

Three-Dimensional Structure and Modeling of a Normal Bifurcated Shock Train from Experimental Measurements

Louis M. Edelman* and Mirko Gamba†
University of Michigan, Ann Arbor, MI, 48109

Robin L. Hunt and Aaron Auslender‡
NASA Langley Research Center, Hampton, VA 23681

An asymmetric, normal bifurcated shock train in a Mach 2 constant area, rectangular duct is investigated in the University of Michigan Direct Connect Isolator facility. High-speed schlieren imaging, wall static pressure measurements, surface oil flow visualization, and particle image velocimetry of the shock train are synthesized into a three-dimensional representation. This visualization is then used to inform the underlying flow physics of the distributed fluid dynamical processes along the shock train. The detailed, three-dimensional morphology of the flow profile entering the shock train is shown to have a significant impact on the separated flow morphology of the shock train. This results in a skewed supersonic core flow with alternating separation bubbles in the surrounding boundary layers. Finally, the efficacy of the prevailing pseudo-shock models in the literature is analyzed with the support of the available flow measurements. None are found to model both the pressure and streamwise Mach number profile of the UMDCI pseudo-shock.

I. Background

A. The Shock Train in Context

The *dual-mode scramjet*, shown schematically in Figure 1, is a high-speed air-breathing engine that operates as a ramjet at lower Mach numbers ($3 \sim 5$) with mostly subsonic flow in the combustor and as a scramjet at higher Mach numbers ($M \geq 5$) with mostly supersonic flow in the combustor. Between the inlet and combustor sits the *isolator*, a constant or nearly constant area duct. The isolator contains the *pseudo-shock*, a complex compressive flow structure that forms as a result of back-pressure in the combustor.

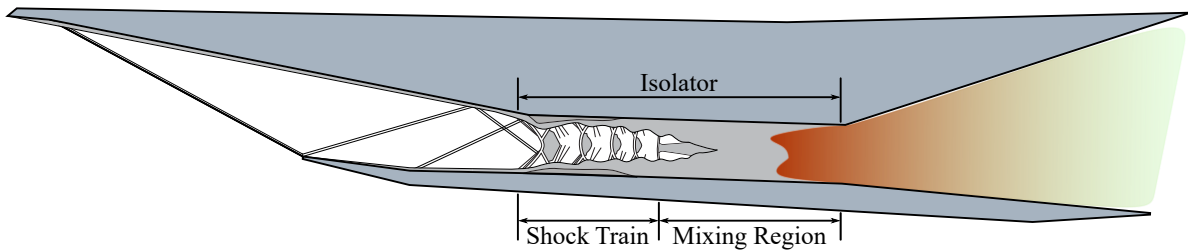


Fig. 1 Schematic of a normal bifurcated shock train in a dual-mode scramjet engine.

Matsuo et al. [1] provide a survey of steady-state pseudo-shock behavior with a more contemporary survey performed by Gnani et al. [2] to include some discussion of passive control. The pseudo-shock is divided into two streamwise regions. Upstream is the *shock train*, a mostly supersonic region dominated by a series of interconnected shock-boundary layer interactions. Two types of shock trains can manifest in dual-mode a scramjet: a normal bifurcated shock train present in ramjet mode or an oblique shock train at the upper edge of performance in the scramjet mode [3]. If the duct is long enough, downstream of the shock train is the *mixing region* where mixed sub/supersonic flow yield further

*Ph.D. Candidate, Dept. of Aerospace Engineering, AIAA Student Member, <https://orcid.org/0000-0001-8862-7534>

†Associate Professor, Dept. of Aerospace Engineering, AIAA Member

‡Research Aerospace Engineer, Hypersonic Airbreathing Propulsion Branch, AIAA Member

compression. Qualitatively, the two regions are separated by the streamwise point at which shocks no-longer dominate the flow field. Typically, this division is quantitatively estimated by the point where 80% of pseudo-shock compression has occurred [4].

Even under nominally constant boundary conditions, the pseudo-shock exhibits *inherent unsteadiness* by which the shock train shifts erratically streamwise about a time averaged position. As such, we refer to this state as *quasi-steady*. The spectral quality of the quasi-steady inherent unsteadiness was studied by Sugiyama et al. [5] and expanded upon by Hunt and Gamba [6] to detail the origin of unsteadiness within the pseudo-shock. As the primary source of compression in the pseudo-shock, this study focuses on the shock train as series of interconnected shock-boundary layer interactions (SBLI) with strong dynamics separated by re-expansion regions called *shock cells*. The ramjet mode is highly sensitive to changes in combustor back pressure [7] and is a state through which a dual-mode scramjet must pass to achieve high Mach, hypersonic flight. As such, it is the system we focus on to understanding the fluid dynamical structures and mechanisms underlying the shock train to enable better prediction of shock train behavior in operational dual-mode scramjet engines.

B. Shock Train Structure

The shock train structure is typically represented as a two-dimensional structure derived from schlieren imaging, or a quasi-1D pressure profile along the streamwise length of the isolator derived from wall static pressure taps. While quantitative and qualitative data can be gleaned from schlieren, it is a line-integrated technique such that only a two-dimensional representation of the complex SBLI structures is resolved. Wall pressure taps are excellent for understanding the time averaged profile of the shock train, however the smearing effect of thick boundary layers and limited spatial resolution abstracts these measurements from the shock structure and filters unsteadiness. These techniques obfuscate unsteady behavior and collapse down the importance of three-dimensional effects such as corner flows in rectangular ducts and boundary layer confinement [4, 8].

Carroll and Craig Dutton [8] investigated these three-dimensional effects on oblique ($M = 2.45$) and normal bifurcated ($M = 1.6$) shocks trains in a symmetric rectangular duct using schlieren imaging, surface oil flow visualization, and laser Doppler velocimetry (LDV). While large corner separation bubbles were observed in the oblique case, only small incipient separation bubbles were observed below the first shock in the bifurcated case. More recently, Morgan et al. [9] performed large-eddy simulations (LES) to match the bifurcated case and observed a strong dependence between boundary layer confinement and the shock structure. They concluded that spanwise confinement and thus corner effects must be included to successfully model shock train behavior. This was further corroborated by Fiévet et al. [10] for a Mach 2 shock train. Geerts and Yu [11] observed a Mach 2.5 shock train with shadowgraph from two orthogonal fields-of-view to elucidate the effect of line-integration on rendering the shock train. They found that the shock structures are highly three dimensional with the structure initialized and controlled by corner separation, even in a high aspect ratio duct. These findings are consistent with the results of Hunt and Gamba [6] on the importance of separated corners in the generation and propagation of unsteadiness within a highly confined shock train.

The structure of the normal bifurcated shock train in the University of Michigan Direct Connect Isolator (UMDCI) facility was initially studied by Hunt and Gamba [12]. Hunt and Gamba made detailed observations of the leading SBLI structure from schlieren imaging and cross-sectional particle image velocimetry (PIV) below the leading lambda foot. They found that the leading SBLI structure becomes more oblique, and thus corner dominated, the further upstream the shock train sat in the isolator. This corresponds primarily to changes in the boundary layer ingested by the shock train.

The present study builds on corner cross-sectional PIV data from Hunt and Gamba [6, 12]. Additional datasets included for this study are high-speed schlieren imaging [13], streamwise particle image velocimetry planes, wall static pressure traces [7], and surface oil flow visualization [6]. The measurements are synthesized to infer a detailed three-dimensional cartoon of the shock train to include the sonic surface bounding the core flow, the shock structures, and the separation bubbles. From this, the influence of incoming flow profile, specifically a strong side wall vortex, is observed to control the separated flow morphology surrounding the shock train core. This offsets and skews the shock train structure to adjust the nature of the compressive pressure profile along the streamwise length of the isolator.

C. Shock Train Modeling

The metric of greatest interest to dual-mode scramjet engine designers is the compressive pressure profile along the isolator as it drives the necessary structure and is relatively easy to measure during flight. These quasi-1D models typically take in a set of *approach conditions*, the freestream and boundary layer properties entering the shock train, and output a monotonic pressure profile estimate. The simplest and most utilized model is the Waltrup-Billig Correlation

developed for cylindrical isolators [14] and later rectangular ducts [15]. The correlation for rectangular ducts is shown in Eqn.1 where M_0 , P_0 , θ , and Re_θ are the centerline Mach number, static pressure, wall momentum thickness, and momentum thickness Reynolds number, respectively, at approach conditions just upstream of the pseudo-shock.

$$50 \left(\frac{P}{P_0} - 1 \right) + 170 \left(\frac{P}{P_0} - 1 \right)^2 = \frac{x(M_0^2 - 1)Re_\theta^{1/5}}{\sqrt{H\theta}} \quad (1)$$

To collapse the velocity profile at each streamwise isolator station while maintaining the effect of displacement blockage and momentum losses, *flux-conserved averaging* or *stream thrust averaging* is used. This applies the mass, momentum, and continuity equations over an imaginary control volume between states 0 and 1 on Figure 2 to conservatively convert a velocity profile with centerline values of M_0 , P_0 , and T_0 into a quasi-1D, uniform profile with values M_1 , P_1 , and T_1 [16].

