

1 Numerical Validation of Composite Panel Impact Tests

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15

16 Abstract

17 Researchers have met the challenge of modeling impact events involving composite targets for a

18 variety of applications in a variety of ways. In this paper, the theory and implementation details of

19 an orthotropic visco-elastic-visco-plastic material model with strain rate dependence are discussed.

20 The model is driven by experimental data from quasi-static as well as high strain rate tests, and the

21 data is used in defining the deformation, damage and failure sub-models. Validation data is

22 generated by shooting a hollow 50 g Al-2024 projectile at different velocities against a flat panel

23 target made of unidirectional composite material in a 16-ply [(0/90/45/-45)₂]_s layup. Explicit

24 dynamic finite element analyses of four high speed tests involving one contained (projectile
25 rebounded) and three uncontained (projectile penetrated) impact tests show that the developed
26 material model and modeling techniques yield reasonable and acceptable predictions.

27

28 **Keywords**

29 Orthotropic composite, viscoplasticity, impact testing, explicit dynamic finite element analysis

30

31 **1. Introduction**

32 Researchers have met the challenge of modeling impact events involving composite targets for a
33 variety of applications in a variety of ways. Two of the more common applications can be found
34 in the defense industry [Bhatnagar, 2016; Chen, 2015] and in the aerospace industry [Roberts et
35 al., 2002; Melis et al., 2018]. In these applications, a robust, efficient and easy-to-use numerical
36 modeling scheme is of great importance as it leads to a quicker and reliable design of composite
37 structural systems. For example, development of advanced computational analysis methods was
38 an important aspect of the NASA Advanced Composites Project (ACP) which was created with
39 the goal of reducing the development and certification timeline for new composite structures used
40 in aeronautics applications [Melis et al., 2018]. Development of such a complex framework
41 requires a convergence of several tools – a constitutive model suitable for a wide variety of
42 composite materials and architecture and one that includes deformation, damage and failure
43 components; a versatile explicit dynamics finite element program with capabilities of modeling
44 impact events involving contact, damage, failure and possibly, probabilistic input; a high-fidelity,
45 laboratory-based material characterization process to generate the data for the constitutive model;

46 and an experimental facility capable of conducting high energy impact testing to create validation
47 data.

48

49 Several studies have been conducted in the past where some or all tools needed for impact
50 simulations are discussed. Impact simulations of aircraft engine containment systems involving an
51 aluminum target and a titanium impactor [Ambur et al., 2001], and a Kevlar dry fabric target and
52 a steel impactor [Stahlecker et al., 2009], show the challenges in conducting and modeling the
53 impact event. First, it is difficult to control both the velocity and orientation of the impactor in an
54 experiment. The experience of the laboratory personnel plays an important role and trial-and-error
55 is required especially if specific test data are required, e.g., threshold velocity, V_{50} , etc. Second,
56 constitutive models require several material parameters and sometimes, these are not easy to obtain
57 experimentally. The number of required parameters increases with the increase in sophistication
58 of the model. The simplest orthotropic model in LS-DYNA [Ansys-LST, 2020], *MAT_002 that
59 supports elastic behavior, requires eleven material constants. More sophisticated models that
60 support plastic deformation, damage, rate and temperature sensitivities, and element erosion, may
61 require tens, if not hundreds of material properties. It is challenging first to obtain and then to use
62 these data. To reduce the number of experiments required, there are approaches that can be used
63 to gage the effects of the parameters and rank their importance. For example, first-order sensitivity
64 analysis can be used to gage the effect of individual parameters on the computed response
65 [Stahlecker et al., 2009] and then additional effort can be focused on the more important ones.

66

67 While there is no established process to study impact modeling of composite structures, it is
68 recognized that the constituents and the composite architecture play important roles. A healthy

69 mix of constituent and composite testing along with finite element analysis (FEA) can be used to
70 infer what the behavior of the composite will be at the structural level. A study of non-woven (2D)
71 composite as well as a 3D woven composite with the same constituents (IM7 carbon fibers,
72 CYCOM PR520 epoxy resin matrix) was carried out involving experiments of the material
73 constituents, the composite at low and high strain-rates, and ballistic tests [Chocron et al., 2019].
74 LS-DYNA mesoscale computer model was developed and used in the numerical simulations. An
75 orthotropic-elastic model was used for the carbon fibers, a Mohr-Coulomb-based material model
76 was used for the matrix, and cohesive mixed mode model was used for the fiber-matrix interface.
77 The authors state that the numerical results indicate that the predictions for the ballistic tests are
78 very good both for ballistic limits and deflections. Not all research solutions require sophisticated
79 models using numerical solution techniques. Researchers [Shaktivesh et al., 2013] have used
80 analytical solution techniques to compute impact-related parameters on a polymer matrix
81 composite target, such as energy absorbed by different mechanisms, kinetic energy of the
82 projectile, contact force, projectile velocity and projectile tip displacement as a function of time.
83 In Section 2 of the paper, the focus is on modeling impact events using explicit dynamics finite
84 element analysis. A previously developed material model that has been implemented in LS-DYNA
85 as *MAT_213 [Goldberg et al., 2016; Hoffarth et al., 2016; Khaled et al., 2017; Shyamsunder et
86 al., 2019] is suitably modified for modeling impact events over a range of velocities. In the next
87 section, the modeling challenges in impact analysis are discussed with respect to the development
88 and usage of the orthotropic visco-elastic-visco-plastic (VEVP) material model. First, the model
89 enhancement details involving VEVP theory and implementation details are presented. This is
90 followed by specifics of how rate dependent stress-strain data are processed and used. Finally,
91 damage and failure modeling details are discussed to complete the details of the constitutive model.

92 In the context of this paper, the term *failure onset* implies a reduction in the load carrying capacity,
93 and the term *erosion* implies deletion of elements from the finite element (FE) analysis. The focus
94 of Section 3 is on test setup and data processing of the impact tests. These tests are conducted to
95 generate data for the validation of the developed constitutive model. The target is a 16-ply layup
96 [(0/90/45/-45)₂]_s unidirectional tape made up of T800/F3900 composite material [Toray, 2020]
97 and is impacted by a hollow 50 g Al-2024 projectile. Out of the 14 tests where the projectile speed
98 varied between 119 ft/s to 530 ft/s, simulation of the four highest speed tests involving one
99 contained and three uncontained impact tests are discussed. These tests provide an insight into the
100 accuracy and efficiency of the developed material model. The numerical models and results are
101 discussed in Section 4. The paper concludes with a summary of the performance of the developed
102 model and possible improvements that can be made to the developed constitutive model and
103 modeling process.

104

105 **2. Modeling Challenges in Impact Analysis**

106 In this section, we discuss three topics alluded to earlier that address the modeling challenges in
107 impact of composite panels. The first topic is enhancing the capabilities of MAT_213 by
108 modifying the deformation model to be able to capture visco-elastic and visco-plastic behaviors.
109 The second topic deals with pre-processing stress-strain curves corresponding to different strain-
110 rates to yield consistent data for use by the constitutive model. The last topic deals with
111 incorporating damage and failure models in the constitutive model framework thus providing the
112 means to track the behavior of the target composite structure for both contained and uncontained
113 impacts.

114

115 2.1 Overview of Material Model

116 Experiments have shown that (a) structural composites exhibit nonlinear stress-strain response
117 both when loaded and unloaded with the unload slope decreasing with increasing strain, and (b)
118 the overall response including initial elastic modulus change with strain rate [Daniel et al., 2011;
119 Khaled et al., 2017; Deshpande et al., 2019]. Examples of stress-strain curves at several strain rates
120 and room temperature for a carbon fiber reinforced polymer composite, T800/F3900 [Toray, 2020]
121 used for the validation tests in the paper are shown in Fig. 1.

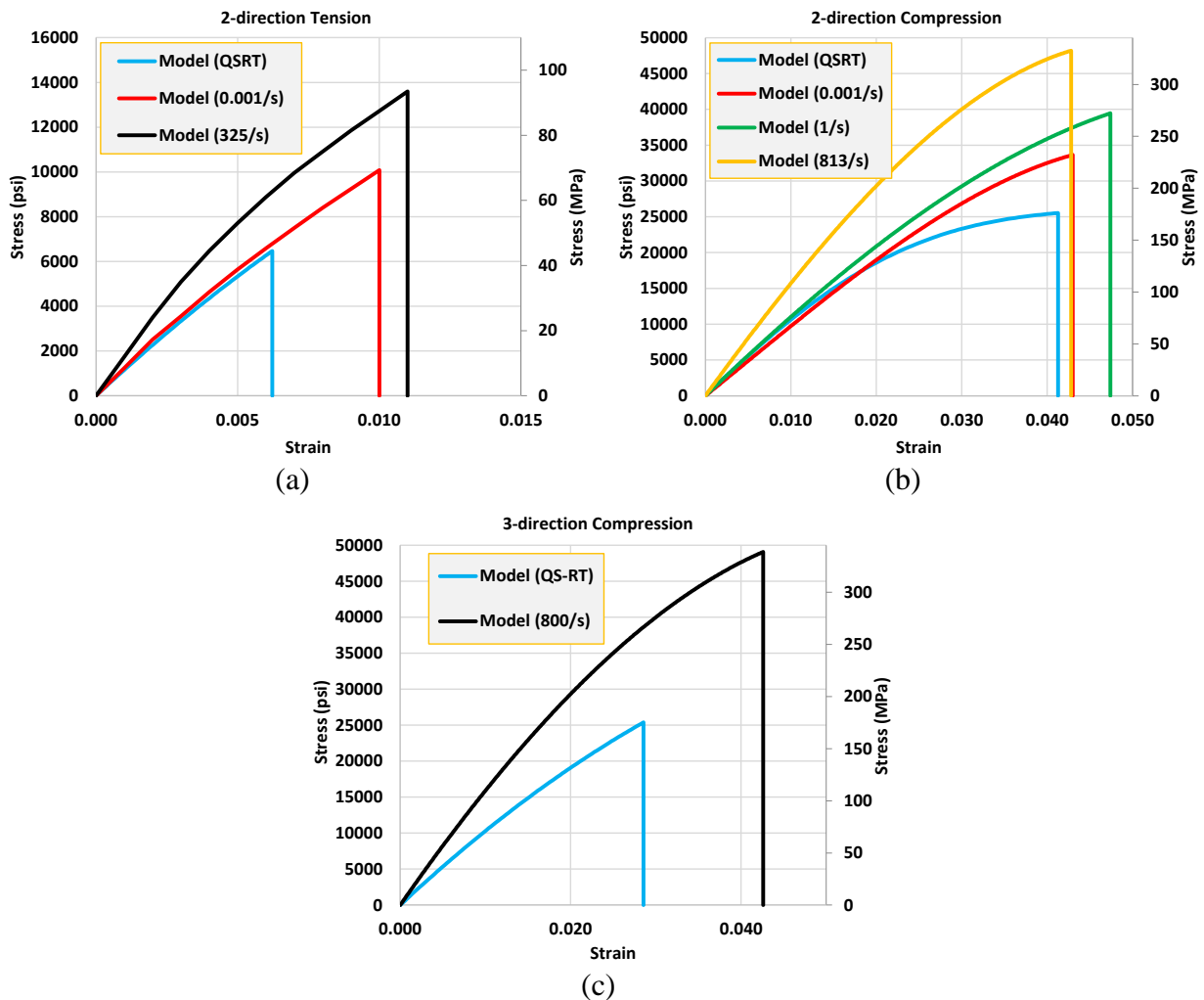


Figure 1. Stress-strain curves for (a) 2-direction tension (b) 2-direction compression (c) 3-direction compression (“1” is along the fiber direction, “2” and “3” refer to the transverse in-plane and through-thickness directions. QS-RT denotes quasi-static room temperature.

