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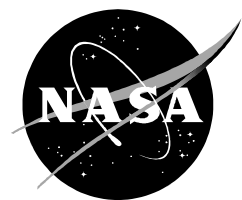
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A New Method for Predicting Turbulent Flow Reattachment Location: Application of Stratford Criterion for Reattaching Flows

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Abstract

A new method for predicting turbulent flow reattachment location has been developed. The method predicts the reattachment location using the surface static pressure distribution and requires only a single empirical parameter. The method is based on the well-known Stratford flow separation criterion, which results from an approximate solution to the equations of motion within the boundary layer. Because of the large pressure rise, the original Stratford separation criterion is not valid for reattaching flows. In order to apply the Stratford criterion for reattaching flows, the reference location was moved to the pressure drop conditions instead of pressure rise conditions. In addition, the pressure distribution was scaled using a technique that was originally introduced for supersonic base flows. A new empirical parameter was also obtained for the modified Stratford criterion. The method has been tested for 30 different separated flow configurations from the literature. The variety of test cases include subsonic flows, transonic flows, 2D flows, and axisymmetric flows with experimental as well as high resolution numerical simulation results. The flow reattachment predictions using the new method agree very well with the reported data. The error is less than 5% and typically about 2%. The new method is an extremely simple and a quick way of finding the reattachment location. Given the complexities of the existing experimental techniques for finding the reattachment location, the method easily and quickly provides the flow reattachment location, which could be used to estimate the physical extent of separation bubbles, as a performance index for flow control methods, or to assess the predictive capability of turbulence models.

Nomenclature

C_p	=	pressure coefficient referenced to freestream
$\overline{C_p}$	=	pressure coefficient referenced to x_0
$C_{p_{inv}}$	=	inviscid pressure coefficient
$C_{p_{eff}}$	=	effective pressure coefficient
$\tilde{C}_{pm}$	=	reduced pressure coefficient
C_μ	=	momentum coefficient
C_f	=	skin friction coefficient
c	=	hump chord
M_∞	=	freestream Mach number
P	=	static pressure
P_∞	=	freestream static pressure
P_o	=	static pressure at x_0
P_T	=	total pressure
Re_x	=	local Reynolds number
$S(x)$	=	Stratford parameter
$S'(x)$	=	first derivative of the Stratford parameter
V	=	streamwise velocity
x_0	=	pressure drop (or rise) location
x_{EL}	=	effective length
x_r	=	reattachment location
β	=	Stratford empirical parameter
ν	=	kinematic viscosity

Introduction

Flow separation is a phenomenon that occurs in many engineering configurations as well as in the natural environment. Because flow separation causes energy loss in the form of lift, drag, total pressure, heat transfer, etc., understanding and alleviating flow separation is essential to the fluids engineering field. According to Cebeci et al. [1], the prediction of flow separation, together with the prediction of viscous drag of a body, are two very important problems in the fluid mechanics. Accurate prediction of flow separation, particularly the location of flow separation, is vital in many design problems of aerodynamics.

There have been numerous studies that explored the prediction of flow separation spanning from high resolution computational fluid dynamic (CFD) simulations to simpler analytical/empirical methods. Because of the mathematical complexities as well as the amount of work required, high resolution CFD simulations are not always feasible for practical applications. Therefore, an analytical/empirical method that could provide a reasonably accurate prediction would be a preferred method. Four well-known methods for flow separation prediction include the Stratford method [2], Cebeci-Smith method [3], Head method [4], and Goldschmid method [5]. The Stratford method is particularly successful as it is simple and only requires a pressure distribution, whereas Cebeci-Smith and Head methods require detailed boundary layer solutions [6]. The Stratford method has been used to design airfoils to achieve desired pressure distributions [6-7].

While the majority of the analytical/empirical methods are focused on the flow separation location, the prediction of flow reattachment location has not received much attention. This is unfortunate because flow separation usually (i.e., when it is closed form) includes a reattachment process, where the free shear layer attaches to a wall forming a closed circulating region. The size of the closed recirculating region (i.e., separation bubble) is very important because it is usually proportional to the form drag, lift loss, or pressure loss. Since the flow separation bubble is the zone between the flow separation and reattachment, the flow reattachment location is as important as the flow separation location. For separation control purposes, if complete elimination of flow separation is not possible, then the location of flow reattachment becomes even more important than the separation location. This is because alleviating or controlling flow separation usually implies reducing the size of the separation bubble by moving the reattachment location upstream. As such, the direct measure of the separation bubble size can be used as a major performance index for flow separation control techniques.

Another aspect of flow reattachment is its usage in the validation of turbulence models. Despite the advancement in simulation techniques, flow reattachment is a very sensitive parameter and difficult to predict accurately. Therefore, it has historically been used to assess the predictive capability of turbulence models [8]. Not only is the reattachment location difficult to predict computationally, it is also very tedious to determine experimentally. Current techniques to find the reattachment location, which are also used for flow separation location, include velocimetry techniques, skin friction measurement, surface flow visualization, and unsteady pressure measurement. All of these techniques have their own complexities and are far from being simple.

A limited number of studies for predicting reattachment location involve the supersonic base flow problem (Fig.1). Although the theoretical works of Chapman [9], Korst [10], Nash [11], and McDonald [12] were mainly focused on the prediction of base pressure using various reattachment criteria, these theories can also be used to find the reattachment location with a given pressure distribution. Roshko et al. [13] used the theory of Chapman in subsonic flows (Fig. 1) and collapsed the pressure data of various bluff bodies by defining a reduced pressure coefficient:

$$\tilde{C}_{pm} = \frac{C_{pm} - C_{pb}}{1 - C_{pb}} = U^{*2} \quad (1)$$

where C_{pb} is the base pressure coefficient (see Figure 1) and U^* is the ratio of centerline velocity to edge velocity of the shear layer that was calculated as 0.587 using the laminar free shear layer assumption [9]. Although the theory of Chapman that leads to equation (1) is questionable (mainly due to the inviscid flow assumption, see Ref. [14] for details), the value of $\tilde{C}_{pm} = 0.34$ is in very good agreement with measurements [14]. In addition to Roshko et al. [13], Ruderich et al. [15] and Hudy et al. [16] also tried to develop with a simple prediction technique by investigating the separated flow over a splitter plate with a fence. They showed that the pressure distributions correlate with each other when the reduced pressure distribution is plotted against the reattachment length (x/x_r). Therefore, they concluded that there is a universal pressure coefficient value at the reattachment location, and this value (0.35) can be used to determine the reattachment length. However, this universality appears to be valid only for specific configurations. In fact, Adams et al. [14] showed that the reattachment pressure value decreases with the boundary layer thickness, and the universal nature of the correlation is only applicable for thin boundary-layers. Note that majority of these studies use a backward facing step (BFS) as the test bed to generate reattaching flows. This is because BFS has the simplest reattaching flow that contains a single separation bubble, and the location of the separation is fixed [17]. There are also other studies that try to correlate the reattachment length to Reynolds number or expansion ratio [14]. For example, for the turbulent flow over a BFS, the reattachment length appears to be near $x_r/H = 6$, but according to Ref. [14], the scatter in the data is in the range of 20% - 50%, which is not an acceptable error margin.

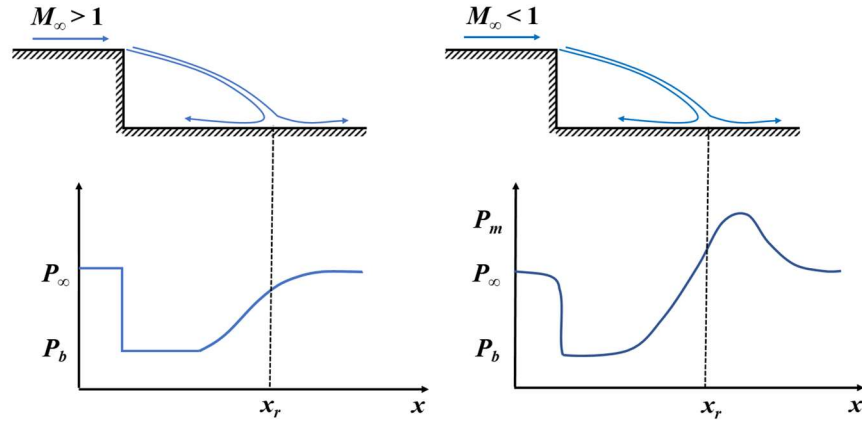


Figure 1. Ideal flow reattachment and pressure distributions for supersonic / subsonic flows [13].

Given the limited number of prediction methods focused specifically on reattachment location and unacceptable error margins in the existing ones, the objective of this study is to develop a new method for predicting the flow reattachment location. This new method is based on the Stratford separation criterion and uses the scaling of Roshko et al. [13], which is originally based on the Chapman-Korst theory of supersonic base flows. After briefly explaining the Stratford separation criterion in the second section, it is applied on the NASA hump model to predict the flow separation location. In the third section, the original Stratford separation criterion is modified and applied to the flow over the hump model to predict the flow reattachment location. In the fourth section, the new method is tested on various models and flow configurations that are available in the literature.

Stratford Separation Criterion

Since the new flow reattachment prediction method is based on the Stratford separation prediction method, explaining the Stratford analysis first will help to understand the new method. Although Stratford also developed a prediction method for laminar flows [18], our focus is on turbulent flow separation. Figure 2a shows a picture of the flow considered in Stratford's analysis. The pressure distribution has a simplified form in which a pressure rise starts abruptly at $x = x_0$ after constant pressure for a distance of x_0 . Stratford criterion is based on the approximate solution of the equations of motion within the boundary layer. The method divides the boundary layer into inner and outer regions. As depicted in Fig. 2b, in the outer region ($y > y_i$), the pressure rise causes a lowering of the dynamic head profile, and the losses due to the shear stresses are almost similar to the flow over a flat plate. In the inner layer ($y < y_i$), the inertia forces are small, and the pressure gradient is balanced by the gradient of shear stress [2].

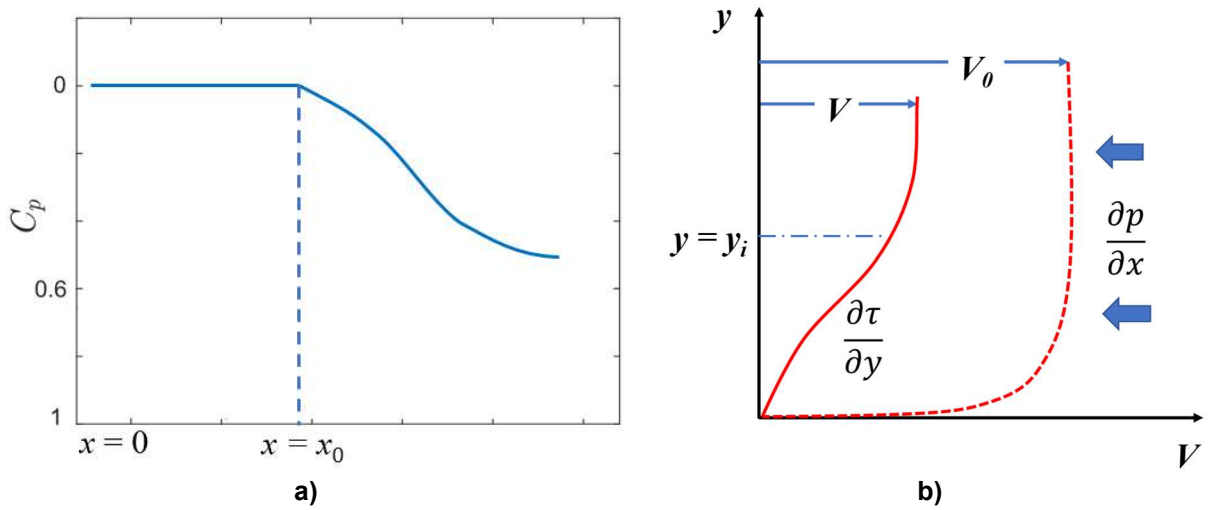


Figure 2. a) Pressure distribution used in the Stratford analysis and b) pressure gradient acting on the inner and outer regions of the velocity profile at flow separation [2].

The outer layer velocity profile is derived using an analogy to a flat plate with a power law velocity profile and associated boundary layer quantities. The inner layer velocity profile is obtained using mixing length theory (as well as dimensional analysis) [2]. By joining the two velocity profiles and implementing the zero-shear stress requirement at the wall, Stratford developed the following definition at flow separation:

$$(2C_p)^{\frac{1}{4}(n-2)} \left(x \frac{dC_p}{dx} \right)^{1/2} = 1.06\beta(10^{-6}Re_x)^{0.1} \quad \left(C_p \leq \frac{n-2}{n+1} \right) \quad (2)$$

where, n is the power of the turbulent flat plate velocity profile, Re_x is the local Reynolds number, and β is the only empirical parameter the Stratford criterion requires. By assuming $n = 6$, $\beta = 0.66$, the separation will not occur when

$$S(x) = C_p \left(x \frac{dC_p}{dx} \right)^{1/2} (10^{-6}Re_x)^{-0.1} \leq 0.35 \quad \left(C_p \leq \frac{4}{7} \right). \quad (3)$$

This definition is valid as long as $C_p \leq 4/7$. When $C_p = 4/7$, the inner layer reaches to the edge of the main boundary layer, and the equation is at the limit of its validity [2]. In order to find the flow separation location, the pressure distribution in the pressure recovery region is tested for this condition.

Application of Stratford Criterion on The NASA Hump Model

The Stratford criterion has been applied to pressure distributions in the past to design airfoil shapes with a desired pressure distribution [6-7]. In order to understand the Stratford method, the separation criterion will be applied on the pressure distribution of the NASA hump model as an example.

The wall-mounted NASA hump has been used as a benchmark case for CFD validation. The model was one of the case studies in the CFD Validation of Synthetic Jets and Turbulent Separation Control Workshop in 2004 [19] and is well documented both experimentally [20-22] and numerically [23] over a wide range of Reynolds numbers ($0.4 \times 10^6 < Re_c < 26 \times 10^6$). The hump model is the suction side of the Glauert Glass II airfoil [24]. The characteristic length is defined as the hump chord, which is $c = 420$ mm. The model is 584 mm wide, and the flow is nominally two-dimensional. The original model had a two-dimensional slot located at $x/c = 0.65$. The experiments were conducted in the NASA Langley Shear Flow Tunnel, which is an atmospheric, open-return wind tunnel. The model was mounted on a splitter plate between two endplates, and the experiments were performed at a freestream Mach number of 0.1, which corresponded to $Re_c = 0.94 \times 10^6$ based on the hump chord.

The hump model has been tested both numerically and experimentally with various flow control methods providing very rich flow separation states. The wind tunnel measurements include steady/unsteady pressure, particle image velocity (PIV), skin friction, and surface flow visualization. An example of the flow field over the hump model from numerical simulations is presented in Fig. 3. As shown in the velocity contours and streamlines, there is massive flow separation in the aft region of the hump model. Since the model is mounted on a splitter plate, the separated flow reattaches downstream of the model trailing edge ($x/c > 1$).

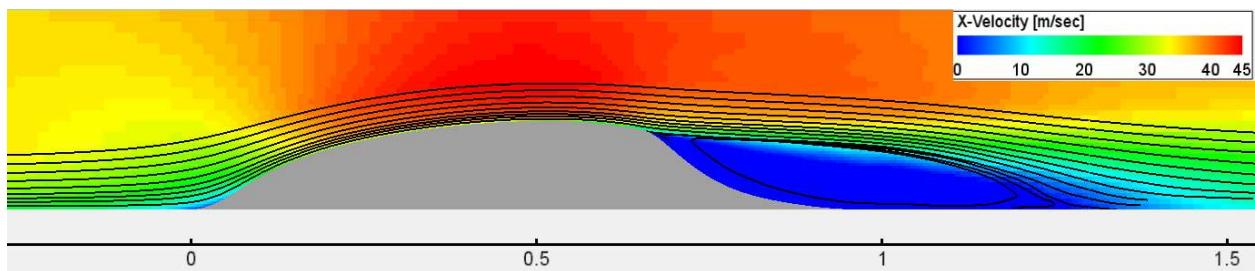


Figure 3. CFD simulation of the flow over the NASA hump model [25].

Surface oilflow visualization (OFV) of the separated flow is presented in Fig. 4. In this figure, the flow direction is from top to bottom. The separation line appears essentially two-dimensional with the exception of the near-endplate regions. Two small corner vortices are observed downstream of the separation line at each side. The OFV image was post-processed to find the flow separation and reattachment locations, which were $x/c = 0.66$ and $x/c = 1.15$, respectively.