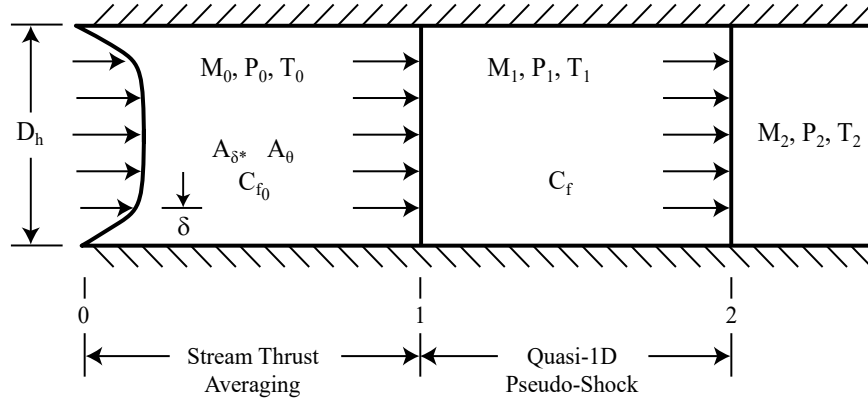


Fig. 2 Diagram of McLafferty's [16] flux-conserved stream thrust averaging procedure to estimate approach conditions (0-1) and compute pressure jump conditions across the pseudo-shock (1-2).

Equations 2-7 detail flux-conservation process for a flow with known displacement thickness blockage, A_{δ^*} , and momentum thickness blockage, A_θ . The value u is x-direction velocity, ρ is the density, a is the speed of sound, R is the specific gas constant, γ is the ratio of specific heats, $\dot{m}$ is the mass flow rate, and T_t is the total temperature. In this study, integration of these values is performed on the full three-dimensional CFD simulations of the isolator without a shock train to estimate approach conditions such as the flux-conserved Mach number, M_1 .

$$F = \iint P dA + \iint \rho u^2 dA = P_0 [A + \gamma M_0^2 (A - A_{\delta^*} - A_\theta)] \quad (2)$$

$$\dot{m} = \iint \rho u dA = P_0 M_0 \sqrt{\frac{\gamma}{RT_0}} (A - A_{\delta^*}) \quad (3)$$

$$u_1 = \frac{\gamma}{\gamma + 1} \frac{F}{\dot{m}} + \sqrt{\left(\frac{\gamma}{\gamma + 1} \frac{F}{\dot{m}} \right)^2 - \frac{2\gamma RT_t}{\gamma + 1}} \quad (4)$$

$$a_1 = \sqrt{\gamma R \left[T_t - \frac{u_1^2}{2R} \left(1 - \frac{1}{\gamma} \right) \right]} \quad (5)$$

$$M_1 = \frac{u_1}{a_1} \quad (6)$$

$$P_1 = \frac{F}{A} - \frac{\dot{m} u_1}{A} \quad (7)$$

McLafferty [16] modeled the pseudo-shock pressure rise as a normal shock jump between the quasi-1D flux-conserved conditions from point 1 to 2 in Figure 2 with frictional momentum loss defined by Δ . The key output of this model is σ , the fraction of static pressure recovered in comparison to an inviscid duct with uniform flow across a M_0 normal shock. Equations 8-13 adapted from [16] defines the computation of σ .

$$K_F = \frac{F}{P} = \frac{1}{A} \iint (1 + \gamma M^2) dA; K_{F_1} = 1 + \gamma M_1^2; K_{F_2} = 1 + \gamma M_2^2 \quad (8)$$

$$K_W = \frac{\dot{m}}{P} = \frac{1}{A} \iint \frac{A_*/A}{P/P_t} dA; K_{W_1} = \frac{A_{*1}/A}{P_1/P_t}; K_{W_2} = \frac{A_{*2}/A}{P_2/P_t} \quad (9)$$

$$\frac{A_*}{A} = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} M \left(1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma+1}{2(\gamma-1)}}; \frac{P}{P_t} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma}{\gamma+1}} \quad (10)$$

$$\frac{K_{W_2}}{K_{F_2}} = \frac{1}{1 - \Delta} \frac{K_{W_1}}{K_{F_1}}; \Delta = \frac{C_{f0} \pi D_h (L - s) (\gamma M_0^2 / 2)}{(\pi/4) D_h^2 (1 + \gamma M_0^2)} \quad (11)$$

$$\frac{P_2}{P_1} = \frac{K_{W_1}}{K_{W_2}} \quad (12)$$

$$\sigma = \left(\frac{P_2}{P_1} \right) \bigg/ \left(\frac{P_2}{P_1} \right)_{NS}; \left(\frac{P_2}{P_1} \right)_{NS} = \frac{2\gamma M_0^2 - (\gamma - 1)}{\gamma + 1} \quad (13)$$

Sullins and McLafferty [4] later applied the flux-conserved conversion as a modification to the Waltrup-Billig correlation, which originally assumed a centerline Mach number, M_0 [14]. Smart [17] took this a step further by modeling the pseudo-shock flow as a quasi-1D stream tube bounded by a fully separated displacement area, A_{δ^*} , such that $C_f = 0$ and all mass flow passed through a core region, $A_c = A_{\delta^*}$. The duct area ratio, A_c/A , is solved simultaneously with M^2 and the physics of shock train compression are imposed on the integration by incorporating the single parameter Ortwerth diffusion model [18]. This model relates the pressure gradient to the dynamic pressure and approach condition wall friction in a scramjet isolator as metrics of the shear the pre-shock train boundary layer can sustain before separating. A single parameter, k , scales the model and defines the rate at which kinetic energy is transferred to internal energy by the pseudo-shock.

Fiévet et al. [19] expanded the flux-conserved approach by adding a flexible set of nine parameters to account for non-uniform kinetic energy transfer along the pseudo-shock, allow for non-zero wall friction within the pseudo-shock, and add scaling by the expected pressure recovery, σ . Leveraging the three-dimensional cartoon of the UMDCI shock train structure developed in this study, the quasi-1D collapse and validity of the assumptions underlying the flux-conserved modeling approach are assessed.

II. Experimental Facilities and Methodology

A. University of Michigan Direct Connect Isolator Facility (UMDCI)

The UMDCI, diagrammed in Figure 3, is a supersonic wind tunnel that draws atmospheric air through an asymmetric, two-dimensional nozzle into a constant-area rectangular test section with a duct height, H , of 69.8 mm and an aspect ratio of 1.2. The top wall forms a Mach 2 converging-diverging nozzle with the coordinate system fixed positive streamwise from the nozzle throat. Following the test section, a diffuser connects the tunnel to a butterfly valve that mechanically chokes the flow venting into the driving vacuum chamber. The pressure ratio across the test section, P_B/P_A , is measured by a pair of absolute capacitance manometers along the top wall. Glass side walls provide full-length optical access. A reconfigurable bottom wall plug provides further optical access through a borosilicate glass window flush to the bottom wall. The windows and bottom wall plug may be replaced with instrumented panels to take wall pressure measurements. For this study, the normal bifurcated shock train in the UMDCI has an isolator pressure ratio of 3.25 and a static pressure recovery ratio, σ , of 0.72.

B. Time Resolved Schlieren Imaging

The shock structure is visualized by a vertical knife edge, Z-type schlieren system that qualitatively resolves the density gradient field along the streamwise, x , direction of the test section. A continuous white LED illuminates the 3.45H diameter mirrors and a high-speed camera captures the frames at 10 kHz with a 2 μs exposure. A second-order mapping function dewarps and converts the image coordinate system to the tunnel coordinate system. The shock train

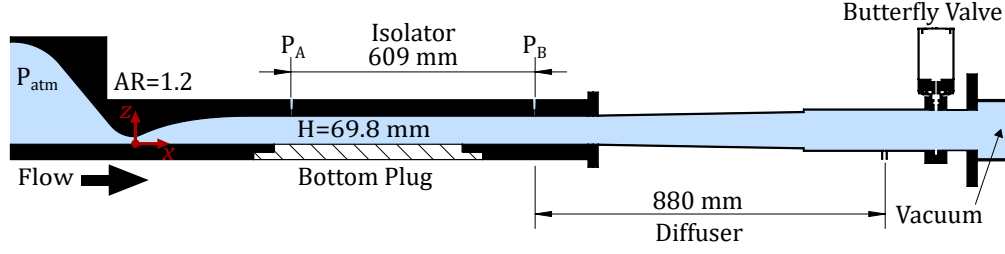


Fig. 3 Diagram of the UMDCI facility.

leading edge, s_1 , is defined as the streamwise location of the leading shock stem as measured by line-scanning intensity along the height $z/H = 0.6$. The subsequent shock features, $s_2 - s_4$, are detected along the same line-scan. Spatial uncertainty due to the pixel pitch of the camera, the lab frame resolution of each pixel, and averaging over the $2 \mu s$ exposure is measured to be approximately $0.014H$. The shock foot location, s_f , is detected visually where the leading lambda foot contacts the boundary layer. An example of an instantaneous schlieren frame is shown in Figure 4.

