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Compressive Sensing Application for Transient Photometric Measurements

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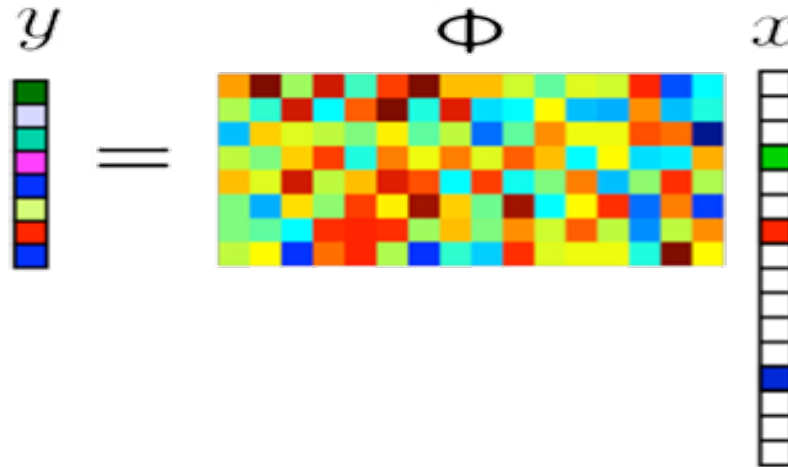
Conclusions and Future work



Compressive Sensing (CS)

- Mathematical theory for simultaneous data acquisition and compression
- **Potentially reduce data samples during acquisition, hence reducing resources required for storing and processing data**
- Ability to sample less than the traditional Nyquist rate samples
- Here, x is the original signal of length N , Φ is the measurement matrix of size $M \times N$, and y are the acquired measurements of length M
 - $M \ll N$

$$y_{M \times 1} = \Phi_{M \times N} x_{N \times 1}$$

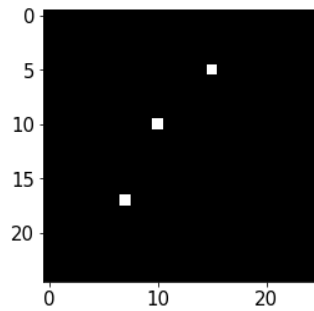


CS Parameters

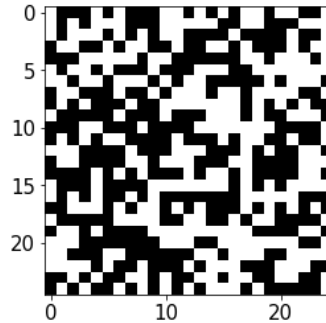
- Measurement Matrix
 - Basis of the measurement matrix has to be as uncorrelated with the basis of the sampling domain
 - Random matrices are universally incoherent → good choice for CS measurement matrix
- Sparsity
 - Number of non-zero elements in a signal
 - Real world applications: Coefficients of a signal must decay at a high rate
- Number of CS measurements needed
 - Depends on sparsity of the signal
 - Amount of error tolerance



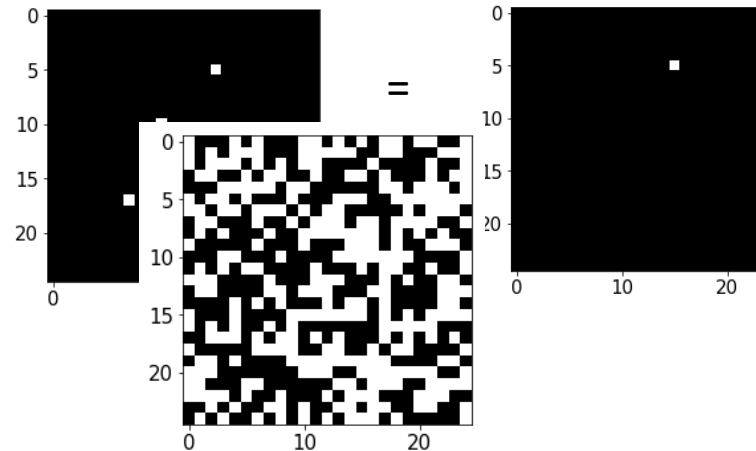
CS Theory



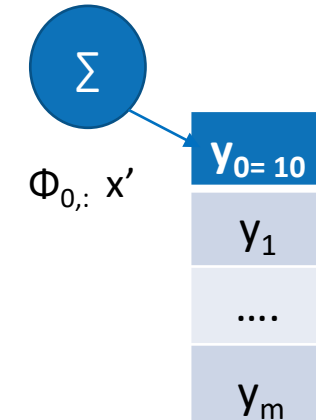
Original image, x
3 elements with
magnitude 10
 $x' = 1D$
transformation
of x



(sub)
measurement
matrix, A
Convert A into a
1D vector: $\Phi_{0,:}$



Projection of (sub) measurement
matrix onto x : element wise
multiplication



- Each sub measurement matrix, A , gets transformed into a 1D signal representing a row in the measurement matrix, Φ .
 - m sub measurement matrices
- Reconstruct original image, x , given y vector and the measurement matrix for each element in y
 - $y_{m \times 1} = \Phi_{m \times n} x'_{n \times 1}$
 - Optimization (L1 minimization) and greedy algorithms
- A unique solution is obtained only if the original image is transformed to a sparse domain



CS Parameters Sparsity

	<u>Sparse Encoding Method</u>	<u>Sparse Domain Transform Method</u>
Acquired Measurements:	$y = \Phi x$	$y = \Phi D x$
Reconstruct by solving:	$y = \Phi \psi s$	$y = \Phi D x$
Transform to sampling domain:	$x = \psi s$ ψ : dictionary, which is set a of basis s : sparse code	$s = D x$ $x = A s$, <i>where A is the pseudoinverse of D</i>