“Model” denotes average of several test replicates.) [Khaled et al., 2017; Deshpande et al., 2019]

122 The stress-strain curves shown in Fig. 1 were generated from physical test of coupons obtained
 123 from multi-ply unidirectional composite panels. There are two approaches to developing a material
 124 model to capture the nonlinear behavior – continuum damage mechanics models [Matzenmiller et
 125 al., 1995; Maimi et al., 2007; Wu and Yao, 2010] and plasticity [Sun and Chen, 1989; Vaziri et al,
 126 1991; Khan and Huang, 1995; Cho et al., 2010]. In the present work, the constitutive model is
 127 divided into three sub-models – deformation [Goldberg et al., 2016] damage [Khaled et al., 2019a]
 128 and failure [Goldberg et al., 2018; Shyamsunder et al., 2019, 2020b]. Adding the visco-elastic
 129 component to the material model facilitates modeling rate-dependent behavior of the material in
 130 the elastic regime in a physically consistent manner, whereas the visco-plastic component is used
 131 to capture the strain-rate effects in the plastic domain which accounts for the permanent
 132 deformation. The combination of the visco-elastic and the visco-plastic approach is referred here
 133 to as visco-elastic-visco-plastic (VEVP) model. The VEVP behavior is captured in the deformation
 134 sub-model. A yield function which is a modified form of the commonly used Tsai–Wu composite
 135 failure model is used as

$$136 \quad f(\sigma) = a + (F_1 \quad F_2 \quad F_3 \quad 0 \quad 0 \quad 0) \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} + \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix}^T \begin{bmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ F_{12} & F_{22} & F_{23} & 0 & 0 & 0 \\ F_{13} & F_{23} & F_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & F_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & F_{66} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} \quad (1)$$

137 where $a = -1$. The yield function coefficients, F_i and F_{ij} , depend on the current yield stress
 138 values and are calculated as

$$\begin{aligned}
139 \quad F_1 &= \frac{1}{\sigma_{11}^T} - \frac{1}{\sigma_{11}^C} & F_{11} &= \frac{1}{\sigma_{11}^T \sigma_{11}^C} & F_{44} &= \frac{1}{\sigma_{12}^2} \\
F_2 &= \frac{1}{\sigma_{22}^T} - \frac{1}{\sigma_{22}^C} & F_{22} &= \frac{1}{\sigma_{22}^T \sigma_{22}^C} & F_{55} &= \frac{1}{\sigma_{23}^2} \\
F_3 &= \frac{1}{\sigma_{33}^T} - \frac{1}{\sigma_{33}^C} & F_{33} &= \frac{1}{\sigma_{33}^T \sigma_{33}^C} & F_{66} &= \frac{1}{\sigma_{31}^2}
\end{aligned} \tag{2}$$

$$\begin{aligned}
140 \quad F_{12} &= \frac{2}{(\sigma_{12}^{45})^2} - \frac{F_1 + F_2}{\sigma_{12}^{45}} - \frac{1}{2}(F_{11} + F_{22} + F_{44}) \\
F_{23} &= \frac{2}{(\sigma_{23}^{45})^2} - \frac{F_2 + F_3}{\sigma_{23}^{45}} - \frac{1}{2}(F_{22} + F_{33} + F_{55}) \\
F_{13} &= \frac{2}{(\sigma_{31}^{45})^2} - \frac{F_1 + F_3}{\sigma_{31}^{45}} - \frac{1}{2}(F_{11} + F_{33} + F_{66})
\end{aligned} \tag{3}$$

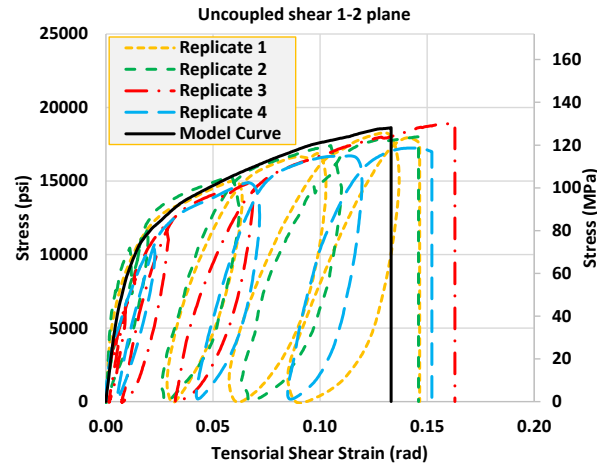
141 where the superscripts T, C and 45 denote data obtained from tension, compression and 45-degree
142 off-axis tests, respectively. The plastic potential function used with a non-associative flow rule is
143 defined as

$$144 \quad h = \sqrt{\frac{H_{11}\sigma_{11}^2 + H_{22}\sigma_{22}^2 + H_{33}\sigma_{33}^2 + 2H_{12}\sigma_{11}\sigma_{22} + 2H_{23}\sigma_{22}\sigma_{33} + 2H_{31}\sigma_{33}\sigma_{11}}{+H_{44}\sigma_{12}^2 + H_{55}\sigma_{23}^2 + H_{66}\sigma_{31}^2}} \tag{4}$$

145 where the H_{ij} terms are a set of constant coefficients which are computed based on the plastic
146 Poisson's ratios, with the coefficients (referred to as flow rule coefficients) defined as input
147 parameters in the model [Hoffarth et al., 2017]. Plasticity has been used to capture nonlinearity in
148 a phenomenological fashion in order to capture the permanent strains in addition to accounting for
149 strain rate effects in a physically consistent manner.

150 The damage sub-model is used for two purposes. First, the damage sub-model, which is synergistic
151 to the plasticity model, tracks the reduction in the unloading slope that takes place with increasing
152 plastic strain. A semi-coupled damage model is used to account for the fact that loading in one
153 direction can lead to stiffness reductions in other material directions. Fig. 2 shows an example

154 stress-strain curve from the in-plane shear test with a cyclic displacement-controlled loading.
155 Damage-related parameters are obtained from this test.



156
157 **Figure 2. 1-2 shear specimen subjected to cyclic loading [Khaled et al., 2017, 2019a]**
158

159 While the deformation sub-model provides the mechanism to simulate the nonlinear stress-strain
160 response, the damage sub-model is used to capture nonlinear unloading and local plastic softening
161 often observed in composites [Barbero, 2013; Goldberg et al., 2016]. In our earlier work [Goldberg
162 et al., 2018], we show that using the strain equivalence assumption specifies that the effective
163 stresses result in the same deformations in the effective, undamaged material as would be caused
164 by applying the actual stresses on the damaged material. In this way, the yield function, flow rule,
165 and constitutive equation, can be written in terms of the effective stresses and applied in effective
166 stress space. The use of effective stresses allows the plasticity calculations to be uncoupled from
167 the damage law.

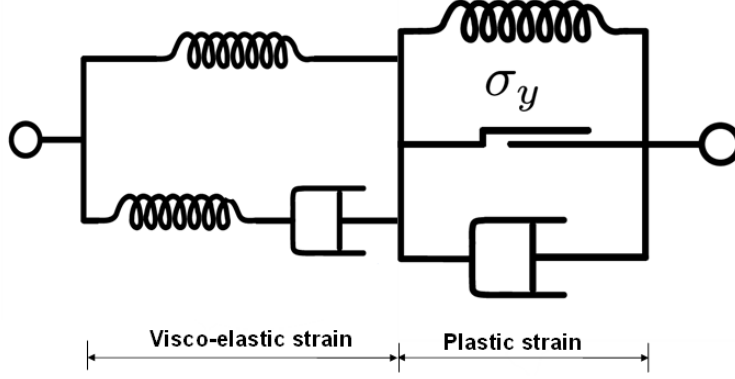
168 Second, the damage sub-model, as shown later, is used to permit the simulation of post-peak stress
169 degradation that is required to account for the load shedding that takes place in actual materials
170 and mitigates the potential of a sudden brittle failure being simulated once the peak stress is
171 reached in the simulation of impact events.

172 Finally, the failure sub-model is used to erode the element when the erosion strain is reached. The
173 motivation for using the generalized tabulated failure model is to allow the failure surface to take
174 on any arbitrary shape and not be driven by specific geometrical shapes defined by mathematical
175 equations. This point cloud of failure surface associated with composites permits complex loading
176 scenarios to be handled correctly, e.g., permitting tension-compression asymmetry, shear-
177 dominated failure, etc. The input needed to drive the material model includes the following: (a) at
178 a minimum, twelve sets of stress-strain curves at a specified temperature-strain rate combination
179 (3 tension tests in the principal material directions (PMDs), 3 compression tests in the PMDs, 3
180 shear tests in the principal material planes, and 3 45^0 off-axis tests) which are input as tabulated
181 data set, (b) damage model-related data obtained from coupled and uncoupled damage tests in the
182 PMDs [Khaled et al., 2019a], and (c) additional data that include mass density, elastic orthotropic
183 material properties, flow rule coefficients, V EVP related parameters, and failure model-related
184 parameters.

185

186 **2.2 Visco-elastic-visco-plastic Considerations**

187 The V EVP behavior can be represented as a spring-dashpot model as shown in Fig. 3. To derive
188 the stresses in the visco-elastic and the visco-plastic component, the model shown in Fig. 3 can be
189 divided into a standard linear solid model and a hardening model, separately. There are at least
190 three different approaches to viscoplasticity [Wang et al., 1997] – overstress visco-plastic models
191 of Perzyna and of Duvaut-Lions, and the consistency visco-plastic approach. In the present work,
192 the consistency approach is used since only with this approach it is possible to use the Kuhn-
193 Tucker form of the loading-unloading conditions in the visco-plastic case.



194

195 **Figure 3. Schematic of V EVP model**

196

197 The stress is divided into equilibrium, $\sigma_{n+1}^{e,eff}$ and viscous, $\sigma_{n+1}^{v,eff}$ components [Du Bois et al., 2017;
 198 Achstetter, 2019] that are computed as

199
$$\sigma_{n+1}^{e,eff} = \sigma_n^{e,eff} + C_\infty \dot{\epsilon}_{n+1}^{avg} \Delta t \quad (5)$$

200
$$\sigma_{n+1}^{v,eff} = \sigma_n^{v,eff} \circ \beta + \left[(C - C_\infty) \circ B \right] \dot{\epsilon}_{n+1}^{avg} \Delta t \quad (6)$$

201 where \circ denotes Hadamard product between the matrices or vectors. These computations take
 202 place in the effective stress space [Khaled et al., 2019a] and hence the superscript *eff* is used. C_∞
 203 is the equilibrium stiffness matrix generated using the moduli corresponding to the quasi-static
 204 stress-strain curves while C is the total stiffness matrix generated using the moduli corresponding
 205 to the current strain rate. In computing the total stiffness matrix, the total strain-rate, $\dot{\epsilon}_{n+1}^{avg}$ and the
 206 temperature is used to determine the moduli for a given time step and Gauss point [Shyamsunder
 207 et al., 2020a]. It is necessary to smoothen out the strain-rate components to avoid numerical
 208 instabilities. This is done using an exponential averaging technique as

209
$$\dot{\epsilon}_{n+1}^{avg} = (1 - FILT) \times \dot{\epsilon}_{n+1} + FILT \times \dot{\epsilon}_n^{avg} \quad (7)$$

210 where n denotes the previous time step, and $0 \leq FILT \leq 1$. Using Eq. 7, the average strain rates at
 211 the current time step, $\dot{\epsilon}_{n+1}^{avg}$ are computed as a function of the non-smooth strain rates at the current

