)$. The chosen profile $P_Q(z)$ is a
 1127 modified Jerlov type I profile (Paulson and Simpson 1977), such that the first e-folding scale is
 1128 increased from 0.35 to 2.0 m in an ad hoc attempt to compensate for missing near-surface mixing
 1129 due to surface gravity waves as in Watkins and Whitt (2020).

1130 There are three new terms on the right side of the equations that are new implementations specific
 1131 for this study and discussed briefly in the main methods section of the manuscript. These terms,
 1132 large-scale tendencies \mathcal{F} , restoring \mathcal{R} and damping \mathcal{D} , are included to make the solution more
 1133 realistic given the limited domain size. First, $\mathcal{F}(t, z)$ includes horizontally-uniform (in the LES)

1134 large-scale tendencies, that is

$$\mathcal{F}_u(z, t) = -\mathbf{u}_{ROMS} \cdot \nabla u_{ROMS} - \frac{1}{\rho} \frac{\partial p_{ROMS}}{\partial x} + D_{u_{ROMS}}, \quad (\text{A14})$$

$$\mathcal{F}_v(z, t) = -\mathbf{u}_{ROMS} \cdot \nabla v_{ROMS} - \frac{1}{\rho} \frac{\partial p_{ROMS}}{\partial y} + D_{v_{ROMS}}, \quad (\text{A15})$$

$$\mathcal{F}_w(z, t) = 0, \quad (\text{A16})$$

$$\mathcal{F}_T(z, t) = -\mathbf{u}_{ROMS} \cdot \nabla T_{ROMS} + D_{T_{ROMS}}, \quad (\text{A17})$$

$$\mathcal{F}_S(z, t) = 0, \quad (\text{A18})$$

1135 where D represents the explicit lateral mixing from ROMS. The restoring \mathcal{R} operates throughout
1136 the entire depth of the LES domain but operates only on the horizontal average:

$$\mathcal{R}_u(z, t) = -(\bar{u} - u_{ROMS})/t_r, \quad (\text{A19})$$

$$\mathcal{R}_v(z, t) = -(\bar{v} - v_{ROMS})/t_r, \quad (\text{A20})$$

$$\mathcal{R}_w(z, t) = 0, \quad (\text{A21})$$

$$\mathcal{R}_T(z, t) = -(\bar{T} - T_{ROMS})/t_r, \quad (\text{A22})$$

$$\mathcal{R}_S(z, t) = 0, \quad (\text{A23})$$

1137 where the over-bar denotes the lateral average and the restoring timescale $t_r = 11.6$ days (10^6 s).

1138 In general, $\mathcal{F} \gg \mathcal{R}$ because t_r is so long.

1139 Since the LES only simulates a small domain, the tendencies associated with larger scales, namely
1140 \mathcal{F} which includes three-dimensional advection, horizontal mixing, and the pressure gradient force
1141 but excludes the Coriolis force and vertical mixing because they are simulated in LES, are obtained
1142 from the 6-hourly-averaged budget diagnostic output of ROMS and are independent of the LES
1143 state. These large-scale tendencies are first averaged over a 3 by 3 array of ROMS grid cells (about
1144 a 16.5 km square) centered on the LES locations, then interpolated using cubic splines from the
1145 ROMS sigma levels (about every 8 m) to the LES vertical levels, and finally linearly interpolated

1146 in time and added as a tendency to the horizontally-averaged components of the LES momentum
 1147 and tracer equations as the LES runs (as expressed in equations above). The analogous large-scale
 1148 interior salinity tendencies are set to zero in the LES for simplicity. Although the omission of
 1149 interior salinity tendencies may complicate the interpretation, temperature is highly correlated with
 1150 buoyancy (initial $r^2 = 0.99$ at both 0° and $3^\circ\text{N}, 140^\circ\text{W}$) and has a three-fold stronger influence on
 1151 buoyancy than salinity in the region. Specifically, the initial bulk 108 m buoyancy differences
 1152 are 0.0029 m/s^2 (for a 0.4 psu salinity difference) and 0.0080 m/s^2 (for a 2.74°C temperature
 1153 difference) at $0^\circ \text{N}, 140^\circ \text{W}$. Thus, the results are expected to be qualitatively unaffected by the
 1154 omission of interior salinity tendencies, but future simulations are required to precisely quantify
 1155 the turbulent response to salinity advection.

1156 Finally, the fluctuations below 84 m depth are slowly damped toward zero:

$$\mathcal{D}_u(z, t) = -\sigma(u - \bar{u})/t_r, \quad (\text{A24})$$

$$\mathcal{D}_v(z, t) = -\sigma(v - \bar{v})/t_r, \quad (\text{A25})$$

$$\mathcal{D}_w(z, t) = -\sigma w/t_r, \quad (\text{A26})$$

$$\mathcal{D}_T(z, t) = -\sigma(T - \bar{T})/t_r, \quad (\text{A27})$$

$$\mathcal{D}_S(z, t) = -\sigma(S - \bar{S})/t_r, \quad (\text{A28})$$

1157 where

$$\sigma(z) = \left(\frac{z + H - L_s}{L_s} \right)^2 \quad \text{for } z < (L_s - H) \text{ and} \quad (\text{A29})$$

$$\sigma(z) = 0 \quad \text{for } z \geq (L_s - H), \quad (\text{A30})$$

1158 where z is the depth from 0 to $-H$, $H = 108 \text{ m}$ is the domain height, $L_s = 24 \text{ m}$ is the thickness
 1159 of the damping layer. It is notable that the timescale t_r is very long; it is about 66 days at 94 m,
 1160 17 days at 104 m, and 12 days at the bottom 108 m. These timescales are much longer than the

1161 time scale of the relevant stratified shear instabilities or internal waves (Smyth et al. 2011; Moum
1162 et al. 2011) and thus the damping has a negligible influence on shear instabilities, internal waves
1163 and turbulence at essentially all depths (the damping is of the order 10^{-12} to 10^{-10} m^2/s^3), and the
1164 DCT never gets within 15 m of the bottom in any case. Despite the slow damping rate and shallow
1165 domain bottom, the bottom 20 m remains strongly stratified with internal wave fluctuations that
1166 are weak compared to DCT. Short tests with a deeper 144 m domain suggested that the shallow
1167 domain bottom does not qualitatively impact the results. The stability analysis of Smyth et al.
1168 (2011) also suggests that the shallow domain depth is unlikely to impact the results since all of the
1169 shear instabilities they identify occur at depths shallower than 100 m and have a thickness of 20-40
1170 m.

1171 This manuscript focuses on the horizontally-averaged dynamics in the LES,

$$\frac{\partial \bar{\mathbf{u}}_h}{\partial t} = -\mathbf{f} \times \bar{\mathbf{u}}_h + \frac{\partial}{\partial z} \overline{\left(\nu_{sgs} \frac{\partial \mathbf{u}_h}{\partial z} - w \mathbf{u}_h \right)} + \mathcal{F}_{\mathbf{u}} + \mathcal{R}_{\mathbf{u}}, \quad (\text{A31})$$

$$\frac{\partial \bar{T}}{\partial t} = \frac{\partial}{\partial z} \overline{\left(\kappa_{sgs} \frac{\partial T}{\partial z} - w T \right)} + I + \mathcal{F}_T + \mathcal{R}_T, \quad (\text{A32})$$

$$\frac{\partial \bar{S}}{\partial t} = \frac{\partial}{\partial z} \overline{\left(\kappa_{sgs} \frac{\partial S}{\partial z} - w S \right)}, \quad (\text{A33})$$

$$\bar{b} = -g(1 - \alpha(\bar{T} - T_0) + \beta(\bar{S} - S_0)). \quad (\text{A34})$$

1172 The right-hand side terms in these budgets are averaged over the duration of the LES simulations
1173 and plotted in Fig. A1 and compared with output from ROMS (the subscript h indicates horizontal,
1174 e.g. the horizontal velocity vector $(\mathbf{u}, \mathbf{v}, 0)$). We define

$$\mathbf{F}_{\mathbf{m}} = \overline{\left(\nu_{sgs} \frac{\partial \mathbf{u}_h}{\partial z} - w \mathbf{u}_h \right)}, \text{ and} \quad (\text{A35})$$

$$F_b = \overline{\left(\kappa_{sgs} \frac{\partial b}{\partial z} - w b \right)} = g(\alpha F_T - \beta F_S), \quad (\text{A36})$$

1175 where F_T and F_S have the same functional form as F_b but operate on temperature (A32) and salinity
1176 (A33). The kinetic and potential energy equations for the horizontally-averaged state are then given

1177 by:

$$\frac{\partial |\bar{\mathbf{u}}_h|^2/2}{\partial t} = \frac{\partial}{\partial z} (\bar{\mathbf{u}}_h \cdot \mathbf{F}_m) - \mathbf{F}_m \cdot \frac{\partial \bar{\mathbf{u}}_h}{\partial z} + \bar{\mathbf{u}}_h \cdot \mathcal{F}_u + \bar{\mathbf{u}}_h \cdot \mathcal{R}_u, \quad (\text{A37})$$

$$\frac{\partial \bar{b}_z}{\partial t} = \frac{\partial}{\partial z} (zF_b) - F_b + z\mathcal{F}_b + z\mathcal{R}_b, \quad (\text{A38})$$

1178 and $\mathcal{F}_b = g(\alpha\mathcal{F}_T - \beta\mathcal{F}_S)$ and similarly for \mathcal{R}_b . On the right hand side, the first terms represent
 1179 vertical redistribution or transport in the interior and sources and sinks at the surface boundary
 1180 (e.g., wind work on the mean flow). The third and fourth terms are interior sources and sinks
 1181 related to the larger-scale dynamics inherited from ROMS (e.g., advection, pressure work, etc.).
 1182 The second term is the sink of mean kinetic energy to turbulence usually referred to as shear
 1183 production $-\mathbf{F}_m \cdot \partial \mathbf{u}_h / \partial z$ and the source of potential energy due to turbulent vertical mixing or
 1184 buoyancy flux $-F_b$.

1185 The governing equation for the horizontally-averaged turbulent kinetic energy (i.e., for $k = \overline{|\mathbf{u}'|^2}/2$
 1186 where $\mathbf{u}' = \mathbf{u} - \bar{\mathbf{u}}$) is given by

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial z} \left(\overline{wp/\rho} + \overline{wk} - \overline{\nu_{SGS} \frac{\partial k}{\partial z}} - \overline{\mathbf{u}' \nu_{SGS} \frac{\partial \bar{\mathbf{u}}}{\partial z}} \right) = -\overline{w\mathbf{u}'_h} \cdot \frac{\partial \bar{\mathbf{u}}_h}{\partial z} + \overline{wb} - \bar{\epsilon} + \overline{\mathbf{u}' \cdot \mathcal{D}_u} \quad (\text{A39})$$

1187 where the dissipation of turbulent kinetic energy is $\bar{\epsilon} = \overline{\nu_{SGS} S'_{ij} S'_{ji}} + \overline{\nu_{SGS} \partial \mathbf{u}'_h / \partial z \cdot \partial \mathbf{u}_h / \partial z}$. In the
 1188 limit that $\nu_t \rightarrow 0$, the shear production and buoyancy flux terms in the turbulent kinetic energy
 1189 equation and the mean kinetic energy/potential energy equations become effectively identical.
 1190 However, because the LES is a filtered approximation of high-Reynolds number flow with finite
 1191 $\nu_t \gg \nu_0$, a finite amount of mean-profile buoyancy flux, shear production and total dissipation
 1192 occur via the subgrid-scales without passing through k . Hence, we plot the total dissipation
 1193 $\overline{\nu_{SGS} S'_{ij} S'_{ji}}$, shear production $\mathbf{F}_m \cdot \partial \bar{\mathbf{u}}_h / \partial z$, and buoyancy flux F_b throughout the manuscript and
 1194 define the deviations from this balance to be transport and transience, i.e.:

$$T = \mathbf{F}_m \cdot \frac{\partial \bar{\mathbf{u}}_h}{\partial z} - F_b - \bar{\epsilon}, \quad (\text{A40})$$

1195 where ϵ is total dissipation. Consistent with the discussion in Osborn (1980), the left hand side is
1196 generally small when averaged horizontally and over a full day at z_{max} in the LES. For reference,
1197 the subgrid-scale parts of F_b and F_m are small relative to the resolved parts where F_b and ϵ are
1198 strong and Ri_g is low, e.g. above H_{Rig} or shallower than about 75 m depth on average. The
1199 subgrid-scale fluxes become relatively large deeper in the thermocline, where $Ri_g > 1$ is relatively
1200 high and F_b and ϵ are relatively weak, e.g. below H_{Rig} or below 75 m on average; results from
1201 these depths should be interpreted more cautiously.

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1435 <https://doi.org/10.6084/m9.figshare.17009660>, roms and les code bases and configura-
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1446 **LIST OF TABLES**

1447 **Table 1.** A glossary table with definitions and sections where key metrics are defined. . . . 70

Metric	Definition (key defining sections)
Q_0^{net}	net surface heat flux (3.a)
$\langle F_Q \rangle^{max}$	maximum (over depth) of the daily-mean downward turbulent heat flux (3.a, 4.a)
F_b	downward turbulent buoyancy flux; roughly proportional to F_Q (4.a, 4.c, Appendix)
F_m	downward turbulent momentum flux (4.b, Appendix)
ϵ	dissipation of turbulent kinetic energy (4.b, Appendix)
SP	shear production of turbulent kinetic energy $F_m \cdot \partial \mathbf{u}_h / \partial z$ (4.d, Appendix)
T	convergence of the vertical transport of turbulent kinetic energy (4.d, Appendix)
z_{max}	depth at which the maximum $\langle F_Q \rangle^{max}$ or $\langle F_b \rangle^{max}$ occurs (3.a, 4.c)
z_{pen}	depth to which DCT penetrates; shallowest depth $\epsilon \leq 2 \times 10^{-8} \text{ m}^2/\text{s}^3$ (4.c)
MLD	mixed layer depth, first depth 0.015 kg/m^3 denser than 0-10 m mean (3.a and 4.c)
H_{Rib}	thickness of the surface layer with bulk $Ri_b = 0.2$ (4.c)
Ri_b	bulk Richardson number of a surface layer (4.c)
H_{Rig}	thickness of the low Ri_g layer, $Ri_g < 0.35$ (4.c)
Ri_g	gradient Richardson number, $Ri_g = \partial b / \partial z / \partial \mathbf{u}_h / \partial z ^2 = N^2 / S^2$ (2.c)
Ri_f	flux Richardson number, $Ri_f = F_b / SP$ (4.d)
Pr_t	turbulent Prandtl number, $Pr_t = Ri_g / Ri_f$ (4.d)
S_b	bulk vertical shear from least-squares fit to the horizontal velocity from H_{Rig} to 5 m depth (4.e, 4.f)

TABLE 1. A glossary table with definitions and sections where key metrics are defined.