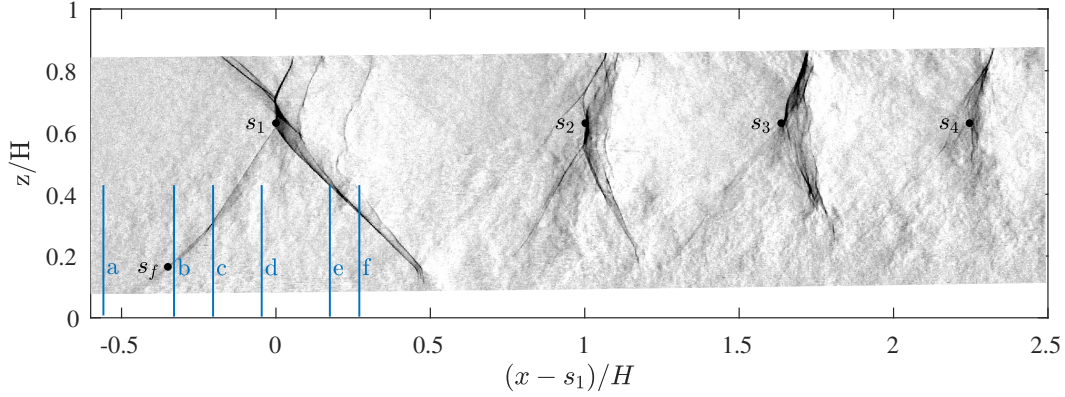


Fig. 4 Instantaneous schlieren frame with shock features and the location of cross-sectional PIV planes a-f.

C. Oil Flow Visualization

Surface oil flow visualization on the side and bottom walls resolves the time averaged boundary layer structure. A thin base coat of 100 cSt silicone oil is seeded with a titanium dioxide-oil mixture. After tunnel startup, the valve is set to position a shock train in the test section and flow pattern development is observed. The procedure is repeated first with cameras focused on the top and bottom walls and then with both cameras aligned streamwise on the side wall. A composite average of the flow structure in the corner is shown in Figure 5.

D. Time Averaged Wall Pressure Measurements

Time averaged, streamwise pressure measurements are taken through 0.8 mm diameter pressure taps along the side and bottom wall centerlines as marked on Figure 5 with a varying spacing of 12.7 mm or 25.4 mm. The taps are connected via flexible tubing to three ± 17.2 kPa and one ± 103 kPa differential pressure scanners for a total of 63 channels. All scanner measurements are referenced to the top wall pressure at the inlet of the isolator, P_A . The root-mean-square (RMS) uncertainty in the pressure scanner measurements are ± 30 Pa and ± 60 Pa for the low and high range pressure scanners, respectively. The higher range scanner channels are restricted to the most downstream pressure taps where the relative pressure differential is high and thus the higher uncertainty is less impactful. The leading shock foot location, s_f , in the time averaged wall pressure measurements is estimated by finding the first peak in the streamwise pressure variance gradient $d\sigma(P)^2/dx$ [7].

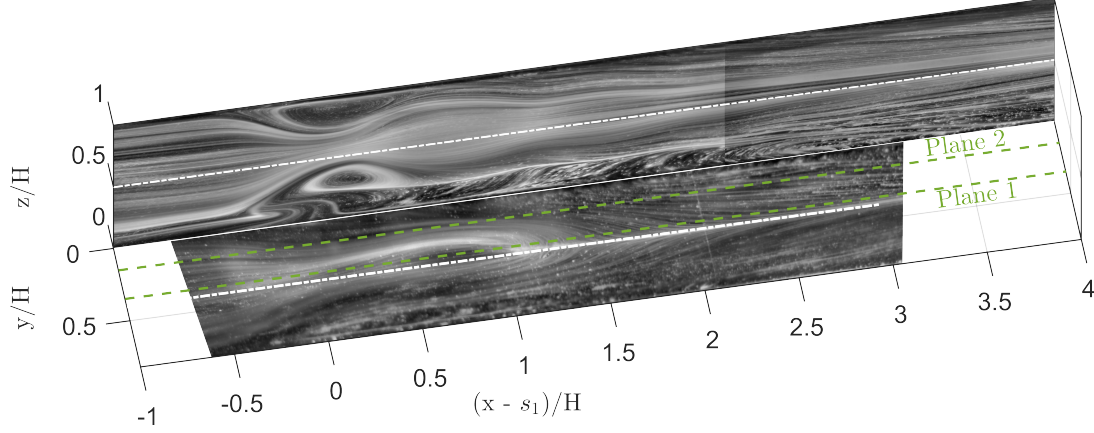


Fig. 5 Oil flow visualization in the UMDCI with flow from left to right. Green lines show the location of streamwise PIV planes and white lines the pressure ports.

E. Particle Image Velocimetry

Two sets of particle image velocimetry (PIV) data are included in this study: streamwise two-component planes and stereo PIV (SPIV) cross-sectional slices. The streamwise PIV renders the streamwise, u , and vertical, w , velocity fields in the $x - z$ plane while the SPIV measurements resolve all three velocity components in $y - z$ planes. Based on the particle relaxation distance across the normal shock stem at s_1 , the worst-case uncertainty in PIV derived shock locations is approximately $0.03H$. This is sufficiently low to track the large-scale motions of the shock structures in a compressible flow with some particle tracking error to be expected in large gradient areas such as shocks [20]. Two individual 532 nm Nd:YAG lasers combined through a Brewster angle mirror output 10 ns, 80 mJ pulses to illuminate the particle field through the sheet forming optics as shown in Figure 6. The time delay between pulses, Δt , is measured by a fast-acting photodiode on a 350 MHz bandwidth digital oscilloscope with an uncertainty of ± 2 ns. Δt is set to $1.5 \mu\text{s}$ and $0.6 \mu\text{s}$ for the streamwise and cross-sectional cases, respectively.

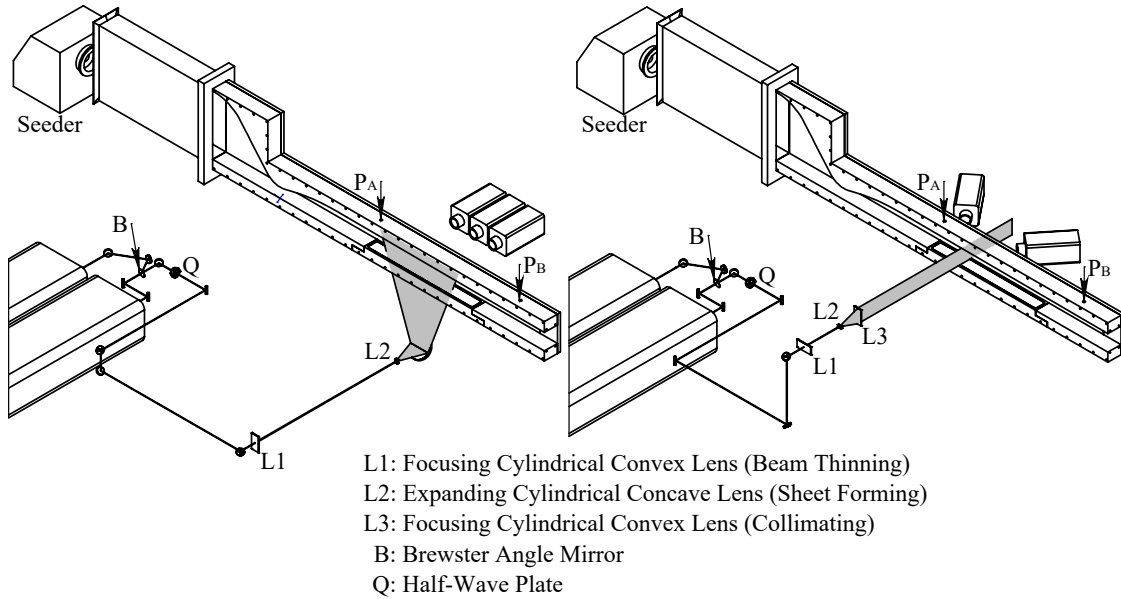


Fig. 6 Diagram of the streamwise PIV (left) and the cross-sectional PIV (right) configurations.

