Extinction of the Stagnation Point Diffusion Flame: Effect of Conductive Heat Loss into Solid Interior

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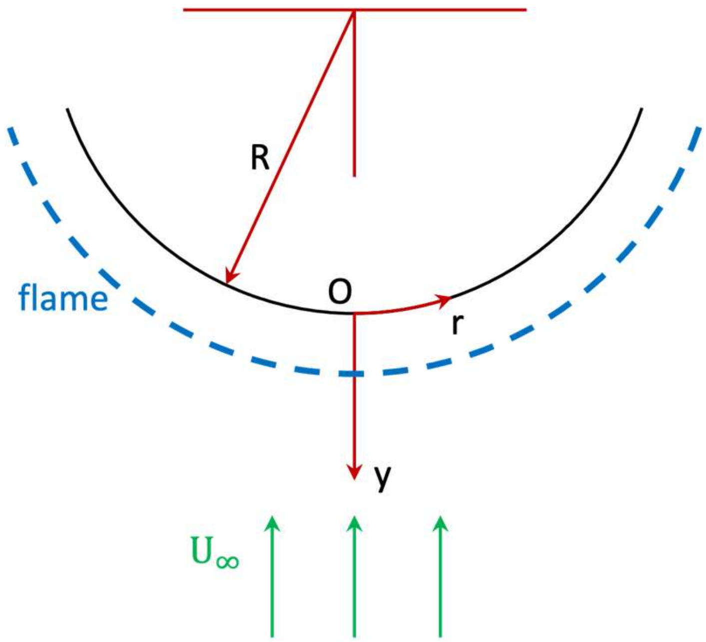
2021 Annual Meeting of American Society for Gravitational and Space Research
Baltimore Maryland, November 3-6, 2021

Backgrounds

- T'ien et al. [1] and T'ien [2] developed a theoretical model to study the near-limit flame in the stagnation point boundary layer of condensed fuel.
- Gas-phase conservations of mass, momentum, energy, and species and one-step second-order gas-phase reaction are considered.
- The solid pyrolysis follows the Arrhenius law.
- Assumptions include constant Prandtl and Schmidt number and unit Lewis number, ideal gas, constant specific heat, and constant product of density and viscosity.
- Gas-phase radiation is neglected.
- One-dimensional solid temperature and the solid phase reaches the steady-state conduction-convection energy balance.
- Model in this study is extended from T'ien [2]



[1] J. S. T'ien, S. N. Singhal, D. P. Harrold, and J. M. Prahl, Combustion and extinction in the stagnation-point boundary layer of a condensed fuel, Combustion and Flame, Vol. 33, pp. 55-68, 1978.
[2] J. T'ien, Diffusion flame extinction at small stretch rates: the mechanism of radiative loss, Combustion and flame, Vol. 65(1), pp. 31-34, 1986.

Backgrounds

- Introducing the Howarth–Dorodnitsyn transformation:

$$\eta \equiv \frac{r^{\epsilon} u_e}{2s^{1/2}} \int_0^y \rho dy \qquad s \equiv \int_0^r \rho_e \mu_e u_e r^{2\epsilon} dr$$

where $\epsilon = 1$ for axisymmetric flow

$\epsilon = 0$ for two-dimensional flow

- The conservation equations become:

$$f''' + ff'' = \frac{1}{1+\epsilon}[(f')^2 - 4\theta]$$

$$(1/Pr)\theta'' + f\theta' = -qw$$

$$(1/Sc)Y_i'' + fY_i' = N_i w$$

where $i = F, O_2, CO_2, H_2O$

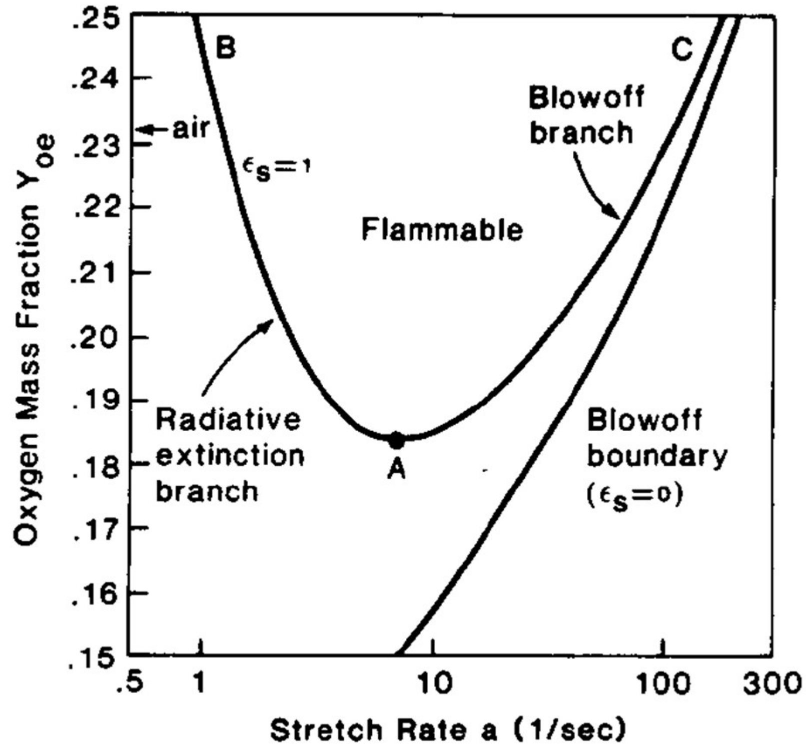
$w = DY_F Y_O e^{-E/\theta}$ and $D = \frac{2}{1+\epsilon}\frac{B}{a}$