Compressive Sensing for Spaceflight Instruments

Science data resolution vs engineering feasibility

- There is a need for higher resolution data to obtain novel science
 - This leads to the “Big data” problem → how do we obtain, store and transmit such large amounts of data from a space flight instrument?
- Limitations on SmallSat type instruments:
 - On-board computing resources
 - Data storage resources
 - Data bandwidth for data transmission

CS proves a potential solution for obtaining high resolution data without compromising on data integrity for SmallSat type instruments



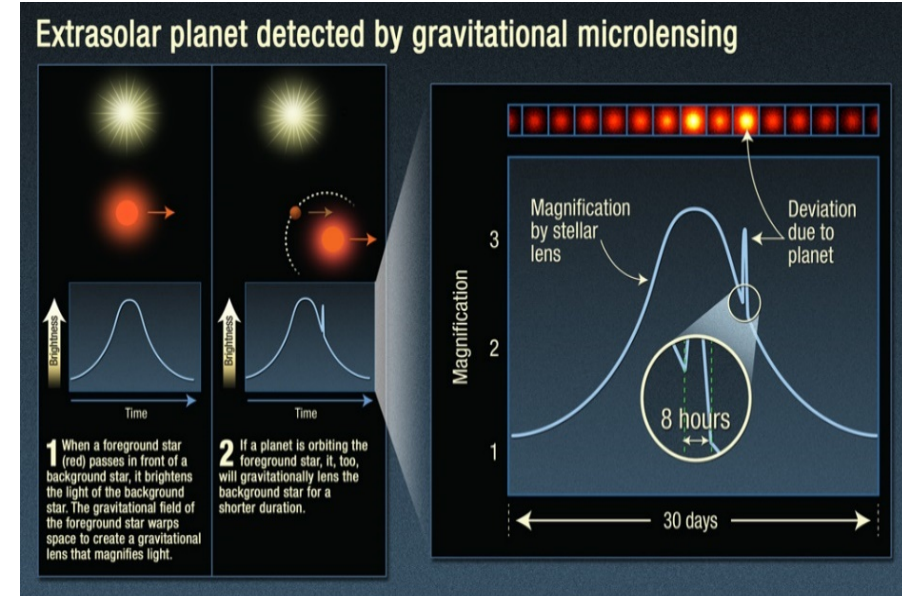
Transient Photometry

Measure light photons as a function of time

Application - Measure variability in light emitted from star sources over time

Light from stars are generally constant, hence such events are typically rare

Example: gravitational microlensing

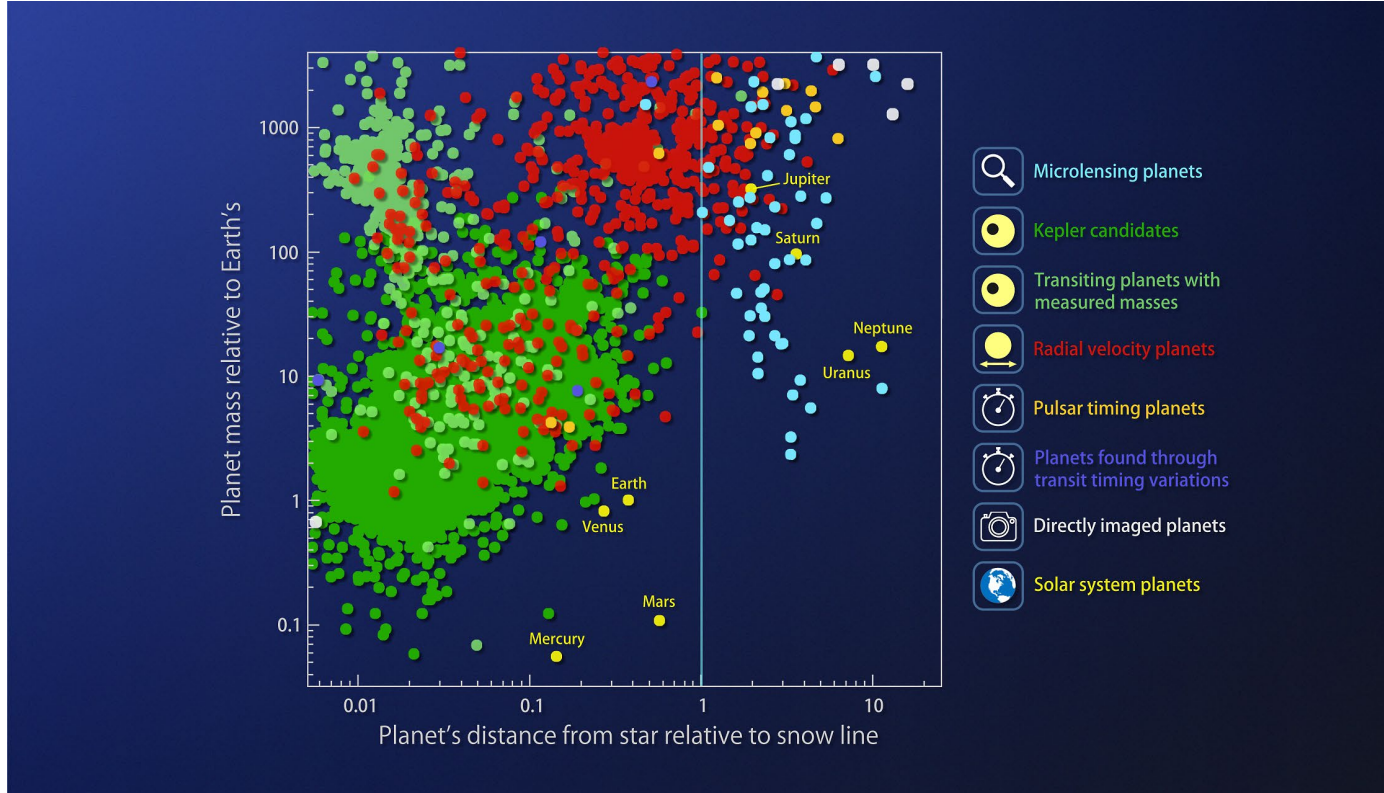


Change in magnification in light curve due to a gravitational microlensing event. It is critical to capture the short peak caused due to a planet. Our CS architecture will sample in the spatial domain, while focusing on accurately reconstructing the photometric curve in time domain.