212 time step, $\dot{\boldsymbol{\epsilon}}_{n+1}$ and the averaged strain rate from the previous time step, $\dot{\boldsymbol{\epsilon}}_n^{avg}$. Vector $\boldsymbol{\beta}$ and the
 213 matrix \mathbf{B} are given by the following expressions [Achstetter, 2019]

214
$$\boldsymbol{\beta} = \left[e^{-\beta_{11}\Delta t} \quad e^{-\beta_{22}\Delta t} \quad e^{-\beta_{33}\Delta t} \quad e^{-\beta_{44}\Delta t} \quad e^{-\beta_{55}\Delta t} \quad e^{-\beta_{66}\Delta t} \right]^T \quad (8)$$

215
$$\mathbf{B} = \begin{bmatrix} \frac{1-e^{-\beta_{11}\Delta t}}{\beta_{11}\Delta t} & \frac{1-e^{-\beta_{12}\Delta t}}{\beta_{12}\Delta t} & \frac{1-e^{-\beta_{13}\Delta t}}{\beta_{13}\Delta t} & 0 & 0 & 0 \\ \frac{1-e^{-\beta_{12}\Delta t}}{\beta_{12}\Delta t} & \frac{1-e^{-\beta_{22}\Delta t}}{\beta_{22}\Delta t} & \frac{1-e^{-\beta_{23}\Delta t}}{\beta_{23}\Delta t} & 0 & 0 & 0 \\ \frac{1-e^{-\beta_{13}\Delta t}}{\beta_{13}\Delta t} & \frac{1-e^{-\beta_{23}\Delta t}}{\beta_{23}\Delta t} & \frac{1-e^{-\beta_{33}\Delta t}}{\beta_{33}\Delta t} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-e^{-\beta_{44}\Delta t}}{\beta_{44}\Delta t} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-e^{-\beta_{55}\Delta t}}{\beta_{55}\Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-e^{-\beta_{66}\Delta t}}{\beta_{66}\Delta t} \end{bmatrix} \quad (9)$$

216 where, β_{ij} 's, $ij = 11, 22, 33, 44, 55, 66, 12, 23$ and 13 , are the decay constants. Currently, there are
 217 no experimental methods to determine these β_{ij} values. These β_{ij} parameters are fitted in such a
 218 way that together with the tabulated definition of the rate dependent Young's modulus, they allow
 219 to simulate the rate dependent slopes of the stress-strain curves measured in the experiments. It
 220 should be noted here that the visco-elastic model that is used is highly non-linear since linear
 221 viscoelasticity will not allow to fit the initial slopes for a wide enough range of strain rates. The
 222 β_{ij} parameters should be small enough not over damp and large enough to allow for viscoelasticity.
 223 Typically, a trial-and-error process is carried out to estimate these values, e.g., inverse analysis.
 224 The components of $\boldsymbol{\sigma}_n^{e,eff}$ and $\boldsymbol{\sigma}_n^{v,eff}$ are given by the following equations

$$\sigma_{11,n}^{i,eff} = \begin{cases} \frac{\sigma_{11,n}^i}{1-c_1^d} & \text{if } \sigma_{11,n} \geq 0 \\ \frac{\sigma_{11,n}^i}{1-c_4^d} & \text{if } \sigma_{11,n} < 0 \end{cases} \quad (10)$$

$$\sigma_{33,n}^{i,eff} = \begin{cases} \frac{\sigma_{33,n}^i}{1-c_3^d} & \text{if } \sigma_{33,n} \geq 0 \\ \frac{\sigma_{33,n}^i}{1-c_6^d} & \text{if } \sigma_{33,n} < 0 \end{cases} \quad (12)$$

$$\sigma_{22,n}^{i,eff} = \begin{cases} \frac{\sigma_{22,n}^i}{1-c_2^d} & \text{if } \sigma_{22,n} \geq 0 \\ \frac{\sigma_{22,n}^i}{1-c_5^d} & \text{if } \sigma_{22,n} < 0 \end{cases} \quad (11)$$

$$\sigma_{12,n}^{i,eff} = \frac{\sigma_{12,n}^i}{1-c_7^d} \quad (13)$$

$$\sigma_{23,n}^{i,eff} = \frac{\sigma_{23,n}^i}{1-c_8^d} \quad (14)$$

$$\sigma_{13,n}^{i,eff} = \frac{\sigma_{13,n}^i}{1-c_9^d} \quad (15)$$

225 where, i can be either e (equilibrium) or v (viscous) and the stress components on the right-hand
 226 side are in the true stress space. An assumption is thus made that damage affects the equilibrium
 227 and viscous stresses in the same way. The true stress space corresponds to the damaged state
 228 whereas the effective space corresponds to the undamaged state. Strain equivalence assumption
 229 allows the damage sub-model to predict the reduction in the stiffness without affecting the
 230 deformation sub-model. The parameter, c_i^d , is referred to as the effective damage parameter and
 231 is a measure of damage in each component in the principal material direction/plane [Shyamsunder
 232 et al., 2020b]. These are given by the following expressions for the normal component,

$$c_i^d = 1 - \left\{ (1-d_{11_r}^{i_r}) (1-d_{11_c}^{i_c}) (1-d_{22_r}^{i_r}) (1-d_{22_c}^{i_c}) \right. \\ \left. (1-d_{33_r}^{i_r}) (1-d_{33_c}^{i_c}) (1-d_{12}^{i_r}) (1-d_{23}^{i_r}) (1-d_{13}^{i_r}) \right\} \quad (16)$$

$$c_{i+3}^d = 1 - \left\{ (1-d_{11_c}^{i_c}) (1-d_{11_r}^{i_r}) (1-d_{22_c}^{i_c}) (1-d_{22_r}^{i_r}) \right. \\ \left. (1-d_{33_c}^{i_c}) (1-d_{33_r}^{i_r}) (1-d_{12}^{i_c}) (1-d_{23}^{i_c}) (1-d_{13}^{i_c}) \right\} \quad (17)$$

235 where $i = 1, 2$ and 3 . The shear components are given as

236
$$c_k^d = 1 - \left\{ (1 - d_{11_r}^{ij})(1 - d_{11_c}^{ij})(1 - d_{22_r}^{ij})(1 - d_{22_c}^{ij}) \right.$$

237
$$\left. (1 - d_{33_r}^{ij})(1 - d_{33_c}^{ij})(1 - d_{12}^{ij})(1 - d_{23}^{ij})(1 - d_{13}^{ij}) \right\}$$
 (18)

238 where, $ij=12, 23$ and 13 , and $k = 7, 8$ and 9 , respectively. d_{ij}^{kl} is referred to as the damage

239 parameter and accounts for damage in the kl direction due to loading in the ij direction. If the ij

240 direction and the kl direction are the same, d_{ij}^{kl} is referred to as an uncoupled damage parameter.

241 Otherwise, it is referred to as coupled damage parameter. For example, in Eq. 16, $d_{22_r}^{22_r}$ is an

242 uncoupled damage parameter obtained by loading the specimen in 2-direction tension and

243 interrogating the reduction in stiffness in the 2-direction tension. Similarly, $d_{22_c}^{22_r}$ is a coupled

244 damage parameter obtained by loading the specimen in 2-direction compression and interrogating

245 the reduction in the stiffness in the 2-direction tension. The conversion from the true stress space

246 to the effective stress space is done separately for the equilibrium and the viscous components.

247 The components of $\sigma_n^{v,eff}$ are taken as zero if the material is assumed to be purely visco-plastic.

248 The total trial stress is computed using the following equation

249
$$\sigma_{n+1}^{t,eff} = \sigma_{n+1}^{e,eff} + \sigma_{n+1}^{v,eff}$$
 (19)

250 The total trial stress is corrected back to the yield surface using a radial return algorithm. The yield

251 function given by Eq. 1 is computed based on the total trial stress, $f(\sigma_{n+1}^{t,eff})$. It has to be noted that

252 the yield function coefficients (F_i and F_{ij}) are computed based on the current yield stresses

253 corresponding to the quasi-static stress-strain curves to predict the onset of plasticity [Hoffarth et

254 al., 2016]. The stress tensor in the effective space is updated as

255
$$\sigma_{n+1}^{eff} = \left(\sigma_{n+1}^{t,eff} - \left[\mathbf{C}_\infty + (\mathbf{C} - \mathbf{C}_\infty) \circ \mathbf{B} \right] \Delta \lambda \frac{\partial h}{\partial \sigma} \Big|_{\sigma_{n+1}^{t,eff}} \right)$$
 (20)

255 where, $\Delta\lambda$ is the plastic multiplier and is taken as zero if the material is assumed to be purely
256 visco-elastic, and h is the plastic potential function given by Eq. 4. For a given simulation, the
257 user input is pre-processed where for each input stress-strain curve, the stress is converted into
258 effective stress, and the strain is converted into plastic strain and finally to effective plastic strain
259 (illustrated in Fig. 4). Each of these curves is assigned an effective plastic strain rate which is
260 computed based on the strain-rate specified for each input curves. During the simulation, $\Delta\lambda$ is
261 computed as a function of the current yield stress obtained from interpolation using the effective
262 plastic strain rate and the temperature. The highest and the lowest rate curves are used if the current
263 value is greater than or less than the given user input curves. It should be noted that even though
264 the implementation supports user input for stress-strain curves at various strain-rate and
265 temperature combinations, only strain-rate has been varied in the current work. The authors do not
266 yet have experimental data for loading and unloading at different combinations of strain-rate and
267 temperature. After the stress is updated using Eq. 20, the equilibrium and the viscous stresses also
268 need to be updated. In order to ensure that the total stress is equal to the sum of the equilibrium
269 and the viscous stresses, these stress tensors are updated as follows:

270 If $f(\boldsymbol{\sigma}_{n+1}^{e,eff}) \leq \delta_{tol}$,

271
$$\boldsymbol{\sigma}_{n+1}^{e,eff} = \boldsymbol{\sigma}_{n+1}^{e,eff} \quad (21)$$

272
$$\boldsymbol{\sigma}_{n+1}^{v,eff} = \boldsymbol{\sigma}_{n+1}^{v,eff} - \left[\mathbf{C}_{\infty} + (\mathbf{C} - \mathbf{C}_{\infty}) \circ \mathbf{B} \right] \Delta\lambda \left. \frac{\partial h}{\partial \boldsymbol{\sigma}} \right|_{\boldsymbol{\sigma}_{n+1}^{e,eff}} \quad (22)$$

273 else

274
$$\boldsymbol{\sigma}_{n+1}^{e,eff} = \left(\boldsymbol{\sigma}_{n+1}^{e,eff} - \mathbf{C}_{\infty} \Delta\lambda \left. \frac{\partial h}{\partial \boldsymbol{\sigma}} \right|_{\boldsymbol{\sigma}_{n+1}^{e,eff}} \right) \quad (23)$$

275
$$\boldsymbol{\sigma}_{n+1}^{v,eff} = \boldsymbol{\sigma}_{n+1}^{v,eff} - \left[(\mathbf{C} - \mathbf{C}_\infty) \circ \mathbf{B} \right] \Delta \lambda \left. \frac{\partial h}{\partial \boldsymbol{\sigma}} \right|_{\boldsymbol{\sigma}_{n+1}^{v,eff}} \quad (24)$$

276 where δ_{tol} is a tolerance used in the analysis, which for the current work is taken as 10^{-6} . Eqs. 21-
 277 24 show that if the yield function value computed from the equilibrium stress is non-positive, the
 278 return to the yield surface is achieved by reducing the viscous stresses only. This ensures that the
 279 stress does not fall below the equilibrium stress in the effective space. Finally, $\boldsymbol{\sigma}_{n+1}^{eff}$, $\boldsymbol{\sigma}_{n+1}^{e,eff}$ and $\boldsymbol{\sigma}_{n+1}^{v,eff}$
 280 are converted into the true stress space using Eqs. 10-15. The updated $\boldsymbol{\sigma}_{n+1}^{e,eff}$ and $\boldsymbol{\sigma}_{n+1}^{v,eff}$ are used in
 281 the next time step for computation of trial stress.

