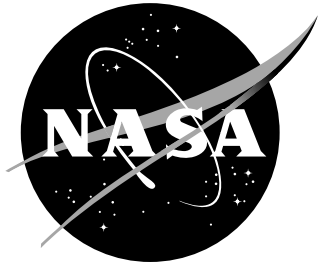


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In-Flight Turbulence Emulation for Fixed-Wing Aircraft using Equivalent Multisine Excitations

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September 2021

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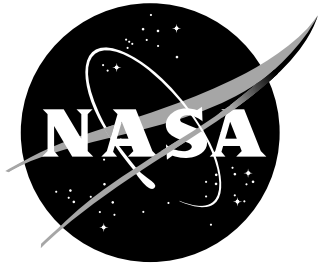
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Abstract

A procedure is discussed for designing control-equivalent turbulence input (CETI) excitations to emulate a fixed-wing aircraft response to turbulence while flying through calm air. These excitations are added to the actuator commands to produce the same motion of the aircraft that would have resulted from flying in actual turbulence. Although CETI models are usually shaping filters driven by white noise inputs, the equivalent control inputs in this report were constructed as multisine excitations, which enables mimicking the response to turbulence with arbitrary power spectra, within hardware limitations. Practical aspects of the approach are discussed, such as actuator dynamics, model uncertainty, actual in-flight turbulence, and others. The procedure is demonstrated using a flight dynamics simulation model of a subscale generic jet transport aircraft.

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Nomenclature

Roman

A	system state matrix
a_k, b_k	multisine amplitudes
a_x, a_y, a_z	accelerometer output components, g
B_δ, B_ξ	system control input matrices
b	wingspan, ft
$\bar{c}$	wing mean aerodynamic chord, ft
∂	partial derivative
f_k	harmonic frequencies, Hz
G	actuator transfer function matrix
g	acceleration due to gravity, ft/s ²
h	altitude, ft
$\Im$	imaginary part
$I_{xx}, I_{yy}, I_{zz}, I_{xz}$	inertia tensor elements, slug-ft ²
j	imaginary number, $= \sqrt{-1}$
k	harmonic number
L, M, N	body-axis moment components, ft-lbf
L_w	turbulence length, ft
m	aircraft mass, slug
N	number of data samples
n	number of elements
p, q, r	body-axis angular rates, rad/s
$\bar{q}$	dynamic pressure, lbf/ft ²
$\Re$	real part
S	wing reference area, ft ²
s	Laplace variable
T	time duration, s
t	time, s
u, v, w	body-axis translational velocities, ft/s
V	true airspeed, ft/s
W	weighting matrix
X	regressor matrix
X, Y, Z	body-axis aerodynamic force components, lbf
X_p, Y_p, Z_p	body-axis thrust components, lbf
x	state vector
x, y, z	body-axis locations, ft
z	measurement vector

Greek

α	angle of attack, rad
β	sideslip angle, rad
Δ	perturbation value
Δt	sample period, s
δ	control surface deflections, rad
μ	flank angle, rad
σ_w	turbulence intensity, ft/s
Φ	power spectrum
ϕ_k, ψ_k	multisine phase angles, rad
ξ	turbulence vector
ω	radian frequency, rad/s

Subscripts

0	reference value
c	commanded value
d	desired value
e	elevator
eq	control equivalent value
f	flap
g	turbulence component
m	measured value

Superscripts

$\cdot$	time derivative
$\wedge$	estimated value
-1	inverse
T	transpose

Acronyms

CETI	control-equivalent turbulence input
GTM	Generic Transport Model
MIMO	multiple input multiple output
SISO	single input single output
STA	Shuttle Training Aircraft

1 Introduction

When flight testing modern research aircraft, excitation inputs are added to various signals in the flight control system to evaluate the design using measured flight test data and system identification analyses. For example, excitations are summed with the command path at the actuator inputs to identify bare-airframe dynamics, at the mixer input for stability margins, at the sensor outputs for disturbance rejection bandwidth or other stability margins, and at the pilot effectors for closed-loop dynamics and handling qualities [1–3].

If a dynamic model of the flight dynamics is known, the same flight control architecture used to inject excitations for system identification can be used to apply other inputs to the command path that produce the same motion of the aircraft flying in calm air as it would experience while flying through turbulence. Such capabilities allow for an in-flight evaluation of the airplane, handling qualities, aeroelastic response, and flight control system in artificial turbulence before actual turbulence is encountered. Applying the same turbulence sequence facilitates characterizing random scatter or direct comparison of control laws, or using different turbulence sequences results in an in-flight Monte Carlo experiment.

One approach for in-flight turbulence emulation was used with the Shuttle Training Aircraft (STA), where a Grumman Gulfstream II airplane was modified and programmed with model-following control laws [4–6]. The reference model included turbulence inputs and the response of the aircraft tracked the reference model response through by using feedback. In this way, the aircraft response to turbulence could be evaluated and used to train future astronauts while flying in calm air. Actuator bandwidths and dynamics, system latencies, controllability, and actual in-flight turbulence degraded the extent to which turbulence was emulated.

For rotorcraft, the standard turbulence models used for fixed-wing aircraft (such as the Dryden or von Kármán continuous turbulence models [7]) are less applicable because the assumption that the turbulence field is fixed in space becomes invalid at low speeds or hover. Instead, a successful approach has been to use control-equivalent turbulence input (CETI) models [8, 9]. These models are identified from flight test data and require a known model of the flight dynamics. The analysis results in linear shaping filters that when driven with white noise inputs (similar to conventional turbulence models for fixed-wing aircraft [7]), produce control inputs that create the same responses in the aircraft motion as turbulence. CETI models have been identified from flight test data [10, 11], used for simulation analysis and control law optimization [12, 13], evaluated hold functions of a control law [14], quantified additional pilot workload during turbulence [15], and tested in flight [9, 12, 16]. During handling qualities testing, pilots have provided favorable remarks about the realism of CETI models [12, 16]. However, most of the work done has focused on single-input single-output (SISO) models for rotorcraft, and has neglected multiple-input multiple-output (MIMO) coupling of the models for turbulence emulation, such as for higher-fidelity rotorcraft models and for fixed-wing aircraft.

In this work, an approach using the CETI model concept is applied to design excitations for control surface actuator commands on a fixed-wing airplane to create similar responses when the aircraft is flying in calm air to when the aircraft is flying through turbulence. MIMO coupling is considered and as a result, multiple control effectors are required to emulate turbulence. The equivalent inputs were constructed as multisines and were added to the command path in the flight control system just before the actuators. Within hardware limitations, these multisines may be designed to have desired bandwidth and power spectra, and may be augmented to compensate for actuator dynamics.

The approach described has several advantages. It can be implemented using the existing excitation system built into flight control systems of modern research aircraft. No additional feedbacks or augmentation to an existing control system is needed for implementation, and the inputs can be applied when flying in an open-loop or closed-loop configuration. The approach could be thought of as an open-loop model following architecture. As a result, the response of the aircraft due to turbulence is not dependent on sensor noise levels. Other multisine excitations, such as those designed for system identification, have been used with many aircraft flight test campaigns [17–19] and are simple to design and analyze. The desired turbulence signals may have arbitrary power spectra and are not limited to traditional models or to those with rational power spectra.

The approach requires information about the linear dynamics of the aircraft, in terms of the static stability derivatives and the control derivatives at a particular condition. Actuator dynamics and bandwidth

are a limiting factor in the turbulence that can be simulated. Another limitation could be the aircraft configuration and the available actuators for use, particularly for quickly changing the lift or side force acting on the airplane. Also, the approach assumes flight in calm air, where any actual turbulence experienced in flight increases the turbulence response of the aircraft.

The remainder of this paper is organized as follows. Section 2 presents an overview of the method for generating equivalent control inputs. In Section 3, these inputs are developed for specific aircraft models, such as the short period approximation. Section 4 describes an approach for generating a random turbulence signal with an arbitrary power spectrum as a multisine. Section 5 discusses practical aspects of the approach for implementation. A simulation example is presented in Section 6 to demonstrate the approach. Conclusions are given in Section 7.

2 Method Overview

Under standard simplifying assumptions [20], the equations of motion for the bare-airframe dynamics of an aircraft flying through a moving volume of fluid may be modeled by the linear system

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B}_\delta \boldsymbol{\delta} + \mathbf{B}_\xi \boldsymbol{\xi} \quad (1a)$$

$$\mathbf{y} = \mathbf{C} \mathbf{x} + \mathbf{D}_\delta \boldsymbol{\delta} + \mathbf{D}_\xi \boldsymbol{\xi} + \mathbf{v} \quad (1b)$$

In Eq. (1), $\mathbf{x}$ is an $(n_x \times 1)$ vector of state perturbations, $\boldsymbol{\delta}$ is an $(n_\delta \times 1)$ vector of control effector perturbations, and $\boldsymbol{\xi}$ is an $(n_\xi \times 1)$ vector of turbulence components. The terms $\mathbf{A}$, $\mathbf{B}_\delta$, $\mathbf{B}_\xi$, $\mathbf{C}$, $\mathbf{D}_\delta$, and $\mathbf{D}_\xi$ are constant matrices for a given flight condition. The outputs of the system are the state and the $(n_y \times 1)$ vector of measured output perturbations $\mathbf{y}$ which include measurement noise $\mathbf{v}$. These dynamics are represented graphically as the simplified block diagram in Fig. 1.

The turbulence input may include any of the translational and rotational velocity components of the air mass. The rotational components p_g , q_g , and r_g are given along the aircraft body axes. The translational components may also be given along the body axes as u_g , v_g , and w_g , or in the wind axes as V_g , α_g , and β_g . In this work, the convention is taken that positive turbulence velocities are in the same directions as the corresponding variables in the body and wind axes [20–23]. For example, $u_g > 0$ is a tailwind, whereas $u_g < 0$ is a headwind. These conventions are illustrated in Fig. 2.