In the streamwise case, 1200 samples are taken for two planes at $y = 0.44W = 0.36H$ and $y = 0.20W = 0.16H$ referred to as Plane 1 and Plane 2, respectively. In the cross-sectional case, the beam is periscoped up to the level of the isolator to project a laser plane at $x = 7.22H$ about which the shock train oscillates due to the inherent unsteadiness. 300

samples with no shock train are averaged for the empty tunnel plane (a) in Figure 4. Cross-sectional planes are located relative to s_1 and sorted into 5 bins of 160 samples each by matching the centerline displacement thicknesses between streamwise Plane 1 and each cross-sectional frame. Interline transfer charged-couple device (CCD), 1280×1024 pixel resolution cameras (LaVision Flowmaster 3) are triggered on every third laser pulse-pair at 3.33 Hz. The streamwise planes are imaged by three partially overlapped cameras and the cross-sections are imaged by a two cameras at $\pm 33^\circ$ from the laser sheet as shown in Figure 6. Both cases are processed by four passes of 32×32 Gaussian weighted windows with a 75% overlap to yield vector field resolutions of 0.63 mm and 0.27 mm for the streamwise and cross-sectional cases, respectively.

III. Datasets and Post-Processing

A. Approach Conditions

Each measurement required different tunnel, data acquisition systems, and optical configurations requiring the shock train to be positioned at different locations along the test section. The effect of this is characterized by the flow properties just upstream of the shock train known as the *approach conditions*. These include the flux-conserved Mach number, M_1 , momentum thickness Reynolds number, Re_θ , and the boundary layer confinement ratio or blockage ratio, $C_\theta = A_\theta/A$. The effect of each factor is collapsed together into the Waltrup-Billig factor, $Q = (M_1^2 - 1)Re_\theta^{1/5}C_\theta^{-1/2}$ [4, 7, 15]. The empty isolator, unbackpressured UMDCI flow is estimated with the NASA Langley VULCAN Reynolds-averaged Navier-Stokes solver [21]. Curves of each approach condition are extracted from these computations and compared to wall pressure measurements from the facility. The approach conditions for each dataset are tabulated in Table 1.

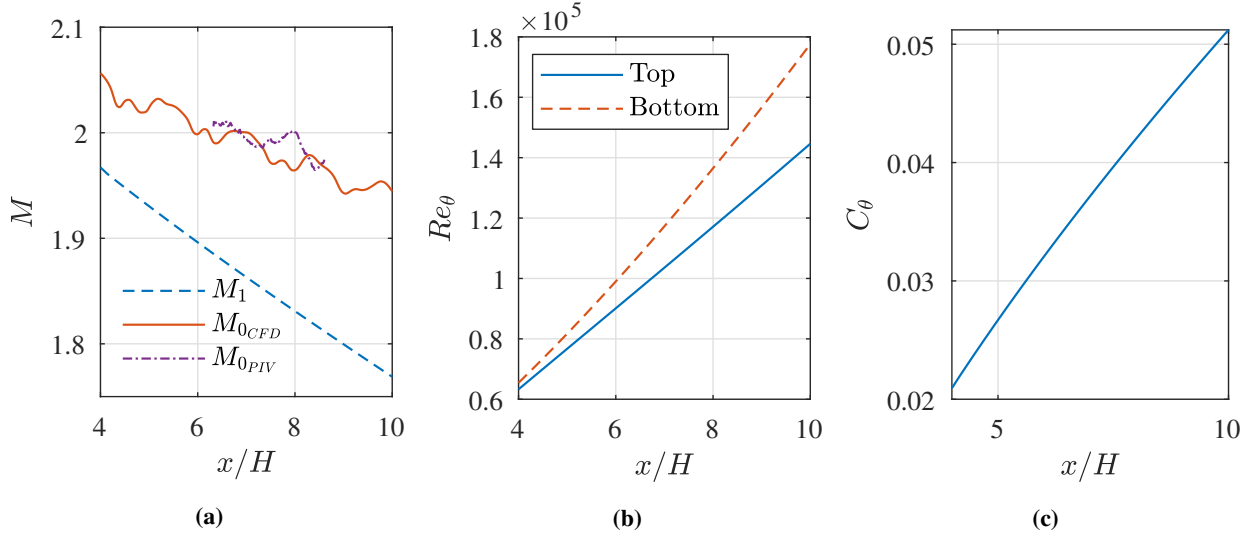


Fig. 7 Approach condition profiles from CFD of the unbackpressured UMDCI with no shock train present.

With the exception of the oil flow visualization, all Waltrup-Billig factors and tunnel positions are sufficiently close to analyze the time averaged structure of the shock train. This is because the structure of the shock train shifts streamwise with an approximately rigid body motion such that the spacing between shock structures is constant under low frequency oscillations [13]. The jump in Q for the oil flow visualization is acceptable to infer boundary layer morphology relative to the general structure of the shock train.

B. Stem-Aligned Reference Frame

To resolve the structure of the shock train independent of the inherent unsteadiness, each frame of schlieren and streamwise PIV data is aligned such that $s_1 = 0$. The shock stem is located in each frame and then shifted to overlap all frames on the same grid yielding an uncertainty on the order of the grid resolution, $\pm \Delta x$. Combined with the uncertainty in the shock stem locations, this yields RMS uncertainties of $0.0142H$ and $0.0313H$ for the schlieren and PIV alignment,

respectively. The cross-sectional SPIV frames are located relative to s_1 by binning the centerline, incompressible displacement thickness against the values measured in the Plane 1 streamwise PIV. The wall static pressure data is aligned by matching the shock foot location, s_f , in the average schlieren imagery to the leading shock foot location detected on the bottom wall by the Hunt variance method [7]. The aligned oil flow visualization is shown in Figure 5. Figure 8 shows the aligned streamwise measurements: time averaged schlieren, streamwise PIV, and wall static pressure.

C. PIV Processing

The Mach number, M , field from the streamwise PIV planes at $y = 0.44$ ($W = 0.36H$) and $y = 0.20$ ($W = 0.16H$) for Plane 1 and Plane 2 are shown in Figures 8b-c. The streamwise location of each cross-sectional SPIV plane, a-f, is shown in Figure 9.

1. Uncertainty

PIV vectors with a correlation peak less than 1.1 are removed and Grubbs' test for outliers [22] is applied at each grid point to eliminate spurious samples. Uncertainties in each frame are computed for each velocity component by the correlation statistics method of Wieneke [23]. Applied using DaVis 8.4, the asymmetry of the correlation function within each interrogation window is related to uncertainty in the computed displacement vector and thus the computed velocity components. This validated methodology [24, 25] accounts for sources of error including random particle motion, out-of-plane motion, camera noise, and shadowgraph effects. The uncertainty of each i th frame, $\tilde{u}_i$, is then cascaded into the average velocity, $\bar{u}$, and average Mach number, $\bar{M}$, fields via root-mean-square uncertainty propagation at each grid point, represented by $\tilde{u}$ and $\tilde{M}$, respectively.

The PIV derived velocity uncertainty for each vector in each frame, u_i , increases as seed particles accumulate in the shocks and separation bubbles [20]. Unsteadiness, such as within separated flow regions, further increases uncertainty in the average profiles. The minimum uncertainty sonic line, $\bar{M} - \tilde{M}$, and maximum uncertainty sonic line, $\bar{M} + \tilde{M}$, are shown as solid and dashed lines, respectively. The divergence of these curves exemplifies the downstream growth of uncertainty in the streamwise dataset.

2. Regions of Probable Reverse Flow

Regions of probable reverse, or separated, flow are estimated by the fraction of PIV frames with instantaneous reverse velocities, $u_i \leq 0$. A region with greater than 50% reversed frames is considered to be region of probable separation. Just as in the computation of the sonic line, uncertainty must be considered in the computation of separated region contours. The least conservative definition being $u_i \leq \tilde{u}_i$ and the most conservative definition being $u_i \leq -\tilde{u}_i$. The average reversed flow region is rendered in cool colors on the cross-sectional PIV frames in Figure 9 with a dashed line to show the least conservative boundary.

In the streamwise PIV planes, no point within the resolved field-of-view has greater than 50% reversed flow as to indicated a stable separated flow region as prominent as the corner separation shown in the oil-flow visualization in Figure 5 and the cross-sectional PIV in Figure 9. Instead, a threshold of at least 10% reversed flow is considered to indicate the presence of intermittent separation significant enough to influence the average flow structure. The color contours of the median definition, $u_i \leq 0$ is shown in Figure 10 with dashed and dotted lines to show uncertainty bounds corresponding to $u_i \leq -\tilde{u}_i$ and $u_i \leq \tilde{u}_i$, respectively.

Table 1 Approach conditions and scaling parameters for each dataset extracted from CFD at the streamwise location of each dataset, s_f/H .

