

Thermal Inspection of a Composite Fuselage Section Using the Method of Proper Generalized Decomposition

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ABSTRACT

Proper Generalized Decomposition (PGD) is a reduced order modeling technique for the simulation of physical systems whose governing equations depend on boundary conditions, initial conditions, material properties, and geometric parameters. It uses separated representations of system covariates combined with an iterative approximation method known as successive enrichment in order to compute an accurate parameter-dependent approximation to the full governing equations. PGD can also be used as an alternative to the Singular Value Decomposition (SVD) of a matrix and therefore as an alternative to PCA thermography. In this paper PGD was used to analyze data derived from the inspection of a composite fuselage forward section using flash thermography, and the results were compared against the standard PCA approach.

Keywords: Thermal nondestructive evaluation, composite fuselage, water intrusion damage, delamination, composite sandwich structure, proper generalized decomposition, principal component analysis

1. INTRODUCTION

The idea of applying principal component analysis (PCA) to active thermographic inspection of composite structures was introduced in [1], where it was also demonstrated that PCA could be used to qualitatively enhance and quantitatively interpret data derived from active thermography. The purpose of this paper is to show that the method of Proper Generalized Decompositions (PGD) [2] can be applied for a similar purpose. For its use in active thermography PGD compression is simple to implement and offers potential benefits over PCA in terms of accuracy and speed, since PCA (or PGD) thermography uses only the first few eigenvectors corresponding to the largest singular values.

PGD was compared against PCA in terms of accuracy and computational expense on image stacks derived from an active thermographic inspection of composite fuselage aircraft forward and aft sections that was carried out in [3]. See Figure 1. Recall that in [3] 477 sub-inspections of the fuselage were executed and that each captured a 23 by 33 cm² region of interest with a frame rate of 60 Hz. In each sub-inspection, a thermal flash of approximately 10°C above the ambient temperature occurred at frame 10 after which 890 additional frames were captured, corresponding to approximately 15 seconds of data. In [3] a fixed eigenvector approach [4] was used to analyze the resulting image stacks in order to observe details of the substructure and identify potential damage such as paint thickness variation, possible water intrusion damage, and delamination.

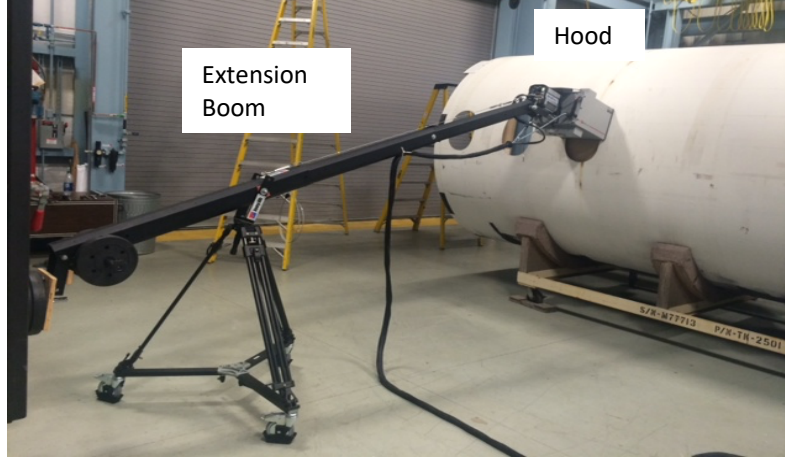


Figure 1. Thermal inspection setup for forward section measurements. From [3].

2. SINGULAR VALUE DECOMPOSITION AND THE PROPER GENERALIZED DECOMPOSITION

Since it plays a central part in PCA thermography, we briefly recall the singular value decomposition (SVD) of a real J by I matrix M of rank r and then explain its relationship with the PGD separated representation of the discretization $M_{ij} = f(x_j, y_i)$ of a two-dimensional field $f(x, y)$.