LIST OF FIGURES

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 1451 W, the observations (horizontal bars) span the inter-quartile ranges of all monthly means
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 1459 N, 140° W for comparison (1988-2018). The observed spectra are calculated in overlapping
 1460 time windows that are the same length as the LES simulations (with 17% of points overlapped
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 1492 and c: the depth where strongest mixing occurs z_{max}). Maps (d-f) quantify the respective
 1493 aseasonal inter-quartile ranges over all latitudes and years 1999-2016. Aseasonal variability
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 1496 signal at each grid point. 80

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 1501 included. For comparison, the observational climatology of $\langle F_Q \rangle^{20-60}$ from chipods (Moum
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 1503 $\langle F_Q \rangle^{max}$ from ROMS and LES (roughly October 1985) as well as the TIWE observations
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 1505 the LES and TIWE are computed as $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$ [W/m²] where ρ , c_p ,
 1506 and α are the reference density, specific heat, and thermal expansion coefficient of seawater,
 1507 respectively, g is the acceleration due to gravity, and F_b is the downward turbulent buoyancy
 1508 flux. Data from two other shorter field experiments (not shown) resulted in means of roughly
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 1510 1985; Moum and Caldwell 1985) (see Fig. 2d of Moum et al. 2009). 81

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 1514 N, 140° W (red) for the 18-year MITgcm simulation (left column) as well as the 34-day LES
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 1517 TIWE are computed based on buoyancy fluxes, e.g. $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$ 82

1518 **Fig. 9.** Time series of zonal and meridional velocity (color), temperature (white contours in °C),
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 1521 $Ri_g < 0.35$ (H_{Rig} , thick black) in the LES at 0° N and 3° N along 140° W. All fields are
 1522 defined from horizontally-averaged profiles. The MLD is defined to be the shallowest depth
 1523 where water is 0.015 kg/m³ denser than the top 10 m in the instantaneous but horizontally-
 1524 averaged density profile. All time tick marks are at 0 UTC; local solar time at 0°N, 140° W
 1525 is about 9 hours behind UTC, so local solar noon is at about 21 UTC. 83

1526 **Fig. 10.** Time series of the net surface heat flux Q_0^{net} (left axis, blue), the magnitude of the wind
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 1528 October-November 1985 in the LES at 0° N (a) and 3° N (b) along 140° W. Overlaid on F_Q
 1529 are the depth at which the bulk Richardson number $Ri_b = 0.2$ (H_{Rib} , thin black line), the
 1530 depth of the maximum daily-mean downward heat flux z_{max} (+ symbols), the daily maximum
 1531 MLD (defined from the horizontally averaged LES density profiles; magenta circles), and
 1532 the base of the low gradient Richardson layer $Ri_g < 0.35$ (H_{Rig} , thick black line). The
 1533 daily-mean meridional velocity averaged from 25 to 75 m depth is in blue; the origin is at
 1534 a depth of 100 m, a 1m spacing corresponds to 10 cm/s, and the peak-to-trough amplitudes
 1535 are about 40 cm/s at 0° N and 90 cm/s at 3° N. For consistency with other results in section
 1536 4, we plot $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$ [W/m²] where ρ , c_p , and α are the reference
 1537 density, specific heat, and thermal expansion coefficient of seawater, respectively, g is the
 1538 acceleration due to gravity, and F_b is the downward turbulent buoyancy flux. All time tick
 1539 marks are at 0 UTC, but local solar time at 0°N, 140° W is about 9 hours behind UTC, so
 1540 local solar noon is at about 21 UTC. Daily mean statistics (e.g., z_{max} indicated by + symbols)
 1541 are calculated from 21 UTC so that the averages begin and end near solar noon. 85

- 1542 **Fig. 11.** As in Fig. 10, but zoomed in on a few days in November and with the addition of the MLD
1543 (dashed magenta) and the DCT penetration depth z_{pen} ($\epsilon \geq 2 \times 10^{-8} \text{ m}^2/\text{s}^3$; thin green). The
1544 MLD is defined to be the shallowest depth where water is $0.015 \text{ kg}/\text{m}^3$ denser than the top
1545 10 m in the instantaneous but horizontally-averaged density profile. 86
- 1546 **Fig. 12.** As in Fig. 11, but plots show (a)-(b) the vertical buoyancy gradient N^2 , (c)-(d) the squared
1547 vertical shear S^2 , (e)-(f) $Ri_g = N^2/S^2$, and (g)-(h) the rate of dissipation of kinetic energy
1548 ϵ . It may be noted that there are a few instances of elevated dissipation $10^{-8} < \epsilon < 10^{-7}$
1549 m^2/s^3 below the deepest depths of DCT (z_{pen} , green line) in (h) where $Ri_g > 1$. However,
1550 these instances of elevated dissipation near the bottom are dominated by dissipation of the
1551 mean-flow kinetic energy, and the turbulent fluxes and energetics depend strongly on the
1552 subgrid-scale parameterization in the LES (A6)-(A7), may be influenced by the bottom
1553 boundary, and should be interpreted with caution. 87
- 1554 **Fig. 13.** As in Fig. 11, but turbulent vertical momentum fluxes projected onto the shear, i.e. $(\mathbf{F}_m \cdot$
1555 $\partial \mathbf{u}_h / \partial z) / |\partial \mathbf{u}_h / \partial z|$ 88
- 1556 **Fig. 14.** In (a), the depth z_{max} of maximum daily mean turbulent heat flux is related to the depth
1557 H_{Rib} at which the bulk Richardson number is 0.2. And in (b), the daily maximum depth
1558 z_{pen} to which DCT penetrates ($\epsilon > 2 \times 10^{-8} \text{ m}^2/\text{s}^3$) is related to the low-gradient Richardson
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- 1560 **Fig. 15.** Relationships between various terms in the daily mean turbulent kinetic energy budget
1561 at the depth z_{max} where the downward turbulent buoyancy flux is maximum ($\langle SP \rangle^{max}$
1562 $+ \langle T \rangle^{max} \approx \langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}$; see the Appendix for details). The depths z_{max} are plotted
1563 as + symbols in Fig. 10. Buoyancy flux $\langle F_b \rangle^{max}$ is plotted against (a) shear production
1564 over buoyancy flux plus dissipation $\langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$ and (b) shear production
1565 over buoyancy flux (i.e., the inverse flux Richardson number $Ri_f^{-1} = \langle SP \rangle^{max} / \langle F_b \rangle^{max}$).
1566 The inverse gradient Richardson number of the horizontally-averaged profile $Ri_g^{-1} =$
1567 $\langle |\partial \mathbf{u}_h / \partial z|^2 \rangle^{max} / \langle \partial b / \partial z \rangle^{max} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ is shown in color on all four panels and
1568 on the y axes in (c)-(d) against Ri_f^{-1} (c) and $Pr_t^{-1} = Ri_f / Ri_g$ (d) (the inverse turbulent Prandtl
1569 number Pr_t^{-1} is the ratio of the turbulent diffusivity of buoyancy over the turbulent viscosity of
1570 momentum). The thick black line (c) is the 1-1 line, the thin solid line is a fit to LES of a coastal
1571 boundary layer under a hurricane by Watkins and Whitt (2020), and the thin dashed line is a
1572 fit to atmospheric boundary layer observations by Anderson (2009), which parameterizes the
1573 subgrid-scale Pr_t^{-1} in the LES. The two days with most anomalously low Ri_f^{-1} (b-c; $Ri_f^{-1} = 0.9$
1574 and 1.6) and high Pr_t^{-1} (d; $Pr_t^{-1} = 0.4$ and 1.8) also have the largest relative non-local sources
1575 of turbulent kinetic energy $\langle T \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}) \approx 1 - \langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$
1576 (i.e., the points with lowest values in a; 0.3 and 0.6). Plus (+) symbols are from LES at 0° N
1577 and circles (o) from 3° N 90
- 1578 **Fig. 16.** Relationship between $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ and $\langle F_b \rangle^{max}$ (a) and $K_b = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$
1579 (b) at z_{max} (i.e., at the depths indicated by the + symbols in Fig. 10). Averaging diffusivity
1580 directly in (b) yields quantitatively different results but qualitatively the same conclusion
1581 that K_b is at best weakly related to Ri_g . Overlaid in (b) are parameterizations of turbulent
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1583 Peters et al. (1988) (PGT), and Large and Gent (1999) (KPP). 91
- 1584 **Fig. 17.** Maximum daily mean turbulent buoyancy flux $\langle F_b \rangle^{max}$ scales with oceanic bulk vertical
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1586 wind stress $|\tau| = u_*^2 \rho$ (b,d). The scalings are obtained via linear regression on the LES output

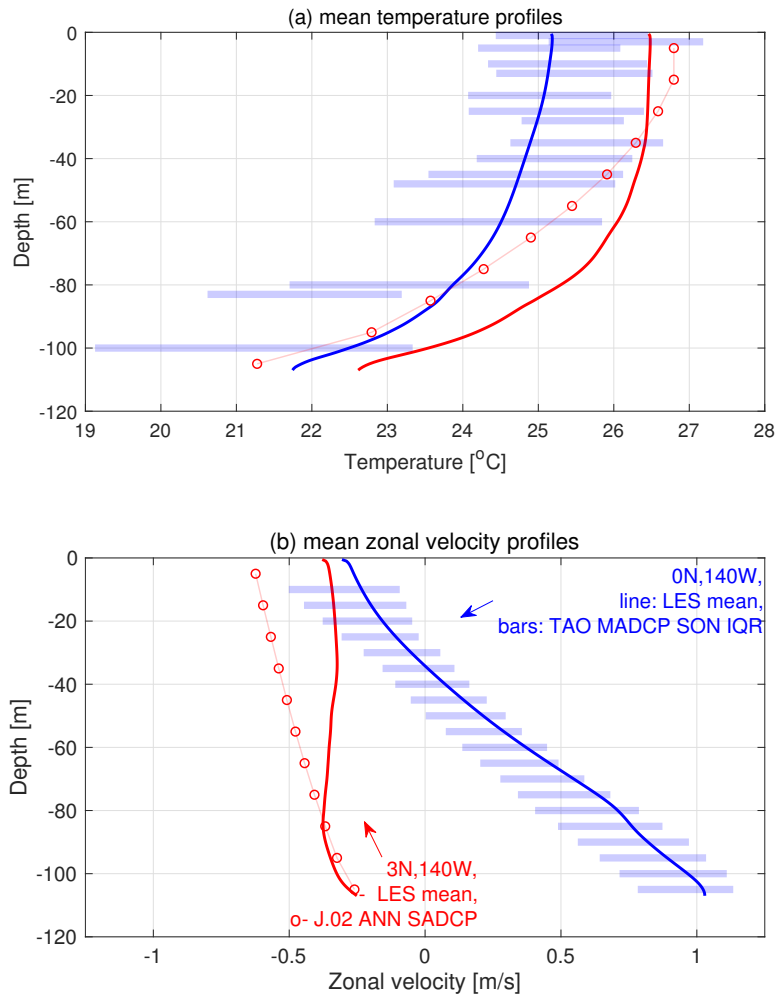
1587 in (a)-(b), which includes 34 days at 3° N (black ○) and 34 days at 0° N (black +), or on
1588 the 68 days of LES output plus 38 days of TIWE data (blue *) in (c)-(d). Hence, the TIWE
1589 observations serve as an independent validation of the regressions in (a)-(b) and constrain
1590 the regressions in (c)-(d). The predictors include S_b , which is derived from a linear fit to
1591 the mean velocity from H_{Rig} to 5 m depth (thick black lines in Fig. 4), and the friction
1592 velocity $u_* = \sqrt{|\tau|/\rho}$. All variables are log-transformed and Pearson's r in the panel titles is
1593 calculated in log space. The various diagonal black lines indicate where the data are along
1594 the 1-1 line, within a factor of 2, and within a factor of 3. With 95% confidence intervals, the
1595 scalings are as follows: $(2 - 6) \times 10^{-6} |S_b|^{(0.7-1.0)}$ (a), $(1 - 200) \times 10^{-2} |S_b|^{(0.9-1.1)} u_*^{(1.6-2.5)}$
1596 (b), $(2 - 6) \times 10^{-6} |S_b|^{(0.8-1.0)}$ (c), and $(0.03 - 1.3) \times 10^{-2} |S_b|^{(0.8-1.0)} u_*^{(0.9-1.6)}$ (d). 92

1597 **Fig. 18.** Various ratios of terms in Eqn. (6) showing how the local energetics of the buoyancy flux
1598 at z_{max} (Fig. 15) relate to the bulk scalings derived via regression (Fig. 17). Circles (○)
1599 are from the LES at 3° N, and pluses (+) are from the LES at 0° N; the color indicates
1600 $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ 93

