Evaluation of a Traditional Regression Model Term Selection Process for Balance Calibration Data Sets

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Key elements of a traditional regression model term selection process for the analysis of wind tunnel strain–gage balance calibration data are discussed. The traditional process is sometimes applied when regression models of the gage outputs are needed and an analyst does not have access to a term selection algorithm. In principle, the traditional process identifies supported regression model terms of a gage output by combining an analyst's subject–matter knowledge with a visual examination of selected plots of the calibration loads. Twelve empirical rules are listed that may be used to select regression model terms. Data from the manual calibration of a force balance is used to illustrate the application of the traditional term selection process. The data example shows that the traditional process can become very complex. Its successful application highly depends on an analyst's experience and skills. Therefore, it is suggested to replace the traditional term selection process with an automated term selection algorithm. Then, objective metrics are systematically applied during the term selection that make the identification of the regression model more repeatable and less dependent on an analyst's personal skills.

Nomenclature

AF	= axial of a force balance
$b_{i,0},\ b_{i,1},\ b_{i,2},\ \ldots$	= coefficients of the regression model of a gage output difference
D_i	= difference between the raw output and the natural zero of a balance gage
F_i	= balance load component with index i
i	= index of a load component, index of a gage output
j	= index of a load component, index of a gage output
k	= index of a load component
m	= total number of supported regressors or regression coefficients
n	= total number of independent load components or independent gage outputs
N_i	= natural zero output of a balance gage with index i
$N1, N2$	= forward and aft normal force components of a force balance
p	= total number of calibration data points
RM	= rolling moment of a force balance
rF_i	= raw output of a balance gage with index i
$S1, S2$	= forward and aft side force components of a force balance
T_ν	= auxiliary summation that is used to define the regression model of a gage output
μ_ν	= total number of terms of a regression model term group
ν	= regression model term group index
ξ	= regression coefficient index
φ	= data point index

I. Introduction

In general, calibration data of a wind tunnel strain–gage balance is analyzed by using multivariate regression models so that balance loads can be predicted from gage outputs during a wind tunnel test (see, e.g., Ref. [1] for more details). Unfortunately, the application of a multivariate regression model to balance

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calibration data can become very complex if the data does not support a set of preferred regression model terms. This situation is often caused by the fact that a calibration facility's equipment may not be able to apply all initially chosen load combinations that a preferred regression model needs. Therefore, the subset of regression model terms that are supported by the given calibration data must be identified. Otherwise, the regression analysis of the calibration data will not lead to reliable load prediction equations.

Different methods may be used to identify the subset of regression model terms supported by the given calibration data. The recommended approach for identifying the supported terms is based on a rigorous analysis of the linear dependencies between the chosen regression model terms. The goal of this analysis is to remove all regression model terms that cause linear or massive near-linear dependencies in the regression model. The author developed a term selection algorithm for strain–gage balance data that uses *Singular Value Decomposition* in combination with the *Variance Inflation Factor* and the *Hierarchy Rule* for the identification of the linearly independent subset of regression model terms that the given calibration data supports (see Ref. [2]). This algorithm was implemented in NASA's BALFIT data analysis tool. It is being used on a regular basis to automatically build regression models for the analysis of strain–gage balance calibration data sets (see Ref. [3]).

Not every data analysis tool has an algorithm that selects regression model terms. Therefore, many analysts still rely on a combination of subject–matter knowledge and past experience in order to choose regression model terms. Let us assume, for example, that an analyst wants to use the *Iterative Method* for the balance load prediction (see Ref. [1]). Then, regression models of the gage outputs are needed so that the load iteration equation can be generated. In this case, the analyst often starts the term selection process by plotting individual load components versus the data point index and comparing them side–by–side. A visual examination of the so–called "combined load plots" of the balance loads is also an element of the traditional term selection process. A combined load plot is a representation of the loads of all calibration data points in a two–dimensional Cartesian coordinate system. The plot is generated for all possible pairs of load components by plotting the load value of one component versus the load value of another component. Consequently, fifteen combined load plots are needed to describe the calibration data set of a typical six–component balance.

It is useful for an analyst to understand the relationship between a combined load plot and the supported regression model terms of a balance calibration data set even if the analyst has access to software that identifies supported regression model terms. Therefore, some basic concepts are described in this paper so that an analyst better understands key elements of the traditional term selection process.

II. Regression Model of Gage Output Differences

The traditional term selection process is typically used to assemble regression models of gage outputs that the *Iterative Method* needs for the analysis of balance calibration data. This method first fits the electrical outputs of balance calibration data as a function of the tare corrected loads. Afterwards, a load iteration equation is constructed from the regression models of the outputs so that loads can be predicted from outputs during a wind tunnel test (see, e.g., Ref. [1] for a detailed description of the method).

The supported regression models of the gage outputs have to be identified. Afterwards, the calibration data can be processed and the load iteration equation can be generated. It is critical for the successful regression analysis of calibration data of an n–component balance to describe both loads and electrical outputs of the balance in a suitable format before the regression models are defined. Ideally, balance loads $F_1,\ F_2,\ \ldots,\ F_n$ should be tare corrected. In other words, they should be corrected for the combined weight of the metric part of the balance and the calibration equipment so that all loads are described relative to the absolute load datum of zero load. Similarly, it is best to describe the electrical outputs of an n–component balance as output differences $D_1,\ D_2,\ \ldots,\ D_n$ between a raw output and the corresponding natural zero of the gage. This recommended output format can be defined by the equation below where D_i is the output