- The energy balance at the solid surface follows:

$$\theta_w' = -f_w Pr\left(\theta_w - \frac{c_s}{c_p} + L\right) + S\theta_w^4$$

where $S = \frac{\sigma \varepsilon_s T_e^3}{\lambda_e}\left(\frac{\mu_e}{\rho_e a}\right)^{1/2}$

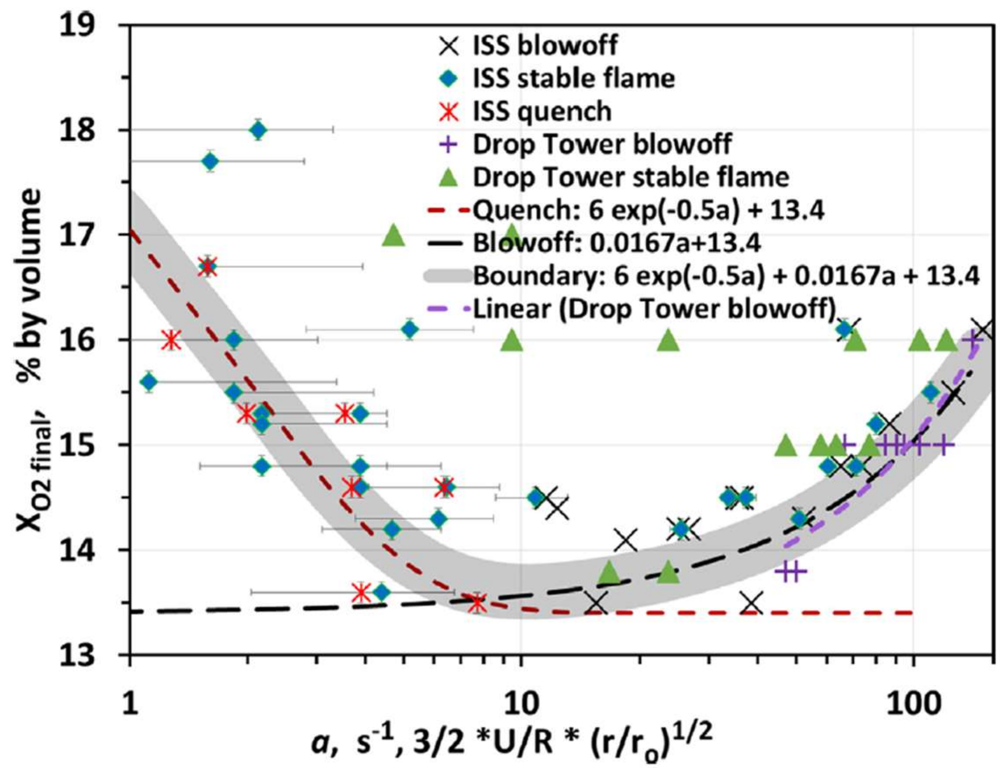
$$-f_w = \left(\frac{2}{1+\epsilon}\frac{b}{\rho_e \mu_e a}\right)^{1/2} \exp\left(-\frac{E_w}{\theta_w}\right)$$



Predicted Flammability Boundary of PMMA

Backgrounds

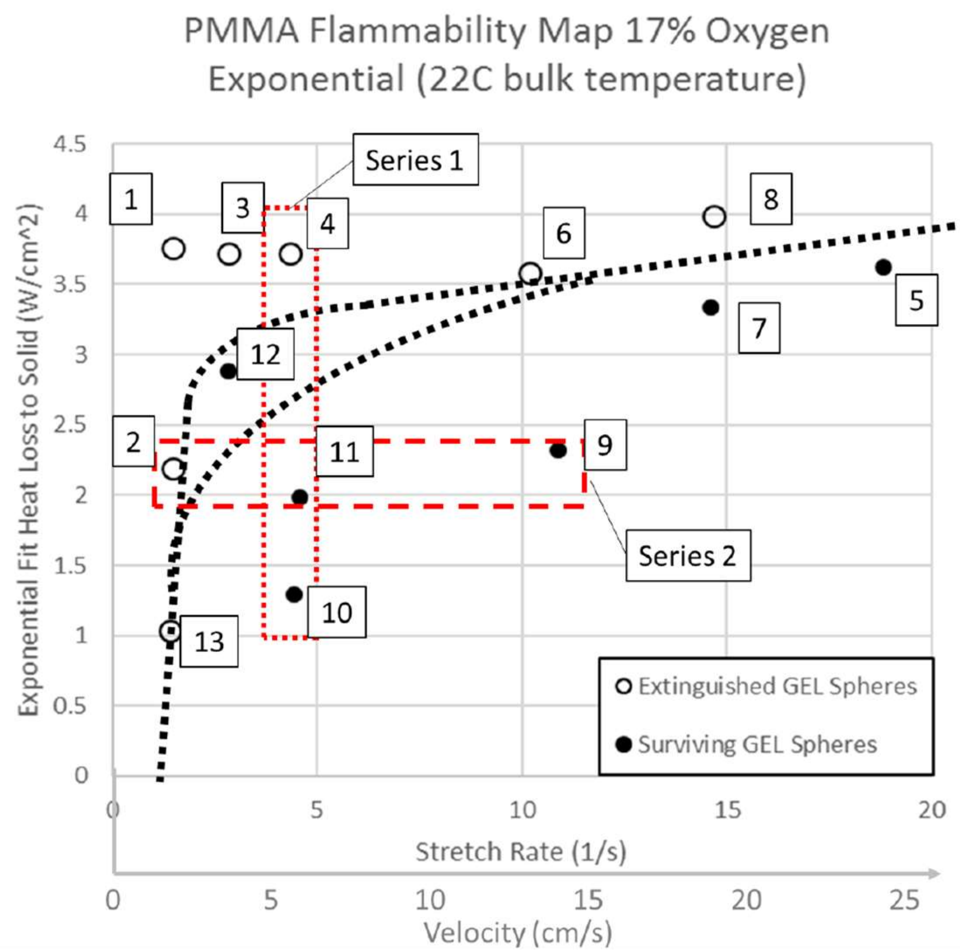
- Olson and Ferkul [1] studied the microgravity material flammability boundary of PMMA rods in BASS-II using a 7.6 cm x 7.6 cm x 17 cm flow duct in the Microgravity Science Glovebox aboard the International Space Station and complementary 5.18s drop tower tests
- The minimum occurs at a stretch rate of 8 – 12 s^{-1} and oxygen concentration of 13.5%.



