

Three-Dimensional Instantaneous Flowfield Reconstruction Using Planar Spectral Proper Orthogonal Decomposition

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A novel method for three-dimensional flowfield reconstruction is presented that utilizes spectral proper orthogonal decomposition (SPOD). The time-resolved PIV data are acquired in multiple two-dimensional spanwise planes, with adequate streamwise spacing to resolve the out-of-plane wavelength. Knowledge of the out-of-plane phase speed is also required, and for the present case this value was obtained using instantaneous hot-wire measurements. The method is validated by comparing phase-averaged results acquired with a forcing signal to the SPOD reconstructed results using the same data. The method is also applied to data with no forcing applied, and the results are shown to be similar to the forced case. This method is particularly useful for cases in which there are multiple frequencies of interest, or cases in which the phenomena occur over a broad range of frequencies. This method allows reconstruction of the frequencies of interest without the need for a forcing signal, so the flow can be studied in its natural (undisturbed) state.

I. Introduction

PARTICLE image velocimetry (PIV) is a well-known and popular technique used to acquire velocity measurements. There are many different flavors of PIV, including both planar and tomographic PIV. In planar PIV, a laser sheet is established and velocity data in a two-dimensional (2D) plane are acquired, consisting of either two (2C) or three (3C) velocity components, depending on whether one or two cameras are used. These planar techniques are well established, and one can achieve well-resolved and accurate measurements of the velocity field. With a tomographic (3D) PIV setup, a volume is illuminated, and multiple cameras (usually four or more) are utilized to capture the three-component velocity field within the measurement volume. In addition, high-speed lasers and cameras can be utilized to capture time-resolved measurements of the velocity field. There are advantages and disadvantages to the different approaches. In particular, complexity of the setup for a 2D2C or 2D3C PIV acquisition is much lower than tomographic PIV, since it requires fewer cameras and less optical access. The processing requirements for tomographic PIV data are also much higher than its 2D counterparts. Additionally, the accuracy of tomographic PIV measurements can suffer with limited camera views, particularly in regions with high velocity gradients [1]. The spatial resolution and accuracy of tomographic PIV is also typically lower than planar PIV due to the reconstruction algorithm. The main advantages of tomographic PIV include the ability to compute multiple components of vorticity, along with the ability to instantaneously measure three-dimensional flow features. Another option for three-dimensional measurements is volumetric PIV, which is conventional photogrammetry-based particle-position analysis. This technique produces more accurate results, but also suffers from a particle density constraint, resulting in lower data density.

Many flows, such as the present case of an unstable and transitional boundary layer, contain organized three-dimensional flow structures. Thus, it is desirable to acquire three-dimensional flow field data. However, it is often challenging to design a volumetric system that simultaneously provides adequate velocity and spatial resolution, along with the required field of view, which, in this case, is a thin layer near the wall. In certain time-dependent flows, phase-locked sampling is a viable approach to construct composite flow data of interest. However, in some cases, phase-locking may not be easily achievable or desirable. Other methods exist that allow three-dimensional reconstruction from two-dimensional measurements [2–4]. Each of these methods require different setups or assumptions. For instance, the method by Chandramouli et al. [2] requires acquisition of two orthogonal planes of PIV data along with the assumption that the flow is homogenous in one direction. To overcome this assumption, it may be possible to acquire multiple parallel planes in one direction rather than assuming homogeneity. For the current experiment, a streamwise laser plane was not feasible due to limited optical access, so only spanwise planes were acquired. Hamdi et al. [3] presented a method that utilizes a series of nonsimultaneous, parallel time-resolved stereo PIV measurement planes, along with proper orthogonal decomposition (POD) reconstruction, to reconstruct the instantaneous three-dimensional

structures shedding from a nozzle. This method is the most similar to the method proposed in the current work, but one notable difference is the fact that Hamdi et al. [3] assumed zero phase speed in the out-of-plane direction to reconstruct the flow. Additionally, the work was performed to study a plane jet, which exhibited features at a very narrow and distinct frequency. This type of flow, in particular, is more suitable for phase-averaging.

On the other hand, the current work involves more broadband phenomena. The current work is focused on understanding the flow downstream of a backward-facing step in a swept-wing flow. This flow consists of several broadband instabilities that occur within three different frequency bands. Time-resolved stereo PIV was utilized to acquire three-component velocity measurements within multiple parallel spanwise planes downstream of the step. A method was devised by which the data that were acquired in the 2D planes could be used to reconstruct the 3D structures corresponding to each of the three instability types. The method utilizes the spanwise planes of data, spectral POD analysis, and a separate measurement of streamwise phase speed to reconstruct the velocity field in three dimensions.

II. Experimental Setup

The experiment was performed in the 2-Foot by 3-Foot Low Speed Boundary-Layer Channel at the NASA Langley Research Center. The tunnel is a closed circuit facility with a 0.61-m high by 0.91-m wide by 6.1-m long test section. The tunnel can reach speeds up to 45 m/s ($Re' = 2.87 \times 10^6/m$), and freestream turbulence intensity levels, $Tu = \frac{1}{U_\infty} \sqrt{\frac{1}{3}(u'^2 + v'^2 + w'^2)}$, were measured to be less than 0.05% for the test speed of 26.5 m/s. This value represents the total energy across the spectrum, high-pass filtered at 0.25 Hz. Thus, this tunnel is considered a low-disturbance facility for purposes of conducting transition experiments [5].

The 0.0127-m thick flat plate model consists of a 0.41-m long leading edge piece, swept at 30° , and a larger downstream piece (see Fig. 1). The model is 0.91 m wide (thus, spanning the width of the test section) and 2.54 m long on the longest edge. The downstream or leading edge pieces can be adjusted relative to each other using precision shims to create either forward-facing or backward-facing 2D steps of different heights, parallel with the leading edge. The leading edge piece was polished to a surface finish of $0.2 \mu m$, and the larger downstream plate had a surface finish of $0.4 \mu m$. A leading-edge contour was designed for the bottom side of the plate in order to make the suction-peak less severe, and therefore, avoid separation, which could potentially cause unsteadiness in the attachment line.

A 3D pressure body along the ceiling was designed to induce a streamwise pressure gradient, which, along with the sweep, causes stationary crossflow growth. A second purpose of the ceiling liner was to simulate infinite swept-wing flow within a midspan measurement region of width 0.3 meters. This was achieved by designing the liner such that the C_p contours were parallel with the leading edge within the measurement region. The ceiling liner was fabricated out of a hard foam using a computer-controlled milling machine.