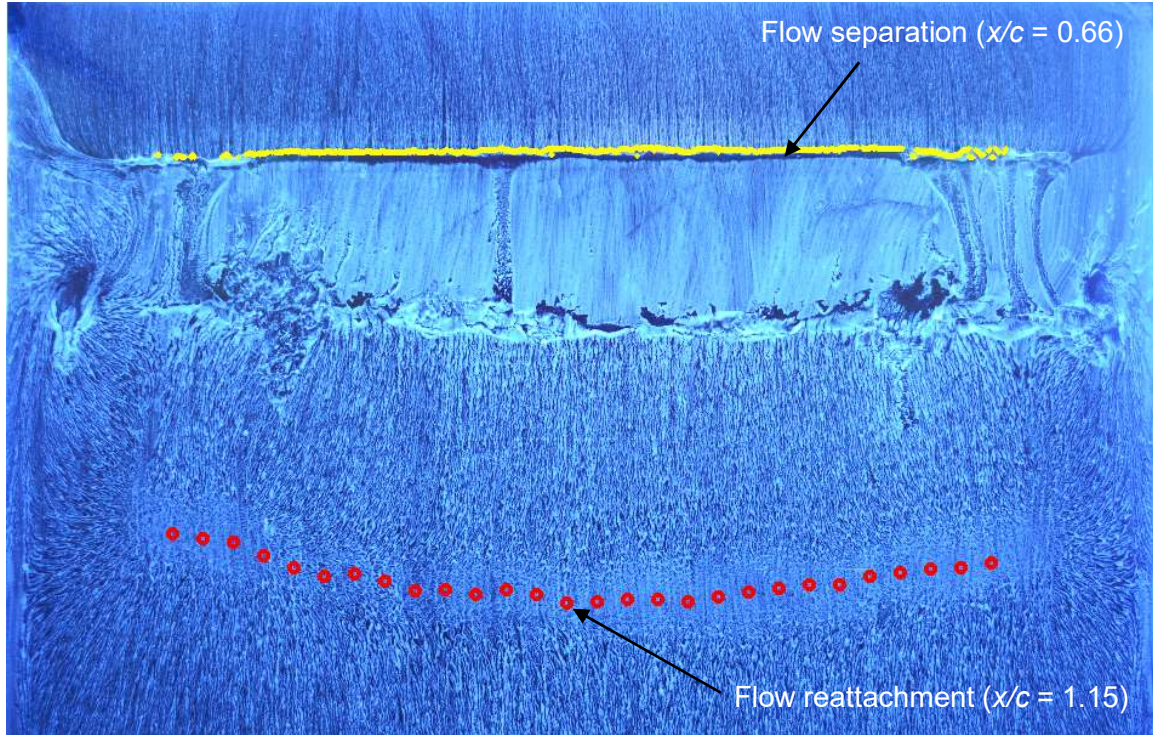


Figure 4. Surface oilflow visualization of the separated flow over the hump model [26].

The centerline pressure (C_p) distribution of the separated flow is presented in Fig. 5a. The decelerated flow upstream of the hump model rapidly accelerates until $x/c = 0.2$ due to the favorable pressure gradient, then the acceleration relaxes, and the pressure distribution reaches peak suction near $x/c = 0.5$. The pressure recovery starts immediately after the suction peak and continues until flow separation occurs near $x/c = 0.66$. After flow separation, flow over the separation bubble reaccelerates as shown by the gradual decrease in the C_p distribution. The pressure appears to be recovered downstream of the model trailing edge near $x/c = 1.3$.

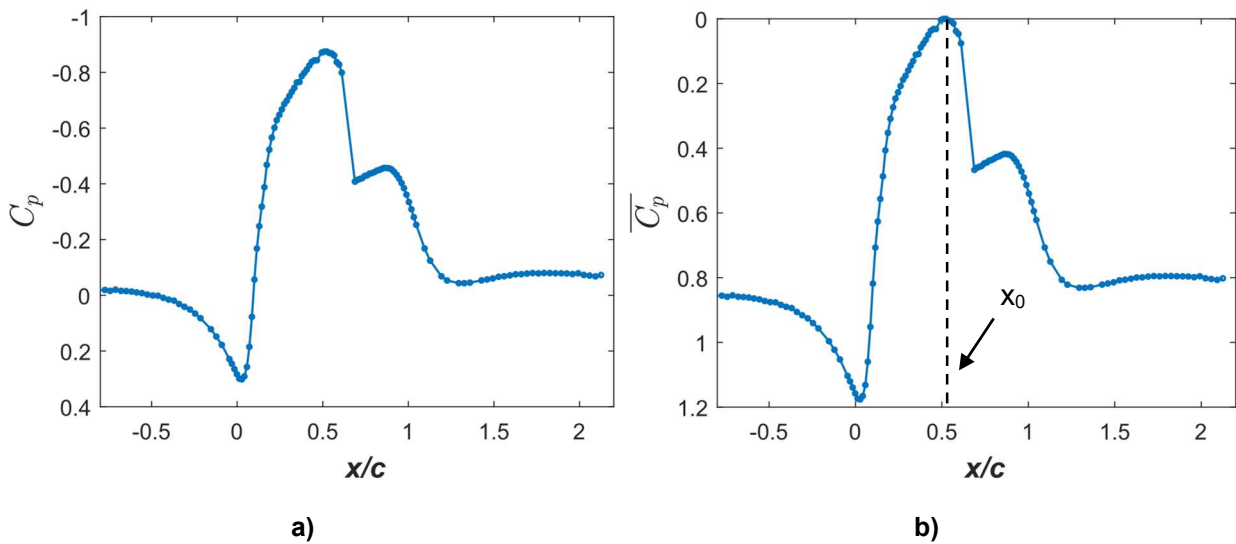


Figure 5. C_p distribution of the flow over the hump model referenced to a) freestream and b) x_0 .

The pressure distribution of the hump model is slightly different than the pressure distribution used by Stratford in his analysis (Fig. 2a). Stratford assumed a constant pressure at the leading edge of the model ($x = 0$) that is followed by a pressure rise at $x = x_0$ (referred to as a canonical C_p distribution [6]). The pressure distribution of the hump model needs to be adjusted to be able to use the Stratford criterion. The C_p distribution of the hump model was referenced to the freestream condition that is upstream of the model (Fig. 5a). On the other hand, Stratford used the pressure rise (x_0) location as the reference location. Therefore, the C_p distribution is adjusted to make x_0 as the reference location (Fig. 5b). Secondly, as shown in the hump geometry as well as in the pressure distribution, there is a favorable pressure gradient (FPG) region upstream of x_0 . To account for the FPG, Stratford suggested an equivalent pressure length (x_{EL}) such that at $x = x_0$, the momentum thickness of the boundary layer of the FPG region of length, L , is equal to that of a constant pressure distribution [2]

$$x_{EL} = \int_0^L \left(\frac{V}{V_0} \right)^3 dx_0, \quad (4)$$

where the new reference location used in equation (3) becomes $x = x_0 + x_{EL}$. Stratford developed this definition of equivalent constant pressure length for a turbulent boundary layer starting from the leading edge of a model (e.g., airfoil). In the wall-mounted hump case, flow does not start from the leading edge of the hump model; therefore, the momentum thickness would be more than that of flow starting from the leading edge of the model. A zeroth-order approximation to the equivalent constant pressure length could be the half of equation (4):

$$x_{EL} = \frac{1}{2} \int_0^L \left(\frac{V}{V_0} \right)^3 dx_0. \quad (5)$$

Since the Stratford method is based on the pressure distribution, x_{EL} should be defined in terms of pressure. A conventional pressure distribution for incompressible flow referenced to x_0 is

$$\overline{C_p} = \frac{P - P_0}{\frac{1}{2} \rho V_0^2} = 1 - \frac{V^2}{V_0^2}. \quad (6)$$

Then, the integral in equation (5) can be written in terms of $\overline{C_p}$ as

$$x_{EL} = \frac{1}{2} \int_0^L (1 - \overline{C_p})^{\frac{3}{2}} dx_0. \quad (7)$$

The $\overline{C_p}$ distribution and the Stratford parameter, $S(x)$, in the pressure recovery region of the hump model are shown in Fig. 6. The $S(x)$ parameter is zero at x_0 and gradually increases over the pressure recovery region until flow reacceleration over the separation bubble. Over the flow reacceleration region, the pressure gradient term in equation (3) changes sign and becomes a complex number; therefore, the real part of $S(x)$ is zero over this region. The flow separation location is defined as the location where $S(x) = 0.35$ and obtained as $x/c = 0.645$ by interpolation. This separation location is found to be slightly upstream ($\approx 2\%$) of the experimental separation location that was obtained using OFV ($x/c = 0.66$). It is known that the Stratford criterion tends to

predict early flow separation [3]. In addition, the limited number of pressure ports near flow separation may also contribute to this deviation.

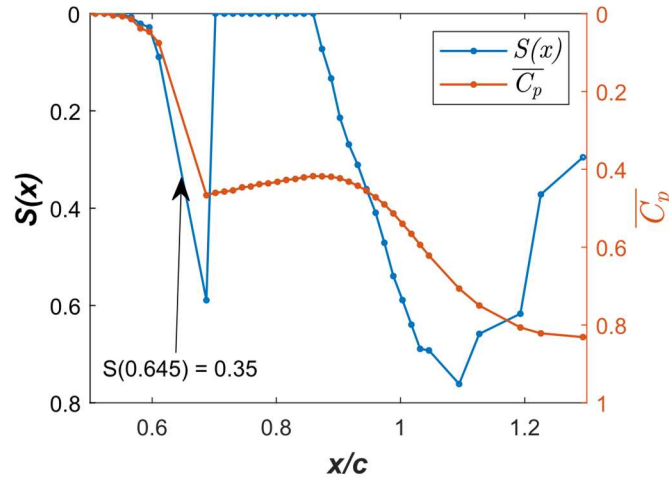


Figure 6. $S(x)$ parameter and $\overline{C_p}$ distribution in the pressure recovery region.

Application of Stratford Criterion for Flow Reattachment

A natural question that arises is whether we can apply the Stratford criterion further downstream to find the reattachment location. This is a legitimate question because the Stratford criterion assumes zero shear stress, and the shear stress is zero both at the separation and reattachment locations. In fact, using this criterion Stratford developed a ramp design (Stratford ramp), which provides a pressure recovery without flow separation, namely a continuously zero-skin friction flow [27].

It is known from the OFV (in Fig. 4) that the reattachment location is at $x/c = 1.15$. As shown in Fig. 6, the location where $S(x) = 0.35$ corresponds approximately to $x/c = 1.25$, which is not acceptable as the reattachment location. Another problem is the validity of the Stratford criterion further downstream. As given in equation (3), the criterion is applicable for $\overline{C_p} \leq 4/7$. Figure 6 indicates that the pressure coefficient is approximately $\overline{C_p} = 0.75$ near flow reattachment, which clearly violates the validity of the Stratford criterion. Therefore, the reference location at the suction peak (see Fig. 7a) is not applicable for flow reattachment. Instead, the maximum-pressure location downstream of flow reattachment could be used. This is because the flow field downstream of flow reattachment is affected by the flow structure at reattachment, and the reattachment zone establishes the initial conditions for the developing flow downstream [28]. As shown in Fig. 7a, the pressure starts to rise from the suction peak at $x/c = 0.5$ until flow separation, drops over the separation bubble slightly, and continues to rise until the maximum pressure near $x/c = 1.3$. Alternately, when the reference location x_0 (or origin) is moved to the maximum-pressure location, and the x-axis is reversed as shown in Fig. 7b, the pressure drops from the maximum pressure to flow reattachment.

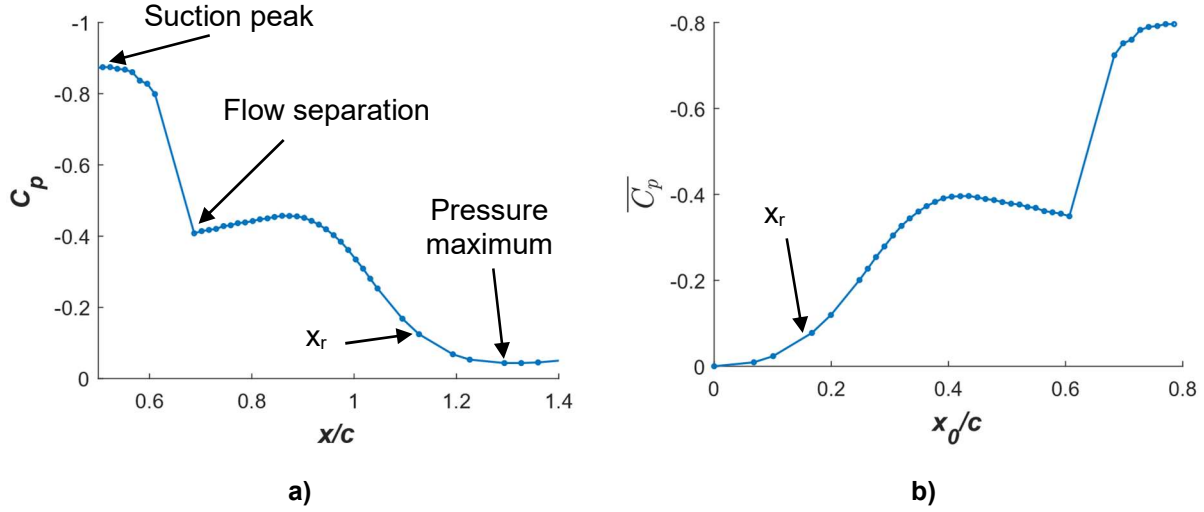


Figure 7. Pressure distributions over the pressure recovery region. a) C_p referenced to freestream with origin at the hump leading edge and b) $\overline{C_p}$ referenced to x_0 with the origin at x_0 .

The Stratford criterion was also used by Liebeck to design airfoils with optimum pressure recovery. We will follow the procedure of Liebeck that is outlined in his thesis [29]. Usually, C_p distributions are computed using the reference conditions upstream of the test section (freestream conditions). The conventional pressure distribution for incompressible flow is

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \rho V_\infty^2} = 1 - \frac{V^2}{V_\infty^2}. \quad (8)$$

On the other hand, the $\overline{C_p}$ distribution referenced to the maximum-pressure location (x_0) is

$$\overline{C_p} = \frac{P - P_0}{\frac{1}{2} \rho V_0^2} = 1 - \frac{V^2}{V_0^2}. \quad (9)$$

These two relations can be combined as

$$\overline{C_p} = \left(\frac{V_\infty}{V_0} \right)^2 (C_p - 1) + 1. \quad (10)$$

The velocity ratio, V_0/V_∞ , can be defined using equations (8-9) as

$$\frac{V_0}{V_\infty} = \sqrt{1 - [C_p]_{x_0}}, \quad (11)$$

where, $[C_p]_{x_0}$ is the C_p value at x_0 location. The equation (10) can be rearranged as

$$\overline{C_p} = \frac{(C_p - 1)}{(V_0/V_\infty)^2} + 1 = \frac{(C_p - 1)}{(1 - [C_p]_{x_0})} + 1 = \frac{C_p - [C_p]_{x_0}}{1 - [C_p]_{x_0}}. \quad (12)$$

It should be noted that this definition is somewhat similar to the normalization technique introduced by Roshko et al. [13] in equation (1) to scale the reattaching flows, except the maximum pressure, $[C_p]_{x_0}$, is used instead of the base pressure, C_{pb} .

