Modeling Shock-Separated Flow with the Active Model Split

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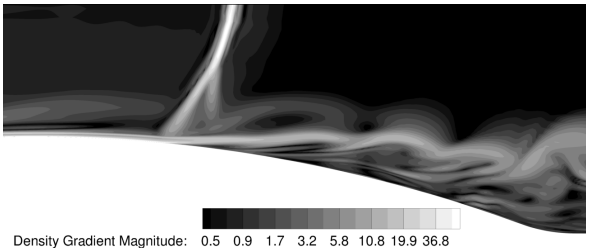




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Motivation

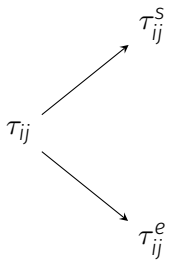
- RANS models have limited accuracy for flows with large-scale unsteadiness.
- Use RANS models where accurate.
- Switch to LES with resolved fluctuations where necessary.
- DES-type hybrid models have problems such as modeled stress depletion.



Density gradients in the Bachalo-Johnson transonic axisymmetric bump.

Active Model Split

Proposed by Haering et al.[1]



- τ_{ij}^s is the mean model stress
  - Contributes to the Reynolds stress
  - Modeled using a rescaled RANS stress
- τ_{ij}^e is the fluctuating model stress
  - Transfers energy from fluctuations to unresolved scales
  - Uses LES-type models
- Reynolds stress is the sum of the resolved plus mean modeled stress: $-\langle \rho u_i'' u_j'' \rangle \approx -\langle \rho u_i^> u_j^> \rangle + \tau_{ij}^s$

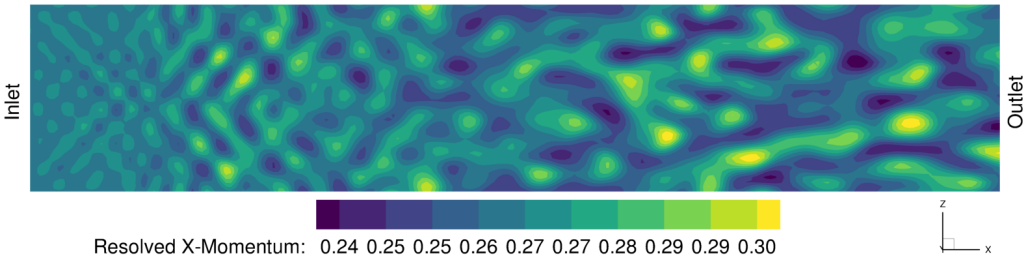
[1] S. Haering, T. A. Oliver, and R. D. Moser, "Towards a Predictive Hybrid RANS/LES Framework," *AIAA Paper 2019-0087*, 2019.

Key Differences from DES-type Models

- Models for average and fluctuating stresses are separate.
- RANS and LES stresses are not blended. Rather, the models coexist and serve different purposes.
- The scaling of the mean stress is dependent on the amount of resolved kinetic energy, rather the grid scale or velocity gradients.
- Only average quantities enter the RANS equations.
- Forcing is applied to introduce fluctuations where the simulation is over-resolved.

Spatially Developing Channel

Supersonic Plane Channel

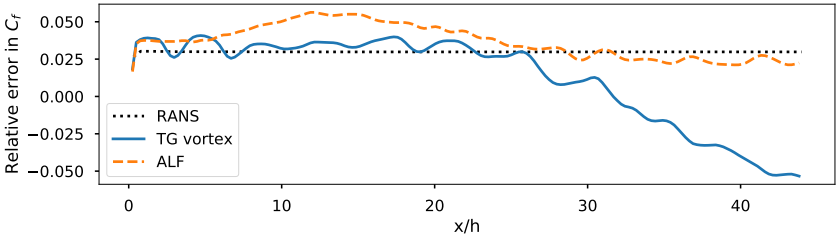


- $M_b = 1.5$, $Re_\tau \approx 1010$
- Set up to match DNS study by Modesti and Pirozzoli [2]

[2] D. Modesti and S. Pirozzoli, "Reynolds and Mach number effects in compressible turbulent channel flow," *International Journal of Heat and Fluid Flow*, vol. 59, pp. 33–49, Jun. 2016