The idea of control-equivalent turbulence inputs is that the control effectors can be moved to create the same motion of the aircraft in calm air as it would have from flying in turbulence. Mathematically, this is achieved by equating the last two terms in Eq. (1a) as the constraint

$$\mathbf{B}_\delta \boldsymbol{\delta} = \mathbf{B}_\xi \boldsymbol{\xi} \quad (2)$$

Solving Eq. (2) for the equivalent control effector positions, now denoted as $\boldsymbol{\delta}_{eq}$, yields

$$\boldsymbol{\delta}_{eq} = \mathbf{B}_\delta^{-1} \mathbf{B}_\xi \boldsymbol{\xi} \quad (3)$$

For a given time history of turbulence velocities and for a linear dynamic model of the aircraft, Eq. (3) is the solution for the equivalent control inputs. These control inputs create the same response for the system defined by Eq. (1) as does the turbulence. All control surfaces move in proportion to the turbulence and are collinear.

Note that because $\mathbf{B}_\delta$ is inverted in Eq. (3), there must be at least the same number of control effectors as states on which the turbulence inputs act. Requirements on the number of control inputs and directionality of these inputs in the state space are discussed more in Section 5. In the rotorcraft literature, the requirement for multiple control surfaces is usually obviated by using SISO models used for the CETI analysis. For fixed-wing aircraft or higher-fidelity rotorcraft models, however, the presence of lifting surfaces requires multiple control surfaces to accurately mimic responses due to turbulence.

The state dynamics may be arbitrarily nonlinear because the constraint in Eq. (2) involves only the control and turbulence inputs to the bare-airframe dynamics. It is only necessary in the present development that

the control inputs and the turbulence inputs appear linearly in the equations of motion. However, for ease of discussion, only linear dynamics in the form of Eq. (1) are discussed in this work.

To emulate the aircraft response to turbulence while flying in calm air, the equivalent turbulence inputs are added to the command path for the control effectors. This is illustrated by the simplified block diagram in Fig. 3. The signal δ_d are the desired positions, which may come from the pilot and/or flight control system and include the effects of feedback control. The total effector position is a summation of the desired positions and the control-equivalent positions. The effect of the equivalent controls must also be added to any outputs for the air data variables, as the relevant sensors are subject to the local moving air mass.

3 Equivalent Control Surface Deflections

This section develops the equivalent control inputs needed to emulate flight in turbulence for several different linearized aircraft models. The short period approximation is first presented to develop the solution. This model is next expanded to include unsteady aerodynamics, and then the entire longitudinal dynamics model. Last, the lateral-directional model is discussed.

3.1 Short Period Model

For small perturbations about a steady flight condition, low frequencies, and low angles of attack, a representation of the dimensional aerodynamic forces and moment perturbations in the body frame for the classical short period approximation using quasi-static stability and control derivatives [23–25] is

$$Z = Z_\alpha (\alpha + \alpha_g) + Z_q q + Z_\delta \delta \quad (4a)$$

$$M = M_\alpha (\alpha + \alpha_g) + M_q q + M_\delta \delta \quad (4b)$$

In Eq. (4), a short-hand notation was used in which δ stands for all control effectors [20].

Only the vertical component of turbulence w_g is considered in this development using the short period model. The method for generating w_g is discussed later in Section 4. The dynamics affected by the lateral component are typically decoupled from the longitudinal dynamics, and the main effect of the forward turbulence component here is to alter the effective airspeed and dynamic pressure. The primary influence of the vertical component of turbulence is to create an angle of attack turbulence component, α_g , which for small perturbations is approximately

$$\alpha_g \simeq -\frac{w_g}{u_0} \quad (5)$$

Substituting the aerodynamic forces in Eq. (4) into the linearized equations of motion [2, 20], the short period model in the form of Eq. (1a) is

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 + Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_\delta \\ M_\delta \end{bmatrix} \delta + \begin{bmatrix} Z_\alpha \\ M_\alpha \end{bmatrix} \alpha_g \quad (6)$$

The constraint equation to be satisfied for this model, in the form of Eq. (2), is

$$\begin{bmatrix} Z_\delta \\ M_\delta \end{bmatrix} \delta = \begin{bmatrix} Z_\alpha \\ M_\alpha \end{bmatrix} \alpha_g \quad (7)$$

This constraint involves two equations but one unknown (control surface deflection, written in short-hand notation). To satisfy the constraint, the short-hand notation must be expanded to account for at least two control effectors, now denoted as δ_1 and δ_2 . With this minimum number of effectors, the system in Eq. (7) is fully determined, and the control input matrix can be inverted if it is not singular. More discussion on the requirements for the effectors and using additional effectors is in Section 5.

The solution for the equivalent control inputs given by Eq. (3) is then

$$\begin{bmatrix} \delta_{1eq} \\ \delta_{2eq} \end{bmatrix} = \begin{bmatrix} Z_{\delta_1} & Z_{\delta_2} \\ M_{\delta_1} & M_{\delta_2} \end{bmatrix}^{-1} \begin{bmatrix} Z_{\alpha} \\ M_{\alpha} \end{bmatrix} \alpha_g \quad (8a)$$

$$= \frac{1}{Z_{\delta_1} M_{\delta_2} - Z_{\delta_2} M_{\delta_1}} \begin{bmatrix} Z_{\alpha} M_{\delta_2} - Z_{\delta_2} M_{\alpha} \\ Z_{\delta_1} M_{\alpha} - Z_{\alpha} M_{\delta_1} \end{bmatrix} \alpha_g \quad (8b)$$

The perturbation outputs for the angle of attack vane, pitch rate gyroscope, and vertical accelerometer at the center of mass using the short period model are

$$\begin{bmatrix} \alpha_m \\ q_m \\ a_{z_m} \end{bmatrix} = \begin{bmatrix} 1 & -x_{\alpha}/V_0 \\ 0 & 1 \\ c_1 Z_{\alpha} & c_1 Z_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ c_1 Z_{\delta_1} & c_1 Z_{\delta_2} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ c_1 Z_{\alpha} \end{bmatrix} \alpha_g \quad (9)$$

where

$$c_1 = \frac{V_0}{g} \quad (10)$$

and where x_{α} is the longitudinal position of the angle of attack sensor relative to the center of mass along the body axes. Because the pitch rate and vertical accelerometer outputs in Eq. (9) are based on the motion of the aircraft, the replicated effects of turbulence are properly reflected in the outputs. However, when applying equivalent control inputs to replicate turbulence, the turbulence signal (or the influence of the equivalent control surface deflections) needs to be added to the air data measurement, as illustrated in Fig. 3 by the quantity y_{eq} .

3.2 Short Period Model with Unsteady Aerodynamics

To increase the fidelity of the short period model considered in Section 3.1, additional terms may be added to the aerodynamic model. The angular rate derivatives may be separated from the angle-of-attack rate derivatives, the turbulence rate terms may be included, and the rotational component of turbulence may be included by deriving [20, 23]

$$q_g = -\frac{\partial w_g}{\partial x} \quad (11a)$$

$$= -\frac{\partial w_g / \partial t}{\partial x / \partial t} \quad (11b)$$

$$\simeq -\frac{\dot{w}_g}{u_0} \quad (11c)$$

$$\simeq \dot{\alpha}_g \quad (11d)$$

Including these terms, the aerodynamic forces and moment perturbations expand [23, 24] as

$$Z = Z_{\alpha} (\alpha + \alpha_g) + Z_{\dot{\alpha}} (\dot{\alpha} + \dot{\alpha}_g) + Z_q (q - \dot{\alpha}_g) + Z_{\delta} \delta \quad (12a)$$

$$M = M_{\alpha} (\alpha + \alpha_g) + M_{\dot{\alpha}} (\dot{\alpha} + \dot{\alpha}_g) + M_q (q - \dot{\alpha}_g) + M_{\delta} \delta \quad (12b)$$

and the equations of motion become

$$\begin{bmatrix} 1 - Z_{\dot{\alpha}} & 0 \\ -M_{\dot{\alpha}} & 1 \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_{\alpha} & 1 + Z_q \\ M_{\alpha} & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta} \\ M_{\delta} \end{bmatrix} \delta + \begin{bmatrix} Z_{\alpha} & Z_{\dot{\alpha}} - Z_q \\ M_{\alpha} & M_{\dot{\alpha}} - M_q \end{bmatrix} \begin{bmatrix} \alpha_g \\ \dot{\alpha}_g \end{bmatrix} \quad (13)$$

Proceeding in the same manner as before, the equivalent control effectors are

$$\begin{bmatrix} \delta_{1eq} \\ \delta_{2eq} \end{bmatrix} = \begin{bmatrix} Z_{\delta_1} & Z_{\delta_2} \\ M_{\delta_1} & M_{\delta_2} \end{bmatrix}^{-1} \begin{bmatrix} Z_{\alpha} & Z_{\dot{\alpha}} - Z_q \\ M_{\alpha} & M_{\dot{\alpha}} - M_q \end{bmatrix} \begin{bmatrix} \alpha_g \\ \dot{\alpha}_g \end{bmatrix} \quad (14a)$$

$$= \frac{1}{Z_{\delta_1} M_{\delta_2} - Z_{\delta_2} M_{\delta_1}} \begin{bmatrix} Z_{\alpha} M_{\delta_2} - Z_{\delta_2} M_{\alpha} & (Z_{\dot{\alpha}} - Z_q) M_{\delta_2} - Z_{\delta_2} (M_{\dot{\alpha}} - M_q) \\ Z_{\delta_1} M_{\alpha} - Z_{\alpha} M_{\delta_1} & Z_{\delta_1} (M_{\dot{\alpha}} - M_q) - (Z_{\dot{\alpha}} - Z_q) M_{\delta_1} \end{bmatrix} \begin{bmatrix} \alpha_g \\ \dot{\alpha}_g \end{bmatrix} \quad (14b)$$

which is similar to the previous solution developed in Section 3.1, except that the expressions are more complicated and the control inputs are proportional to both the turbulence component and its rate. Note that when solving for the equivalent inputs, the matrix on the left side of Eq. (13) involving $Z_{\dot{\alpha}}$ and $M_{\dot{\alpha}}$ cancels during the algebra and does not affect the solution. Again, at least two control effectors are needed to emulate the aircraft response to turbulence because the turbulence is modeled as acting on two state variables.

