Inertia; Inertial Frame

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Definition

Inertia can be described as a property or tendency of an object that resists any change to its state of motion. Subsets of the gravity field harmonics and the librational parameters allow the computation for the moments of inertia of the Moon with the Moon being the central point of reference. The Earth-Moon system using the Moon as an inertial frame requires a Lunar Coordinate system.

A white paper from the Lunar Reconnaissance Orbiter Project and the Lunar Geodesy and Cartography Working Group (LRO & LGCWG 2008) defines a reference system as a system including a physical environment and associated theories that define positions on a specific body (or in space). A reference frame is the reality of that reference system where from observational data the specific numerical location of given points are in a reference system (Kovalevsky & Mueller 1988).

Moment of Inertia

Harmonics of the gravity field and librational parameters can be used as computational subsets to determine the lunar moments of inertia (Bills 1995). However, it has been noted that the discrepancies involved in non-rigid librational motion are either from an elastic toroidal strain or an interaction of a fluidized core coupled with a rigid mantle (Bills 1995), much like how moments of inertia are determined for other planetary bodies.

The Moon may have been in either a synchronous rotation or in a 3:2 resonance of spin and mean motion (Garrick-Bethell & Zuber 2007), with the plausibility of past high eccentricity and tidal interactions. From Garrick-Bethell & Zuber (2007), the 3:2 resonance can be achieved if a magma ocean density was present shortly after lunar accretion. This past high eccentricity orbit may have also affected the Moon's ability to develop lithospheric strength from tidal dissipation. These internal and orbital dynamics of the Moon, whether past or present, requires the coordination of the Moon's inertial reference frame.

Lunar Reference System

The use of such inertial frames with the Moon being the central component has led to creating a lunar-designated coordinate system. This system was originally created by the Lunar Reconnaissance Orbiter (LRO) mission, and has since been used for numerous working groups (e.g., Lunar Geodesy and Cartography Working Group; Archinal et al. 2008a, 2008b), and is still widely used amongst international lunar missions.

The inertial frames have fixed primary origins. For example, the Earth Inertial (EI) frame $(\hat{X}_e, \hat{Y}_e, \hat{Z}_e)$ is an inertially fixed frame with Earth as the origin point of reference. Such a reference frame can also be denoted as the Earth Mean Equator or Equinox of J2000 (Wilson 1998). The Moon Inertial (MI) frame $(\hat{X}_m, \hat{Y}_m, \hat{Z}_m)$ has its origins at the center of the Moon, where each axis is parallel to the corresponding EI frame. Note that uppercase unit vector letters denote inertial reference frames, while lowercase represents rotating coordinates, and the symbol (^) denotes unit vectors while an overbar (¯) indicates a vector of variable length. The Earth-Moon Barycentric Rotating (EMBR) frame $(\hat{x}_b, \hat{y}_b, \hat{z}_b)$ is a rotating frame with the center being the Earth-Moon barycenter. The $\hat{z}_b$ axis is normal to the plane of motion of the Earth and Moon, while the $\hat{x}_b$ axis is always directed from the Earth to the Moon. The EI, MI, and EMBR frames are illustrated in Figure 1.

The barycenter (*b*) of the Earth-Moon system position vector to a spacecraft (denoted as *s*) is written as:

$$\bar{r}_{bs} = x\hat{x}_b + y\hat{y}_b + z\hat{z}_b$$

The location of the spacecraft could also be defined in MI coordinates as:

$$\bar{R}_{ms} = X_m\hat{X}_m + Y_m\hat{Y}_m + Z_m\hat{Z}_m$$

It should be noted that the EI reference system is used for planetary data archiving purposes, and the MI used for LRO instrument teams and others for specific applications. Because the Moon is not truly a synchronously rotating triaxial ellipsoid, the MI and EI rotation axes do not coincide (LRO & LGCWG 2008), which translates to a difference of approximately 1 km at the lunar surface.

The derivation of rotation from the MI to the EI reference frame is given in LRO & LGCWG (2008), where Q is a vector of Cartesian coordinates (in the EI reference frame) and P is a coordinate vector in the MI reference frame:

$$Q = R_x(-0.3'')\ R_y(-78.56'')\ R_z\ (-67.92'')\ P$$

As the angles are in seconds of arc and the rotations are the *x, y, z* axes.

References

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Figures and Tables:

Figure 1: The reference rotating frames of the Earth (EI), Moon (MI) and the Barycenter (EMBR). Note that the vector Rem relates the location of the Moon relative to the Earth in inertial coordinates. Adapted from Wilson (1998).