Credit: NASA, ESA and STSci



Why Gravitational Microlensing?



Snow line: where water freezes solid

Microlensing is sensitive to smaller mass planets beyond the snow line

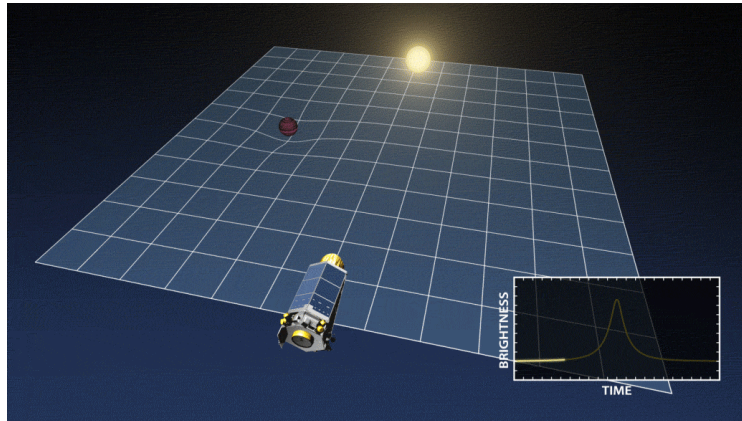
Credit: NASA Goddard Space Flight Center



Time Series Photometry

Gravitational Microlensing

- Time-series photometry- Measurement of apparent brightness of an astronomical object as a function of time
- Gravitational microlensing – A type of time-series photometric measurement
 - A method to detect exoplanets
 - Exoplanet – any planet outside of our solar system
- Gravitational microlensing takes advantage of any massive object bending space-time, hence deflecting the path of light



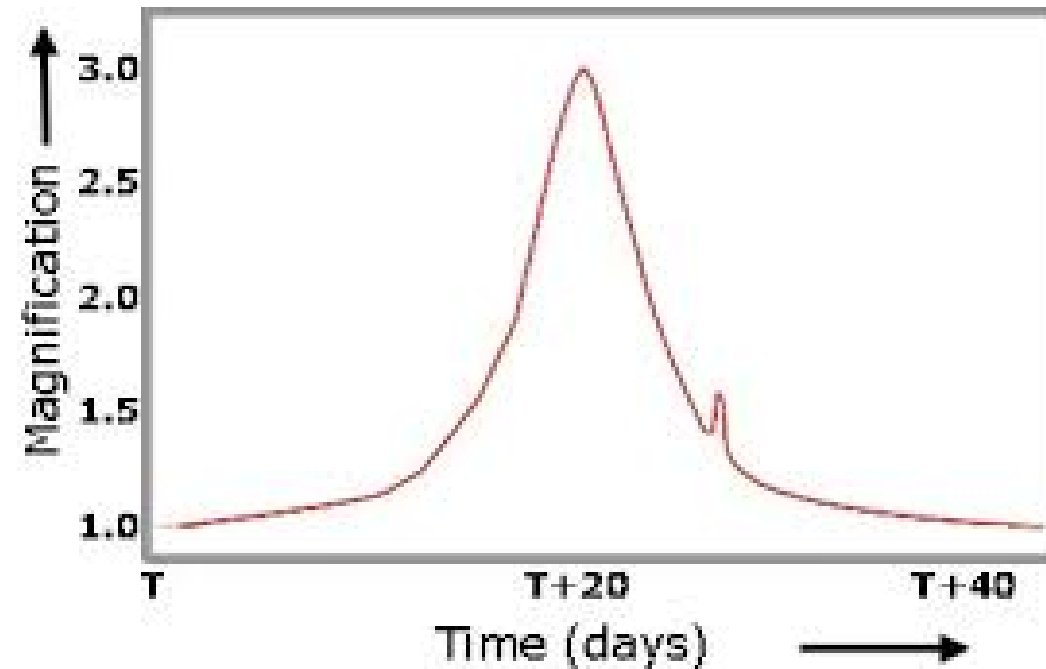
Credit: NASA Ames/JPL-Caltech/T. Pyle

- Strong lensing- more than one image might be visible due to the lensing effect
 - **Microlensing** – lensing effect is very minute such that the multiple images are not resolved, but only a magnification in the light is observed



Time-Series Photometry Generalization

Task: Reconstruct such photometric curves with accurate magnification at any given time



Use microlensing application as a means to create real-world scenarios for time-series curves



Why use CS for gravitational microlensing?

- **Resolution in spatial domain**

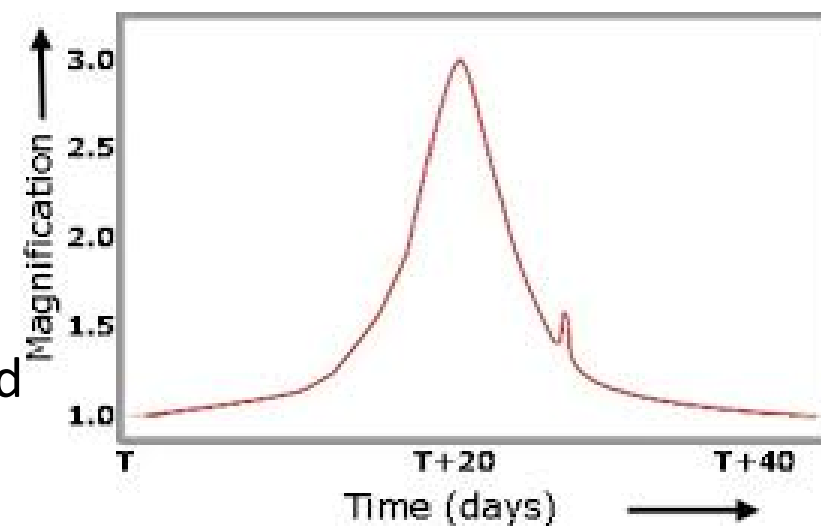
- Less error and uncertainty in magnification values
- More data samples for one spatial image for high resolution images