The output equations for this case are

$$\begin{aligned} \begin{bmatrix} \alpha_m \\ q_m \\ a_{z_m} \end{bmatrix} &= \begin{bmatrix} 1 & -x_{\alpha}/V_0 \\ 0 & 1 \\ c_1 c_2 Z_{\alpha} & c_1 c_3 Z_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ c_1 c_2 Z_{\delta_1} & c_1 c_2 Z_{\delta_2} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \\ &+ \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ c_1 c_2 Z_{\alpha} & c_1 c_2 (Z_{\dot{\alpha}} - Z_q) \end{bmatrix} \begin{bmatrix} \alpha_g \\ \dot{\alpha}_g \end{bmatrix} \end{aligned} \quad (15)$$

where

$$c_2 = 1 + \frac{Z_{\dot{\alpha}}}{1 - Z_{\dot{\alpha}}} \quad (16)$$

$$c_3 = 1 + \frac{Z_{\dot{\alpha}}}{1 - Z_{\dot{\alpha}}} \frac{1 + Z_q}{Z_q} \quad (17)$$

3.3 Longitudinal Model

To expand the model discussed in Section 3.1 to the full decoupled longitudinal dynamics, the drag force is included as well as the airspeed state and airspeed gust contributions. For small perturbations and low angles of attack, the airspeed gust perturbation is approximately

$$V_g \simeq u_g \quad (18)$$

The relevant quasi-static aerodynamic forces and moment perturbations in the body frame for a given propulsion power level are

$$X = X_V (V + V_g) + X_{\alpha} (\alpha + \alpha_g) + X_q q + X_{\delta} \delta \quad (19a)$$

$$Z = Z_V (V + V_g) + Z_{\alpha} (\alpha + \alpha_g) + Z_q q + Z_{\delta} \delta \quad (19b)$$

$$M = M_V (V + V_g) + M_{\alpha} (\alpha + \alpha_g) + M_q q + M_{\delta} \delta \quad (19c)$$

The equations of motion are then

$$\begin{bmatrix} \dot{V} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_V & g + X_{\alpha} & X_q & -g \\ Z_V & Z_{\alpha} - \frac{X_{p0}}{mV_0} & 1 + Z_q & 0 \\ M_V & M_{\alpha} & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta} \\ Z_{\delta} \\ M_{\delta} \\ 0 \end{bmatrix} \delta + \begin{bmatrix} X_V & X_{\alpha} \\ Z_V & Z_{\alpha} \\ M_V & M_{\alpha} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ \alpha_g \end{bmatrix} \quad (20)$$

for low angles of attack and a zero initial flight path angle [2].

Proceeding, the constraint equations to be satisfied are

$$\begin{bmatrix} X_{\delta_1} & X_{\delta_2} & X_{\delta_3} \\ Z_{\delta_1} & Z_{\delta_2} & Z_{\delta_3} \\ M_{\delta_1} & M_{\delta_2} & M_{\delta_3} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} X_V & X_{\alpha} \\ Z_V & Z_{\alpha} \\ M_V & M_{\alpha} \end{bmatrix} \begin{bmatrix} V_g \\ \alpha_g \end{bmatrix} \quad (21)$$

with the equivalent control inputs

$$\begin{bmatrix} \delta_{1_{eq}} \\ \delta_{2_{eq}} \\ \delta_{3_{eq}} \end{bmatrix} = \begin{bmatrix} X_{\delta_1} & X_{\delta_2} & X_{\delta_3} \\ Z_{\delta_1} & Z_{\delta_2} & Z_{\delta_3} \\ M_{\delta_1} & M_{\delta_2} & M_{\delta_3} \end{bmatrix}^{-1} \begin{bmatrix} X_V & X_{\alpha} \\ Z_V & Z_{\alpha} \\ M_V & M_{\alpha} \end{bmatrix} \begin{bmatrix} V_g \\ \alpha_g \end{bmatrix} \quad (22)$$

For this case, at least three control effectors were needed because the turbulence inputs acted on three state variables. The same number of control effectors would be needed for fewer or additional turbulence inputs, for example from neglecting α_g or including q_g . As before, the control surfaces all move in a correlated manner.

The output equations for this case are

$$\begin{bmatrix} V_m \\ \alpha_m \\ q_m \\ \theta_m \\ a_{x_m} \\ a_{z_m} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -x_\alpha/V_0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ c_1 X_V & c_1 X_\alpha & c_1 X_q & 0 \\ c_1 Z_V & c_1 Z_\alpha & c_1 Z_q & 0 \end{bmatrix} \begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_1 X_{\delta_1} & c_1 X_{\delta_2} & c_1 X_{\delta_3} \\ c_1 Z_{\delta_1} & c_1 Z_{\delta_2} & c_1 Z_{\delta_3} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ c_1 X_V & c_1 X_\alpha \\ c_1 Z_V & c_1 Z_\alpha \end{bmatrix} \begin{bmatrix} V_g \\ \alpha_g \end{bmatrix} \quad (23)$$

Note here that the turbulence contributions are needed for the airspeed measurements, as well as the angle of attack measurements.

3.4 Lateral-Directional Model

The decoupled lateral-directional model can be developed in a similar manner. For small perturbations and low angles of attack, the lateral velocity perturbation component of turbulence can be approximated in terms of sideslip as

$$\beta_g \simeq \frac{v_g}{u_0} \quad (24)$$

The aerodynamic forces and moment perturbations using quasi-steady aerodynamics are expanded as

$$Y = Y_\beta (\beta + \beta_g) + Y_p p + Y_r r + Y_\delta \delta \quad (25a)$$

$$L = L_\beta (\beta + \beta_g) + L_p p + L_r r + L_\delta \delta \quad (25b)$$

$$N = N_\beta (\beta + \beta_g) + N_p p + N_r r + N_\delta \delta \quad (25c)$$

The equations of motion are

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & I_{xx} & -I_{xz} & 0 \\ 0 & -I_{xz} & I_{zz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_\beta & Y_p & Y_r - 1 & g/V_0 \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} Y_\delta \\ L_\delta \\ N_\delta \\ 0 \end{bmatrix} \delta + \begin{bmatrix} Y_\beta \\ L_\beta \\ N_\beta \\ 0 \end{bmatrix} \beta_g \quad (26)$$

at low angles of attack [2]. If the minimum number of three control effectors are available, the equivalent input is

$$\begin{bmatrix} \delta_{1_{eq}} \\ \delta_{2_{eq}} \\ \delta_{3_{eq}} \end{bmatrix} = \begin{bmatrix} Y_{\delta_1} & Y_{\delta_2} & Y_{\delta_3} \\ L_{\delta_1} & L_{\delta_2} & L_{\delta_3} \\ N_{\delta_1} & N_{\delta_2} & N_{\delta_3} \end{bmatrix}^{-1} \begin{bmatrix} Y_\beta \\ L_\beta \\ N_\beta \end{bmatrix} \beta_g \quad (27)$$

Note that, similar to the case in Section 3.2, the generalized mass matrix on the left side of Eq. (26) did not affect the solution for the equivalent control inputs. If the matrix were absorbed into the right side of the equation and the derivatives were written using the primed notation [20], as is usually done for integrating the equations of motion, the analogous solution would be

$$\begin{bmatrix} \delta_{1_{eq}} \\ \delta_{2_{eq}} \\ \delta_{3_{eq}} \end{bmatrix} = \begin{bmatrix} Y_{\delta_1} & Y_{\delta_2} & Y_{\delta_3} \\ L'_{\delta_1} & L'_{\delta_2} & L'_{\delta_3} \\ N'_{\delta_1} & N'_{\delta_2} & N'_{\delta_3} \end{bmatrix}^{-1} \begin{bmatrix} Y_\beta \\ L'_\beta \\ N'_\beta \end{bmatrix} \beta_g \quad (28)$$

The outputs for this model are

$$\begin{aligned} \begin{bmatrix} \beta_m \\ p_m \\ r_m \\ \phi_m \\ a_{y_m} \end{bmatrix} &= \begin{bmatrix} 1 & z_\beta/V_0 & -x_\beta/V_0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ c_1 Y_\beta & c_1 Y_p & c_1 Y_r & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_1 Y_{\delta_1} & c_1 Y_{\delta_2} & c_1 Y_{\delta_3} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ c_1 Y_\beta \end{bmatrix} \beta_g \end{aligned} \quad (29)$$

where x_β and z_β are the forward and vertical locations of the side slip sensor relative to the aircraft center of mass along the body axes. Similar to the longitudinal case, the side slip turbulence is added to the air data measurement, but no further modification is needed for the inertial measurements.

4 Turbulence Generation

The most common approach for simulating continuous turbulence is to pass a zero-mean, unit-intensity, and band-limited white noise signal through a shaping filter with a transfer function derived from the modeled turbulence power spectrum [7, 26–28]. The transfer function for that shaping filter can come from the Dryden or von Kármán models of turbulence, CETI models identified from flight test data, or other sources. The resulting filter output will have the desired statistical properties of the modeled turbulence.

A different approach for simulating turbulence is to synthesize the turbulence signal in the time domain from a frequency-domain description of the power spectrum and randomized phase angles [29]. This process produces a time history with power at a set of discrete and harmonic frequencies over a bandwidth of interest, similar to the multisine excitation inputs [2, 17] used for system identification maneuvers during flight tests of many aircraft [17–19]. As the number of frequencies increases (by either extending the time duration of the simulated turbulence or by increasing the bandwidth), the simulated turbulence becomes more representative of the actual modeled turbulence.