Similarly, the Reynolds number is written as

$$Re_\infty = \frac{V_\infty c}{\nu}, \quad Re_0 = \frac{V_0}{V_\infty} \frac{x_0}{c} Re_\infty. \quad (13)$$

Using the relation in equation (11), the Reynolds number can be written in terms of $[C_p]_{x_0}$ as

$$Re_0 = \sqrt{1 - [C_p]_{x_0}} \frac{x_0}{c} Re_\infty. \quad (14)$$

The modified Stratford criterion can be obtained by plugging equations (12) and (14) into equation (2). It should be noted that the modified criterion seeks a reattachment point (x_r) of the separation bubble, whereas the Stratford criterion for flow separation seeks a condition to indicate whether flow separates or not. Therefore, the inequality in equation (2) can be converted to equality. The modified Stratford criterion is:

$$S(x) = \overline{C_p} (x d\overline{C_p}/dx)^{\frac{1}{2}} (10^{-6} Re_{0x})^{-0.1} = 1.06 \beta/2 \quad (\overline{C_p} \leq 4/7). \quad (15)$$

Note that in this equation the reference location or the origin of x-axis is at x_0 , and the direction of the x-axis is reversed as depicted in Fig. 7b. Next step is the determination of the empirical parameter, $1.06\beta/2$, in equation (15). Stratford determined the empirical parameter (β) by comparing the equation (3) results to the experimental results. In his experiments, flow was maintained just at the separation condition, and the empirical parameter was adjusted such that the equation (3) held true. A similar procedure can be followed for the reattachment location. The reattachment location for the separated flow over the hump model is reported as $x/c = 1.11$ in Ref. [21] and $x/c = 1.15$ in Ref. [26] using OFV technique. Using the mean reattachment location ($x/c = 1.13$) in equation (15) results in $S(x/c = 1.13) = -0.035$ as shown in Fig. 8. The $S = -0.035$ (or $\beta = -0.066$) value can be used as the new empirical parameter for reattaching flows. The negative sign comes from the fact that in the original Stratford criterion, the pressure is zero at $x = x_0$ and rises downstream towards the flow separation location. In the current x_r prediction method, the pressure again is zero at $x = x_0$ but drops upstream towards the flow reattachment location (i.e., direction reversed). Interestingly, the new empirical parameter of 0.035 came out one order of magnitude smaller than the empirical parameter (0.35) of the Stratford separation criterion. This is also in line with the $\overline{C_p}$ value (-0.06) at the flow reattachment location being approximately one order of magnitude smaller than the C_p value (-0.5) at the flow separation location. With a new empirical parameter, the modified Stratford criterion that is applicable to reattaching flows is

$$S(x) = \overline{C_p} (x d\overline{C_p}/dx)^{\frac{1}{2}} (10^{-6} Re_{0x})^{-0.1} = -0.035 \quad (\overline{C_p} \leq 4/7). \quad (16)$$

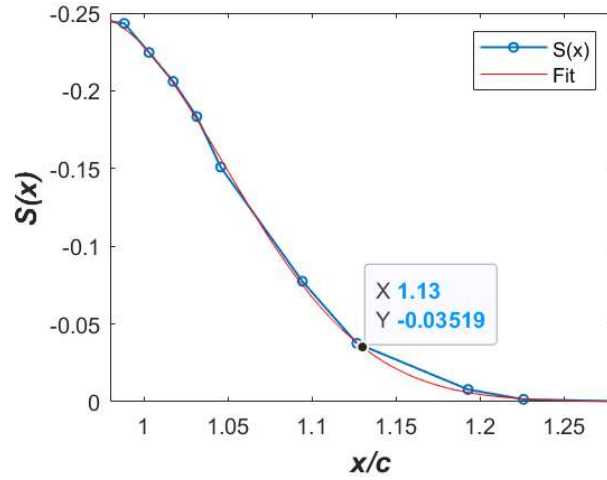


Figure 8. $S(x)$ parameter near flow reattachment.

To evaluate the reattachment predicting method, first we must ascertain that the flow considered involves separation. Otherwise, the result is misleading, and there is no base for assessment. Generally, the shape of the pressure distribution can be used to identify the existence of flow separation. Typical separating flows have a peculiar pressure distribution over the flow separation bubble [3]. This is possibly because the separation bubble changes the apparent aerodynamic shape of the flow surface, which results in a different pressure recovery profile. The effect of separation bubble sometimes manifests itself as a flattened pressure distribution profile, where the pressure is relatively constant over the separated flow region. This type of characteristic (i.e., flattening) can be observed for the separated flows over airfoils [3]. For the other cases, for example the separated flow over the hump model used in this study (Figs. 5-7), the pressure distribution is more than being flattened and shows FPG due to flow reacceleration. On the other hand, there are also other cases, which will be shown later in the testing section, where the effect of separation bubble is relatively small on the pressure distribution. This type of flow separation does not cause enough pressure drop to flatten the pressure distribution but changes the slope of the pressure distribution near flow separation. The slope change in the pressure distribution is the hardest to identify compared to two former types. Since the $S(x)$ parameter involves the pressure distribution as well as the pressure gradient terms (see equation 16), it is possible to use the $S(x)$ and $S'(x)$ parameters to check the existence of flow separation similar to checking the flattening or slope change in the C_p distribution. The $S(x)$ and $S'(x)$ parameters are especially useful when the slope change in the C_p distribution is visually hard to identify. The pressure gradient term in the $S(x)$ parameter amplifies the changes; therefore, it is easier to identify compared to checking the pressure distribution alone.

Figure 9a shows the typical $\overline{C_p}$ and $S(x)$ distributions in the pressure recovery region of the hump model that were calculated using equations (12) and (16). As shown in this figure, the $S(x)$ parameter is negative except for the $0.66 < x/c < 0.86$ region due to the negative valued $\overline{C_p}$. Flow appears to reaccelerate over the $0.66 < x/c < 0.86$ region as indicated by pressure drop in the C_p distribution. As mentioned before, the flow reacceleration and the pressure drop in the pressure recovery region indicates flow separation. In this region, since the dC_p/dx is negative and the $S(x)$ parameter is complex, the real part of $S(x)$ becomes zero. After $x/c > 0.86$, $S(x)$ first decreases, reaches a peak near the trailing edge of the model, and exponentially increases to zero at the maximum-pressure location.

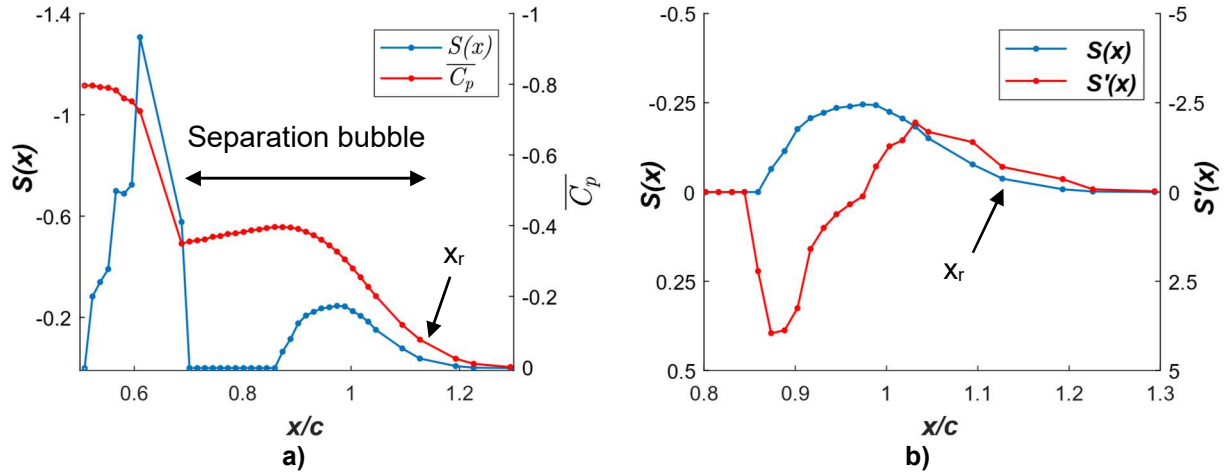


Figure 9. a) $\overline{C_p}$ and $S(x)$ parameters in the pressure recovery region and b) close up view of $S(x)$ and $S'(x)$ parameters near the flow reattachment region.

In Fig. 9b, close-up view of the $S(x)$ parameter and its first derivative are presented near the trailing edge of the model. The $S'(x)$ parameter shows the variation of $S(x)$ over the flow separation region. While the C_p distribution indicates slight pressure drop due to flow separation, the effect of flow separation is more noticeable in the $S(x)$ and $S'(x)$ distributions. Therefore, the $S(x)$ and $S'(x)$ parameters could be used to check the existence of flow separation, in addition to checking the C_p distribution, when the slope change in the C_p distribution is not visually noticeable.

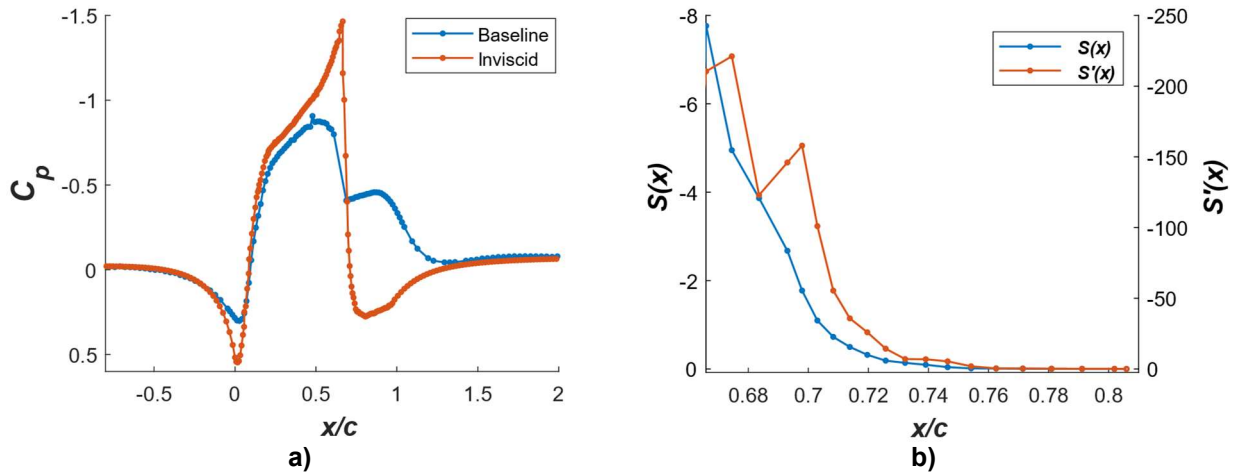


Figure 10. a) Comparison of the C_p distributions for the separated and inviscid flow over the hump model and b) $S(x)$ and $S'(x)$ distributions over the pressure recovery region for inviscid flow.

Now let's look at an inviscid flow over the hump model to see how the Stratford parameter and its derivative vary for a non-separated flow case. Figure 10a shows the inviscid CFD solution of the flow over the three-dimensional hump model together with the baseline separated flow. The inviscid flow over the hump model results in more flow deceleration near the leading edge of the hump model and more acceleration afterward in the absence of flow separation. There is a

consistent increase in the suction pressures reaching up to $C_p = -1.5$ at $x/c = 0.66$, where flow normally separates for the baseline configuration. After the peak, the suction pressure suddenly drops and recovers to the tunnel pressure downstream. The $S(x)$ distribution and its first derivative for the inviscid pressure distribution are given in Fig. 10b for the pressure recovery region ($x/c > 0.66$). Starting from the suction peak at $x/c = 0.66$, the $S(x)$ parameter gradually increases and reaches zero at x_0 ($x/c = 0.81$). The first derivative of $S(x)$ also gradually increases to zero without showing any drastic changes that were seen in the separated flow configuration in Fig. 9b.

Testing of The New Reattachment Prediction Method

Now that the new flow reattachment criterion has been developed and a new empirical parameter obtained, the next step is to test the new x_r prediction method with the data available in the literature. First, the method will be tested on the separated flow over the NASA hump model. One of the main advantages of the NASA hump model is that the model has been tested rigorously both in low and high Reynolds number facilities. There are also numerous flow control configurations that generated different separation bubble sizes and associated pressure distributions. In addition to the hump model, the method will be tested with various models and flow configurations existing in the literature.

Test Case 1: High Reynolds Number Testing of The NASA Hump Model

High Reynolds number testing of the NASA hump model was conducted in the 0.3-meter Transonic Cryogenic Wind Tunnel at the NASA Langley Research Center. The tunnel is a closed loop facility running with gaseous nitrogen, and the experimental results were reported in [20]. The hump model is geometrically similar to the hump model discussed in the previous section but physically smaller by a factor of 2 (the chord length is 200 mm vs 420 mm). The installations differed in that the model was mounted on a splitter-plate for low Reynolds number ($Re_c = 0.94 \times 10^6$), whereas it was mounted on the tunnel floor [21] for high Reynolds number ($Re_c = 16 \times 10^6$).

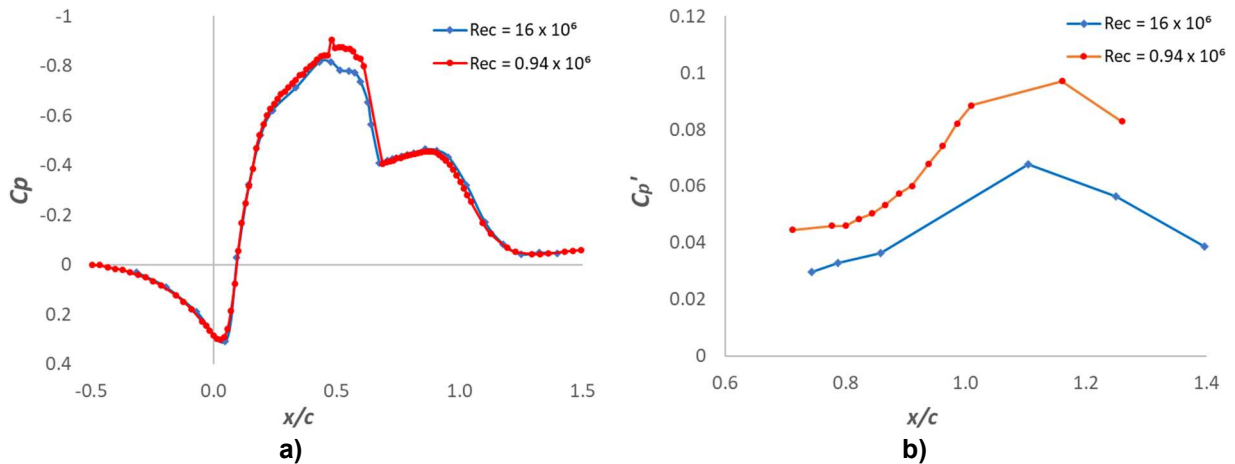


Figure 11. Comparison of the a) steady and b) unsteady pressure distributions of high and low Reynolds number tests on the NASA Hump Model.

The centerline pressure (C_p) distribution of the high Reynolds number test is compared to that of the low Reynolds number test in Fig. 11a. As shown in this figure, the C_p distributions are very close for most of the regions. The main difference between the high and low Reynolds number

results is near the suction peaks where the high Reynolds number flow generates lower suction pressures. This is attributed to the installation differences where the low Reynolds model was installed on a splitter plate that produced a net circulation over the model [21]. Figure 11b shows the unsteady pressure distributions in the pressure recovery region of the model as an indication of reattachment location. Although the magnitudes of the unsteady pressures are different, their peak locations appear to be close, within the spatial resolution of the sensors. This indicates that the flow reattachment locations are similar as well (Table 1). Since the new prediction method uses the pressure distribution, and the pressure distributions are very close, especially near the reattachment region, one can expect similar reattachment locations. The new method predicted the reattachment location as $x_r = 1.13$, which is 3% off from the reattachment location obtained by the C_p' -peak method ($x/c = 1.1$) for both configurations. Note that the C_p' -peak method usually predicts slightly early flow reattachment.

Table 1. Comparison of the reattachment locations for Case 1.

Configurations	C_p' -peak	$S(x)$	Error (%)
$Re_c = 0.94 \times 10^6$	1.1	1.13	3.1
$Re_c = 16 \times 10^6$	1.1	1.13	3.1

Test Case 2: Wall Resolved Simulation of The Flow over The NASA Hump Model

The separated flow over the NASA hump model was also studied using CFD simulations. The numerical modeling has included Reynolds Averaged Navier Stokes (RANS) simulations, wall modelled Large-Eddy Simulation (LES), and wall resolved LES simulations. The latter (wall resolved LES) results were selected for testing the current x_r prediction method because the numerical predictions are closest to the experimental results. The wall resolved LES results were reported in [30] and are available on the NASA Turbulence Modeling Resource website [31]. The flow conditions are similar to those of the experiment ($M = 0.1$, $Re_c = 0.94 \times 10^6$). Despite being highly resolved LES computations, there is still disagreement between the experimental and numerical results, especially over the separated flow region (Fig. 12a). Many parameters were studied to investigate the discrepancy between the experimental and numerical results, such as the effect of domain span, subgrid-scale model, tunnel back pressure, upstream boundary layer conditions, grid refinement, upstream Mach number, and top wall contour [30]. The discrepancies between the experimental and numerical results are out of scope of this study, and we only focused on the testing of the new x_r prediction method using the results obtained by wall resolved LES simulations. The data used in this test case is referred to as third variant in Ref. [31].

Figure 12b compares the $\overline{C_p}$ distributions that are referenced to the maximum-pressure location, x_0 . Despite the improved agreement between the numerical and experimental pressure distributions, especially over separated flow region, a slight shift on the x-axis is due to the shorter separation bubble obtained by the numerical simulations ($x_r = 1.08$ in LES vs. $x_r = 1.15$ in experiment, Table 2). Using the pressure distribution presented in Fig. 12b, the new method predicted the reattachment as $x_r = 1.09$, which is an excellent agreement with the LES computations where the error is 1% (Table 2).