Definition of the Gage Output Difference

$$D_i \;=\; rF_i \;-\; N_i \quad ; \quad 1 \le i \le n \qquad (1)$$

difference, rF_i is the raw output of the balance gage, and N_i is the natural zero of the gage (natural zero $\equiv$ electrical output of a gage in an assumed "weightless" condition). A strain–gage balance experiences a series of finite load states during its calibration that can be described by using either the tare corrected balance loads or the measured gage output differences. These loads and output differences are the input data set for the regression analysis of the balance calibration data that results in the load prediction equations. Then, assuming that the calibration data set has a total of "p" data points, i.e., load states, the input data set for the regression analysis of the calibration data of an n–component balance can be summarized as follows:

$$D_i,\ F_i \quad \Longrightarrow \quad \underbrace{D_i(\varphi),\ F_i(\varphi) \quad where \quad 1 \leq i \leq n \quad and \quad 1 \leq \varphi \leq p}_{balance\ data\ set\ for\ the\ multivariate\ regression\ analysis}$$

Now, the regression model of the gage output difference of each balance gage can be defined. This regression model is summarized by the generic equation below. It includes all possible regression model

Regression Model of a Gage Output Difference

$$D_i \quad = \quad b_{i,0} \ + \ b_{i,1} \cdot F_1 \ + \ b_{i,2} \cdot F_2 \ + \ \ldots \quad = \quad b_{i,0} \ + \ \sum_{\nu=1}^{10} T_\nu \qquad (2)$$

terms that are used in the aerospace testing community to define the regression model of a gage output (the appendix of the paper has additional details). The symbol D_i represents the output difference of the gage with index i, $F_1,\ F_2,\ \ldots$ are the balance load components, $b_{i,0},\ b_{i,1},\ b_{i,2},\ \ldots$ are the regression coefficients, and T_ν are auxiliary summations that are defined in the appendix.

It is important for the reader to realize that not all terms listed in Eq. (2) above are typically supported by a given calibration data set. Therefore, either a term selection algorithm (e.g., the algorithm described in Ref. [2]) or the traditional term selection process has to be used to identify the subset of terms that the calibration data supports. The traditional term selection process often applies empirical "rules" during a "visual" examination of plots of the calibration loads that help identify supported terms. These "rules" are an important part of an analyst's subject–matter knowledge. Therefore, they are summarized in the next section. Afterwards, data from the manual calibration of a force balance is used to demonstrate how these "rules" are applied to real–world balance calibration data.

III. Empirical Term Selection Rules

The traditional term selection process uses a set of empirical rules that are applied during the examination of the combined load plots of the balance calibration data. Regression model terms are investigated one by one. A term is either selected or rejected depending on an analyst's subject–matter knowledge and the result of the "visual" examination of the balance load plots. The analyst's subject–matter knowledge, the application of empirical rules to the combined load plots, and the examination of other load plots are the traditional process' equivalent to the more rigorous numerical investigation of linear and near–linear dependencies between regression model terms that a term selection algorithm performs.

In theory, an analyst's subject–matter knowledge is the most important element of the traditional term selection process. Unfortunately, it cannot easily be described because it highly depends on an analyst's approach and past experiences. Nevertheless, subject–matter knowledge is the basis of all empirical rules that are described in this section. For example, it is known that data of single–piece balances does not need absolute value terms. On the other hand, outputs of TASK/ABLE balances are bi–directional. Therefore, their regression models need absolute value terms (see Ref. [4] for more details). It is also known that the cross–product terms $N1 \cdot AF$ and $N2 \cdot AF$ of force balance data can only simultaneously be used in the regression model of an output if the normal forces are separately applied at the forward and aft normal force gages while an axial force is acting. These examples show how subject–matter knowledge is often applied during the traditional term selection process.

In principle, the regression model of a gage output difference can consists of terms that are taken from ten term groups. The terms of each group are constructed from the balance loads (the paper's appendix provides more details). The ten term groups can be split into four groups that are associated with linear and quadratic terms, two groups that are associated with cubic terms, and four groups of cross–product terms. First, empirical rules for the four linear and quadratic term groups are discussed. The generic terms of these groups can be described as follows where the symbol F_i repre–

$$\textit{four linear \& quadratic term groups} \implies F_i \;\; ; \;\; |F_i| \;\; ; \;\; F_i^2 \;\; ; \;\; F_i \cdot |F_i|$$

sents a single balance load component. The following empirical rules may be used during the selection of terms from the four linear and quadratic term groups:

Rule 1 $\implies$ The term F_i is supported if either positive or negative values of the load component (or both) were applied during the calibration.

Rule 2 $\implies$ The term $|F_i|$ is supported if (i) both positive and negative values of the load component were applied and (ii) the related primary gage output rF_i is bi–directional.

Rule 3 $\implies$ The term F_i^2 is supported if (i) either positive or negative values of the load component (or both) were applied during the calibration and (ii) at least three load values exists between 0.0 % and $\pm$100 % of load capacity.

Rule 4 $\implies$ The term $F_i \cdot |F_i|$ is supported if (i) both positive and negative values of the load component were applied during the calibration, (ii) the related primary gage output rF_i is bi–directional, and (iii) at least three load values exists between 0.0 % and $\pm$100 % of load capacity.

In the next step, empirical rules for the two cubic terms are described. These terms are defined by the two expressions that are listed below.

$$\textit{two cubic term groups} \implies F_i^3 \;\; ; \;\; |F_i^3|$$

Rule 5 $\implies$ The term F_i^3 is supported if (i) either positive or negative values of the load component (or both) were applied during the calibration and (ii) at least three load values exist between 0.0 % and $\pm$100 % of load capacity.

Rule 6 $\implies$ The term $|F_i^3|$ is supported if (i) both positive and negative values of the load component were applied, (ii) the related primary gage output rF_i is bi–directional, and (iii) at least three load values exist between 0.0 % and $\pm$100 % of load capacity.