282

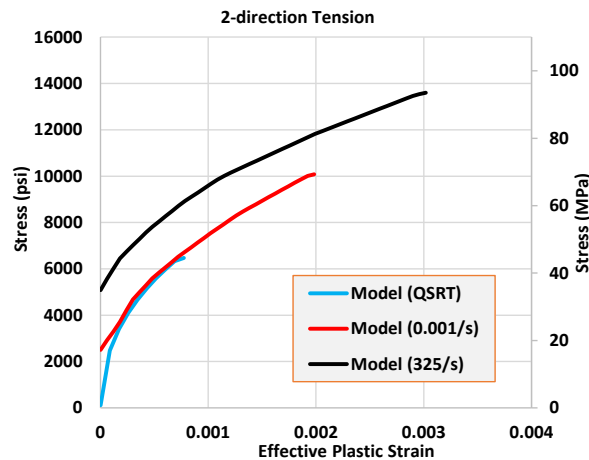
283 **2.3 Processing Rate Data**

284 Careful processing of rate-dependent stress-strain data before it is used in finite element analysis
 285 is necessary to ensure that the data for various rates are used consistently in the constitutive model.
 286 Input to the material law consists of yield curves corresponding to different values of the equivalent
 287 plastic strain rates. The difference in yield stress between two curves corresponds to viscous stress.
 288 For a positive increment of the equivalent plastic strain rate the effective viscous stress increment
 289 must be positive, as viscosity is necessarily positive. Consequently, yield curves at higher strain
 290 rates must necessarily be above yield curves at lower strain rates. And yield curves at different
 291 strain rates cannot intersect.

292

293 Processing of the input stress-strain curves is illustrated using Fig. 1(a) and Fig. 4 as an example.
 294 The term *QS-RT* refers to testing carried out at quasi-static ($\sim 10^{-4}$ /s), room temperature conditions,
 295 and *Model* refers to the curve generated from the average values obtained from multiple test
 296 replicates and used as input to the constitutive model. Fig. 1(a) shows the curves for the 2-direction

297 tension component at three different strain rates [Khaled et al., 2017; Deshpande et al., 2019]. The
298 corresponding effective stress-effective plastic strain curves are shown in Fig. 4. Care must be
299 taken to avoid intersecting effective stress-effective plastic strain curves as intersecting curves will
300 likely result in either inaccurate results or numerical instability. Often, intersecting curves result
301 from selecting an incorrect initial yield strain value.

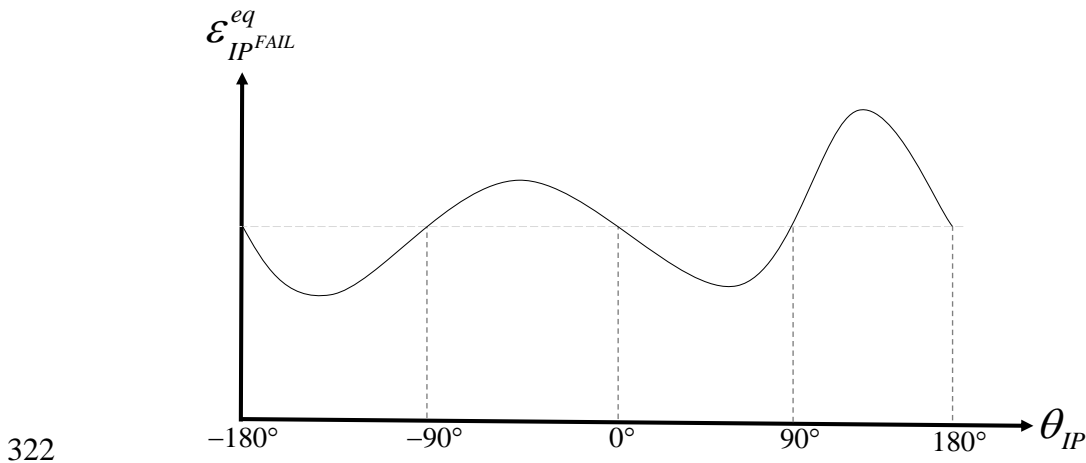


302
303 **Figure 4. Effective stress versus effective plastic strain curves for 2-direction tension data for**
304 **the T800-F3900 composite constructed from data in Fig. 1(a).**

305
306 **2.4 Failure Modeling**

307 A failure model where an arbitrarily shaped failure surface in the stress space can be used to predict
308 the failure of a composite is highly desirable [Goldberg et al., 2018]. In this paper, the Generalized
309 Tabulated Failure Criterion (GTFC) implemented in MAT_213 is used [Shyamsunder et al.,
310 2020b]. The current implementation is designed for unidirectional, laminated composites. The
311 failure state is divided into two components– in-plane failure taking place in the 1-2 plane and out-
312 of-plane failure involving the out-of-plane stress/strain components. Failure surfaces need to be
313 defined for each of these failure states in the equivalent failure strain and failure angle space. Fig.

314 5 shows the in-plane failure surface where $\varepsilon_{IP^{FAIL}}^{eq}$ is a function of the failure angle, θ_{IP} . It must be
 315 noted that the term failure angle is unrelated to the fracture plane angle in a composite. The data
 316 to construct this surface can be obtained from laboratory testing or from virtual testing. Typically,
 317 the testing process starts with uniaxial testing in the 1- and in the 2-directions with tensile and
 318 compressive loadings, and a pure shear test. This would yield the final states of stress and strain
 319 (ignoring relatively small magnitudes) as $(\sigma_1^t, 0, 0)$, $(\sigma_2^t, 0, 0)$, $(\sigma_1^c, 0, 0)$, $(\sigma_2^c, 0, 0)$, $(\varepsilon_1^t, 0, 0)$, $(\varepsilon_2^t, 0, 0)$,
 320 $(\varepsilon_1^c, 0, 0)$, $(\varepsilon_2^c, 0, 0)$, $(0, 0, \sigma_{12})$, and $(0, 0, \varepsilon_{12})$. This can be followed by biaxial testing (laboratory or
 321 virtual) yielding a richer set of $(\sigma_1, \sigma_2, \sigma_{12})$ and $(\varepsilon_1, \varepsilon_2, \varepsilon_{12})$ data lying on the failure surface.



323 **Figure 5. General form of in-plane failure surface [Shyamsunder et al., 2020b]**

324 The equivalent strain, ε_{IP}^{eq} and failure angle, θ_{IP} can then be computed for each failure point as

325
$$\varepsilon_{IP}^{eq} = \sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 + 2\varepsilon_{12}^2} \quad (25)$$

326
$$\theta_{IP} = \cos^{-1} \left(\frac{\sigma_{22}}{\sqrt{\sigma_{22}^2 + \sigma_{12}^2}} \right) \quad (26)$$

327 thereby permitting the creation of $(\varepsilon_{IP^{FAIL}}^{eq}, \theta_{IP})$ input tabular data. Similarly, an out-of-plane failure

328 surface can be constructed with $(\varepsilon_{OOP^{FAIL}}^{eq}, \theta_{OOP})$ input tabular data with the values computed as

$$329 \quad \varepsilon_{OOP}^{eq} = \sqrt{\varepsilon_{33}^2 + 2\varepsilon_{13}^2 + 2\varepsilon_{23}^2} \quad (27)$$

$$330 \quad \theta_{OOP} = \cos^{-1} \left(\frac{\sigma_{13}}{\sqrt{\sigma_{13}^2 + \sigma_{23}^2}} \right) \quad (28)$$

331 During the finite element analysis, the failure detection at each Gauss point takes place as follows.

332 First, the in-plane failure state e_1 is computed as

$$333 \quad e_1 = \frac{\varepsilon_{IP}^{eq}}{\varepsilon_{IP^{FAIL}}^{eq}} \quad (29)$$

334 where, ε_{IP}^{eq} is computed for each Gauss point and time step and $\varepsilon_{IP^{FAIL}}^{eq}$ is what is specified as input

335 failure surface. Similarly, the out-of-plane failure state e_2 is computed as

$$336 \quad e_2 = \frac{\varepsilon_{OOP}^{eq}}{\varepsilon_{OOP^{FAIL}}^{eq}} \quad (30)$$

337 An element is eroded if $e \geq 1$ where

$$338 \quad e = \begin{cases} \max(e_1, e_2) & \text{if } n = 0 \\ \sqrt[n]{(e_1)^n + (e_2)^n} & \text{if } n > 0 \end{cases} \quad (31)$$

339 where n is a user defined interaction parameter that can be used to couple the in-plane state of

340 stress/strain to the out-of-plane state of stress/strain. The failure surfaces specified are used for

341 predicting the element erosion rather than the failure onset.

342

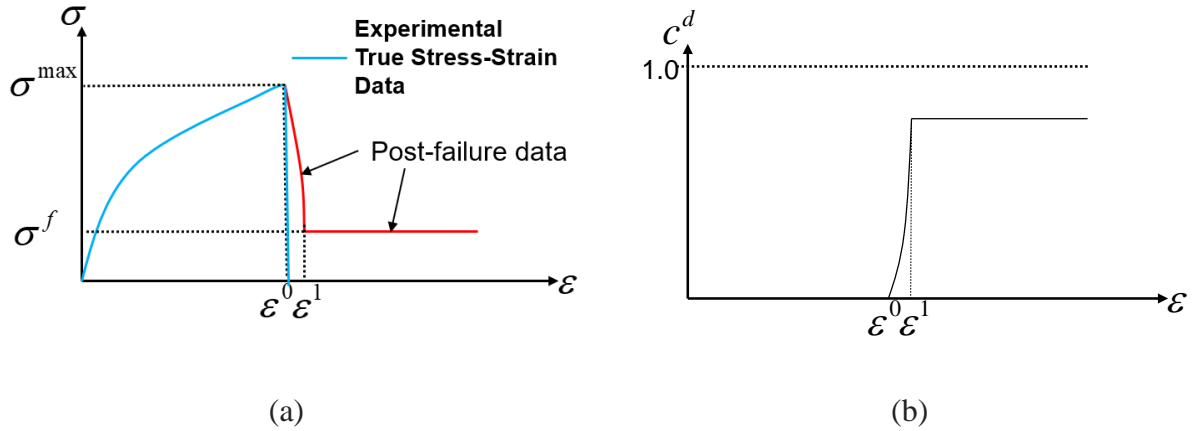
343 Each stress-strain curve obtained from principal material direction/plane testing needs to be

344 augmented to have a post-failure regime to handle the post-failure behavior via the damage sub-

345 model. It should be noted that handling the post-failure behavior correctly is crucial to obtaining a
346 numerically stable solution since a finite element cannot be eroded (deleted) the instant peak stress
347 is reached in the principal material direction/plane component. Fig. 6(a) shows a typical stress-
348 strain input curve with a brittle failure where the post-failure data is added to experimentally
349 obtained pre-peak data. The strain corresponding to peak stress is denoted ε^0 , and ε^1 is the strain
350 at the beginning of the residual strength regime in the post-failure region. Residual strength is
351 expressed as a fraction (c^{RS}) of the peak stress that the material has when the element is finally
352 eroded, i.e. $\sigma^f = c^{RS} \sigma^{\max}$, $0 \leq c^{RS} \leq 1.0$ (σ^f is the residual strength). There are a total of 9 residual
353 strength parameters: $c_i^{RS}, i=1, \dots, 6$ for the normal tensile (1-3)/compressive (4-6) stresses in the
354 PMDs, $c_i^{RS}, i=7, 8, 9$ for the three principal shear components. For the deformation sub-model to
355 carry out plasticity-related computations, the input stress in the effective stress space cannot have
356 a negative slope. To ensure that this condition is satisfied, an effective damage parameter curve is
357 used as input (Fig. 6(b)). This effective damage parameter data is computed for the respective
358 component by keeping the effective stress constant and equal to the peak stress, σ_{\max} .
359 Implementation details included a detailed algorithm can be found in our earlier publication
360 [Shyamsunder et al., 2020b].