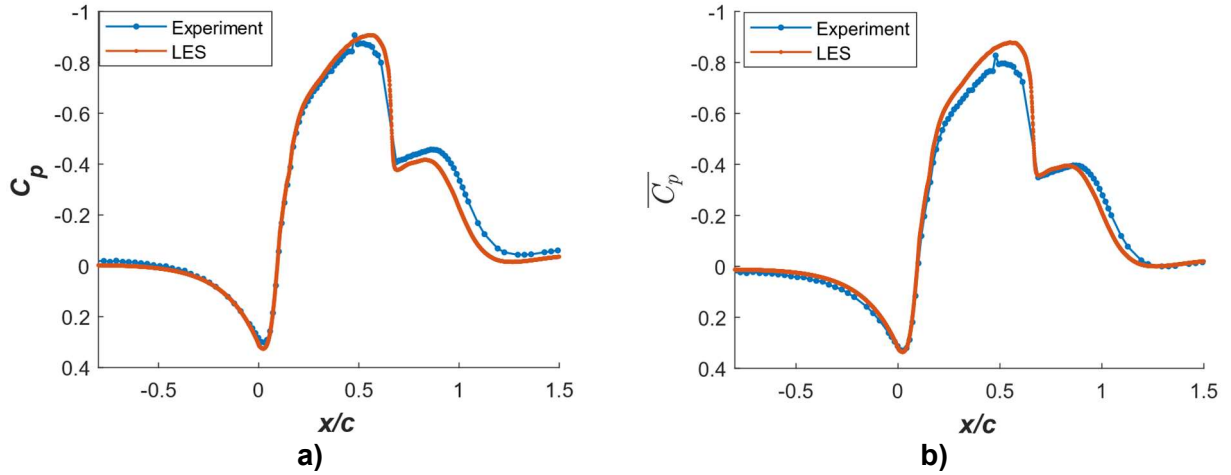


Figure 12. Comparison of the experimental and numerical pressure distributions for the NASA Hump Model a) referenced to freestream and b) referenced to x_0 .

Table 2. Comparison of the reattachment locations for Case 2.

Configurations	x_r	S(x)	Error (%)
Exp.	1.15	1.13	-1.7
LES	1.08	1.09	1

Test Case 3: The NASA Hump Model with Steady Suction

The flow separation control over the hump model with steady suction was reported in Ref. [21] as part of the CFD validation workshop [19]. Steady suction was applied from a two-dimensional slot at $x/c = 0.65$ with a nominal slot width of 0.78 mm. The experiments were conducted in the NASA Langley Shear Flow Tunnel at a freestream Mach number of 0.1, which corresponds to $Re_c = 0.94 \times 10^6$ based on the hump chord.

The pressure (C_p) distributions with various levels of flow control using 2D steady suction are presented in Fig. 13a. Starting from the baseline separated flow configuration ($C_\mu = 0$), the application of steady suction accelerates the flow upstream of the slot and increases the pressure recovery by reducing the flow separation bubble size. These different levels of flow control provide excellent test cases for the new x_r prediction method as they generate different separation bubble sizes. In Fig. 13b, the unsteady pressure (C_p') distributions are given for the associated flow control configurations. Consistent with C_p distributions, the C_p' -peak pressures move upstream indicating smaller separation bubble as the suction rate increases.

As given in Table 3, the C_p' -peak and hence the flow reattachment location gradually moves from $x/c = 1.08$ for the baseline configuration to $x/c = 0.84$ for the steady suction control with $C_\mu = 0.73\%$ configuration. The reattachment locations obtained using the new method are presented in the next column. As shown in the table, the obtained reattachment locations follow the trend, where the reattachment locations move upstream as the momentum coefficient increases. The maximum error between the C_p' -peak method and the current prediction method is less than 5% with the current pressure transducer resolution.

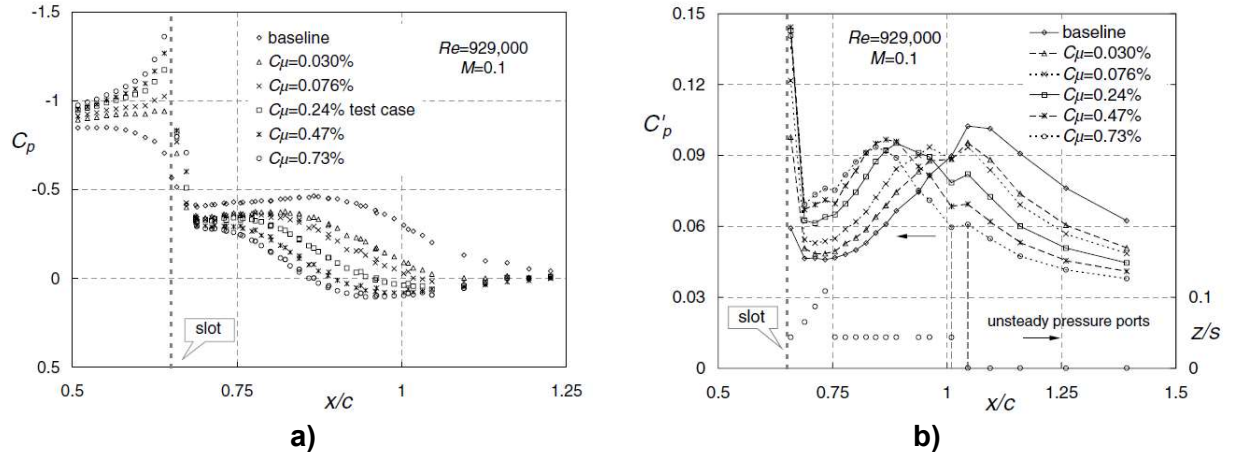


Figure 13. a) Steady and b) unsteady pressure distributions for different suction rates on the NASA Hump Model [21].

Table 3. Comparison of the reattachment locations for Case 3.

C_μ (%)	C_p' -peak	$S(x)$	Error (%)
0	1.08	1.11	3.1
0.03	1.04	1.01	-2.6
0.08	0.99	0.99	-0.4
0.24	0.90	0.93	3.8
0.47	0.86	0.90	4.6
0.73	0.84	0.88	4.8

It is known that the C_p' -peak method predicts early flow reattachment. This is clearly seen when the reattachment locations obtained by a different technique (OFV) are compared for the same configuration. The new x_r prediction method compares very well with the reattachment location obtained by OFV, and the error is less than 1%.

Table 4. Comparison of the reattachment locations obtained by different methods.

C_μ (%)	C_p' -peak	OFV	$S(x)$	Error (%)
0	1.08	1.11	1.11	0.3
0.24	0.90	0.94	0.93	-0.6

Test Case 4: The NASA Hump Model with ZNMF Actuators

Experiments with zero-net-mass-flow (ZNMF) actuators to control flow separation on the same hump model were conducted as part of the CFD validation workshop [19]. The ZNMF actuators were voice-coil based actuator modules that produced unsteady (synthetic) jets out of the same two-dimensional slot located at $x/c = 0.65$. The experimental configurations ($Re_c = 0.94 \times 10^6$ and Mach = 0.1) were the same, and the results were reported in Ref. [22]. The ZNMF actuator and the simulated flow field are depicted in Fig. 14a. The new x_r prediction method was

evaluated for two momentum coefficients, where the flow reattachment locations are reported (using PIV) in the reference dataset [22].

The C_p distributions for the two momentum coefficients (baseline with $C_\mu = 0$, and ZNMF control with $C_\mu = 0.11\%$) are presented in Fig. 14b. Flow control with ZNMF actuators increases the flow acceleration upstream of the slot and the pressure recovery downstream. Different from the steady suction case in Fig. 13a, the ZNMF actuators increase the circulation of the separation bubble as shown by a sudden decrease in the pressure (shift up) immediately downstream of the slot. The reattachment locations for the baseline and ZNMF flow control configurations were reported as 1.11 and 0.98 using PIV [22], respectively. The corresponding reattachment locations are obtained as 1.12 and 1.02 using the new prediction method, where the deviations are less than 5% compared to PIV results (Table 5).

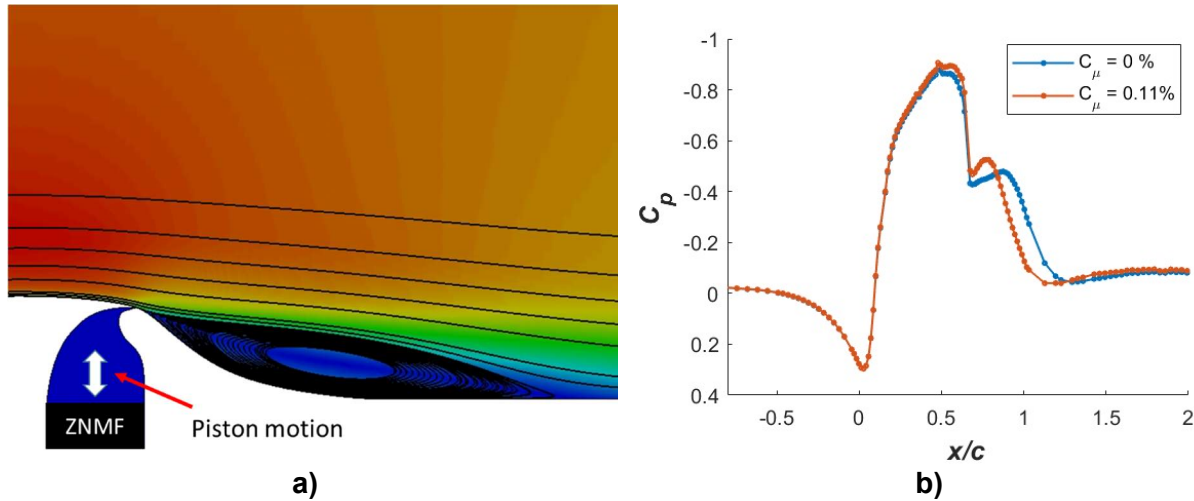


Figure 14. a) ZNMF flow field [19] and b) C_p distributions for Case 4.

Table 5. Comparison of the reattachment locations for Case 4.

C_μ (%)	PIV	S(x)	Error (%)
0	1.11	1.12	1.8
0.11	0.98	1.02	4.2

Test Case 5: The NASA Hump Model with Steady Discrete Blowing

The same hump model was also equipped with steady discrete nozzles to control flow separation (Fig. 15). Experiments were conducted in the same wind tunnel facility with the same experimental conditions ($Mach = 0.1$, $Re_c = 0.94 \times 10^6$), and the results were reported in Ref. [32]. There were 31 discrete nozzles with $1 \text{ mm} \times 5.6 \text{ mm}$ cross section (Fig. 15 inset) at the nozzle exit. The nozzles were placed at the same streamwise location ($x/c = 0.65$) with 16.5 mm spacing.

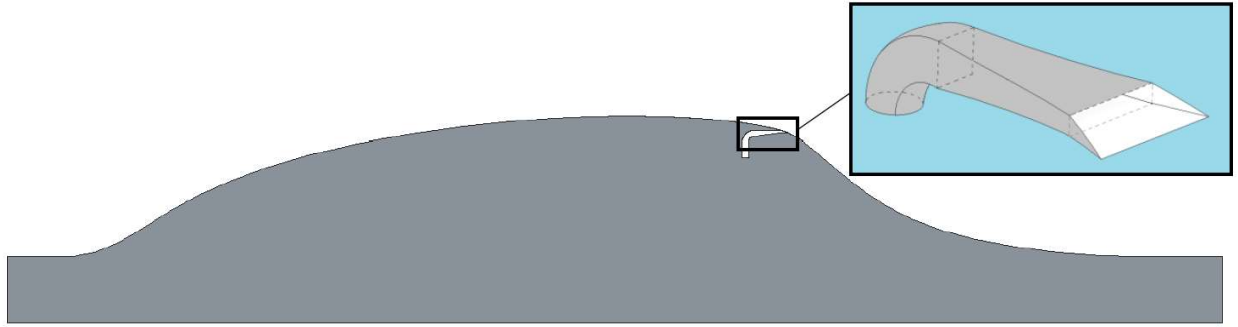


Figure 15. Steady blowing nozzles installed on the NASA Hump Model [25].

Unlike the two previous cases (steady suction and ZNMF actuators with 2D actuator exit slot), the steady blowing has discrete nozzles that create 3D flow structures downstream. This is especially challenging when using the C_p' -peak as an indication of flow reattachment location. This effect is shown in Fig. 16a where the C_p' pressures in the pressure recovery region are plotted for different values of C_μ . While there is a peak in the C_p' distribution near $x/c = 1.1$ for the baseline configuration, the peak moves to $x/c = 0.73$ for the flow control configurations and does not change with flow control amplitude. As clearly shown in numerical simulations (Fig. 16b), the discrete jets interact with the incoming tunnel flow and generate 3D flow structures. Therefore, the higher fluctuating pressures observed by the pressure transducers near the actuator exits are not due to the flow reattachment process but due to the 3D flow structures generated downstream of actuator exits. Although it is possible to notice another peak near $x/c = 0.90$ for the low momentum coefficient, it is hard to discern a second peak for the higher C_μ values as the generated 3D flow structures become more dominant. This plot clearly shows the limitation of the C_p' -peak method for indicating flow reattachment location for certain cases, especially for flow fields that involve 3D flow structures.

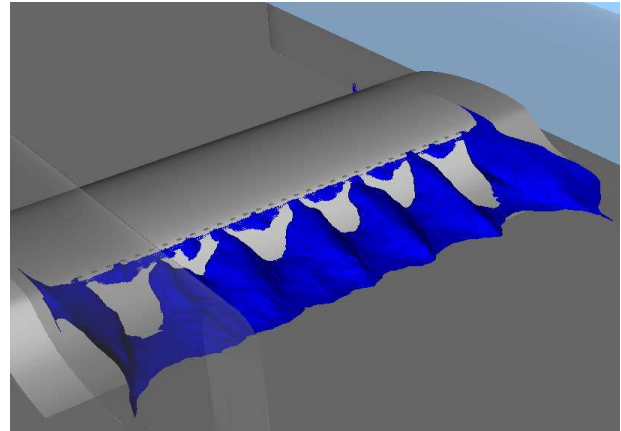
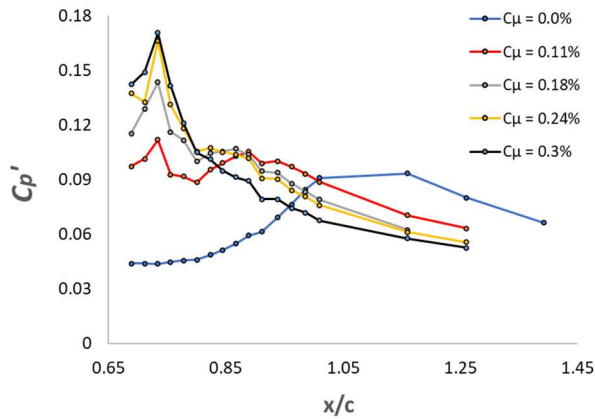


Figure 16. a) C_p' distribution and b) simulated flow field with steady blowing [32].

Figure 17 shows the C_p distributions for the two momentum coefficients (baseline with $C_\mu = 0$, and the steady blowing with $C_\mu = 0.24\%$) where flow reattachment locations are available. As

expected, the steady blowing from discrete nozzles increases the suction pressure upstream and pressure recovery downstream by reducing the separation bubble size. Although only a slight slope change appears in the C_p distribution near $x/c = 0.7$, the $S'(x)$ distribution in the pressure recovery region reveals this change which indicates the existence of flow separation (Fig. 17b). As mentioned before, the reattachment location for the baseline configuration was found as $x/c = 1.15$ using OFV. Flow control with steady discrete blowing resulted in a smaller separation bubble ($x_r = 0.89$ using OFV) as reported in Ref. [32]. The comparison of the reattachment locations obtained with the current prediction method to that of the OFV are presented in Table 6. The reattachment location for the baseline configuration is predicted as $x/c = 1.127$, which is 2% smaller compared to that of OFV. On the other hand, the reattachment location for the flow control configuration is predicted as $x/c = 0.90$ with the new method, which equates to a 1.2% error compared to the OFV result.