Supersonic Plane Channel: Skin friction

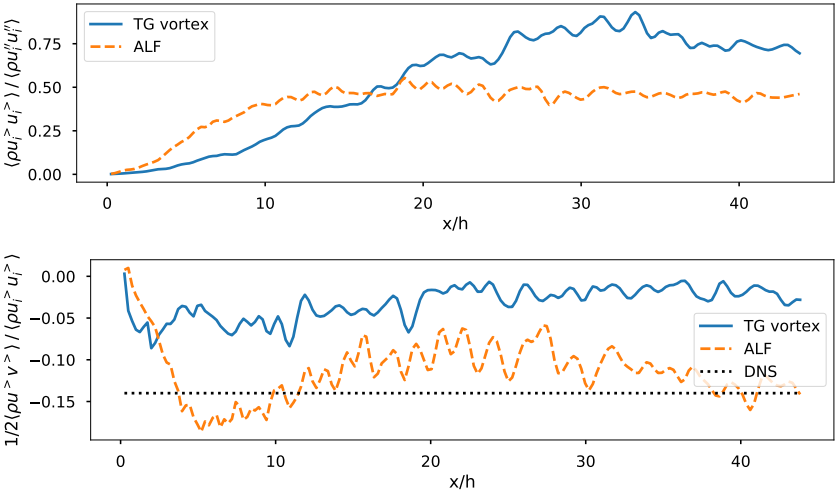
- Examined two different types of forcing:
  - Taylor-Green vortices by Haering et al. [3]
  - Anisotropic Linear Forcing by de Laage de Meux et al. [4]



[3] S. Haering, T. A. Oliver, and R. D. Moser, "Towards a Predictive Hybrid RANS/LES Framework," *AIAA Paper 2019-0087*, 2019

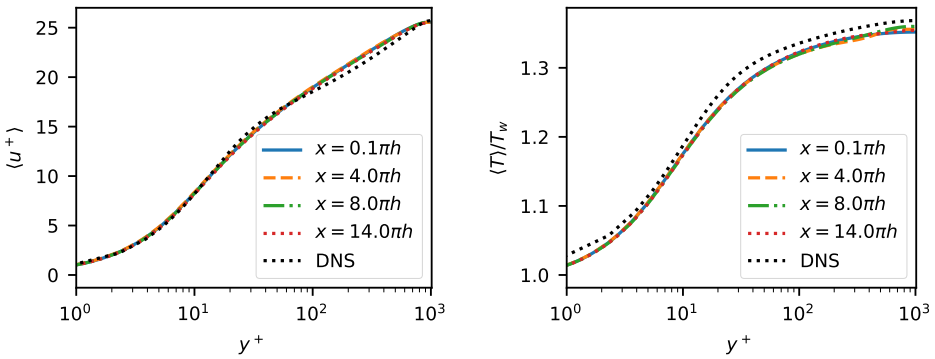
[4] B. de Laage de Meux, B. Audebert, R. Manceau, *et al.*, "Anisotropic linear forcing for synthetic turbulence generation in large eddy simulation and hybrid RANS/LES modeling," *Physics of Fluids*, vol. 27, no. 035115, pp. 1–34, Mar. 2015

Supersonic Plane Channel: Anisotropy



Evolution of the resolved turbulent kinetic energy and resolved shear stress in the streamwise direction.

Supersonic Plane Channel



Profiles for the mean streamwise velocity and the average temperature.

Bachalo-Johnson Transonic Bump

Bachalo-Johnson Transonic Bump

Shock-induced boundary-layer separation over an axisymmetric bump

- $M_\infty = 0.875$, incoming $Re_\theta \approx 6600$
- Original experiments performed by Bachalo and Johnson [5].
- Wall-resolved LES performed by Uzun and Malik [6].