[1] S. L. Olson, and P. V. Ferkul, Microgravity flammability boundary for PMMA rods in axial stagnation flow: experimental results and energy balance analyses, Combustion and Flame, Vol. 180, pp. 217-229, 2017.

Backgrounds

- Johnston [1] mapped the 4-cm-diameter PMMA sphere flammability boundary in terms of the conductive heat loss into the solid interior using the NASA Zero-G Facility 5.18s drop tower in support of the Growth and Extinction Limit (GEL) project.
- Within the test range (17% Oxygen by volume, 2 – 25 cm/s concurrent flow), the conductive heat loss increases with the stretch rate.
- $a = \frac{3}{2}\frac{U}{R}$



[1] M. C. Johnston, Growth and Extinction Limits: Ground Based Testing of Solid Fuel Combustion in Low Stretch Conditions in Support of Space Flight Experiments, Ph.D. dissertation, Case Western Reserve University, 2018.

Objective and Approach

- The planned microgravity experiment Growth and Extinction Limit (GEL) is designed to study the material flammability and extinction of thermally thick solid with an emphasis to quantify the effect of conductive heat loss into the solid interior by measuring the in-depth temperature profile.

- To obtain a more detailed understanding of the extinction phenomena, the one-dimensional axisymmetric stagnation-point diffusion flame model developed by T'ien (1986) is expanded to account for the excessive heat loss into the solid beyond the steady burning state.

Model Configuration – Gas Phase

Continuity $$\frac{1}{r^{\epsilon}}\frac{\partial(r^{\epsilon}\rho u)}{\partial r}+\frac{\partial(\rho v)}{\partial y}=0 \quad (1)$$

Momentum $$\rho\left(u\frac{\partial u}{\partial r}+v\frac{\delta u}{\delta y}\right)=-\frac{\delta p}{\delta r}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (2)$$

Species $$\rho u\frac{\partial Y_i}{\partial r}+\rho v\frac{\partial Y_i}{\partial y}=\frac{\partial}{\partial y}\left(\rho D\frac{\partial Y_i}{\partial y}\right)+N_i\widehat{w} \quad (3)$$

where $i=F,O_2,CO_2,H_2O;\ N_F=-1,N_{O_2}=-1.92,N_{CO_2}=2.2,N_{H_2O}=0.72$

Energy $$\rho uc_p\frac{\partial T}{\partial r}+\rho vc_p\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\lambda}{c_p}\frac{\partial T}{\partial y}\right)-\frac{\partial}{\partial y}\hat{q}_r''+\hat{q}\widehat{w} \quad (4)$$

Two-flux gas-phase radiation model (consider non-scattering gray media):

$$\hat{q}_r''=\hat{q}^+-\hat{q}^- \quad (5)$$

$$\begin{cases}\frac{1}{2K}\frac{d\hat{q}^+}{dy}+\hat{q}^+=\hat{q}_b\\ -\frac{1}{2K}\frac{d\hat{q}^-}{dy}+\hat{q}^-=\hat{q}_b\end{cases} \quad (6)$$

where $K=C\cdot\kappa_p$, $C=-0.0346-0.152\ln\left(\int_0^\infty \kappa_p\,dy\right)$ [1], and
κ_p the Plank mean absorption coefficient $\kappa_p=p_e\cdot\left(X_{CO_2}\cdot\kappa_{p,CO_2}+X_{H_2O}\cdot\kappa_{p,H_2O}\right)$

[1] J. Rhatigan, H. Bedir, and J. T'ien, Gas-phase radiative effects on the burning and extinction of a solid fuel, Combustion and Flame, Vol. 112, pp. 231-241, 1998.

Model Configuration – Solid Surface

Energy Balance $$\lambda_g \frac{\partial T}{\partial y}\bigg|_g = \lambda_s \frac{\partial T}{\partial y}\bigg|_s + \dot{m}''\hat{L} + \hat{q}''_{r,out} \qquad (7)$$

For semi-infinite thick solid, the steady state energy balance:

$$\rho_s \bar{r} c_s \frac{\partial \bar{T}}{\partial y} - \lambda_s \frac{\partial^2 \bar{T}}{\partial y^2} = 0 \qquad (8)$$

y=0 on moving surface, $\bar{r}$ is the steady surface regression rate ($\bar{\dot{m}}'' = \rho_s \bar{r}$), bars denotes steady state