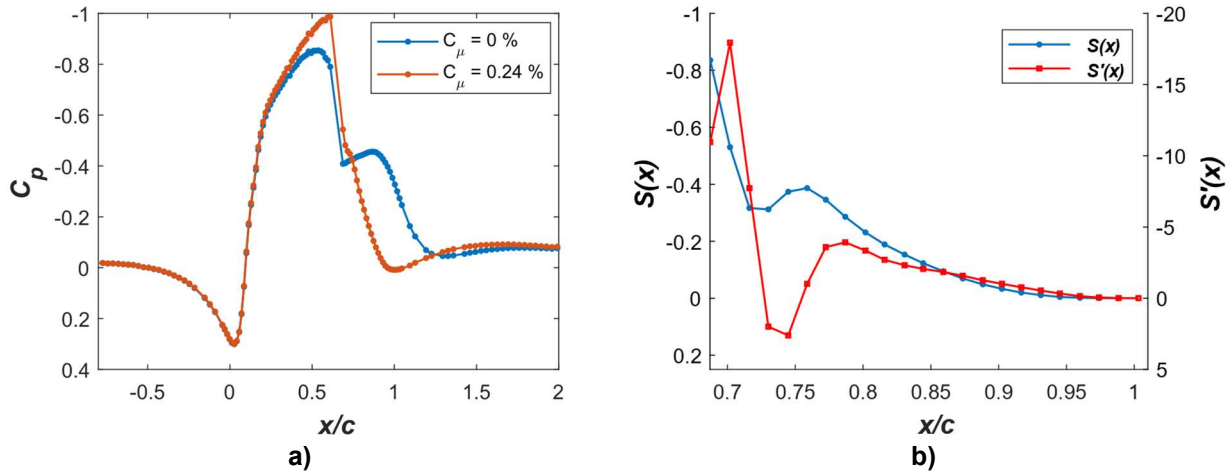


Figure 17. a) C_p distributions for baseline and flow control configurations [32] and b) $S(x)$ and $S'(x)$ distributions for the flow control configuration.

Table 6. Comparison of the reattachment locations for Case 5.

C_μ (%)	OFV	$S(x)$	Error (%)
0	1.15	1.127	-2.0
0.24	0.89	0.90	1.2

Test Case 6: The NASA Hump Model with SWJ Actuators

Flow separation behind the NASA hump model was also controlled by using sweeping jet (SWJ) actuators. The SWJ actuator is a type of fluidic actuator that emits a spatially and temporally unsteady jet at the actuator exit without having any moving or mechanical parts (Fig. 18a). Experiments were conducted in the same wind tunnel with the same experimental conditions ($M = 0.1$, $Re_c = 0.94 \times 10^6$), and the results were reported in Ref. [26]. The SWJ actuator array consisted of 17 actuators with 33 mm spacing. The SWJ actuators had a throat dimension of 1 mm by 2 mm, and they were placed at the same streamwise location as the other active flow control (AFC) actuators (i.e., $x/c = 0.65$).

The SWJ actuators are also discrete actuators similar to the steady blowing nozzles, which generate unsteady 3D flow structures. These 3D flow structures make it challenging to use the C_p' -peak method to identify the flow reattachment location. This is seen in the C_p' distributions

given in Fig. 18b for different momentum coefficients. Although there is a noticeable peak for the baseline ($C_\mu = 0\%$) configuration, there is no distinct C_p' peak in the reattachment region for the flow control configurations. Similar to the steady blowing case in Fig. 17a, there is a peak near the actuators exit that does not correlate with the AFC effects and hence does not indicate a reattachment location. Again, these peaks are possibly due to the 3D unsteady flow structures generated near the actuator exits (see OFV in Ref. [26]).

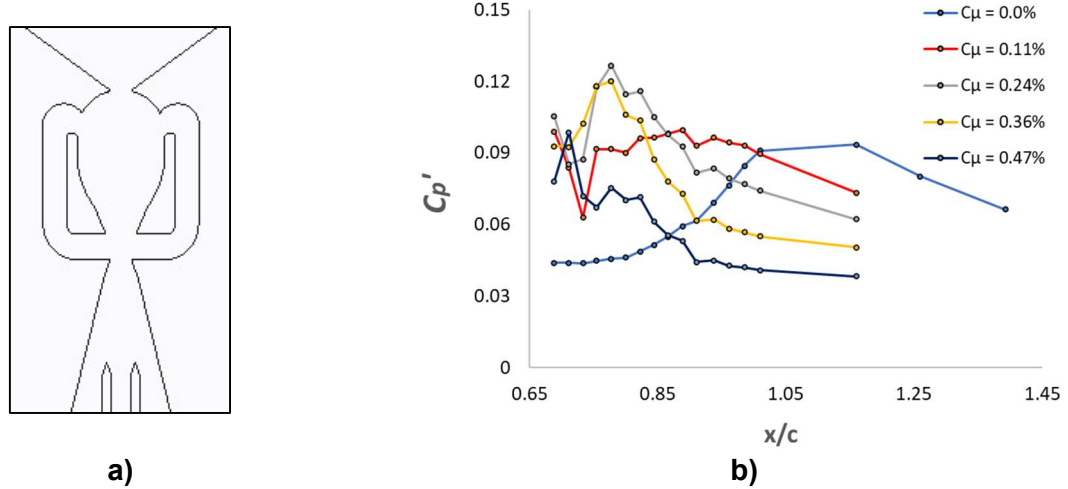


Figure 18. a) SWJ actuator geometry and b) C_p' distributions for Case 6 [26].

The corresponding pressure distributions are presented in Fig. 19 with different levels of flow control authority. As expected from AFC, the flow control increases the suction pressure upstream and pressure recovery downstream. Similar to the steady blowing case, the SWJ actuators also strengthen the circulation of the separation bubble as shown by a sudden decrease in the pressure immediately downstream of the actuator exits. Different levels of flow control resulted in different pressure recovery due to the different separation bubble sizes (Table 7). The slope changes in the pressure distributions indicate flow separation, which could be confirmed by $S'(x)$ parameter over the pressure recovery (not shown here).

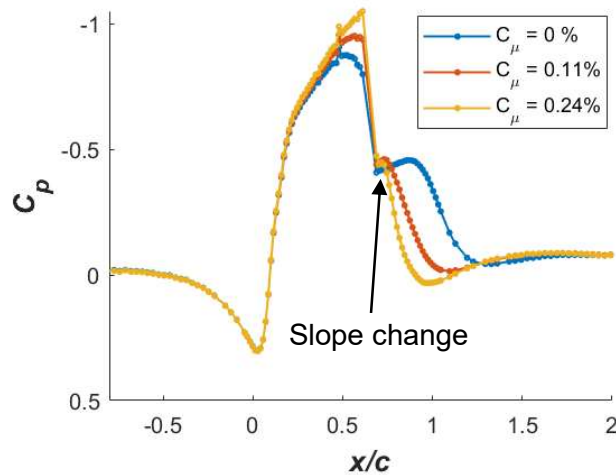


Figure 19. C_p distributions with different SWJ flow control authority [26].

The baseline configuration has a reattachment location at $x/c = 1.15$ as shown previously in OFV (Fig. 4). The reattachment location predicted by the new method is $x/c = 1.13$. This corresponds to 1.7% error compared to the OFV result. Application of SWJ actuators at low excitation ($C_\mu = 0.11\%$) resulted in flow reattachment at $x/c = 0.96$ obtained using OFV. The predicted reattachment location with the new method ($x/c = 0.95$) agrees very well with that of OFV with an error of less than 1%. Increasing the flow control authority ($C_\mu = 0.24\%$) further reduces the separation bubble size. The reattachment location is estimated to be at $x/c = 0.85$ using OFV, whereas the new method predicts the reattachment location as $x/c = 0.86$, which is only 1.2% off from the OFV result.

Table 7. Comparison of the reattachment locations for Case 6.

C_μ (%)	OFV	$S(x)$	Error (%)
0	1.15	1.13	-1.7
0.11	0.96	0.95	-0.7
0.24	0.85	0.86	1.2

Test Case 7: Axisymmetric Transonic Bump

The previous cases for testing the new x_r prediction method involved the NASA hump model with various flow control configurations. Next, the new x_r prediction method will be tested using published data on other configurations with flow separation. Most of the test cases were obtained from the Turbulence Modeling Resource website [31] where detailed experimental results are provided for turbulence modeling and CFD validation purposes.

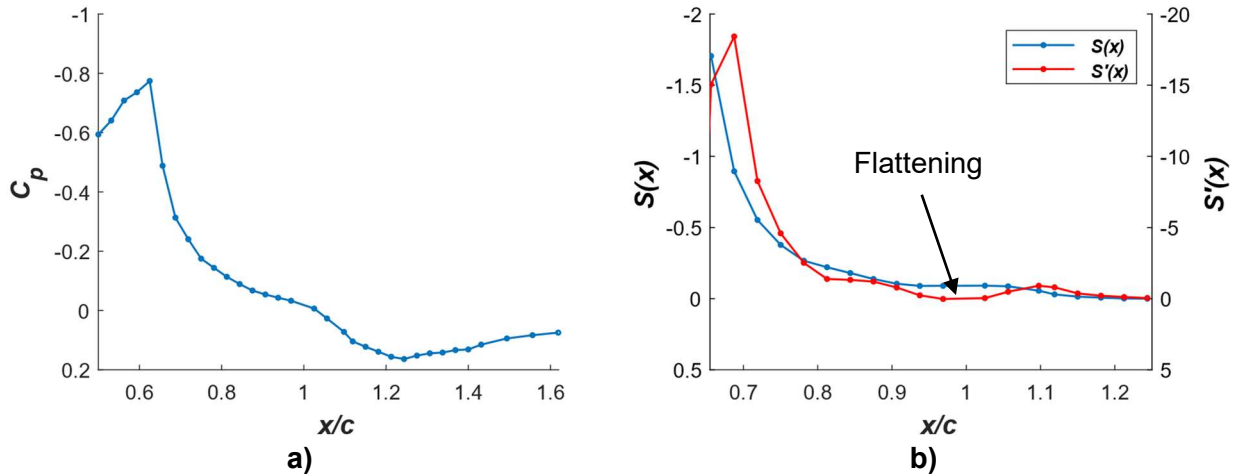


Figure 20. a) C_p distribution [33] and b) $S(x)$ and $S'(x)$ parameters.

The first non-hump case is the well-known axisymmetric transonic bump model that was experimented by Bachalo and Johnston [33]. The axisymmetric model consists of an annular circular-arc bump attached to a cylinder. The data were acquired at a freestream Mach number of $M = 0.875$ and a unit Reynolds number of $Re/m = 13.6 \times 10^6$. At these conditions, a shock occurs near $x/c = 0.66$. The combination of the shock and trailing-edge adverse pressure gradient cause flow separation (i.e., shock induced boundary layer separation). Although flow separation is difficult to be identified by looking at the C_p distribution (Fig. 20a), the flattening especially in

the $S'(x)$ distribution (Fig. 20b) indicates the existence of flow separation. The flow subsequently reattaches downstream of the bump at $x/c = 1.1$ as shown by OFV (Fig. 21). The reattachment location predicted by the current method is 1.11, which is an excellent agreement with the experimental data. Even though the error is 1%, this is due to the truncation error in the OFV result (Table 8).

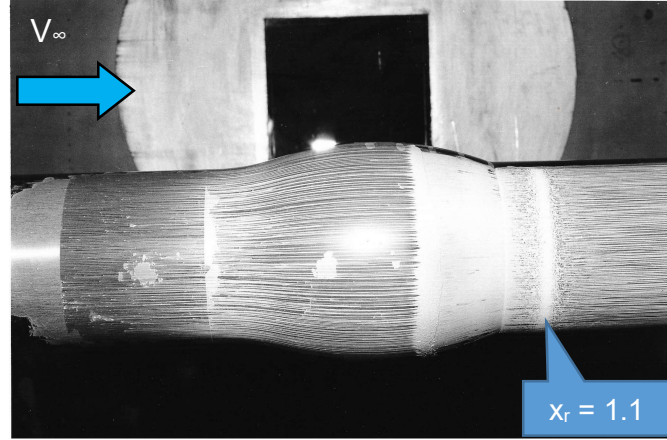


Figure 21. a) Oilflow visualization of transonic flow over the bump [33].

Table 8. Comparison of the reattachment locations for Case 7.

OFV	$S(x)$	Error (%)
1.1	1.111	1

Test Case 8: Wall Resolved Simulation of Axisymmetric Transonic Bump

As a well-known CFD validation case, the transonic flow over the axisymmetric bump has been studied with various CFD simulation techniques and turbulence models. Although there are many CFD results published in the literature, the new method will be tested with compressible CFD data that is available on the Turbulence Modeling Resource website [31]. The wall resolved CFD simulations were performed using a compressible LES code with more than 20 billion grid points. The flow conditions are similar to the experiment ($M = 0.875$, $Re_c = 2.76 \times 10^6$), and the results are reported in Ref. [34].

The flow field of the time-averaged transonic flow over the axisymmetric bump is presented in Fig. 22a. As presented in the streamwise velocity contours, the local Mach number exceeds sonic conditions and generates a shock near $x/c = 0.6$. Interaction of the shock and the adverse pressure gradient causes boundary layer separation over the aft region of the bump surface. The flow reattaches further downstream and the reattachment location was provided as $x_r/c = 1.162$ based on the streamwise velocity. The comparison of the C_p distributions from the experiment and the simulations are given in Fig. 22b. Although there are some discrepancies, the wall resolved LES simulations overall agreed well with the experimental results. Using the C_p distribution from the LES simulations, the new method predicted the reattachment location to be at $x_r/c = 1.161$, which is in perfect agreement with the CFD results, having only 0.1% error (Table 9).

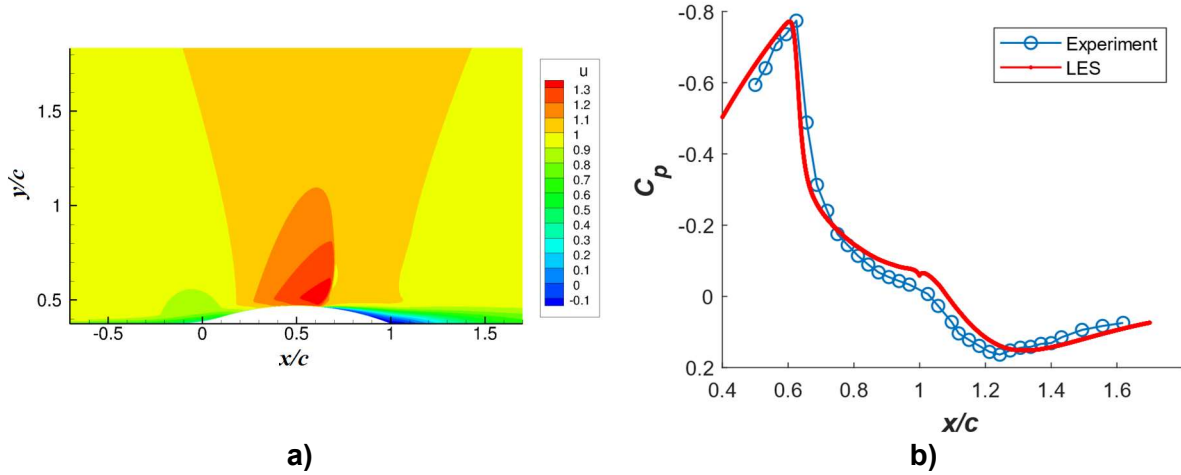


Figure 22. a) LES simulation of the flow field over an axisymmetric transonic bump [31] and b) comparison of C_p distributions.

Table 9. Comparison of the reattachment locations for Case 8.

CFD	S(x)	Error (%)
1.162	1.161	-0.1

Test Case 9: Sajben Transonic Diffuser

Another transonic flow case used to test the new prediction method is the transonic flow through a converging-diverging diffuser that was experimented by Bogar et al. [35]. The data are available in Ref. [36]. This diffuser was designed to mimic the supersonic inlet flows of aircraft engines. Figure 23 shows the schematic of the two-dimensional diffuser geometry. The freestream Mach number was reported as $M = 0.9$. The freestream Reynolds number based on the throat height (H) was reported to vary between $6.4 \times 10^5 - 8.4 \times 10^5$ due to the air supply temperature variations. Two-dimensional supersonic flow was maintained using the suction slots placed on the side walls of the constant area sections and along the top corners of the diffuser. The time-averaged static pressure distribution on the diffuser top wall was measured using pressure transducers, and the separation and reattachment locations were obtained using oilflow visualization.

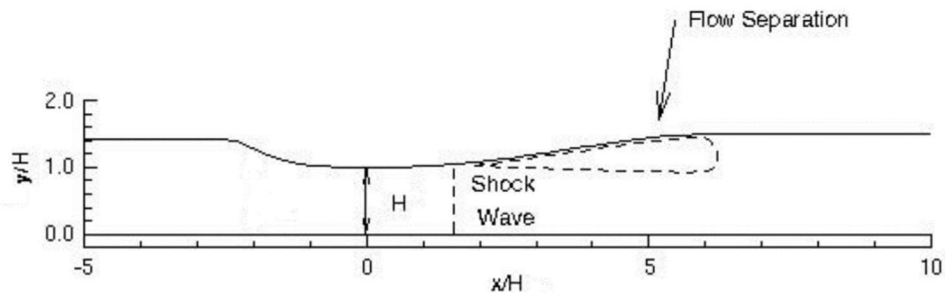


Figure 23. Schematic of the Sajben diffuser [36].