- **Resolution in time domain**

- To better capture short-lived microlensing events
 - Solar mass size lens - ~30 days
 - Jupiter mass size lens - ~ few days
 - Earth mass size lens - ~ few hours
- Higher cadence - rate at which each spatial image is captured
 - Need to sample faster in time

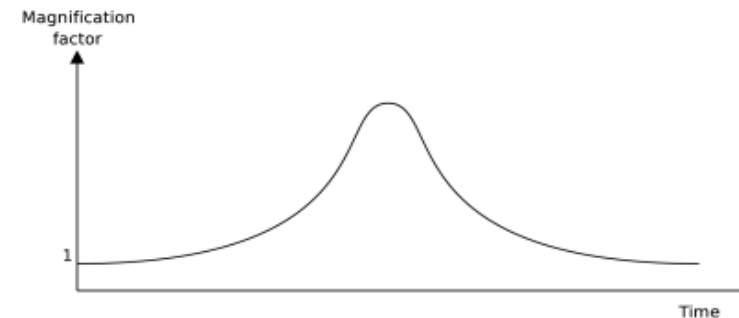
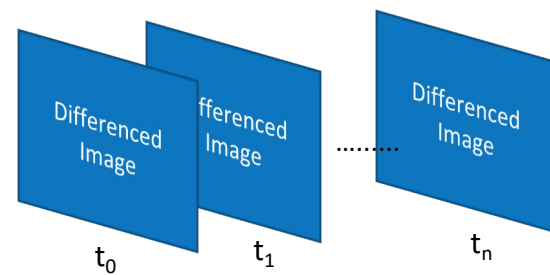
- **Tradeoff between data volume and image resolution**

- Increase in spatial resolution → more data samples to capture an image



Spatial Images → Time-series photometry

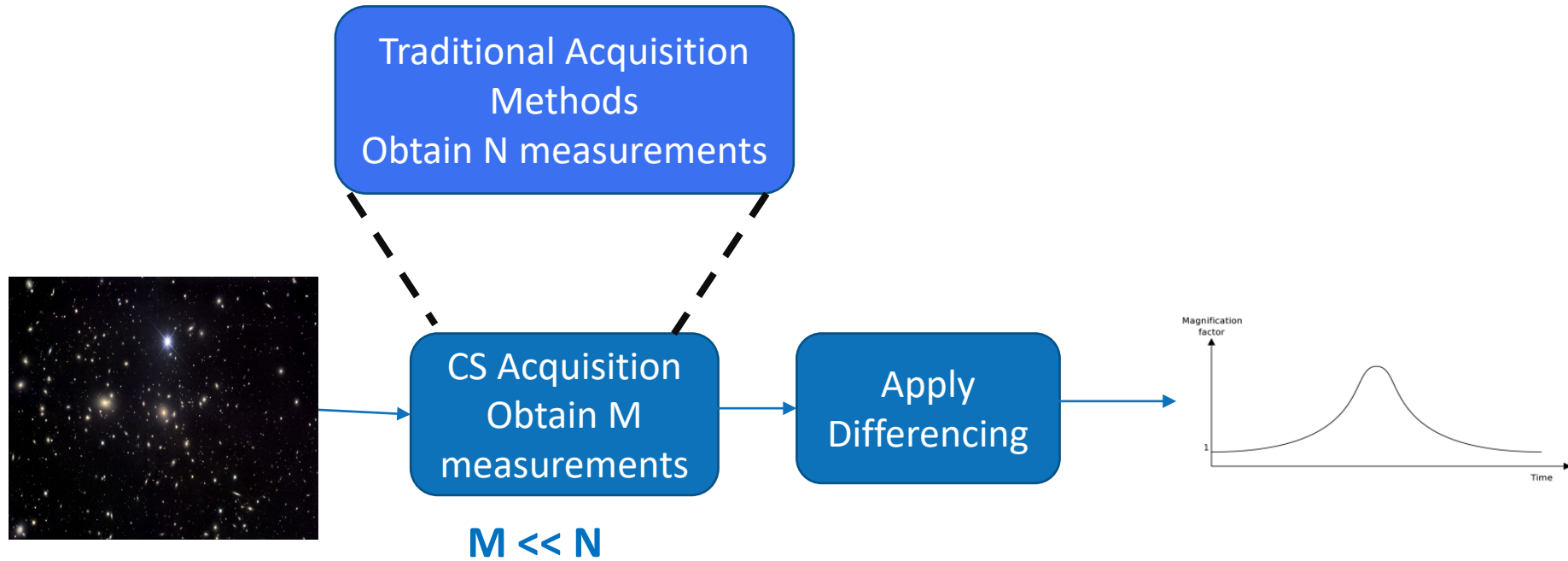
- Observed spatial image is “differenced” with a reference image
- Using such differenced images, a time-series photometric curve can be generated for each pixel, a superpixel or a cluster of pixels known to be a star source