The SVD representation of a matrix M of rank r takes the form:

$$M = U \cdot \Sigma \cdot V^T \quad (1)$$

where U and V are $I \times I$ and $J \times J$ orthogonal matrices, and Σ is an $I \times J$ matrix with non-negative entries σ_i on the diagonal and zeros elsewhere. The numbers σ_i are the singular values of M and the columns U_i and V_i of U and V are the left and right singular eigenvectors of M , i.e., the eigenvectors of MM^T and $M^T M$. If we denote the outer product of U_i and V_i^T by $U_i \cdot V_i^T$, then equation (1) can be written

$$M = \sum_{i=1}^r M_i = \sum_{i=1}^r \sigma_i \cdot U_i \cdot V_i^T. \quad (2)$$

Following [2] we observe that retaining N terms ($N \ll r$) in equation (2) gives a compressed representation of M . Indeed, such an approximation of M requires data on the order of $N \cdot (I + J)$, which is quite small when compared against $I \times J$, i.e., the order of data in M .

The PGD approximation of a two-dimensional field $f(x, y)$ takes the form of a so-called *separated representation*

$$f(x, y) \approx u^n(x, y) = \sum_{k=1}^n X_k(x) \cdot Y_k(y), \quad (3)$$

where $X_k(x)$ and $Y_k(y)$ are each functions of a single variable. The method of weighted residuals combined with an alternating direction scheme is used to iteratively solve the nonlinear problem defined by equation (3) for the functions X_k and Y_k . The details can be found in [2].

In the case when the function f is evaluated at discrete points $x_j, j = 1, \dots, J$ and $y_i, i = 1, \dots, I$ such as for an image, equation (3) is written in the form:

$$M_{ij} = f(x_j, y_i) \approx \sum_{k=1}^n X_k \cdot Y_k, \quad (4)$$

where $X_k = (X_k(x_1), \dots, X_k(x_J))$ and $Y_k = (Y_k(y_1), \dots, Y_k(y_I))$.

A numerical example based on a Python implementation of the PGD approximation of an image $M_{ij} = f(x_j, y_i)$ is shown in Figure 2 for equation (4) when $n = 1, 40$, and 150 .