Two configurations were tested with the current prediction method. These two configurations, namely strong and weak shock configurations, were obtained by varying the exit static pressure in the experiment. The strong shock configuration results in separated flow on the top wall of the diffuser as depicted in Fig. 23. The reattachment location for the strong shock configuration was reported as $x/H = 6$ in Refs. [35-36]. On the other hand, the weak shock configuration does not cause flow separation. The static pressure distributions (P/P_T) for the two configurations are presented in Fig. 24a.

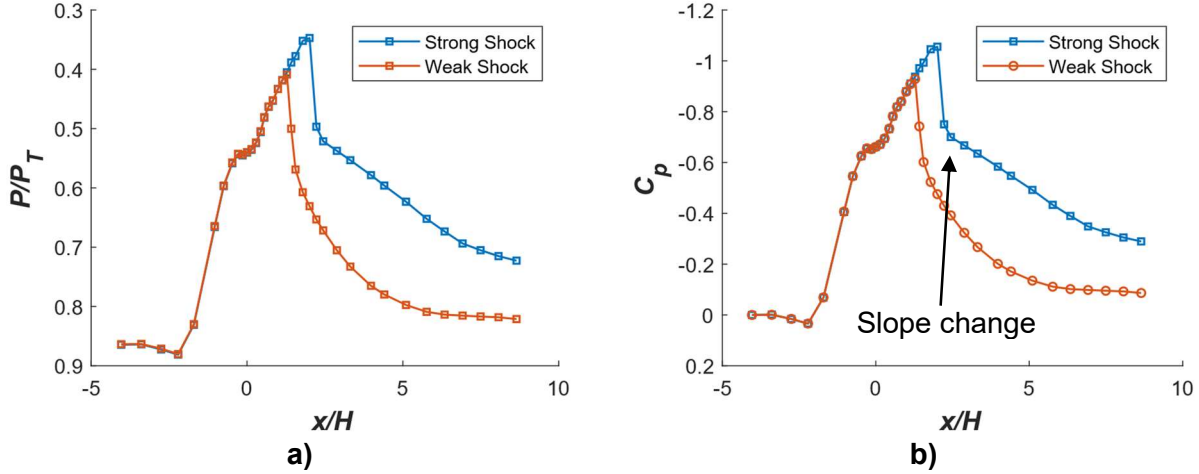


Figure 24. a) Static pressure and b) C_p distributions of the transonic diffuser.

The static pressure distributions along the top wall of the Sajben diffuser were given in terms of P/P_T rather than a typical pressure coefficient. Therefore, the pressure distribution should be converted into the pressure coefficient (C_p) in order to apply the new prediction method. For compressible flows, one can write the pressure coefficient (C_p) in terms of pressure ratio (P/P_T) as follows:

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \rho V_\infty^2} = \frac{2}{\gamma M_\infty^2} \left(\frac{P}{P_\infty} - 1 \right) = \frac{2}{\gamma M_\infty^2} \left(\frac{P}{P_T} \frac{P_T}{P_\infty} - 1 \right). \quad (17)$$

The nondimensional pressure can be converted into the C_p form using the equation (17) with $\gamma = 1.4$, $M_\infty = 0.9$, and $P_\infty/P_T = 0.86$ (Fig. 24a). Application of the new prediction method using the C_p distribution in Fig. 24b resulted in a reattachment location at $x/H = 6.0$ that is in perfect agreement with the reported reattachment location (Table 10).

Table 10. Comparison of the reattachment locations for Case 9.

OFV	$S(x)$	Error (%)
6.0	6.0	0

The weak shock configuration does not generate flow separation; therefore, there is no flow reattachment location for the new method to predict. The weak shock configuration is presented here to highlight the differences in the Stratford parameter between the separated and attached

flow configurations. The $S(x)$ parameter has a different slope for the strong shock configuration (i.e., separated flow) when compared to the weak shock case (Fig. 25a). The slope difference is also visible in the C_p distributions in Fig. 24b. The slope difference in the pressure recovery region is amplified in the $S'(x)$ distribution and seen as a flattened curve for the strong shock case, while it smoothly increases to zero for the weak shock case (Fig. 25b).

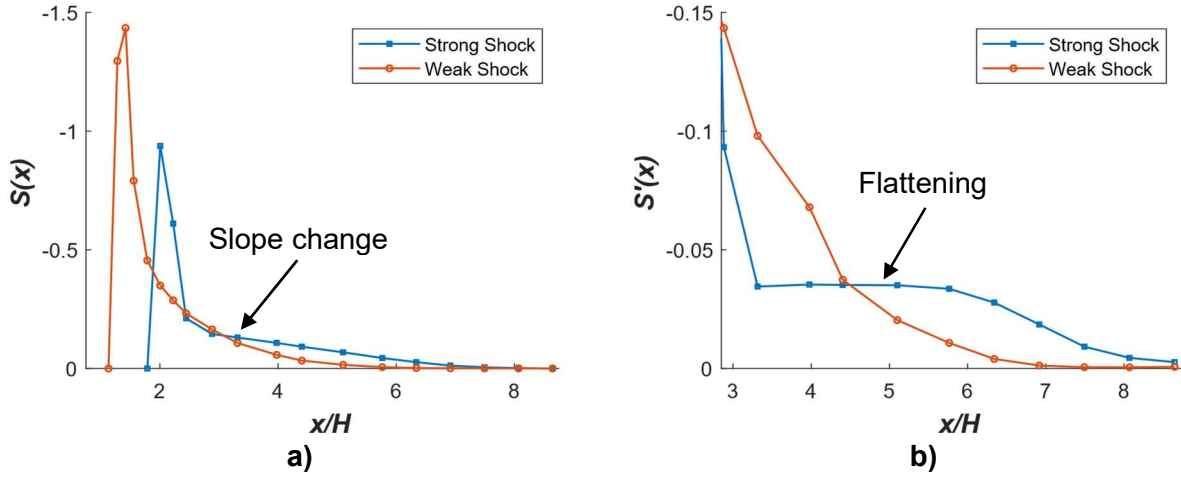


Figure 25. a) $S(x)$ and b) $S'(x)$ parameters for the strong and weak shock configurations.

Test Case 10: Buice-Eaton 2D Plane Diffuser

The next test case for the x_r prediction method is the planar diffuser, which is known as Buice-Eaton diffuser in the literature [37]. The Buice-Eaton diffuser (Fig. 26) is a 2D diffuser in which incompressible, fully developed turbulent flow undergoes smooth body separation. The diffuser has an aspect ratio of 40, and its lower wall slopes down at 10° and opens the diffuser up to 4.7 times the entrance height (H). The Mach and Reynolds numbers are approximately 0.058 and 20,000, respectively.

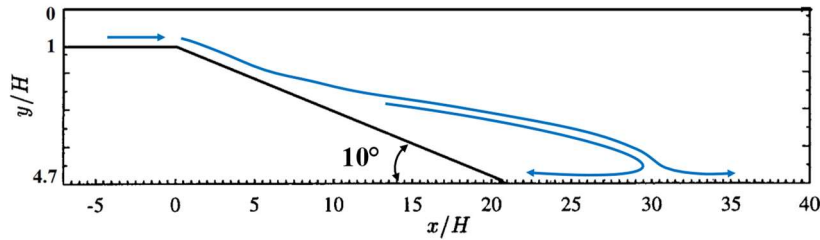


Figure 26. Schematic of the 2D diffuser [37].

Although a slope change appears in the pressure distribution (Fig. 27a), the existence of flow separation can be identified by the anomalies in the $S(x)$ and $S'(x)$ distributions (Fig. 27b). The skin friction measurements using oil flow interferometry (OFI) indicated that flow reattachment occurs at $29.2H$ downstream of the diffuser entrance. As shown in Table 11, the new method predicts the reattachment location at $x/H = 29.6$, which agrees well with the C_f measurements (error is less than 2%).

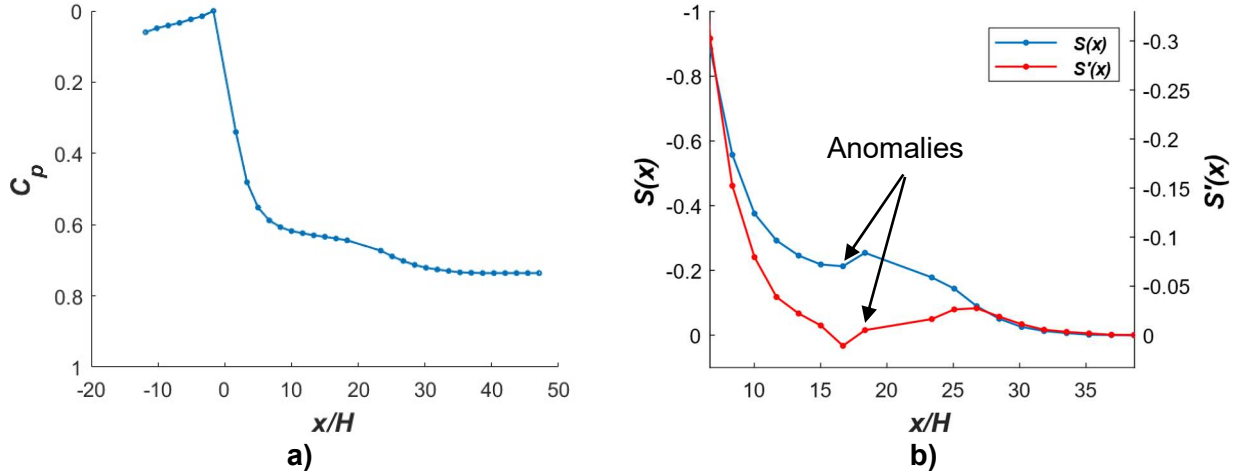


Figure 27. a) C_p distribution and b) $S(x)$ and $S'(x)$ parameters for Case 10.

Table 11. Comparison of the reattachment locations for Case 10.

C_f	$S(x)$	Error (%)
29.2	29.62	1.4

Test Case 11: 2D Convex Ramp

The 11th test case is smooth body flow separation over a convex ramp (Fig. 28). The convex ramp geometry is a 5th order polynomial contour with a ramp height of $H = 0.2$ m. The experiments were conducted at freestream Mach number of 0.2 ($Re/m = 4.1 \times 10^6$) in a quiet tunnel, and the experimental results are reported in Ref. [38]. A key feature of the experiment was the ability to vary the streamwise pressure gradient via modification of the top wall shape, allowing the flow over the ramp to vary from fully attached (Case C) to small scale separation (Case B) and to large-scale separation (Case A).

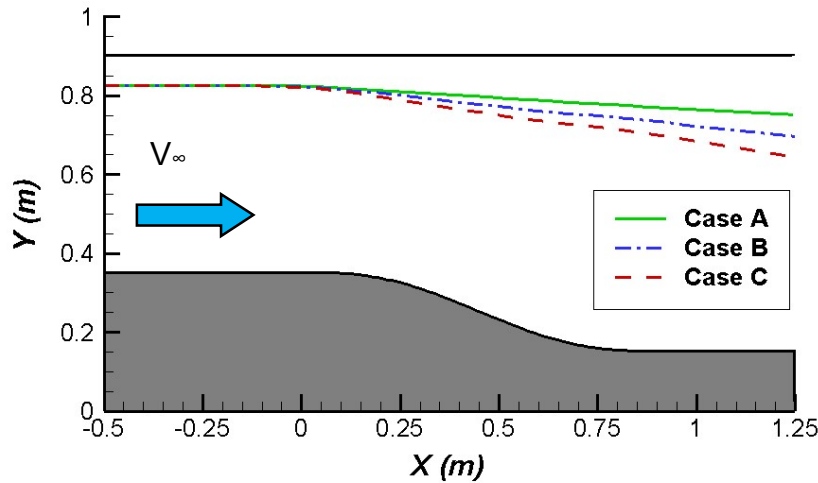


Figure 28. Description of the convex ramp geometry [38].

The pressure distributions of the three different configurations are presented in Fig. 29a. As shown in this figure, introducing a FPG by top wall deflection resulted in increased suction peak and pressure recovery. Figure 29b shows the variation of the $S'(x)$ parameter in the pressure recovery region. As shown by the $S'(x)$ plots, while the separated flow configurations (Case A and B) have flattening as an indication of flow separation, the $S'(x)$ parameter smoothly increases to zero for the attached flow configuration (Case C).

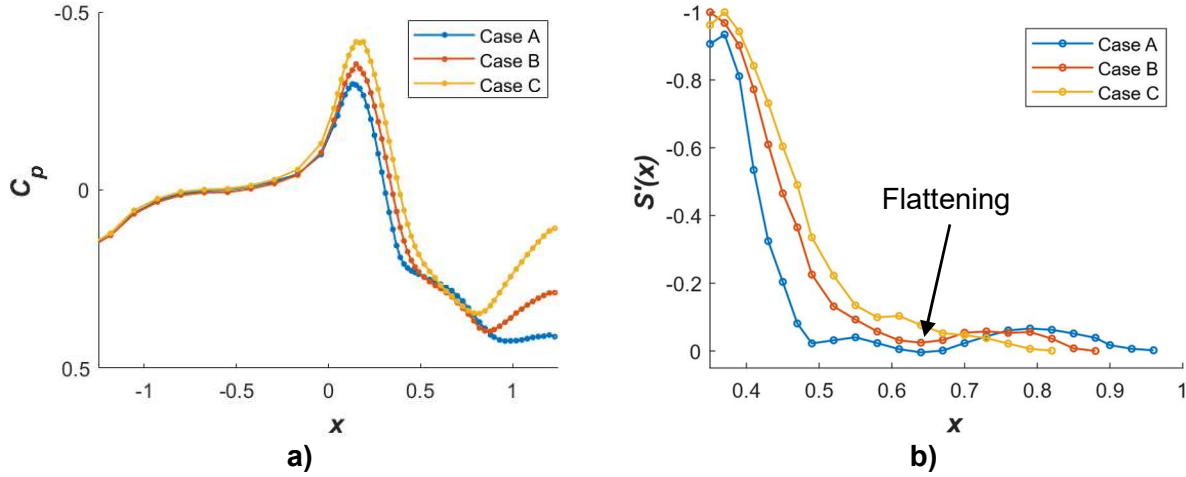


Figure 29. a) C_p distributions and b) $S'(x)$ parameter for different top wall deflections.

For the ZPG configuration (Case A), a large-scale separation bubble forms, and the reattachment location was reported as $x = 0.83$ using OFV. The reattachment location with the new method is predicted as $x = 0.83$, which is very good agreement with the reported data (Table 12). On the other hand, the reattachment location for the smaller scale separation bubble in Case B was reported as $x = 0.73$ using OFV. The reattachment location of Case B is predicted as $x = 0.75$, which corresponds to 2.3% error compared to the OFV result.

Table 12. Comparison of the reattachment locations for Case 11.

Configuration	OFV	$S(x)$	Error (%)
A	0.83	0.83	-0.6
B	0.73	0.75	2.3

Test Case 12: 2D Backward Facing Step

Another well-known experiment that has been used in CFD validation studies is the turbulent separated flow behind a 2D backward facing step (BFS). Experiments were conducted by Driver and Seegmiller in an incompressible, high Reynolds number ($Re_H = 50,000$) flow, and the results were reported in Ref. [8]. The test configuration has a large aspect ratio (12) to minimize three-dimensional effects and a small expansion ratio (1.125) to minimize the freestream pressure gradient [8]. The 2D BFS also has an option for a diverging channel flow, as shown in Fig. 30a. Channel divergence was accomplished by deflecting the top wall, which introduces a pressure gradient on the reattaching boundary layer. A laser interferometry skin-friction apparatus was used to provide C_f measurements along the bottom wall.