It is the author's experience that cubic terms should be <u>avoided</u> in regression models of the outputs. Their use will most likely cause near–linear dependencies. In addition, they could lead to unreliable load predictions if the given calibration data set is very sparse. It must be pointed out that the combined load plots for linear, quadratic, and cubic terms should show data points on or close to the two coordinate system axes whenever the potential use of a single load component F_i is being investigated.

The situation becomes more complex as soon as cross–product terms, i.e., the products of two different load components, are assessed. These terms are defined by the following four generic expressions. The symbols F_i and F_j describe the values of a pair of load components that defines the two axes of the combined

$$\textit{four cross}-\textit{product term groups} \implies F_i \cdot F_j \;\; ; \;\; |F_i \cdot F_j| \;\; ; \;\; F_i \cdot |F_j| \;\; ; \;\; |F_i| \cdot F_j$$

load plot. Then, the following empirical rules for the selection of cross–product terms of the load pair $\{F_i, F_j\}$ can be defined:

Rule 7 $\implies$ A data point of the load pair $\{F_i, F_j\}$ is assumed to be located <u>inside</u> of a quadrant of the related combined load plot if the absolute value of load F_i and the absolute value of load F_j <u>simultaneously</u> exceed 20 % of load capacity.

Rule 8 $\implies$ Cross–product terms obtained from the load pair $\{F_i, F_j\}$ <u>cannot</u> be used if data points <u>cannot</u> be found <u>inside</u> of the four quadrants of the combined load plot of the load pair.

Rule 9 $\Longrightarrow$ The cross–product term $F_i \cdot F_j$ is most likely supported if an analyst can find data points inside of at least one of the four quadrants of the combined load plot of the load pair.

Rule 10 $\Longrightarrow$ The cross–product term $|F_i \cdot F_j|$ is most likely supported, if (i) data points can be found inside of all four quadrants of the combined load plot and (ii) the related gage outputs rF_i and rF_j are bi–directional.

Rule 11 $\Longrightarrow$ The cross–product term $F_i \cdot |F_j|$, is most likely supported if data points can be found in (i) the first & third quadrants or the second & fourth quadrants or the first & fourth quadrants or the second & third quadrants and (ii) the related gage output rF_j is bi–directional.

Rule 12 $\Longrightarrow$ The cross–product term $|F_i| \cdot F_j$, is most likely supported if data points can be found in (i) the first & second quadrants or the third & fourth quadrants and (ii) the related gage output rF_i is bi–directional.

It is important to mention that **Rule 10**, **Rule 11**, and **Rule 12** were independently verified by using subsets of the machine calibration data of a six–component force balance. The listed twelve rules are by no means complete. They are simply a reflection of the author's personal observations and conclusions. Other analysts may use similar or alternate rules that are based on their own experiences.

The "visual" examination of the combined load plots is "helpful" but not "sufficient" when it comes to using the traditional term selection process for the identification of a reliable regression model of a gage output difference. In most cases, additional subject–matter knowledge must be applied in order to gain confidence in the chosen regression model. – A balance calibration data example is discussed in the next section in order (i) to illustrate the application of the traditional term selection process and (ii) to demonstrate complexities that could surface during the selection of cross–product terms.

IV. Balance Calibration Data Example

Data from a manual calibration of NASA's MK4 force balance is used in this section to illustrate the application of the traditional term selection process. The MK4 balance is a six–component force balance that was manufactured by the *TASK/ABLE Corporation*. It has a diameter of 4.0 inches and measures five forces and one moment ($N1$, $N2$, $S1$, $S2$, AF, RM). Table 1 below lists capacities of its load components.

Table 1: Load capacities of the MK4 force balance (*TASK/ABLE* design).

$N1,\ lbs$	$N2,\ lbs$	$S1,\ lbs$	$S2,\ lbs$	$AF,\ lbs$	$RM,\ in{-}lbs$
6000	6000	3000	3000	1000	16000

The balance is a typical TASK/ABLE design. Therefore, the normal and side force gages of the balance are known to be bi–directional. Consequently, absolute value terms of the normal and side forces are potentially supported in the regression models of the gage outputs.

A manual calibration data set of the MK4 balance was obtained in June 2016 in the manual calibration rig of *CALSPAN Force Measurement Systems*. The final calibration data set had a total of 615 data points that were distributed across 48 load series. Between one to three load components were simultaneously applied during the calibration. Calibration hardware and load rig limitations did not allow for the application of all desired load combinations. NASA's BALFIT software package was used to perform an initial analysis of the calibration data set so that key characteristics of the data and the balance could be understood (see Ref. [3] for details about the BALFIT software).

All fifteen possible combined load plots of the MK4 balance were generated during the initial analysis of the data. A subset of five plots is used in this section in order to discuss the traditional term selection process. Figure 1 below shows the combined load plot of the forward normal and side force. Gray regions can be seen in the plot that are defined by a load threshold of ± 20 % of capacity. BALFIT's term selection algorithm uses these regions for the detection of points that are located inside of a quadrant. Any data point located outside of the gray regions is interpreted as a point that is located inside of a quadrant.