The Fourier-based design procedure begins by establishing an array of time samples

$$t_i = (i - 1)\Delta t, \text{ for } i = 1, 2, \dots, N \quad (30)$$

where Δt is the sample period,

$$N = \frac{T}{\Delta t} + 1 \quad (31)$$

is the number of samples, and T is the time duration of the turbulence. The time duration then defines the harmonic frequencies

$$f_k = \frac{k}{T}, \text{ for } k = 1, 2, \dots, N/2 \quad (32)$$

of the associated fundamental time period up to the Nyquist frequency. A random sequence for a gust velocity component is then synthesized as the Fourier series

$$\xi(t_i) = \sum_{k=1}^{N/2} a_k \sin(2\pi f_k t_i + \phi_k) \quad (33)$$

The coefficients a_k are discussed next and define the power spectrum of the input. The phase angles are drawn randomly from a uniform distribution over the interval $[0, 2\pi)$. Different realizations of the phase angles create different turbulence sequences but with the same power spectra. The bandwidth of the turbulence signal can be selected by discarding values of k in the generation of Eq. (33). Unlike the multisines applied for

system identification [2, 17–19], which are designed so that multiple inputs are mutually orthogonal, here all associated harmonic frequencies are applied to all turbulence components.

The amplitude coefficients a_k in Eq. (33) define the power spectrum of the signal. For example, selecting equal values of a_k would produce a band-limited white noise signal that could be used to emulate sensor measurement noise. In conventional turbulence models for fixed-wing aircraft, the power spectra $\Phi(j\omega)$ are specified [7]. After sampling this power spectrum at frequencies corresponding to the harmonic frequencies in Eq. (32), the coefficients are computed as

$$a_k = \sqrt{2\Delta\omega \Phi(j\omega_k)} \quad (34)$$

where

$$\Delta\omega = \frac{2\pi}{T} \quad (35)$$

is the resolution of the radial frequency distribution. The power spectra can be arbitrary (for example, from empirical data) and does not need to be approximated by rational polynomials such as a transfer function.

If unsteady aerodynamics are considered, as in Section 3.2, the gust rates must also be available. Because the turbulence is constructed as a summation of harmonic sinusoids, the gust component rates can be obtained exactly by analytically differentiating Eq. (33) as

$$\dot{\xi}(t_i) = -2\pi \sum_{k=1}^{N/2} a_k f_k \cos(2\pi f_k t_i + \phi_k) \quad (36)$$

If a filtering approach is used to construct the turbulence instead of using multisines, then a second filter implementing a derivative is needed, or numerical derivatives must be applied, which can introduce kinematic errors between the turbulence component and its derivative.

5 Further Discussion

To emulate flying in turbulence on an aircraft, a number of practical issues must be considered. The first issue is that computer-generated excitations must be added to the command path, ideally at the actuators just before any software position and rate limiters are applied. The equivalent commands could conceivably be injected at other places within the flight control system. While analog legacy aircraft would need to be augmented, this setup is becoming standard for modern research aircraft [17–19, 24].

Another issue is the minimum number of control effectors needed. The short period model requires two control effectors, whereas the full longitudinal model and the lateral-directional model each require three. Some aircraft may need to be outfitted with additional control effectors to accurately emulate flight in turbulence, which was the case for the STA in-flight trainer [4–6]. If fewer effectors are available, the problem can not be solved as posed. Instead, a simpler dynamic model would be needed, involving fewer state variables, and would result in a less accurate emulation of turbulence in flight.

Extra control effectors, beyond the minimum number needed, can also be used to emulate flight in turbulence. Using multiple control effectors can reduce control activity and control rate requirements. In this case a turbulence mixer is needed to distribute the equivalent control surface commands because there are more unknowns than equations or constraints, so that the problem is underdetermined. For the short-period model considered in Section 3.1, a simple allocation law given by the weighted pseudo-inverse approach [30] for three control surfaces is

$$\delta_{eq}(t) = \mathbf{W}^{-1} \mathbf{X}^T (\mathbf{X} \mathbf{W}^{-1} \mathbf{X}^T)^{-1} \mathbf{z}(t) \quad (37a)$$

where

$$\mathbf{X} = \begin{bmatrix} Z_{\delta_1} & Z_{\delta_2} & Z_{\delta_3} \\ M_{\delta_1} & M_{\delta_2} & M_{\delta_3} \end{bmatrix} \quad (37b)$$

$$\mathbf{z} = \begin{bmatrix} Z_{\alpha} \\ M_{\alpha} \end{bmatrix} \alpha_g(t) \quad (37c)$$

and where $\mathbf{W}$ is a (3×3) weighting matrix. For simplicity, the weighting matrix is usually designed to be a diagonal matrix. The matrix inversion needed in Eq. (37a) needs to be computed only once for a given model and weighting matrix. Similar to the case with the minimum number of effectors, each of the equivalent control inputs are proportional to each other and the desired turbulence sequence.

In addition to needing a minimum number of unique control effectors, the effectors must also be able to force each of the state equations in a non-collinear manner. For example in Section 3.1, if both effectors only produced pitching moments but created negligible vertical forces, then $Z_{\delta_1} = Z_{\delta_2} = 0$ and the vertical force constraint in Eq. (7) could not be enforced, which would prohibit an accurate emulation of in-flight turbulence. Furthermore, if the control effectiveness of different controls is the same for both pitch moment and vertical force, then the matrix inversion used in Eq. (8) to calculate the equivalent control inputs is ill-conditioned because the determinant $Z_{\delta_1}M_{\delta_2} - Z_{\delta_2}M_{\delta_1}$ is zero. This situation would happen, for example, if inboard and outboard pairs of symmetric elevators were to be used for in-flight turbulence emulation. This issue is related to the concept of controllability for linear systems. A more successful approach could be to use inboard flaps or a forward canard with aft elevators. The challenge for fixed-wing aircraft is to have a control effector that can move quickly and impact the lift on the airplane. This approach is made difficult because the faster conventional control surfaces on fixed-wing aircraft are primarily used to impart moments, rather than direct forces, on the aircraft.

Another issue is that although the equivalent control effector positions were designed, only the actuator commands

$$\delta(j\omega) = \mathbf{G}(j\omega) \delta_c(j\omega) \quad (38)$$

can be manipulated, in which $\mathbf{G}$ is the $(n_\delta \times n_\delta)$ diagonal transfer function matrix for the actuators and δ_c is the effector command. For aircraft applications, the actuators typically have significant dynamics that limit the ability to replicate turbulence at higher frequencies. The constraint to be enforced in this case changes from Eq. (2) to

$$\mathbf{B}_\delta \mathbf{G}(j\omega) \delta_c(j\omega) = \mathbf{B}_\xi \xi(j\omega) \quad (39)$$

with the equivalent control command

$$\delta_{ceq}(j\omega) = [\mathbf{B}_\delta \mathbf{G}(j\omega)]^{-1} \mathbf{B}_\xi \xi(j\omega) \quad (40)$$

Because the turbulence components were generated using multisines, Eq. (40) can be implemented in the time domain using multisines with augmented amplitudes and phase angles. For the short period model in Section 3.1, the simplified equations for the equivalent control effector commands are

$$\begin{bmatrix} \delta_{c1eq} \\ \delta_{c2eq} \end{bmatrix} = \frac{1}{Z_{\delta_1}M_{\delta_2} - Z_{\delta_2}M_{\delta_1}} \begin{bmatrix} (Z_\alpha M_{\delta_2} - Z_{\delta_2}M_\alpha) \sum_{k=1}^{N/2} b_{1k} \sin(2\pi f_k t_i + \psi_{1k}) \\ (Z_{\delta_1}M_\alpha - Z_\alpha M_{\delta_1}) \sum_{k=1}^{N/2} b_{2k} \sin(2\pi f_k t_i + \psi_{2k}) \end{bmatrix} \quad (41a)$$

with

$$b_{jk} = a_k |1/G_{jj}(j\omega_k)| \quad (41b)$$

$$\psi_{jk} = \phi_k + \arctan \left[\frac{\Im\{1/G_{jj}(j\omega_k)\}}{\Re\{1/G_{jj}(j\omega_k)\}} \right] \quad (41c)$$

for $j = 1, 2, \dots, n_\delta$. In Eq. (41a), the angle of attack turbulence component multisine has been absorbed into the vector on the right side of the equation, unlike in Eq. (8), and two sets of augmented amplitudes and phases are needed to compensate for the effects of potentially different actuator models. The equivalent turbulence input is amplified at the higher frequencies to compensate for the actuator roll-off so that the resulting control surface deflections match those needed to replicate the motion in flight.

When considering actuator dynamics, a limiting factor in the turbulence that can accurately be emulated is the actuator position and rate limits. The combination of these limits will determine the combinations of maximum turbulence level and frequency content that can be emulated in flight.

Although the equivalent control inputs were developed for models based on dimensional stability and control derivatives, the nondimensional versions could also be used. For example in Section 3.1, Eq. (8) can

also be written as

$$\begin{bmatrix} \delta_{1eq} \\ \delta_{2eq} \end{bmatrix} = \frac{1}{C_{Z_{\delta_1}} C_{m_{\delta_2}} - C_{Z_{\delta_2}} C_{m_{\delta_1}}} \begin{bmatrix} C_{Z_\alpha} C_{m_{\delta_2}} - C_{Z_{\delta_2}} C_{m_\alpha} \\ C_{Z_{\delta_1}} C_{m_\alpha} - C_{Z_\alpha} C_{m_{\delta_1}} \end{bmatrix} \alpha_g \quad (42)$$

where the derivatives are related by

$$Z_\alpha = \frac{\bar{q}S}{mV} C_{Z_\alpha} \quad Z_{\delta_e} = \frac{\bar{q}S}{mV} C_{Z_{\delta_e}} \quad M_\alpha = \frac{\bar{q}S\bar{c}}{I_{yy}} C_{m_\alpha} \quad M_{\delta_e} = \frac{\bar{q}S\bar{c}}{I_{yy}} C_{m_{\delta_e}} \quad (43)$$

If the nondimensional form is used, the same turbulence input could be scaled according to the flight condition (for example, using different weights or trim angles of attack) and repeated.