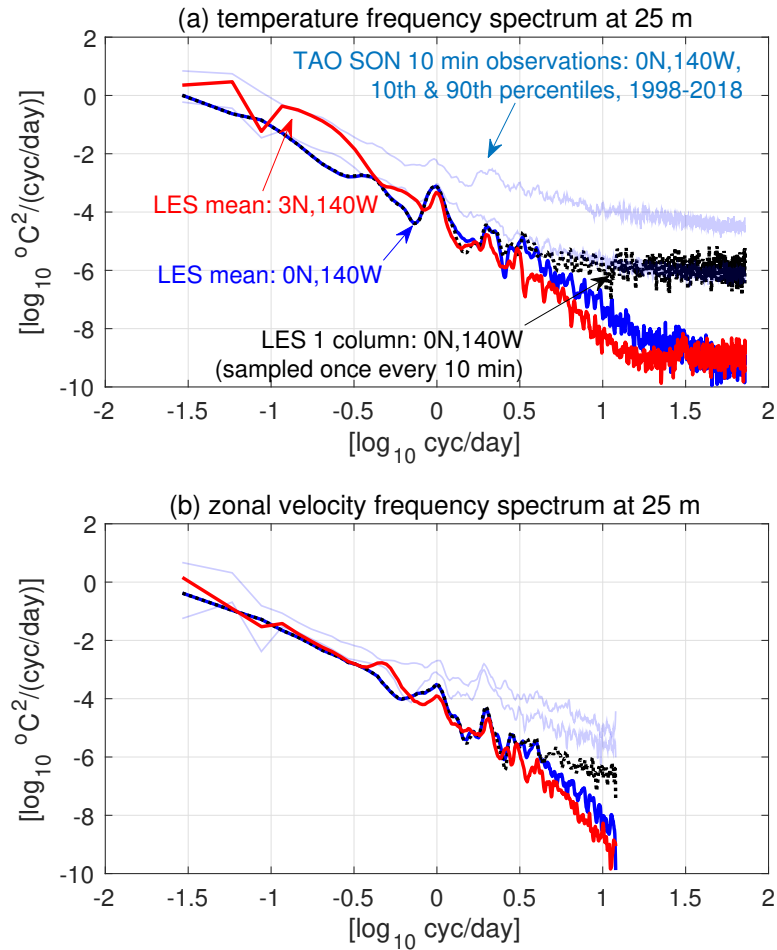
1601 **Fig. 19.** Comparisons between the LES and ROMS (KPP) at the LES locations (+ at 0° N and ○ at 3° N
1602 along 140° W): (a) The maximum turbulent heat flux $\langle F_Q \rangle^{max}$, (b) the turbulent diffusivity of
1603 heat K at z_{max} , and (c) the depth z_{max} at which $\langle F_Q \rangle^{max}$ occurs. Note, however, that the LES
1604 results are derived from the buoyancy dynamics whereas the ROMS results are derived from
1605 the temperature dynamics. That is, the LES results are $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 \langle F_b \rangle \approx \langle F_Q \rangle$
1606 [W/m²] in (a) and $K = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$ in (b), and z_{max} is calculated from from $\langle F_b \rangle$
1607 profiles. 94

1608 **Fig. 20.** Daily averaged net vertical heat flux $\langle Q \rangle$ (including turbulent F_Q as in Fig. 10 plus penetrative
1609 radiative P_Q components) at 0° N, 140° W (left column) and 3° N, 140° W (right column) as
1610 simulated by the LES [(a)-(b)] and as parameterized based on horizontally-averaged velocity
1611 and density profiles and net surface buoyancy and momentum fluxes [(c)-(d)]. For reference,
1612 the the piecewise linear flux profiles with $\langle Q \rangle(z = 0)$ and $\langle Q \rangle(z = z_{max})$ from LES are
1613 shown in e-f. In addition, the vertical heat fluxes (penetrating shortwave plus turbulent)
1614 from the parent ROMS model are shown in the bottom row [(e)-(f)]. Note the different
1615 colorbar ranges in the left and right columns. For consistency with earlier results, (a)-(f)
1616 plot $(\rho c_p / g \alpha) \langle B \rangle \approx \langle Q \rangle$ where $\langle B \rangle$ is the daily-averaged vertical buoyancy flux including
1617 the parts due to turbulence and penetrative shortwave radiation. 95

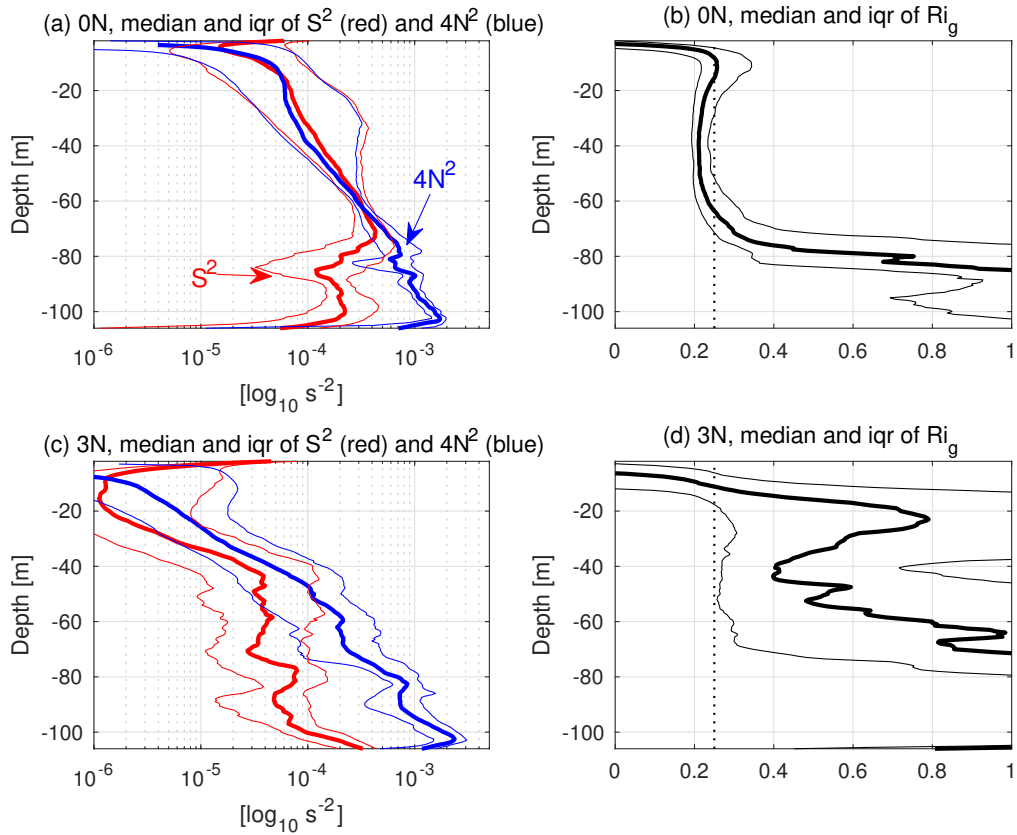
1618 **Fig. A1.** Time-means of various terms in the horizontally-averaged tracer and momentum budgets
1619 from ROMS (solid lines) and LES (dashed lines) at 0° N, 140° W (top) and 3° N, 140° W
1620 (bottom). The blue lines represent the time-mean convergence of vertical transport of (a,e)
1621 temperature, (b,f) zonal momentum and (c,g) meridional momentum and (d,h) salinity due
1622 to turbulence (and solar radiation in the case of temperature). The black lines represent all
1623 other tendencies of horizontally-averaged momentum and tracers as diagnosed from ROMS,
1624 i.e. \mathcal{F} (plus Coriolis in the case of momentum), and as diagnosed in LES, i.e. $\mathcal{F} + \mathcal{R}$ (plus
1625 Coriolis in the case of momentum). See the Appendix for the budget formulas. 96



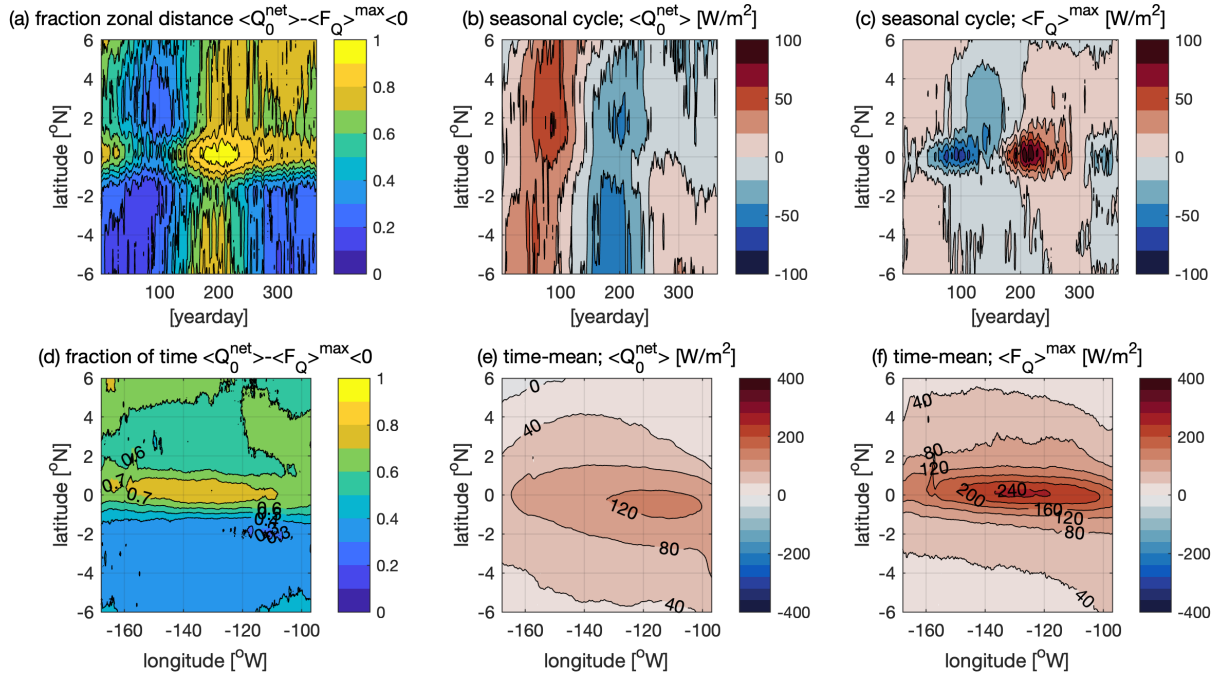
1626 FIG. 1. A comparison between the simulated (LES, solid lines) and observed mean temperature (a) and zonal
 1627 velocity (b) profiles at 0° (blue) and 3° N (red) along 140° W. At 0° N, 140° W, the observations (horizontal bars)
 1628 span the inter-quartile ranges of all monthly means (September-October-November only) from the TAO mooring
 1629 (1988-2018). At 3° N, 140° W, a ship-based annual climatology is plotted (Johnson et al. 2002), but these are
 1630 more for reference than for validation since there is significant seasonal and inter-annual variability.



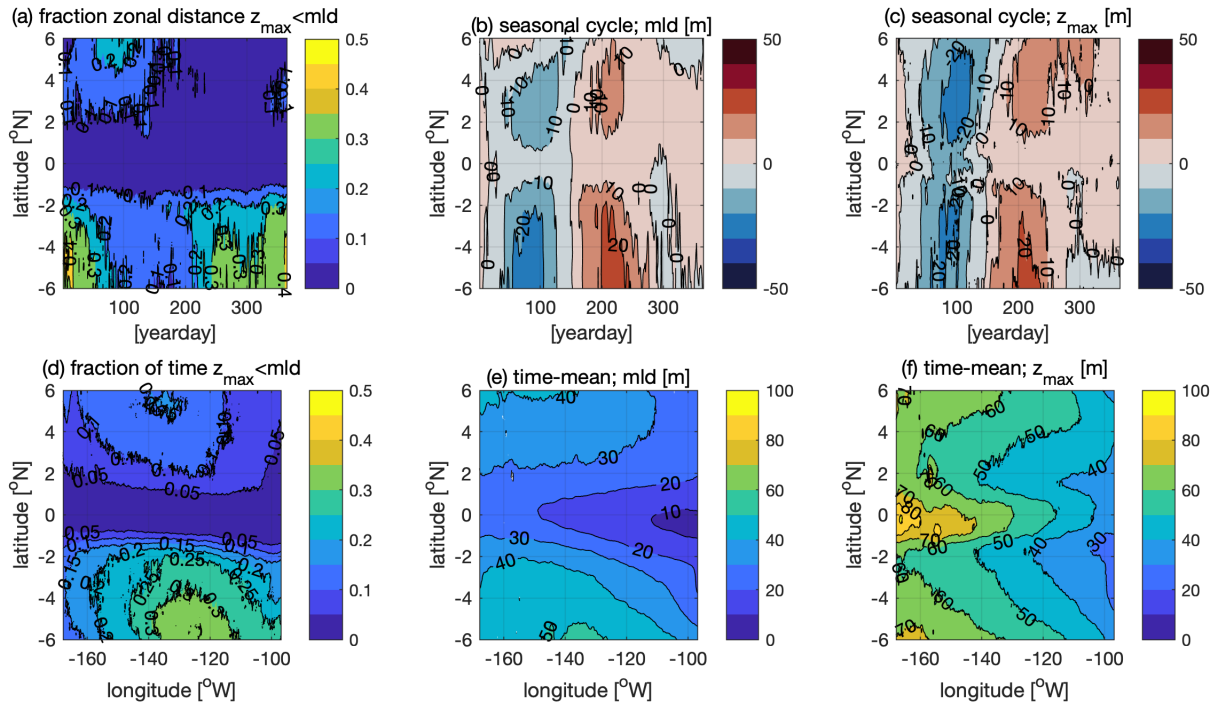
1631 FIG. 2. Simulated (LES) and observed frequency spectra of temperature (a) and zonal velocity (b) at 25 m depth
 1632 at 0° (blue) and 3° N (red) along 140° W. Observed spectra are calculated from the moored temperature sensor
 1633 (10 minute instantaneous sampling) and current meter (1 hour average sampling) from the months September-
 1634 October-November on the TAO mooring at 0° N, 140° W for comparison (1988-2018). The observed spectra
 1635 are calculated in overlapping time windows that are the same length as the LES simulations (with 17% of points
 1636 overlapped in each window). The 10% and 90% quantile at each frequency (across all of the spectra windows)
 1637 is plotted in light blue. The black dotted and blue lines are derived from LES: the sampling is instantaneous
 1638 (averaged over a single time step) every 10 min (a) or 1 hour (b) and averaged spatially over a single grid
 1639 cell/virtual mooring (black dotted) or the entire horizontal extent of the domain (blue). The spectrum from the
 1640 virtual mooring (black) flattens similarly to the observations from the TAO mooring at frequencies higher than
 1641 3 cyc/day due to aliasing in (a).