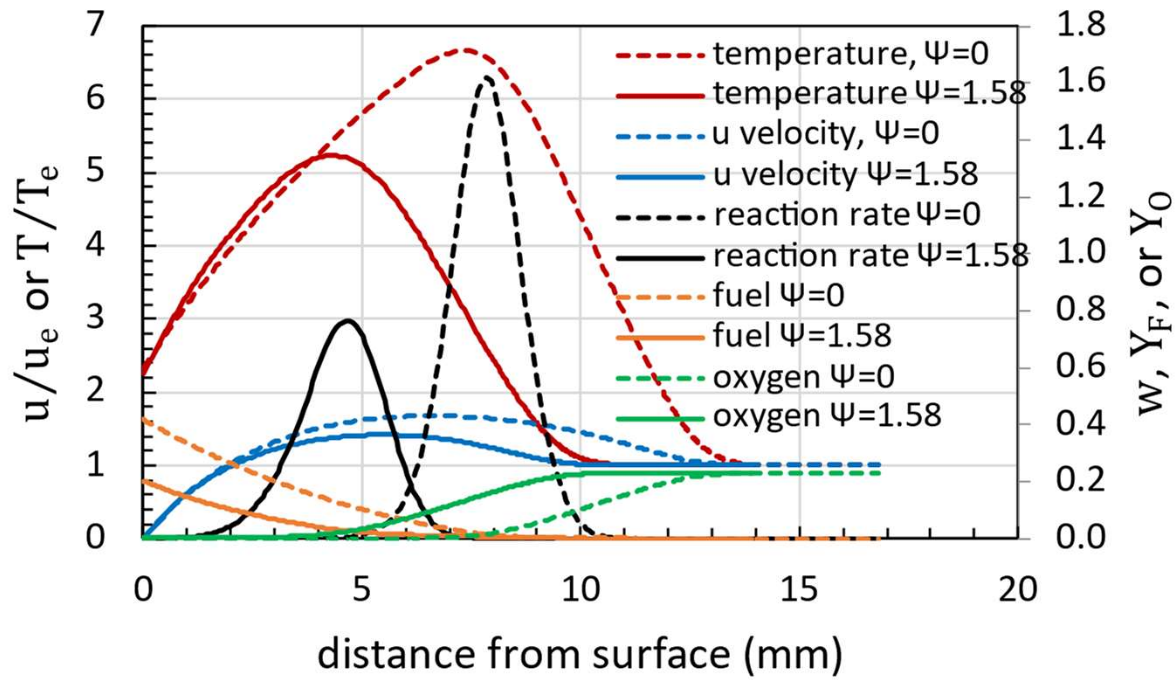
From (8): $$\lambda_s \frac{\partial \bar{T}}{\partial y}\bigg|_s = \bar{\dot{m}}'' c_s (\bar{T}_w - T_\infty) \qquad (9)$$

From (7): $$\lambda_g \frac{\partial \bar{T}}{\partial y}\bigg|_g = \bar{\dot{m}}'' \left[c_s (\bar{T}_w - T_c) + \hat{L} \right] + \hat{q}''_{r,out} \text{ (Steady State)} \qquad (10)$$

Define $$\Psi = \frac{\lambda_s \frac{\partial T}{\partial y}\Big|_s - \lambda_s \frac{\partial \bar{T}}{\partial y}\Big|_s}{\lambda_s \frac{\partial \bar{T}}{\partial y}\Big|_s}$$ (dimensionless excessive conductive heat loss) (11)

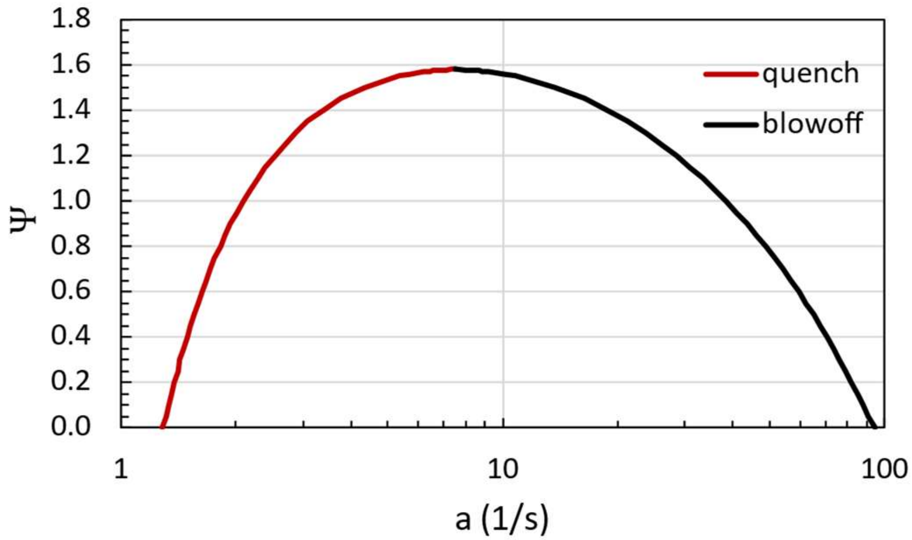
From (7): $$\lambda_g \frac{\partial T}{\partial y}\bigg|_g = (1 + \Psi) \bar{\dot{m}}'' c_s (\bar{T}_w - T_\infty) + \dot{m}''\hat{L} + \hat{q}''_{r,out} \text{ (Transient)} \qquad (12)$$

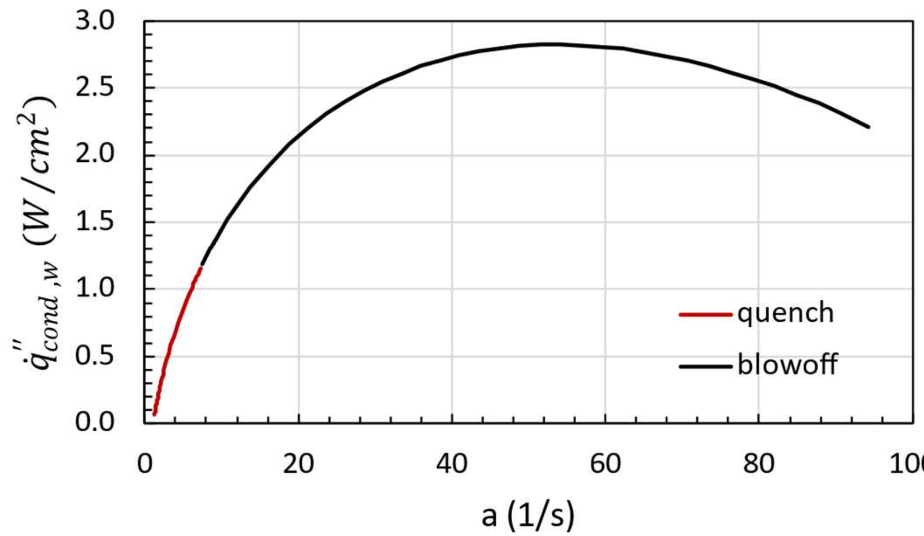
Flame Structure at a Representative Case
$(a = 7.5s^{-1}, P_e = 1atm, Y_{O_2} = 23.1\%)$