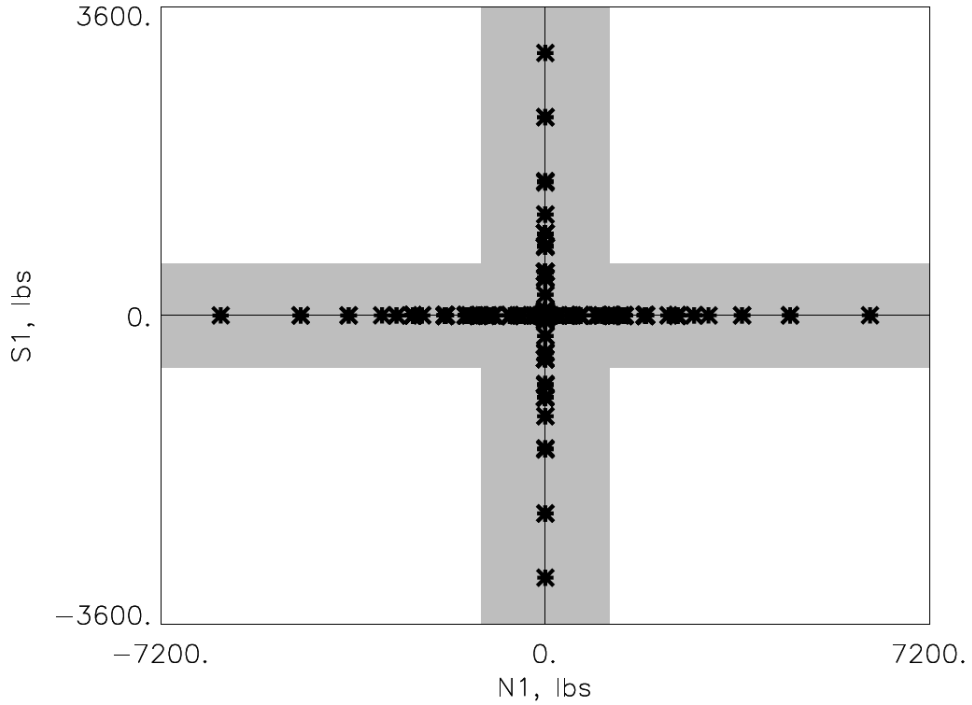


Fig. 1 Combined load plot of the forward side force ($S1$) versus the forward normal force ($N1$).

It can be seen that positive and negative values are located on both coordinate system axes. Therefore, after using **Rule 1** and **Rule 3** of the previous section, it is likely that the given data set supports the following linear and quadratic terms (cubic terms are ignored):

$$\textit{most likely supported} \quad \Longrightarrow \quad N1 \;\; ; \;\; S1 \;\; ; \;\; N1^{\,2} \;\; ; \;\; S1^{\,2}$$

It is also known that the normal and side force gage outputs of the balance are bi–directional. Therefore, after using **Rule 2** and **Rule 4** of the previous section, the following related linear and quadratic terms with absolute values are most likely supported:

$$\textit{most likely supported} \quad \Longrightarrow \quad |N1| \;\; ; \;\; |S1| \;\; ; \;\; N1 \cdot |N1| \;\; ; \;\; S1 \cdot |S1|$$

Furthermore, it is observed in Fig. 1 above that no data points can be found inside the four quadrants. Therefore, after using **Rule 7** and **Rule 8** of the previous section, the calibration data set cannot support the following four cross–product terms:

$$\textit{not supported} \quad \Longrightarrow \quad N1 \cdot S1 \;\; ; \;\; |N1 \cdot S1| \;\; ; \;\; N1 \cdot |S1| \;\; ; \;\; |N1| \cdot S1$$

Figure 2 below shows the combined load plot of the forward normal force ($N1$) and the aft normal force ($N2$) that was obtained from the calibration data.

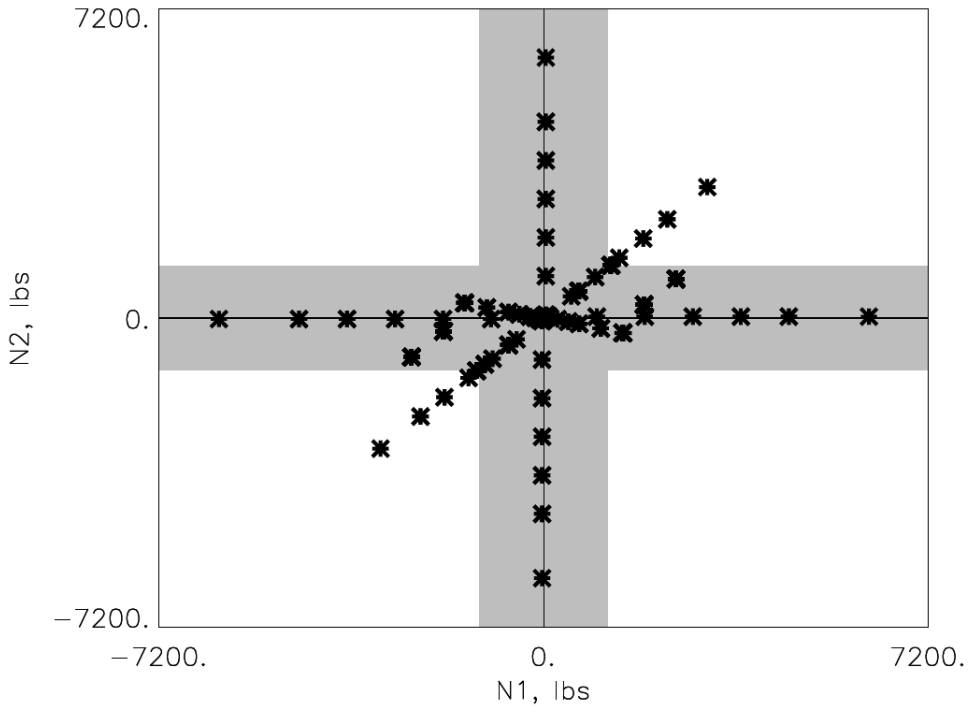


Fig. 2 Combined load plot of the aft normal force ($N2$) versus the forward normal force ($N1$).