Data Type	s_1/H	M_1	Re_θ	C_θ	$Q = (M_1^2 - 1)Re_\theta^{1/5}C_\theta^{-1/2}$
Schlieren	6.34	1.88	6613	0.0339	80.6
Wall Pressure	7.07	1.86	7301	0.0375	75.3
Oil Flow	8.80	1.81	8944	0.0458	65.2
Streamwise PIV	6.89	1.87	7131	0.0366	76.5
Cross-Sectional PIV	7.22	1.86	7442	0.0383	74.3
Quasi-2D DNS [19]	N/A	1.72	7560	0.0323	63.8

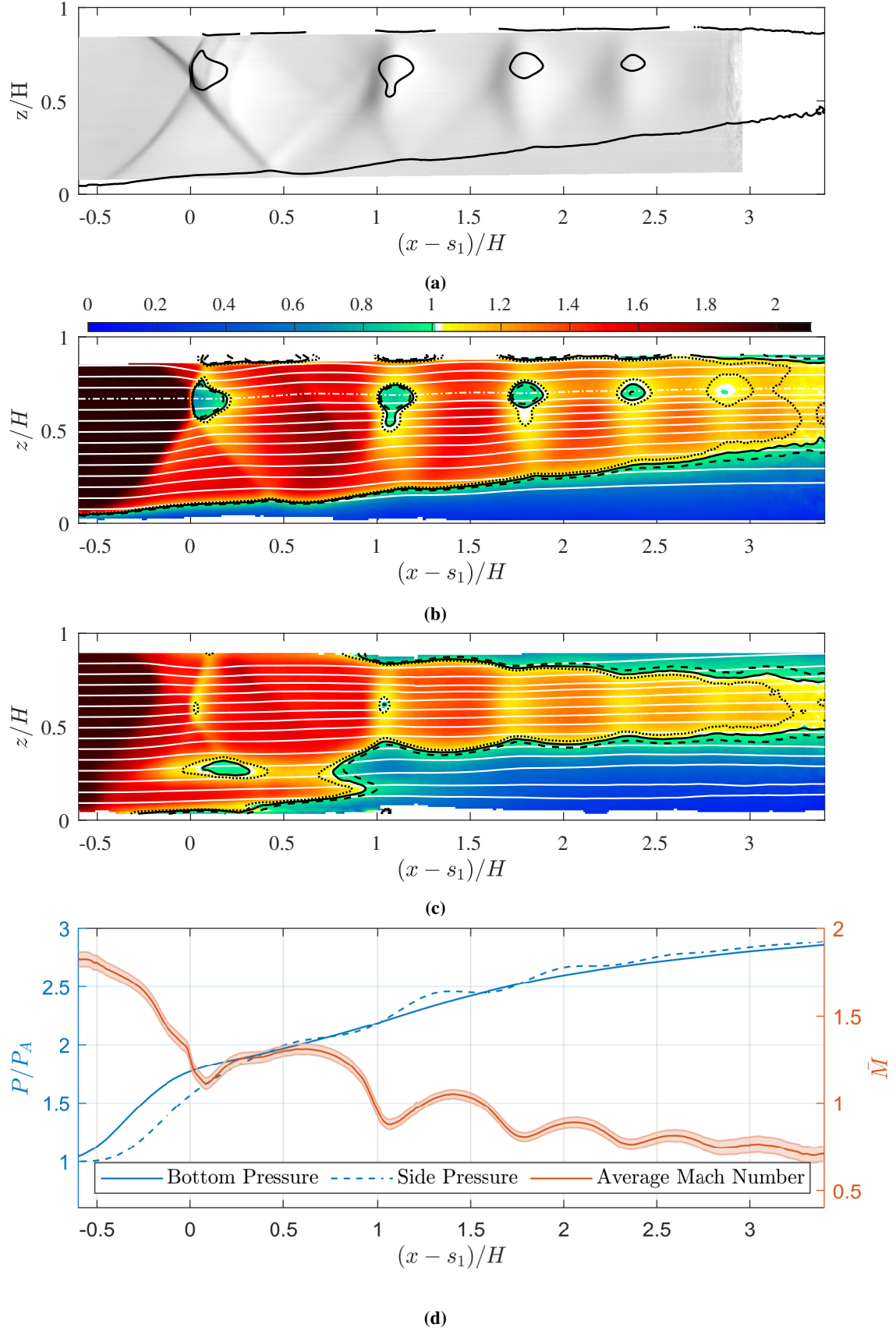


Fig. 8 Aligned a) average schlieren and b-c) Plane 1-2 PIV average Mach contours overlaid with white streamlines. Solid Line: $\bar{M} = 1$, Dotted Line: $\bar{M} - \tilde{M} = 1$, Dashed Line: $\bar{M} + \tilde{M} = 1$. d) Wall pressure (blue) and 1D average PIV Mach number (red).

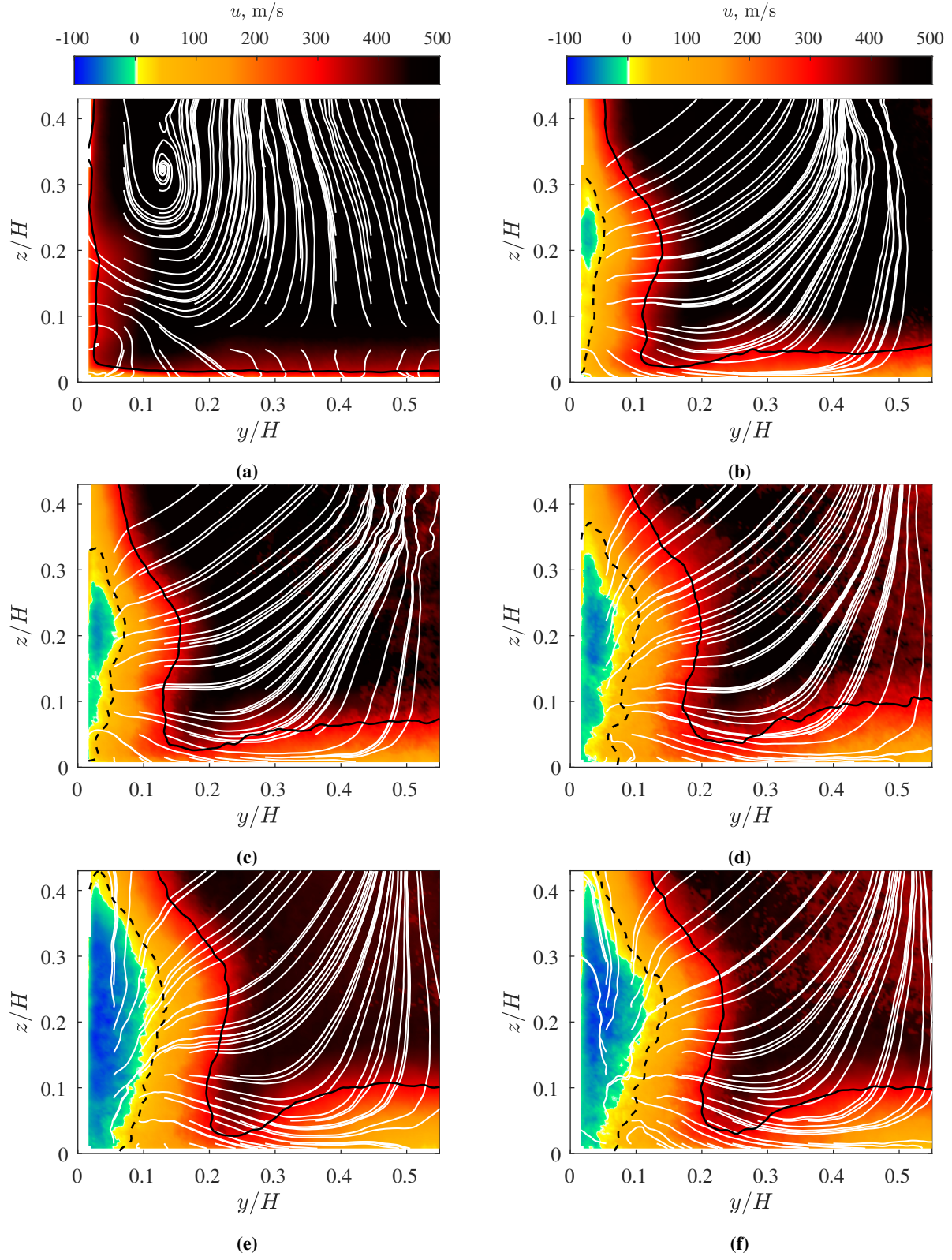


Fig. 9 Cross-sectional PIV planes of streamwise velocity overlaid with in-plane streamlines. Dashed Line: 50% region of probable reverse flow, Solid Line: $\bar{M} = 1$.