- Flame standoff distance decreases to increase the convective heat input to the solid when near extinction.
- Flame becomes fatter (reaction width increases) when near extinction.
- Slightly more fuel vapor and oxygen are left over at the flame standoff distance when near extinction.

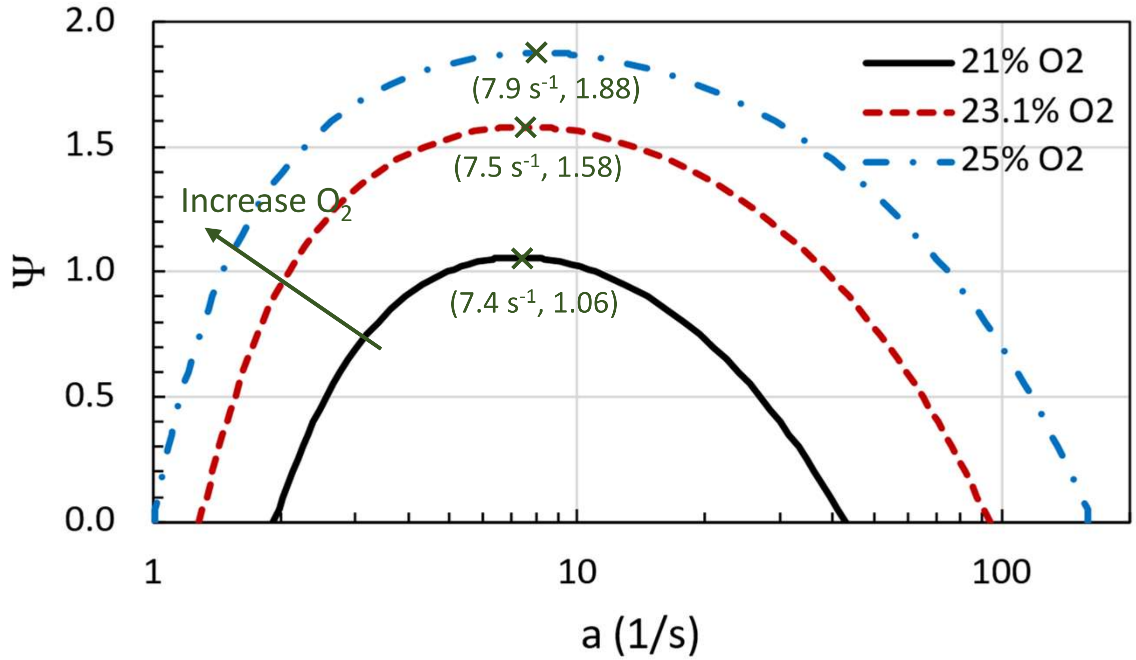
Flammability Boundary – Ψ vs a $(P_e = 1atm, Y_{O_2} = 23.1\%)$





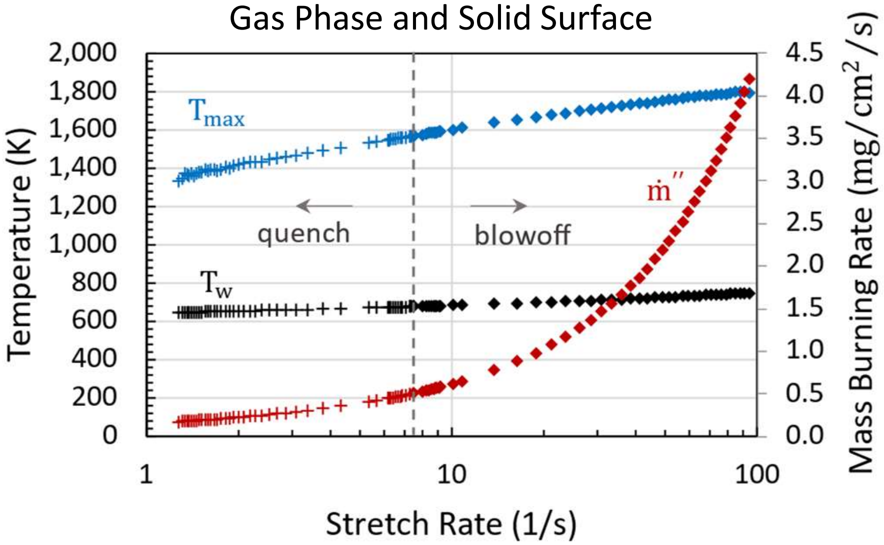
- The dimensionless excessive conductive heat loss (Ψ) increases with the stretch rate on the quenching branch but decreases with the increase of stretch rate on the blowoff branch.
- The maximum Ψ equals to 1.58 with a stretch rate of 7.5 s^{-1}.
- The conductive heat loss into the solid interior at the solid surface increases with the stretch rate on the quenching branch but increases and then decreases on the blowoff branch.