Results shown here are from coarse-grid tests. The grid had 170 times fewer nodes than a similar study by Spalart et al. [7]

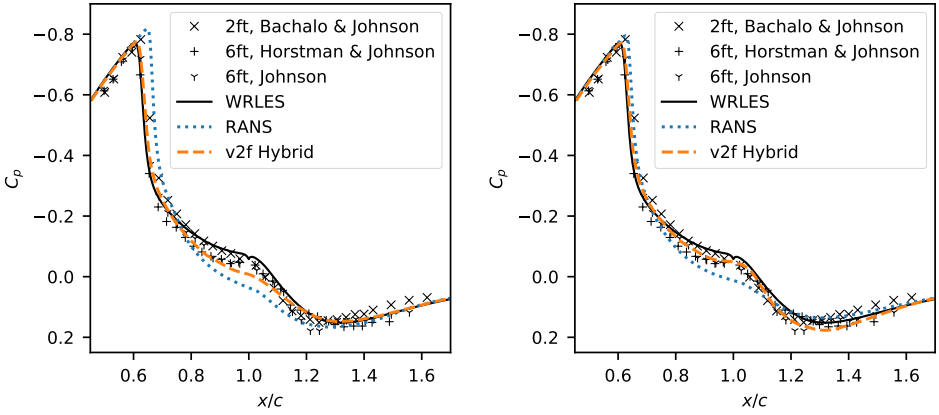
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[5] W. D. Bachalo and D. A. Johnson, "Transonic, turbulent boundary-layer separation generated on an axisymmetric flow model," *AIAA Journal*, vol. 24, no. 3, pp. 437–443, Mar. 1986

[6] A. Uzun and M. R. Malik, "Wall-Resolved Large-Eddy Simulations of Transonic Shock-Induced Flow Separation," *AIAA Journal*, vol. 57, no. 5, pp. 1955–1972, May 2019

[7] P. R. Spalart, K. V. Belyaev, A. V. Garbaruk, *et al.*, "Large-Eddy and Direct Numerical Simulations of the Bachalo-Johnson Flow with Shock-Induced Separation," *Flow, Turbulence and Combustion*, vol. 99, no. 3-4, pp. 865–885, Dec. 2017

Bachalo-Johnson

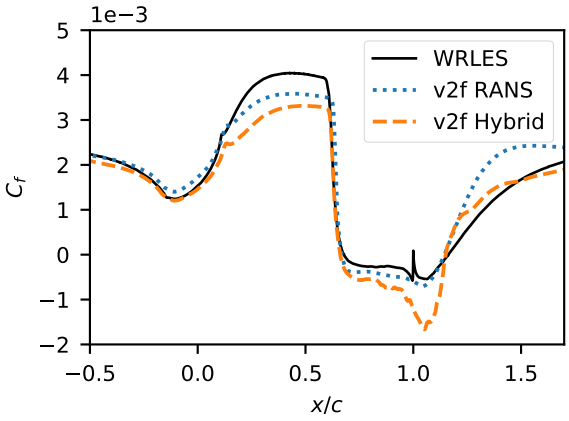


(a) ALF Forcing

(b) TG Forcing

Pressure coefficient for the Bachalo-Johnson axisymmetric bump

Bachalo-Johnson: Skin Friction



Skin friction coefficient for the Bachalo-Johnson axisymmetric bump

Conclusions

- The AMS framework maintains a consistent mean flow, while supporting resolved turbulence.
- The results are sensitive to the anisotropy of the forcing. Anisotropy of the resolved Reynolds stress changes slowly.
- On the Bachalo-Johnson bump, predictions of shock location and the pressure coefficient in the separated region are improved by moving from RANS to AMS.
- Skin friction is still a challenging quantity of interest.

References

S. Haering, T. A. Oliver, and R. D. Moser, "Towards a Predictive Hybrid RANS/LES Framework," *AIAA Paper 2019-0087*, 2019.

D. Modesti and S. Pirozzoli, "Reynolds and Mach number effects in compressible turbulent channel flow," *International Journal of Heat and Fluid Flow*, vol. 59, pp. 33–49, Jun. 2016.

B. de Laage de Meux, B. Audebert, R. Manceau, and R. Perrin, "Anisotropic linear forcing for synthetic turbulence generation in large eddy simulation and hybrid RANS/LES modeling," *Physics of Fluids*, vol. 27, no. 035115, pp. 1–34, Mar. 2015.