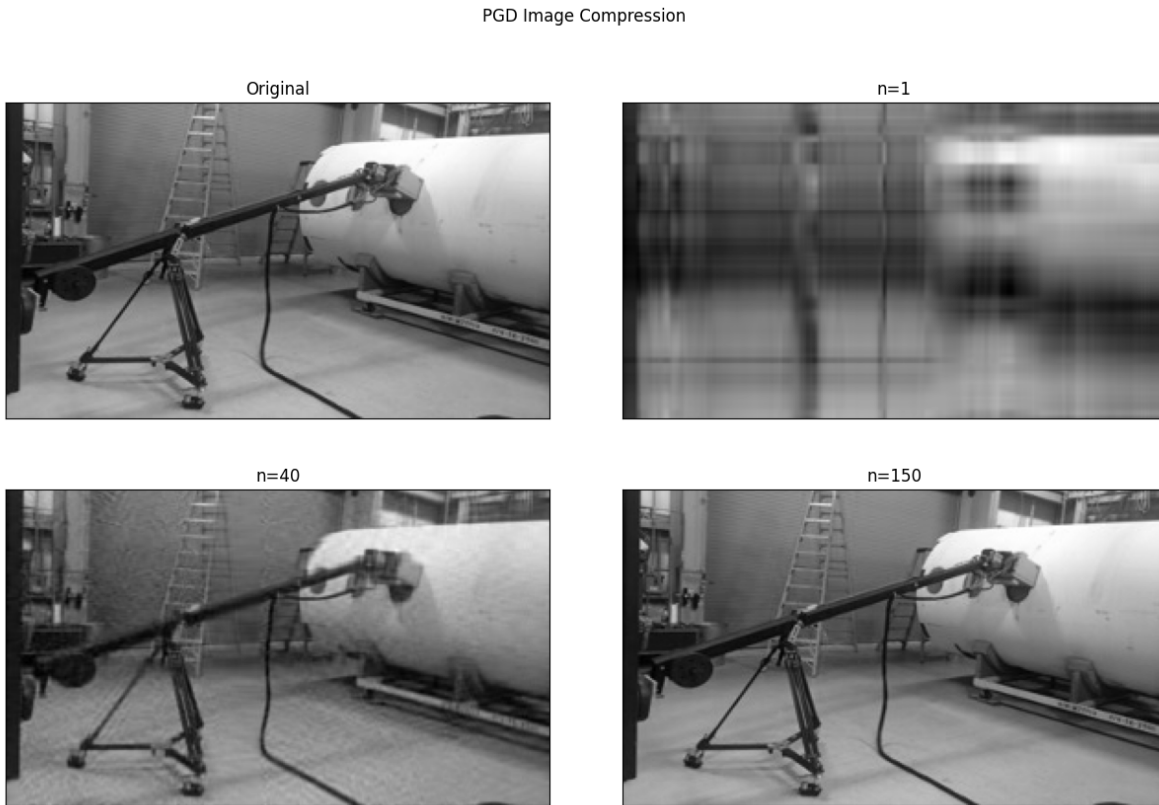


Figure 2. PGD approximation of an image with $n = 1, 40$, and 150 enrichment steps.

Figure 3 shows a comparison of PCA versus PGD in compressing the upper left image in Figure 2. Equation (2) is solved using Scipy's iterative solver `scipy.linalg.sparse.svds` for $r = 1, \dots, 25$. On the other hand, a naïve Python implementation of the PGD method is used to solve equation (4)

for $n = 1, \dots, 25$. The left plot in Figure 3 compares the PCA and PGD approximations to the original image with respect to the (normalized) Euclidean norm.

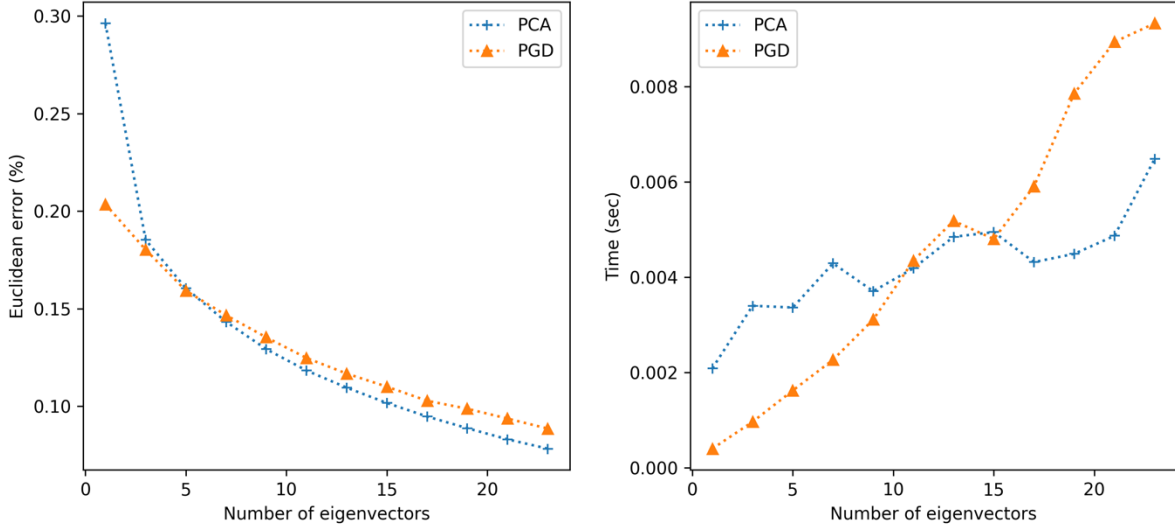


Figure 3. Comparison of PCA and PGD in terms of accuracy and running time for the compression of the grayscale image in Figure 2. Note that PGD outperforms PCA for small numbers of eigenvectors. This is advantageous for thermography applications where only 1 or 2 eigenvectors are often needed.