All measurements were performed at a freestream velocity of 26.5 m/s ($Re' = 1.69 \times 10^6/m$). The current experiment utilized a single leading-edge roughness configuration consisting of discrete roughness elements (DREs) with a diameter of 4.4 mm. These DREs correspond to the large DRE case studied previously [6]. The DREs were applied with a spanwise spacing, λ_z , of 11 mm and were approximately $20 \mu m$ thick. The spacing of the DREs (11 mm) corresponds to the most amplified stationary crossflow wavelength calculated for the baseline case with no step. For more details of the experiment setup, refer to Eppink [7].

A high-speed double-pulsed Nd:YLF laser provided the laser sheet for the PIV measurements (see Fig. 1). The laser sheet was set up parallel with the leading edge and the backward-facing step. Two high-speed 4-megapixel cameras were used to acquire the time-resolved PIV (TRPIV) measurements and were placed downstream of the step. One was placed on the outboard side of the test section at approximately 30° to the laser sheet, and the second camera was placed on the inboard side (in backward scattering) at an angle of approximately 45° to the laser sheet (Fig. 1). To achieve the desired field of view and resolution, 300 mm lenses were utilized, resulting in a total possible measurement area of approximately 60 mm x 30 mm. The cameras and laser were all mounted on the same traversing system, which allowed measurements at multiple locations. An oil-based fog machine generated the seeding, which was introduced downstream of the test section.

To demonstrate the spectral POD reconstruction technique, results are included for two different backward-facing step (BFS) heights of approximately 1.218 mm ($h/\delta = 0.49$) and 1.83 mm ($h/\delta = 0.73$). For the smaller step height, the acquisition rate was set at 4.7 kHz, and the area of interest was reduced to approximately 35 mm x 7 mm. This area allowed acquisition of slightly more than three wavelengths of the stationary crossflow instability in a single frame, while still acquiring approximately 15 points (using 75% overlap and 32x32 pixel interrogation size) inside the boundary layer. For these measurements, 10,000 image pairs were acquired at each streamwise station, and the spacing between each streamwise station was approximately 6.35 mm. For the larger step height, acoustic forcing was supplied using

speakers installed in the settling chamber of the wind tunnel. The beginning of the PIV measurements were triggered off the same signal supplied to the speakers, so that phase-averaged results could be computed. Measurements were acquired with forcing at two different frequencies, including 500 and 1 kHz. A higher acquisition rate of 8 kHz was used for the larger step height data to allow for more time steps within the phase-averaged results. The higher sampling rate required a smaller acquisition region of approximately 15 mm x 5 mm. Data were acquired starting just downstream of the step, and moving downstream in 2 mm increments. Due to the larger number of planes acquired, and to reduce the acquisition and processing time, the number of image pairs was reduced to 6,000 at each station.

A previous measurement campaign [8] utilized hot-wire measurements to study the instabilities downstream of a backward-facing step on the same model. For some limited cases, simultaneous measurements were acquired from a fixed and a traversing hot wire, allowing measurements of the phase speed and wave direction of the instabilities. From this measurement campaign, three different families of unsteady instabilities were identified downstream of the BFS. The lower frequency instabilities ($f \approx 80$ to 200 Hz) corresponded to a primarily spanwise-traveling instability, and this instability was found to be insensitive to acoustic forcing. The middle frequency band ($f \approx 200$ to 800 Hz) was identified as a Tollmien-Schlichting instability, while the higher-frequency band ($f \approx 800$ to 1500 Hz) corresponded to a shear-layer instability. Both the middle and higher-frequency instabilities were found to be sensitive to acoustic forcing, and thus, were ideal for a phase-locking study. Therefore, the current study focuses on these two families of instabilities. Due to the model sweep and the streamwise pressure gradient, stationary crossflow instabilities were also present in the boundary layer. The stationary vortices appear as streamwise-oriented vortices with a spanwise wavelength of 11 mm. The growth of these stationary vortices was found to be relatively unaffected by the BFS. The main contribution of the stationary crossflow vortices to the development of the flow was to cause a spanwise modulation of the unsteady instabilities.

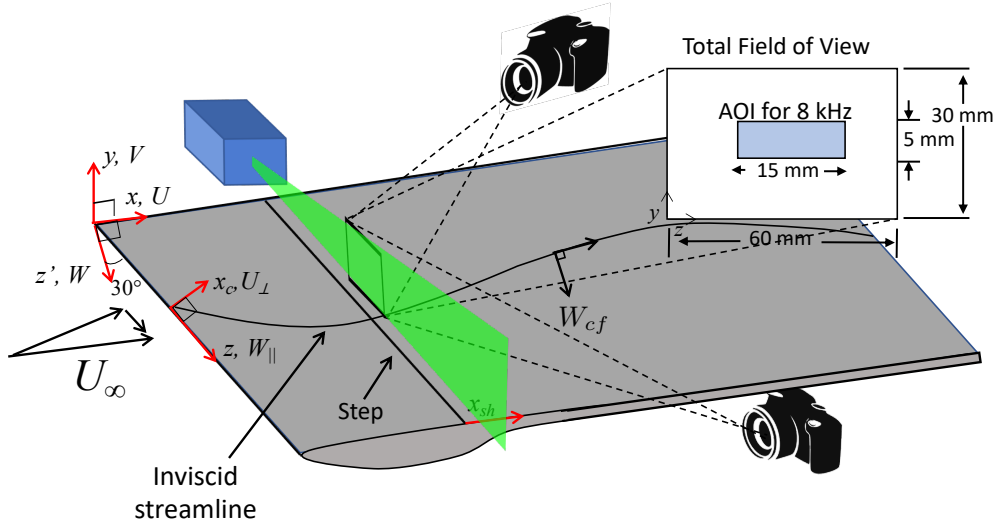


Fig. 1 Sketch of model coordinate system and TRPIV setup.