Microlensing curve for each pixel or superpixel

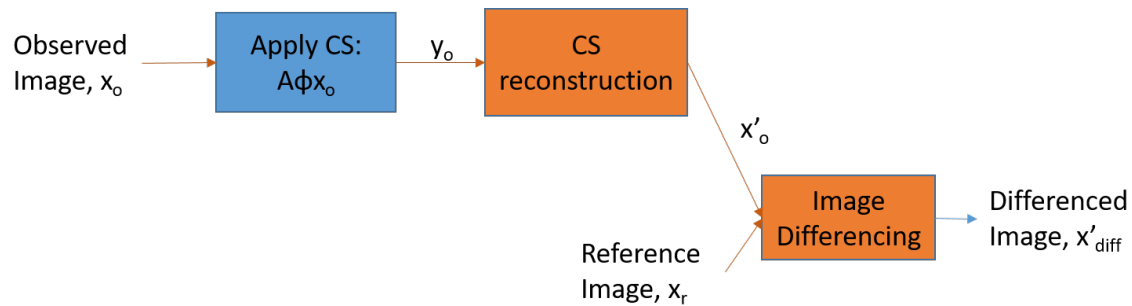


Compressive Sensing (CS)



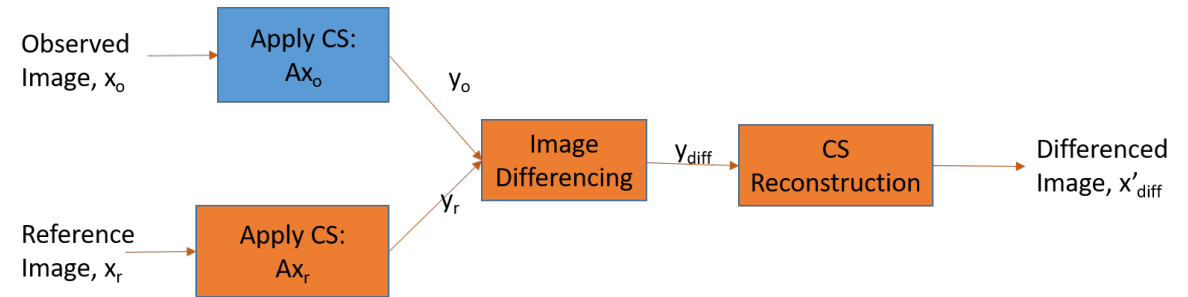
CS Architecture

Traditional Architecture



Differencing is applied on the CS reconstructed image

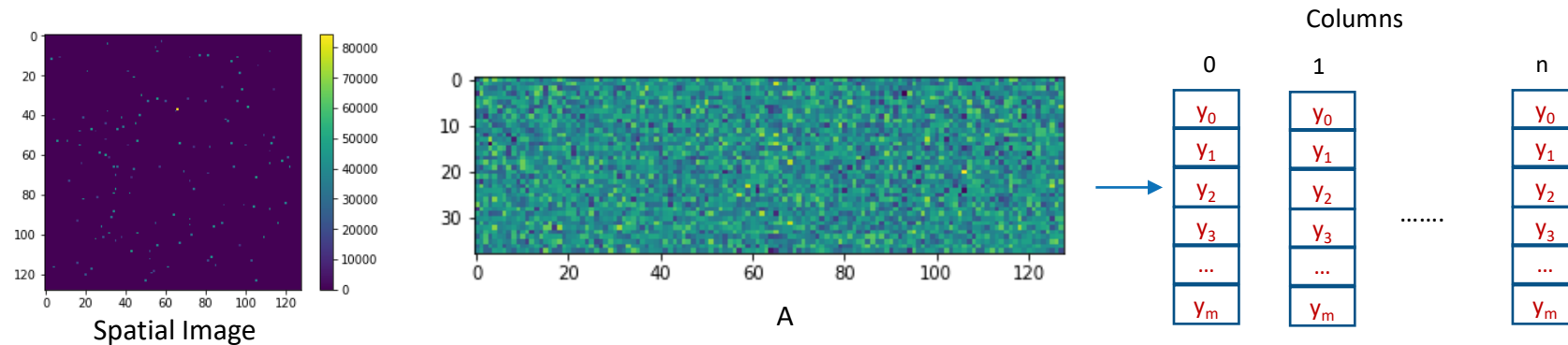
Novel Architecture



Differencing is applied on the CS measurements, resulting in only reconstructing the differenced image