Note when n and r are small PGD offers a more accurate approximation than PCA in less computational time. Because the application of PCA to thermography that is described in the next section requires only 1 or 2 eigenvectors, Figure 3 shows the potential advantages in terms of accuracy and speed that PGD thermography offers over traditional PCA.

3. PCA AND PGD THERMOGRAPHY METHODS

Now consider a collection of $I \times J$ -dimensional images f_k indexed by time $k = 1, \dots, K$. From the collection of images, a single data matrix A of dimension $(I \cdot J) \times K$ is constructed to have columns consisting of the flattened arrays f_k . Following [1] we normalize A as follows:

$$\hat{A}_{mn} = \frac{A_{mn} - \mu_m}{\sigma_m}, \quad (5)$$

where

$$\mu_m = \frac{1}{K} \sum_{n=1}^K A_{mn} \quad (6)$$

is the column-wise mean of A , and

$$\sigma_n^2 = \frac{1}{K-1} \sum_{n=1}^K (A_{mn} - \mu_n)^2 \quad (7)$$

is the variance of A along its columns.

To every right singular eigenvector V_l of $\hat{A}$ (corresponding to the columns of the matrix V of equation (2)) there is an associated PCA image P_l . It is defined by multiplying each row of $\hat{A}$ with V_l and then reshaping the result into an array of dimension $I \times J$, i.e.

$$P_{ij}^l = \hat{A} \cdot V_{ik+j}^l$$

Similarly, each X_l term in the PGD representation of the matrix $\hat{A}$ can be used to define a PGD image Q_l by multiplying pixel-by-pixel with the temporal mode X_l , i.e.

$$Q_{ij}^l = \hat{A} \cdot X_{im+j}^l.$$

Figure 4 shows PCA and PGD images corresponding to the second eigenvector V_2 and PGD mode X_2 from the fuselage inspection carried out in [3]. Note that substructure details can be observed using either approach.

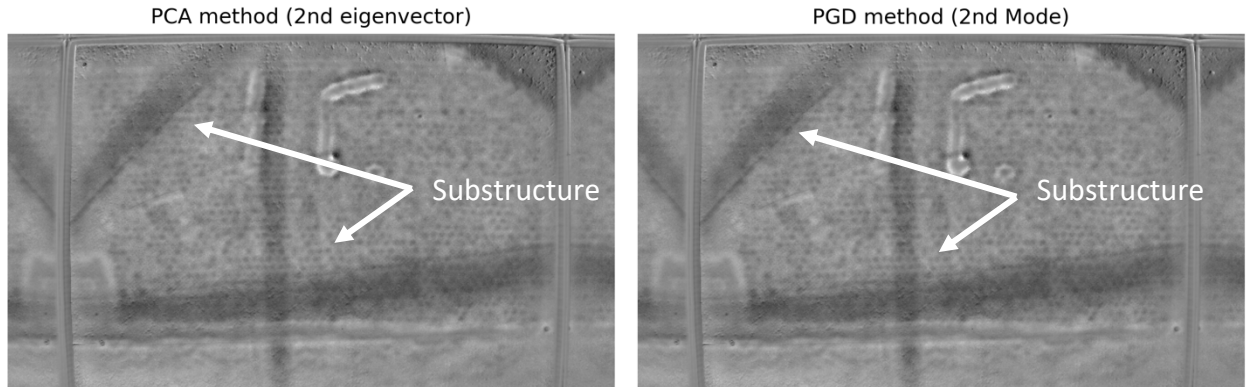


Figure 4. PCA and PGD images obtained from thermographic data collected in [3]. In both cases the second eigenvector is used to derive the displayed images. Both methods show substructure detail and defect indications. The PGD image is obtained with only two sub-enrichment steps.

Figure 5 shows suspected water damage beneath the windows of the fuselage section.