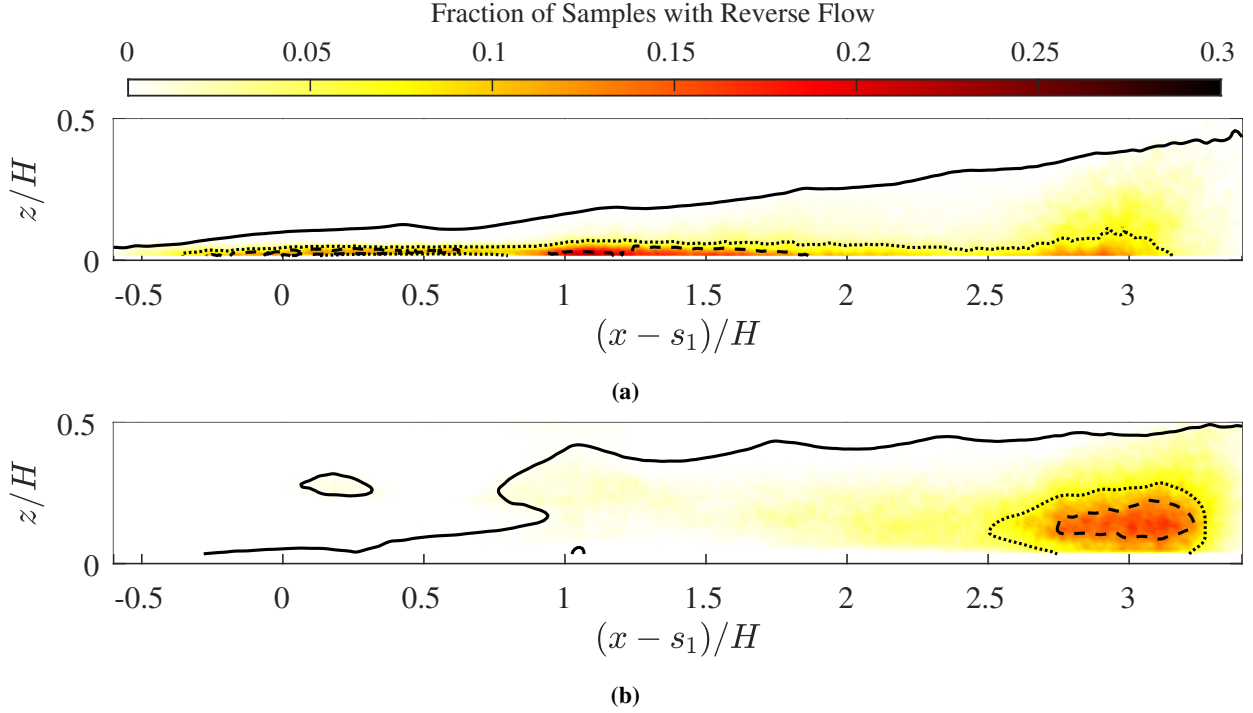


Fig. 10 Aligned a) Plane 1 PIV fraction of frames with reverse flow, b) Plane 2 PIV fraction frames frames with reverse flow. Solid Line: $\bar{M} = 1$, Dashed Line: $10\% u_i \leq -\tilde{u}_i$, Dotted Line: $10\% u_i \leq \tilde{u}_i$.

IV. Data Synthesis and Analysis

A. Three Dimensional Cartoon of an Asymmetric Normal Bifurcated Shock Train

The datasets described above are compared and synthesized to produce a detailed three-dimensional cartoon of the UMDCI shock train in Figure 11. First, the sonic surface, shown as a translucent blue envelope, is constructed from the streamwise and cross-sectional PIV measurements. The average streamwise PIV sonic lines from Figure 8b-c are intersected with the sonic lines in the cross-sectional SPIV on Figure 9. Lateral symmetry is inferred from the oil flow rendered separation bubble symmetry at $-0.5 \leq x/H \leq 1.5$ in Figure 5. Due to laser reflections from the test section walls, information in the top-wall boundary layer is limited. Extrapolation from the unbackpressured CFD and the side-wall oil-flow visualization is used to infer the upper edge of the sonic surface. High uncertainty in the PIV derived sonic line forces us to estimate the downstream closure of the sonic surface at $x/H \approx 4$. The shape of this closure is inferred from the convergence of oil-flow streamlines and the morphology of the CFD sonic line uncertainty bounds.

Next, the series of shocks comprising the shock train are modeled. The centerline shock profiles for the first four structures are extracted from the instantaneous and average schlieren images in Figures 4 and 8a, respectively. Two additional shock structures are evident from the Plane 1 PIV data in Figure 8b. The lateral extent of the shock stems and their sweep toward the side-wall are extracted from the Plane 2 PIV data in Figure 8c. The profiles are lofted between these anchor profiles and bounded by the sonic surface. The six shock structures are rendered in Figure 11 as red surfaces.

Finally, regions of strong, separated flow are shown as green bubbles between the walls and sonic surface in Figure 11. The strongest separated flow regions form around the leading shock structure. The corner bubbles are extracted from the oil flow visualization in Figure 5 and the regions of probable reverse flow in the cross-sectional PIV in Figure 9. A weaker, intermittent bottom wall separation bubble is evident from the oil flow visualization and the regions of probable intermittent separation from the Plane 1 PIV in Figure 10a. The bottom wall separation at $x/H = 1$ is the first significant bottom-wall separation. Closure of this bubble at $x/H \approx 2$ is inferred from the oil flow pattern and the uncertainty bounds of the probable regions of intermittent separation in the streamwise PIV. An additional downstream corner separation is evident from the oil flow pattern and aligns well to the spot of probable intermittent separation impinging on the Plane 2 PIV in Figure 10b. A final corner separation bubble is observed in the upper corners at $1.0 \leq x/H \leq 1.5$. No further upper-wall corner separation is evident from the oil flow pattern on the side-wall. Additional, top wall

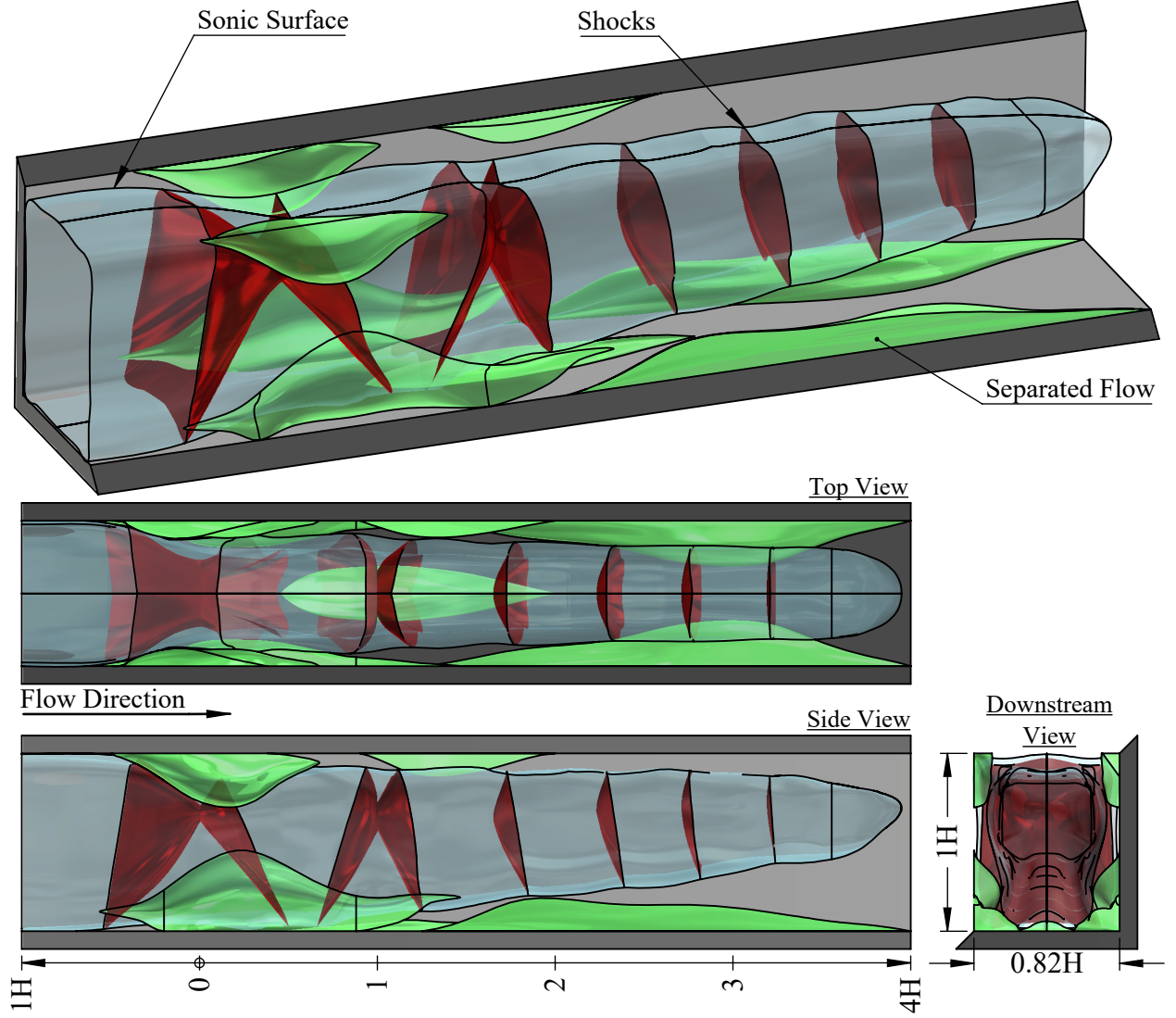


Fig. 11 Cartoon of the shock train structure.

separations may be present but are not observed due to the bias of available data toward the more complex bottom corner regions. However, since the bottom wall separations are weak and intermittent, the presence of significant separations on the top wall is unlikely due to the stronger attachment on that surface. The cause of this asymmetry and the influence of the separated flow morphology on the core structure are discussed in the following sections.