This approach requires an accurate model of the stability and control derivatives at the flight condition(s) of interest. For the quasi-steady models, only the static stability derivatives and control derivatives are needed, and these terms are relatively easily obtained by analytical and experimental methods in comparison to the rate derivatives. For the unsteady model in Section 3.2, special wind tunnel tests or flight maneuvers [31–33] are typically needed to obtain the translational acceleration derivatives and rate derivatives. Uncertainty in these parameters results in errors in the equivalent control positions and undesired motions of the aircraft. Unlike with the model-following control law used with the STA, there is no feedback mechanism here to ensure the aircraft flies as desired.

The approach assumes that the airplane is flying in calm air. In the real case, the airplane is flying in some level of actual turbulence, while emulating the effect due to other simulated turbulence. As these excitations are applied to the command path, in contrast to the model-following approach where feedback is used to follow the model responses, this can result in additional responses from the combined simulated and actual turbulence inputs. If a real-time estimate of the turbulence $\hat{\xi}$ were available during flight [23], the equivalent input (for example, Eq. (3) from Section 3.1) could be diminished by this amount as

$$\delta_{eq} = \mathbf{B}_\delta^{-1} \mathbf{B}_\xi [\xi - \hat{\xi}] \quad (44)$$

6 Simulation Example

The Generic Transport Model (GTM) is a subscale, dynamically-scaled model of a conventional jet transport airplane that was used during the NASA Aviation Safety Program [34]. During the project, wind tunnel tests and flight tests were performed using approximately the same aircraft shape. For example, Fig. 4 shows the wind tunnel test article installed in the NASA Langley Research Center 14×22 Ft Subsonic Tunnel, and Fig. 5 shows the flight test vehicle, tail number T-2, in flight.

One simulation model of the GTM is the GTM PolySim [35], which is a nonlinear, 6-DOF, rigid-body simulation of the bare-airframe dynamics written in MATLAB[®]. The aerodynamic model uses a buildup of the nondimensional aerodynamic coefficients, based on static aerodynamics with increments due to angular rates and control deflections. The aerodynamic model uses tabulated data from global polynomial functions identified from static and forced oscillation data taken in the NASA Langley Research Center 14×22 Ft Subsonic Tunnel, and rotary balance data taken in the NASA Langley Research Center 20 Ft Vertical Spin Tunnel. The aerodynamic data were valid over large ranges of air flow angles and were useful for flight research in loss of control scenarios. The control surfaces modeled in this simulation include elevator, aileron, rudder, left and right spoilers, stabilizer, flaps, and landing gear. Nominal geometry and mass properties of the airplane are given in Table 1. More information on the simulation can be found in the documentation associated with Ref. [35].

The GTM PolySim was trimmed for straight and level flight at a nominal flight condition described by the parameters listed in Table 2. Central finite-differences were used to numerically linearize the aerodynamics and produce the nondimensional stability and control derivatives listed in Table 3. Only the short period dynamics were investigated using the quasi-static aerodynamics and the vertical component of turbulence, as discussed in Section 3.1.

The control surfaces selected to emulate flight in turbulence for this example were the symmetric flaps δ_f on the inboard section of the wing, and the ganged elevator δ_e on the horizontal tail. The primary use of the flaps here is to generate lift whereas the elevator was used to generate pitching moment. Although for conventional aircraft flaps are moved slowly between a few discrete settings, for the T-2 airplane the flaps could be commanded to arbitrary values and at higher rates, similar to the elevator. The actuator model for both the flap and elevator, identified from laboratory testing with the T-2 airplane, was

$$G_f(s) = G_e(s) = \frac{1}{0.0318s + 1} e^{-0.01s} \quad (45)$$

Position limits for these surfaces are listed in Table 1. The simulation was trimmed with flaps deflected to half the limit (15 deg) so they could be moved equally in both directions.

A block diagram of the simulation setup used for this example is shown in Fig. 6. The simulation was run in an open-loop configuration without feedback control. The recorded outputs were angle of attack, pitch rate, and vertical accelerometer signals at the aircraft center of mass. No measurement noise was added to the sensor outputs, for a clearer comparison. In the case that equivalent control inputs for turbulence were run, the synthetic angle of attack gust was added to the angle of attack sensor measurements.

The vertical gust component was simulated using the von Kármán turbulence spectrum [7], which is defined as

$$\Phi_{ww}(j\omega) = \sigma_w^2 \sqrt{\frac{L_w}{\pi V}} \frac{1 + 8/3 (1.339 L_w \omega / V)^2}{[1 + (1.339 L_w \omega / V)^2]^{11/6}} \quad (46)$$

The turbulence scale $L_w = 2500$ ft and turbulence intensity $\sigma_w = 5$ ft/s were used, which corresponds to moderate turbulence at this altitude. This power spectrum was sampled and constructed as a multisine, as described in Section 4, having a 10 s time duration and frequencies 0.1, 0.2, $\dots$, 10 Hz. A realization of this turbulence, converted to an angle of attack gust, is shown in Fig. 7.

Following the development described in Section 3.1, the equivalent flap and elevator perturbations that achieve the same dynamic response as the angle of attack gust are shown in Fig. 8 as red lines. The blue lines are the corresponding multisine excitations added to the command path before the actuators. A comparison of the amplitude and phase spectra are given in Figs. 9 and 10 for the flap and elevator channels, respectively. Note these signals have larger high-frequency dynamics and lead the desired control surface position data in order to compensate for the actuator dynamics. The flap is used with an order of magnitude more amplitude than the elevator due to the differences in the control effectiveness, as given in Table 3. The control surface positions remained within the limits and actuator rates were less than 50 deg/s for the flap and 2 deg/s for the elevator.

The simulation was run twice. In the first instance, only the angle of attack turbulence was applied. These inputs, states, and outputs are shown in Fig. 11 through 13 as the blue lines. In the second instance, only the equivalent flap and elevators were applied, and these are plotted with red lines. The excitations were applied 2 s into the simulations. Errors between the desired outputs due to turbulence and the outputs from the equivalent control surface deflections were small and difficult to see in Fig. 13. Maximum differences were 0.093 deg in angle of attack, 0.043 deg/s in pitch rate, and 0.067 g for the accelerometer. These maximum errors occurred at the initial onset or when the excitation ended, due to transient responses in the actuators.

7 Conclusions

A method for designing perturbation inputs to simulate flight in turbulence while flying in calm air was presented. The excitations are multisines, similar to those used in several past programs for obtaining data for a system identification analysis, and can be designed for the Dryden or von Kármán turbulence models, or with an arbitrary power spectra. The excitations are added to the command path at the actuators and do not require any alterations to the flight control laws. The excitations can be designed using any flight dynamics models, several of which were developed in the paper. A simulation demonstration using the Generic Transport Model was presented as a demonstration of this approach.

The main findings of this report can be summarized as the following:

1. For fixed-wing aircraft, turbulence acts on several state variables and therefore requires multiple redundant control effectors to accurately emulate flight in turbulence.
2. Due to the directionality of the dynamic model, control effectors chosen for turbulence emulation must be able to couple in to the forced states to remain within actuator position and rate limits.
3. The excitations can be augmented to account for the effect of actuator dynamics, which limit the extent to which flight in turbulence can be emulated.
4. Using multisines, any arbitrary turbulence profile can be emulated within hardware constraints and without affecting existing control laws or without adding model-following control architectures.

8 Acknowledgements

This research was supported by the NASA Low Boom Flight Demonstrator (LBFD) project. Technical discussions with Randall Bailey, Tom Berger, Ondrej Juhasz, Eugene Morelli, and Jerry Newsom are acknowledged and appreciated.

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Tables

Table 1: GTM geometry and nominal mass properties.

Parameter	Value	Unit
b	6.8488	ft
$\bar{c}$	0.9153	ft
S	5.9018	ft ²
m	1.6395	slug
I_{xx}	1.1830	slug-ft ²
I_{yy}	4.6514	slug-ft ²
I_{zz}	5.5780	slug-ft ²
I_{xz}	0.2122	slug-ft ²
x_α	0	in
δ_f	[0, +30]	deg
δ_e	[-30, +20]	deg

Table 2: GTM flight condition.

Parameter	Value	Unit
V	94.0	ft/s
α	5.0	deg
h	1000	ft
δ_f	15.0	deg
δ_e	-2.8	deg

Table 3: GTM stability and control derivatives.

Parameter	Value
C_{Z_α}	-4.77
C_{Z_q}	-29.1
$C_{Z_{\delta_f}}$	-1.33
$C_{Z_{\delta_e}}$	-0.477
C_{m_α}	-1.68
C_{m_q}	-40.8
$C_{m_{\delta_f}}$	-0.401
$C_{m_{\delta_e}}$	-1.90

Figures

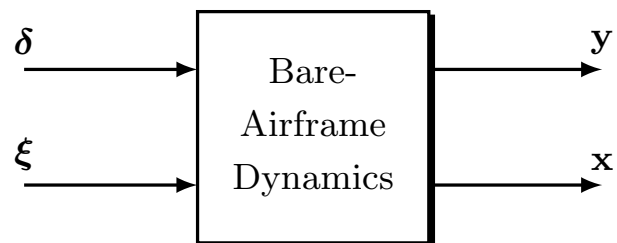


Figure 1: Simplified block diagram of the bare-airframe dynamics.