III. Data Analysis Techniques

A. Phase Averaging

For analysis of the instantaneous three-dimensional flowfield when acoustic forcing was applied, the start of the PIV acquisition was phase-locked using the 500 Hz or 1 kHz signal as the reference. The data were first bandpass-filtered in a 200-Hz band centered around the frequency of interest (i.e., from 400-600 Hz or 900-1100 Hz). Then, the bandpass-filtered data were decomposed using proper-orthogonal decomposition, and the time series was reconstructed using the first five POD modes and their coefficients. It was found from trial and error that this number of modes was sufficient to maintain the large structures of the flow, while removing the background noise. This preprocessing was useful primarily as a means of smoothing the results for better visualization. Finally, the processed time series (which each consisted of 6000 time steps) were divided into segments, with the length of the segments dependent upon the

period of the supplied acoustic forcing. For instance, for the 1 kHz forcing, each period consisted of 8 samples, since the sampling frequency was 8 kHz. Thus, the 6000 snapshots were divided into 750 segments of 8 samples. Finally, the 750 segments were averaged together to obtain the phase-averaged results.

B. SPOD Reconstruction Method

Spectral proper orthogonal decomposition (SPOD) is a modal decomposition technique that is similar to the snapshot POD method, but provides the added benefit of including frequency information. The approach used in this paper is the one described by Schmidt and Colonius [9], in which POD is applied to the FFT of the data. The SPOD results presented in section IV were computed using a block size of 500 frames, with 50% overlap. All three velocity components were included in the computation. The results from this specific SPOD analysis consisted of 23 modes for each of the 251 frequency bands, with a frequency resolution of $\Delta f = 16$ Hz. Because of the presence of the instabilities in the flow downstream of the step, the data are generally found to be low-rank, which means that the data can be represented by a single or a small number of basis vectors. This can be inferred from the SPOD spectra (or eigenvalues), which are also returned for each mode as part of the SPOD analysis. It is generally found that, shortly downstream of the step, the first mode contains significantly more energy than the rest of the modes. Farther downstream, the second modes at some frequencies tend to become more important.

The SPOD modes that are returned as a result of the analysis are in the form of complex numbers, thus, they include phase and amplitude information and can be reconstructed throughout a cycle for a given frequency. For instance, the reconstructed velocity for SPOD mode m , U_m , corresponds to $U_m = A_m \sin(\Phi_m + \phi)$, where A_m , a 2D matrix, is the magnitude of the SPOD mode at each (y, z) location, Φ_m , also a 2D matrix, is the phase of the SPOD mode, and ϕ corresponds to the constant phase shift applied to the mode, which would cycle from 0 to 2π in the full cycle reconstruction of the mode.

The SPOD results are then used to reconstruct the results in three dimensions. The basic idea of this approach was used by Schmidt and Colonius [9] to reconstruct data in multiple streamwise planes by matching the phase of the modes at the boundaries of the planes. In the current paper, the data are acquired in spanwise planes, so reconstruction in three dimensions relies on a few assumptions. The first assumption is that the streamwise phase speed of the mode at a given frequency is already known. In the current case, we can rely on hot-wire measurements that were performed previously for the same setup [8], in which the phase speed and wave direction were computed by using simultaneous measurements from a fixed and traversing hot-wire. Given that the PIV results were already shown to compare favorably with the hot-wire measurements [10], it is likely safe to assume that the use of those measured phase speeds is valid in the current study. The second assumption is that the consecutive PIV measurement planes were acquired with a sufficiently small streamwise spacing to properly resolve the streamwise wavelength of the instability. Additionally, the primary SPOD mode under consideration must not change significantly between the consecutive stations.

The reconstruction is illustrated in Fig. 2 and is performed as follows. First, the SPOD analysis is performed on the velocity data at each of the stations, independently. The SPOD mode of interest (i.e., the first or second mode at a given frequency) is then extracted for each plane. Next, it is necessary to determine the phase shift, ϕ , to apply to the SPOD modes at each station, since we do not have an absolute phase reference. A sequential analysis is performed in which the SPOD mode at station $i+1$ is compared to the SPOD mode at station i to find the correct phase shift to apply to the mode at station $i+1$ so that the reconstructed velocities match between the two stations (i.e., this would physically correspond to a streamwise phase speed of 0). The reconstructed velocity of the SPOD mode at station $i+1$, U_m^{i+1} , is computed for each ϕ from 0 to 2π . Then, the difference is computed between each of these U_m^{i+1} , and the previous result at station i , U_m^i , (which is fixed at its previously-computed ϕ^i), to find the ϕ^{i+1} that produces the minimum difference between the two modes: $\min(U_m^{i+1}(\phi = 0 \dots 2\pi) - U_m^i(\phi^i))$. This difference is computed by comparing each grid point of the reconstructed velocities at the two stations and computing the sum over the entire plane. This process is repeated until the optimal ϕ is found at each station. Again, this process requires that the modes do not change substantially between each streamwise station, otherwise, it would not be possible to find a good match. This should be the case if the flow is adequately resolved in the streamwise direction. Finally, the previously measured phase speed is utilized to determine the additional phase shift required between each pair of adjacent stations to achieve the proper phase speed in the reconstructed results. Using the approach described above, we can reconstruct an entire cycle of the mode in three dimensions. These results are presented as single snapshots in time for illustrative purposes.

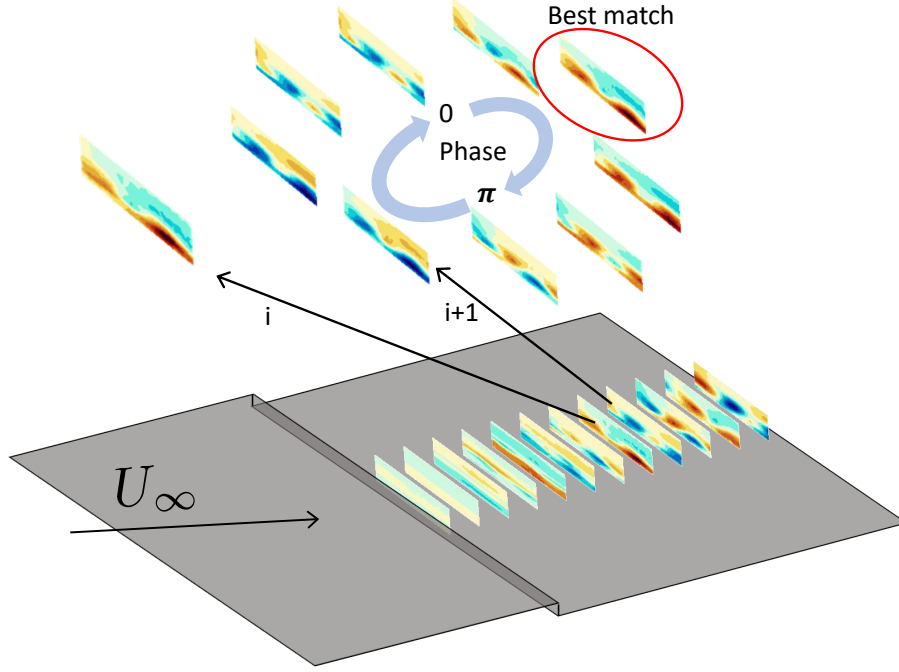


Fig. 2 Conceptual sketch of the phase-matching process.