B. Influence of Incoming Flow Vortex Structure on Asymmetry

A distinct asymmetry in the UMDCI shock train is observed. One might attribute this asymmetry to the asymmetric nozzle of the UMDCI. However, schlieren imaging of shock trains in symmetric and asymmetric nozzle configurations by Geerts and Yu [26] show no such distinct asymmetry in the shock train. The asymmetry of the UMDCI nozzle yields a momentum thickness difference between the flat bottom wall and the contoured upper wall of only 20%, as shown by the divergence of top and bottom wall momentum thickness Reynolds number, R_θ , in Figure 7b. This difference does not account for the 200% difference in the centerline boundary layer thickness after the shock train at $x/H = 4$ in Figure 7b.

Figure 9a shows the undisturbed corner flow $\bar{u}$ field just upstream of the shock train overlaid with streamlines of the in-plane flow. The dominant flow feature is a side wall vortex with a core at $z/H = 0.32$ pulling streamwise momentum from the side wall boundary layer at $0.1 \leq z/H \leq 0.25$. This corresponds to a thickened side wall boundary layer

in this region and thus an area pre-conditioned to separate. Previously observed by Morajkar [27], this vortex is a result of the UMDCI's nozzle, consistent with the observations of Sebacher and Lee [28] and more recently Sabnis et al. [29]. Using oil flow visualization and RANS CFD, Sabnis et al. [29] observed the formation of separated flow region, or a spanwise-aligned vortex, at the junction between the tunnel inlet and the supersonic nozzle block of the University of Cambridge Supersonic Wind Tunnel No 1. Since it isn't energetically favorable for this vortex to terminate on the wall [30], this vortex advects into a horseshoe vortex extending downstream along the side walls. Two smoothing fillets were applied to the inlet-nozzle interface. The initial fillet reduced the strength of the separation, as measured by oil flow visualization, but not the apparent strength of the vortex in the test section. A larger fillet that did eliminate the separation and consequently the vortex indicating the direct causal relationship between the two phenomena. The morphological similarity between the UMDCI vortex observed in Figure 9a and that of Sabnis et al. [29] indicates the probability that the side-wall vortex in this study has a similar origin upstream of the facility nozzle.

The observed influence of this vortex on the side wall boundary layer dwarfs the effect of momentum thickness asymmetry from the nozzle. Thus, the initial offset and skew in the UMDCI shock train core is induced by the vortex pre-conditioning that causes the strong asymmetry observed in the UMDCI shock train. While the source of the vortex is an experimental aberration of the UMDCI's nozzle, a similar vortex may form within the inlet of an operational dual-mode scramjet engine. For example, at the mechanical joints between moving components of an adjustable inlet nozzle plate. As such, the impact of isolator flow morphology containing strong, near-wall vortices on the structure and response of the shock train must be assessed.

C. Core Confinement by the Separated Flow Morphology

The flow separation grows downstream from the initial separation point to establish distinct separation bubbles that interact with the adverse pressure gradient to yield the complex interconnected series of shock boundary layer interactions, or shock train. The bottom wall corner separations grow to confine the core, shock containing flow. Due to their upward shifted initiation on the side wall, the flow reattaches along the side wall, as indicated by the slip line in Figure 5. The shape of the bubble is also skewed upward and away from the corner bisector symmetry common for a more traditional SBLI in a low aspect ratio duct like the UMDCI [31]. The result is a streamwise upward shift and skew to the entire shock containing core structure as shown in Figure 11. The upper corner separation bubble above the leading SBLI, where no additional side wall vortex is present, shows a traditional structure originating and closing in the corner. The size and intensity of this separation is also markedly weaker than that observed in the bottom corners indicating that the flow below the shock train core is deprived of more energy and momentum than the flow above the core.

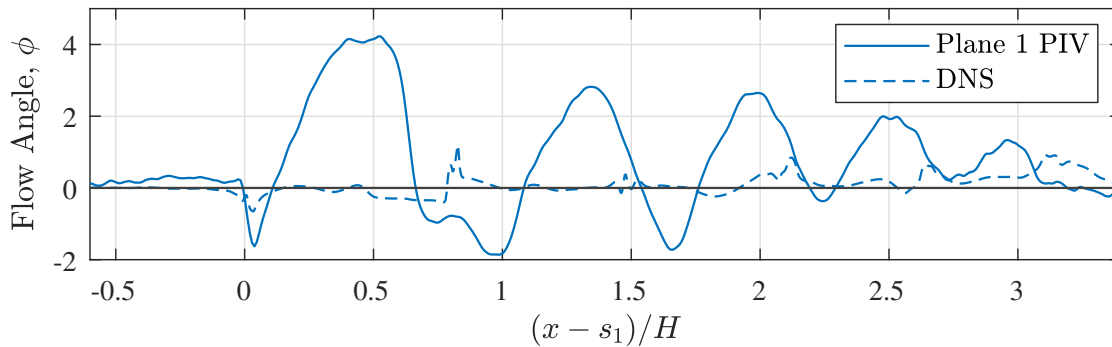


Fig. 12 Flow angle along the Plane 1 PIV streamline marked as a dashed line in Figure 8b compared against symmetric DNS data.

It is this difference in momentum extraction between the upper and lower SBLI components that drives the core flow at an upward angle, cascading the initial asymmetry induced by the side-wall vortex into a substantially different structure than a symmetric shock train like that shown in Figure 1 [1]. The fluid dynamical influence of this upward shift is observed by following a streamline originating from $y/H = 0.65$ through the strongest portion of each SBLI, marked as a dashed line in Figure 8b. This is compared to DNS computations performed previously by Fiévet and Raman [32]. The DNS was performed without a nozzle by initializing the turbulent boundary layer thickness and flow to match the inflow of the UMDCI. As such, it produces a symmetric shock train without the presence of a side-wall vortex.

Figure 12 shows the streamline flow angle, ϕ , where significant upward flow turning is observed at each SBLI as the

flow is confined by the separated flow. This effect is strongest for the first SBLI and weakens with each subsequent interaction. The weaker adverse pressure gradient imposed by successively weaker shocks and relieving expansion waves in the shock cells are unable to restore symmetry to the flow angle. Taking the average flow angle downstream of the shock stem gives a bulk upward tilt of the shock train core of 0.85° . No significant flow turning is observed in the DNS, which lacks any large separated regions.

The skew modifies the strength of the SBLI's, which then modifies the compressive pressure recovery of the shock train. Furthermore, the difference in local boundary layer thickness diffuses the core adverse pressure gradient into the wall profile measured by the pressure taps in Figure 8d. This combines to divergence in the wall pressure profile, of primary interest to dual-mode scramjet designers, from the equivalent symmetrical case.

D. The Assumptions and Performance of Flux-Conserved Models

With the structure of the UMDCI shock train established above, it becomes a tool to assess the underlying assumptions and predictive capability of the prevailing pseudo-shock models in the literature detailed previously. To generate curves comparable in the quasi-1D frame of the flux-conserved models, the PIV data is extrapolated to the wall using a $1/7$ power law profile. Linear interpolation is then applied between the two planes, and symmetry is exploited to create a profile representative of the full cross-section. Integration of these profiles at each streamwise grid point along the isolator yields reference curves for the average Mach number and the displacement area, A_C , shown as black curves with uncertainty bands in Figure 13b and Figure 13c, respectively.

The simplest model is the empirical Waltrup-Billig correlation in Equation 1 which estimates the pressure profile from the state of the incoming boundary layer and the centerline Mach number. While the Waltrup-Billig correlation provides insight into the most relevant approach conditions, its empirical nature makes it poorly suited to address variation in isolator geometry, approach flow morphology, and approach Mach numbers outside its limited calibration data. This is exemplified in Figure 13a where the blue Waltrup-Billig curve quickly departs from a representative pressure curve in black.