361

362

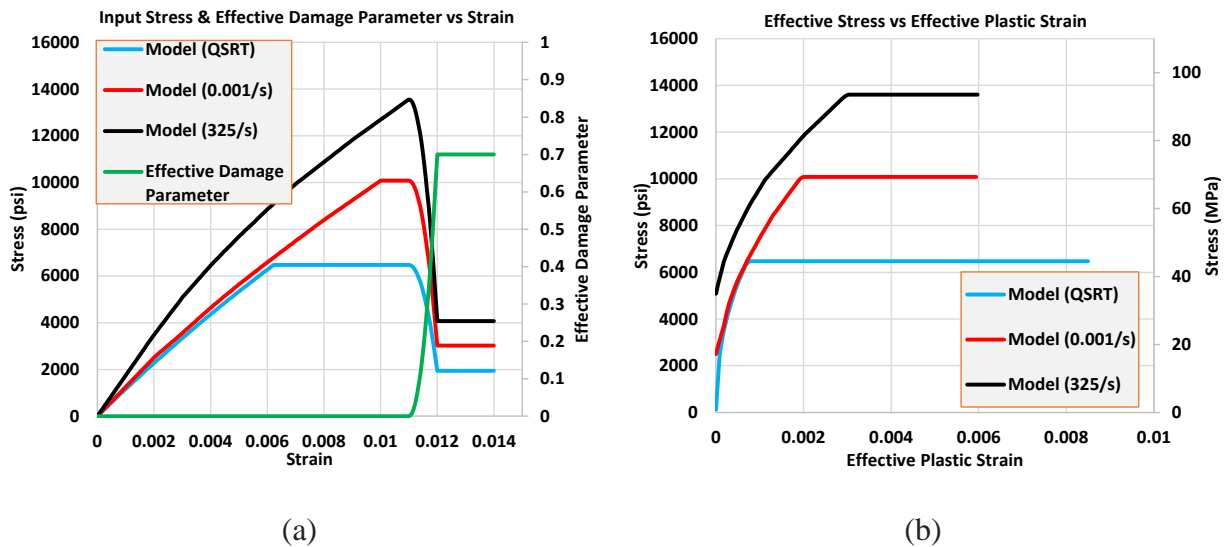


363 **Figure 6. (a) Stress-strain input augmented with post-failure data (b) Effective damage**
 364 **parameter input for handling post-failure behavior [Shyamsunder et al., 2020b]**

365

366 Fig. 7(a) shows the post-failure augmented stress-strain curves for 2-direction tension. The figure
 367 also shows the 2-direction tension effective damage parameter curve (see secondary axis). In the
 368 current implementation, for each of the twelve stress-strain components only a single effective
 369 damage parameter curve can be used. This implies that for a given component, the effective
 370 damage parameter curve must be compatible with all the stress-strain curves. If there is no
 371 compatibility, there would be a numerical issue - plastic strain computed in the pre-processing step
 372 will turn out to be negative. The compatibility is maintained by making sure that all the stress-
 373 strain curves (3 in this case) have the same ultimate strain values. As can be seen in Fig. 1(a)/Fig.
 374 7(a), *Model (325/s)* has the highest ultimate strain. Hence, the *Model (QS-RT)* and *Model (0.001/s)*
 375 are modified to have an additional data point with a stress value equal to the respective peak stress
 376 and a strain value equal to the ultimate strain of the *Model (325/s)* curve. The stress is gradually
 377 brought down to a residual strength of 30% of the respective peak stresses for all the 3 curves. The
 378 residual strength of 30% is a calibrated value obtained from the impact validation tests and is
 379 explained further in the numerical results section. Since the plasticity algorithm requires a non-

380 negative slope in the effective stress-effective plastic strain curve, the effective stress is kept
381 constant as shown in Fig. 7(b).



382 **Figure 7. 2-direction tension data for T800/F3900 composite (a) Input stress and effective**
383 **damage parameter versus strain curves (b) Effective stress versus effective plastic strain**
384 **curves constructed from data in (a)**

385

386 3. Impact Test Setup and Protocol

387 Data for validating the model was obtained by conducting impact tests on flat composite panels
388 according to the ASTM D8101 standard test method [ASTM, 2018] titled “Standard Test Method
389 for Measuring the Penetration Resistance of Composite Materials to Impact by a Blunt Projectile”.

390 This test method is designed for measuring the response of composite materials under free-flight
391 impact conditions when impacted in a normal direction. The test method utilizes a flat composite
392 panel specimen whose thickness is small compared to its lateral dimensions. The specimen is
393 clamped with 28 bolts between two thick metal plates with a 10-inch (25.4 cm) diameter circular
394 aperture, as shown in Fig. 8(a). The test utilizes a cup-shaped aluminum 2024 projectile with a
395 complex radiused nose and a mass of 1.77 oz (50 g). A cross-section of the projectile is shown in

404 volume of 1900 cubic inches. Fourteen tests were conducted at velocities that spanned the range
405 from no detectable non-destructive inspection (NDI) damage to full penetration by the projectile.
406 The test articles were Toray® T800/F3900 laminated angle-ply composites with a 16-ply
407 $[(0/90/45/-45)_2]_S$ layup.

408

409 Pairs of high-speed cameras and digital image correlation (DIC) software were used to compute
410 the full-field displacements on both the front (impacted side) and rear of the specimen.
411 Additionally, two pairs of cameras were located forward and behind the test specimen to measure
412 the projectile velocity and orientation (pitch and yaw) before and after impact using
413 photogrammetry software. The projectile orientation was specified by a set of three Euler angles
414 (roll, pitch and yaw) about a local coordinate system fixed to the projectile. The coordinate system,
415 in the un-rotated state, had an x-direction in the direction of the gun barrel, a y-direction
416 horizontally to the right when looking at the front of the specimen, and a z-direction vertically
417 downward. The roll angle was not reported due to the symmetrical nature of the projectile.

418

419 The front side DIC cameras operated at a rate of 40,000 frames per second (fps) with a resolution
420 of 384x320 pixels per inch in horizontal and vertical direction respectively, and the rear side DIC
421 cameras operated a rate of 80,000 fps with a resolution of 512x424 pixels per inch in horizontal
422 and vertical directions. Both pairs of photogrammetry cameras operated at a rate of 20,000 fps
423 with a resolution of 608x200 pixels per inch in the horizontal and vertical directions. Due to the
424 tumbling of the projectile after impact, only the absolute values of post-penetration or rebound
425 velocities are reported.

426

427 For the purpose of validating the damage and failure components of the MAT_213 model, only
 428 the four highest velocity tests from the series were used. Table 1 lists the test conditions.

429 **Table 1. Validation impact test conditions**

Test ID	Specimen	Projectile					Result
	Panel Thickness in inches (cm)	Mass in grams	Impact Velocity in ft/s (m/s)	Post-Impact Velocity in ft/s (m/s)	Pitch Angle in degrees	Yaw Angle in degrees	
LVG1075	0.122 (0.3)	49.65	385.0 (117.3)	-46.4 (-14.14)	0.08	-1.4	Rebound
LVG1074	0.122 (0.3)	49.96	417.0 (127.1)	25.4 (7.7)	0.38	-2.7	Penetration
LVG1076	0.122 (0.3)	49.55	453.3 (138.2)	114.3 (34.8)	-0.69	-1.2	Penetration
LVG1063	0.122 (0.3)	49.84	534.8 (163)	262.9 (80.1)	0.45	-0.4	Penetration

430
 431 *Computation of Strains and Strain Rates:* The tests considered in this study involved impact
 432 velocities near to or above the threshold penetration velocity for the test specimens. As a result,
 433 there was considerable damage or penetration in all cases, causing a loss of the paint speckle
 434 pattern used for computing deformations in the region of the impact location. Therefore,
 435 deformations and strains could not be computed at the impact point soon after impact. However,
 436 it was evident that, for the short period when they could be computed, deformations, strains and
 437 strain rates were highest at the impact location itself.

438
 439 **4. Numerical Results**

440 The developed constitutive model is used to model the four tests discussed in the earlier section.
 441 Two major components of the FE models used are described in detail in this section – (a) the

442 material data for various materials used in describing the various parts of the model, and (b) the
443 finite element types and mesh, and boundary conditions.

444

445 *Material Data:* Two material models were used – MAT_213 for the composite panel and
446 MAT_024 for the aluminum projectile. MAT_213 requires at least 12 QS-RT stress-strain curves
447 [Khaled et al., 2017]. Two sets of simulations were carried out – (1) rate-independent cases which
448 uses only the aforementioned 12 QS-RT stress-strain curves (2) rate-dependent cases which uses
449 in addition to the QS-RT curves, the following stress-strain curves at room temperature: (a) 2-
450 direction tension at $10^{-3}/s$ and $325/s$, (b) 2-direction compression at $10^{-3}/s$, $1/s$ and $813/s$, and (c)
451 3-direction compression at $800/s$ [Deshpande et al., 2019]. $FILT=0.0$ in Eq. 7 since no numerical
452 instabilities were detected. A numerical calibration process was used to estimate the decay
453 constants β_{ij} (Eqns. 8 and 9) and a uniform value of 0.001 was found to be a suitable value for
454 modeling the short-duration impact events considered in the paper.

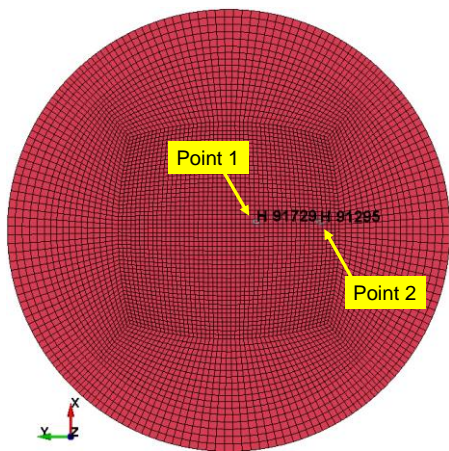
455 *FE Model:* A detailed mesh convergence study was carried out to find an optimal mesh with a
456 view to satisfactorily balance accuracy and computational time [Achstetter et al., 2020]. The
457 element aspect ratio, number of elements in the through-thickness direction, hourglass control, and
458 the element integration order were varied to gage the effect of each parameter on the maximum
459 out-of-plane displacement and the time at which the maximum displacement occurred. These two
460 computed values were compared against the experimental results. Based on the study, the
461 following FE model parameters were selected for use in this paper: (a) 8-node reduced integration
462 hexahedral elements were used to model the panel and the projectile, (b) each ply in the panel was
463 modeled using one element through the thickness, and (c) while the results were not sensitive to
464 different hourglass control, viscous hourglass control (IHQ=1) led to low hourglass energy values

465 and hence was used. The optimal mesh used in the simulations has a total of 91,728 elements to
466 model the panel, and 17,040 elements to model the projectile. It should be noted that solid elements
467 are appropriate for modeling this structure since the small diameter of the impactor compared to
468 the total thickness of the target plate could cause out-of-plane shear failure which cannot be
469 modeled with shell elements. Two points marked on the back side of the panel (Fig. 10) were used
470 for the comparison of strain-rates with the experimental data. These points are chosen since
471 experimental data were available at these points.