It can be seen that both positive and negative values are located on both axes. Therefore, after using **Rule 1** and **Rule 3** of the previous section, it is likely that the given data set supports the following linear and quadratic terms (again, cubic terms are ignored):

$$\textit{most likely supported} \quad \Longrightarrow \quad N1 \;\; ; \;\; N2 \;\; ; \;\; N1^2 \;\; ; \;\; N2^2$$

It is known that the forward and aft normal force gage outputs of the balance are bi–directional. Therefore, after using **Rule 2** and **Rule 4** of the previous section, the following linear and quadratic terms with absolute values are most likely supported:

$$\textit{most likely supported} \quad \Longrightarrow \quad |N1| \;\; ; \;\; |N2| \;\; ; \;\; N1 \cdot |N1| \;\; ; \;\; N2 \cdot |N2|$$

It is also observed in Fig. 2 above that data points can be found inside the first and third quadrants. Therefore, after using **Rule 9** and **Rule 11** of the previous section, the calibration data set does only support two of the four possible cross–product terms:

$$\textit{most likely supported} \quad \Longrightarrow \quad N1 \cdot N2 \;\; ; \;\; N1 \cdot |N2|$$

The remaining two cross–product terms are not supported as the load pair values located inside the first and third quadrants always have the same sign. Therefore, all values of $|N1 \cdot N2|$ equal corresponding values of $N1 \cdot N2$. Similarly, all values of $|N1| \cdot N2$ equal corresponding values of $N1 \cdot |N2|$. Then, we get:

$$\textit{not supported} \quad \Longrightarrow \quad |N1 \cdot N2| \;\; ; \;\; |N1| \cdot N2$$

Figure 3 below shows the combined load plot of the forward normal force and the rolling moment that were applied during the calibration.

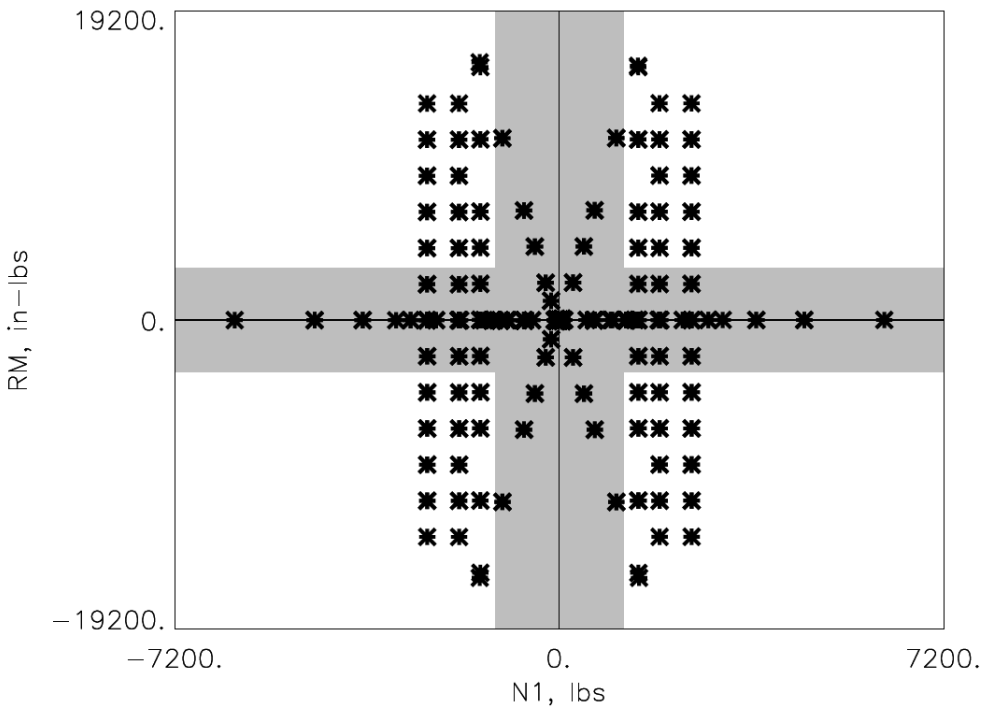


Fig. 3 Combined load plot of the rolling moment (RM) versus the forward normal force ($N1$).

It can be seen that both positive and negative values are located on or near both axes. Therefore, after using **Rule 1** and **Rule 3** of the previous section, it is likely that the given data set supports the following linear and quadratic terms (cubic terms are ignored):

$$\textit{most likely supported} \quad \Longrightarrow \quad N1 \;\; ; \;\; RM \;\; ; \;\; N1^{\,2} \;\; ; \;\; RM^{\,2}$$

It is known that the forward normal force gage output is bi–directional. The rolling moment gage output, on the other hand, is not bi–directional. Therefore, after using **Rule 2** and **Rule 4** of the previous section, the following related linear and quadratic terms with absolute values are most likely supported:

$$\textit{most likely supported} \quad \Longrightarrow \quad |N1| \;\; ; \;\; N1 \cdot |N1|$$

It is also observed in Fig. 3 above that data points can be found inside all four quadrants. Therefore, the calibration data set most likely supports all possible cross–product terms that do not use $|RM|$:

$$\textit{most likely supported} \quad \Longrightarrow \quad N1 \cdot RM \;\; ; \;\; |N1| \cdot RM$$

Again, the rolling moment gage output is not bi–directional. Therefore, the following two terms are most likely not supported:

$$\textit{not supported} \quad \Longrightarrow \quad |N1 \cdot RM| \;=\; |N1| \cdot |RM| \quad ; \quad N1 \cdot |RM|$$

Figure 4 below shows the combined load plot of the forward side force and the axial force that were applied during the calibration.