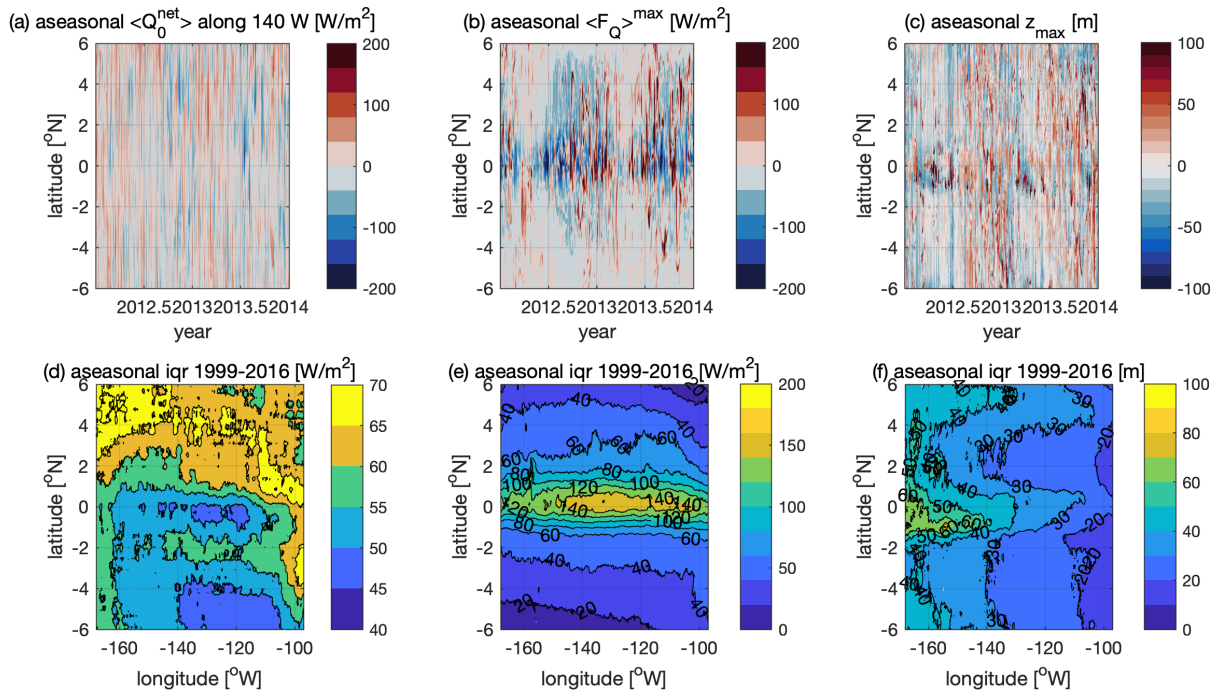
1642 FIG. 3. Profiles of the median (thick lines) and inter-quartile range (iqr, thin lines) of the squared vertical shear
 1643 of horizontal velocity S^2 , the vertical buoyancy gradient N^2 , and the gradient Richardson number $Ri_g = N^2/S^2$
 1644 (all of the horizontally-averaged profiles). The top row show results from the LES at 0°N and the bottom row
 1645 the results from the LES at 3°N . The dotted vertical line in (b) and (d) indicates $Ri_g = 0.25$ for reference.



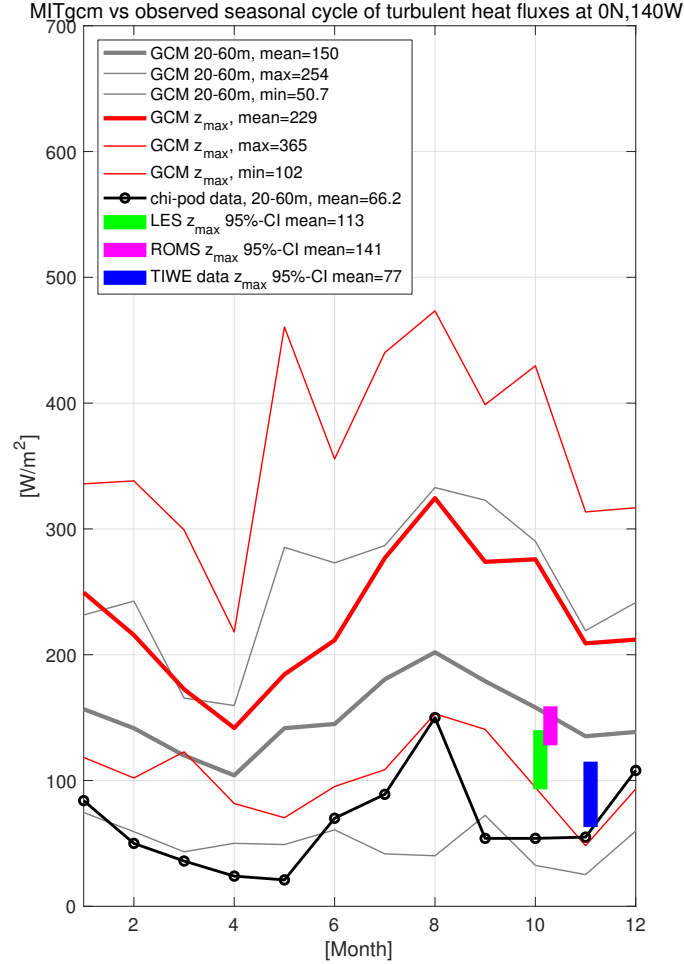
1646 FIG. 4. Climatological spatial structure and seasonal cycle of downward heat fluxes in a regional ocean model
 1647 of the equatorial Pacific Ocean cold tongue forced by atmospheric reanalysis from 1999-2016. The net air-sea
 1648 flux $\langle Q_0^{net} \rangle$ is in (b) and (e), and the maximum flux due to ocean mixing $\langle F_Q \rangle^{max}$ is in (c) and (f). b-c are
 1649 the zonal means from 95-170° W with the time-mean subtracted, and e-f are the time-means. In addition, we
 1650 quantify the fraction of the zonal distance (a) and time (d) over which there is net cooling of the surface ocean
 1651 due to air-sea exchange and ocean mixing, that is $\langle Q_0^{net} \rangle - \langle F_Q \rangle^{max} < 0$. The flux due to ocean mixing $\langle F_Q \rangle^{max}$
 1652 (c,f) is defined as the maximum (over depth) of the daily-mean downward turbulent heat flux, so the zonal and
 1653 time means are calculated at a depth that varies in time and space that is plotted in Fig. 5.



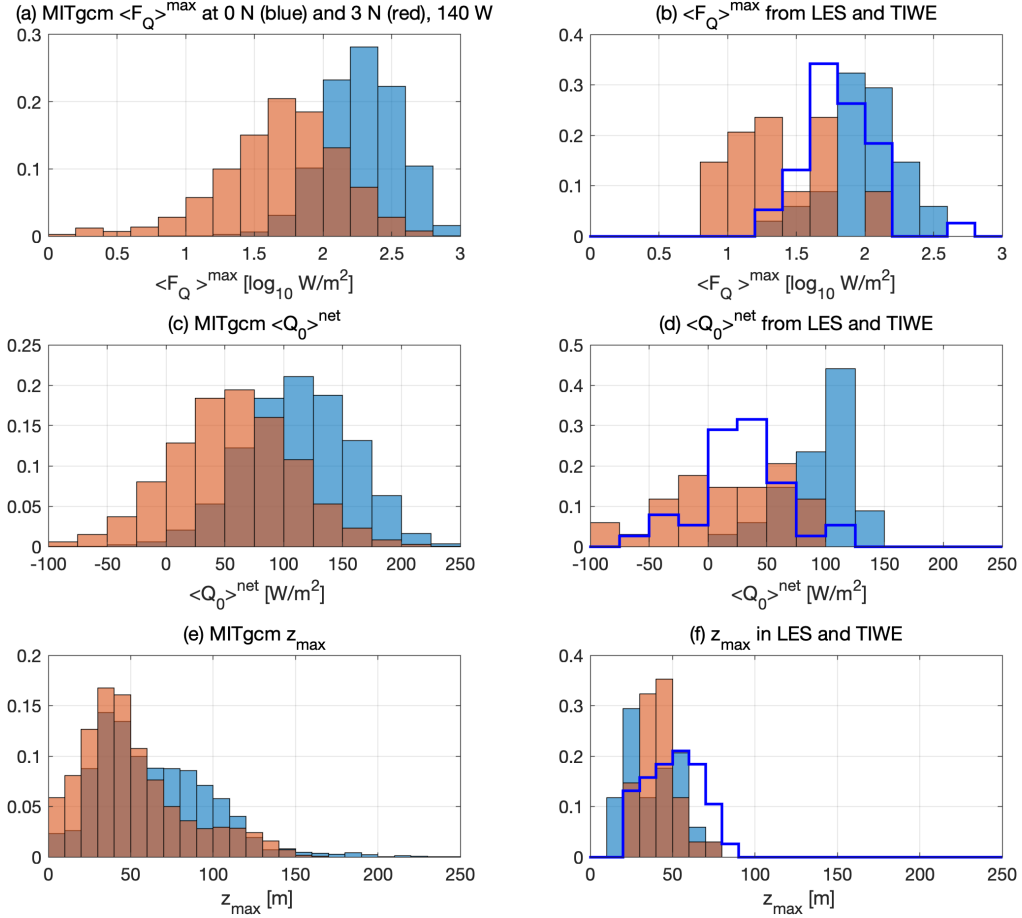
1654 FIG. 5. Climatological comparisons between mixed layer depth (MLD, b,e) and the depth z_{max} (c,f) where the
 1655 downward turbulent heat flux is maximum (i.e., the depth where $\langle F_Q \rangle^{max}$ plotted in Fig. 4 occurs). As in Fig. 4,
 1656 b-c are the zonal mean anomalies from the time mean, and e-f are the time-means. In addition, we quantify the
 1657 fraction of the zonal distance (a) and time (d) over which the the MLD is deeper than z_{max} . The MLD is defined
 1658 to be the shallowest depth where water is 0.015 kg/m^3 denser than the top 10 m in the daily-mean density profile
 1659 (since higher-frequency output is not available).



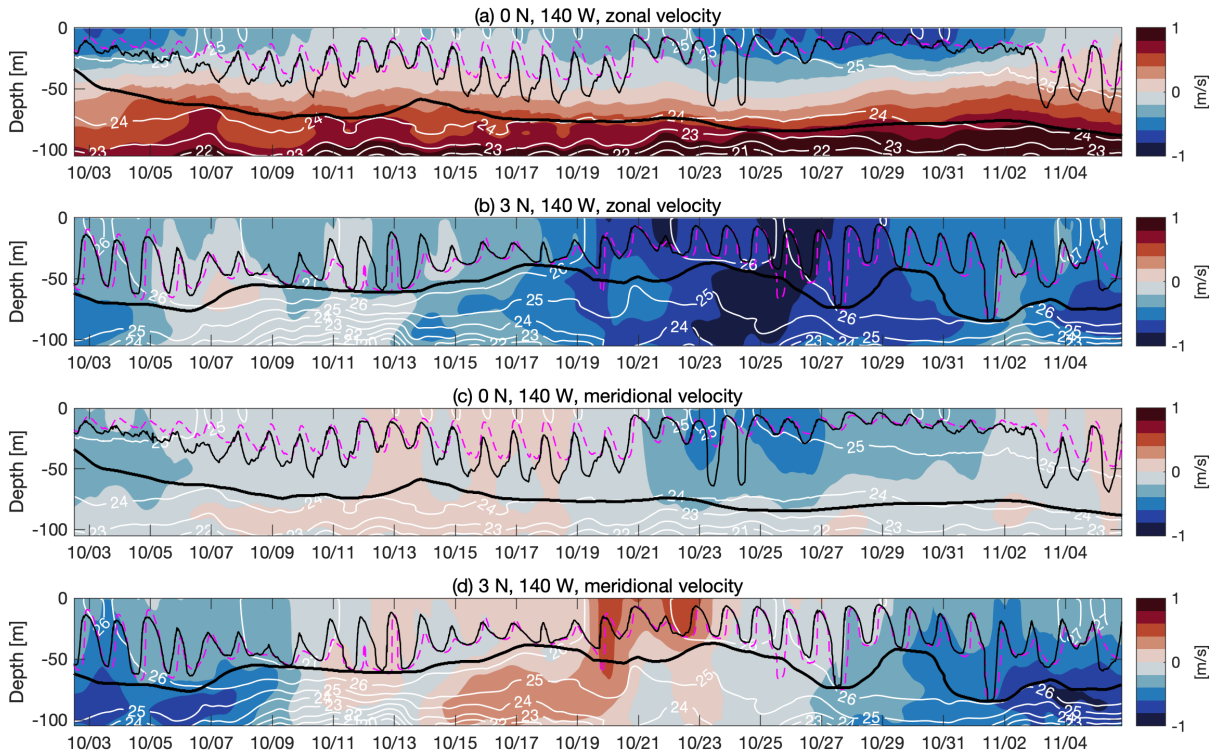
1660 FIG. 6. The top row shows the hindcast aseasonal daily-mean vertical heat fluxes during 2012 and 2013 along
 1661 the 140° W meridian (a: net surface flux $\langle Q_0^{net} \rangle$, b: ocean mixing $\langle F_Q \rangle^{max}$ and c: the depth where strongest
 1662 mixing occurs z_{max}). Maps (d-f) quantify the respective aseasonal inter-quartile ranges over all latitudes and
 1663 years 1999-2016. Aseasonal variability is defined by subtracting the mean seasonal cycle (i.e., a daily annual
 1664 climatology, which is averaged over 18 years and then smoothed with a 15-day moving average), from the total
 1665 signal at each grid point.