CS Theory

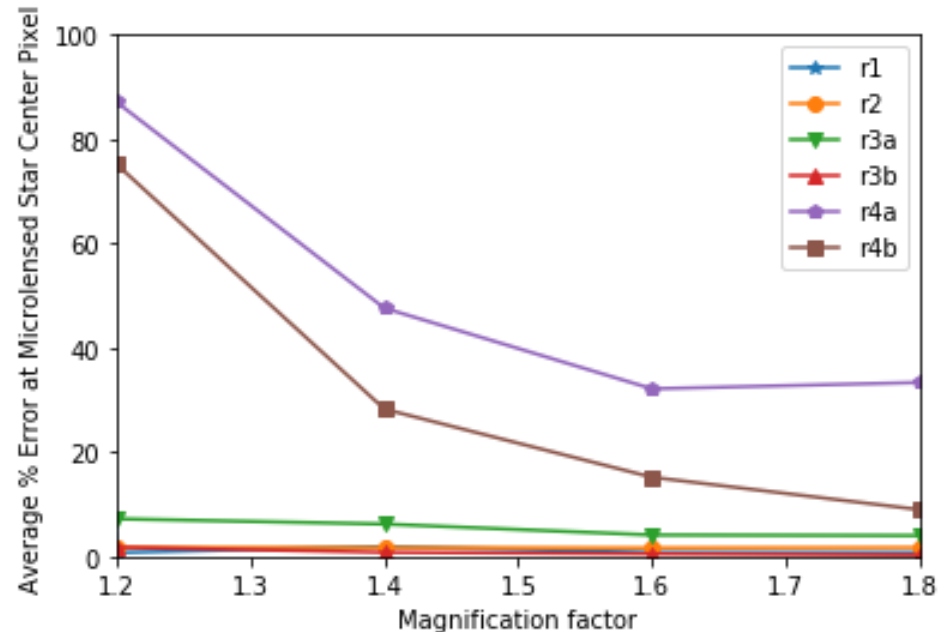


- 1) Obtain CS based measurements, y_o , for a spatial image.
 - CS can be applied by projecting a matrix, A , onto the region of interest, x_o
 - Column-by-Column basis for a $n \times n$ spatial region, x_o . Thus, for 2D images, y_o and A are of size $m \times n$, where $m \ll n$.
- 2) Given A and a clean reference image, x_r , construct measurements matrix y_r , where $y_r = Ax_r$
- 3) Apply differencing algorithm on y_o and y_r to obtain a differenced image, y_{diff} , and the corresponding convolution kernel, M , which is used to match the observed and reference CS measurement vectors, y_o and y_r .
- 4) Reconstruct the differenced image, x'_{diff} using CS algorithms, given A and y_{diff} .



Simulation Results

Apply CS on reference and observed images with Gaussian PSF standard deviation as shown in table.



Average % error as a function of the magnification of a source star in the observed image

CS is able to reconstruct images with fairly narrow PSFs, and which contain star sources experiencing magnification in their flux, using only **10% of traditional Nyquist rate measurements** with **less than 2% error**.

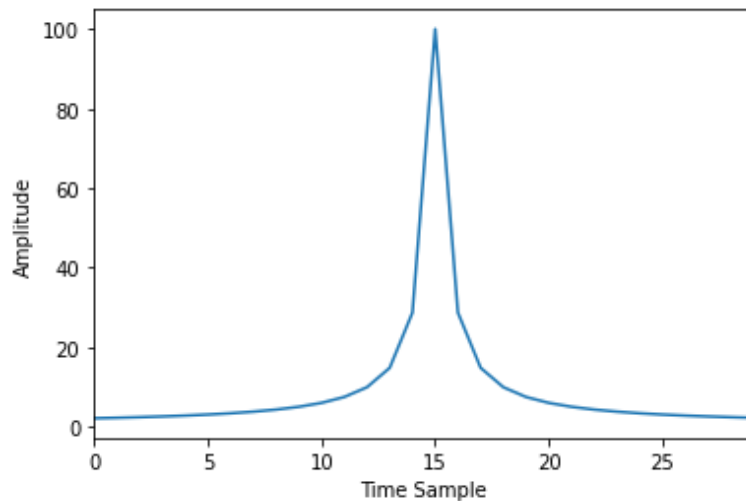
Marker Name	Table Reference	Description
r1	Table I	Convolution Kernel is known 10% of Nyquist rate measurements used Observed image PSF = 0.3 Reference image PSF = 0.1
r2	Table II	Convolution Kernel is unknown 10% of Nyquist rate measurements used Observed image PSF = 0.1 Reference image PSF = 0.1
r3a	Table III	Convolution Kernel is unknown 10% of Nyquist rate measurements used Observed image PSF = 0.3 Reference image PSF = 0.1
r3b	Table III	Convolution Kernel is unknown 30% of Nyquist rate measurements used Observed image PSF = 0.3 Reference image PSF = 0.1
r4a	Table IV	Convolution Kernel is unknown 10% of Nyquist rate measurements used Observed image PSF = 0.5 Reference image PSF = 0.1
r4b	Table IV	Convolution Kernel is unknown 50% of Nyquist rate measurements used Observed image PSF = 0.5 Reference image PSF = 0.1



Single and Binary Microlensing

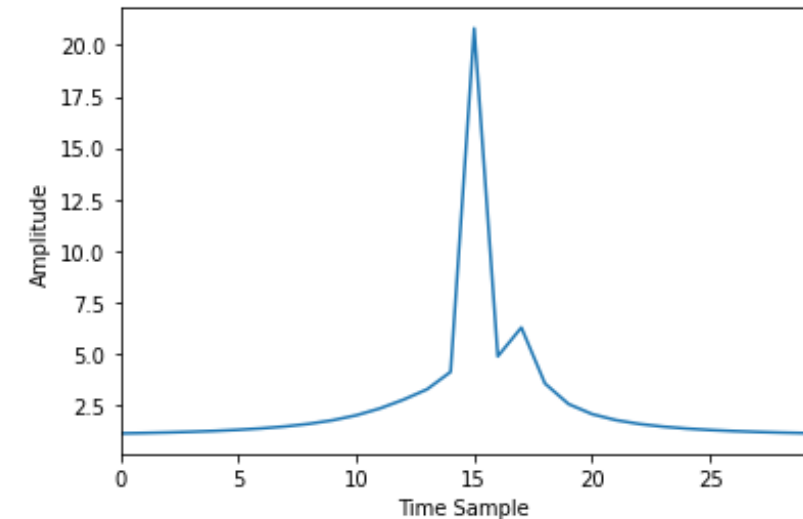
Single Microlensing:

Lensing system consists of only one lensing object- typically a lensing star



Binary Microlensing:

Lensing system consists of two lensing bodies- typically a planet orbiting a lensing star