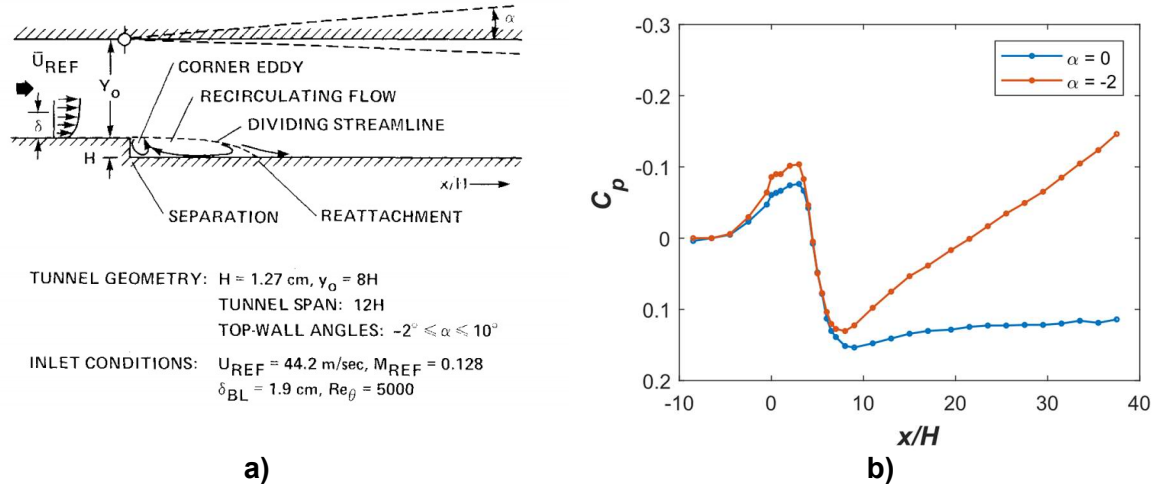


Figure 30. a) Description of the BFS flow [8] and b) C_p distributions, $\alpha = -2$ and $\alpha = 0$ deg.

The first two configurations tested with the current x_r prediction model are the FPG ($\alpha = -2$) and ZPG ($\alpha = 0$) configurations. The pressure distributions for these two configurations are presented in Fig. 30b. As shown, the flow accelerates until the flow separation location at the beginning of the step and then suddenly increases due to the flow separation bubble. The pressure reaches a local maximum (i.e., x_0), and the significant portion of the pressure rise takes place ahead of reattachment [8]. Further downstream, the pressure drops gradually in both configurations, but the FPG configuration has a steeper gradient than the ZPG configuration due to upper wall deflection.

The downstream FPG requires a similar treatment that Stratford used in his prediction method as explained in the previous section. To account for the FPG, Stratford suggested an equivalent pressure length such that at $x = x_0$, the momentum thickness of the boundary layer of the FPG region is equal to that of the constant pressure distribution as given in equation (7). The reattachment locations using C_f measurements were given as 5.82 and 6.26 for the $\alpha = -2$ deg and $\alpha = 0$ deg. configurations, respectively (Table 13). The new method with the FPG effect predicted the reattachment locations as 5.90 and 6.27, respectively. These numbers correspond to %1.4 error for the $\alpha = -2$ deg. configuration and perfect agreement (%0.2) for the $\alpha = 0$ deg. configuration.

Table 13. Comparison of the reattachment locations for Case 12.

α (deg.)	C_f	$S(x)$	Error (%)
-2	5.82	5.90	1.4
0	6.26	6.27	0.2
6	8.30	8.22	-0.9
10	10.18	10.34	1.5

The second group of testing involves the outward deflection (positive α) of the top wall so that the top wall generates additional APG on the flow and hence on the separation bubble. The additional APG configuration is challenging for the current x_r prediction method because the new method requires a local pressure maximum (x_0) downstream of flow reattachment. As shown in

Fig. 31a, the original C_p distribution increases continuously without reaching a local maximum. To overcome this problem, the effect of APG generated by the top wall deflection is subtracted from the C_p distribution. Assuming inviscid flow and using the conservation of mass, the APG generated by the top wall can be approximated as

$$C_{p_{inv}} = 1 - \frac{1}{\left(1 + \frac{x \tan \alpha}{Y_o + H}\right)^2}. \quad (18)$$

The approximated pressure distribution ($C_{p_{inv}}$) also gradually increases downstream similar to the original C_p distribution. When the effect of top wall deflection is subtracted from the C_p distribution, the resultant distribution, C_{peff} , now has a maximum pressure (near $x/H = 10$) that could be used as a reference location (Fig. 31b). However, the C_p distribution was altered too much from the original C_p distribution showing an artificial FPG region downstream of the maximum pressure (x_0). We will try to minimize this artificial FPG region by multiplying the original C_p distribution with an arbitrary coefficient, k , and then subtract the effect of top wall deflection

$$C_{peff} = kC_p - C_{p_{inv}}. \quad (19)$$

An optimum k value should provide a downstream pressure distribution with ZPG-like pressure distribution (or minimal artificial FPG). Note that this downstream pressure region can also be an APG region depending on the value of k as shown by the C_{peff} distributions with various k values in Fig. 31b. As mentioned before, the original C_p distribution gradually increases indicating an APG region. The APG becomes less severe for $k = 2$. On the other hand, the C_{peff} distribution with $k = 1$ indicates FPG. The optimum k -value (1.66) generates a ZPG-like pressure distribution downstream of x_0 . Using this effective pressure distribution in the new prediction method resulted in a $x_r = 8.22$ that is less than 1% error compared to that of the C_f measurement (Table 13).

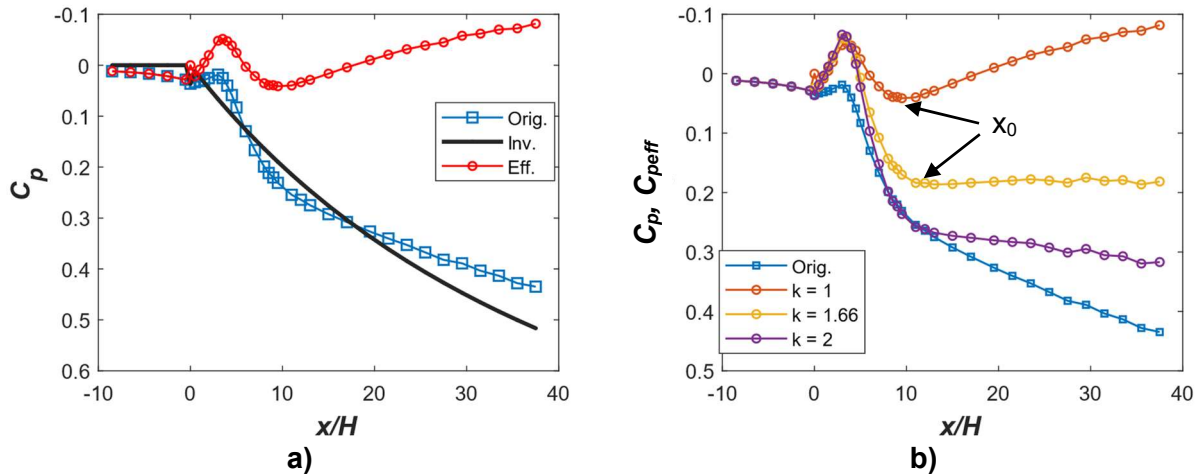


Figure 31. C_p distributions with top wall deflection, $\alpha = 6$ deg.

Similarly, the $\alpha = 10$ deg. configuration also has a pressure distribution that gradually increases downstream without having a local pressure maximum (Fig. 32a). Subtracting the effect of top wall ($k = 1$) resulted in an artificial FPG region similar to the $\alpha = 6$ deg. configuration. On the other hand, a larger k value ($k = 3$) generated a more severe APG region downstream compared to the original C_p distribution (Fig. 32b). The optimum value ($k = 2.11$) provided a ZPG-like pressure distribution with a local pressure maximum near $x/H = 15$. The new prediction method with this $C_{p\text{eff}}$ distribution resulted in a reattachment location at $x_r = 10.34$, which agrees very well (%1.5 error) with the C_f measurement results of $x_r = 10.18$.

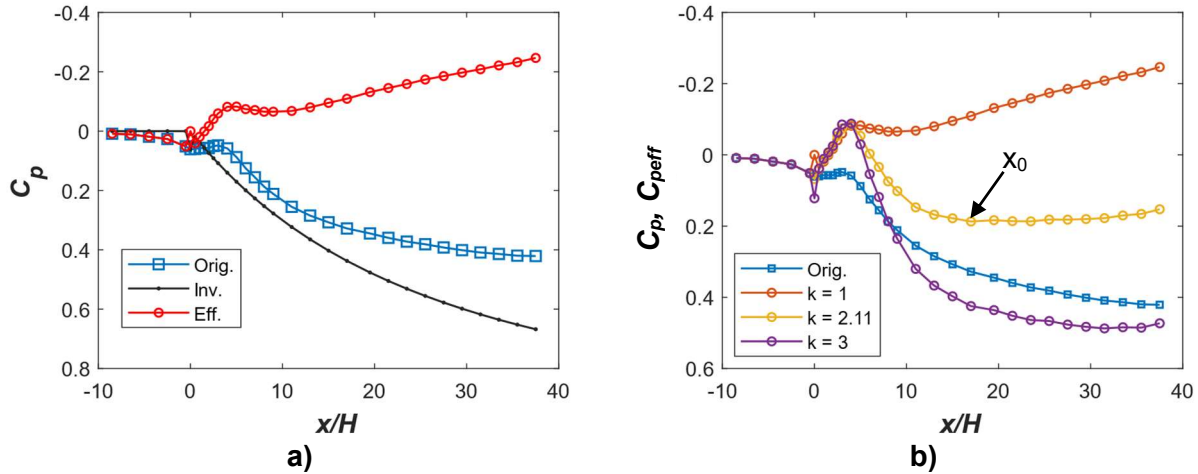


Figure 32. C_p distributions with top wall deflection, $\alpha = 10$ deg.

Test Case 13: 2D Curved Backward Facing Step

The 13th test case for the new method for predicting reattachment location is the 2D curved backward facing step from the Turbulence Modeling Resource website [31]. This geometry is somewhat similar to the Buice-Eaton diffuser or 2D convex ramp. The main difference is that the ramp surface is concave, which was convex in the 2D ramp and an inclined wall in Buice-Eaton diffuser. Another difference is that the results are from an LES simulation rather than an experiment. The LES simulations were performed using an incompressible code, and the results are documented in Ref. [39]. The incoming flow velocity is 6.5 m/s, and the Reynolds number is 13700.

The time-average flow field over the curved step is shown in Fig. 33a. As shown in this figure, flow separates over the ramp near $x/H = 0.83$, and the separation bubble extends on the flat plate portion. The corresponding C_p distribution is given in Fig. 33b. As typical, the flow accelerates upstream of the step following by a sudden increase in the pressure due to flow separation. The slope change due to flow separation in the C_p distribution is also visible. Further downstream, the pressure reaches a local maximum near $x/H = 6.01$ and then gradually decreases over the flat plate portion of the model. The LES simulation predicted the reattachment location as $x/H = 4.35$. The new method predicted the flow reattachment location to be at $x/H = 4.45$. This corresponds to about 2% error compared to the LES result (Table 14).

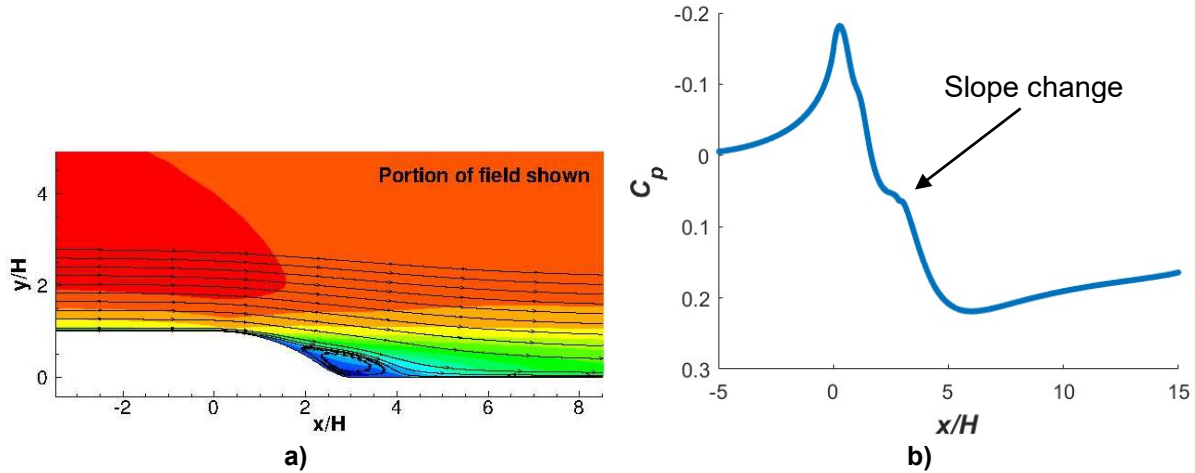


Figure 33. a) LES Simulated flow field and b) C_p distribution over the curved step [39].

Table 14. Comparison of the reattachment locations for Case 13.

LES	S(x)	Error (%)
4.355	4.452	2.2

Test Case 14: 2D Periodic Hill

The 14th test case for the new x_r prediction method is the flow over a 2D periodic hill. This case has been studied both numerically and experimentally in the literature [31] and is a challenging case for CFD simulations. The most interesting features that differentiate it from the previous test cases are its streamwise periodicity and alternating adverse (flow deceleration) and favorable (flow acceleration) pressure gradient effects. The typical flow field over the 2D periodic hill is depicted in Fig. 34. The x_r prediction method is tested for two different configurations, where the flow over the periodic hill was simulated with LES [40] and with Direct Numerical Simulations (DNS) [41]. Except from the simulation methods, the main difference between the two configurations is the Reynolds number based on the hill height (H), which are $Re_H = 10,575$ and $Re_H = 2,800$ for LES and DNS configurations, respectively. Different Reynolds numbers resulted in different sized separation bubbles.

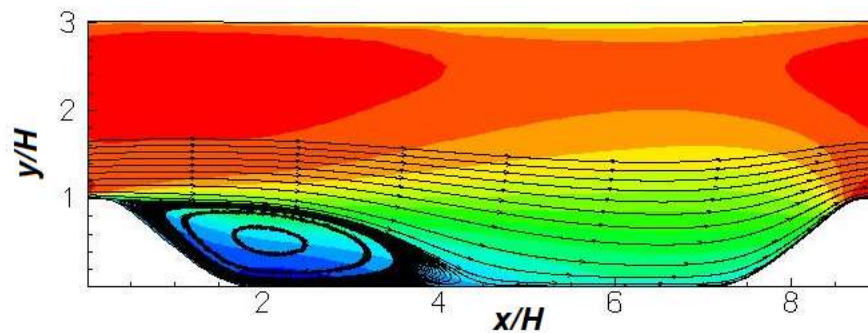


Figure 34. LES simulation of flow field over the periodic hill [40].

The periodic hill configuration is also a challenging case for the x_r prediction method. This is mainly due to the favorable pressure gradient region downstream of the hill. The existence of the forward-facing hill increases the pressure and does not allow natural flow relaxation downstream of flow separation. Therefore, there is no local pressure maximum downstream of reattachment to be used in the new prediction method (Fig. 35). Note that there is a local pressure maximum near $x/H = 7.64$, but it also has the FPG effect of the forward hill and cannot be used as a reference location. To overcome this problem, the maximum pressure location on the top wall (Fig. 35) is treated as the location of x_0 . Since there is no actual local pressure maximum, the normalization $(1 - [C_p]_{x_0})$ term in equation (12) is not evaluated. The pressure distribution used in the prediction method [equation (12)] is simply the pressure drop from x_0 .

$$\overline{C_p} = C_p - [C_p]_{x_0}. \quad (20)$$

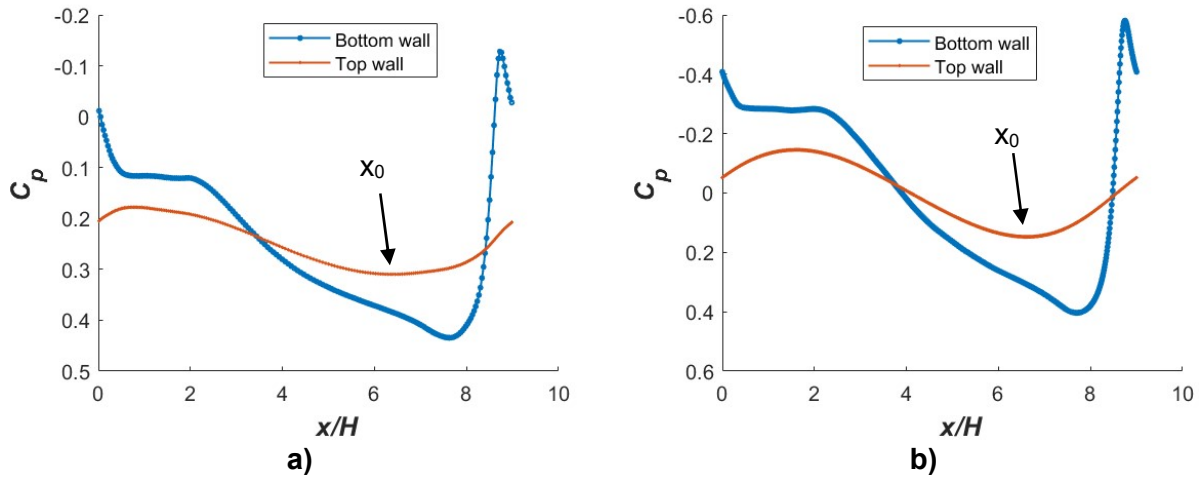


Figure 35. C_p distributions over the periodic hill a) LES simulation [40] and b) DNS simulation [41].