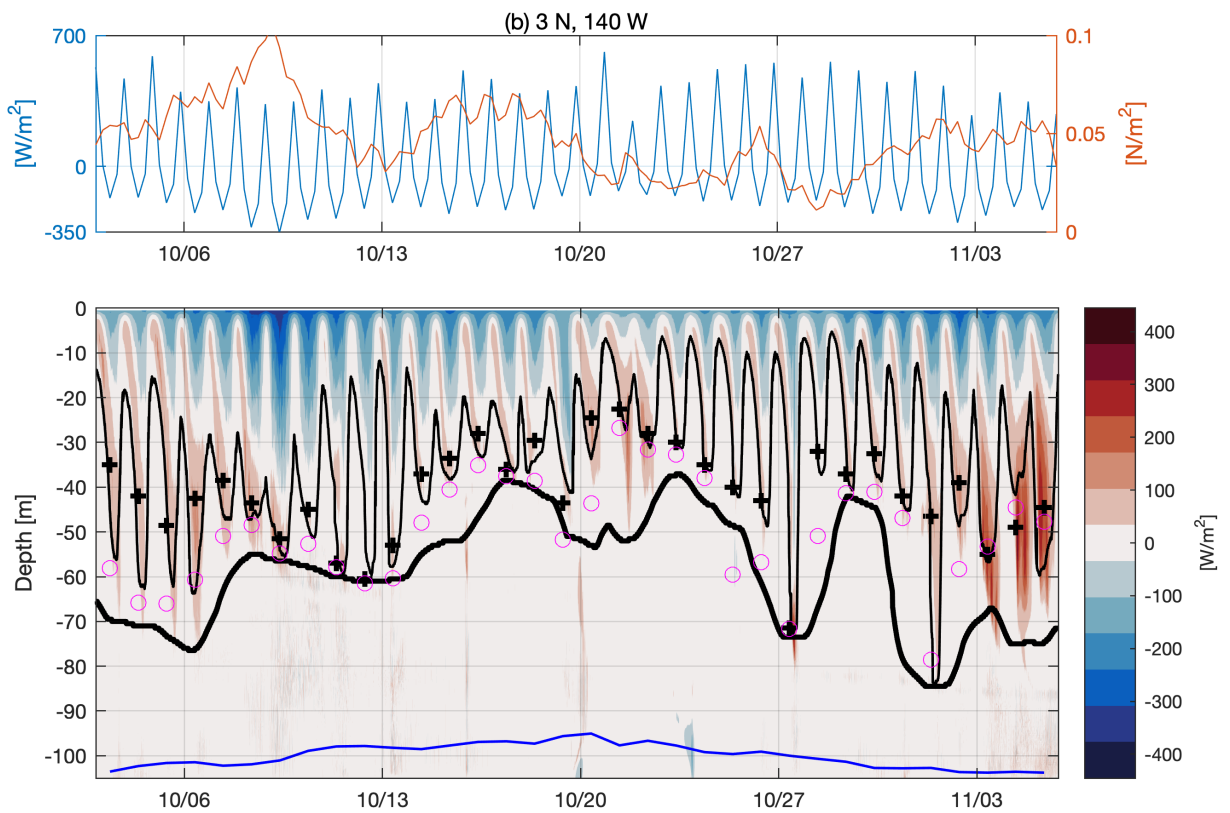
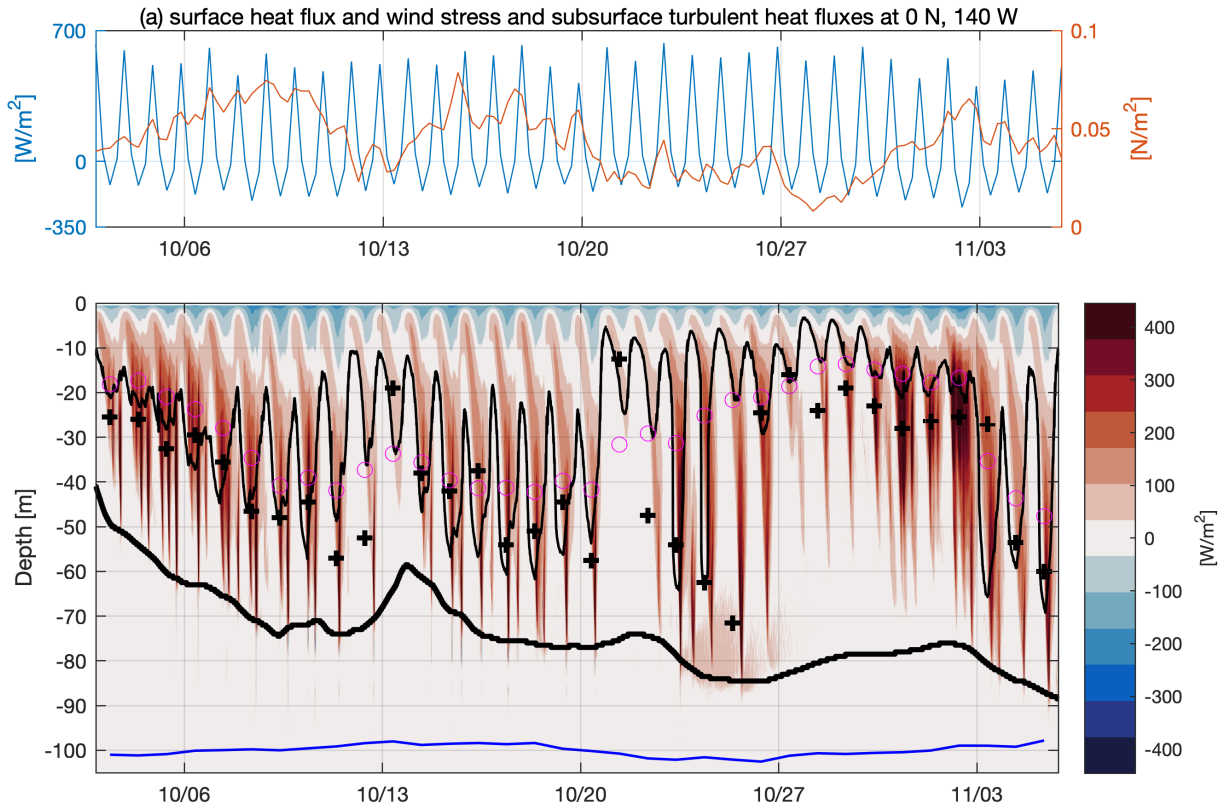
1666 FIG. 7. Climatological annual cycle of the downward turbulent heat flux at 0° N, 140° W in the MITgcm
 1667 regional ocean model, including monthly means at z_{max} ($\langle F_Q \rangle^{max}$, thick red) as well as monthly means from
 1668 20-60 m depth $\langle F_Q \rangle^{20-60}$ (thick gray). Corresponding minima and maxima of monthly $\langle F_Q \rangle^{20-60}$ (thin gray) and
 1669 $\langle F_Q \rangle^{max}$ (thin red) from 1999-2016 are included. For comparison, the observational climatology of $\langle F_Q \rangle^{20-60}$
 1670 from chipods (Moum et al. 2013) is plotted in black circles. The 95% confidence intervals for the monthly mean
 1671 $\langle F_Q \rangle^{max}$ from ROMS and LES (roughly October 1985) as well as the TIWE observations (roughly November
 1672 1991) are in magenta, green and blue respectively. Note, however, that the LES and TIWE are computed
 1673 as $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$ [W/m²] where ρ , c_p , and α are the reference density, specific heat, and
 1674 thermal expansion coefficient of seawater, respectively, g is the acceleration due to gravity, and F_b is the downward
 1675 turbulent buoyancy flux. Data from two other shorter field experiments (not shown) resulted in means of roughly
 1676 400 W/m² in Oct/Nov 2008 (Moum et al. 2009) and 100 W/m² in Nov 1984 (Gregg et al. 1985; Moum and
 1677 Caldwell 1985) (see Fig. 2d of Moum et al. 2009).



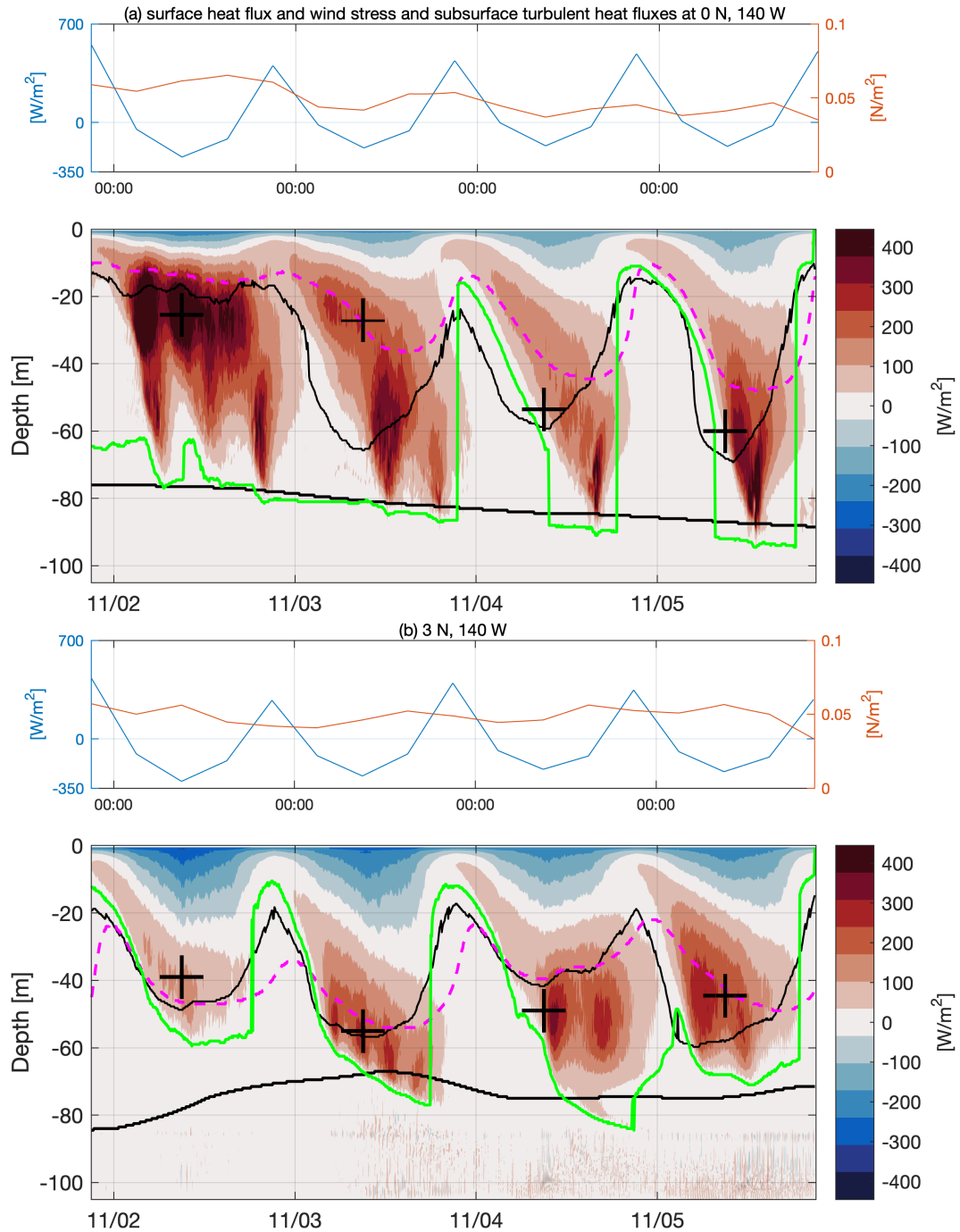
1678 FIG. 8. Relative probability distributions of the maximum daily-mean turbulent heat flux due to ocean mixing
 1679 $\langle F_Q \rangle^{max}$ (a-b), the daily-mean net surface heat flux $\langle Q_0^{net} \rangle$ (c-d), and the depth z_{max} at which $\langle F_Q \rangle^{max}$
 1680 occurs (e-f). Histograms are included for both 0° N, 140° W (blue) and 3° N, 140° W (red) for the 18-year MITgcm
 1681 simulation (left column) as well as the 34-day LES in October 1985 (red and blue histograms) and the 38-day
 1682 TIWE experiment at 0° N, 140° W in November 1991 (right column, dark-blue edged bars). Note that the data
 1683 from LES and TIWE are computed based on buoyancy fluxes, e.g. $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$.