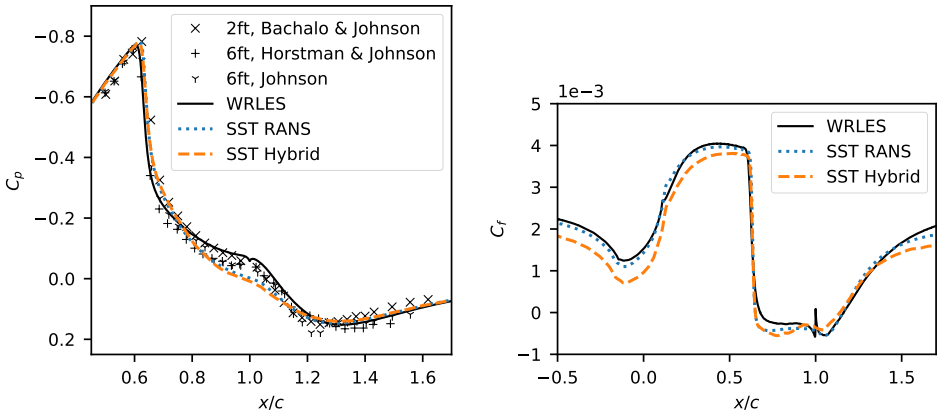
W. D. Bachalo and D. A. Johnson, "Transonic, turbulent boundary-layer separation generated on an axisymmetric flow model," *AIAA Journal*, vol. 24, no. 3, pp. 437–443, Mar. 1986.

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P. R. Spalart, K. V. Belyaev, A. V. Garbaruk, M. L. Shur, M. K. Strelets, and A. K. Travin, "Large-Eddy and Direct Numerical Simulations of the Bachalo-Johnson Flow with Shock-Induced Separation," *Flow, Turbulence and Combustion*, vol. 99, no. 3-4, pp. 865–885, Dec. 2017.

Backup Slides

Bachalo-Johnson: Pressure coefficient



Pressure coefficient and skin friction using SST as the underlying RANS model on the Bachalo-Johnson axisymmetric bump.

Mean modeled stress

$$\tau_{ij}^{s} = f(\beta)\left[2\mu_t\left(\langle S_{ij}\rangle_f - \frac{1}{3}\frac{\partial\langle u_k\rangle_f}{\partial x_k}\delta_{ij}\right)\right] - \beta\left(\frac{2}{3}\langle\rho\rangle k_{tot}\delta_{ij}\right) \tag{1}$$

where:

- $\langle\cdot\rangle$ and $\langle\cdot\rangle_f$ are Reynolds and Favre-averages, respectively
- k_{tot} is the total turbulent kinetic energy from RANS
- α is the ratio:

$$\alpha = 1 - \frac{\langle u_k^{>} u_k^{>}\rangle_f}{2k_{total}} \approx \frac{\text{Unresolved turbulent kinetic energy}}{\text{Total turbulent kinetic energy}} \tag{2}$$

- $f(\beta) = \beta^{1.7}(2 - \beta^{1.7})$ is an empirical function representing:

$$\beta \approx \frac{\text{Unresolved deviatoric mean stress}}{\text{Deviatoric Reynolds stress}} \tag{3}$$

- μ_t is the eddy viscosity from a RANS model.

Fluctuating modeled stress

$$\tau_{ij}^{e} = \mu_{ik}\left(\frac{\partial u_j^{>}}{\partial x_k} - \frac{1}{3}\frac{\partial u_m^{>}}{\partial x_m}\delta_{jk}\right) + \mu_{jk}\left(\frac{\partial u_i^{>}}{\partial x_k} - \frac{1}{3}\frac{\partial u_m^{>}}{\partial x_m}\delta_{ik}\right) \tag{4}$$

where:

$$\mu_{ij} = C(\mathcal{M})\langle\rho\rangle\varepsilon^{1/3}\mathcal{M}_{ij}^{4/3} \tag{5}$$

and:

- $u^{>} \equiv \tilde{u} - \langle u\rangle_f$ is the resolved portion of the fluctuating velocity
- $\mathcal{M}$ is a tensor generalization of the local grid size
- ε is the total dissipation from a turbulence model
- $C(\mathcal{M})$ is both the model constant and a correction for anisotropic resolutions

Supersonic Channel, $Re_\tau = 1010$

- Isothermal walls, with $T_b/T_w = 1.3$
- $M_b = 1.5$
- $\Delta x+ = 130$, $\Delta z+ = 81$
- $\min(\Delta y+) = 1.0$
- $\max(\Delta y+) = 56$
- $\max(\Delta y/h) = 0.056$
- DNS data from Modesti and Pirozzoli [2]