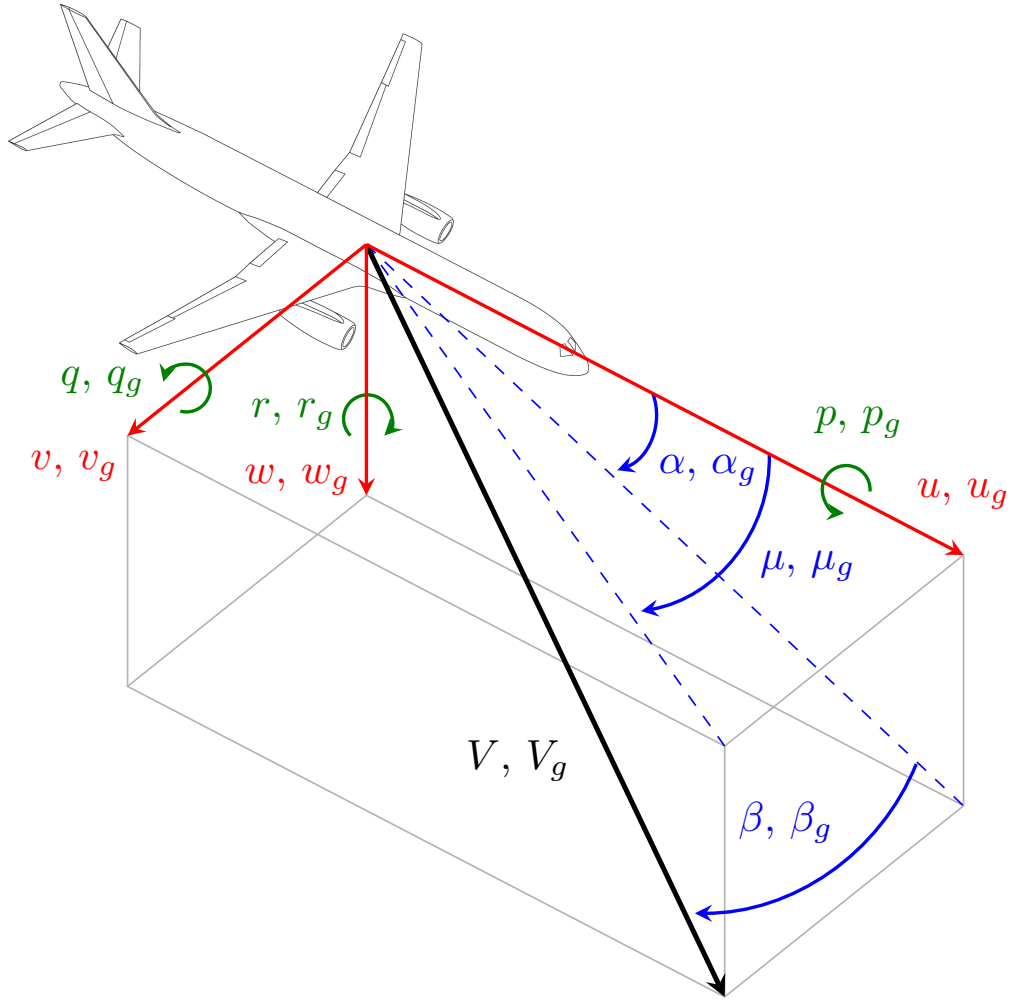


Figure 2: Aircraft definitions and sign conventions.

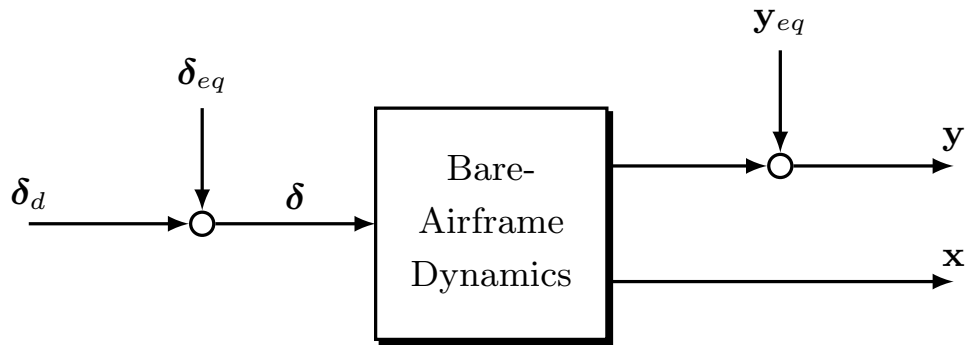


Figure 3: Simplified block diagram of aircraft response to equivalent control inputs.



Figure 4: GTM wind tunnel test article installed in the NASA Langley Research Center 14×22 Ft Subsonic Tunnel (credit: NASA Langley Research Center).



Figure 5: T-2 airplane in flight (credit: NASA Langley Research Center).

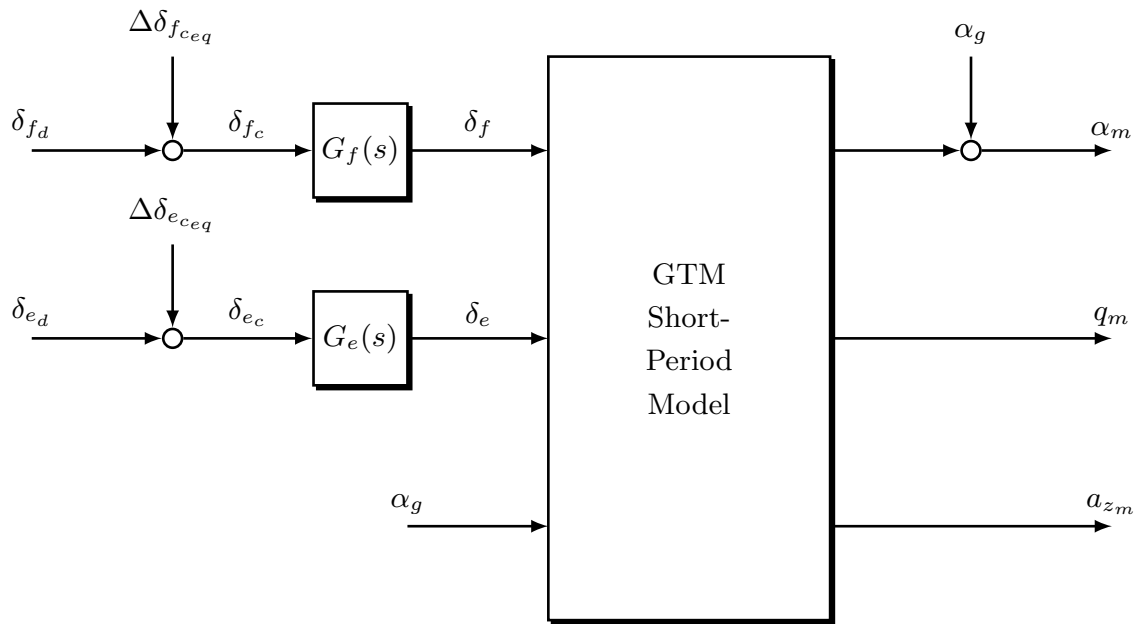


Figure 6: Block diagram for the GTM simulation example.

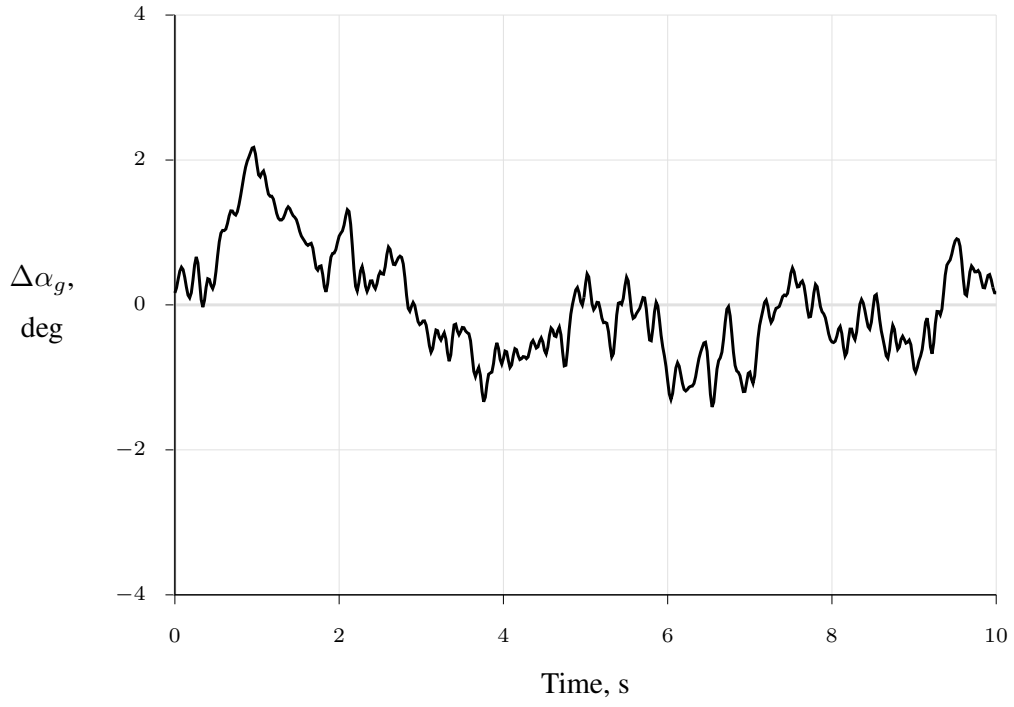


Figure 7: Simulated angle of attack turbulence sequence.