With these modifications, the predicted reattachment location agrees well with the highly resolved numerical simulations. As shown in Table 15, the reattachment location of the LES simulation is between 4.6–4.7 [31]. The new method predicted the reattachment location to be at $x/H = 4.67$, which deviates less than 1% from the CFD prediction. Similarly, while DNS results indicated the reattachment location at $x/H = 5.50$ for lower Reynolds number configuration, the new prediction method resulted in $x/H = 5.61$, which is 2% off from the DNS result.

Table 15. Comparison of the reattachment locations for Case 14.

Method	CFD	$S(x)$	Error (%)
LES	4.65	4.67	0.5
DNS	5.5	5.61	2.0

Test Case 15: Axisymmetric Separated Boundary Layer

The last test case for the new x_r prediction method is the axisymmetric subsonic flow that was reported in Ref. [42]. The experiment utilized a circular cylinder in a tunnel where an APG was imposed over a portion of the flow by diverging the four tunnel side walls (Fig. 36a). Each wall

was deflected by 0.045 m resulting in an area expansion ratio of about 1.6. The flow conditions are given as a Mach number of $M = 0.088$ and a Reynolds number of $Re/m = 2 \times 10^6$. In the experiment, the tunnel sidewall boundary layers were thinned via boundary layer suction. The resulting APG was severe enough that the flow on the cylinder is separated (Fig. 36a). Because of the divergence of the side walls and boundary layer suction on the side walls, the generated C_p distribution is too aggressive downstream of flow separation. In fact, the streamlines of the resultant flow appear to be a diverging (APG) – converging (FPG) channel similar to the periodic hill cases. Therefore, a procedure similar to the periodic hill case is followed, where the equation 20 (i.e., pressure drop from the local pressure maximum, x_0) is used in the x_r prediction method. This resulted in a predicted reattachment location of $x = 0.217$ that is in very good agreement with the reattachment location obtained by C_f measurements (Table 16). Although this corresponds to about 1.3% deviation from the experiment, this difference is possibly due to the round off error in the experiment ($x_r = 0.22$).

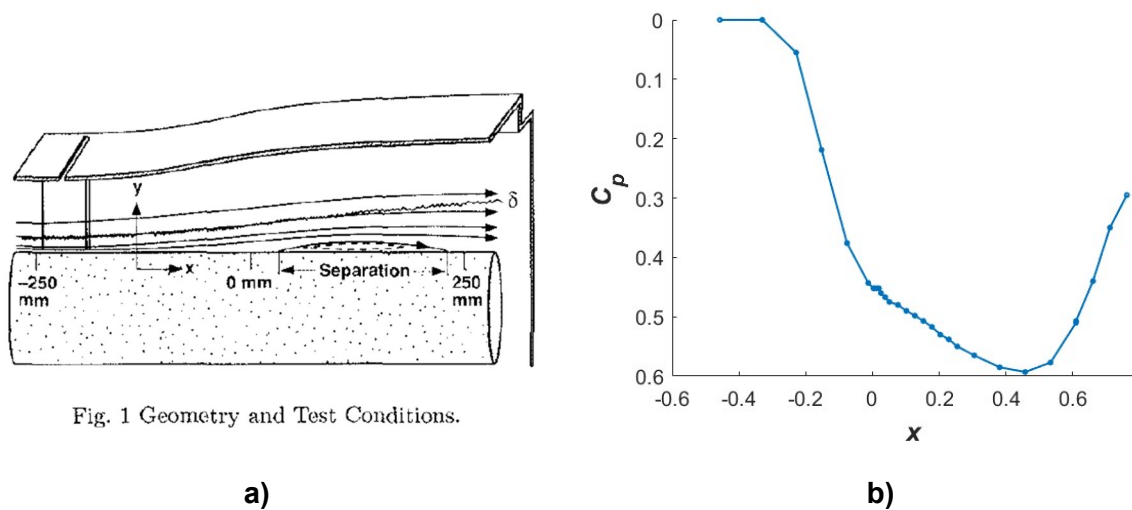


Figure 36. a) Description of axisymmetric separated flow case [42] and b) its C_p distribution.

Table 16. Comparison of the reattachment locations for Case 15.

C_f	$S(x)$	Error (%)
0.22	0.217	1.3

Summary

A new method for predicting turbulent reattachment location has been developed. The method predicts the reattachment location using the surface static pressure distribution and requires only a single empirical parameter. The method is based on the well-known Stratford flow separation criterion, which results from an approximate solution to the equations of motion within the boundary layer.

Starting from a brief explanation of the Stratford separation criterion, the criterion is applied to the pressure distribution of the NASA hump model to predict the flow separation location. The separation location predicted by the Stratford criterion agreed reasonably well with the separation location reported in the literature. Then, the Stratford separation criterion was modified, and a new empirical parameter was obtained for reattaching flows. In order to apply the Stratford

criterion, the reference location is moved to the maximum-pressure location downstream of flow reattachment. The pressure distribution is scaled using the method introduced by Roshko et al. [13] that was originally developed by Chapman [9] for supersonic base flow problems.

The new method has been tested with 30 different flow configurations. Most of these test cases are obtained from the Turbulence Modeling Resource website [31], where detailed experimental results were provided. In addition to the typical low Reynolds number tests, the new method was successfully tested on the high Reynolds number flows, 2D planar flows, axisymmetric flows, shock boundary layer interaction. The test models include the NASA hump model with various AFC configurations, transonic bump, transonic diffuser, plane diffuser, backward facing step with inclined wall, 2D convex ramp, curved backward facing step, 2D periodic hill, and axisymmetric cylinder with APG. Most of the test cases are experimental; however, highly resolved numerical (LES and DNS) results were also used in testing the new method.

Figure 37 is the summary of the testing results of the new x_r prediction method. Overall agreement is very good with the reported data. The error is less than 5% and typically around 2%. Note that the error in the first six test cases appears slightly more than typical 2% error margin. This is because the method is compared to the C_p' -peak method, which is known to predict early flow reattachment.

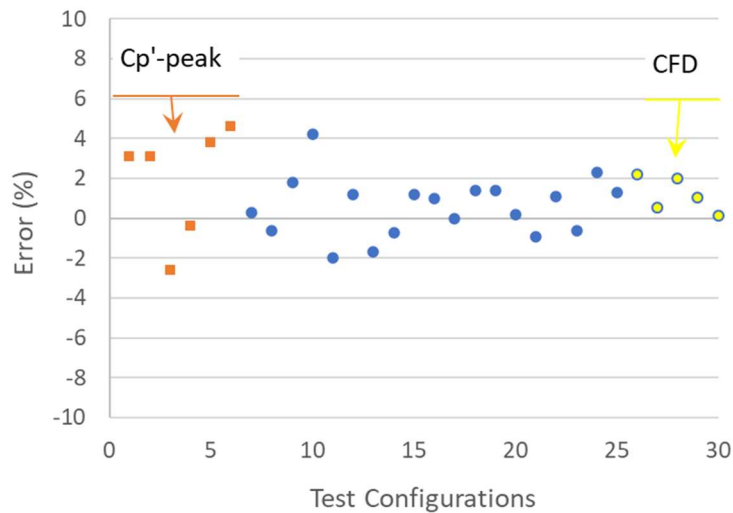


Figure 37. Summary of the test results.

Since the reattachment location is one of the most important parameters for the assessment of turbulent models in separated flows, turbulent modeling efforts would benefit from more precise prediction of the reattachment location. In addition, since one of the main goals of flow control methods is to reduce flow separation, accurately locating flow reattachment results in a better performance index in assessing the flow control methods.

The method developed is extremely simple and provides a quick way, especially for experimental researchers, to find the flow reattachment location just using the pressure distribution. Considering the pressure measurement is more common and relatively easier, for example compared to skin friction measurement, obtaining the flow reattachment location using the pressure distribution is a great benefit to the fluid dynamics community.

References

1. Cebeci, T., Mosinskis, G. J., and Smith, A. M. O., "Calculation of Viscous Drag and Turbulent Boundary-Layer Separation on Two-Dimensional and Axisymmetric Bodies in Incompressible Flows," Rept. MDC J0973-01, 1970, Douglas Aircraft Co.
2. Stratford, B. S., "The Prediction of Separation of The Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 5, 1959, pp. 1-16.
3. Cebeci, T., Mosinskis, G. J., and Smith A. M. O, "Calculation of Separation Points in Incompressible Turbulent Flows," *Journal of Aircraft*, Vol. 9., Sept. 1972, pp. 618-624.
4. Head, M. R., "Entrapment in The Turbulent Boundary Layer," R & M No. 3152, 1960, Aeronautical Research Council, England.
5. Goldschmied, F. R., "An Approach to Turbulent Incompressible Separation Under Adverse Pressure Gradients," *Journal of Aircraft*, Vol. 2, No. 2, Feb. 1965, pp. 108-115.
6. Smith, A.M., "High-Lift Aerodynamics," *Journal of Aircraft*, Vol. 12, No. 6, 1975, pp.501-530.
7. Liebeck, R.H. and Ormsbee, A.I., "Optimization of Airfoils for Maximum Lift," *Journal of Aircraft*, Vol. 7, No. 5, 1970, pp.409-416.
8. Driver, D. M. and Seegmiller, H. L., "Features of Reattaching Turbulent Shear Layer in Divergent Channel Flow," *AIAA Journal*, Vol. 23, No. 2, Feb 1985, pp. 163-171.
9. Chapman, D. R., Kuehn, D. M. and Larson, H. K.: "Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on The Effect of Transition," NACA Report 1356, 1958.
10. Korst, H. H., "A Theory for Base Pressures in Transonic and Supersonic Flow," *Journal of Applied Mechanics*. Vol. 23, 1956, pp. 593-600.
11. Nash, J. F.: "An Analysis of Two-Dimensional Turbulent Base Flow Including the Effect of The Approaching Boundary Layer," National Physical Laboratory Aero Report 1036, July 1962.
12. McDonald, H., "Turbulent Shear Layer Reattachment with Special Emphasis on The Base Pressure Problem," *Aeronautical Quarterly*, Vol. 15, August 1964.
13. Roshko, A and Lau, J. C., "Some Observations in Transition and Reattachment of a Free Shear Layer in Incompressible Flow," Proc. Heat Transfer and Fluid Mechanics Institute, 1965, pp. 157-167. Stanford, CA: Stanford Univ. Press.
14. Adams, E.W. and Johnston, J.P., "Effects of The Separating Shear Layer on The Reattachment Flow Structure Part 1: Pressure and Turbulence Quantities," *Experiments in Fluids*, Vol. 6, No. 6, 1988, pp.400-408.
15. Ruderich, R. and Fernholz, H.H., "An Experimental Investigation of a Turbulent Shear Flow with Separation, Reverse Flow, and Reattachment," *Journal of Fluid Mechanics*, 163, 1986, pp.283-322.
16. Hudy, L.M., Naguib, A.M. and Humphreys Jr, W.M., "Wall-Pressure-Array Measurements Beneath A Separating/Reattaching Flow Region," *Physics of Fluids*, Vol. 15 No. 3, 2003, pp.706-717.
17. Eaton, J.K. and Johnston, J.P., "A Review of Research on Subsonic Turbulent Flow Reattachment," *AIAA Journal*, Vol. 19, No. 9, 1981, pp.1093-1100.
18. Stratford, B.S. and Gadd, G.E., 1954. "Flow in The Laminar Boundary Layer Near Separation," Aeronautical Research Council, London, R & M. No. 3002.
19. <https://cfdval2004.larc.nasa.gov/case3.html>. Accessed July 01, 2021.
20. Seifert, A., and Pack, L.G. "Active Flow Separation Control on Wall-Mounted Hump at High Reynolds Numbers," *AIAA Journal*, 2020, Vol. 40, pp.1363–1372.
21. Greenblatt, D., Paschal, K.B., Yao, C.S., Harris, J., Schaeffler, N.W., and Washburn, A.E. "Experimental Investigation of Separation Control Part 1: Baseline and Steady Suction," *AIAA Journal*. 2006, Vol. 44, pp. 2820–2830.
22. Greenblatt, D., Paschal, K.B., Yao, C.S., and Harris, J., Experimental Investigation of Separation Control Part 2: Zero Mass Efflux Oscillatory Blowing. *AIAA Journal*, Vol. 44, No. 12, 2006b, pp. 2831–2845.
23. Rumsey, C., Gatski, T., Sellers, W., Vatsa, V., and Viken, S. "Summary of the 2004 CFD Validation Workshop on Synthetic Jets and Turbulent Separation Control," AIAA Paper 2004-2217, 28 June–01 July 2004.
24. Glauert, M.B. "The Design of Suction Aerofoils With A Very Large C_L -Range," Aeronautical Research Council: London, R&M 2111, UK, 1945.
25. Koklu, M. "A Numerical and Experimental Investigation of Flow Separation Control Over a Wall-Mounted Hump Model," AIAA Paper 2018-1280, 8–12 January 2018.
26. Koklu, M., "Application of Sweeping Jet Actuators on The NASA Hump Model and Comparison with CFDVAL2004," Experiments. AIAA Paper 2017-3313, June 2017.
27. Stratford, B.S., "An Experimental Flow with Zero Skin Friction Throughout Its Region of Pressure Rise," *Journal of Fluid Mechanics*, 1959. Vol. 5, No. 1, pp.17-35.
28. Adams, E.W. and Johnston, J.P., "Effects of The Separating Shear Layer on The Reattachment Flow Structure Part 2: Reattachment Length and Wall Shear Stress," *Experiments in Fluids*, 1988. Vol. 6, No. 7, pp.493-499.
29. Liebeck, R.H., 1968. "Optimization of Airfoils for Maximum Lift," Doctoral Dissertation, University of Illinois At Urbana-Champaign.
30. Uzun A. and Malik M. R., "Wall-Resolved Large-Eddy Simulation of Flow Separation Over NASA Wall-Mounted Hump," AIAA Paper 2017-0538, January 2017.
31. <https://Turbmodels.Larc.Nasa.Gov/Index.Html>. Accessed July 01, 2021.

32. Koklu, M., "Steady and Unsteady Excitation of Separated Flow Over the NASA Hump Model," AIAA Paper 2018-4016, June 2018.
33. Bachalo, W.D. and Johnson, D.A., "Transonic, Turbulent Boundary-Layer Separation Generated on an Axisymmetric Flow Model," *AIAA Journal*, 1986. Vol. 24, No. 3, pp.437-443.
34. Uzun A. and Malik M. R., "Wall-Resolved Large-Eddy Simulations of Transonic Shock-Induced Flow Separation," AIAA Paper 2018-3854, January 2018.
35. Bogar, T. J., Sajben, M., and Kroutil, J. C. "Characteristic Frequencies of Transonic Diffuser Flow Oscillations," *AIAA Journal*, Vol. 21, No. 9, pp. 1232-1240, September 1983.
36. <https://www.grc.nasa.gov/www/wind/valid/transdif/transdif.html>. Accessed July 01, 2021.
37. Buice, C. U. and Eaton, J. K., "Experimental Investigation of Flow Through an Asymmetric Plane Diffuser," *Journal Fluids Engineering*, Vol. 122, June 2000, pp. 433-435.
38. Simmons, D. J., Thomas, F. O., and Corke, T. C., "Smooth Body Flow Separation Experiments and Their Surface Topology Characterization," AIAA Paper 2019-3085, June 2019.
39. Bentaleb, Y., Lardeau, S., and Leschziner, M. A., "Large-Eddy Simulation of Turbulent Boundary Layer Separation from A Rounded Step," *Journal of Turbulence* 2012. Vol. 13, No. 4, pp. 1-28.
40. Frohlich, J., Mellen, C. P., Rodi, W., Temmerman, L., and Leschziner, M. A., "Highly Resolved Large-Eddy Simulation of Separated Flow in A Channel with Streamwise Periodic Constrictions," *Journal Fluid Mechanics*. 526, 2005, pp. 19-66.
41. Balakumar, P., "DNS/LES Simulations of Separated Flows at High Reynolds Numbers," AIAA Paper 2015-2783, June 2015.
42. Driver, D. M., "Reynolds Shear Stress Measurements in A Separated Boundary Layer Flow," AIAA Paper 91-1787, June 1991.