High-Mach Channel

- Computational domain is $16\pi h \times 2h \times 3\pi$ in the streamwise, wall-normal, and spanwise directions.
- $Re_b = 17000$
- $M_t = 0.065$
- $B_q = -0.038$

Resolution Tensor

- One commonly known weakness of DES is its reliance on a scalar grid measure.
- We use a tensor version, such that the grid size in a direction $\boldsymbol{e}$ can be represented as $\Delta_e = e_i \mathcal{M}_{ij} e_j$
- For rectangular cuboids aligned with the principle coordinates,

$$\mathcal{M} = \begin{bmatrix} \Delta_1 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{bmatrix} \tag{6}$$

Resolution Adequacy

$$r_{\mathcal{M}} = \rho_s(L_{ik}^{-1}\mathcal{M}_{kj}) \tag{7}$$

where:

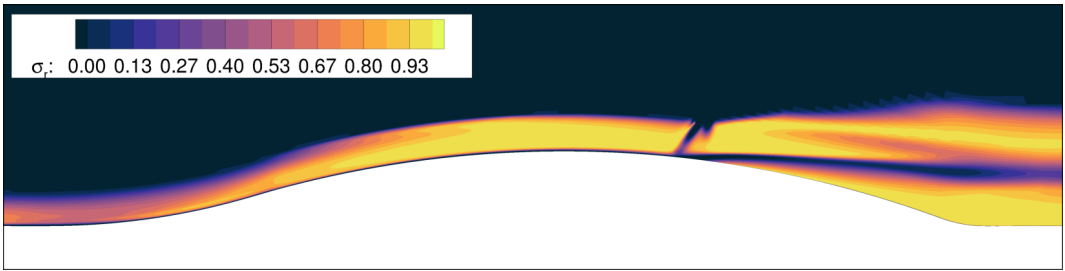
- $\rho_s()$ means the spectral radius of a tensor
- $\mathcal{M}$ is the resolution tensor
- L_{ij}^{-1} is the inverse length scale of the resolved turbulence:

$$L_{ij}^{-1} = \frac{1}{2(3/2\alpha\langle v^2\rangle)^{3/2}}\left(\tau_{ij}\frac{\partial\tilde{u}_k}{\partial x_j} + \tau_{jk}\frac{\partial\tilde{u}_k}{\partial x_i}\right) \tag{8}$$

Forcing Regions

The forcing is attenuated in coarse regions by multiplying the force by the function:

$$\sigma_r = \max(\tanh(1/\sqrt{r_{\mathcal{M}}} - 1), 0) \tag{9}$$



Compressibility

Total energy equation:

$$\frac{\partial \bar{\rho}\tilde{E}}{\partial t}+\frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_j\tilde{H}) = \frac{\partial}{\partial x_j}(\tilde{u}_i\breve{\sigma}_{ij}+\tilde{u}_i\tau_{ij})+\frac{\partial}{\partial x_j}\left(\frac{C_p\mu}{Pr}\frac{\partial \tilde{T}}{\partial x_j}\right)-\frac{\partial(Q_j^S+Q_j^E)}{\partial x_j}+\frac{\partial}{\partial x_j}\left(\left(\mu+\frac{\beta\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right) \tag{10}$$

Modeled heat flux from mean stress:

$$Q_j^s = -C_p\frac{\beta\mu_t}{Pr_t}\frac{\partial\langle T\rangle_f}{\partial x_j} \tag{11}$$

Modeled heat flux from fluctuating stress:

$$Q_j^e = -C_p\frac{\mu_{ij}}{Pr_{sgs}}\frac{\partial T^{>}}{\partial x_i} \tag{12}$$

Averaging

We use an exponentially-weighted moving average (EWMA). The time evolution can be expressed as:

$$\frac{\partial \langle \phi \rangle}{\partial t} = \frac{\phi - \langle \phi \rangle}{T_{ave}} \tag{13}$$

where T_{ave} is a suitable averaging timescale. Using Backward Euler, the average at a timestep $n+1$ can be represented as:

$$\langle \phi \rangle^{n+1} = w\phi^n + (1-w)\langle \phi \rangle^n \tag{14}$$

where:

$$w = \Delta t / (T_{ave} + \Delta t) \tag{15}$$