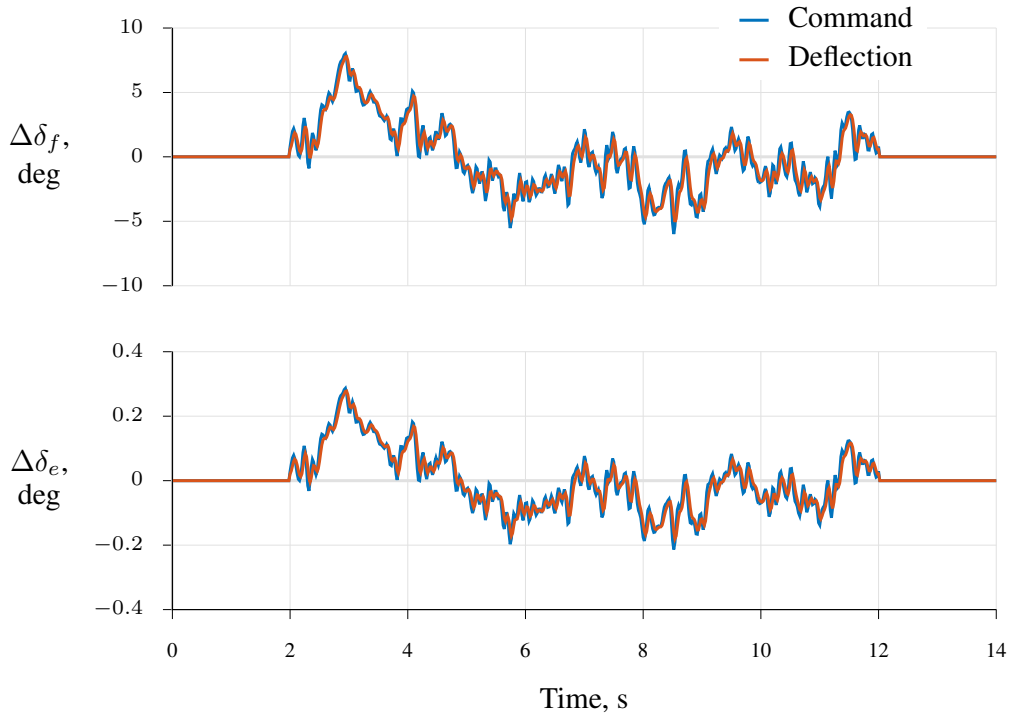


Figure 8: Designed control surface deflections and actuator commands.

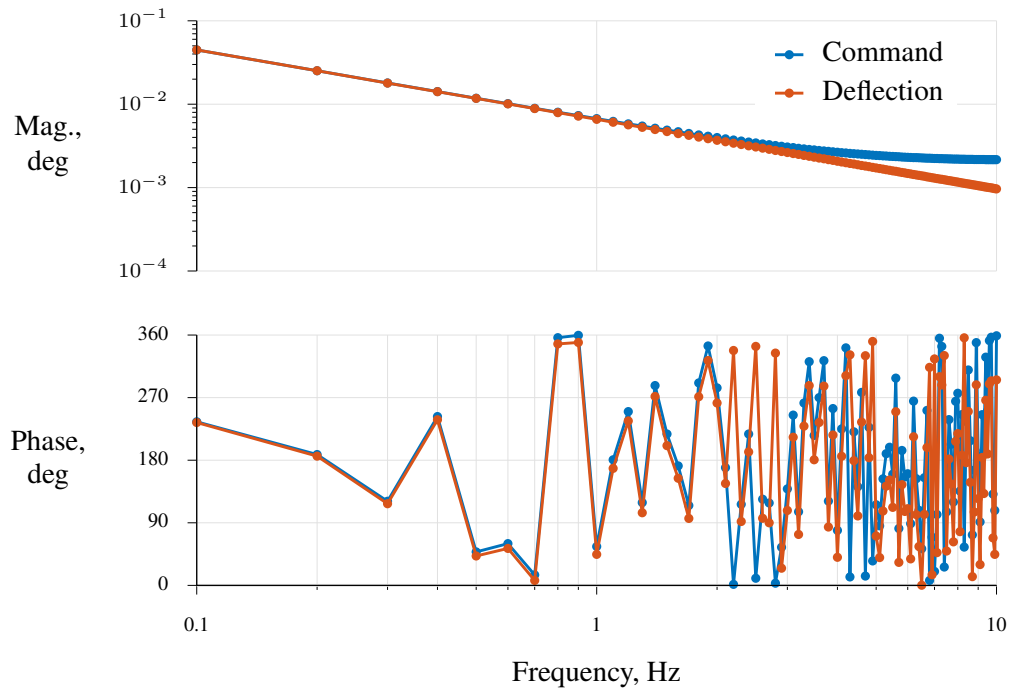


Figure 9: Equivalent flap multisine design parameters.

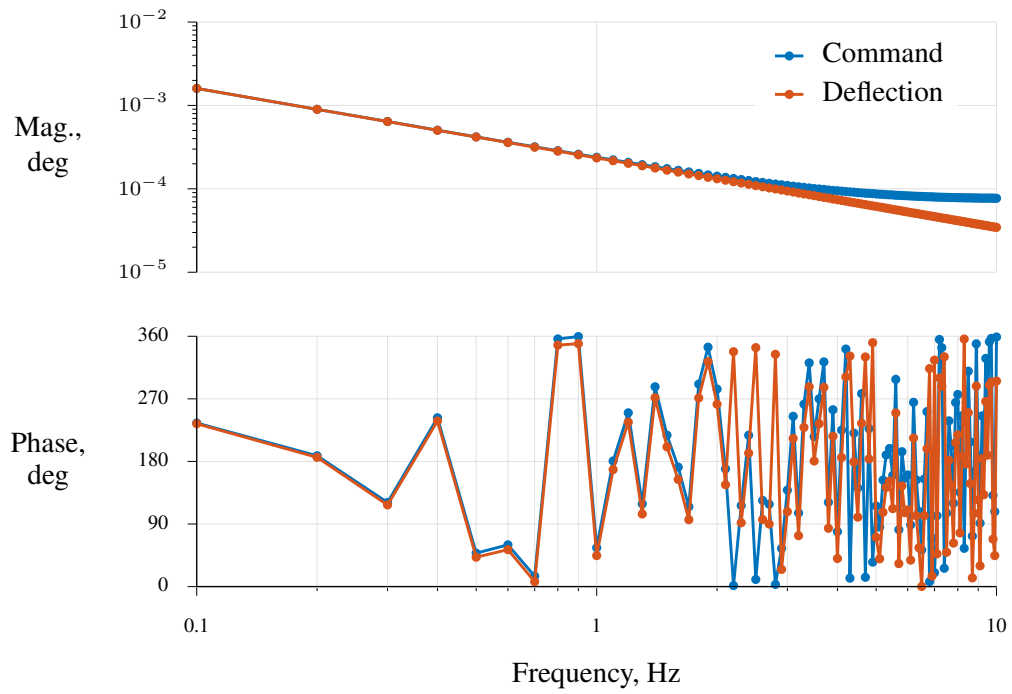


Figure 10: Equivalent elevator multisine design parameters.

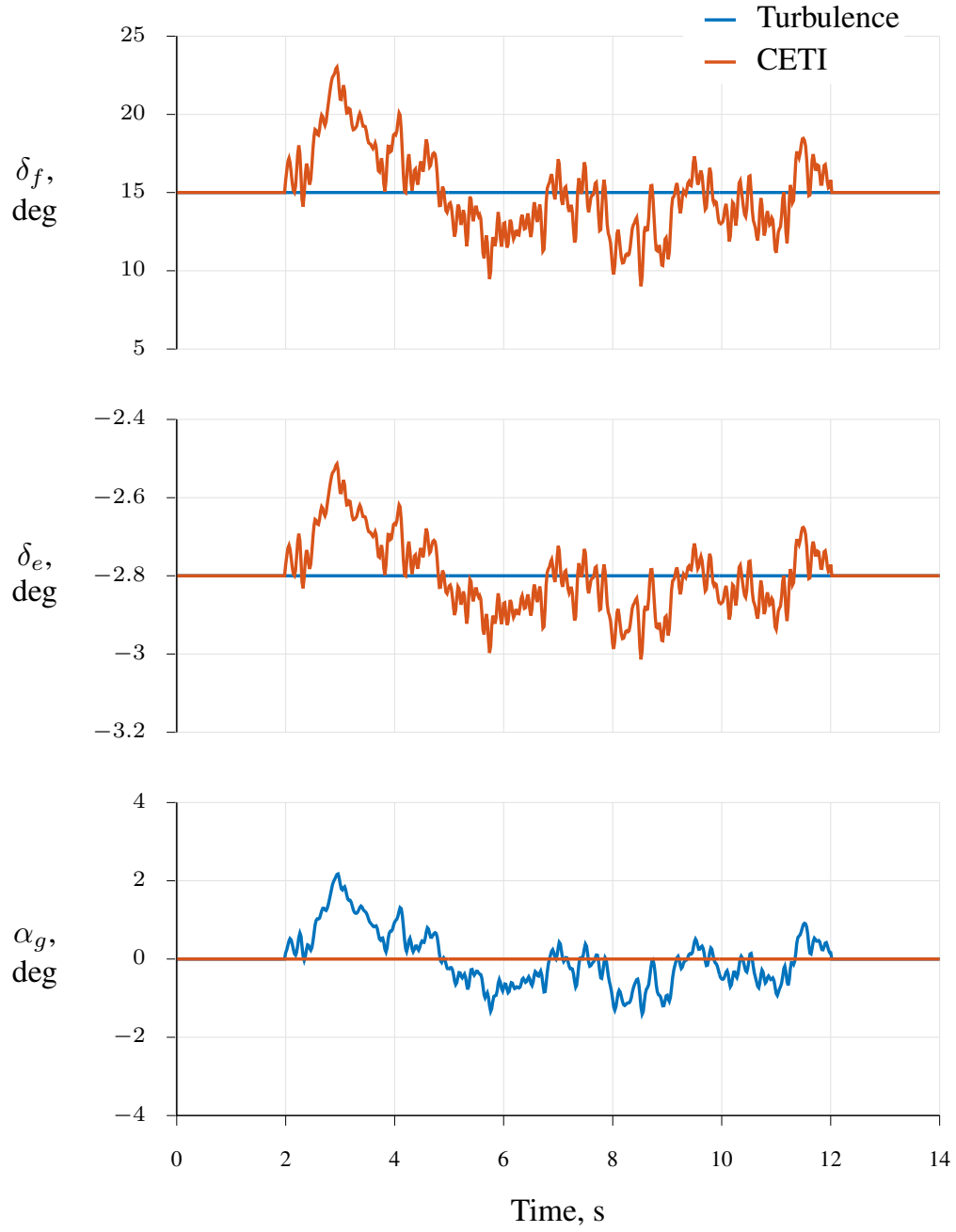


Figure 11: GTM simulation input time histories.