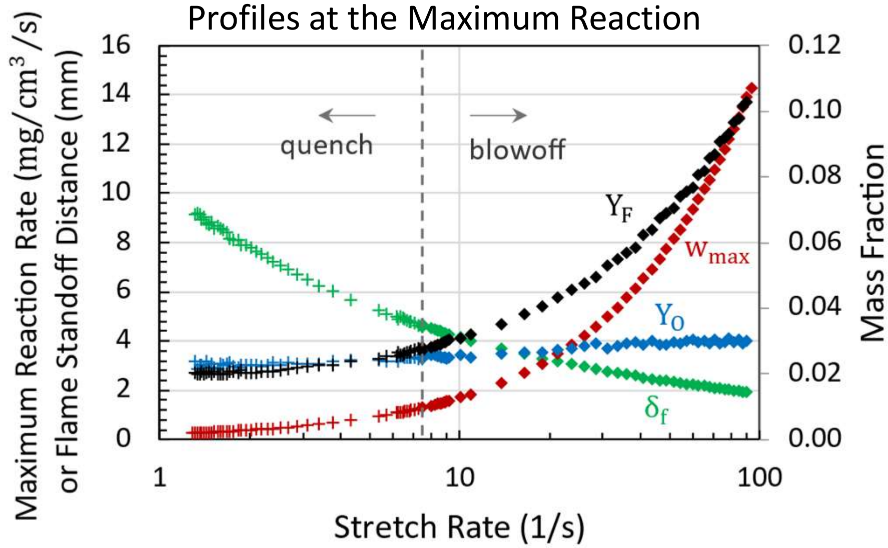
Flammability Boundary – $\Psi\ vs\ a\ (P_e = 1atm)$

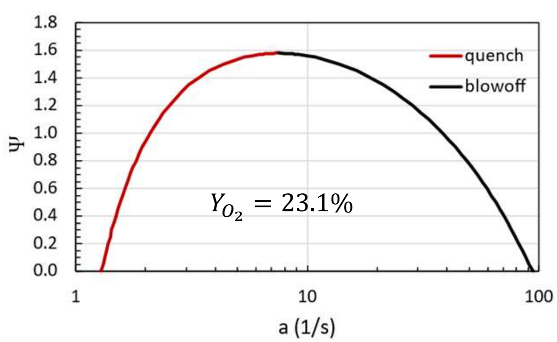


- The material flammability boundary enlarges with the increase of oxygen concentration.
- The stretch rate of maximum dimensionless excessive conductive heat loss (Ψ_{max}) slightly increases with the oxygen concentration.

Flame Profiles along the Flammability Boundary
$(P_e = 1atm, Y_{O_2} = 23.1\%)$







- Reaction intensity and hence the maximum flame temperature increase with stretch rate.
- Flame standoff distance decreases with the increase of stretch rate.
- Mass burning rate at the solid surface increase with stretch rate.
- On the blowoff branch, more fuel vapor is left over in the reaction zone and taken away to sides by the convection flow.
- Oxygen mass fraction in the reaction zone slightly increases but is almost invariant.

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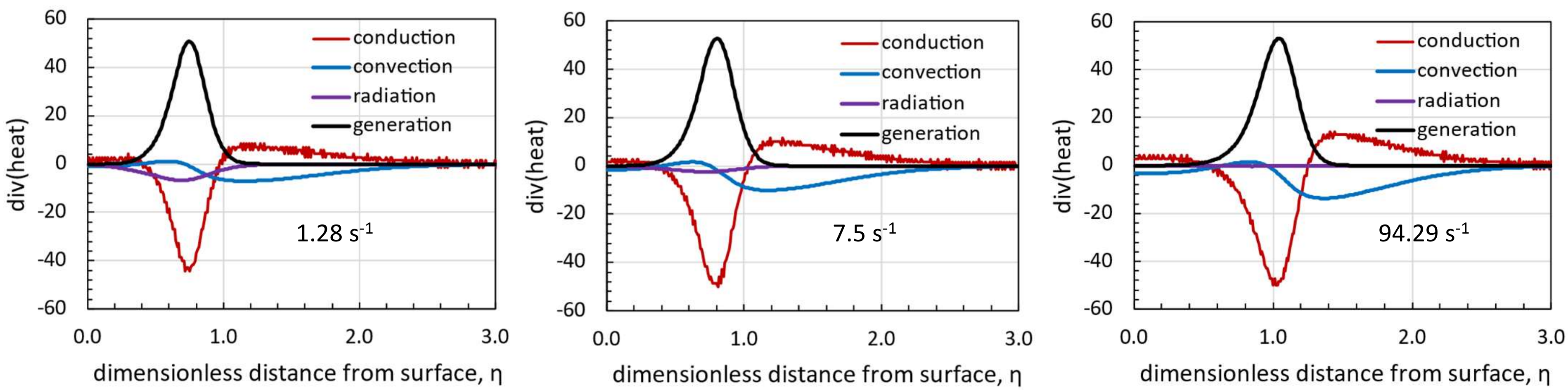
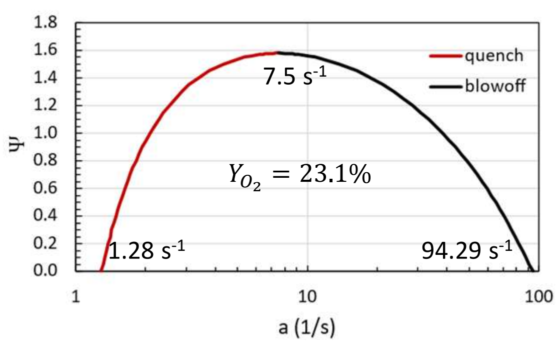
Flame Profiles along the Flammability Boundary
$(P_e = 1atm, Y_{O_2} = 23.1\%)$