IV. Results and Discussion

A. With Forcing

This reconstruction technique was tested by applying it to the data that were acquired downstream of the $h/\delta = 0.73$ step. Since these data were phase-locked, it is possible to perform the SPOD reconstruction for the 500 Hz or 1 kHz mode and compare with the phase-averaged results. For this analysis, we compare the SPOD reconstruction using the phase as measured (i.e., relying on the phase-locking), and using the streamwise phase speed that was measured for the 500 Hz and 1 kHz instabilities using two hot-wire probes downstream of the $h/\delta = 0.49$ BFS. Relying on the phase-locked measurements, we can compute the streamwise phase shift at each streamwise location by performing the SPOD phase-matching process as described above. These results are illustrated in Fig. 3 for both frequencies tested. The results for the 500 Hz instability (Fig. 3a) are extremely linear, and the linear best-fit line gives a slope (i.e., phase speed) of approximately 0.4 rad/mm, which is remarkably close to what was measured during the hot-wire campaign for the smaller BFS. Note that for this case, the amplitude of the 500 Hz instability at the first station was too low to obtain a meaningful phase shift between the first two stations, so the first point ($x - x_s = 0$) was not included in the figure. The phase speed computed for the 1 kHz mode (Fig. 3b), 0.669 rad/mm, is also very similar to what was measured during the hot-wire campaign (0.65 rad/mm), although the 1 kHz results are clearly not as linear as those for the 500 Hz mode.

The results from the SPOD reconstruction are illustrated in Fig. 4 for the 500 Hz mode, and Fig. 5 for the 1 kHz mode. For the 500 Hz mode, the SPOD reconstructed results using the phase shift of 0.4 rad/mm (Fig. 4a) and the phase as measured (Fig. 4b) are very similar, which is not surprising given the great agreement between the measured phases. In addition, these results also match very well with the phase-averaged results (Fig. 4c).

However, at the higher frequency of 1 kHz, there are some noticeable differences between the SPOD reconstructed result using the hot-wire measured phase speed (Fig. 5a), and both the reconstructed result using the measured phase from the phase-locked results (Fig. 5b), and the phase-averaged data (Fig. 5c). This is particularly true when comparing the streamwise wavelength and the angle of the oblique structures farther downstream. However, the overall qualitative behavior of the instability is still captured well by the SPOD reconstruction, namely, the transition from the 2D wavefront near the step to the more oblique structure farther downstream. Given the rather significant difference in step height and the nonlinear phase shift development seen in Fig. 3b, some disagreement in results is expected. However, better agreement with the phase-averaged results is achieved by using the correct phases (Fig. 5b) to perform the SPOD reconstruction. This illustrates that with the correct phase, one can use this technique to adequately reconstruct the

flowfield in three dimensions in order to obtain a qualitative picture of the downstream development of the instability.

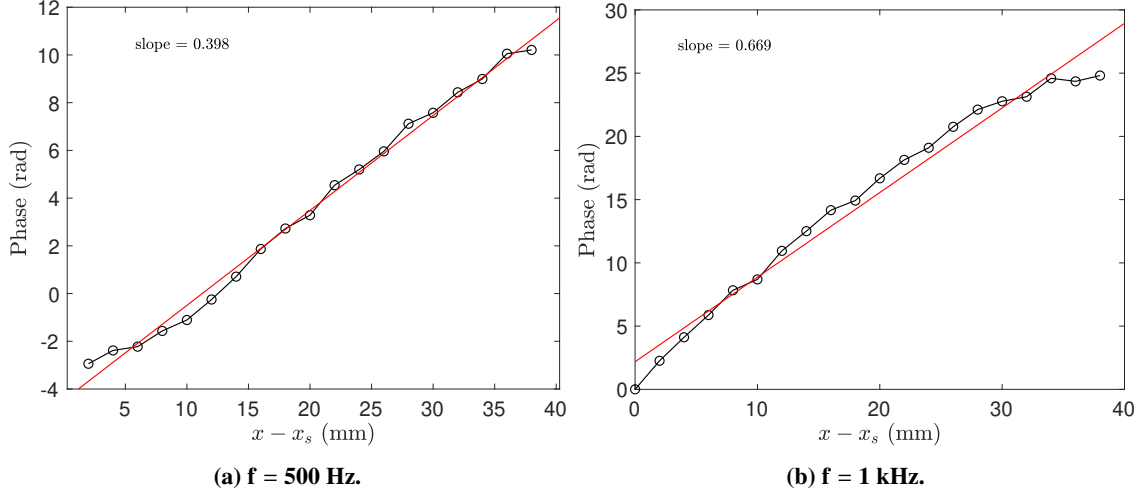


Fig. 3 The cumulative sum of the phase shift found from SPOD phase-matching process at each x -location with forcing at two different frequencies. Linear best-fit line included in red.