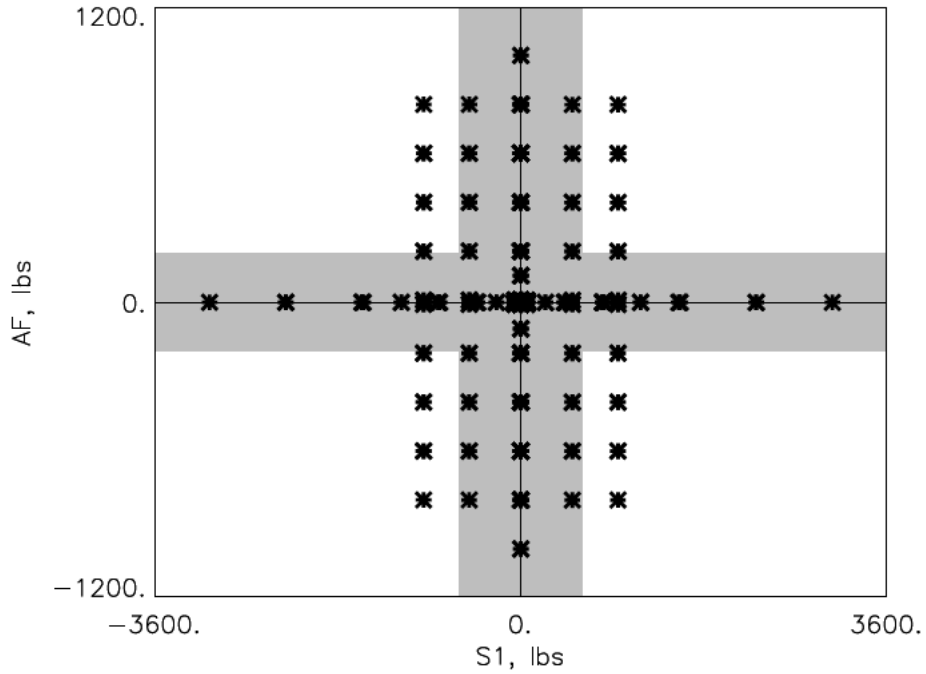


Fig. 4 Combined load plot of the axial force (AF) versus the forward side force ($S1$).

It can be seen that both positive and negative values are located on both axes. Therefore, after using **Rule 1** and **Rule 3** of the previous section, it is likely that the given data set supports the following linear and quadratic terms (cubic terms are ignored):

$$\textit{most likely supported} \quad \Longrightarrow \quad AF \;\; ; \;\; S1 \;\; ; \;\; AF^{\,2} \;\; ; \;\; S1^{\,2}$$

The forward side force gage output of the balance is known to be bi–directional. The axial force gage output, on the other hand, is not bi–directional. Therefore, after using **Rule 2** and **Rule 4** of the previous section, the following related linear and quadratic terms with absolute values are most likely supported:

$$\textit{most likely supported} \quad \Longrightarrow \quad |S1| \;\; ; \;\; S1 \cdot |S1|$$

It is also observed in Fig. 4 above that data points can be found inside all four quadrants. Therefore, the calibration data set most likely supports all possible cross–product terms that do not use $|AF|$ as the axial force gage output of the balance is not bi–directional:

$$\textit{most likely supported} \quad \Longrightarrow \quad S1 \cdot AF \;\; ; \;\; |S1| \cdot AF$$

Again, the axial force gage output is not bi–directional. Therefore, the following terms are not supported:

$$\textit{not supported} \quad \Longrightarrow \quad |S1 \cdot AF| \;=\; |S1| \cdot |AF| \;\; ; \;\; S1 \cdot |AF|$$

Figure 5 below shows the combined load plot of the aft side force and the axial force that were applied during the calibration. It is concluded, after comparing Fig. 5 with Fig. 4, that the combined load plot of AF & $S2$ "looks" more or less identical to the combined load plot of AF & $S1$. Therefore, superficially

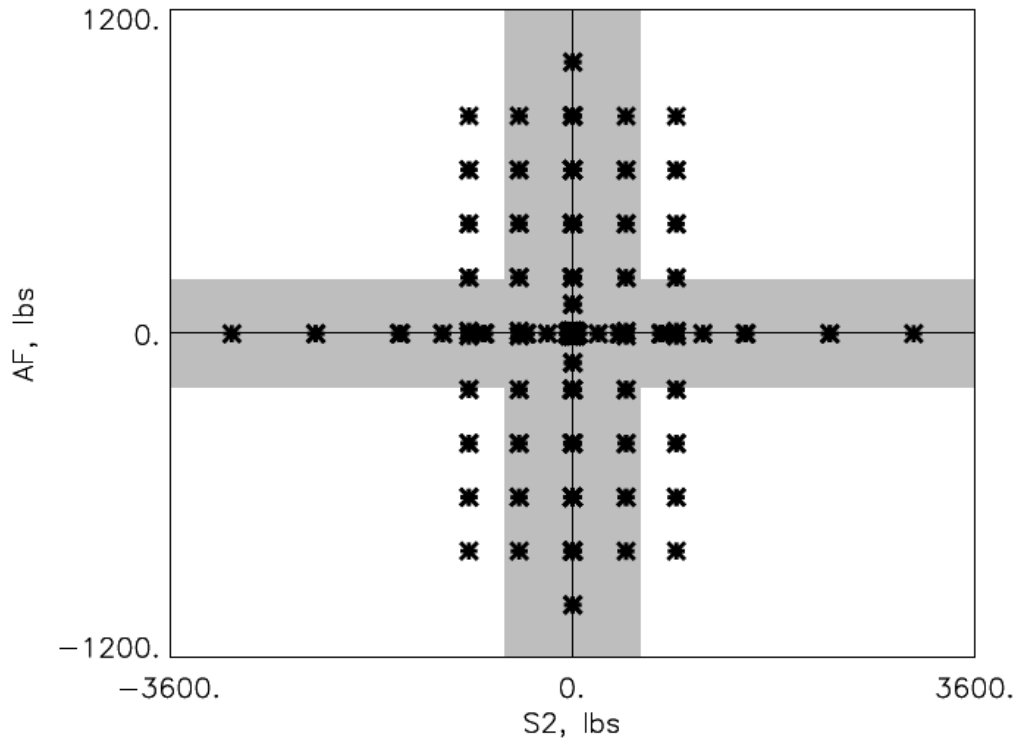


Fig. 5 Combined load plot of the axial force (AF) versus the aft side force ($S2$).