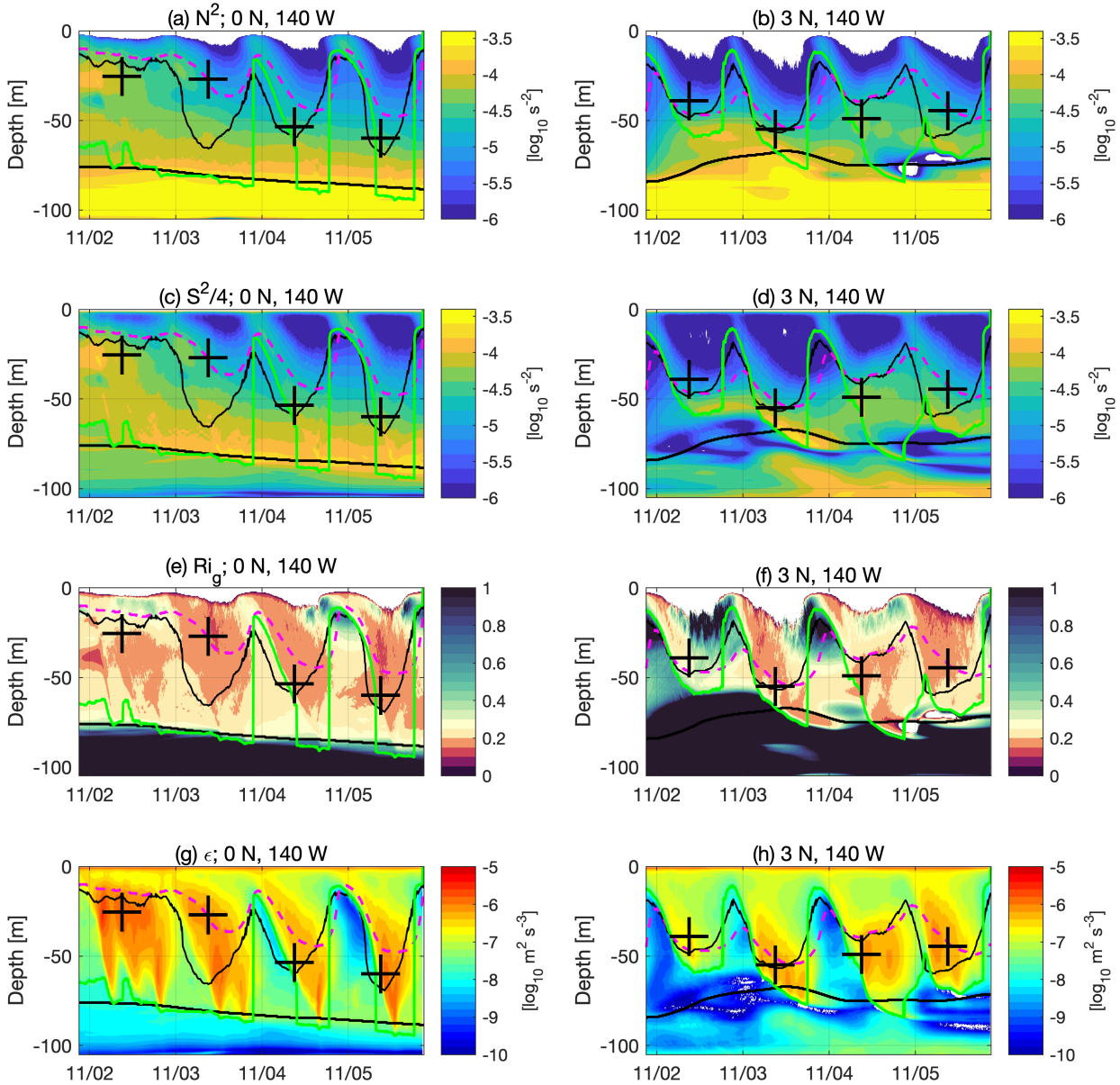
1684 FIG. 9. Time series of zonal and meridional velocity (color), temperature (white contours in $^{\circ}\text{C}$), mixed layer
 1685 depth (MLD, dashed magenta), the depth where the bulk Richardson number $Ri_b = 0.2$ (H_{Rib} , thin black), and
 1686 the base of the low-gradient Richardson number layer $Ri_g < 0.35$ (H_{Rig} , thick black) in the LES at 0° N and 3° N
 1687 along 140° W. All fields are defined from horizontally-averaged profiles. The MLD is defined to be the shallowest
 1688 depth where water is 0.015 kg/m^3 denser than the top 10 m in the instantaneous but horizontally-averaged density
 1689 profile. All time tick marks are at 0 UTC; local solar time at 0°N , 140° W is about 9 hours behind UTC, so local
 1690 solar noon is at about 21 UTC.



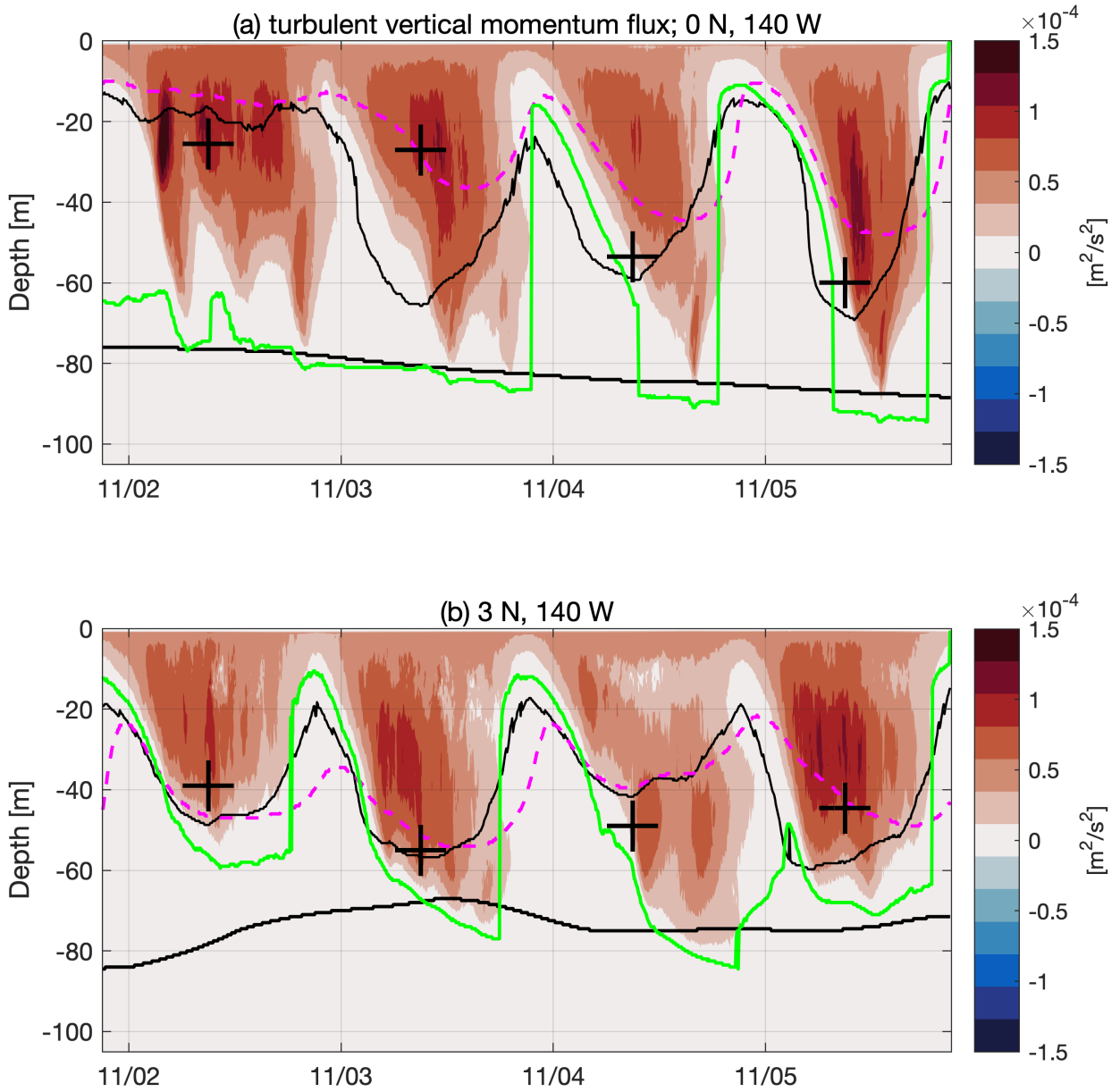
1691 FIG. 10. Time series of the net surface heat flux Q_0^{net} (left axis, blue), the magnitude of the wind stress $|\tau|$
 1692 (right axis, red), and the subsurface downward turbulent heat flux F_Q profiles from October-November 1985 in
 1693 the LES at 0° N (a) and 3° N (b) along 140° W. Overlaid on F_Q are the depth at which the bulk Richardson number
 1694 $Ri_b = 0.2$ (H_{Rib} , thin black line), the depth of the maximum daily-mean downward heat flux z_{max} (+ symbols),
 1695 the daily maximum MLD (defined from the horizontally averaged LES density profiles; magenta circles), and
 1696 the base of the low gradient Richardson layer $Ri_g < 0.35$ (H_{Rig} , thick black line). The daily-mean meridional
 1697 velocity averaged from 25 to 75 m depth is in blue; the origin is at a depth of 100 m, a 1 m spacing corresponds to
 1698 10 cm/s, and the peak-to-trough amplitudes are about 40 cm/s at 0° N and 90 cm/s at 3° N. For consistency with
 1699 other results in section 4, we plot $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 F_b \approx F_Q$ [W/m²] where ρ , c_p , and α are the reference
 1700 density, specific heat, and thermal expansion coefficient of seawater, respectively, g is the acceleration due to
 1701 gravity, and F_b is the downward turbulent buoyancy flux. All time tick marks are at 0 UTC, but local solar time
 1702 at 0° N, 140° W is about 9 hours behind UTC, so local solar noon is at about 21 UTC. Daily mean statistics (e.g.,
 1703 z_{max} indicated by + symbols) are calculated from 21 UTC so that the averages begin and end near solar noon.



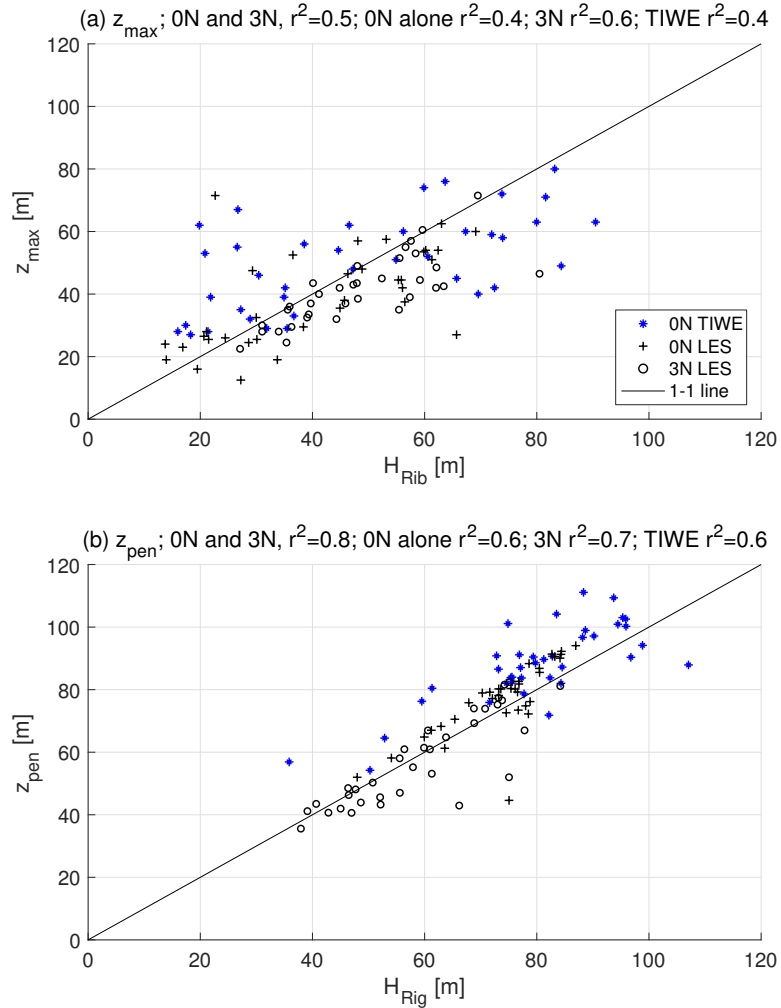
1704 FIG. 11. As in Fig. 10, but zoomed in on a few days in November and with the addition of the MLD
 1705 (dashed magenta) and the DCT penetration depth z_{pen} ($\epsilon \geq 2 \times 10^{-8} \text{ m}^2/\text{s}^3$; thin green). The MLD is defined
 1706 to be the shallowest depth where water is 0.015 kg/m^3 denser than the top 10 m in the instantaneous but
 1707 horizontally-averaged density profile.



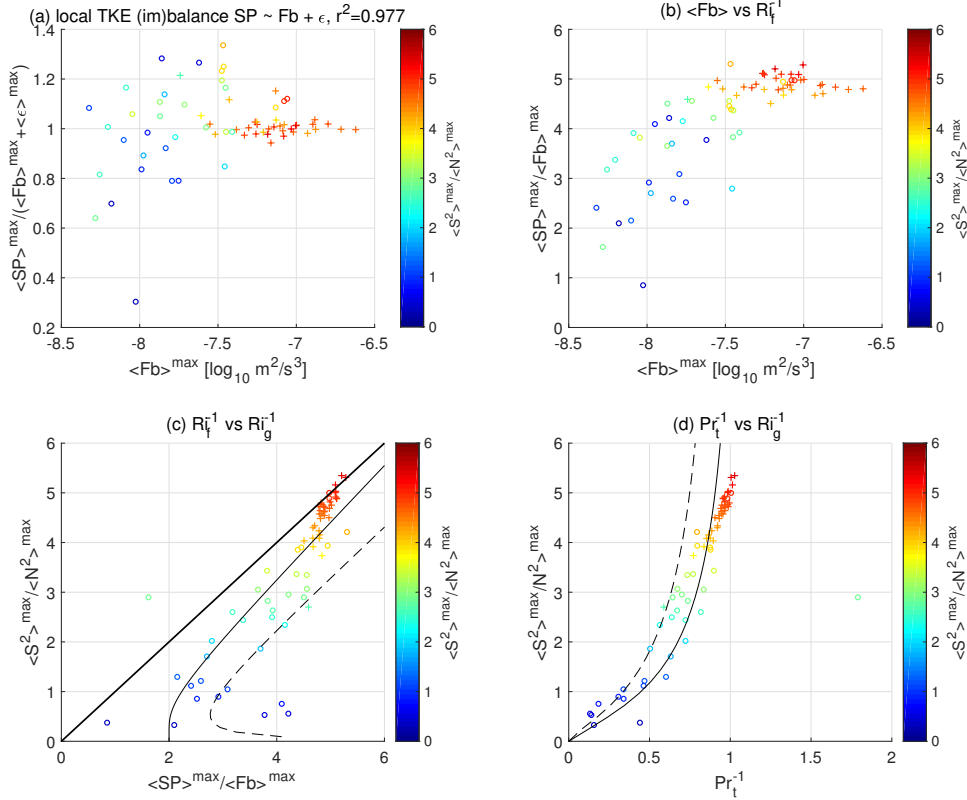
1708 FIG. 12. As in Fig. 11, but plots show (a)-(b) the vertical buoyancy gradient N^2 , (c)-(d) the squared vertical
 1709 shear S^2 , (e)-(f) $Ri_g = N^2/S^2$, and (g)-(h) the rate of dissipation of kinetic energy ϵ . It may be noted that there
 1710 are a few instances of elevated dissipation $10^{-8} < \epsilon < 10^{-7} \text{ m}^2/\text{s}^3$ below the deepest depths of DCT (z_{pen} , green
 1711 line) in (h) where $Ri_g > 1$. However, these instances of elevated dissipation near the bottom are dominated
 1712 by dissipation of the mean-flow kinetic energy, and the turbulent fluxes and energetics depend strongly on the
 1713 subgrid-scale parameterization in the LES (A6)-(A7), may be influenced by the bottom boundary, and should be
 1714 interpreted with caution.