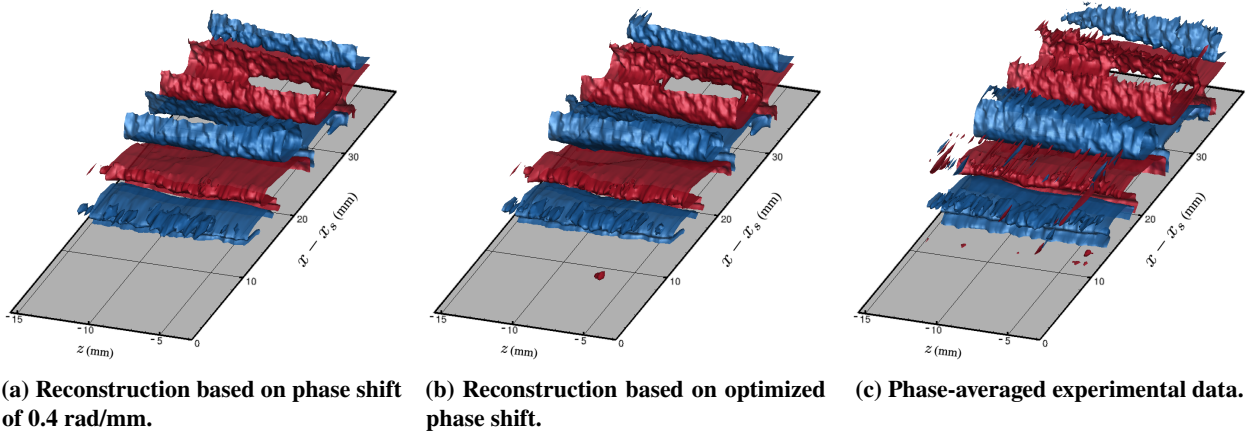
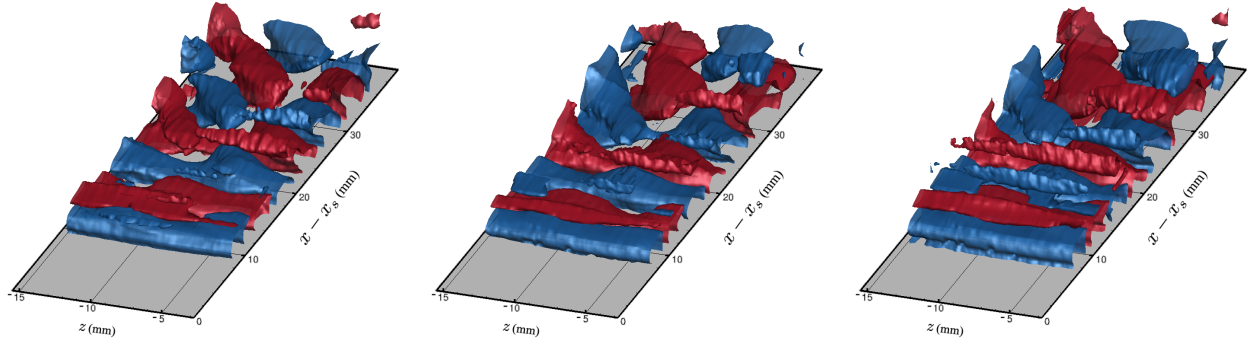


Fig. 4 Reconstructed SPOD and phase-averaged results at $f = 500$ Hz, $h/\delta = 0.73$. Positive (red) and negative (blue) isosurfaces of $u'_{\perp} = \pm 0.2$ m/s.



(a) Reconstruction based on phase shift of 0.65 rad/mm.

(b) Reconstruction based on optimized phase shift.

(c) Phase-averaged experimental data.

Fig. 5 Reconstructed SPOD and phase-averaged results at $f = 1$ kHz, $h/\delta = 0.73$. Positive (red) and negative (blue) isosurfaces of $u'_{\perp} = \pm 0.2$ m/s.

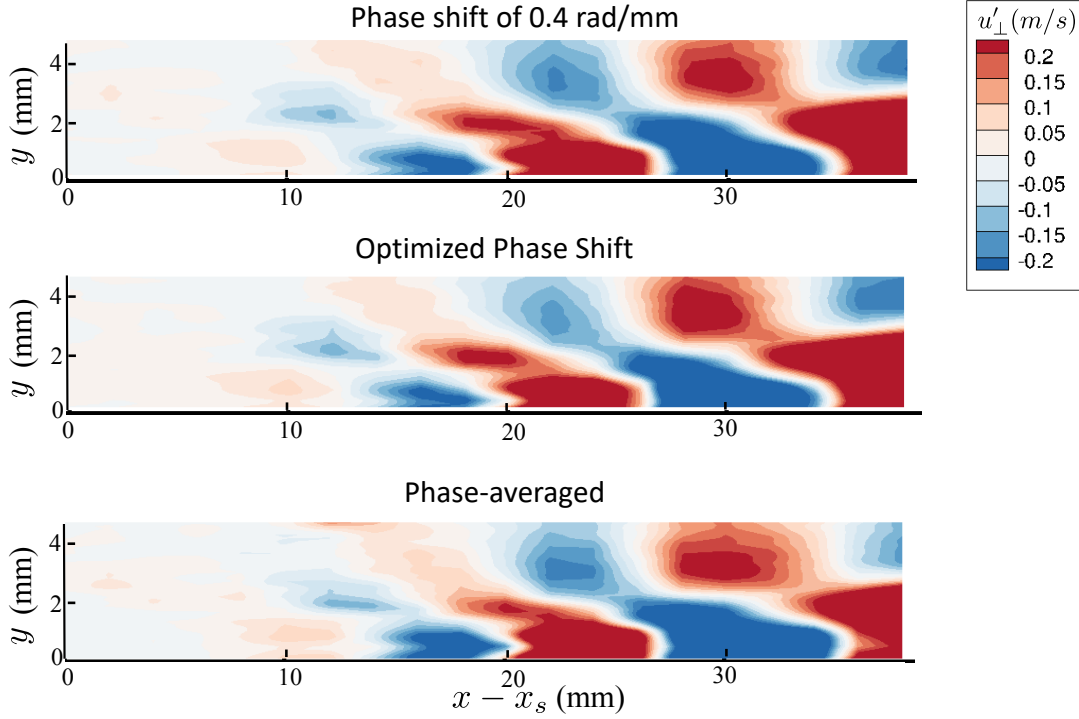


Fig. 6 Reconstructed SPOD and phase-averaged results at $f = 500$ Hz, $h/\delta = 0.73$. Streamwise plane at $z = -10$ mm, contours of u' -velocity.

Figures 6 and 7 show streamwise cuts taken from Figs. 4 and 5 near $z = -10$ mm. Once again, in Fig. 6, both SPOD reconstructed results show great agreement with the phase-averaged results for the 500 Hz mode. For the 1 kHz mode in Fig. 7, the discrepancy in streamwise wavelength becomes very clear when comparing the SPOD-reconstructed results with the constant phase speed of 0.65 rad/mm (top) with the other two results. This again is a direct result of the discrepancy between the actual measured phase shift (Fig. 3b), and the assumption of a constant phase speed (or linear fit to the phase vs. x plot). However, when the correct phases are applied to the SPOD modes, the reconstruction matches much better with the phase-averaged results.