472

473 The nodes at the location of the bolts are restrained in-plane (Fig. 11(a)), and the nodes at the
474 clamps were restrained in the out-of-plane direction (Fig. 11(b)). Eroding single surface
475 (*CONTACT_ERODING_SINGLE_SURFACE) and eroding surface-to-surface
476 (*CONTACT_ERODING_SURFACE_TO_SURFACE) control cards were used. The numerical
477 examples use the deformation, damage and the failure sub-models in MAT_213.

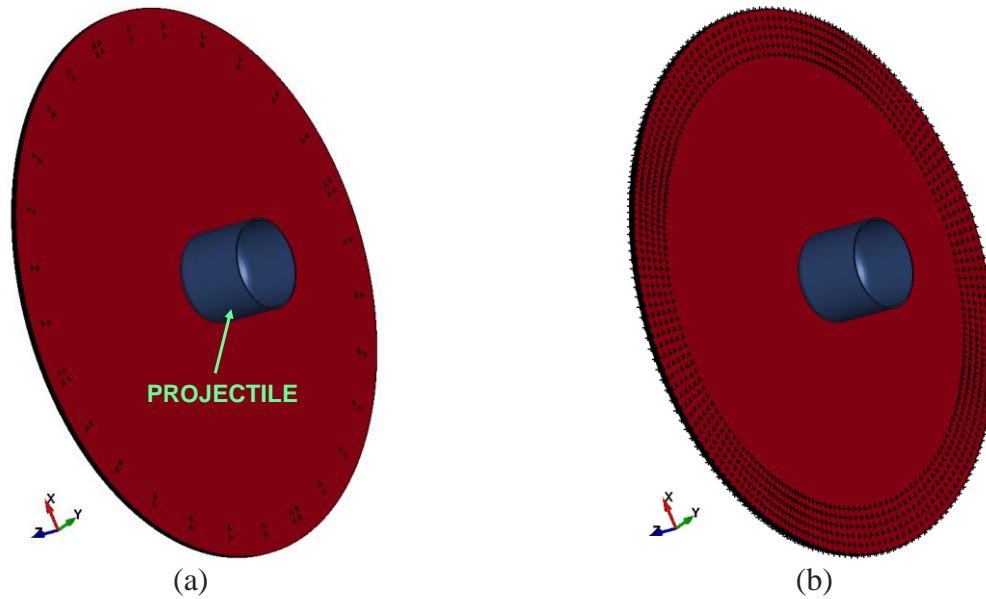
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479

480 **Figure 10. FE model used for the simulation**

481



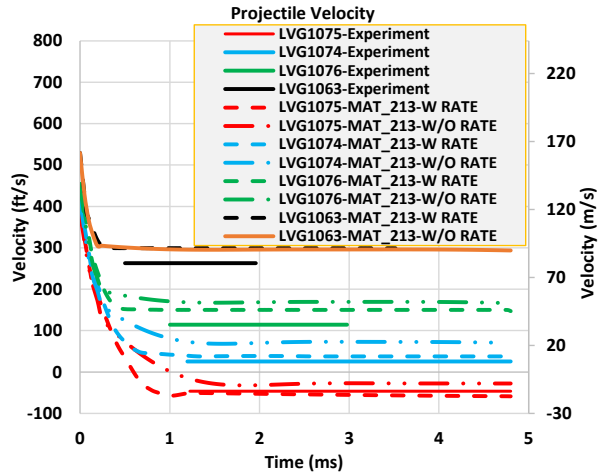
482 **Figure 11. Nodes which have translational restraints: (a) in-plane displacement (b) out-of-**
 483 **plane displacement**

484 *Calibration of the Residual Strength Values:* While all other input to MAT_213 were obtained
 485 from experimental data, the residual strength parameters need to be calibrated since these
 486 parameters are not directly linked to any experimental data. To aid in the calibration process, a
 487 sensitivity analysis was carried out with three sets of failure parameters that are not directly related
 488 to experimentally obtained data. The in-plane residual strength ($c_1^{RS} = c_2^{RS} = c_7^{RS}$), out-of-plane
 489 residual strength ($c_3^{RS} = c_8^{RS} = c_9^{RS}$) and equivalent failure strain ($\varepsilon_{IP}^{eq} = \varepsilon_{OOP}^{eq}$) were varied, and the
 490 exit/rebound projectile velocities were monitored. It was observed that the out-of-plane residual
 491 strength values most significantly affect the projectile exit/rebound velocity. As we will show later,
 492 this is also evident from Fig. 14 where it can be seen that the out-of-plane failure state values are
 493 higher than the corresponding in-plane values. A projectile impacting a plate perpendicularly is
 494 likely to show that the through-thickness failure is the dominant mode of failure. Thus, increasing
 495 the out-of-plane residual strength would make the panel stiffer and stronger, and thereby decrease
 496 the exit velocity or increase the rebound velocity. While the calibration exercise could have been

497 done with all the four test cases, the lowest velocity uncontained test (LVG1074) was chosen as
 498 this is the test with the impact velocity that is closest to V_0 , the threshold penetration velocity. In
 499 the absence of experimental data, it was assumed that the two equivalent failure strains are equal
 500 ($\varepsilon_{IP^{FAIL}}^{eq} = \varepsilon_{OOP^{FAIL}}^{eq} = 0.80$) and are a constant function of the failure angle. The interaction term, n was
 501 taken as 2 to couple the in-plane and the out-of-plane states of stress that are expected in an impact
 502 event. All the compression residual strength parameters were set equal to the respective peaks in
 503 the stress-strain input curves, $c_i^{RS} = 1.0, i = 4, 5, 6$, and the in-plane values as 30% of the peak values:
 504 $c_1^{RS} = c_2^{RS} = c_7^{RS} = 0.3$. Hence, with the objective of matching the FE simulation final (rebound)
 505 velocity to the experimental value, the only parameters that were obtained through a simple trial-
 506 and-error procedure were the out-of-plane components that were collectively (this was taken as a
 507 one-parameter calibration exercise) found to be 15%: $c_3^{RS} = c_8^{RS} = c_9^{RS} = 0.15$.

508 *Results:* The projectile exit/rebound velocity are shown in Fig. 12. The rate-dependent simulations
 509 and the rate-independent simulations are represented by the suffixes *MAT_213-W RATE* and
 510 *MAT_213-W/O RATE*, respectively. In this figure, a positive residual velocity implies a penetration
 511 of the projectile, and a negative residual velocity implies a rebound. The qualitative comparison
 512 of the crack patterns is shown in Fig. 13. It must be noted that the crack patterns shown are captured
 513 at the last frame from each simulation corresponding to the last point in Fig. 12. Table 2 shows the
 514 exit/rebound velocity error with respect to the experimental values. It also shows the normalized
 515 energy absorbed based on the kinetic energy and the difference with respect to the experimental
 516 values. Fig. 14 shows e_1 and e_2 contours on the back side of the panel. These images were captured
 517 at the same point of time for each of the test cases when e_1 and e_2 values were significantly large

518 before large through cracks were formed. The results shown in Figs. 13, 14, 15, 16, 17 and Table
 519 2 correspond to the rate-dependent simulations.

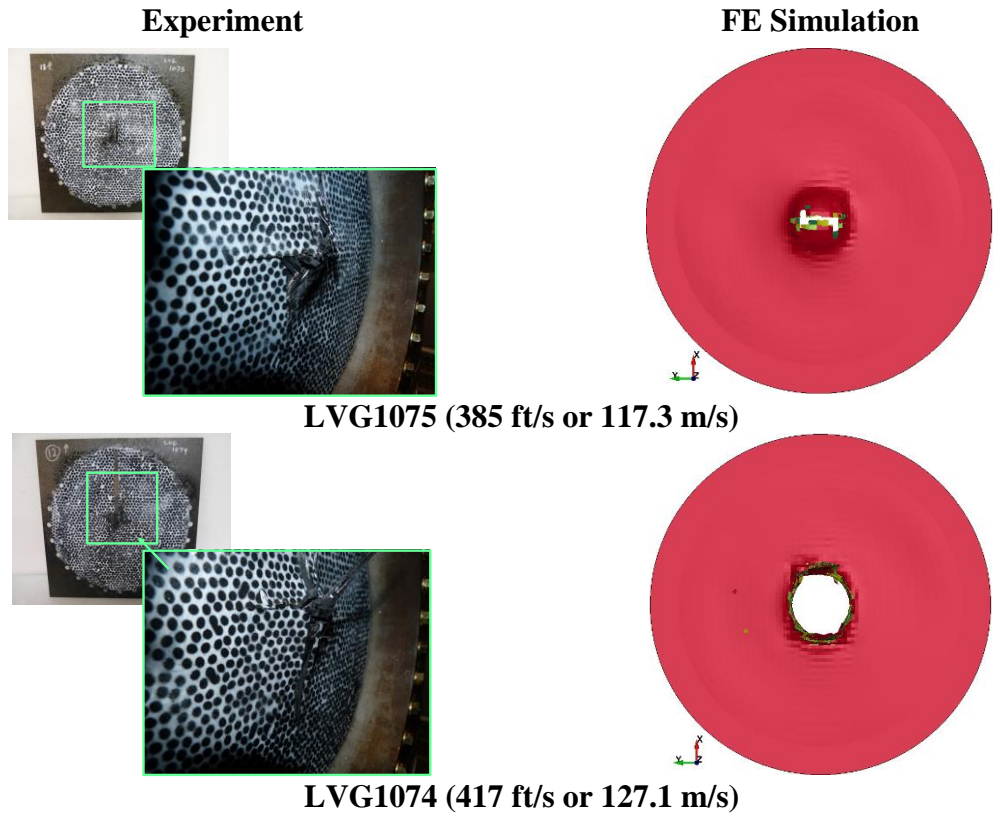


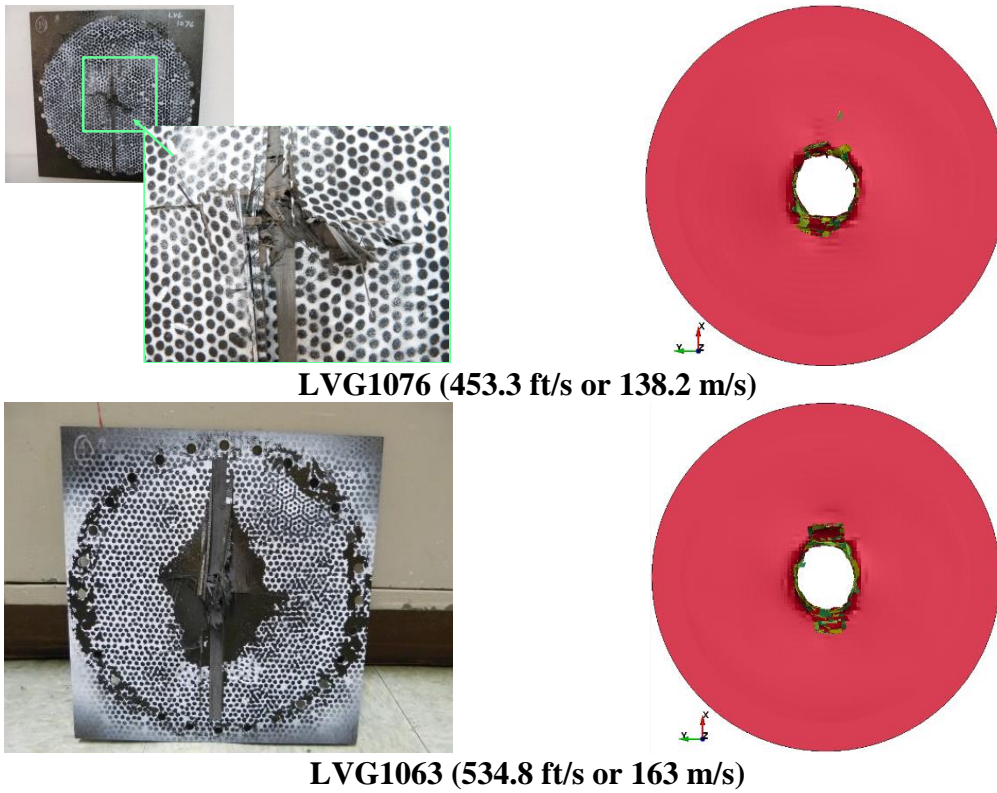
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521 **Figure 12. Comparison of projectile velocity for the impact tests**