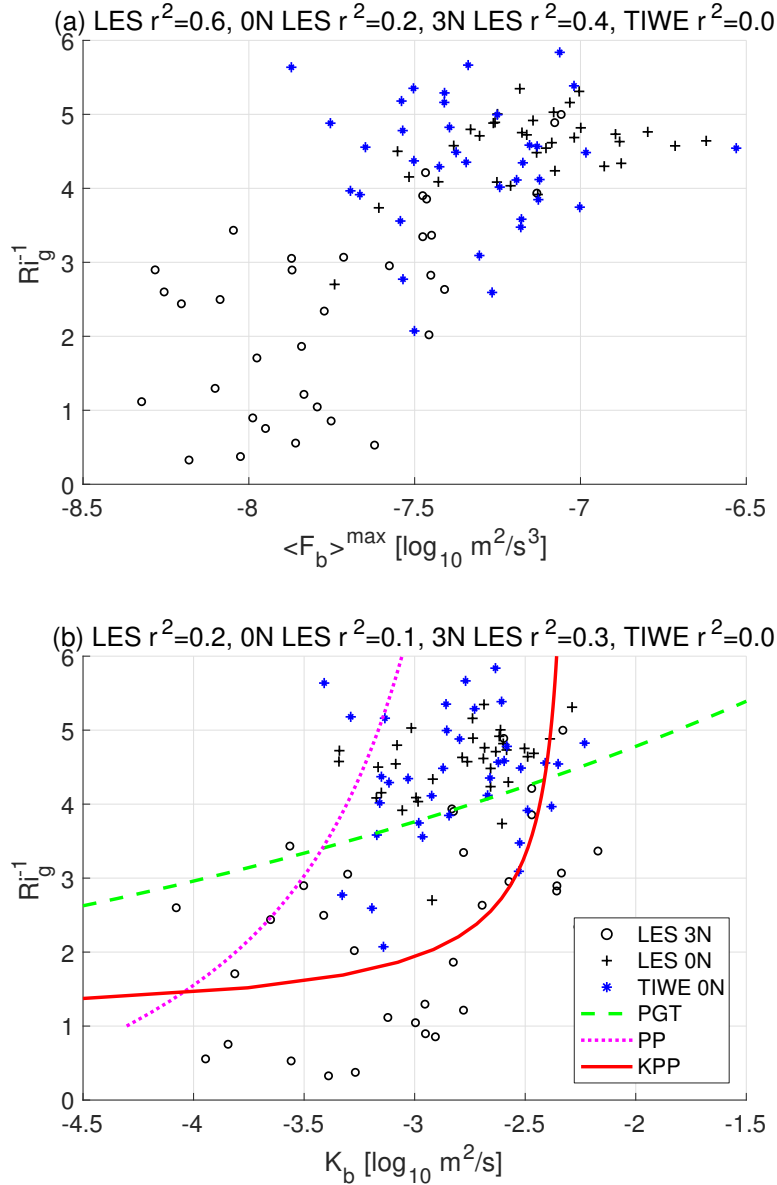
1715 FIG. 13. As in Fig. 11, but turbulent vertical momentum fluxes projected onto the shear, i.e. $(\mathbf{F}_m \cdot$
 1716 $\partial \mathbf{u}_h / \partial z) / |\partial \mathbf{u}_h / \partial z|$.



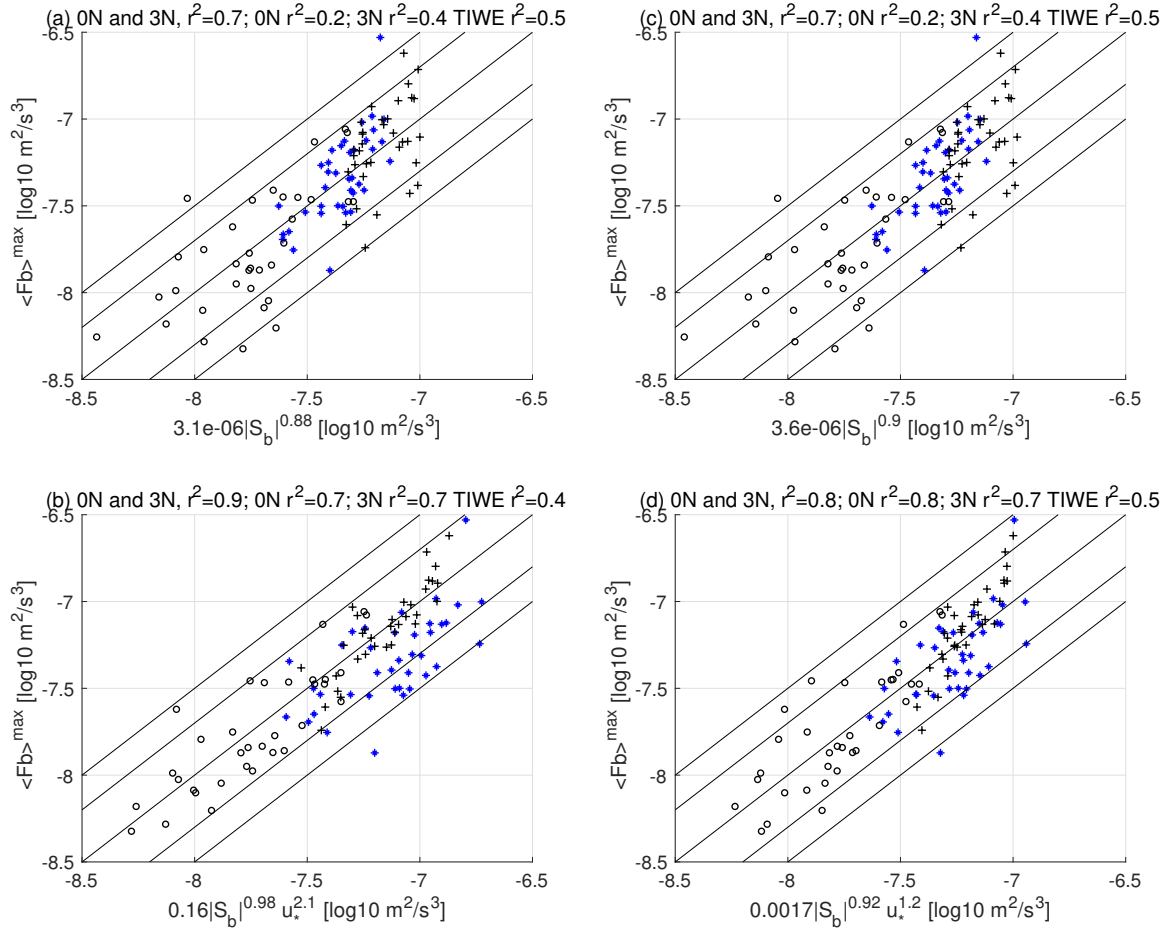
1717 FIG. 14. In (a), the depth z_{max} of maximum daily mean turbulent heat flux is related to the depth H_{Rib} at
 1718 which the bulk Richardson number is 0.2. And in (b), the daily maximum depth z_{pen} to which DCT penetrates
 1719 ($\epsilon > 2 \times 10^{-8} \text{ m}^2/\text{s}^3$) is related to the low-gradient Richardson layer depth H_{Rig} (above which $Ri_g < 0.35$).



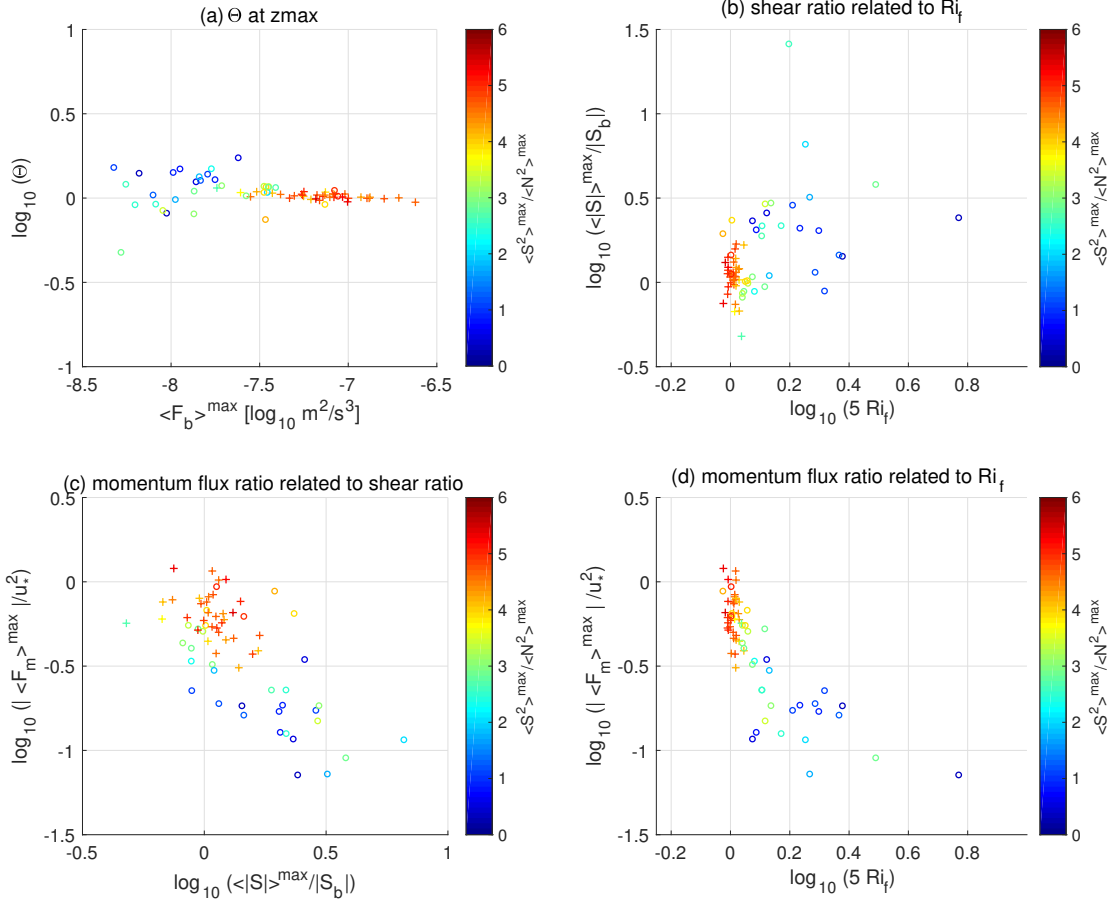
1720 FIG. 15. Relationships between various terms in the daily mean turbulent kinetic energy budget at the depth
 1721 z_{max} where the downward turbulent buoyancy flux is maximum ($\langle SP \rangle^{max} + \langle T \rangle^{max} \approx \langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}$; see
 1722 the Appendix for details). The depths z_{max} are plotted as + symbols in Fig. 10. Buoyancy flux $\langle F_b \rangle^{max}$ is
 1723 plotted against (a) shear production over buoyancy flux plus dissipation $\langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$ and (b)
 1724 shear production over buoyancy flux (i.e., the inverse flux Richardson number $Ri_f^{-1} = \langle SP \rangle^{max} / \langle F_b \rangle^{max}$). The
 1725 inverse gradient Richardson number of the horizontally-averaged profile $Ri_g^{-1} = \langle |\partial \mathbf{u}_h / \partial z|^2 \rangle^{max} / \langle \partial b / \partial z \rangle^{max} =$
 1726 $\langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ is shown in color on all four panels and on the y axes in (c)-(d) against Ri_f^{-1} (c) and $Pr_t^{-1} =$
 1727 Ri_f / Ri_g (d) (the inverse turbulent Prandtl number Pr_t^{-1} is the ratio of the turbulent diffusivity of buoyancy over
 1728 the turbulent viscosity of momentum). The thick black line (c) is the 1-1 line, the thin solid line is a fit to LES
 1729 of a coastal boundary layer under a hurricane by Watkins and Whitt (2020), and the thin dashed line is a fit to
 1730 atmospheric boundary layer observations by Anderson (2009), which parameterizes the subgrid-scale Pr_t^{-1} in
 1731 the LES. The two days with most anomalously low Ri_f^{-1} (b-c; $Ri_f^{-1} = 0.9$ and 1.6) and high Pr_t^{-1} (d; $Pr_t^{-1} = 0.4$
 1732 and 1.8) also have the largest relative non-local sources of turbulent kinetic energy $\langle T \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max}) \approx$
 1733 $1 - \langle SP \rangle^{max} / (\langle F_b \rangle^{max} + \langle \epsilon \rangle^{max})$ (i.e., the points with lowest values in a; 0.3 and 0.6). Plus (+) symbols are from
 1734 LES at 0° N and circles (o) from 3° N.