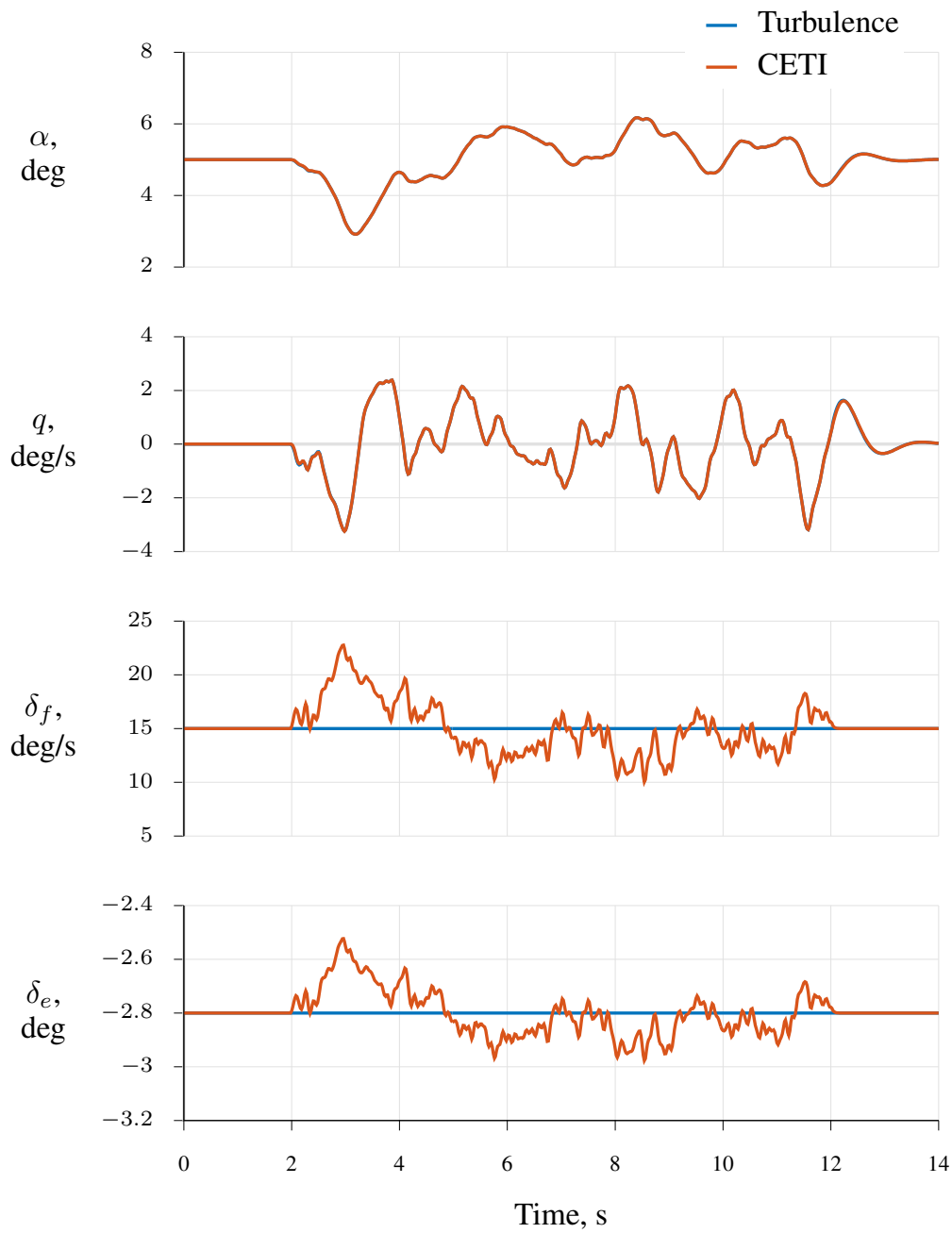


Figure 12: GTM simulation state time histories.

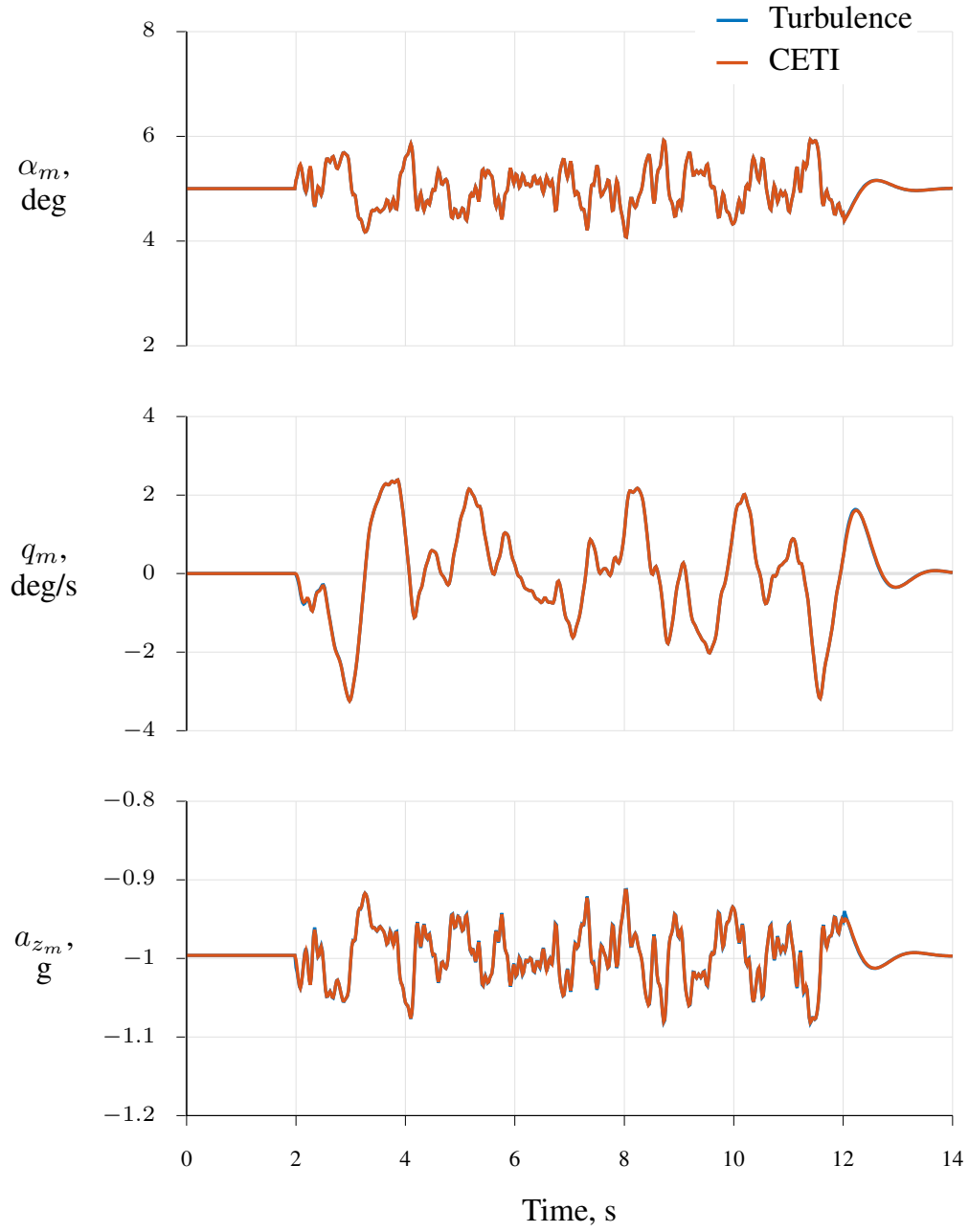


Figure 13: GTM simulation output time histories.

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1. REPORT DATE (DD-MM-YYYY) 01-09-2021		2. REPORT TYPE Technical Memorandum			3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE In-Flight Turbulence Emulation for Fixed-Wing Aircraft using Equivalent Multisine Excitations				5a. CONTRACT NUMBER		
				5b. GRANT NUMBER		
				5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) Jared A. Grauer Langley Research Center, Hampton, Virginia				5d. PROJECT NUMBER		
				5e. TASK NUMBER		
				5f. WORK UNIT NUMBER 339442.04.07.02		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) NASA Langley Research Center Hampton, Virginia 23681-2199					8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546-0001					10. SPONSOR/MONITOR'S ACRONYM(S) NASA	
					11. SPONSOR/MONITOR'S REPORT NUMBER(S) NASA/TM-20210020347	
12. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified-Unlimited Subject Category 08 Availability: NASA STI Program (757) 864-9658						
13. SUPPLEMENTARY NOTES						
14. ABSTRACT A procedure is discussed for designing control-equivalent turbulence input (CETI) excitations to emulate a fixed-wing aircraft response to turbulence while flying through calm air. These excitations are added to the actuator commands to produce the same motion of the aircraft that would have resulted from flying in actual turbulence. Although CETI models are usually shaping filters driven by white noise inputs, the equivalent control inputs in this report were constructed as multisine excitations, which enables mimicking the response to turbulence with arbitrary power spectra, within hardware limitations. Practical aspects of the approach are discussed, such as actuator dynamics, model uncertainty, actual in-flight turbulence, and others. The procedure is demonstrated using a flight dynamics simulation model of a subscale generic jet transport aircraft.						
15. SUBJECT TERMS Frequency response, aircraft system identification, maximum likelihood estimation, output error parameter estimation, multisine inputs						
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES 34	19a. NAME OF RESPONSIBLE PERSON STI Help Desk (email: help@sti.nasa.gov)	
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER (Include area code) (757) 864-9658	