Conservation of Energy: $\frac{1}{Pr}\theta'' + f\theta' - \frac{4}{1+\epsilon}\frac{SD_k}{Pr}\left(\theta^4 - \frac{q^+ + q^-}{2}\right) = qw$

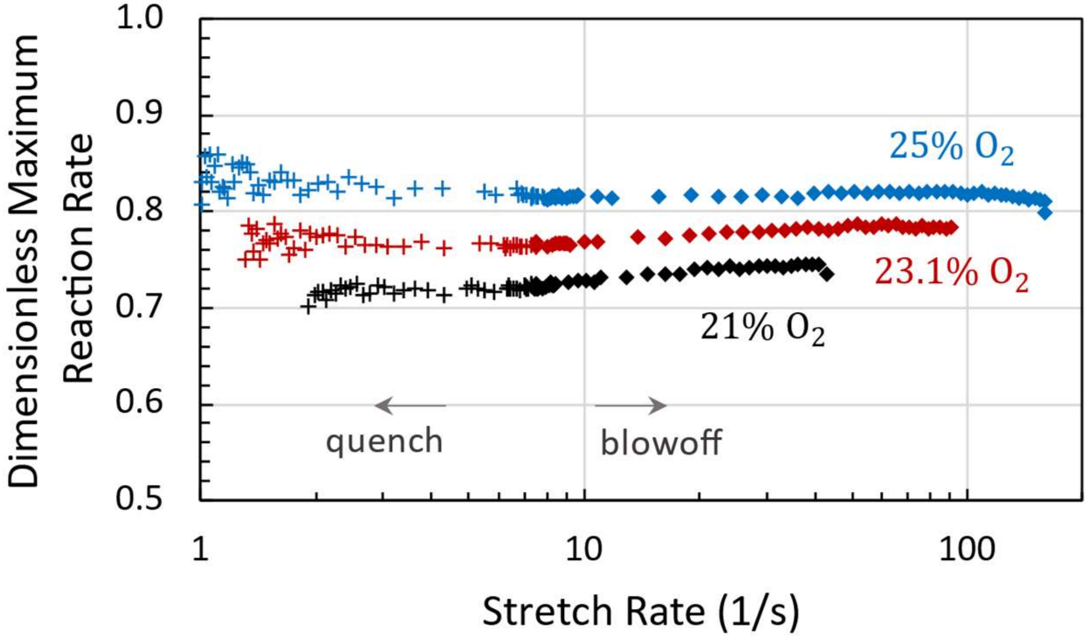
conduction convection radiation generation

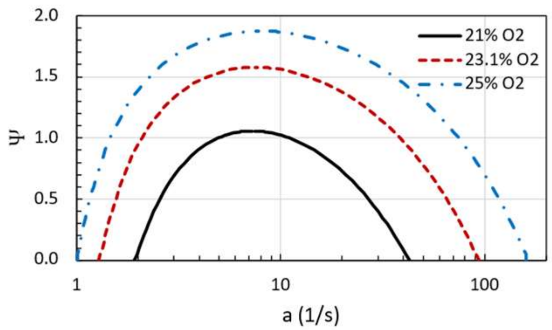




- Reaction width is about 0.5η at different stretch rate. Considering $\frac{dy}{d\eta} = \sqrt{Sc\frac{D}{a}}$, it scales up with diffusion distance.
- Increasing stretch rate leads to decrease of the relative importance of radiation but increase of the relative importance of convection.

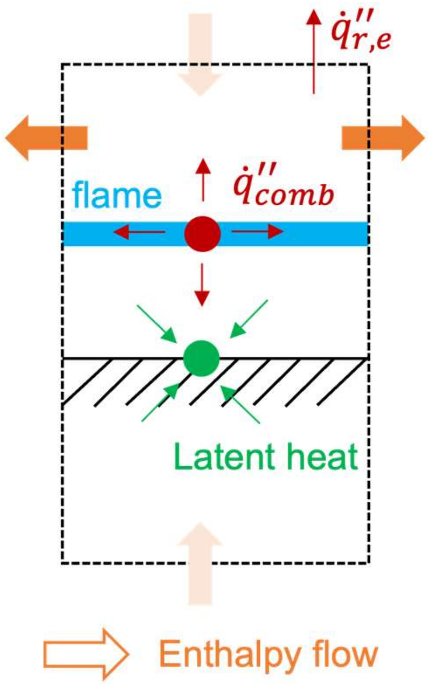
Dimensionless Maximum Reaction Rate ($P_e = 1atm$)