viewed, it appears as if the supported term types of the load pair AF & $S2$ should be identical with the supported term types of the load pair AF & $S1$. However, it is known from BALFIT's initial analysis of the calibration data that a massive near–linear dependency would exist in the regression model of the data if the cross–product terms $AF \cdot S1$ and $AF \cdot S2$ would simultaneously be used. The reported *Variance Inflation Factors* of the two terms are on the order of 7000. Similarly, the same analysis showed that a massive near–linear dependency would exist in the regression model of the data if the cross–product terms $AF \cdot |S1|$ and $AF \cdot |S2|$ would simultaneously be used. In this case, the reported *Variance Inflation Factors* of the two terms are on the order of 9000. Unfortunately, these important connections cannot be "seen" in the combined load plots that are shown in Fig. 4 and Fig. 5 above. Therefore, additional information and/or data plots must be used to evaluate the term choices if an analyst's data analysis software tool cannot compute *Variance Inflation Factors* of a chosen regression model term combination.

Separate plots of the three load components $S1$, $S2$, and AF versus the data point index and the load series number provide additional critical insight that help better evaluate possible cross–product term choices for the load components $S1$, $S2$, and AF. Information contained in these three plots will have to be used as additional input for the traditional term selection process in order to pick the cross–product terms that can be obtained from the three components. For example, Fig. 6a below shows the forward side force plotted

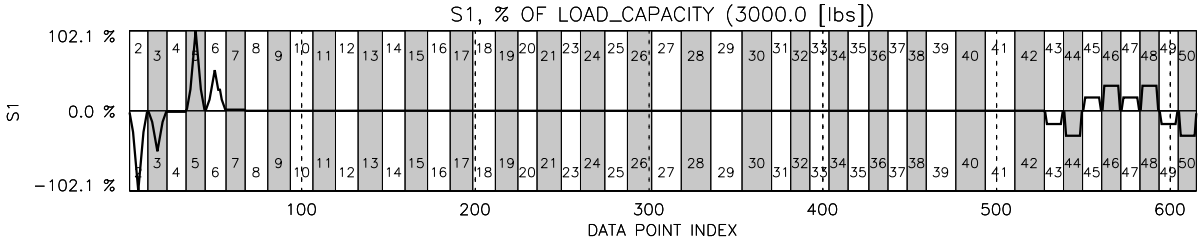


Fig. 6a Forward side force plotted versus data point index and load series number.

versus the data point index and the load series number. Similarly, Fig. 6b below shows the aft side force plotted versus the data point index and the load series number.

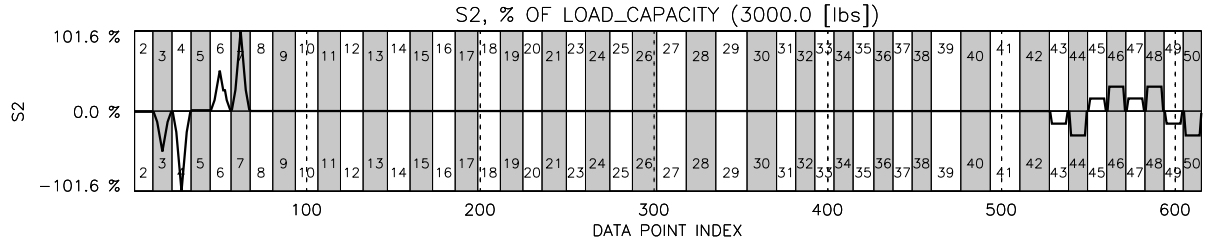


Fig. 6b Aft side force plotted versus data point index and load series number.

Finally, Fig. 6c below shows the axial force plotted versus the data point index and the load series number. Now, after comparing Fig. 6a and Fig. 6b with Fig. 6c, it is concluded that only the loads from

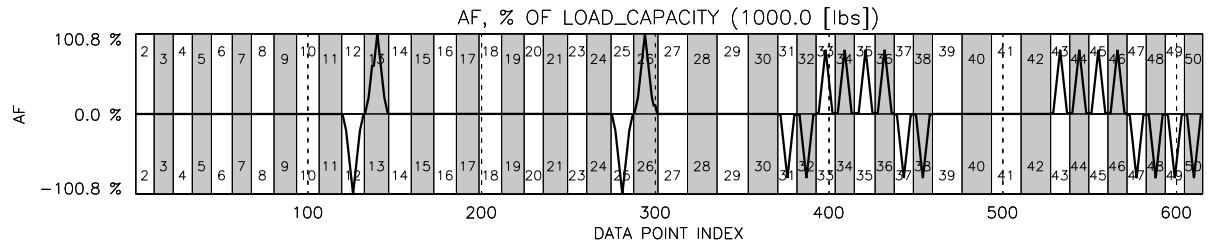


Fig. 6c Axial force plotted versus data point index and load series number.

load series 43 to load series 50 will result in non–zero values of the cross–product terms that can be defined by using the axial force and any one of the two side force components. In addition, after comparing the side forces $S1$ and $S2$ of series 43 to 50 side–by–side, it is observed that the magnitude of the side force loadings of this load series range is the same for all practical purposes. Therefore, it is concluded that the term $AF \cdot S1$ cannot be independent of the term $AF \cdot S2$. Consequently, an analyst must use either $AF \cdot S1$ or $AF \cdot S2$ in the regression model of an output in order to avoid unwanted linear dependencies. Similarly, $AF \cdot |S1|$ cannot be independent of $AF \cdot |S2|$. Therefore, an analyst must either use $AF \cdot |S1|$ or $AF \cdot |S2|$ in the regression model of an output in order to avoid unwanted linear dependencies.