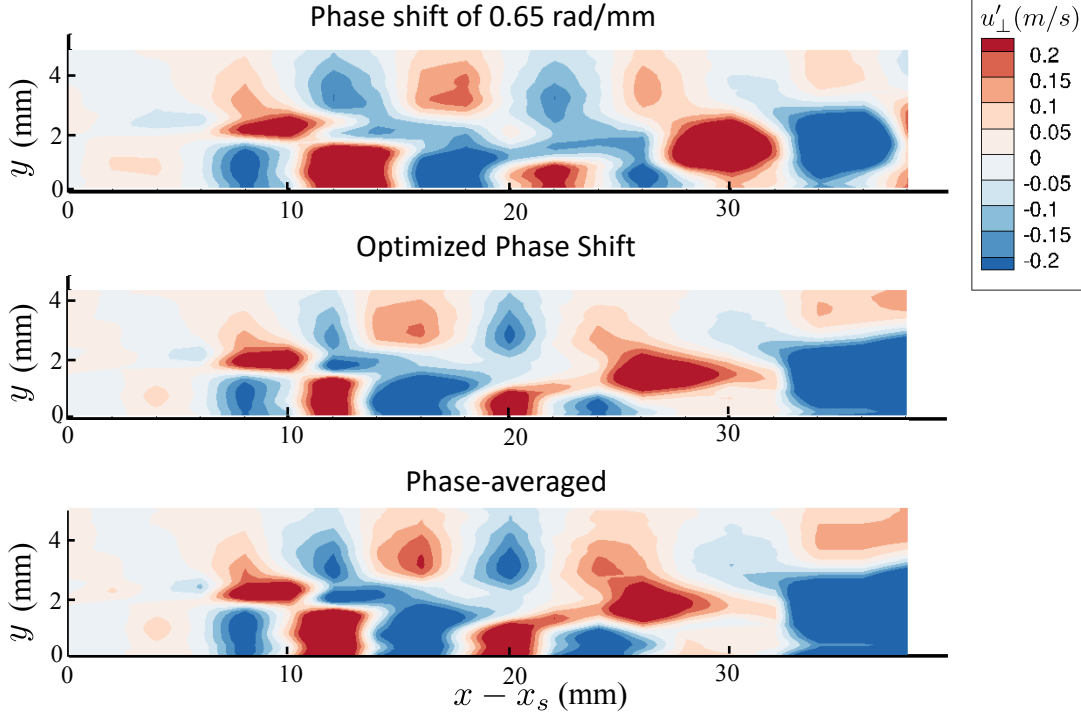


Fig. 7 Reconstructed SPOD and phase-averaged results at $f = 1$ kHz, $h/\delta = 0.73$. Streamwise plane at $z = -10$ mm, contours of $u'_\perp$ -velocity.

B. No Forcing

In this section, results acquired downstream of a smaller BFS ($h/\delta = 0.49$) with no acoustic forcing applied are presented. These data were acquired in a similar manner to those presented in the previous section. However, no phase-locking was possible due to the lack of acoustic forcing. Additionally, the streamwise spacing of the planes was significantly sparser, with a streamwise spacing of 6.35 mm between each plane. This streamwise resolution puts a lower limit on the streamwise wavelengths that can be resolved. Based on the previously acquired hot-wire measurements, the streamwise wavelength of the 260 Hz instability is approximately 21 mm, which results in at least 3 points per wavelength with the given streamwise resolution. Therefore, this frequency is chosen for the reconstruction of the data within the middle frequency band. Higher frequencies resulted in streamwise wavelengths that were too short to resolve adequately. The SPOD spectra are shown for two different locations in Fig. 8. At the first location, $x - x_s = 50$ mm, the first mode contains almost all of the energy. However, shortly downstream, at $x - x_s = 56$ mm, there is significant energy contained in the second mode as well.

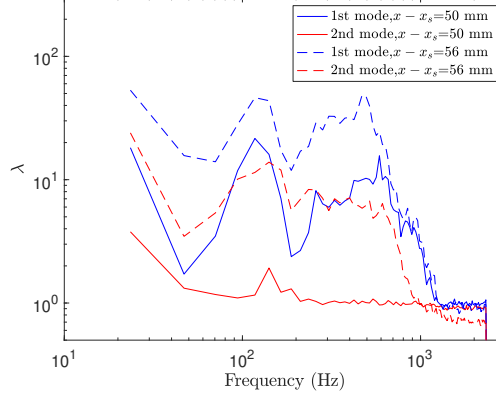


Fig. 8 SPOD eigenvalues (λ) for the $h/\delta = 0.49$ BFS.

Snapshots from the first and second SPOD mode for the chosen frequency of 260 Hz are shown in Fig. 9 at $x - x_s = 56$ mm downstream of the step. As shown in Fig. 8, the energy for the lower ($f \approx 80$ to 100 Hz) and middle ($f \approx 200$ to 750 Hz) frequency band ranges was no longer contained within just the first SPOD mode, as significant energy was observed to be present in the second SPOD mode as well. The second SPOD modes look qualitatively similar to the first SPOD modes for the higher frequency band, as can be observed by comparing Figs. 9b and 10. Thus, it appears that the higher frequency band instability is spreading to lower frequencies farther downstream, and this is reflected by the SPOD analysis as a separate mode. Therefore, we are able to use the SPOD reconstruction technique on the second mode of this lower frequency (260 Hz) instability, and it may give us valuable insight into the behavior of the higher-frequency instability.