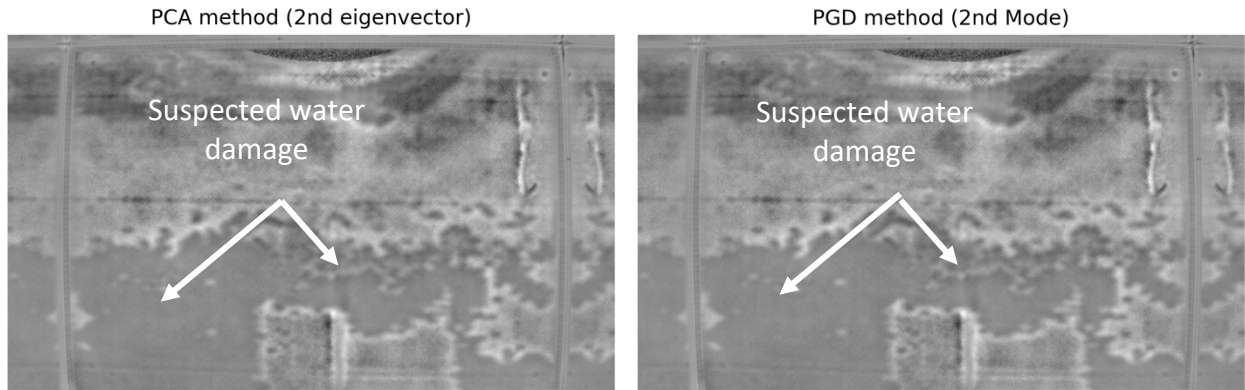


Figure 5. Suspected water damage identified with both PCA and PGD method.

The first and second right eigenvectors corresponding to the largest singular values are most useful for analyzing data derived from active thermography since they are known to contain energy dissipation and contrast information [1]. Figure 6 shows these first two eigenvectors obtained through both methods. Note that the eigenvectors obtained through the PGD method closely agree with the PCA counterparts even though only two sub-enrichment steps are used in their derivation.

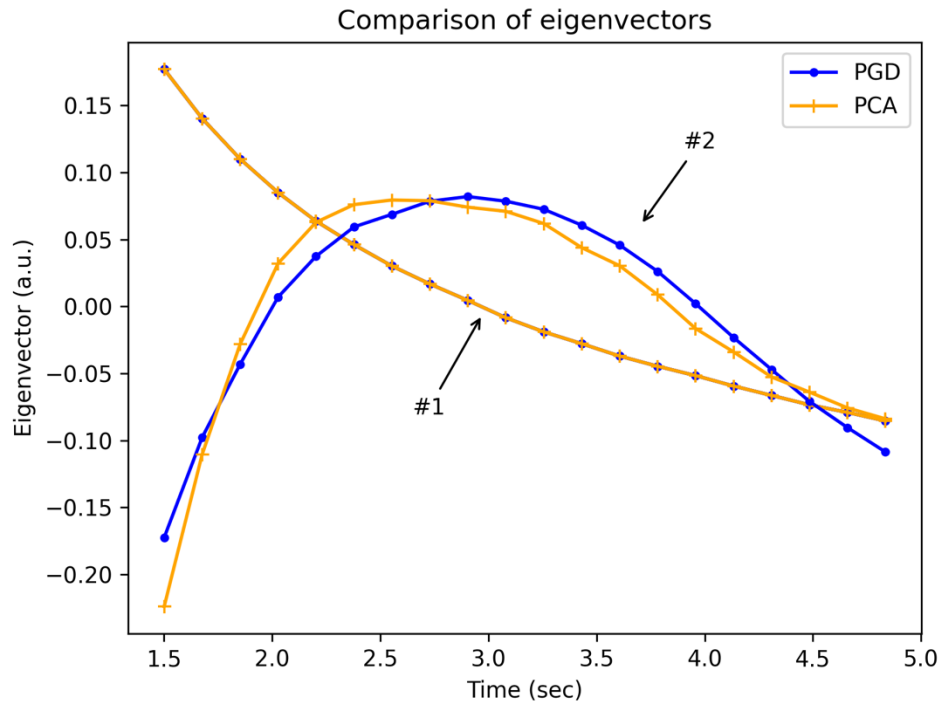


Figure 6. First and second eigenvectors obtained through the PCA and PGD analysis of the thermographic data from [3].