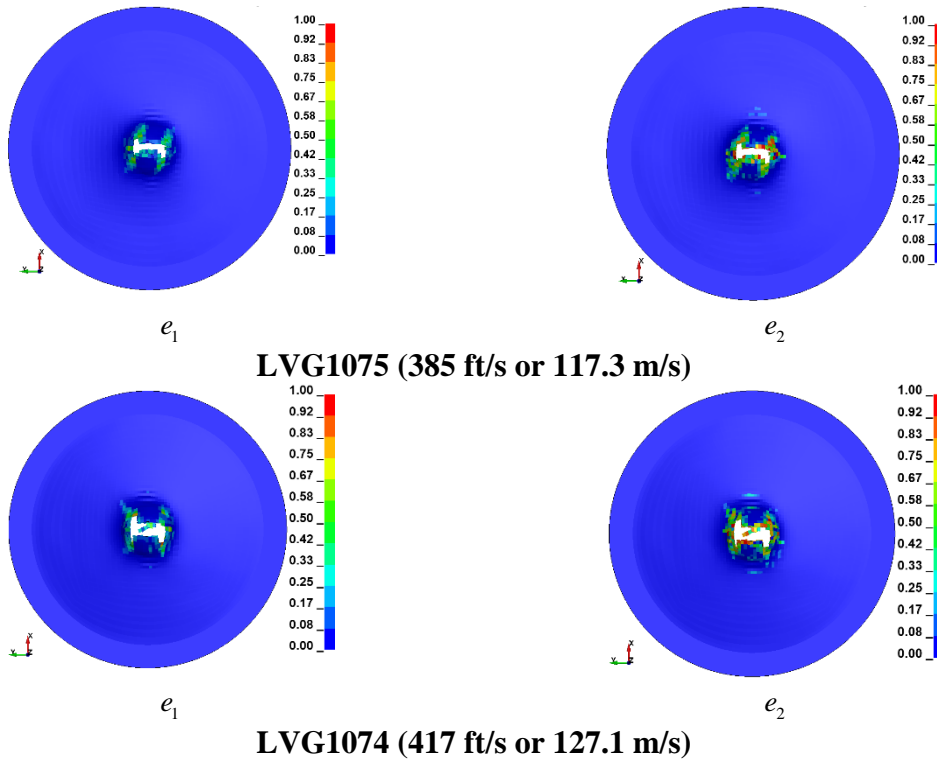
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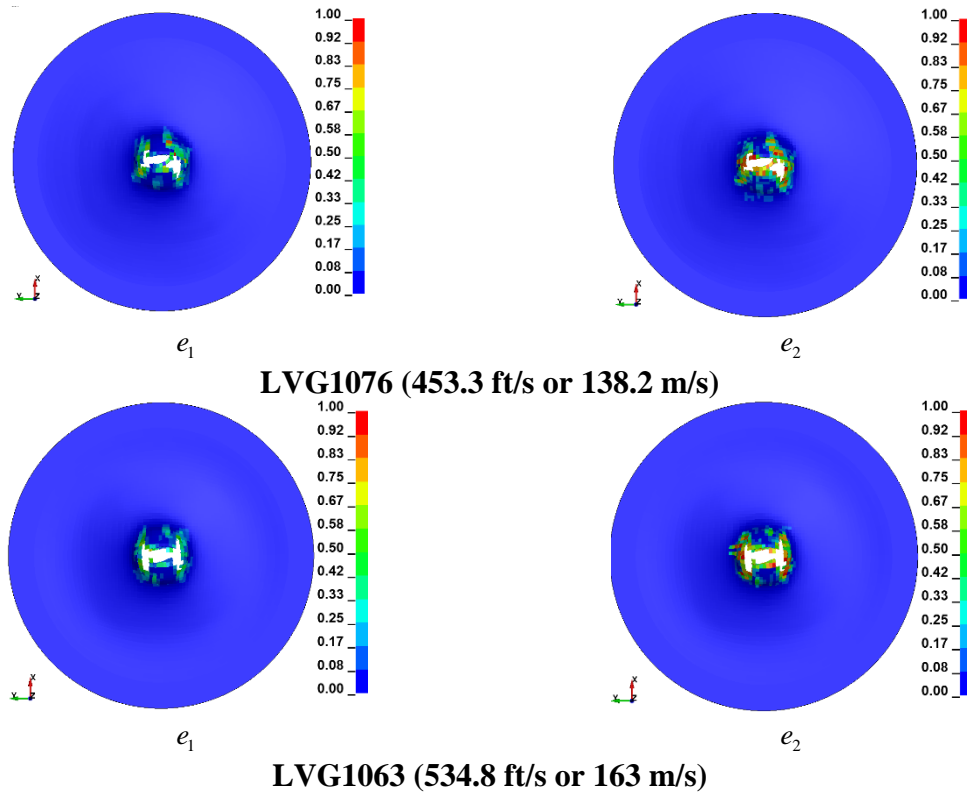
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524 **Figure 13. View of the back side of the tested composite panels and from last frame of the**
 525 **FE simulation**
 526
 527





528 **Figure 14. FE simulation generated in-plane e_1 and out-of-plane e_2 failure state contours on**
 529 **the back side of the panels**

530
 531 **Table 2. Impact validation tests: FEA vs Experiment comparison**

Test	Impact Velocity, V_{IMP} in ft/s (m/s)	Final Velocity, V_f in ft/s (m/s)			Normalized Absorbed Energy $\left(\frac{V_{IMP}^2 - V_f^2}{V_{IMP}^2} \right)$		
		Exp.	FEA	Diff. (%)	Exp.	FEA	Diff. (%)
				$\left(\frac{FEA - Exp}{Exp} \right)$			$FEA - Exp$
LVG1063	534.8 (163)	262.9 (80.1)	300.0 (91.4)	14	0.75	0.68	-7
LVG1076	453.3 (138.2)	114.2 (34.8)	150.0 (45.7)	31	0.94	0.89	-5

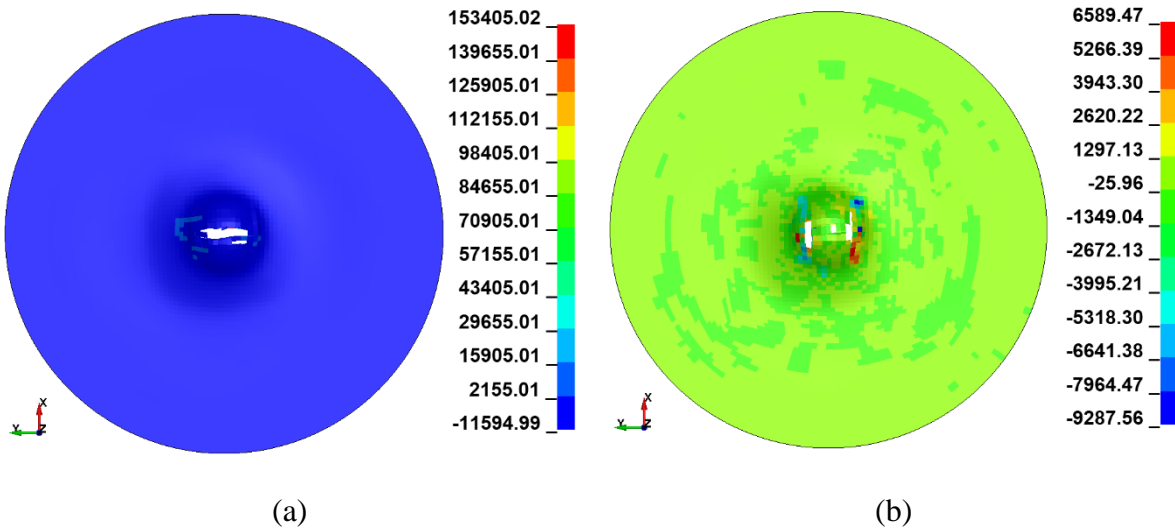
LVG1074	417 (127.1)	25.4 (7.7)	37.6 (11.5)	48	0.99	0.99	0
LVG1075	385 (117.3)	-46.4 (-14.14)	-58.8 (-17.9)	26	0.99	0.98	-1

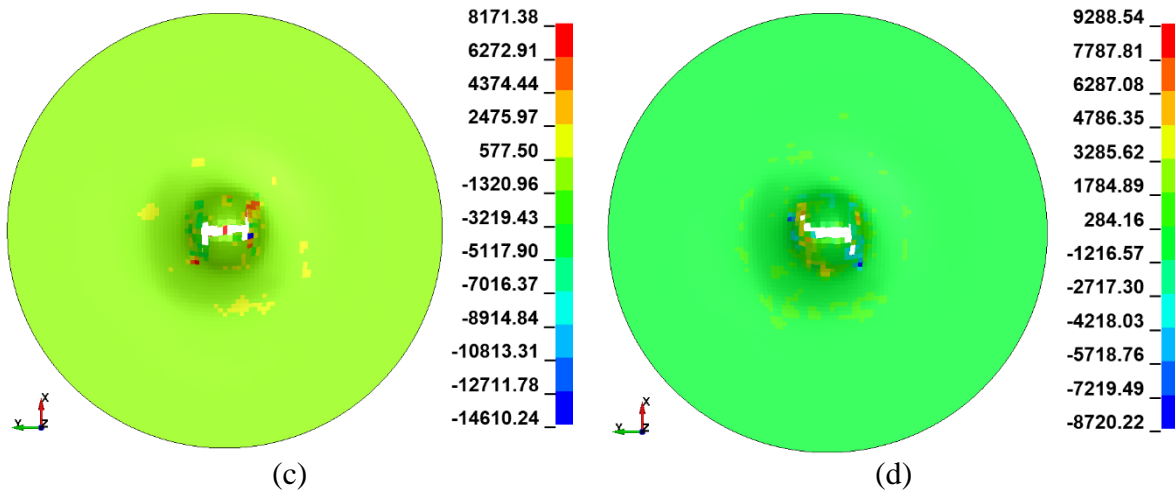
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533 *Discussion:* The presence of rate-dependent stress-strain curves make the panel response stiffer
534 compared to the rate-independent counterpart (Fig. 12) for all tests except LVG1063. In the
535 LVG1063 simulation, the strain-rates in the panel during the impact are much higher (e.g., $\sim 10^4/s$
536 in 2-direction tension/compression components) than the highest input rate-dependent stress-strain
537 curve. Overall, considering all the rate-dependent simulations, the prediction of the projectile
538 exit/rebound velocities are closer to the experimental results compared to the models where rate
539 data has not been used. With using just one-parameter calibration exercise with one model, the
540 results show consistent conservative predictions for all the four tests. Other than LVG1075, the
541 predicted crack patterns did not compare well to the experimental results. There are few major
542 factors affecting this behavior. First, the mesh size affects the crack pattern since in the failure sub-
543 model, there is no mesh regularization technique. Second, as discussed earlier, there are no rate
544 data available for the 3-direction tension component or the shear components. It is likely that these
545 behaviors are rate dependent. Fig. 15 shows the 1-3 shear strain rate for all the different plies at
546 the instant when the projectile makes contact with the panel for the LVG1075 test. The 1-3 shear
547 strain-rate magnitude is very high (in the order of 10^4) in some of the elements in the 0° ply. Fig.
548 16 shows the effective plastic strain rate contour at the same point of time as that the contours
549 shown in Fig. 15. The effective plastic strain rate magnitudes in some of the elements (mostly at
550 the center) are also in the order of 10^4 which is much higher than the input that are of the order of