The flux-conserved integral method introduced by Smart [17] improves on the limitations of the Waltrup-Billig correlation with a single parameter, physics based approach. By integrating the stream-tube equations with a pressure gradient sub-model, the local Mach number, pressure, and stream tube area are output such that the flow state is fully defined along the length of the isolator. This enables comparison against flow properties derived from the PIV and yield useful quantities for combustor modeling and control. Two parameterizations of the model are displayed on Figure 13 against the pressure, profile averaged Mach number, and stream tube area profiles. A value of $k = 35$ is fit to best model the initial, high gradient region of the shock train while a value of $k = 15$ is fit to model the total pressure rise across the isolator. An engine designer using these curves to determine the appropriate isolator structure would underestimate the necessary structure with $k = 15$ and overestimate it using $k = 35$. While the overestimation of $k = 35$ would produce a safe engine, it constitutes excess weight in an operational engine.

Furthermore, the Ortwerth pressure sub-model closure causes a mismatch between accurate prediction of the pressure profile and the underlying Mach number and stream tube area states. This is best exemplified by the $k = 35$ case in Figure 13b which produces a reasonable estimate of the Mach number profile while over-predicting the pressure after the initial, high gradient region. Thus, closure with the Ortwerth model misses critical flow physics that dominate downstream in the highly confined ramjet-like flow observed.

Fiévet et al. [19] observed this issue and proposed to replace the single parameter derivation of the Ortwerth model with a nine parameter model that allows the value of k to monotonically decrease as a power function of the local dynamic pressure, q . The nine model parameters are re-optimized using successive genetic algorithm minimization for this study against pressure profiles from the UMDCI at isolator pressure ratios of 2.5-3.25. The result is shown by the solid yellow curves in Figure 13. The re-optimized Fiévet model is flexible such that it can perfectly model the pressure profile in Figure 13a. However, the Mach number and stream tube area predicted on Figures 13b-c diverge from the data after $x/H \approx 1$. Just as with the Smart model, this indicates missing flow physics that cannot be resolved through the Ortwerth pressure gradient sub-model. Additionally, the need to re-optimize indicates that the high-dimensional Fiévet parameter set lacks universality, making it difficult to implement as a predictive design tool.

The cartoon of the UMDCI shock train in Figure 11 indicates the reason for the Ortwerth-based flux-conserved models breaking down after $x/H \approx 1$. This is the point at which the series of SBLI's underpinning the shock train weaken to the point of losing their bifurcation. Each successive shock and re-acceleration region in the shock cell is weaker such that it contributes less compression. As the strength of the shocks drops off, an additional mechanism that begins to dominate the growth of the pressure profile and is not represented in the form of the stream tube equations used

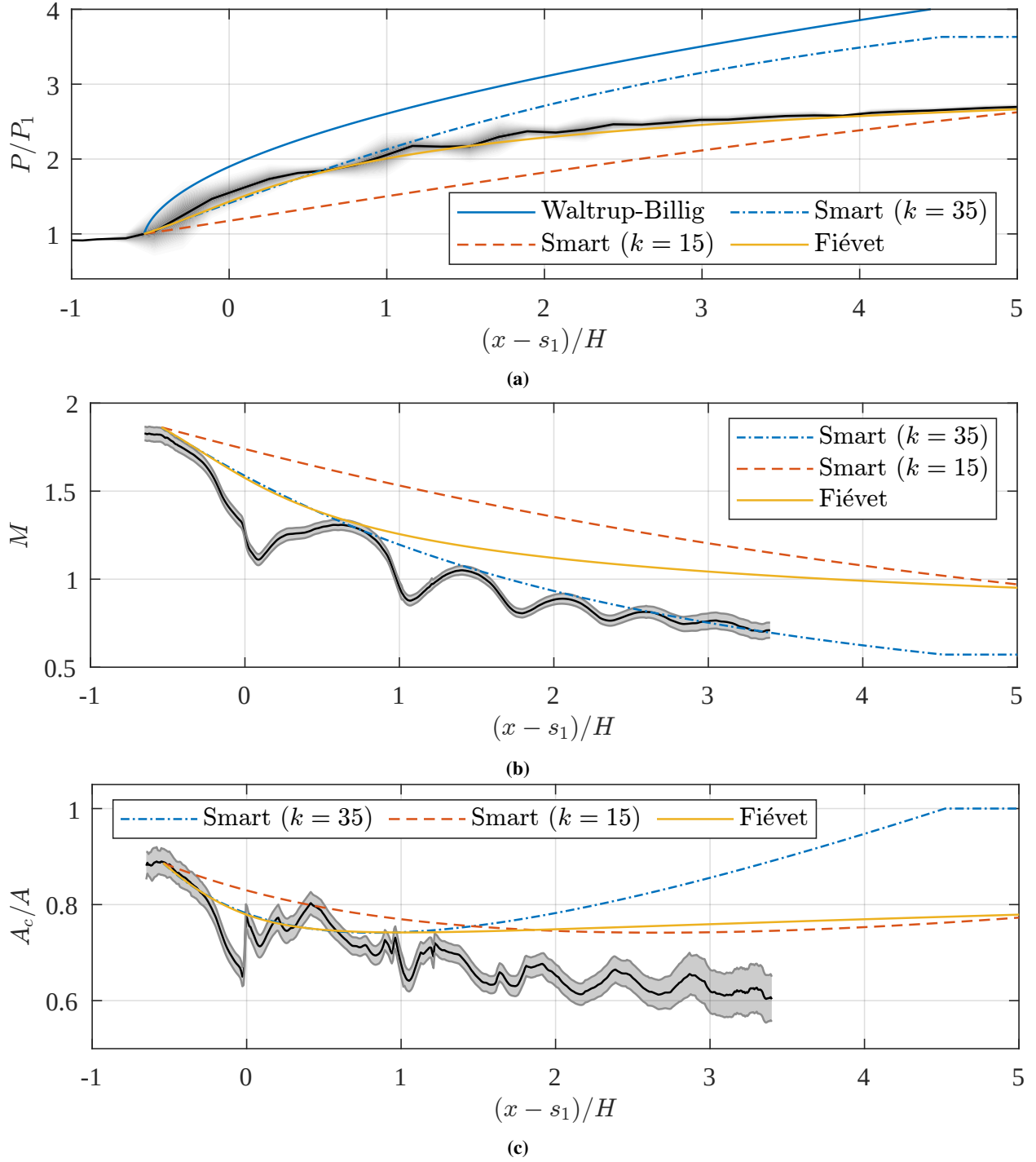


Fig. 13 Comparison of quasi-1D models against PIV profile averaged values with uncertainty bounds.

by Smart and Fiévet. The structure of the flow indicates that this mixing region compression may stem from confinement of the core flow by the growth of the separated flow structure and transonic turbulent mixing within the core.

V. Conclusions

Schlieren imaging, particle image velocimetry, oil flow visualization, and wall pressure measurements were obtained in the University of Michigan Direct Connect Isolator facility to assess various features of an asymmetric, normal bifurcated shock train in a low-aspect ratio isolator. Aligning this data in the reference frame of the shock train enables the inference of a three-dimensional shock train structure, including the location of shocks, separated flow regions, and the supersonic core. A cartoon of the UMDCI normal bifurcated shock train is constructed to yield a tool in assessing the underlying flow physics of the system.

The undisturbed flow in the UMDCI test section reveals the presence of a strong, side wall vortex extending downstream from the asymmetric facility nozzle. This vortex extracts momentum from the side wall boundary layer, preconditioning it to separate before the corner. This is observed to shift the separation under the adverse pressure gradient of the shock train upward, skewing the structure of the shock boundary layer interactions. The separated flow morphology throughout the shock train is thus biased downward, growing the bottom wall boundary layer faster than that of the top surface boundary layer. This skews the angle of the supersonic core upward, adjusting the strength of each subsequent SBLI. In turn, the streamwise pressure gradient underlying the quasi-1D pressure profile is changed. That profile is critical for selecting the length of the isolator in the design optimization of engine flow path and structure. The analysis of shock train structure in this study reveals the importance of the incoming flow morphology on the structure of a normal bifurcated shock train, present during ramjet operation of a dual-mode scramjet engine.

Following from this finding, three prevailing models in the literature: the Waltrup-Billig correlation [14], the Smart flux-conserved model [17], and the Fiévet flux-conserved model [19] are evaluated to assess their efficacy in predicting the shock train state and pressure profile of key interest to engine designers. First, the Waltrup-Billig correlation is observed to be poorly calibrated to the ramjet-like flow present in the UMDCI. The Smart model yields a conservative engineering model and the high-dimensional parameter space of the Fiévet flux-conserved model produces a good fit to the pressure profile. However, in both cases closing the system of ordinary differential equations with a pressure gradient sub-model creates a disconnect between the pressure profile and the underlying fluid dynamical state as defined by the Mach number and the stream tube area. The divergence of the models from the UMDCI data indicates a transition from the shock dominated compression upstream to boundary layer confinement and turbulent mixing dominated compression downstream. Modifying the flux-conserved system of equations to better match the observed fluid dynamical state and comparing it against data from additional facilities is an area of continuing work.

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