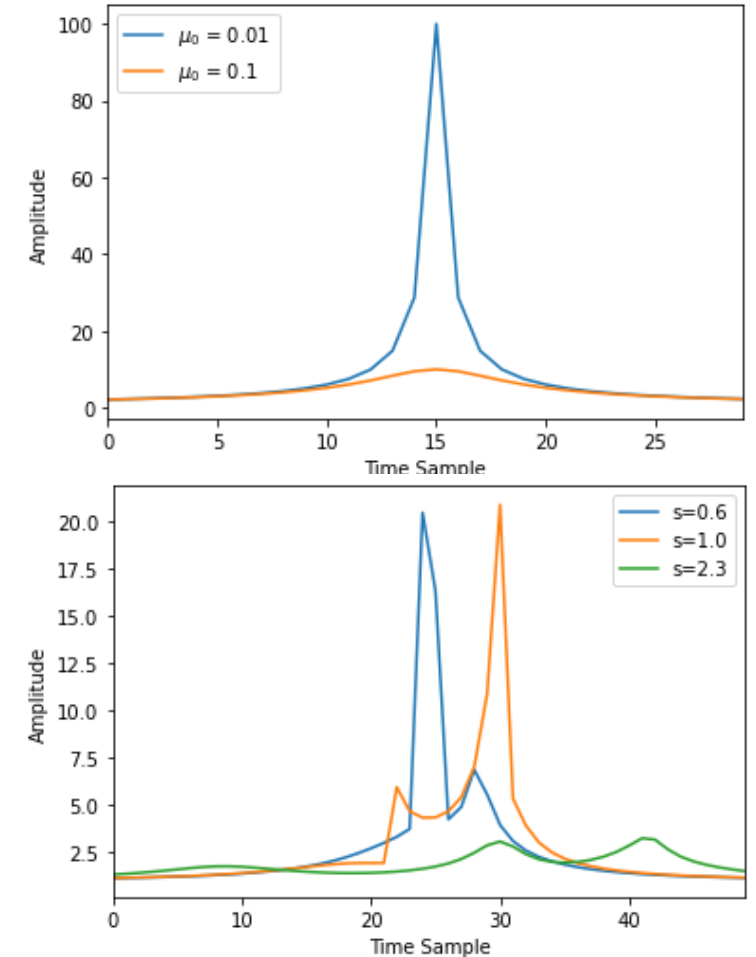
How does CS reconstruction affect Single and Binary microlensing event detection?

From Gravitational Microlensing light curves we can obtain parameters of the lensing system including

- Einstein's ring radius, θ_E , the mass and distance parameter of the lensing system can be obtained
- Single Lens events:
 - Amplification $\rightarrow$ impact factor, $\mu_0 \rightarrow \theta_E$
- Binary Lens events:
 - Amplification $\rightarrow$ caustics generated by mass ratio, q , and lens separation, $s \rightarrow \theta_E$

We want to show error sensitivity of CS reconstruction-

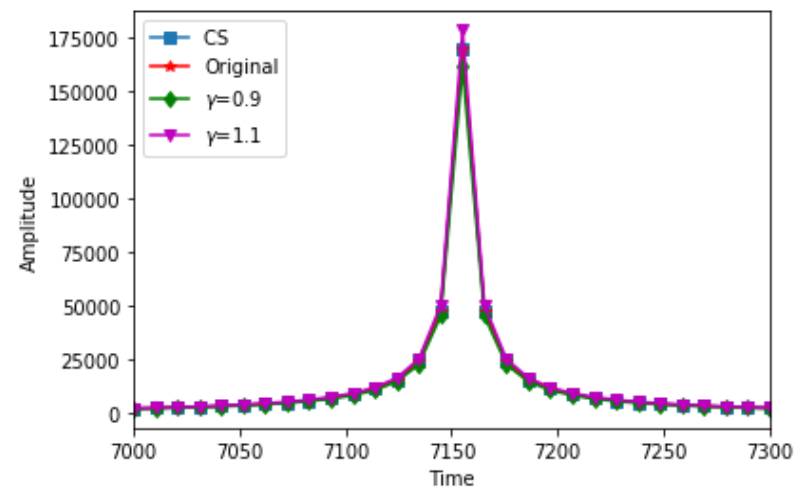
Would the error caused due to CS reconstruction give us incorrect gravitational microlensing parameters?



Single Lens Microlensing

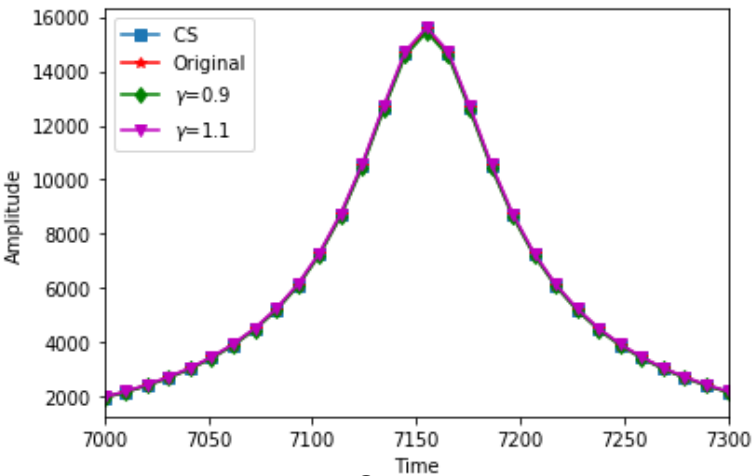
25% CS measurements

$$\tilde{\theta}_E = \gamma \theta_E$$



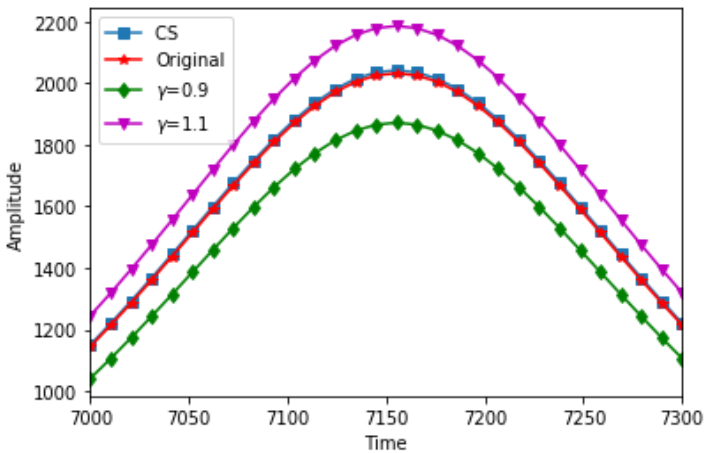
$\mu_0=0.01$

Single lens event with $\mu_0 = 0.01$	Average % error
CS	0.49
$\gamma = 0.9$	6.49
$\gamma = 1.1$	6.19