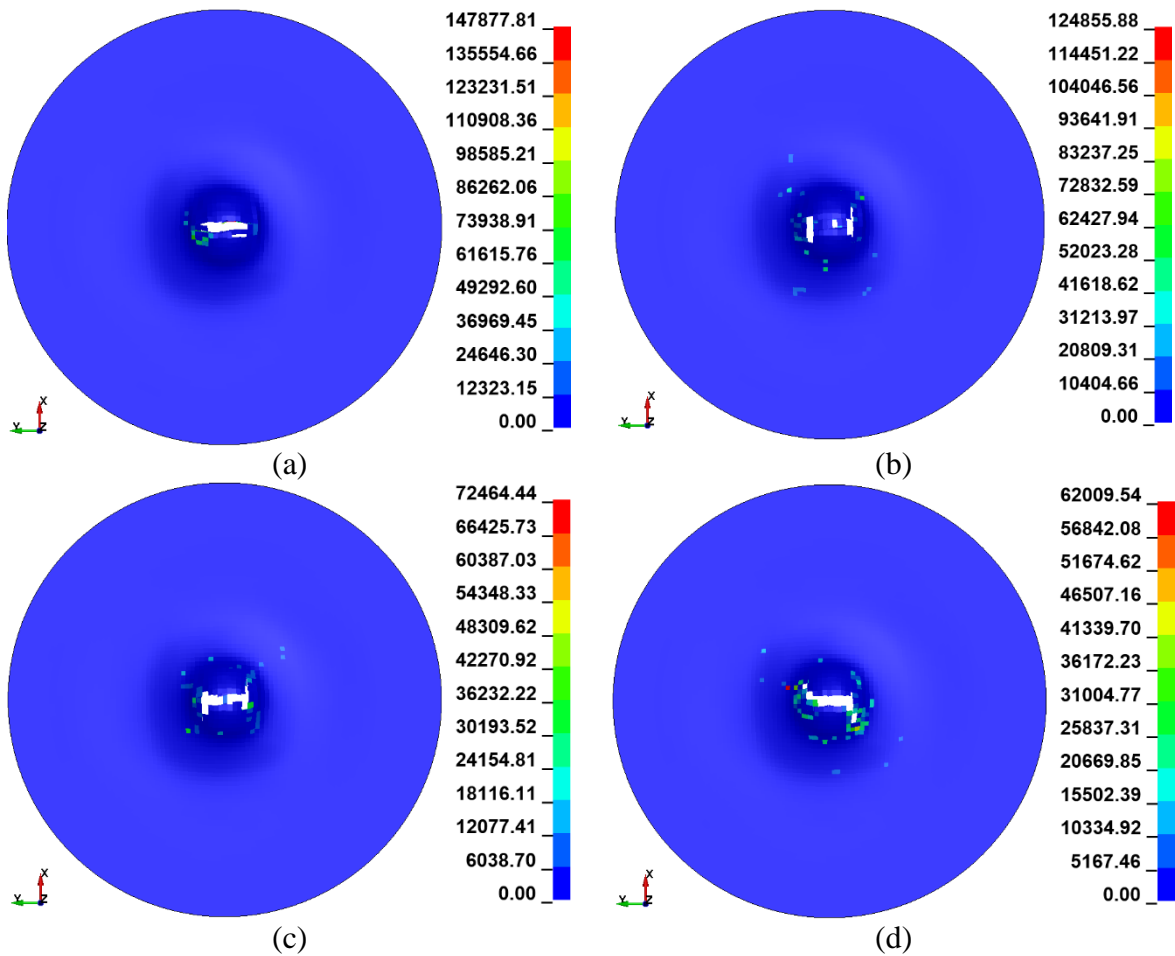
551 10^2 . Having additional rate dependent input stress-strain curve would probably help improve the
552 predictions. Third, the boundary conditions at and around the bolt holes play an important role. In
553 the current model, it appears as if they are not helping promote vertical cracks over the surface
554 area of the panel as seen in Fig. 13 for LVG1076 and LVG1063. This is also evident from Fig. 17
555 which shows the absolute values of the strain-rate in 1-direction component plotted against time
556 at Point 1 and Point 2 (see Fig. 10 for the location of these points). The strain is more localized at
557 the vicinity of the impact in the simulation compared to the experiment. At this stage, due to
558 absence of input stress-strain curves at different temperatures, it is not clear as to what effect the
559 additional temperature dependent stress-strain data would have on the accuracy of the predictions
560 since the increase in the temperature around the point of impact is likely to be small.

561

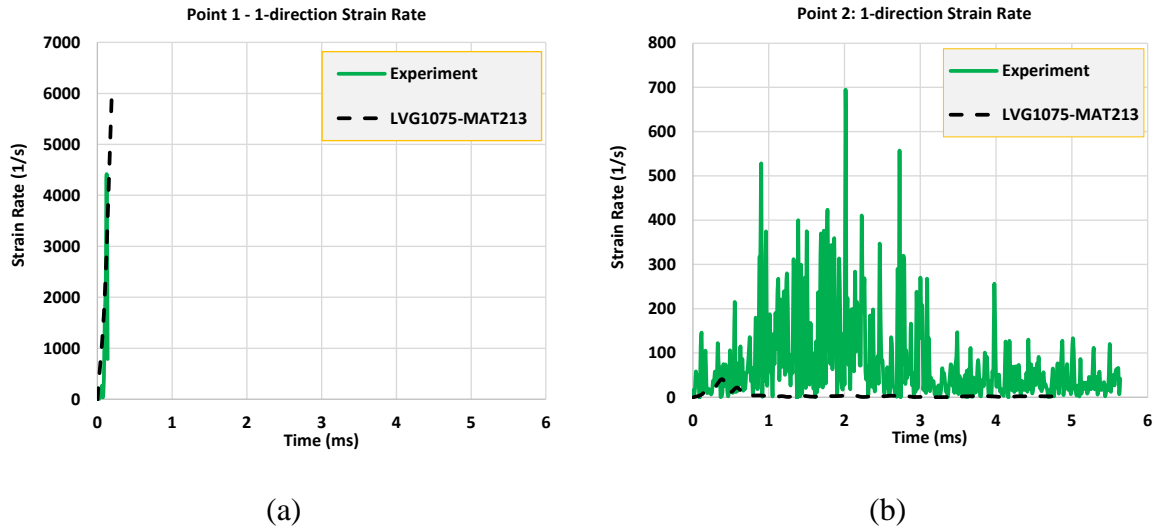




562 **Figure 15. Shear strain rate contour in the 1-3 plane from the LVG1075 simulation for (a)**
 563 **0° plies (b) 90° plies (c) 45° plies and (d) -45° plies (view shown from back side of panel)**
 564



565 **Figure 16. Effective plastic strain rate contour from the LVG1075 simulation for (a) 0°**
 566 **plies (b) 90° plies (c) 45° plies and (d) -45° plies (view shown from back side of panel)**
 567



568 **Figure 17. Strain-rate in 1-direction (a) at point 1 (elements around point 1 are eroded**
 569 **quickly after impact) (b) at point 2 (see Fig. 10 for the location of point 1 and point 2)**
 570

571 **5. Concluding Remarks**

572 Details of a constitutive model that supports orthotropic visco-elastic-visco-plastic material
 573 behavior with temperature and strain rate dependence are presented in this paper. The constitutive
 574 model is implemented in LS-DYNA as *MAT_213 and has three sub-models supporting
 575 computations related to elastic and inelastic deformations, unloading and reloading, and detection
 576 of failure with the ability to erode the element from further computations. The focus is on (a) the
 577 development of V EVP behavior with rate and temperature dependence, (b) the processing the rate
 578 dependent curves to allow physically consistent predictions from FE analysis, and (c) using the
 579 Generalized Tabulated Failure Criterion to obtain numerically consistent failure predictions. A
 580 rich set of experimentally obtained impact validation data has been obtained from well
 581 instrumented tests that yield high fidelity data of the projectile as well as the impacted panel.
 582

583 Tying a failure criterion to an explicit dynamics finite element analysis is difficult for a number of
584 reasons. First, is the challenge in detecting when an element has met the failure criterion. A
585 homogenized material model is typically deficient in its ability to differentiate between different
586 modes of composite failures since the architecture of the underlying composite is missing. Second,
587 most failure models have a large number of failure-related parameters, all of which are not easy to
588 obtain experimentally. The analyst must spend time not only understanding the role of these
589 parameters but also calibrating the values to obtain valid numerical responses. Third, each stress-
590 strain curve obtained from principal material direction/plane testing needs to be augmented to have
591 a post-failure regime so as to handle the post-failure behavior. This is required even if the
592 composite exhibits brittle and sudden failure. This post-failure behavior is crucial to obtaining a
593 numerically stable solution since a finite element cannot be eroded (deleted) the instant when the
594 failure criterion is satisfied.

595

596 Simulations of four ballistic impact test cases are presented. The difference between the predicted
597 normalized absorbed energy and the experimental data is less than 10% with excellent match for
598 the two lowest velocity tests (less than 1% difference). Consistently, the difference is conservative
599 with the difference increasing with increasing projectile velocity. The prediction of the crack
600 pattern of the panel could be improved. It should be noted that using cohesive zone elements
601 (CZEs) (LS-DYNA's MAT_186) to model inter-laminar failure based on our prior experimental
602 and modeling work [Khaled et al., 2019b] did not yield significant change in both the out-of-plane
603 displacement prediction as well as the crack pattern in the simulations discussed in this paper. As
604 stated earlier, it is likely that mesh size, the boundary conditions used, and lack of a complete suite

605 of rate-dependent stress-strain curves play a more important role. Efforts are underway to
606 investigate these issues.

607

608 **Acknowledgements**

609 Authors Shyamsunder, Khaled and Rajan gratefully acknowledge the support of the Federal
610 Aviation Administration through Grant #12-G-001 titled “Composite Material Model for Impact
611 Analysis” and #17-G-005 titled “Enhancing the Capabilities of MAT213 for Impact Analysis”,
612 William Emmerling and Dan Cordasco, Technical Monitors.

613

614 **Conflict of Interest**

615 None declared.

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Author Statement “Numerical Validation of Composite Panel Impact Tests”

Loukham Shyamsunder: Methodology, Software, Validation, Writing – Original Draft, Data Curation.

Bilal Khaled: Methodology, Software, Resources, Investigation, Data Curation.

Subramaniam D. Rajan: Methodology, Supervision, Project Administration, Funding Acquisition, Writing – Review and Editing.

J. Michael Pereira: Validation, Investigation, Data Curation, Writing – Review and Editing.

Paul DuBois: Conceptualization, Methodology, Writing – Review and Editing.

Gunther Blankenhorn: Software, Data Curation.