In terms of computational resources, the PGD method is well-suited to run on computing resources that measure favorably in terms of Size, Weight, and Power. For comparison PCA and PGD representations are computed on the data from [3] using the PGD method (implemented in Python) and Scipy's implementation (`scipy.linalg.sparse.svds`) of the Implicitly Restarted Arnoldi Method [5]. The tests were evaluated on a 2019 Macbook pro and results in terms of runtime and accuracy are shown in Figure 7. In particular, Figure 7 shows PCA and PGD compression of the input data corresponding to the image in Figure 5 above.

The left-hand graph of Figure 7 shows that when using a small number of eigenvectors, a more accurate representation of the thermography data from [3] can be computed with the PGD method, while the results shown in the right-hand graph demonstrate that PGD can be solved more quickly than PCA.

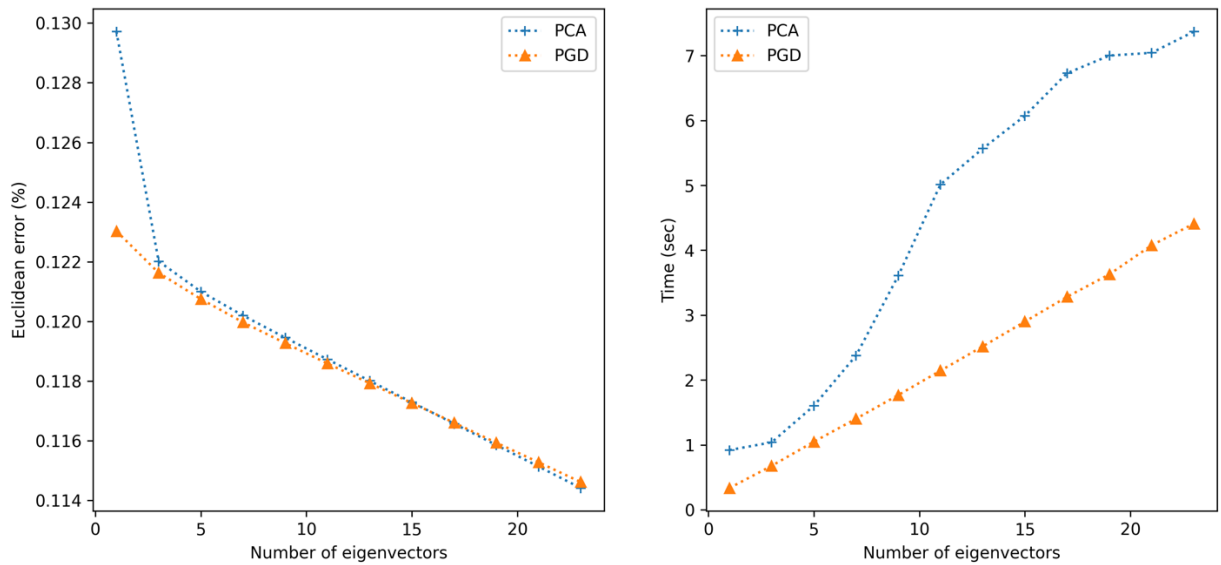


Figure 7. Comparing error and runtime between a Python implementation of the PGD method and Scipy's sparse singular value decomposition function on the fuselage data from [3].

4. CONCLUSIONS

The application of the PGD method to data analysis connected with active thermography was evaluated on composite fuselage data from [3], and like its PCA counterpart, PGD thermography can be used to observe substructure and damage indications. Although a local eigenvector approach has been considered here, a fixed eigenvector version of PGD is also possible. In fact, the full impact of the PGD method applies to approximating the solution to parametrized partial differential equations and could be used to synthesize data as in the fixed eigenvector approach from [4]. PGD offers potential benefits over PCA in thermography since for a small number of eigenvectors, PGD computes a more accurate compression of the data, which can also be evaluated more quickly. Understanding the tradeoffs between PGD and PCA on more complex data sets and in terms of the choices in their algorithmic implementation is the subject of ongoing analysis.

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