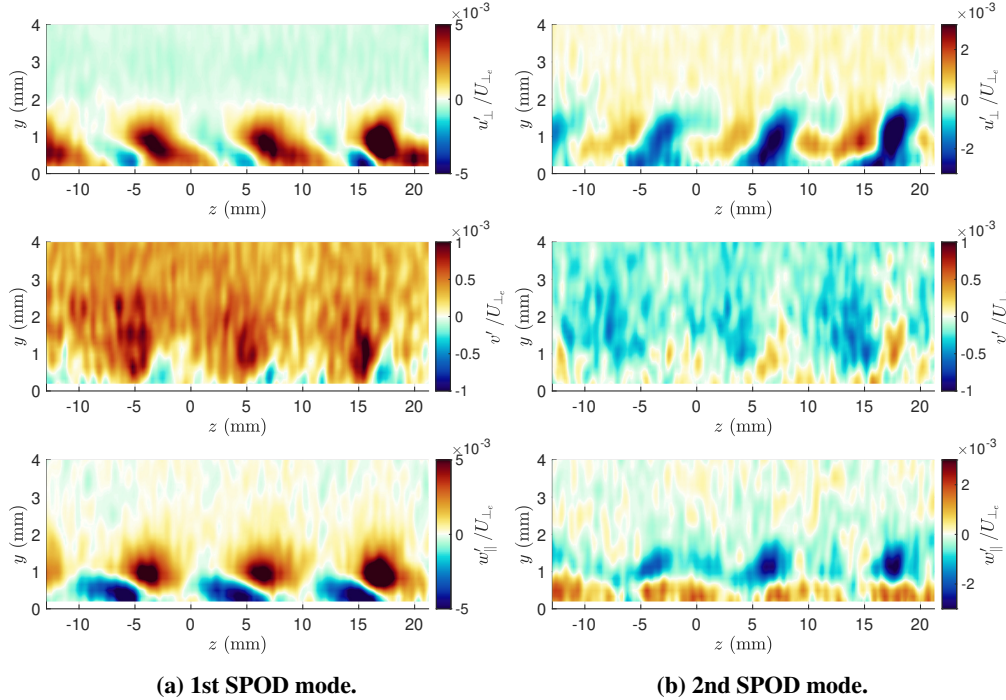


Fig. 9 First and second SPOD modes at $x - x_s = 56$ mm, $f = 260$ Hz, $h/\delta = 0.49$ BFS.

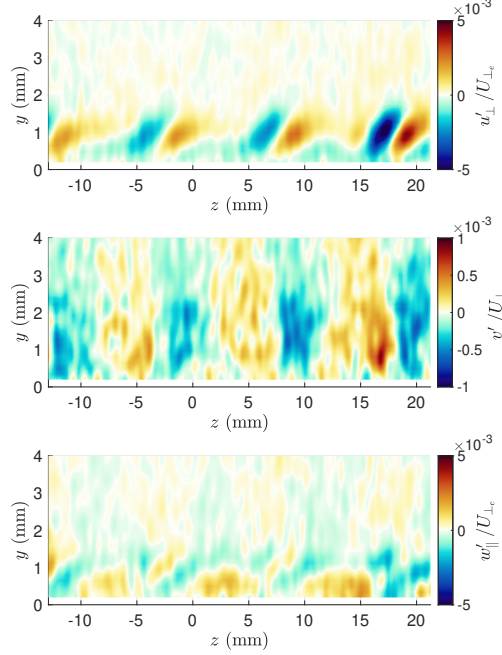


Fig. 10 First SPOD mode at $x - x_s = 56$ mm, $f = 1000$ Hz, $h/\delta = 0.49$ BFS.

The SPOD reconstruction was performed on these two modes using the hot-wire measured streamwise phase shift of 0.28 rad/mm for the 260 Hz instability. The results are shown in Fig. 11. In order to better illustrate the flow structures close to the step, the SPOD modes used to compute the isosurfaces shown in Fig. 11 are normalized at each streamwise location by the maximum amplitude. The first mode (Fig. 11a) initially appears as two-dimensional structures that are parallel with the step. Near approximately 40 mm downstream of the step, the isosurfaces begin to undergo modulation due to interactions with the stationary-crossflow vortices that are present in the boundary layer. The stationary vortices essentially cause regions of the instability to lift up, exposing them to higher velocity flow, causing portions of the wave front to become inclined relative to the step. This inclination angle increases as the instability progresses downstream. The results are expected to be qualitatively similar to the results for the 500 Hz mode shown in Fig. 4, since these two frequencies belong to the same family of instabilities, even though the step heights are different. The results for the 500 Hz mode were only acquired a short distance (40 mm) downstream of the step, and, therefore, do not exhibit the spanwise modulation that is evident farther downstream for the 260 Hz mode. However, near the step, the two instabilities exhibit the same two-dimensional behavior. To further examine the similarities, a streamwise plane of data is shown in Fig. 12 for the SPOD reconstructed data at $f = 260$ Hz. These data were computed with the SPOD amplitudes included, thus, this figure represents the actual amplitude growth of the mode as it travels downstream. In comparison with Fig. 6, the results look qualitatively similar near the step ($x - x_s < 40$ mm), although the streamwise wavelength of the 260 Hz mode is larger than the 500 Hz mode, as expected. Both figures illustrate the phase shift in the wall-normal direction that is characteristic of a Tollmien-Schlichting instability.

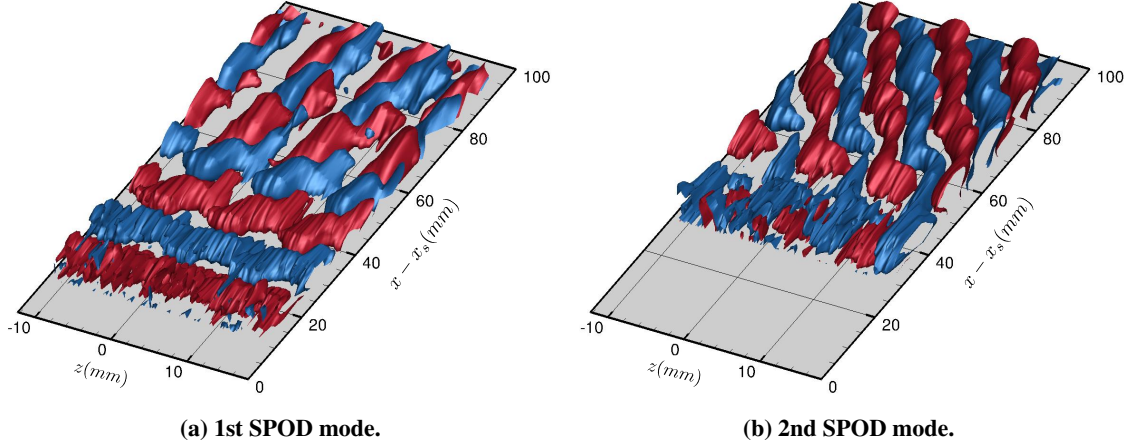


Fig. 11 Reconstructed SPOD results, isosurfaces of $u'_\perp$, $f = 260$ Hz for $h/\delta = 0.49$ BFS.