The above example illustrates how complex the application of the traditional term selection process to real–world balance calibration data can become. In most cases, a "visual" examination of the combined load plots alone is insufficient when cross–product terms are assessed. It takes time and effort for an analyst to master the traditional term selection process. BALFIT's automated term selection algorithm, on the other hand, avoids all these complexities by using *Singular Value Decomposition* in combination with the *Variance Inflation Factor* for both numerical detection and removal of harmful linear or massive near–linear dependencies in a regression model of balance calibration data.

V. Summary and Conclusions

Key elements of a traditional regression model term selection process for wind tunnel strain–gage balance calibration data were discussed. This process is sometimes used to assemble regression models of the balance gage outputs that the *Iterative Method* needs for the definition of the load iteration equation. The traditional term selection process selects regression model terms by combining an analyst's subject–matter knowledge with a "visual" examination of different types of load plots. Data from the manual calibration of a six–component force balance was used to illustrate the application of the traditional term selection process.

It is concluded that the traditional term selection process can become very complex if cross–product terms are examined. Its successful application highly depends on an analyst's experience and skills. An important goal of the traditional term selection process is the identification of a regression model for the balance calibration data analysis that is free of unwanted linear or massive near–linear dependencies. Therefore, the traditional process can be replaced by an automated term selection algorithm that uses, for example, *Singular Value Decomposition* and the *Variance Inflation Factor* for detection and removal of harmful linear

or massive near–linear dependencies between regression model terms. The application of a term selection algorithm also has the advantage that it uses objective metrics for the detection of term dependencies instead of simply relying on an analyst's expertise. Consequently, a term selection algorithm makes it possible for even a novice or less experienced analyst to identify reliable multivariate regression models for the analysis of strain–gage balance calibration data.

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Appendix
Regression Model of a Gage Output Difference

The appendix lists all regression model terms that are traditionally used to define the regression model of an electrical output of a strain–gage balance. This type of regression model is needed for the generation of the load iteration equation that the *Iterative Method* uses for the balance load prediction during a wind tunnel test (see, e.g., Ref. [1] for more details).

The author recommends to express the electrical output as the difference between a raw output and the natural zero of the gage. In addition, terms of the regression model of an electrical output are organized in ten groups (Ref. [1], p. 12, Eq. (3.1.3)). Then, the regression model can be summarized by the following relationship where D_i is the output difference, $b_{i,0}$ is the intercept term, and T_ν are auxiliary summations.

$$\textit{Gage Output Difference} \implies D_i = b_{i,0} + \sum_{\nu=1}^{10} T_\nu \tag{3}$$

Table 2 below lists auxiliary summations and coefficient index ranges for each one of the ten term groups where the symbols F_j and F_k represent balance load components.

Table 2: List of all possible terms of the regression model of a gage output difference.

ν *(group index)*	T_ν *(auxiliary summation)*	$\ldots \leq \xi \leq \ldots$ *(coefficient index range)*	μ_ν *(number of terms)*
1	$\sum_{j=1}^{n} b_{i,\xi} \cdot F_j$	$1 \leq \xi \leq \mu_1$	n
2	$\sum_{j=1}^{n} b_{i,\xi} \cdot \lvert F_j \rvert$	$1 + \mu_1 \leq \xi \leq \sum_{\nu=1}^{2} \mu_\nu$	n
3	$\sum_{j=1}^{n} b_{i,\xi} \cdot F_j^2$	$1 + \sum_{\nu=1}^{2} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{3} \mu_\nu$	n
4	$\sum_{j=1}^{n} b_{i,\xi} \cdot F_j \cdot \lvert F_j \rvert$	$1 + \sum_{\nu=1}^{3} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{4} \mu_\nu$	n
5	$\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} b_{i,\xi} \cdot F_j \cdot F_k$	$1 + \sum_{\nu=1}^{4} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{5} \mu_\nu$	$\frac{n \cdot (n-1)}{2}$
6	$\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} b_{i,\xi} \cdot \lvert F_j \cdot F_k \rvert$	$1 + \sum_{\nu=1}^{5} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{6} \mu_\nu$	$\frac{n \cdot (n-1)}{2}$
7	$\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} b_{i,\xi} \cdot F_j \cdot \lvert F_k \rvert$	$1 + \sum_{\nu=1}^{6} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{7} \mu_\nu$	$\frac{n \cdot (n-1)}{2}$
8	$\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} b_{i,\xi} \cdot \lvert F_j \rvert \cdot F_k$	$1 + \sum_{\nu=1}^{7} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{8} \mu_\nu$	$\frac{n \cdot (n-1)}{2}$
9	$\sum_{j=1}^{n} b_{i,\xi} \cdot F_j^3$	$1 + \sum_{\nu=1}^{8} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{9} \mu_\nu$	n
10	$\sum_{j=1}^{n} b_{i,\xi} \cdot \lvert F_j^3 \rvert$	$1 + \sum_{\nu=1}^{9} \mu_\nu \leq \xi \leq \sum_{\nu=1}^{10} \mu_\nu$	n
—	—	—	$\Sigma = 2 \cdot n \cdot (n+2)$