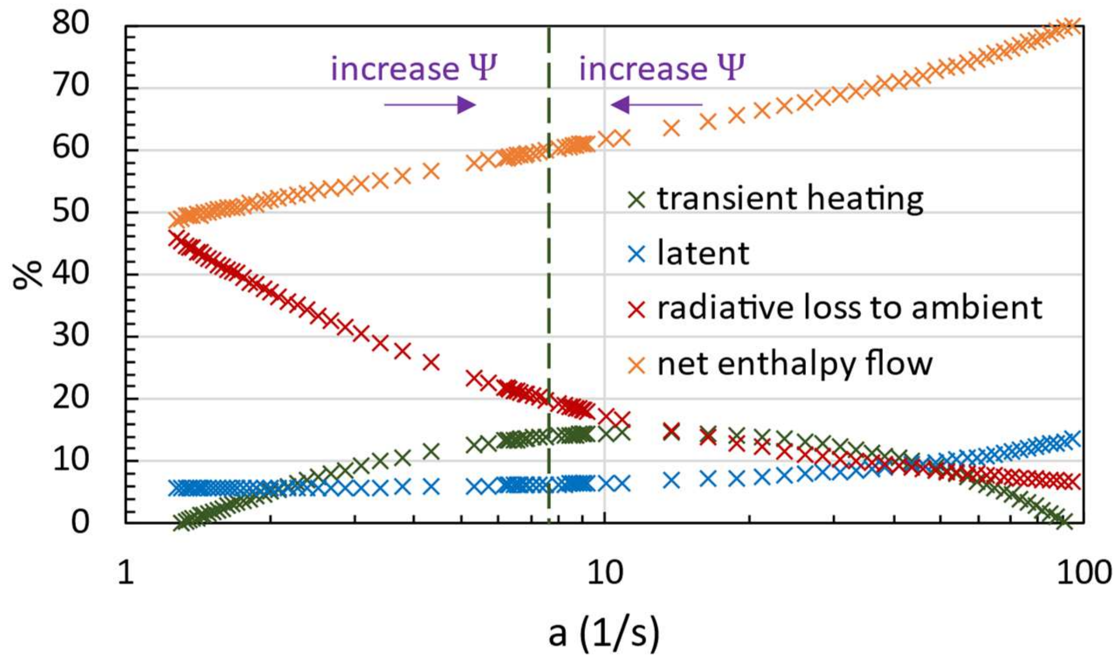


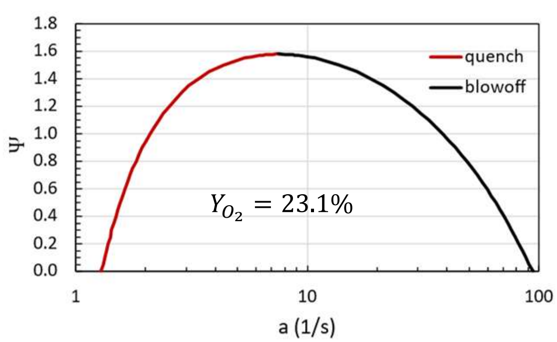


- $w = \frac{\hat{w}}{\rho a} = \frac{B}{a} Y_F \cdot Y_{O_2} e^{-\frac{E}{\theta}}$
- It is the ratio of convection time to the reaction time.
- It has the classical Damhkoler number character.
- It is of the order 1, but slightly increases with oxygen mass fraction.

Heat Transfer Analysis ($P_e = 1atm, Y_{O_2} = 23.1\%$)







- Combustion Heat = Latent Heat + Enthalpy Flow + Radiative Loss + Transient Heating of the Solid
- $\% = \frac{\dot{q}''}{\dot{q}''_{comb}}$
- Radiative heat loss is important on the quenching branch. It explains up to 46% of the total heat generation from combustion.
- Net enthalpy flow to sides increases with stretch rate.

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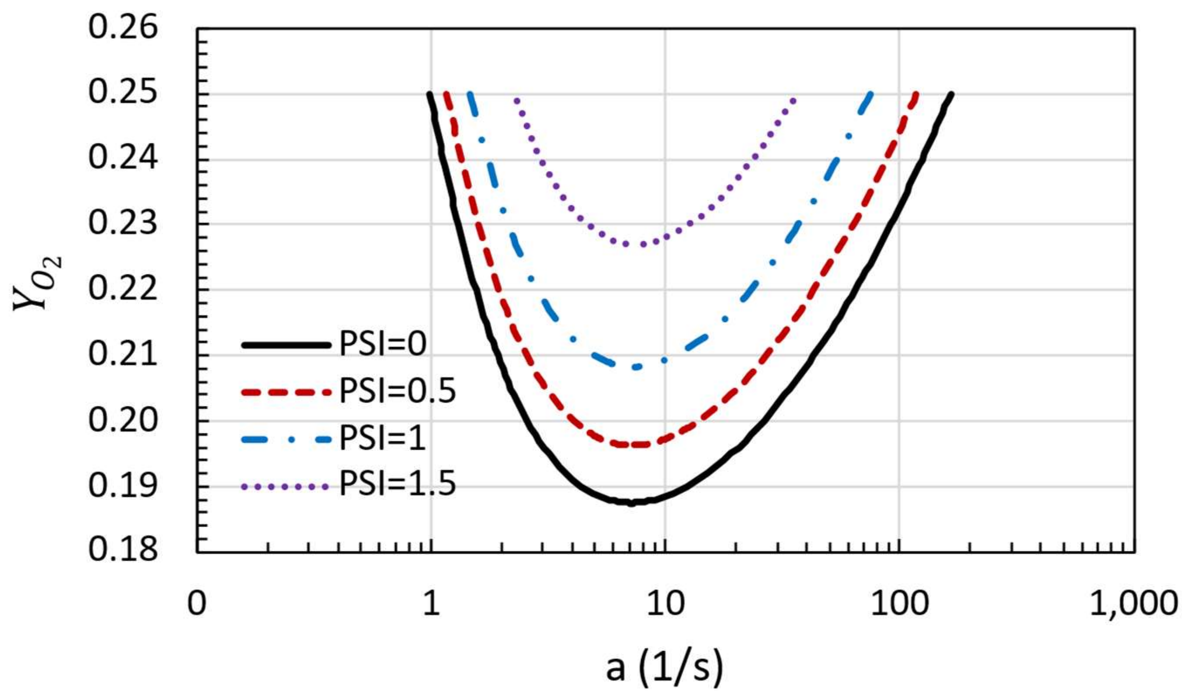
Flammability Boundary – Y_{O_2} vs a $(P_e = 1atm)$



- The classical U-shape flammability boundary is a function of excessive conductive heat loss into the solid interior.

Acknowledgement

- This work is supported by NASA Glenn Research Center under grant number NNX16AE84A.
- We also thank the fruitful discussions with Dr. Paul Ferkul, Dr. Sandra Olson and other members and students in the Solid Fuel Ignition and Extinction (SoFIE) project scientific team.

Thank you!

This slide is only used to generate the plots and will be removed in the final version.