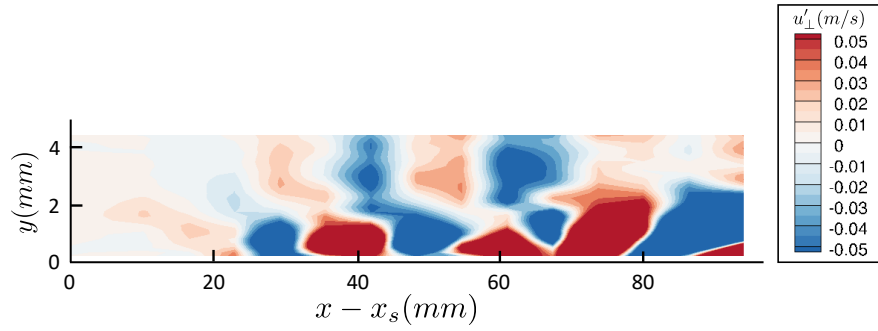


Fig. 12 Reconstructed SPOD results of $f = 260$ Hz 1st mode, $h/\delta = 0.49$ BFS. Streamwise plane at $z = 2$ mm.

The second SPOD mode (Fig. 11b) shows a markedly different behavior to the first mode. This mode does not become coherent until approximately 40 mm downstream of the step. From that point on, an oblique wavefront is evident, inclined in the opposite direction to the first SPOD mode, approximately 60° to the streamwise (x) direction. Since this mode is believed to be representative of the higher frequency instability, it is worthwhile to compare these results to the phase-locked results that were acquired for the larger step height.

Fig. 5 illustrates that, near the step, the 1 kHz instability is two-dimensional and parallel with the step. Farther downstream (near $x - x_s = 25$ mm), it begins to develop into a mode with an obliquely-oriented wavefront. Even nearer to the step, there is some evidence of an oblique mode in the data. In order to examine these two components independently, a streamwise wavelength filter was applied to the phase-averaged results. The oblique wavefront exhibits a much larger streamwise wavelength compared to the two-dimensional wavefront, thus, this filter allows us to easily separate the two modes. The results are presented in Fig. 13. When the smaller wavelengths are filtered out, we can clearly see the oblique wavefront that is evident as close as 10 mm from the step (Fig. 13a). These oblique structures look very similar to what is seen with the reconstruction of the second SPOD mode for the $h/\delta = 0.49$ BFS (Fig. 11b). In fact, the wave angle for this case is remarkably similar, at approximately 62° relative to the x -direction. This is especially interesting considering the differences in step height and the differences in how the three-dimensional results were acquired.

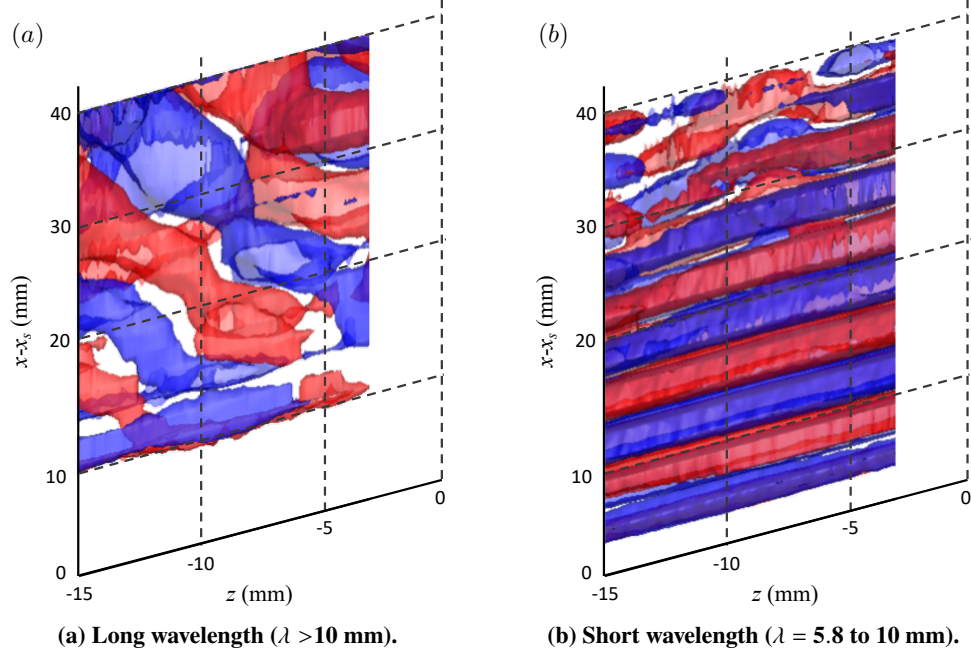


Fig. 13 Phase-averaged results for the $h/\delta = 0.73$ step with 1 kHz forcing. Isosurfaces of positive (red) and negative (blue) u' -velocity. Results are filtered based on streamwise wavelength.

V. Summary and Concluding Remarks

A new method was presented for reconstructing three-dimensional flowfields from two-dimensional planes of time-resolved PIV data using spectral proper orthogonal decomposition. Prior knowledge of the out-of-plane phase speed of the frequency of interest is applied in order to perform the reconstruction. For the present work, this phase speed was acquired using simultaneous hot-wire measurements from a previous experiment using the current model. Time-resolved data could be acquired in one streamwise plane (or in a plane normal to the planes used for reconstruction), and the phase speed could be extracted from that plane for any frequency of interest. If that is not an option, forcing could be used (as demonstrated in the present work) to allow phase-locking and extraction of the phase speed using the phase-averaged data. These phase speeds could then be applied to data acquired with no forcing, assuming the fundamental behavior of the instability does not change when forcing is applied. In the present work, it was found that the phase speed acquired from the simultaneous hot-wire measurements agreed very well with that acquired using the phase-locking approach. This illustrates that the forcing did not fundamentally change the behavior of the instability, and therefore, the phase speed acquired using this approach could also be applied to unforced cases. However, this may not be the case for every situation. It may also be possible to estimate phase speeds from computations.

In the present work, results were presented for a phase-locked measurement scenario, as well as a scenario with no phase locking or acoustic forcing. For the phase-locked case, the SPOD reconstruction method was evaluated by comparing those results with the phase-averaged results. In every case, the SPOD reconstruction results captured the qualitative three-dimensional behavior of the instability very well. In one case, due to the assumption of a constant phase speed, the reconstructed results did not exactly capture the correct streamwise wavelength of the instability farther downstream of the step. The utility of this approach comes in its applicability to broadband phenomena or to cases where multiple frequency bands are of interest, such as the present work. This approach allows the separation of particular frequencies of interest within those broad frequency bands without the need for any external forcing that could potentially affect the instabilities and their interactions. This capability was demonstrated by performing the SPOD reconstruction on a flow where no external acoustic forcing was applied. These results showed remarkably similar behavior to the phase-averaged results of the larger step height and helped to elucidate some physical aspects of the flow that were not evident simply by examining the two-dimensional planes.

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