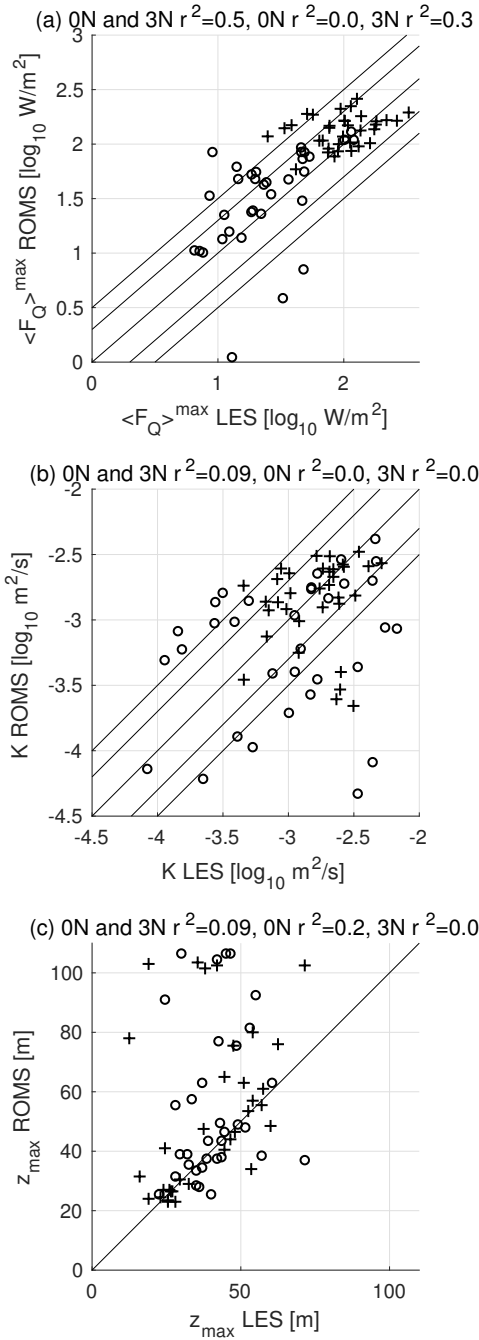
1735 FIG. 16. Relationship between $Ri_g^{-1} = \langle S^2 \rangle^{max} / \langle N^2 \rangle^{max}$ and $\langle F_b \rangle^{max}$ (a) and $K_b = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$ (b) at
 1736 z_{max} (i.e., at the depths indicated by the + symbols in Fig. 10). Averaging diffusivity directly in (b) yields
 1737 quantitatively different results but qualitatively the same conclusion that K_b is at best weakly related to Ri_g .
 1738 Overlaid in (b) are parameterizations of turbulent diffusivity as a function of Richardson number from Pacanowski
 1739 and Philander (1981) (PP) Peters et al. (1988) (PGT), and Large and Gent (1999) (KPP).



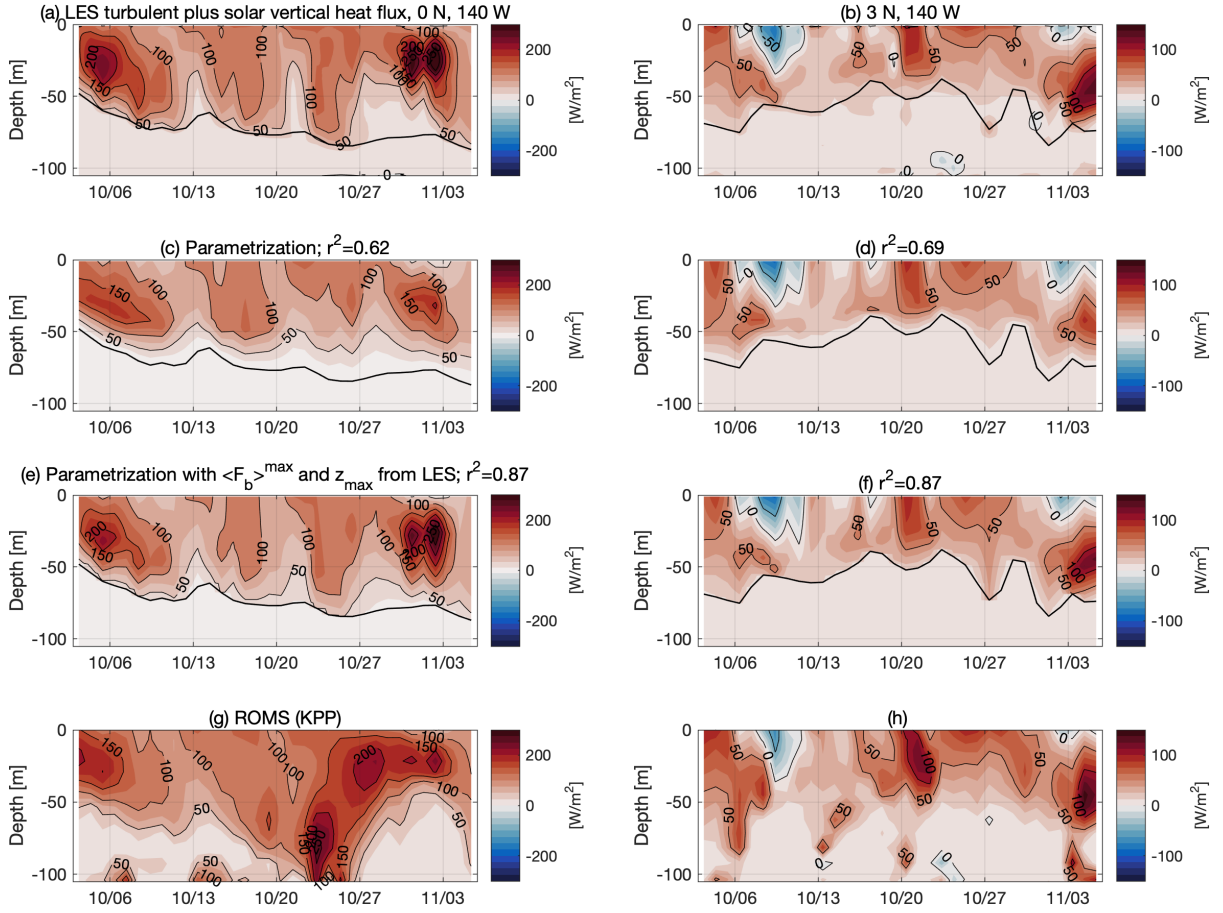
1740 FIG. 17. Maximum daily mean turbulent buoyancy flux $\langle F_b \rangle^{\max}$ scales with oceanic bulk vertical shear S_b
 1741 (a,c) and even more closely with a product of S_b and the magnitude of the surface wind stress $|\tau| = u_*^2 \rho$ (b,d).
 1742 The scalings are obtained via linear regression on the LES output in (a)-(b), which includes 34 days at 3° N
 1743 (black \circ) and 34 days at 0° N (black $+$), or on the 68 days of LES output plus 38 days of TIWE data (blue $*$)
 1744 in (c)-(d). Hence, the TIWE observations serve as an independent validation of the regressions in (a)-(b) and
 1745 constrain the regressions in (c)-(d). The predictors include S_b , which is derived from a linear fit to the mean
 1746 velocity from H_{Ri_g} to 5 m depth (thick black lines in Fig. 4), and the friction velocity $u_* = \sqrt{|\tau|/\rho}$. All variables
 1747 are log-transformed and Pearson's r in the panel titles is calculated in log space. The various diagonal black lines
 1748 indicate where the data are along the 1-1 line, within a factor of 2, and within a factor of 3. With 95% confidence
 1749 intervals, the scalings are as follows: $(2-6) \times 10^{-6} |S_b|^{(0.7-1.0)}$ (a), $(1-200) \times 10^{-2} |S_b|^{(0.9-1.1)} u_*^{(1.6-2.5)}$ (b),
 1750 $(2-6) \times 10^{-6} |S_b|^{(0.8-1.0)}$ (c), and $(0.03-1.3) \times 10^{-2} |S_b|^{(0.8-1.0)} u_*^{(0.9-1.6)}$ (d).



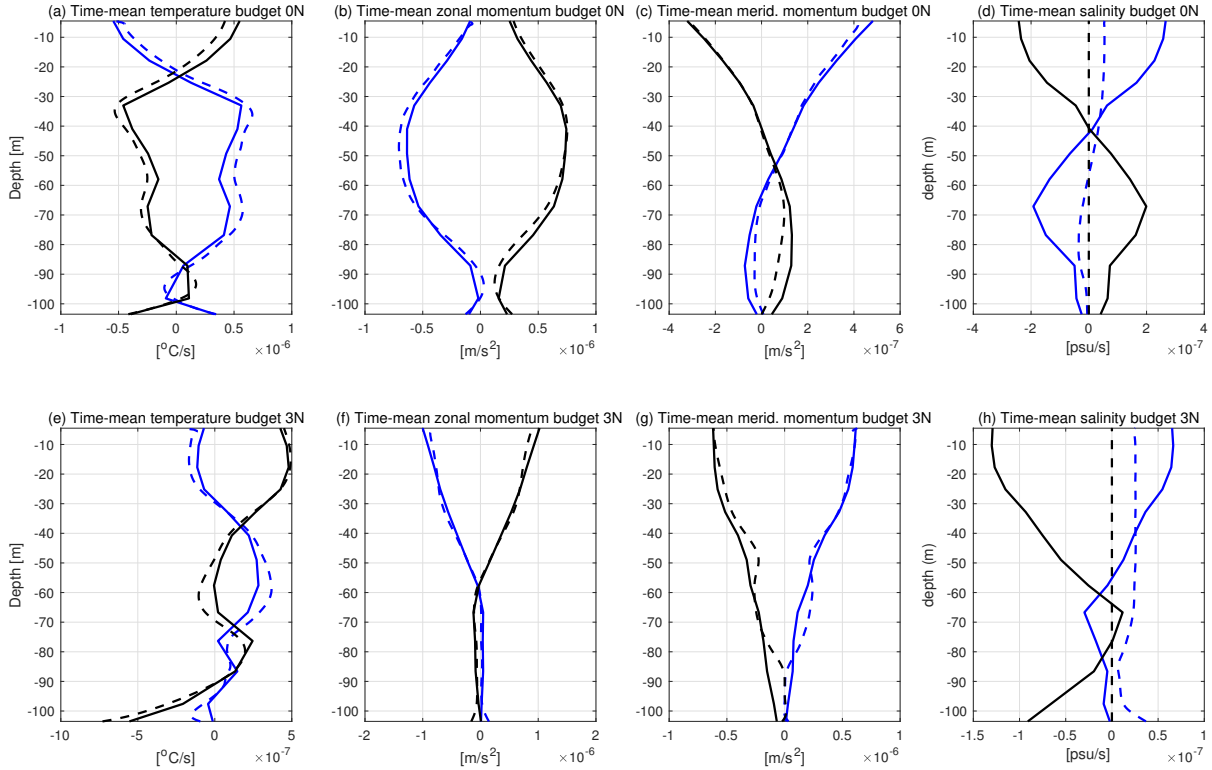
1751 FIG. 18. Various ratios of terms in Eqn. (6) showing how the local energetics of the buoyancy flux at z_{max}
 1752 (Fig. 15) relate to the bulk scalings derived via regression (Fig. 17). Circles (o) are from the LES at 3° N, and
 1753 pluses (+) are from the LES at 0° N; the color indicates $Ri_g^{-1} = \langle S^2 \rangle_{max} / \langle N^2 \rangle_{max}$.



1754 FIG. 19. Comparisons between the LES and ROMS (KPP) at the LES locations (+ at 0° N and ○ at 3° N along
 1755 140° W): (a) The maximum turbulent heat flux $\langle F_Q \rangle^{max}$, (b) the turbulent diffusivity of heat K at z_{max} , and (c)
 1756 the depth z_{max} at which $\langle F_Q \rangle^{max}$ occurs. Note, however, that the LES results are derived from the buoyancy
 1757 dynamics whereas the ROMS results are derived from the temperature dynamics. That is, the LES results are
 1758 $(\rho c_p / g \alpha) F_b = 1.37 \times 10^9 \langle F_b \rangle \approx \langle F_Q \rangle$ [W/m²] in (a) and $K = \langle F_b \rangle^{max} / \langle N^2 \rangle^{max}$ in (b), and z_{max} is calculated
 1759 from from $\langle F_b \rangle$ profiles.



1760 FIG. 20. Daily averaged net vertical heat flux $\langle Q \rangle$ (including turbulent F_Q as in Fig. 10 plus penetrative
 1761 radiative P_Q components) at 0° N, 140° W (left column) and 3° N, 140° W (right column) as simulated by the
 1762 LES [(a)-(b)] and as parameterized based on horizontally-averaged velocity and density profiles and net surface
 1763 buoyancy and momentum fluxes [(c)-(d)]. For reference, the the piecewise linear flux profiles with $\langle Q \rangle(z = 0)$
 1764 and $\langle Q \rangle(z = z_{max})$ from LES are shown in e-f. In addition, the vertical heat fluxes (penetrating shortwave plus
 1765 turbulent) from the parent ROMS model are shown in the bottom row [(e)-(f)]. Note the different colorbar
 1766 ranges in the left and right columns. For consistency with earlier results, (a)-(f) plot $(\rho c_p / g \alpha) \langle B \rangle \approx \langle Q \rangle$ where
 1767 $\langle B \rangle$ is the daily-averaged vertical buoyancy flux including the parts due to turbulence and penetrative shortwave
 1768 radiation.



1769 Fig. A1. Time-means of various terms in the horizontally-averaged tracer and momentum budgets from
 1770 ROMS (solid lines) and LES (dashed lines) at 0° N, 140° W (top) and 3° N, 140° W (bottom). The blue lines
 1771 represent the time-mean convergence of vertical transport of (a,e) temperature, (b,f) zonal momentum and (c,g)
 1772 meridional momentum and (d,h) salinity due to turbulence (and solar radiation in the case of temperature). The
 1773 black lines represent all other tendencies of horizontally-averaged momentum and tracers as diagnosed from
 1774 ROMS, i.e. \mathcal{F} (plus Coriolis in the case of momentum), and as diagnosed in LES, i.e. $\mathcal{F} + \mathcal{R}$ (plus Coriolis in
 1775 the case of momentum). See the Appendix for the budget formulas.