$\mu_0=0.1$

Single lens event with $\mu_0 = 0.1$	Average % error
CS	0.36
$\gamma = 0.9$	0.65
$\gamma = 1.1$	0.65



$\mu_0=0.5$

Single lens event with $\mu_0 = 0.5$	Average % error
CS	0.77
$\gamma = 0.9$	8.30
$\gamma = 1.1$	7.99



Binary Microlensing Caustics

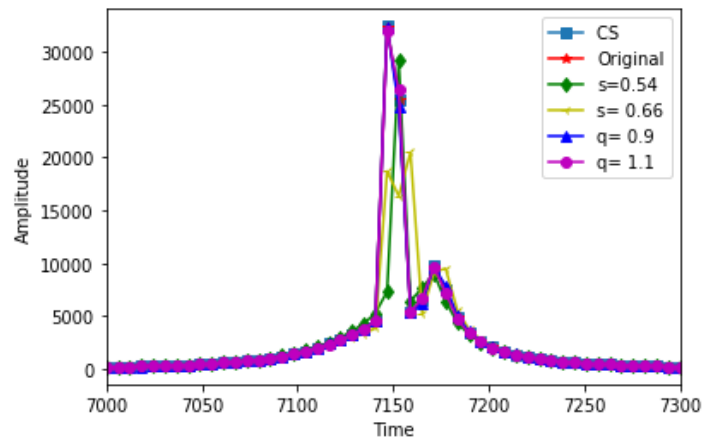
- Amplification curves are dependent on the caustics generated due to gravitational microlensing effects
- Binary Microlensing caustics fall under three categories:
 - Close, Intermediate and Wide
 - Generate different caustic curves
- Caustics are dependent on:
 - separation between the two lenses, s
 - mass ratio of the two lenses, q



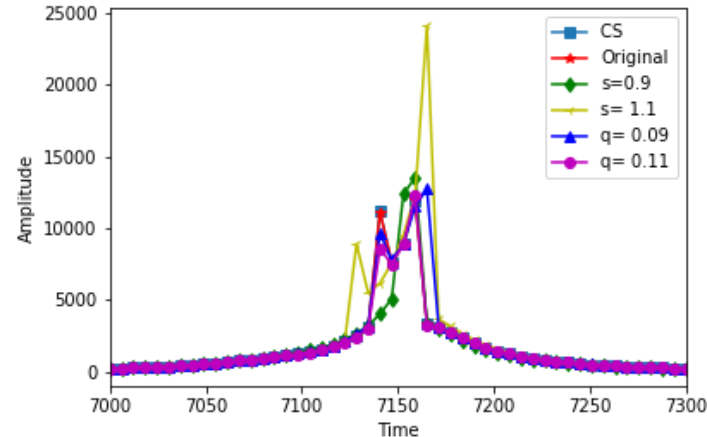
Binary Microlensing Curves

25% Measurements

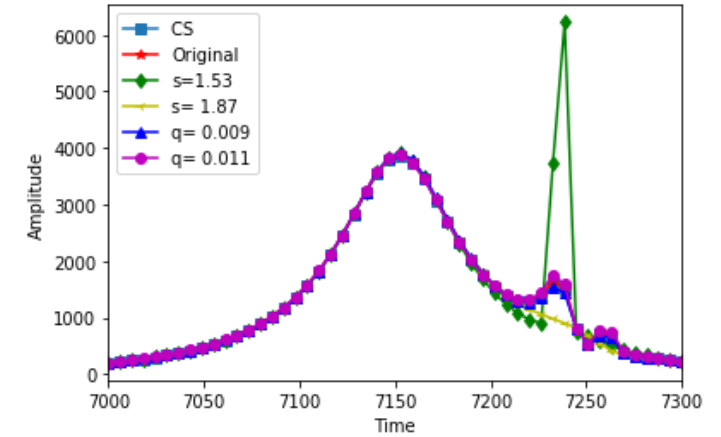
Caustic	Original s	$\pm 0.1s$	Original q	$\pm 0.1q$
Close	0.6	0.54, 0.66	1	0.9, 1.1
Intermediate	1	0.9, 1.1	0.1	0.09, 0.11
Wide	1.7	1.53, 1.87	0.01	0.009, 0.011



Closed curve



Intermediate



Wide



Binary Microlensing Results

Close Caustic	Average % error
CS	0.76
s= 0.54	0.52
s=0.66	10.47
q=0.9	1.11
q=1.1	1.07

Intermediate Caustic	Average % error
CS	0.61
s= 0.9	7.74
s=1.1	25.86
q=0.09	6.76
q=0.11	1.13

Wide Caustic	Average % error
CS	0.97
s= 1.53	13.64
s=1.87	5.54
q=0.009	0.96
q=0.011	0.97

Average % error for CS reconstruction: < 1%

For 10% deviation in s and q values, CS can reconstruct using 25% of measurements within 1% error



Conclusions and Future work

Simulation Results Conclusions

- CS works well for reconstructing transient photometric curves
- Obtain error less than 1% using only 25% Nyquist rate measurements for single and binary microlensing curves
- For magnification factors as low as 1.2, we show CS reconstruction using 10% measurements with accuracy error less than 2%

General CS Summary

- CS works well for sparse data sets
 - Data that is not sparse and needs to be transformed in a sparse domain works as well as the sparse domain transformations
 - Add a margin of error in the sparse transformations, in addition to any error caused due to CS reconstruction
- Trade-off between data sparsity and number of measurements needed

Future Work

- Analyze effects of noise during CS acquisition and reconstruction
- Study implementation techniques of a CS system on a space flight instrument



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